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Professor Wolfgang Drobetz,  
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Maastricht, October 26, 2011

Dear Professor Drobetz,

Please find enclosed my working paper “Higher-moment risk exposures in Hedge Funds” that I have co-written with Professor G. Hübner (HEC-Management School of the University of Liège, Belgium) and Professor N. Papageorgiou (HEC-Montréal).

I will be grateful if you could consider it for the European Financial Management symposium on asset management to be held in April 2012. Of course, I will be willing to serve as a discussant and/or session chair.

I would also like my paper to be considered for publication in the special issue of the *European Financial Management*.

Sincerely,

Marie Lambert

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# Higher-moment risk exposures in Hedge Funds\*

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## Abstract

*The paper singles out the key roles of US equity skewness and kurtosis in the determination of the market premia embedded in Hedge Fund returns. We propose a conditional higher-moment asset pricing model with location, trading and higher-moment factors in order to describe the dynamics of the Equity Hedge (Market Neutral, Short Selling and Long/Short strategies), Event Driven, Relative Value, and Funds of Hedge Funds styles. The volatility, skewness and kurtosis implied in the US options markets are used by Hedge Fund managers as instruments to anticipate market movements. Managers should adjust their market exposure in response to variations in the implied higher moments. We show that higher-moment premia improve a conditional asset pricing model both in terms of explanatory power (R-squares and Schwarz criterion) and specification errors across all Hedge Fund styles.*

**Keywords:** *Hedge Funds; Implied higher-moments; Conditioning factors*

**JEL classification:** *G10; G12*

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# Higher-moment risk exposures in Hedge Funds

## 1. Introduction

The premise that many Hedge Fund strategies generate nonlinear payoffs, resulting in return distributions that exhibit significant higher-order moments (than the variance statistics) has been substantiated by a number of papers since the late 90's (Fung and Hsieh, 1997; Ackermann et al., 1999; Leland, 1999; Agarwal et al., 2009). Hedge Funds' use of leverage and financial derivatives as well as the risks inherent in their quasi-arbitrage activities are all potential contributors to the negative skewness and fat-tails that characterize their return distribution. In general, the introduction of Hedge Funds into a diversified portfolio will improve the mean-variance efficiency, however it will also lead to a greater overall level of negative skewness and a superior level of exposure to extreme negative events (Lo, 2001; Brooks and Kat, 2002; Favre and Galeano, 2002; Signer and Favre, 2002; Amin and Kat, 2003; Gueyié and Amvella, 2006; Davies, Kat and Lu, 2009). The hypothesis that co-skewness and co-kurtosis with the market portfolio are priced by rational risk-averse investors is theoretically and empirically supported by numerous studies (see Kraus and Litzenberger, 1978; Dittmar, 2002; Fang and Lai, 1997 for the most referenced articles).

To date, an extensive stream of literature has favoured the use of time series of option returns to capture these non-linear dependencies of Hedge Funds on the market returns (e.g., Mitchell and Pulvino, 2001; Agarwal and Naik, 2001, 2004; Fung and Hsieh, 2001, 2002a,b; 2004a,b). Alternatively, the Hedge Funds' exposures to higher-order moments of the US equity market have also been tested for their ability to capture their non-normalities. Ranaldo and Favre (2005) and Ding and Shawky (2007) for instance use the levels of variance, skewness, and kurtosis attained in the US markets to capture Hedge Fund systematic moment exposures. Besides, Spurgin et al. (2001) and Chen and Passow (2003) extend the market

model using the square and the cube of the S&P 500. These authors show that Hedge Funds present significant loadings on the two factors that capture respectively their coskewness and cokurtosis with the US market portfolio. Such factor exposures could however be difficult to disentangle from the market timing measures of Treynor and Mazuy (1966), as observed by Kat and Miffre (2006). Therefore, in order to capture Hedge Fund's alternative sources of risk, these authors form six two-dimensional portfolios (for different levels of, respectively, covariance and coskewness, and covariance and cokurtosis) in a way similar to what Fama and French (1993) do for size and book-to-market. They define a coskewness premium as the relative performance of low coskewness stocks over high coskewness stocks and a cokurtosis premium in a similar fashion. These two new risk premia improve the asset pricing models used for capturing Hedge Fund return variation. Finally, Agarwal et al. (2009) follow Kat and Miffre (2006) and construct covariance, coskewness and cokurtosis premia using a similar approach. Instead of considering the US stock market and 6 portfolios however, Agarwal et al. (2009) form 27 portfolios of Hedge Funds according to their levels of covariance, coskewness and cokurtosis with the US market portfolio. Their paper also demonstrates that such premia could strongly improve the explanatory power of multifactor models for Hedge Funds.

Our paper attempts to extend this stream of literature on higher-moments adapted to Hedge Funds. Unlike previous research that introduces risk premia aiming to capture market-wide skewness and kurtosis risks, we focus on how Hedge Fund managers account for higher moments in their investment strategies. This strategy-based analysis drives the way skewness and kurtosis enter the return generating process. We argue that, by dynamically monitoring their asset allocations and leverage, Hedge Fund managers act as if they run their portfolios according to skewness and kurtosis targets. This view would imply that they aim to capture an excess rate of return associated to these risk dimensions in a systematic way and through a

voluntary and steady exposure. The main question we ask is: what kind of targets do they set? The answer should not focus on systematic comovements with the market, which reflects a point of view based on maximization of a representative agent's expected utility: Hedge Fund managers hardly care about such considerations. Rather, when considering their risk targets, we have to focus on the anticipation of extreme market risks, whose information derives from option-implied moments. This is the perspective adopted in the paper.

The reasoning is similar to the implied volatility estimation, but adapted to skewness and kurtosis. Indeed, the market prices of index options convey precious information about the overall market conditions. The levels of the expected market volatility, skewness, and kurtosis for the next 30 trading days could be extracted from a cross-sectional series of out-of-the-money option prices. These parameters are interpreted as follows: a decrease in the skewness of the market portfolio means an increase in the probability of experiencing extreme negative returns, while an increase in the kurtosis of the market corresponds to an increase in the likelihood of extreme return variation. Hedge Fund managers, who attempt to collect the risk premia corresponding to the preference for skewness and aversion to kurtosis, should rationally allocate or re-allocate their portfolios over time according to the levels of the third and fourth co-moments with market returns<sup>1</sup>.

The choice of this kind of instrument as a way to predict market movements is not the only source of divergence from previous attempts to integrate higher moments in the Hedge Fund literature. We also argue that this way of introducing higher moments in an asset pricing specification makes sense in the context of a conditional model. The study of Feunou et al. (2008) indeed reveals that the level of the equity risk premium is conditional on current expected risk-neutral skewness. We therefore consider a multifactor model composed of

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<sup>1</sup> We consider the US equity market as we consider a set of US-based Hedge Funds.

asset- and option-based factors but where buy-and hold factors are conditioned on the values of some higher-order US equity moments.

Our focus on skewness and kurtosis does not lead us to discard the location and trading factors emphasized in the seminal study of Fung and Hsieh (2001). Their strong explanatory power has been proven and refined by Fung and Hsieh (2004b). Higher-moment risk factors are likely to complement these types of premia, and are not meant to replace them. The location factors reflect the fund's linear exposures to a set of risk premia representative of the markets in which it operates. The trading factors, which translate into a systematic exposure to option returns, represent a heuristic, but rather direct way of assessing the departure from normality in the distribution of a fund's returns. This kind of factors is also a proxy for the systematic co-skewness and co-kurtosis.

To sum up, our paper singles out the key roles of skewness and kurtosis in the determination of the market premia embedded in Hedge Fund returns. Our objective is to improve models that have traditionally been used for evaluating Hedge Fund returns. Specifically, we want to dissociate the returns that are due to exposures to systematic risk from those that are due to manager skills (alpha returns). The selection of asset- and option-based factors that constitute control variables in our multifactor approach has been borrowed from the existing literature. We consider the *Hedge Fund Research, Inc. (HFR)* database and more specifically the following styles: Event Driven, Relative Value, Funds of Funds and Equity Hedge, with a special focus on Market Neutral, Short Bias, and Quantitative Directional funds.

Our paper provides evidence that there is significant information embedded in option prices that has not yet been exploited. We find that our conditional higher-moment asset pricing model performs very well at describing the dynamics of Hedge Fund returns. In particular, we show that higher-moment premia improve a conditional asset pricing model in

terms of explanatory power (R-squares and Schwarz criterion) and specification errors across all Hedge Fund styles. R-squares are improved by up to 10% for Relative Value Funds, while the specification error of the model is decreased by up to 33% for the Funds of Funds portfolio. The inclusion of the implied moments as conditioning variables appears to be an important contribution to the performance of the Hedge Fund asset pricing models.

The rest of the paper is structured as follows. Section 2 presents the model-free estimates of the implied moments extracted from a set of out-of-the-money put and call options. Their evolution over the last years is discussed. In Section 3, we describe the Hedge Fund data used in this study and form the dependent dataset. Given the opportunistic behaviour of most of the Hedge Funds, we expect to find different sensitivities to the different sets of factors exposed above. Therefore, we survey the higher-moment exposures per Hedge Fund style. Section 4 provides technical details regarding the asset pricing models performed in the empirical analysis, i.e. a conditional asset pricing model, a conditional higher-moment pricing model, a conditional option pricing model and finally a conditional higher-moment option pricing model. Section 5 reports our main empirical results. Section 6 concludes.

## **2. Model-free estimates of implied moments**

Useful information regarding the moments of the S&P 500 return distribution can be extracted from the option markets. Provided that investors attribute a certain importance to higher-order moments, the implied volatility, skewness and kurtosis of the index return distribution can be interpreted as forward-looking measures of the investor sentiment about equity market risk.

Bakshi et al. (2003) introduce a method to retrieve the intrinsic values of the risk-neutral variance, skewness and kurtosis payoffs from option prices. Their evaluations express risk-neutral skewness and kurtosis as functions of volatility, cubic, or quartic contracts whose

payoffs are defined by stocks' continuously compounded return taken to the 2<sup>nd</sup>, 3<sup>rd</sup>, and 4<sup>th</sup> power respectively. And as any payoff function can be spanned by a continuum of out-of-the-money (OTM) calls and puts (Bakshi and Madan, 2000), they build a model-free connection between the prices of OTM and higher-moment equity risk prices.

The theoretical values of the risk-neutral skewness (*NSK*) and kurtosis (*NKU*) for options with time-to-maturity  $\tau$  are defined as follows (Bakshi et al., 2003, Theorem 1):

$$NSK(\tau) = \frac{e^{r\tau}W(\tau) - 3\mu(\tau)e^{r\tau}V(\tau) + 2\mu(\tau)^3}{[e^{r\tau}V(\tau) - \mu(\tau)^2]^{3/2}} \quad (1)$$

$$NKU(\tau) = \frac{e^{r\tau}X(\tau) - 4\mu(\tau)e^{r\tau}W(\tau) + 6e^{r\tau}\mu(\tau)^2V(\tau) - 3\mu(\tau)^4}{[e^{r\tau}V(\tau) - \mu(\tau)^2]^2} \quad (2)$$

where  $V(\tau)$ ,  $W(\tau)$  and  $X(\tau)$  are the prices of the volatility, the cubic, and the quartic contracts, respectively, and are given by the following expressions:

$$V(\tau) = \int_{s(0)}^{\infty} \frac{2(1 - \ln[\frac{K}{s(0)}])}{K^2} C(\tau; K) dK + \int_0^{s(0)} \frac{2(1 + \ln[\frac{K}{s(0)}])}{K^2} P(\tau; K) dK \quad (3)$$

$$W(\tau) = \int_{s(0)}^{\infty} \frac{6\ln[\frac{K}{s(0)}] - 3(\ln[\frac{K}{s(0)}])^2}{K^2} C(\tau; K) dK - \int_0^{s(0)} \frac{6\ln[\frac{s(0)}{K}] + 3(\ln[\frac{s(0)}{K}])^2}{K^2} P(\tau; K) dK \quad (4)$$

$$X(\tau) = \int_{s(0)}^{\infty} \frac{12\ln[\frac{K}{s(0)}]^2 - 4(\ln[\frac{K}{s(0)}])^3}{K^2} C(\tau; K) dK + \int_0^{s(0)} \frac{12\ln[\frac{s(0)}{K}]^2 + 4(\ln[\frac{s(0)}{K}])^3}{K^2} P(\tau; K) dK \quad (5)$$

while  $\mu(\tau)$  is given by:

$$\mu(\tau) = e^{r\tau} - 1 - \frac{e^{r\tau}}{2}V(\tau) - \frac{e^{r\tau}}{6}W(\tau) - \frac{e^{r\tau}}{24}X(\tau) \quad (6)$$

Similarly to the procedure adopted by Dennis and Mayhew (2002), we discretize the integrals of equations (3) to (5) through piecewise trapezoidal approximation.



For the estimation, we use a sample of daily prices of put and call options written on the S&P 500 index. The sample period ranges from November 1997 to the beginning of 2010. For each first trading day of the month (corresponding to the estimation date), we record the prices of the out-of-the-money (OTM) puts and calls for the option series maturing during the following month. This choice of option maturities yields a very high degree of liquidity for most options as well as a high range of strike prices for the OTM puts and calls. Every day, we record the prices for up to eleven different series of OTM calls and puts. The intervals between adjacent strike prices range from \$5 to \$15.<sup>2</sup> In case of missing values in the option time-series, the unavailable data are replaced by values from up to 3 days later. If the data are still not available, the observation is simply excluded from the analysis.

Each month, we estimate the implied risk-neutral skewness and kurtosis for the first three trading days of the month, and then take a simple average of the daily values. This procedure allows us to limit the microstructure issues (thin trading, limited number of options, and estimation error due to the trapezoidal approximation) as well as to diversify away a part of the measurement risk due to the very nonlinear structure of the skewness and kurtosis functions.

Note that to parameterize the trapezoidal estimation of the integrals in equations (3) to (5), we need two unobservable inputs: the option premium corresponding to an exactly at-the-money (ATM) option, and the strike price corresponding to an option premium that is not distinguishable from zero. This strike price is taken as the bound for the support of the integral.

The ATM option premium is simply obtained by taking a linear interpolation between the prices of the closest out-of-the-money (OTM) and in-the-money (ITM) options with respect

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<sup>2</sup> Most of the time, options with closer strike prices do not display sufficient liquidity to yield usable prices. Our choices of strike intervals aim at tracking the market tendency to consider only a subset of options that are actively traded at each date.

to their implied volatilities. The most extreme strike price is obtained by a linear extrapolation between the current index price and the strike price of the deepest OTM option taken in the sample. Given the very low value taken by the deepest OTM option price, the numerical error incurred by this approximation is very low.

Risk-neutral volatility, skewness, and kurtosis correspond to the actual value of the payoffs of one contract over respectively the US market volatility, skewness, and kurtosis. We use the percentage of change in risk-neutral implied skewness and kurtosis as candidate variables in our regression-based analysis. We call these variables the “risk-neutral moments” DNSK and DNKU, respectively. In order to remain consistent with the existing literature, we use the VIX index or DNVIX (collected on the CBOE website) as a proxy for the change in market implied volatility.

Figures 1 to 3 display the evolution of the implied volatility, skewness, and kurtosis over the period 1997-2010. The period is rich in information for all the three types of risk as the sample covers the technology bubble of the late nineties, the severe market deflation of the early 2000s, and the major market crises that tumbled on financial markets (Peso Crisis, Russian Crisis, LTCM collapse, Asian Crisis, Terrorist Attacks as well as the recent world financial crisis).

< Insert Figures 1 to 3 >

Figures 1 to 3 present strong variations in the levels of volatility, skewness and kurtosis over the period. The strongest variations appear successively to some liquidity events in the market. While the skewness statistics stays significantly negative from November 1997 to beginning 2010, it becomes even more negative in mid 2002, 2004-2005, and at the end of 2009, showing a strong increase in the probability of experiencing a huge negative return in the US market at those periods. As with the skewness parameter, we observe large variations in kurtosis in 1998, and significant spikes in 2002, 2004-2005, and early 2006. On average

however, the market kurtosis remains around the value of 3 throughout the period. Interestingly the decrease in skewness in 1998-1999, 2002 and at the end of 2009 is accompanied by a huge increase in the volatility of the market for the same points in time.

### **3. Hedge Fund styles and higher-moment exposures**

Hedge Funds strategies exhibit higher-order moments in their return distribution through their intensive use of leverage (increasing the likelihood of an extreme loss and of extreme return variation), of derivatives (introducing asymmetry in their payoffs) and also by their net-of-fees return structure which creates asymmetry in the return payoff. The degree and the direction of these exposures vary however from Hedge Fund styles according to their diverse trading strategies.

Equity-oriented Hedge Fund styles for instance exhibit strong moment risks in their return payoffs. Most of them are indeed long volatility, short skewness and long kurtosis (Agarwal et al., 2009). Negative skewness exposition means that these Hedge Funds tend to present a left-asymmetry in their return distribution towards highly negative returns. A long position in kurtosis reflects a high probability of displaying extreme variation in returns. While Long/Short Equity funds show significant exposures to all three kind of risks, Event Driven funds are both short skewness and short kurtosis as well as short volatility to a less extent (Mitchell and Pulvino, 2001; Fung and Hsieh, 2003). In other words, Event Driven funds hedge their risk exposures against variability risk but still present a high probability of experiencing one loss event. Finally, the literature also gives some insight about the Relative Value strategy by relating this style to a long exposure in kurtosis (Ranaldo and Favre, 2005).

The addition of each of this Hedge Fund style into a diversified portfolio of a rational investor (for instance the market portfolio) will change the moment properties of the portfolio: increasing for instance the portfolio kurtosis if the kurtosis of the Hedge Fund

portfolio is superior to the one of the portfolio. In this case, the insertion of these funds into the market portfolio will strengthen the likelihood of extreme returns. Omitting these higher-moment exposures is likely to worsen the performance of the multifactor model in explaining Hedge Fund returns.

This study focuses exclusively on equity moment risks. These higher-moment equity premia are less relevant for styles in which equity risks are not the primary exposure, which explains that we deliberately do not cover all investment styles encompassed by Hedge Fund strategies.

### 3.1. *Hedge Fund data*

We carry out our analysis using data from the *Hedge Fund Research (HFR* hereafter) database. We use the monthly net-of-fee returns of individual Hedge Funds over the period January 1994 to March 2009. Excluded in our analysis are funds that do not report on a monthly basis and that do not report in USD<sup>3</sup>. Our resulting sample covers 4713 individual Hedge Funds. Our sample period includes the technology bubble of the late nineties, the severe market deflation of the early 2000s, and the major market crises that tumbled on financial markets (Peso Crisis, Russian Crisis, LTCM collapse, Asian Crisis, Terrorist Attacks and the recent quant crisis).

In order to mitigate the impact of any reporting biases, we also remove from the database all the funds that reported results for less than 12 consecutive months. (1,328 funds were further excluded due to this restriction). To control for backfill bias, the first 18 months of each fund were also removed from the analysis (computed from the date the fund has been added to the database). As a consequence, the sample period could only start in November 1997. Besides, all observations with a monthly return superior to 100% or equal to 0 around

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<sup>3</sup> 2388 funds were excluded from our analysis due to these restrictions.

the date of observation (lead and lag of one month), indicating possible reporting errors, were excluded from the final dataset.

Our research rests upon the hypothesis that Hedge Fund returns present a very different return profile from traditional assets. The success of any model also varies across the different Hedge Fund categories: Hedge Funds styles differ with respect to the trading strategies, the financial assets they invest in and their performance objectives (Jaeger and Wagner, 2005). Our first focus is on the equity managers who maintain both long and/or short positions in primarily equity and equity-derivative securities. We especially consider the following sub-strategies: Market Neutral, Short Bias and Quantitative Directional which combine both long and short positions. We also analyze the return of Event Driven funds that maintain positions in securities of companies involved in corporate-related events like mergers, restructurings or financial distress. In addition to ‘Equity Hedge’ and ‘Event Driven’ funds, we consider the returns of Relative Value funds that maintain positions in which the investment thesis is predicated on the realization of a valuation discrepancy in the relationship between multiple securities. The last strategy to be included consists of Funds of Funds (FF) that invest in numerous managers’ funds within a strategy or across different strategies. The goal of these funds is to diversify the risk of investing in one individual fund<sup>4</sup>.

The monthly mean return of each strategy is then obtained by computing the equally-weighted average return of all funds belonging to that category during the given month.

### *3.2. Descriptive analysis of the sample*

Table 1 reports the descriptive statistics from the Hedge Fund sample over the period November 1997-March 2009.

< Insert Table 1 >

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<sup>4</sup> See the *Hedge Fund Research Inc.* Strategy Classification System

Each category represents the equally-weighted portfolio of the funds that make up the category. These return series provide a complete representation of each style and would not have been obtained through the use of synthetic indexes available from database providers.

Over the almost 12 years considered in our sample period, the best performing portfolio of Hedge Funds has been the Equity Hedge-Quantitative Directional portfolio with an average monthly return of 0.8105%. The other Hedge Funds portfolios also post very attractive returns: all of them outperform the Russell 3000 taken as benchmark index for the US stock market. While the dispersion in volatility levels between the different Hedge Fund portfolios is very high (underlining the heterogeneity in Hedge Fund strategies), the volatility risk of all strategies but the Equity Hedge – Short Bias is lower than that of the Russell 3000. However, as there is no free lunch, the return distributions of all portfolios of Hedge Funds display significantly fatter tails over the period than the Russell 3000 does. Compounded by negative skewness, fat tails in the Event Driven funds, Relative Value funds and Funds of Hedge Funds imply that more severe losses are expected during downturns. Except the Funds of Funds, these Hedge Fund styles display lower levels of skewness than the Russell 3000. The decrease in volatility, in skewness and the increase in kurtosis imply that potential upside is traded off against downside protection in normal market conditions (and catastrophe risk in highly volatile markets).

This nonlinear profile is more or less pronounced across the strategies. The most nonlinear risks are to be found in Equity Hedge- Market Neutral strategies (regarding the levels of their kurtosis), Event Driven and Relative Value Hedge Funds, while the highest levels of volatility are expected in Equity Hedge –Short Bias and Quantitative Directional styles.

## 4. Conditional pricing models

Our objective is to evaluate the effects of an increase or a decrease of the expected levels of US equity volatility, skewness and kurtosis risks on the risk factors' exposures of Hedge Funds and on their asset allocation.

To answer this question, we consider an asset-based model where risk exposures are conditioned on the values of traditional financial indicators but also on the implied moments. We compare the results of this model to a model made only of traditional information variables with either market factors or both market and option-based factors.

### 4.1. *Asset-based factor models*

We identify a set of asset-based factors that aim to capture the exposures of Hedge Funds to the risks of a broad set of asset classes.

To account for the fact that Hedge Funds may hold a variety of asset classes, we consider the following indexes: the Russell 3000, the three empirical factors of Fama and French and Carhart (SMB, HML, and UMD), the FTSE World Stock Index excluding the US market, the Citigroup World Government Bond Index excluding US and the JP Morgan Emerging Bond Market.

To account for the fact that Hedge Funds may apply sophisticated instruments and dynamic trading, we consider a set of option-based strategies. First, we consider the primitive trading strategies defined in the Fung and Hsieh paper (2004b), i.e. lookback straddles<sup>5</sup> on bonds, stocks, commodities, foreign exchanges and interest rates futures. These strategies are intended to capture trend-following strategies in Hedge Funds trading activity. Besides, we

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<sup>5</sup> A lookback call option allows the investor to buy the underlying asset at the lowest price, while a lookback put option allows him to sell it at the highest price. A straddle is a combination of a long put and a long call. Taking long positions in both lookback options, the owner can make profit whether there is a decreasing or an increasing trend in the asset returns.

measure the monthly returns of at-the-money (ATM) and out-of-the-money (OTM) puts and calls by following a small variation of the rationale put forward by Agarwal and Naik (2004). Their factor construction methodology has been proved to have a greater explanatory power for Hedge Fund returns than the straddle strategy returns proposed by Fung and Hsieh (2004b), as the latter premia mostly apply to trend-followers (see Bailey, Li and Zhang, 2004). The primary information that must be extracted from a data set of option prices is the time series of returns of both calls and puts for a given level of “moneyness”. Agarwal and Naik (2004) retain for valuing ATM and OTM options the option presenting the closest value of moneyness (i.e., 1 and 0.95). To ensure the time consistency of the series of options used, as options are never perfectly ATM, we approximate each option closing price on the first trading day of the month with a linear interpolation of the closest in-the-money (ITM) and OTM option prices with respect to their implied volatilities. The next month, we use the same technique to obtain the closing price. Again, the option series used to build these premia involves options maturing during the next month. We build two series of actual returns of ATM put and call options, and we use the same technique for OTM puts and calls, where the strike price is 5% away from the current value of the index. Indeed, according to De los Rios and Garcia (2011), the strike price of synthetic call options that best explain Hedge Fund returns is slightly higher than 1.05 times the current index price on aggregate, while this multiple is slightly lower than 1.05 for individual Hedge Fund strategies.

The variables corresponding to these series of options are denoted AMC, AMP, OMC and OMP. We also consider the lagged call and put option strategies in order to capture a part of the stale pricing effects in Hedge Fund returns. Stale pricing occurs when Hedge Funds invest in securities that are not actively traded and for which market prices are not always available (see Getmansky et al.; 2004; Jagannathan et al., 2009). Indeed when, for monthly reporting purpose, fund managers have to price these illiquid securities, they use either the



last available traded price or a smoothing evolving model to estimate the actual price. Some part of the asset returns is thus reported contemporaneously, while another part shows up in future returns.

#### *4.2. The conditional approach*

Modelling dynamic trading strategies via static positions in asset-based risk factors is dealt with as an “errors-in-variables” problem. Hedge Fund return expectations are conditioned onto predetermined variables (information variables) for which the empirical predictability has been tested. In this way, it is possible to reflect the importance of the timing of publicly available information (Brealey and Kaplanis, 2001). Such a conditional performance analysis has been advocated by Chen and Knez (1996), Ferson and Schadt (1996), Christopherson et al. (1998), and Christopherson et al. (1999). These models have been shown to explain a significant portion of portfolio returns, but have mostly been applied to Mutual Funds.

Some recent studies have applied this methodology to Hedge Funds. Kat and Miffre (2006) consider conditional performance measures consistent with the semi-strong form of efficiency. They find that the inclusion of conditioning factors significantly alters the measurement of performance. However, neither Kazemi and Schneeweis (2003) nor Chen and Liang (2007) provide evidence to support this conclusion.

Similarly to Kazemi and Schneeweis (2003), we employ the following information variables ( $Z$ ) to construct our premia<sup>6</sup>: the corporate bond spread (the difference between the BBB and AAA ten-year corporate bond yields), the term yield spread (ten-year minus one-year treasury bills) and the value of the VIX index. The choice of the VIX index is motivated

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<sup>6</sup> Several papers on conditional performance evaluation have employed similar instruments, see Ferson and Schadt (1996), Christopherson et al. (1998,1999), Busse (1999), Amenc et al. (2003), and Chen and Liang (2007).

by the importance of volatility trading in the optional investment strategies carried by Hedge Funds (Busse, 1999). For our application, this variable is likely to provide a greater explanatory power than the dividend yield on the S&P index originally proposed by Ferson and Schadt (1996). We also complete this analysis by considering the timing of the Hedge Fund risk exposures towards the level of skewness and kurtosis of the market by using the implied moments define above.

The inclusion of  $L$  information variables in the  $K$ -factor linear pricing model results in the creation of  $L*K$  conditioning risk premia  $\mathbf{Z}'_{t-1}\mathbf{X}_t$ , where  $Z'_{t-1} = Z_{t-1} - \bar{Z}$ . The number of conditioning factors should therefore be restricted. We limit ourselves to the set of factors in the original Carhart (1997) model, using the excess return on the Russell 3000 and the FTSE World Stock Index and the JP Morgan Emerging Market Bond Index as the market proxies. This leaves us with  $L = 5$  and  $K = 6$ . Furthermore, to avoid multicollinearity problems, we choose at most one instrument per factor. Thus, there are no more than six conditioning risk premia in a regression.

If beta exposures vary over time and change with market conditions, the fund's abnormal performance can do likewise. If the Hedge Funds portfolio weights are still correlated to future returns given the public information, then the abnormal performance must be expressed as a function of the same information variables (Christopherson et al., 1998; Christopherson et al., 1999). Kat and Miffre (2006) introduce, for instance, both conditional alpha and beta in their factor model. Therefore, we also condition the values of the alpha on the same set of information variables as used in the model.

For each instrument, we denote the corresponding variable  $Z$  followed by the initial of the instrument ('C' for Credit, 'S' for Slope, 'V' for VIX, 'Sk' for implied skewness, and 'Ku' for implied kurtosis) and the name of the variable. Thus, for the product of the lagged credit spread with the momentum risk premium, the corresponding variable is ZCUMD.

## 5. Empirical results

This section decomposes the net-of-fee returns of the different strategies into the returns that could be replicated using the set of asset-based factors defined above. It considers a conditional multifactor approach where the risk exposures are conditioned onto a set of financial indicators.

We consider four asset-pricing models for capturing Hedge Funds return variability. Results are first presented for the five Hedge Fund investment styles by looking only at traditional buy-and-hold asset-based factors (RUS, SMB, HML, UMD, WEX, WGBI, and EMB). We account for time-varying risk exposures to the Russell 3000 index, the SMB, HML and UMD factors of Fama and French (1993) and Carhart (1997), the Emerging Bond Market Index and the World Equity Index (ex. US) by conditioning the exposures to these risk factors onto the value of traditional information variables such as the term and default spread (see Table 2 below). In the second model, we consider the same sets of buy-and-hold factors that are used in Model 1 and select the set of factors that best fit the Hedge Fund regressions using not only the default and term spread as conditioning factors but also the expected levels of volatility, skewness and kurtosis of the US stock market. This corresponds to a conditional higher moment multifactor model (see Table 3). The third model presents the “optimal” factor model which best fits (over our sample period) each Hedge Fund style using a larger set of asset-based factors. We introduce in Model 1 the returns of a rolling-over ATM and OTM call and put options. These strategies buy, at the beginning of each month, call and put options that expire the next month and sells them at the end of the month. We also consider the primitive option-trading strategies defined in Fung and Hsieh (2004b) which consist in lookback straddles<sup>7</sup> on stocks, interest rates, commodities, bonds and foreign exchanges. Model 3 corresponds to an option-based multifactor model. In this model, the

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<sup>7</sup> Data are downloadable on Fung and Hsieh’s website: <http://faculty.fuqua.duke.edu>

exposures to the buy-and-hold factors are also conditioned onto the same set of information variables used in Model 1 (see Table 4). In the last model, we consider all asset-based factors (including option-based ones) and condition the exposures to the equity factors and the Emerging Bond market factor on the values of both the traditional financial indicators and the three implied moments (see Table 5). Finally, the marginal values of each family of non-directional factors, i.e. either the optional or the (conditional) distributional factors with regard to directional factors are displayed (see Table 6).

For each specification, we consider all the possible combinations of variables, and finally select the one that maximizes (in absolute value) the Schwarz Information Criterion for the linear regression<sup>8</sup>. Our methodology departs from what has generally been done in the literature. Indeed, when confronted with such a large number of variables, most of the studies favour a stepwise regression approach (Liang, 1999; Agarwal and Naik, 2004) in order to select the explanatory variables. Such an approach starts with a general model that incorporates all variables and gradually eliminates the least relevant. Given the complex correlation matrix and the multicollinearities between explanatory factors (see Table A.1 in Appendix), the stepwise regression procedure could not necessarily deliver an optimal output as it may result in the erroneous elimination of some interesting explanatory variables. Some significance can indeed be masked by complex multicollinearity across variables and could lead to wrongly eliminate one relevant factor. It also does not even inform about which explanatory variables are redundant and which are in fact relevant among the non-significant variables. Therefore, even though multicollinearity does not interfere with the reliability of the model estimates, it almost surely affects the precision of our estimates. We thus prefer to perform all the possible regressions and identify the one that best describes each strategy.

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<sup>8</sup> The Schwarz criterion measures the goodness of fit of the model. It is defined as a negative function of the log likelihood of the model and it penalises the model for its number of factors.

Moreover, to deal with stale pricing problem, we applied the following formula for “de-lagging” the serially correlated Hedge Fund returns as in Okunev and White (2003):

$$R_t = \frac{R_t^* - \alpha R_{t-1}^*}{1 - \alpha} \quad (7)$$

where  $R_t$  is the smoothed returns,  $R_t^*$  unsmoothed returns and  $\alpha$  the first-order autocorrelation of the fund returns. We also perform Newey-West estimates for all regressions in order to take into account the impact of autocorrelation and heteroskedasticity on the coefficient estimates.

### 5.1. *Conditional asset pricing model*

We consider a conditional asset pricing model where the risk exposures to equity factors and the Emerging Market bonds are conditional on the values of the default and term spread.

< Insert Table 2 >

The adjusted R-squares range from a low 46.05% for the portfolio of Relative Value funds to a high 79.61% for the portfolio of Equity Long/Short Hedge Funds. This is consistent with the descriptive statistics presented in Table 1, which has identified Relative Value funds as the strategy that displays the most non-directional risk exposures. It is therefore not entirely surprising to find, for this strategy, poor explanatory power from a set of directional factors.

Equity factors and especially the Russell 3000, the relative performance of small over large cap stocks, the relative performance of winner over loser stocks, and the World Equity Index (ex. US) are the most common risk factors independently of the strategy. The sign and magnitude are also coherent with our expectations. Indeed, Market Neutral Hedge Funds display close to zero exposures to the US equity-based indexes. Short Selling Hedge Funds

on the contrary present negative exposures to the same risk factors. Long/Short Equity Hedge Funds present broad exposures to equity factors (in and out the US market) as well as to the Emerging Market Bond Index.

As expected from Agarwal and Naik (2004) and Jaeger and Wagner (2005), Event Driven funds display significant exposures to the relative performance of small over large cap stocks. The strategy returns are linearly explained at about 69% by the Russell 3000, the SMB, the WEX and the EMB factors. As expected from Fung and Hsieh (2007) and Lucas et al. (2009), the returns on Emerging Market bonds explain a significant part of the strategy returns as this factor captures the returns on distressed firms. Relative Value style is poorly explained by buy-and-hold factors. The World Equity index, the relative performance of small over large cap stocks and of value over growth stocks appear however to be significant in explaining the Hedge Fund returns when taking into account the dynamics in asset allocation, as expected from Agarwal and Naik (2004), Jaeger and Wagner (2005), and Gatev et al. (2006). Finally, more than 68% of the funds of Hedge Funds return variability is captured through equity-related risk premia.

The last two lines report the alpha coefficients under two alternative specifications. The static alpha (penultimate line) represents the intercept of the regression that only uses the directional and conditioning risk premia defined above (lagged instruments x risk premia) as dependent variables, while the dynamic alpha (last line) represents the intercept of the same regression with the use of time-varying alphas to account for market timing of managers, as in Christopherson et al. (1998):

$$\alpha' = \alpha_0 + (Z_{t-1} - \bar{Z})\alpha_1 \quad (8)$$

where  $\alpha_1 = 0$  for static  $\alpha$  and  $\alpha_1 \neq 0$  for the dynamic estimate of  $\alpha$ .

The difference between the two series of alphas is very small, meaning that the full static alpha can be attributed to manager skills rather than to market timing.

Significant abnormal returns are found in the Market Neutral, Short Selling and Event Driven Hedge Funds. The positive and significant alpha coefficients would indicate that these portfolios offer superior returns. The magnitude varies from 31 basis points per month for the Event Driven funds to 77 basis points per month for the Equity Hedge Short Selling Funds.

### *5.2 Conditional higher-moment asset pricing model*

This sub-section evaluates the relevance of a higher-moment analysis of Hedge Fund returns. Table 3 presents, for each Hedge Fund (style) portfolio, the results of a conditional higher-moment multifactor model using the subset of equity and bond-like factors that best describe the returns of each strategy.

< Insert Table 3 >

Conditioning the fund risk exposures onto the expected levels of volatility, skewness and kurtosis in the US market improves the specification of the asset pricing model for all Hedge Fund investment strategies. Adjusted R-squares are improved from a low 1.18% for Event Driven funds to almost 10% for Relative Value funds (which also display the strongest nonlinear return structure). The improvements in R-squares are coupled with an improvement of the Schwarz criterion. We therefore conclude to an improvement in the specification of the models.

Among the equity-based strategies, funds following a Market Neutral or a Short Selling strategy time their risk exposures according to the expected volatility of the US market, while Quantitative Directional funds reallocate their investment portfolios according to the expected levels of both skewness and kurtosis in the US stock market. Given the levels of skewness and kurtosis of the Long/Short strategy displayed at Table 1 (i.e. a positive

skewness and a kurtosis similar to the one of the US stock market portfolio), we conclude that these funds hedge their investments against skewness and kurtosis risks by reviewing their allocation to distress and growth firms and to winner and loser stocks. It appears that the HML mimicking strategy contains a high level of skewness, while the UMD factor a high level of kurtosis risks. When the levels of the US equity skewness are expected to decrease, the fund managers indeed increase their investment in distress firms and decrease their investment in growth firms. An investment in growth firms appear therefore to present strong skewness risk. When the levels of US equity kurtosis are expected to increase however, they decrease their exposure to winner stocks. The momentum strategy could capture the kurtosis risk embedded in the US equity market.

In Table 1, we note that Event Driven funds present significant non-linearities in their risk-return profile. They display a negative skewness inferior to the one of the Russell 3000 and a high level of kurtosis, also superior to the level embedded in the US stock market. From Table 3, the funds appear to have some kurtosis risk objectives as they time the exposures to the world equity market according to the expected level of US equity kurtosis.

Relative Value funds present exposures to all three implied moments. Out of the analysis reproduced at Table 3, the funds appear to allocate money to the US stock market according to the levels of expected volatility in this market. Their funds moreover allocate risk to the Emerging Market bonds or to the worldwide stock market according to respectively the skewness and kurtosis of the US market. We expect the performance of these markets (and therefore the levels of volatility, skewness and kurtosis in these markets) to be related to the levels of skewness and kurtosis in the US market.

Both the Event Driven and the Relative Value strategies rebalance their investments to the world equity markets (excluding US) according to the expected levels of kurtosis. Both strategies seem to buy kurtosis risk by investing in the world equity market when the levels of



kurtosis are expected to decrease in the US market. The kurtosis of these strategies seems thus to be related to the kurtosis and/or the skewness of the world equity markets.

Finally, Funds of Funds present exposures to the return asymmetry risk of the US market. As for the Quantitative Directional strategy, the Fund of Funds portfolio hedge skewness risk by reallocating frequently their exposures to value and growth stocks according to the expected levels of skewness for the US equity market. When the levels of skewness is decreasing, the fund managers indeed invest more in distress firms but take also more short positions in growth firms.

Compared to the conditional asset-based multifactor model reproduced at Table 2, the conditional higher-moment model strongly improves the proportion of return that can be explained by the model and also strongly decrease the abnormal performance of all Hedge Fund style portfolios but the Event Driven and Relative Value strategies.

### *5. 3. Conditional option pricing model*

In order to capture the non-linear return structure of Hedge Fund returns, Table 4 considers a conditional option pricing model. Instead of considering the dynamic exposures of Hedge Fund style to asymmetry and extreme risk variation, we consider risk trading strategies that are able to replicate some part of the Hedge Fund returns. Among the available strategies, we consider a rolling-over ATM and OTM put and call option strategies as well as the primitive trading strategies of Fung and Hsieh (2004) which replicate market timing strategies in Hedge Funds.

< Insert Table 4 >

Because they implement market timing strategies, the returns on Market Neutral funds are, not surprisingly, significantly explained by the returns on lookback straddles on stock

and interest rate futures. On the contrary, Event Driven funds and Relative Value funds rely on convergence-based strategies. As a consequence, they present significant negative exposures to straddles on interest rate futures. Event Driven funds also present a short position in lookback straddles on bonds. The payoffs of put and call option-based strategies also replicate a part of the return structure of the Event Driven funds. Funds of Funds and Event Driven funds present a long position in an OTM put option while the Short Selling funds are significantly replicated by a long position in an OTM call option.

Stale pricing in Market Neutral, Quantitative Directional, Event Driven and Relative Value funds can be detected from the significance of the lagged option-based factors.

Compared to the conditional asset-based model displayed at Table 2, the conditional option-based model captures a larger part of the Hedge Fund return variations. The levels of specification error of the model are however superior to the ones displayed by a simple multifactor model for 4 out of the 6 portfolios. The specification errors of the models are even superior to the ones displayed by a conditional higher-moment pricing model for all but the Short Selling funds. The levels of abnormal performance delivered by the higher-moment pricing model are even half the levels displayed by the option-based model for the Market Neutral and the Funds of Funds styles.

#### *5.4. Conditional higher-moment option pricing model*

The following table combines the explanatory power of both the option trading strategies and the higher-moment premia in a conditional asset pricing model in order to explain the Hedge Fund particular return structure.

< Insert Table 5 >

Compared to Tables 2-4, Table 5 offers a better explanation of the return variation of all funds given the values of the R-squares or of the Schwarz criterion. Except for the Relative

Value strategy however, Table 3 still provides less specification error. Moreover, except for Event Driven funds, the higher-moments are not subsumed by the option-like trading factors. For the Event Driven funds however, option straddles and put/call option payoffs are more able than higher-moment premia to capture the style return variability.

Having identified, using the different types of factors, the composite model that best explains Hedge Fund risk exposures for the different styles, we will now focus on the residual returns which cannot be explained by systematic exposures to risk factors. Under the assumption that the model is well-specified, this excess return is generally attributed to manager skill. The Equity Long/Short portfolio outperforms all the other alternative strategies. The worse performer is our portfolio of Funds of Funds. This is not surprising as these funds operate a double-fee structure.

#### *5.5. Incremental significance of each set of factors*

A final important issue needs to be addressed regarding the relevance in Hedge Fund asset pricing model of the new set of conditional moment-related factors. We need to decompose the marginal values of each family of factors in the composite model. Table 6 displays the incremental significance of all the moment and option-based factors over directional factors.

< Insert table 6 >

Column (5) of Table 6 shows that conditional moment-related factors significantly improve the explanatory power of a simple directional model. The strongest model improvement is found in the portfolio of Relative Value funds. We record an increase in the adjusted R-square of about 20% when adding these non-directional factors in the asset-based model used for explaining this category of Hedge Fund. The improvement in R-square is also particularly high for Short Selling Hedge Funds and Funds of Hedge Funds.

Column (6) presents the improvement in explanatory power of the asset-based model when option-like factors are introduced in the models. The improvements are not as good as with moment-related factors. The strongest increase in R-squares is found in the Event Driven strategy. The model seems to outperform the conditional higher-moment multifactor model for Market Neutral and Event Driven funds as the percentage of unexplained variance captured by optional factors are superior to the one explained by higher moments for these Hedge Funds.

## **6. Conclusions**

Even though it makes sense from a fundamental point of view, the association of conditional asset pricing approaches with the use of the information content of market skewness and kurtosis has never been implemented. In this paper, we argue that this association is relevant for several Hedge Fund strategies. Just as the equity option-implied volatility has long been recognized as a useful instrument to anticipate market movements and thus adapt a fund's exposures, the implied skewness and kurtosis of index options can serve a similar purpose. Our paper provides empirical support validating this idea.

The conditional asset pricing model that we propose represents an adequate framework to apply this principle. Unlike previous attempts to integrate skewness and kurtosis risks in Hedge Fund research, our study assumes that higher moments are risks that managers expose themselves to strategically rather than external factors that they are submitted to involuntarily. There are two benefits to this framework. Firstly, this justifies the inclusion of higher-moments risk premia in order to explain Hedge Funds returns. Secondly, this framework does not aim to replace, but rather to complement, asset pricing specifications that have resulted from a careful and progressive stream of research initiated and crowned by the work of Fung and Hsieh (1997).

This attempt to graft skewness and kurtosis appears to be successful. For most strategies studied in this paper, the conditional specification we propose improves existing state-of-the-art multi-factor models. The introduction of implied moments as instruments increases the models' explanatory power and reduces specification error for all tested strategies. The extent of the improvement appears to be superior to the added value of option-based risk premia.

The application provided in this paper uses implied moments retrieved from U.S. equity markets. Even though this market probably has the largest influence on Hedge Fund returns, our encouraging results suggest that the enlargement of the framework to other market types and locations would bring some extra explanatory power. As the ultimate goal of these efforts is to distinguish alternative alphas from alternative betas in Hedge Fund returns, such an extension would certainly be desirable, and is part of our ongoing research agenda.

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**Table 1**  
Descriptive statistics of Hedge Fund style portfolios <sup>a</sup>

Category	Symbol	Nr of Fds	% of the category	% of the total	Mean (%)	Median (%)	Max. (%)	Min. (%)	S.D. (%)	Skew.	Kurt.	J-B	Sharpe ratio
<i>HFR Classification</i>													
Equity Hedge – Market Neutral	MN	133	11.2	4.7	0.6456	0.6269	8.0311	-3.5080	1.4689	1.0451	7.6680	149.33***	0.4395
Equity Hedge – Short Bias	SB	20	87.2	0.7	0.2054	0.2700	20.0367	-21.3567	5.8902	0.0564	4.7157	16.8750***	0.0349
Equity Hedge – Quantitative Directional	QD	1038	1.6	36.8	0.8105	1.1277	11.9715	-8.8689	3.2635	0.1458	4.5275	13.8041***	0.2484
Event driven	ED	269		9.5	0.6103	0.8619	5.3403	-8.5162	1.7993	-1.7813	9.7871	335.4033***	0.3392
Relative Value	RV	407		14.4	0.5242	0.7727	3.2640	-9.6470	1.5034	-3.6000	22.9423	2566.099***	0.3487
<i>Fund of Funds</i>	FF	952		33.8	0.4593	0.5990	7.2596	-7.1737	1.9884	-0.6511	6.6174	84.3754***	0.2310
<b>Total</b>		2819											
Russell 3000	SP				0.0382	0.9105	8.5341	-17.8746	4.8072	-0.8284	4.1250	22.8952***	0.0079

<sup>a</sup> Table 1 reports the mean, median, maximum, minimum, standard deviation (S.D.), skewness, kurtosis, and the Jarque-Bera (J-B) statistics for the 6 *Hedge Fund Research* Hedge Fund styles, and for the Russell 3000, taken as benchmark. The database composition is also described. \*, \*\* and \*\*\* stand for significant at 10%, 5%, and 1%, respectively.

**Table 2**  
Conditional asset pricing models <sup>b</sup>

	HFR Classifications					FF
	EH		ED	RV		
	MN	SB				
<i>Assets</i>						
RUS	0.1956***	-1.0809***	0.3394***	0.2643***		0.1644**
SMB		-0.4806***	0.3286***	0.1239***		0.1821***
HML	-0.0591***					
MOM	0.1511***	-0.3453***	0.1860***			0.1575***
WEX			0.4536***	0.2389***	0.3987***	0.3516***
WGBI						
EMB			-0.1367***			
<i>Instruments</i>						
ZCRUS				-4.9207**		
ZSRUS			-4.4291			
ZSSMB			1607204***		24.9939***	11.8312***
ZSHML		20.4544**			-10.9310*	
ZCUMD		7.3092***				
ZCEMB				5.6457***		2.1486**
ZSWEX			-6.2302		-7.9153**	-6.1659
Adj. R <sup>2</sup> (%)	48.55	77.16	79.61	68.61	46.05	68.15
Schwarz	-5.9886	-3.9415	-4.8118	-5.2422	-4.7783	-5.0900
Static $\alpha$ (%)	0.3110***	0.3009	0.7737***	0.3244**	0.1229	0.2007
Dynamic $\alpha$ (%)	0.3110***	0.3038	0.7316***	0.3272**	0.1233	0.2060

<sup>b</sup>Table 2 reports the estimated coefficients for the conditional asset pricing models made of the subset of directional and conditional factors that best describes each Hedge Fund style. The static and the dynamic alphas are estimated using Equation (8). \*, \*\* and \*\*\* stand for significant at 10%, 5%, and 1%, respectively.

**Table 3**  
Conditional higher-moment asset pricing models <sup>c</sup>

	HFR Classifications					FF
	EH		ED	RV	MN	
	SB	QD				
<i>Assets</i>						
RUS	0.2009***	-1.2329***	0.3560***	0.2720***		0.1676***
SMB		-0.4524***	0.3021***	0.1306***		0.1479***
HML						
MOM	0.1626***	-0.3332***	0.1847***			0.1452***
WEX			0.4533***	0.2140***	0.3380***	0.3691***
WGBI						
EMB			-0.1380***			
<i>Instruments</i>						
ZCRUS				-4.7130*		-5.4461**
ZVRUS		1.5316***			-0.6137**	
ZCSMB					12.3695***	
ZSSMB			20.6526***			12.4843***
ZCHML	-6.4364***					
ZSHML		22.2500***				
ZSkHML			-0.1179***			-0.0890***
ZSUMD	-0.0254***					
ZKuUMD		0.1325	-0.0885*			
ZCEMB				5.5004***		5.2196***
ZVEMB	-0.6910***					
ZSkEMB					0.0696***	
ZSWEX			-11.7504***			-8.7631***
ZKuWEX				-0.0412***	-0.1294***	
Adj. R <sup>2</sup> (%)	52.78	79.95	83.13	69.79	56.79	72.27
Schwarz	-6.0176	-4.0434	-4.9451	-5.2521	-4.9720	-5.1720
Static $\alpha$ (%)	0.2854***	0.2703	0.6875***	0.3633**	0.2667*	0.1337
Dynamic $\alpha$ (%)	0.3268***	0.2415	0.6976***	0.3738**	0.2761*	0.1454

<sup>c</sup> Table 3 reports the estimated coefficients for the conditional higher-moment asset pricing models made of the subset of directional, conditional and distributional factors that best describes each Hedge Fund style. The static and the dynamic alphas are estimated using Equation (8). \*, \*\* and \*\*\* stand for significant at 10%, 5%, and 1%, respectively.

**Table 4**  
Conditional option pricing models <sup>d</sup>

	HFR Classifications					FF
	EH			ED	RV	
	MN	SB	QD			
<i>Assets</i>						
RUS	0.2011***	-1.0774***	0.3817***	0.4132***		0.1757***
SMB		-0.4432***	0.3214***	0.1531***		0.1753***
HML	-0.0565***					
MOM	0.1569***	-0.3351***	0.1902***			0.1678***
WEX			0.4354***		0.3536***	0.3467***
WGBI						
EMB			-0.1241***			
<i>Options</i>						
AMC						
AMP						
OMC		0.0033*				
OMP				0.0018***	0.0028***	0.0022***
LAMC				-0.0043***		
LAMP						
LOMC				0.0021**	-0.0025**	
LOMP	0.0011***		0.0025***			
PTFSIR	-0.0097***			-0.0140***	-0.0209**	
PTFSBD				-0.0397***		
PTFSSTK	0.0238**					
<i>Instruments</i>						
ZCSMB					16.8931***	
ZSSMB			13.4664***			12.4042***
ZCHML	-5.7653**					
ZSHML					-7.7373*	
ZSUMD		20.8589**				
ZCEMB	1.9921					
ZSEMB		0.1793*		3.7160***		
ZSWEX						-5.4333***
Adj. R <sup>2</sup> (%)	55.73	78.35	79.56	73.80	51.54	68.94
Schwarz	-6.0253	-3.9668	-4.8094	-5.2767	-4.8288	-5.1151
Static $\alpha$ (%)	0.4824***	0.1141	0.7554***	0.3798**	0.3075**	0.3087**
Dynamic $\alpha$ (%)	0.4825***	0.1116	0.7590***	0.3726**	0.3152**	0.3169**

<sup>d</sup>Table 4 reports the estimated coefficients for the conditional option pricing models made of the subset of directional, conditional and optional factors that best describes each Hedge Fund style. The static and the dynamic alphas are estimated using Equation (8). \*, \*\* and \*\*\* stand for significant at 10%, 5%, and 1%, respectively.

**Table 5**  
Conditional higher-moment asset- and option-based pricing models <sup>e</sup>

	HFR Classifications					FF
	EH			ED	RV	
	MN	SB	QD			
<i>Assets</i>						
RUS	0.1871***	-1.2401***	0.3719***	0.4132***		0.1868***
SMB		-0.4645***	0.2988***	0.1531***	0.0953***	0.1503***
HML	-0.0613***					
MOM	0.1565***	-0.3295***	0.1909***			0.1541***
WEX			0.4545***		0.3476***	0.3608***
WGBI					0.4228***	
EMB			-0.1402***			
<i>Options</i>						
AMC		0.0050*				
AMP						
OMC						
OMP			0.0022***	0.0018***	0.0034***	0.0018***
LAMC				-0.0043***		
LAMP						
LOMC				0.0021**	-0.0022**	
LOMP						
PTFSIR	-0.0108***			-0.0140***		
PTFSSTK	0.0198**					
PTFSBD				-0.0397***	-0.0278***	
<i>Instruments</i>						
ZCRUS						-3.7113*
ZVRUS		1.4985***			-0.6256**	
ZCSMB					12.3465***	
ZSSMB			20.4437***			13.6684***
ZCHML	-6.1101***					
ZSHML		21.0800***				
ZSkHML			-0.1236***			-0.0897***
ZKuUMD		0.1264	-0.0782			
ZCEMB						4.1665***
ZVEMB	-0.5924***					
ZSkEMB					0.0679***	
ZSWEX			-11.0530***			-8.3029***
ZKuWEX					-0.1321***	
Adj. R <sup>2</sup> (%)	57.58	80.37	83.90	73.80	65.91	73.16
Schwarz	-6.0680	-4.0361	-4.9631	-5.2767	-5.0675	-5.1764
Static $\alpha$ (%)	0.4599***	0.3141	0.7449***	0.3798**	0.2327*	0.1967
Dynamic $\alpha$ (%)	0.4651***	0.2899	0.7733***	0.3726**	0.2506*	0.2257

<sup>e</sup> Table 5 reports the estimated coefficients for the conditional higher-moment asset- and option-based pricing models made of the subset of directional, conditional, optional, and distributional factors that best describes each Hedge Fund style. The static and the dynamic alphas are estimated using Equation (8). \*, \*\* and \*\*\* stand for significant at 10%, 5%, and 1%, respectively.

**Table 6**Incremental significance of moment- and option-based factors regarding directional factors <sup>f</sup>

		Directional and Non-Directional Factors								
		+ assets	+ implied moments		+ options		+ options and implied moments		IUP <sub>moment</sub>	IUP <sub>option</sub>
		(1)	(2)		(3)		(4)		(5)	(6)
				$\Delta$		$\Delta$		$\Delta$		
R <sup>2</sup> <sub>(%)</sub>	MN	48.55	52.78	4.23	55.73	7.18	57.58	9.03		
Schwarz		-5.9886	-6.0176	-0.119	-6.0253	-0.0367	-6.0680	-0.0794	8.22	13.96
$\alpha$ (%)		0.3110***	0.2854***	-0.0256	0.4824***	0.1714	0.4599***	0.1489		
R <sup>2</sup> <sub>(%)</sub>	SB	77.16	79.95	2.79	78.35	1.19	80.37	3.21		
Schwarz		-3.9414	-4.4034	-0.462	-3.9668	-0.0254	-4.0361	-0.0947	12.22	5.21
$\alpha$ (%)		0.3009	0.2703	-0.0306	0.1141	-0.1868	0.3141	0.0132		
R <sup>2</sup> <sub>(%)</sub>	QD	79.61	83.13	3.52	79.61	0	83.90	4.29		
Schwarz		-4.8118	-4.9451	0.1333	-4.8118	0	-4.9631	-0.1513	4.90	0
$\alpha$ (%)		0.7737***	0.6875***	-0.0862	0.7737***	0	0.7449***	-0.0288		
R <sup>2</sup> <sub>(%)</sub>	ED	68.61	69.79	1.18	73.80	5.19	73.80	5.19		
Schwarz		-5.2422	-5.2521	-0.0099	-5.2767	-0.0345	-5.2767	-0.0345	3.76	16.53
$\alpha$ (%)		0.3244**	0.3633**	0.0389	0.3798**	0.0554	0.3798**	0.0554		
R <sup>2</sup> <sub>(%)</sub>	RV	46.05	56.79	10.74	51.54	5.49	65.91	19.86		
Schwarz		-4.7783	-4.9720	-0.1937	-4.8288	-0.0505	-5.0675	0.2892	19.91	10.18
$\alpha$ (%)		0.1229	0.2627*	0.1398	0.3075**	0.1846	0.2327*	0.1098		
R <sup>2</sup> <sub>(%)</sub>	FF	68.15	72.27	4.12	68.94	0.79	73.16	5.01		
Schwarz		-5.0900	-5.1720	-0.082	-5.1151	-0.0251	-5.1764	-0.0864	12.94	2.48
$\alpha$ (%)		0.2007	0.1337	-0.067	0.3087**	0.108	0.1967	-0.004		

<sup>f</sup>Table 6 reports the incremental explanatory power of adding successively asset-based factors, implied moments and optional factors. The set of variables of Colum (2) corresponds to those selected in Table 3; the set of variables of Colum (3) corresponds to those selected in Table 4; the set of variables of Colum (4) corresponds to those selected in Table 5. The table reports in column (5) the change in the adjusted R-square due to the introduction of the conditional higher-moment factors with regard to the

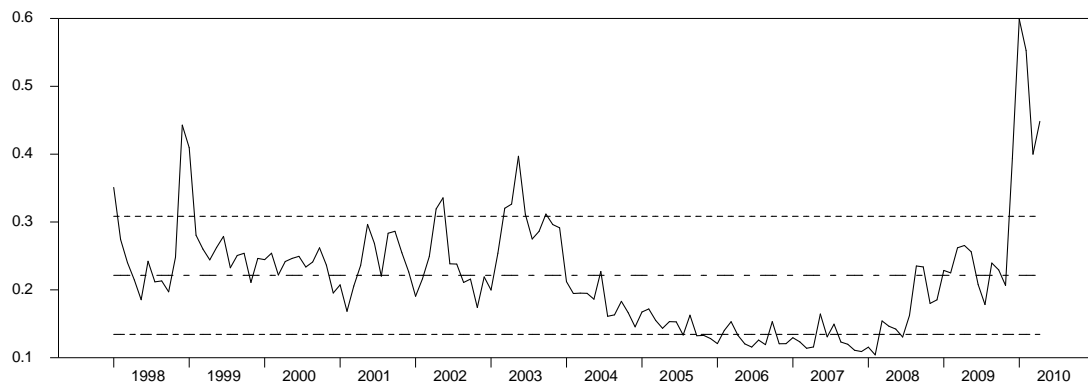


directional factors. Column (6) reports the change in the adjusted R-square due to the introduction of the option-based factors with regard to the directional factors. This

information is summarized in the Incremental Unexplained Proportion (IUP) defined as

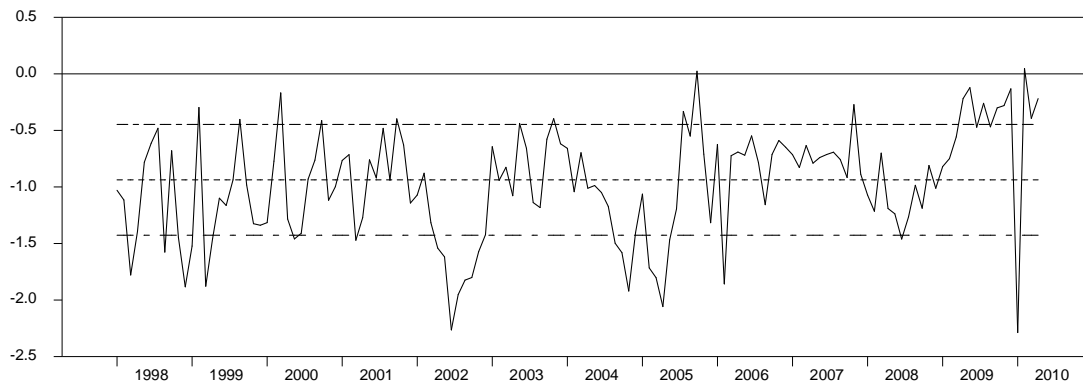
$$IUP = \frac{\bar{R}_{with}^2 - \bar{R}_{no}^2}{1 - \bar{R}_{no}^2}$$

**Figure 1** – Evolution of the US implied volatility



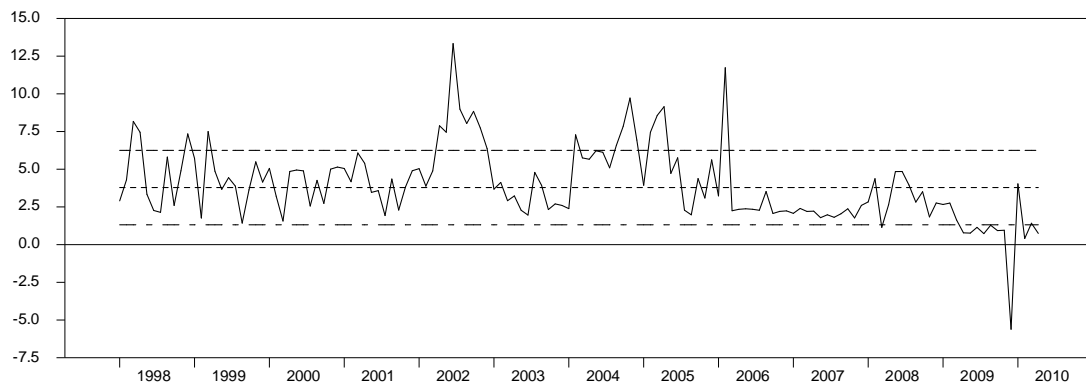
The figure displays the time-series evolution of the US implied stock volatility over the period November 1997 to May 2010. The series has been collected from the CBOE website and corresponds to the market expectations of near-term volatility conveyed by S&P 500 stock index option prices.

**Figure 2** – Evolution of the US implied skewness



The figure displays the time-series evolution of the US implied stock skewness over the period November 1997 to May 2010. The series corresponds to the market expectations of near-term skewness conveyed by S&P 500 stock index option prices. The theoretical values of the risk-neutral skewness has been computed from the methodology of Bakshi et al. (2003) on CBOE S&P 500 stock index option prices collected from the CBOE.

**Figure 3** – Evolution of the US implied kurtosis



The figure displays the time-series evolution of the US implied stock kurtosis over the period November 1997 to May 2010. The series corresponds to the market expectations of near-term kurtosis conveyed by S&P 500 stock index option prices. The theoretical values of the risk-neutral kurtosis has been computed from the methodology of Bakshi et al. (2003) on CBOE S&P 500 stock index option prices collected from the CBOE.

## Appendix

**Table A.1**  
Correlation matrix of asset- and option-based factors

	Asset-based factors							Optional factors								
	RUS	SMB	HML	UMD	WEX	WGBI	EMB	AMC	AMP	OMC	OMP	PTFSBD	PTFSFX	PTFSCOM	PTFSIR	PTFSSTK
RUS	Medium Grey															
SMB	Light Grey	Medium Grey														
HML	Light Grey	Light Grey	Medium Grey													
UMD	Light Grey	Medium Grey	Medium Grey	Medium Grey												
WEX	Black	Medium Grey	Medium Grey	Medium Grey	Medium Grey											
WGBI	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey										
EMB	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey									
AMC	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Dark Grey	Light Grey								
AMP	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey							
OMC	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey						
OMP	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey					
PTFSBD	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey				
PTFSFX	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey			
PTFSCOM	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Dark Grey			
PTFSIR	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	
PTFSSTK	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey	Medium Grey

Table A.1 reports the ranges of linear (Pearson) correlation coefficient  $\rho$  among asset- and option-based risk factors. Color codes for correlations are: strong positive correlation ( $\rho > 70\%$ ) in black (■), moderate positive correlation ( $30\% < \rho < 70\%$ ) in dark grey (■), weak correlation ( $-30\% < \rho < 30\%$ ) in medium grey (■), moderate negative correlation ( $-70\% < \rho < -30\%$ ) in light grey (■), and strong negative correlation ( $\rho < -70\%$ ) in white (□).