An empirical investigation of idiosyncratic risk and stock returns relation in heteroskedasticity corrected predictive models

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Abstract

This paper investigates stock returns behaviour as a function of lagged idiosyncratic risk in the Fama-French three-factor model using two approaches to estimating idiosyncratic risk. The application of ordinary least squares and quantile regression methods to heteroskedasticity corrected data in a panel structure reveals that the form of relationship does not change with the method of estimating idiosyncratic risk and is indeed dynamic. The relationship curves resemble the shapes of the utility curves of risk-seeking and risk-aversion, and not the risk-neutral attitude. The findings are robust to the choice of FF three-factor and capital asset pricing one-factor models, and to the choice of estimation window. Our results help explain some of the basis of conflicting results reported in the literature on the form of idiosyncratic risk –return relation. The form of the relationship is too dynamic to support idiosyncratic risk as a 'priced' item.

Key Words: Idiosyncratic risk, quantile regression; GARCH model; heteroskedasticity, panel data.

JEL classification: C14, C21, C22, C23

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Introduction

The Capital Asset Pricing Model (CAPM) of Sharpe (1964), Lintner (1965) and Black (1972) depicts systematic risk representing contemporaneous positive linear relationship between excess stock returns and excess market returns. The model advocates that the systematic risk is sufficient to explain the expected stock returns in the cross-section. Risk arising from any specific events, the idiosyncratic risk, can be diversified away by holding a portfolio of stocks and that investors will not be compensated for such risk.

The theoretical asset pricing model of Merton (1987) in markets with incomplete information shows a positive relation between idiosyncratic risk and stock returns. In recent years there has been considerable interest in exploring the behaviour of idiosyncratic risk due to the belief that higher idiosyncratic risk may be responsible for generating higher returns, i.e., idiosyncratic risk is a priced factor. Empirical studies in this area use different measures of estimating idiosyncratic risk and models to investigate the idiosyncratic risk-return relationship at the portfolio or stock level. However, the findings provide conflicting evidence about the form of this relation and many refer to this relation as a '*puzzle*'. This makes the study of IRSK very interesting. Many corporate and financial decisions require some knowledge of the expected returns from an investment on an individual stock or a portfolio of risky assets. Some traders and institutional investors perceive stock volatility as an important determinant in their decision to select stocks. Thus, understanding the nature of idiosyncratic risk-return relationship is important. As all stock market investments are exposed to systematic as well as idiosyncratic risk to a certain degree, and in some cases the idiosyncratic risk may constitute the major portion of the total risk, examining idiosyncratic risk-return relationship issue is not only relevant for asset pricing theory, but also of interest to portfolio managers. All studies to date have explored the idiosyncratic risk-return relationship at the mean level via the use of ordinary least squares (OLS). Our study uses quantile regression technique that allows investigation of this relation at various quantiles of the conditional distribution of returns, and thus provides a deeper insight into the form of the relationship.

Malkiel and Xu (1997), Goyal and Santa-Clara (2003), Fu 2009 and many others find a positive relation between idiosyncratic risk and stock returns. Bali et al. (2005) and Wei et al. (2005) independently replicate Goyal and Santa-Clara (2003) study using an extended sample and find no positive relation. Wei et al. (2005) find the relationship to be negative and not very significant. Ang et al. (2006, 2009) and Jiang et al. (2009) observe a negative relation between idiosyncratic risk and future stock returns. All of the above studies are based on US data. Drew et al. (2007) replicate Malkiel and Xu (1997) study using New Zealand data and find a positive relation. Bollen et al. (2009) build their study model on Bali et al. (2005) framework but use the Australian data and observe that idiosyncratic risk is not priced in Australia. Nartea et al. (2010) apply Ang et al. (2006, 2009) model to explore the role of idiosyncratic risk in five South East Asian markets of Malaysia, Singapore, Thailand, Indonesia and the Philippines. They find positive relation between idiosyncratic risk and stock returns in Malaysia, Singapore, Thailand and Indonesia, and no significant relation in the Philippines.

Some studies have investigated idiosyncratic risk-return relationship in models representing contemporaneous relationship (Fu 2009), while others use predictive models (Ang et al. 2006, 2009).

This paper is related most closely to studies that support the notion of predictive models, i.e., IRSK and other explanatory variables are estimated or recorded in a period prior to the period of realising stock returns. From a decision or a policy maker's perspective, models with predictive element have a lot more appeal than static single or same period models depicting contemporaneous relationships. A model that does not offer opportunity to investors to weigh their investment strategy in short to medium term time frame is of limited use. The time between buying and selling of stock (s) to realise returns on an investment is not zero, and therefore a model representing contemporaneous relations cannot serve well. Like Ang et al. (2006, 2009) we observe the idiosyncratic risk to be highly persistent that justifies the use of lagged idiosyncratic risk for predicting stock returns in this paper.

Our study uses Australian data for a number of good reasons. Australia has the most regulated market trading system and the available financial market data is very genuine and reliable. Its economy and the banks enjoy good health; it survived the GFC of 2008 and is weathering quite well the European sovereign debt crisis. Australia, being well poised between the developed financial markets of the West and the fast developing and established markets of the Asia-Pacific region, reflects a good balance between outcomes of financial events of the East and the West. There is also a lot of foreign investment in Australian Stock Market. A study of idiosyncratic risk-return relationship based on Australian data would not only be relevant to asset pricing theory, but also important for the investment community.

This paper investigates idiosyncratic risk and stock returns relationship using two approaches of estimating idiosyncratic volatility, a proxy for idiosyncratic risk. The relationship is explored by applying the quantile regression method in a panel data structure after correcting data for heteroskedasticity. The method estimates parameters of the explanatory variables at various quantiles of the conditional distribution of returns that allows the marginal effects of regressors to change at different quantiles, facilitating parameter heterogeneity across different types of stock returns. The ordinary least squares (OLS) method is used for bench marking. It is observed that the stock returns and idiosyncratic risk relationship is indeed dynamic, of parabolic nature and depends on the quantile of the conditional distribution of returns. The form of the parabolic curves indicate that investors may be risk-averse or risk-seekers rather than being risk-neutral. The findings are invariant to the choice of capital asset pricing one-factor and FF three-factor models. The results hold when the size of estimation window, rolling window and the length of time of holding a stock to realize returns are changed. Thus, our results explain some of the basis of mixed results reported on the form of the idiosyncratic risk-return relationship puzzle.

The rest of paper is organised as follows. Section 2 describes the data used and the empirical modelling framework of the paper. Findings of the analysis together with a likely form of the idiosyncratic risk-return relationship are reported in Section 3, confirming the stability of the results via robustness checks. Finally, Section 4 concludes the paper by summarising the main contributions.

2 Empirical Framework

2.1 Data Description

We use daily end of the day prices of stocks trading on the Australian Securities Exchange (ASX) together with end of the day All Ordinaries Index as a representative of a general market portfolio from Datastream data bank for the 10 years period of January 2001 to June 2010. Data on stock's market capitalisation (firm size), book-to-market (BM) and 30-day Cash rate as set by Reserve Bank of Australia's Board at each monthly meetings as a proxy for daily risk free rate of return are also downloaded from Datastream. Stocks that did not trade for 20% or more days during the sampling period or had missing data on one or more important variables or did not trade for 100 consecutive days were excluded from the data. Due to recent global financial crisis many of the companies did not meet this criterion and the sample ended up with 207 companies that traded during the study period representing 35 of the 40 industry sectors. Our sample consists of a good mixture of small, medium and large stocks, and should provide a good insight into the nature of investor attitude to risk. The days of public holidays were eliminated from the sample for all stocks. This resulted in daily data on the 207 stocks available for the entire length of the sampling period.

2.2 Setting up data Panels for predictive models

Observing a lot of persistence in estimated idiosyncratic risk for the current data, the use of a predictive model is justified. The majority of the empirical studies listed above form portfolios of stocks on explanatory variables size, book-to-market or idiosyncratic risk to study idiosyncratic risk-return relation. Studies indicate that in practice many investors do not hold a portfolio of stocks (Goetzmann and Kumar (2004)) let alone holding a perfectly diversified portfolio. As per Fama and French (1992), the main reason for studies to form portfolios is to get better estimates, or to control the effect of an explanatory variable. Although the forming of portfolios may improve the precision of estimates, it also reduces the number of observations available for estimation. Also the process of aggregation via portfolios may smooth out the underlying pattern, whereas the disaggregation may present better understanding of the pattern (Ferson and Harvey (1999)). Besides, the aggregation of stocks into portfolios may induce the effect of diversifying away the idiosyncratic risk as was the belief of many researchers in the past. In the light of the above, we move away from forming portfolios of stocks. The choice of forming portfolios each month or rolling the estimation window each month by many researchers in the past is just a convenience matter. We use a model of rolling windows where the choice of the size of the estimation window, rolling forward of a window and the period of observation of returns of stocks is all flexible. This creates a panel data structure where panels can be formed using any length of the estimation window and rolling forward of a window by any number of time periods (days or months). The framework can be set up as a contemporaneous or predictive model.

In the following, we provide details of the empirical framework, and clearly set up the models as predictive, i.e., explanatory variables estimated or recorded in a previous time period are linked to the future stock returns. As per Ang et al. (2009) remark that "FF three-factor loadings might not account for all variation in expected returns compared to firm-level characteristics" and foot note 3 in Ferson and Harvey (1999), variables size and book-to-market are used as stock specific important explanatory variables in all models. As past returns may be perceived as an important indicator of a stock's performance, the average of past returns over the estimation period is included in the list of explanatory variables. All results are obtained within the Fama and French (1993) three-factor model (referred to as FF three-factor). Later the same modelling framework is evaluated for the CAPM one-factor model to show robustness and the invariant nature of the findings.

We apply two different methods of OLS and GARCH(1,1) to time series of each stock for estimating parameters of the FF three-factors and idiosyncratic risk for checking the sensitivity of idiosyncratic risk-return relationship to the method of estimation. To allow a stock to have time varying betas, idiosyncratic risk and other important stock specific characteristics, the two-stage procedure of Fama and MacBeth (1973) is adapted. The first stage involves using rolling windows for estimation of parameters and calculation and/or recording of information on important stock specific characteristics for each stock. The estimates from the first stage are used in the second stage for testing purposes and evaluating relationships by running cross-sectional regressions (OLS and quantile regression) in each panel. Specifically, FF three-factor model is fitted using time series of 100 days' excess returns of a stock, excess market returns and Fama-French daily factors SMB, small minus big, and HML, high minus low, via OLS and GARCH(1,1) methods. The estimates of FF three-factor betas for each stock as well as idiosyncratic risk obtained (details in Section 2.3) using OLS and GARCH(1,1) methods are fed into two separate models. Information on median stock size (*lnsize*), median book-to-market (*lnBM*) and average excess return, *R1*, over these 100 days is also recorded in this phase. Next a stock's average excess return, *R*, over the next 25 days is computed and used as the response variable. The process is repeated for each of the 207 stocks; this makes one panel for each of the OLS and GARCH(1,1) based models. The panel's structure is in line with the literature that the dependent variable is constructed in a way that it is non-overlapping over time with the estimation period to avoid possible inter-temporal correlation across panels. By rolling the window 25 days forward, the process is repeated over entire data of 2473 days which creates 91 panels comprising information on dependent and independent variables for each of the 207 stocks. This completes the first stage of the procedure. This first stage of the procedure is referred to as the *E/H/R* plan, where *E* represents the size of estimation sample, *H* the length of the period of holding stocks to realise excess returns and *R* the rolling size of the window. So our first stage *E/H/R* plan as explained above is 100/25/25, where all numbers represent days.

2.3 Measuring idiosyncratic risk: FF three-factor model

Daily Fama and French factors for Australian data are constructed using each day's data for the study sample over 10 years. Each day, six intersecting portfolios are formed based on size and book-to-market value. A stock with size less than or equal to the median size is classified as Small (S), otherwise Big (B). The three groupings on book-to-market values are created using the smallest 30%, Low (L), the top 30%, High (H), and the middle 40%, Medium (M), percentiles. The factors SMB and HML are calculated as given in Fama and French (1993).

Let P_{id} and M_d be the price of a stock i (i = 1, 2, …, N) and the market index on day d, and r_f the daily risk free rate of return. We define excess return on stock i and market index on day d as,

$y_{id} = log(P_{id}/P_{id-1}) - r_f$ and $r_{md} = log(M_d/M_{d-1}) - r_f$. Two different measures of idiosyncratic risk are detailed below.

2.3.1 Using Ordinary Least Squares

Assuming parameters of the FF three-factor model to be constant over the estimation period, idiosyncratic risk for stock *i* (= 1, 2, … , 207) is estimated with respect to OLS estimation of model

$$y_{id} = \alpha_i + \beta_i r_{md} + \beta_{SMB,i}\, SMB_d + \beta_{HML,i}\, HML_d + \varepsilon_{id} \;, \qquad (1)$$

as standard deviation of residuals, referred to as IRSK estimate. The betas in model (1) are the factor loadings of the FF three factor model. The recording of stock specific variables, the average excess return *R1*, *lnsize* and *lnBM* along with idiosyncratic risk and FF three-factor loadings over 100 days, and average excess stock return, *R,* over the next 25 days for each of the207 stocks completes one panel. The process is repeated rolling the window forward by 25 days until all data is covered. This completes the first stage of the two-step procedure, producing a panel data structure of $T = 91$ *panels* where each panel comprises information on dependent and explanatory variables for each of the 207 stocks.

The second step involves estimating cross-sectional relation between average excess return R and explanatory variables *R1, lnsize, lnBM, factor loadings from model (1) and idiosyncratic risk* in each panel *via the use of* ordinary least squares and quantile regression methods and pooling estimates from all the panels.

We observe significant correlation between idiosyncratic risk and explanatory variables *R1*, *lnsize*, *lnbm* and *the factor loading* of FF three-factor model (1) in each panel. This can potentially create a multicollinearity problem, making the conclusions not sound at all. The Breusch-Pagan-Godfrey tests applied to panels reveals the presence of heteroskedasticity in 86 of the 91 panels generated by fitting FF three-factor model via OLS. Detailed analysis suggests that this heteroskedasticity is mainly caused by idiosyncratic risk estimates. Thus to obtain heteroskedasticity-consistent standard errors of the estimates from the fitted models, it seems important to correct data for heteroskedasticity. Multiplying each data row by a factor '1/(idiosyncratic risk estimate)' corrects heteroskedasticity in 78 of the 91 total panels, and the remaining panels show mild heteroskedasticity due to other occasional explanatory variables. We take no further action for correcting heteroskedasticity. After correcting data for heteroskedasticity, we apply the OLS and quantile regression methods to the predictive models set up above. Specifically, the predictive model encompassing the FF three-factor loadings is

$$R_{it} = \gamma_{0t} + \gamma_{1t}\, E_{t'}[\beta_{it}] + \gamma_{2t}\, E_{t'}[\beta_{SMB,it}] + \gamma_{3t}\, E_{t'}[\beta_{HML,it}] + \gamma_{4t}\, E_{t'}[IRSK_{it}] + \sum_{k=5}^{K} \gamma_{kt}\, E_{t'}[X_{kit}] + u_{it}\,,$$
$$i = 1, 2, \ldots, N,\; t = 1, \ldots, T \qquad (2)$$

where the dependent variable R_{it} represents the realised average excess returns for stock *i* in panel *t,* X_{kit} represents explanatory variables (*lnsize* and *lnbm* in our model) in addition to FF three-factor betas and idiosyncratic risk (IRSK) for stock *i* and panel t, and notation $E_{t'}[\cdot]$ represents estimated variable value conditional on the information set available in the estimation phase at time *t'* prior to recording excess returns R_{it}. For example, $E_{t'}[IRSK_{it}]$ is an estimate of *IRSK* for stock *i* in panel *t* conditional on information set available during the estimation phase at time *t'*.

The use of model (2) allows a detailed examination of the contribution of the FF three-factor model. Like Ferson and Harvey (1999) and Ang et al. (2009), the assumption here is that if the FF factor loadings (betas) explain expected excess returns in the cross section then γ_2 and γ_3 would be significantly different from zero at most quantiles and γ coefficients corresponding to *lnsize* and *lnBM* would be virtually zero. Estimates from equation (2) are pooled using Ferson and Harvey (1999) method.

2.3.2 Using GARCH model

In estimating idiosyncratic risk as standard deviation of residuals from fitting OLS method to FF three-factor model (1), it was observed that idiosyncratic volatility estimates of stocks showed a lot of persistence over time and that idiosyncratic volatility estimate was the main factor in producing cross-sectional heteroskedasticity panels. We needed an estimate that was better in capturing the time-varying dynamics of idiosyncratic volatility. Following Fu (2009), we decided to use an autoregressive conditional heteroskedasticity (ARCH) family of models. Since our study builds model on daily data and uses an estimation window of size 100 days rather than confining estimation to calendar months like many past studies, we decided to use the generalised autoregressive conditional heteroskedasticity (GARCH) model of Bollerslev (1986) to capture the finer dynamics over past 100 days. Specifically, we use GARCH(1,1) defined as

$$\begin{aligned} y_{it} &= \alpha_i + \beta_i r_{mt} + \beta_{SMB,i}\, SMB_t + \beta_{HML,i}\, HML_t + \varepsilon_{it}\ , \\ \varepsilon_{it} &\sim N(0, \sigma_{it}^2), \\ \sigma_{it}^2 &= \nu + \alpha\varepsilon_{it}^2 + \beta\sigma_{i,t-1}^2 \end{aligned} \qquad (3)$$

and estimate idiosyncratic risk as detailed below.

Foster and Nelson (1996) investigate strategies used for estimating time-varying variances and covariances. They comment that the use of rolling regression approach of Officer (1973) and Fama and MacBeth (1973) is one way of overcoming this problem. They derive analytically optimal window lengths for standard rolling regressions and optimal weights for weighted rolling regressions and show that the optimal weight (*w*) and length of the estimation window (*E*) in standard rolling regression that minimizes asymptotic variance has the relation $wE = \sqrt{3}$. They comment that GARCH models are one-sided weighted rolling regressions and propose the use geometrically declining weights $w' = we^{-wk}$, where *k* is the time index. For a chosen *E, the* weight *w and therefore* declining weights w' can be obtained easily. Thus, for each panel idiosyncratic risk for *ith* stock (*i = 1, 2, , 207)* is estimated using weighted sum of past GARCH variances as $\sum_{k=1}^{E} w_k' \sigma_{ik}^2$ such that $\sum_{k=1}^{E} w_k' = 1$. We also obtain an average of the GARCH variances over estimation window 'E' of past 100 days as an estimate of idiosyncratic risk. A comparison of the average GARCH variances against the weighted GARCH variances reveals that the two estimates are almost identical during tranquil periods but the weighted variance is more amplified otherwise. Thus, we decided to use the weighted GARCH variance to estimate idiosyncratic risk. This estimate is referred to as IRSK-G.

Once again, we observe significant correlation between idiosyncratic risk and explanatory variables *R1*, *lnsize*, *lnbm* and *the factor loading* of FF three-factor model (3). The Breusch-Pagan-Godfrey tests applied to panels reveal the presence of heteroskedasticity in 87 of the 91 panels which once again is mainly caused by idiosyncratic risk estimates. Thus to obtain heteroskedasticity-consistent standard errors of the estimates from the fitted models, we correct data for heteroskedasticity. Dividing each data row by its corresponding factor $\left(\sum_{k=1}^{E} w_k^{'} \sigma_{ik}^2\right)^{1/2}$, the square root of the idiosyncratic risk as estimated from the GARCH model, corrects heteroskedasticity in 74 of the 91 total panels, and heteroskedasticity in the remaining panels is present due to an occasional other explanatory variable. The number of panels affected is very small indeed and we take no further corrective action. After correcting data for heteroskedasticity, we apply the OLS and quantile regression methods to the predictive model (2) encompassing GARCH model based estimates of FF three-factor loadings and idiosyncratic risk.

2.4 The Quantile Regression Method

To date, most of the idiosyncratic risk-return related studies have used ordinary least squares estimation method in the second stage of the Fama and MacBeth (1973) method. The contribution of idiosyncratic risk and all explanatory variables used in a model for explaining expected returns dynamics is based on OLS estimates. The method of ordinary least squares explains the conditional distribution of returns given a vector of explanatory variables at the mean level. Thus, the conclusions in these studies about the stock returns and idiosyncratic risk relationship apply only at the mean level. A relaionship significant or insignificant at the mean level may not necessarily be so in other parts of the conditional distribution of returns. This could be one reason for the literature reporting mixed results on idiosyncratic risk-return relationship.

Also, the tests based on ordinary least squares method require the model errors to follow a normal distribution, and this may not be the case with stock returns distribution. As per Buchinsky (1998), the quantile regression estimators do not require strong distributional assumptions and are more efficient than ordinary least squares in the absence of normality and are not sensitive to the presence of extreme values. Furthermore, the linear programming representation of quantile regression method makes estimation easier; different solutions at distinct quantiles may be interpreted as differences in the response of the dependent variable to changes in the regressors at various points in the conditional distribution of the dependent variable. The technique has been used in many areas of empirical economics and applied econometrics (Buchinsky (1997,1998), Koenker and Bassett (1978), Koenker and Hallock (2001)). Financial applications include the study of Engle and Manganelli (1999) to the problem of Value at Risk , and Morillo (2000) application to options pricing. Barnes and Hughes (2002), Weng and Wang (2008) and Li (2009) used this method to study the behaviour of beta risk and to test the CAPM. To date, it seems that we are the first ones to use quantile regression technique to explore the stock returns and idiosyncratic risk relationship.

2.5 Pooling estimates: the Ferson-Harvey (1999) method

The application of ordinary least squares and quantile regression methods to cross-sectional data in each panel via estimation equation (2) produces *T* ordinary least squares estimates of each γ_k coefficient, and *T* quantile regression estimates of each γ_k for each of the 9 quantiles (τ = 0.1, 0.2, 0.3, …, 0.9) along with standard errors of the estimates. In the next step, the time-series of these *T* parameter estimates are pooled before testing to see if γ_k , *k = 0, 1, 2, …., K,* is zero or significantly different from zero.

We use Ferson and Harvey (1999) method for combining these estimates. The Ferson-Harvey estimator $\gamma_{k(FH)}$ of γ_k is defined as,

$$\gamma_{k(FH)} = T^{-1} \sum_{t=1}^{T} w_{kt}^{*} \gamma_{kt} \tag{4}$$

where, $w_{kt}^{*} = [\mathrm{var}(\hat{\gamma}_{kt})]^{-1} \Big/ \sum_{t=1}^{T} [\mathrm{var}(\hat{\gamma}_{kt})]^{-1}$ (5)

and sample variance of $\gamma_{k(FH)}$ is obtained as

$$s^2(\gamma_{k(FH)}) = (1/T)(T^{-1} \sum_{t=1}^{T} (w_{kt}^{*} \gamma_{kt})^2 - (T^{-1} \sum_{t=1}^{T} w_{kt}^{*} \gamma_{kt})^2) \tag{6}$$

3 Empirical Findings

The main aim of the paper is to study (i) the form of the idiosyncratic risk-return relationship in a predictive model in the cross section, (ii) whether the form of the relationship depends on the measure used for estimating idiosyncratic risk, and (iii) if the use of quantile regression methodology helps explain conflicting findings of past researchers.

The ordinary least squares regression model assesses the effects of explanatory variables at the mean level; however, the nature of these effects may be quite different in the tails of the conditional distribution. As we below the quantile regression model provides a richer specification than the OLS model, revealing large amount of variation over the range of quantiles that was not expected. Although the quantile regression coefficient at a given quantile indicates the effect on excess returns of a unit change in an explanatory variable, assuming that the other explanatory variables are held fixed, viewing and interpreting a quantile regression coefficient in isolation may provide misleading impressions about the relationship. In the following, OLS (for bench marking) and quantile regression based results are reported and discussed.

Tables 1(a) and 2(a) report OLS and quantile regression estimates from fitting model (2) when idiosyncratic risk is estimated as standard deviation of residuals from fitting Model (1) via OLS method (the IRSK estimate) and weighted variance from fitting Model (3) via GARCH(1,1) method (the IRSK-G estimate), respectively. Observing a significant relationship between stock returns and

Table 1 (a): Idiosyncratic risk and stock returns relation in a predictive FF three-factor model (2).

Quantile	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	OLS
Intercept	-6.9E-06	7.03E-07	3.46E-06	4.19E-06	6.13E-06	8.13E-06	1.04E-05	1.61E-05	1.68E-05	8E-06
t_Interce	-1.68435	0.213184	1.169548	1.424413	1.961489	2.365847	2.705222	3.772515	3.029903	2.581072
R1	0.001537	0.000854	0.00038	0.000165	7.62E-05	-8E-05	4.37E-05	0.000155	-0.00025	0.00043
t_R1	3.964099	2.683041	1.265441	0.631097	0.301581	-0.3007	0.147085	0.380624	-0.44266	1.429532
lnsize	3.55E-07	3.42E-08	-4.6E-09	-1.2E-07	-2E-07	-4E-07	-6.6E-07	-1.2E-06	-1.6E-06	-5.7E-07
t_lnsize	0.655605	0.089956	-0.01377	-0.38067	-0.63088	-1.11231	-1.7709	-2.82559	-2.89665	-1.80735
lnbm	1.11E-06	1.28E-06	1.3E-06	1.55E-06	1.47E-06	1.45E-06	1.01E-06	1.46E-06	1.49E-06	1.24E-06
t_lnbm	1.386321	1.985058	2.268421	2.642962	2.451206	2.206236	1.455973	1.899263	1.334744	2.147129
beta	-1.1E-06	-5.1E-07	-1.4E-07	6.8E-07	9.48E-07	1.04E-06	1.38E-06	1.34E-06	1.28E-06	7.26E-07
t_beta	-0.7656	-0.46859	-0.13204	0.680546	0.988444	0.970602	1.303802	1.01399	0.796964	0.703957
c-smb	-7.8E-05	1.89E-06	3.25E-05	5.22E-05	4.74E-05	4.34E-05	4.69E-05	1.67E-05	4.43E-05	3.83E-05
t_c-smb	-1.84541	0.055812	1.032424	1.647085	1.368824	1.225841	1.198517	0.379948	0.780537	1.129644
c-hml	0.000113	9.24E-05	5.79E-05	4.34E-05	3.86E-05	2.56E-05	-5.3E-06	-5.7E-05	-4.2E-05	4.5E-05
t_c-hml	2.569483	2.463994	1.672588	1.246856	1.136012	0.766068	-0.13877	-1.25822	-0.73555	1.321532
IRSK	-0.00139	-0.00107	-0.00078	-0.0005	-0.00023	8.37E-05	0.000438	0.000904	0.001511	-7.2E-05
t_IRSK	**-11.1268**	**-10.7266**	**-9.35924**	**-6.78941**	**-3.08703**	1.125762	**5.449013**	**9.698813**	**10.00558**	-0.89336
R^2	0.098963	0.077307	0.071677	0.070314	0.069403	0.070488	0.073541	0.080645	0.096737	0.112009
Adj-R^2	0.067427	0.045013	0.039186	0.037775	0.036832	0.037955	0.041115	0.048467	0.065123	0.080929

Table 1 (b): Evidence of quadratic form of idiosyncratic risk and stock returns relation in a predictive FF three-factor model (2).

Quantile	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	OLS
Intercept	-8.1E-07	2.89E-06	3.97E-06	3.9E-06	4.96E-06	5.09E-06	3.3E-06	7.95E-06	9.81E-06	7.35E-06
t_Interce	-0.18396	0.806141	1.232501	1.293666	1.526616	1.333876	0.851557	1.920479	1.866867	2.398375
R1	0.00143	0.000744	0.000391	0.000209	8.32E-05	-8.2E-05	1.48E-05	0.00017	-7.4E-06	0.000456
t_R1	3.902753	2.296476	1.306684	0.789216	0.325127	-0.31027	0.049729	0.412992	-0.01332	1.510523
lnsize	3.75E-07	-1.9E-07	-1.5E-08	-8E-08	-1.4E-07	-2.1E-07	-1.8E-07	-8.8E-07	-1.5E-06	-5.4E-07
t_lnsize	0.789636	-0.53473	-0.04472	-0.25975	-0.44218	-0.58023	-0.49424	-2.21128	-3.06131	-1.7824
lnbm	1.25E-06	1.18E-06	1.73E-06	1.59E-06	1.57E-06	1.28E-06	1.4E-06	1.34E-06	2.28E-06	1.28E-06
t_lnbm	1.530488	1.81036	3.025567	2.657088	2.549212	1.879595	2.054397	1.796099	2.059724	2.213829
beta	-5.4E-07	-2.9E-07	-3.9E-07	3.91E-07	5.95E-07	6.15E-07	5.85E-07	9.47E-07	9.97E-07	5.96E-07
t_beta	-0.36186	-0.26112	-0.38044	0.388011	0.656778	0.620262	0.591684	0.772007	0.705856	0.590442
c-smb	-0.0001	-1.3E-05	1.78E-05	5.24E-05	5.43E-05	5.93E-05	6.73E-05	4.19E-05	8.76E-05	3.99E-05
t_c-smb	-2.609	-0.34724	0.525677	1.599442	1.519789	1.614023	1.670523	0.940124	1.766173	1.15171
c-hml	7.45E-05	8.66E-05	4.93E-05	4.96E-05	3.99E-05	3.23E-05	-1.2E-06	-2E-05	-1.7E-07	4.54E-05
t_c-hml	1.828545	2.418397	1.434911	1.435651	1.202711	0.938563	-0.031	-0.45645	-0.00314	1.342278
IRSK	-0.002	-0.00139	-0.00092	-0.00054	-7.1E-05	0.000432	0.001052	0.001631	0.002519	2.29E-05
t_IRSK	**-7.62033**	**-7.09029**	**-5.61519**	**-3.67562**	-0.4575	**2.624876**	**5.400875**	**7.476419**	**6.246846**	0.148101
$IRSK^2$	0.011856	0.0068	0.004174	0.001478	-0.0014	-0.00439	-0.00832	-0.01102	-0.01617	-0.00076
t_$IRSK^2$	**5.13097**	**3.759928**	**2.129787**	0.957211	-0.92904	**-2.68145**	**-4.34205**	**-4.95238**	**-4.09152**	-0.447
R^2	0.105139	0.08245	0.07655	0.074307	0.07317	0.074369	0.078016	0.086453	0.10426	0.117589
Adj-R^2	0.069165	0.045564	0.039426	0.037093	0.03591	0.037158	0.040952	0.049727	0.06825	0.082115

Table 2 (a): Idiosyncratic risk and stock returns relation in a predictive FF three-factor model (2), using GARCH based estimates.

Quantile	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	OLS
Intercept	-3.6E-05	-2.2E-05	-1.1E-05	-4.4E-06	4.11E-06	1.21E-05	2.23E-05	3.39E-05	5.08E-05	7.5E-06
t_Interce	-7.40607	-7.00635	-4.16237	-1.70282	1.393007	3.662164	6.171699	8.311416	9.64049	2.437664
R1	0.001557	0.000665	0.000417	5.94E-05	-9.3E-05	-0.00013	-0.00017	9.44E-05	-0.0002	0.000291
t_R1	4.305861	2.198091	1.520889	0.230855	-0.35104	-0.45744	-0.5522	0.238304	-0.3337	0.965921
lnsize	2.41E-06	1.67E-06	8.51E-07	3.99E-07	-2.3E-07	-8.5E-07	-1.6E-06	-2.5E-06	-3.9E-06	-6E-07
t_lnsize	3.643077	4.089076	2.487304	1.325863	-0.70066	-2.25775	-4.52585	-5.66499	-7.1134	-1.86778
lnbm	2.52E-06	1.45E-06	1.42E-06	1.71E-06	1.3E-06	1.11E-06	7.36E-07	3.18E-07	3.37E-07	1.18E-06
t_lnbm	2.983105	2.27661	2.541898	3.011766	2.162985	1.604049	0.982676	0.462843	0.282116	2.091417
beta	-2.3E-06	-2.8E-06	-1.2E-06	-1.6E-07	3.76E-07	1.22E-06	2.99E-06	3.39E-06	5.53E-06	7.95E-07
t_beta	-1.66882	-2.17401	-1.07345	-0.14788	0.338783	1.099734	2.62039	2.632437	3.493824	0.700837
c-smb	-9.4E-05	4.12E-06	9.46E-06	4.13E-05	3.61E-05	3.76E-05	4.04E-05	1.98E-05	4.66E-06	2.01E-05
t_c-smb	-2.25488	0.104036	0.258525	1.130676	0.939109	1.001066	0.943541	0.441283	0.078701	0.537101
c-hml	0.000162	0.000129	9.37E-05	6.79E-05	4.12E-05	4.14E-06	-3.4E-05	-4.6E-05	-5.9E-05	3.16E-05
t_c-hml	3.214245	3.036034	2.428392	1.741054	1.152008	0.112691	-0.81009	-1.08119	-1.14518	0.859818
IRSK-G	-0.00873	-0.0083	-0.00643	-0.00462	-0.00271	-0.00038	0.001396	0.004312	0.010389	-0.00121
t_IRSK-G	**-9.69431**	**-11.1026**	**-10.4403**	**-7.72262**	**-4.5389**	-0.57985	1.655489	**4.917583**	**8.799571**	-1.45564
R^2	0.091452	0.072954	0.069468	0.06908	0.069143	0.070184	0.073225	0.077709	0.088122	0.111593
Adj-R^2	0.059494	0.040344	0.036736	0.036334	0.036399	0.037477	0.040625	0.045266	0.056046	0.080343

Table 2 (b): Evidence of quadratic form of idiosyncratic risk and stock returns relation in a predictive FF three-factor model (2), using GARCH based estimates.

Quantile	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	OLS
Intercept	-3.2E-06	9.51E-07	3.7E-06	4.29E-06	5.97E-06	7.64E-06	1.02E-05	1.23E-05	1.76E-05	8.61E-06
t_Interce	-0.72631	0.271435	1.079829	1.36166	1.765706	1.99168	2.42693	2.78115	3.352411	2.730936
R1	0.001528	0.000703	0.000323	9.42E-05	-1.4E-05	-9.2E-05	-1.9E-05	0.000151	4.35E-05	0.000368
t_R1	4.120143	2.148704	1.080595	0.356896	-0.05411	-0.3233	-0.06116	0.382153	0.077735	1.237045
lnsize	4.93E-07	3.73E-08	-7E-08	-1E-07	-1.3E-07	-3.4E-07	-8.6E-07	-1.2E-06	-2.2E-06	-6.5E-07
t_lnsize	0.779862	0.105964	-0.19546	-0.32373	-0.42265	-0.96488	-2.2347	-2.806	-3.92609	-2.12818
lnbm	9.17E-07	1.02E-06	1.31E-06	1.46E-06	1.19E-06	1.3E-06	9.44E-07	8.06E-07	2.03E-06	1.18E-06
t_lnbm	1.072416	1.644726	2.298346	2.49056	1.900082	1.937177	1.376105	1.183793	1.768279	2.103572
beta	-7.1E-07	-1.2E-06	-8.1E-07	-8E-09	4.83E-08	5.71E-07	1.64E-06	1.53E-06	2.86E-06	6.4E-07
t_beta	-0.45015	-1.01606	-0.72863	-0.00759	0.049284	0.555586	1.503922	1.252286	1.942747	0.62366
c-smb	-0.00013	-1.4E-05	9.77E-06	3.06E-05	4.57E-05	4.24E-05	4.32E-05	3.94E-05	3.71E-05	1.86E-05
t_c-smb	-3.36215	-0.34598	0.257359	0.855616	1.15492	1.140605	0.992372	0.830378	0.664969	0.501453
c-hml	9.97E-05	0.000108	6.02E-05	4.95E-05	5.53E-05	1.93E-05	-8.3E-06	-1.5E-05	-1.4E-05	3.47E-05
t_c-hml	2.355352	2.677725	1.708626	1.353389	1.577101	0.558811	-0.20967	-0.38582	-0.25165	0.98727
sqrt_IRSK	-0.00191	-0.00136	-0.00089	-0.00055	-0.00012	0.000325	0.000769	0.001291	0.002367	-2.9E-05
t_sqrt_IR	**-7.6044**	**-7.26638**	**-6.11078**	**-4.29892**	-0.91702	**2.252248**	**5.176119**	**6.736574**	**6.937181**	-0.20833
IRSK-G	0.010848	0.007031	0.003935	0.002209	-0.00059	-0.00344	-0.00633	-0.00948	-0.01574	-0.00014
t_IRSK-G	**5.407259**	**4.368044**	**2.860052**	1.892939	-0.52709	**-2.90266**	**-4.78464**	**-5.2688**	**-5.1699**	-0.09732
R^2	0.106586	0.083552	0.077496	0.075071	0.074107	0.075992	0.081054	0.088972	0.10669	0.119839
Adj-R^2	0.070488	0.046523	0.040224	0.0377	0.036697	0.038658	0.043925	0.052163	0.070596	0.084277

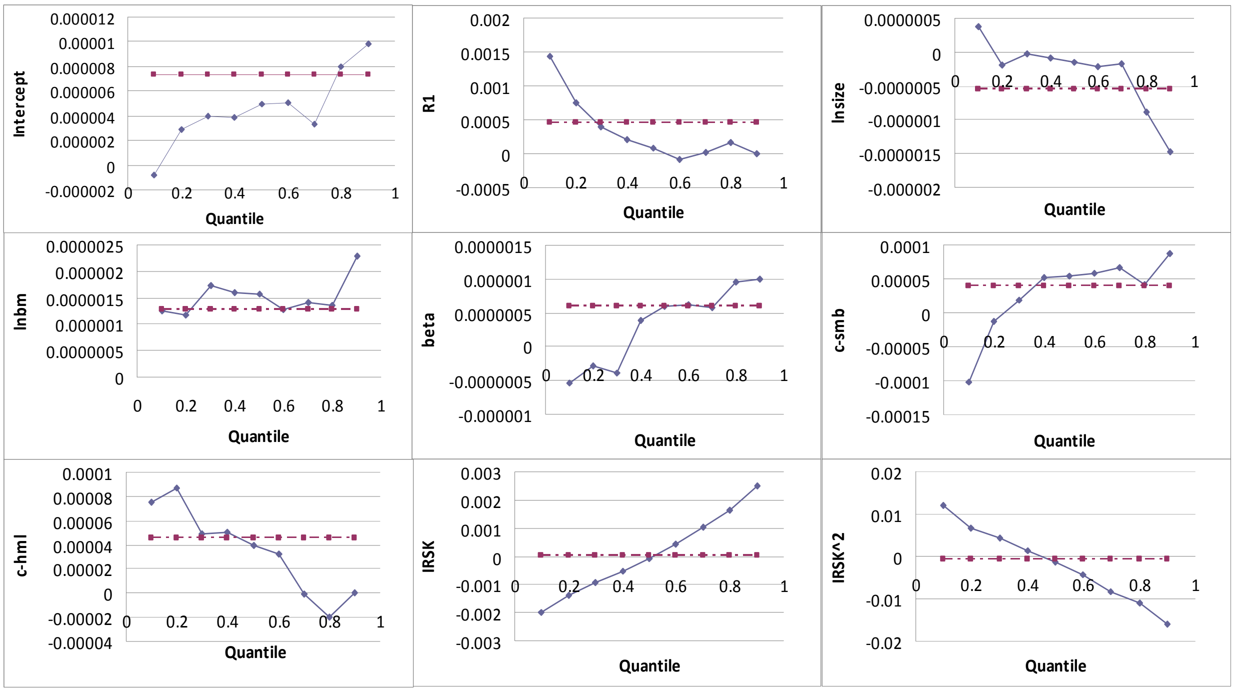

Figure 1: Evidence of quadratic form of idiosyncratic risk and stock returns relation in a predictive FF three-factor model (2). The curves represent the quantile regression based estimates and the dashed horizontal line the OLS estimates.

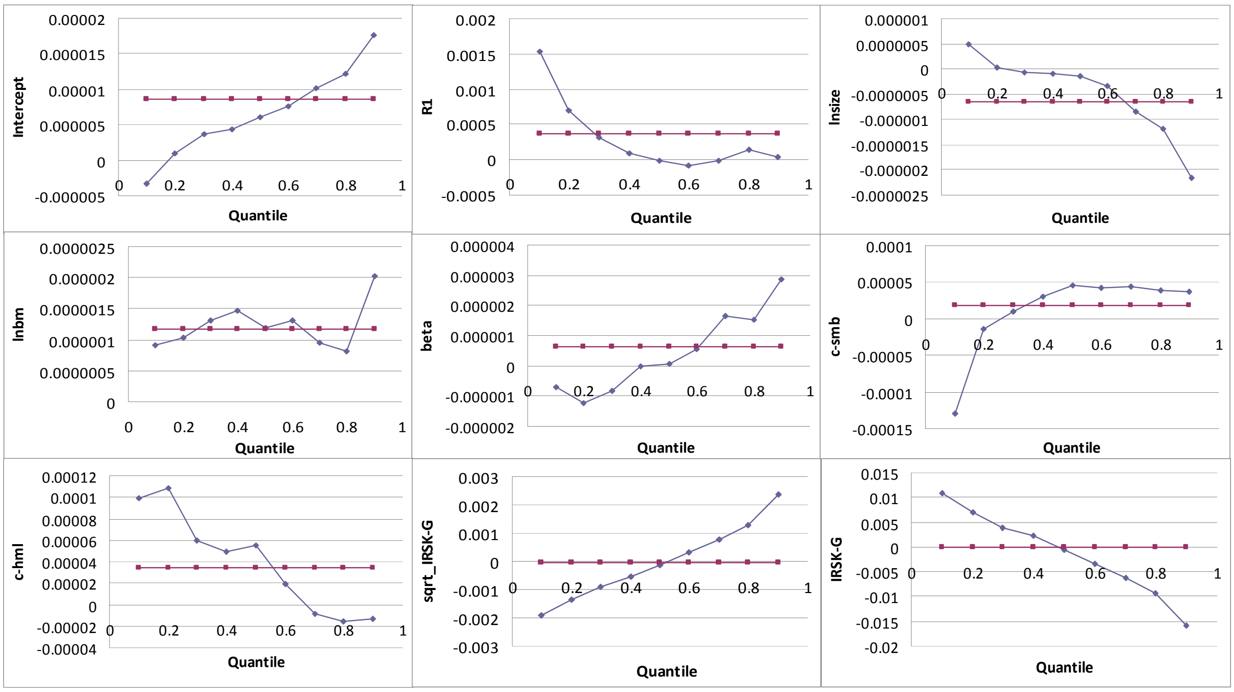

Figure 2: Evidence of quadratic form of idiosyncratic risk and stock returns relation in a predictive FF three-factor model (2), using GARCH based estimates. The curves represent the quantile regression based estimates and the dashed horizontal line the OLS estimates.

lagged idiosyncratic risk obtained as standard deviation as well as variance, we decided to include variance and standard deviation terms on idiosyncratic risk measured via the application of Models (1) and (3) in the fitting of Model (2). This not only produced highly significant idiosyncratic risk coefficients at many quantiles but also improved the R^2 and Adj-R^2 values. Tables 1(b) and 2(b) report the results from fitting the expanded models and Figures 1 and 2 display coefficient estimates graphically.

Graphs in Figures 1 and 2 display striking resemblance in quantile regression based estimates of coefficients obtained from the two models. The dynamic nature of the covariate effects on the conditional distribution of realised excess returns is obvious. In the following we interpret patterns in coefficients of each explanatory variable and the intercept.

3.1 Interpreting Quantile regression coefficients: Tables 1(b) and 2(b), and Figures 1 and 2

Intercept: It represents the unanticipated returns at various quantiles and average level of the conditional distribution of returns when all predictors in the model have a zero value. The unanticipated returns, γ_0, are significant under the ordinary least squares estimation for both measures of idiosyncratic risk. However, the quantile regression based intercept estimates are not significant when IRSK estimate is used, but are significant and positive at quantiles 0.7 to 0.9 with the IRSK-G estimate. A general upward trend in coefficients at quantiles 0.1 to 0.9 is evident.

Average returns R1: The ordinary least squares based estimates of the coefficient of R1, the average excess returns over past 100 days, reveal a positive non-significant relation with expected future returns for both models. The quantile regression based estimates change from being large positive at lower quantiles to small positive at higher quantiles, and negative at quantiles 0.5 to 0.7. The coefficients are significant only at quantiles 0.1 and 0.2.

Lnsize: The pattern in ordinary least square regression estimates of coefficients of *lnsize* are in line with the literature (Fama and French (1992), Fu (2009), and references therein) that small size firms have higher returns and vice-a-versa, meaning a negative linear relationship. Although the ordinary least squares estimates for our data are negative for both models, significance is achieved only in the case of IRSK-G based model (Table 2(b)). As per Figures 1 and 2, the relationship between size and excess returns is negative. For estimates based on quantile regression we observe that *lnsize* coefficients are significant at higher quantiles only confirming that small firms do tend to contribute higher returns than larger firms.

LnBM: The ordinary least squares regression estimates of coefficient of *lnBM* are positive and significant and agree with reported findings in the literature. Quantile regression estimates of *lnBM* are all positive but not all significant. As the coefficients change with quantiles means that the relationship is not static and depends on the quantile of the returns distribution. Thus, our results confirm that the relationship between stock returns and *lnBM* is positive, but the marginal effect changes with quantile levels.

Beta: The coefficients of beta risk show increasing trend for quantiles 0.1 to 0.9 but its quantile regression based as well as OLS estimates are not significant in either model.

c-smb: The c-smb coefficients change from being small negative to small positive from lower quantiles to higher quantiles. With the exception of quantile 0.1, the quantile regression and OLS based c-smb coefficients are not significant for either model.

c-hml: The c-hml coefficients show a decreasing trend from quantile 0.1 to 0.9 but estimates are mostly insignificant in both models. The OLS estimates are not significant either.

IRSK/sqrt_IRSK-G: The ordinary least squares estimate of coefficient of predictor IRSK/sqrt_IRSK-G is positive (negative) and insignificant. The quantile regression based coefficients show increasing trend, are significant at the extreme quantiles of the conditional distribution of excess returns, and change from being very negative to very positive values, passing through a zero point around the median (Figures 1 and 2).

IRSK^2/IRSK-G: The ordinary least squares estimates of coefficients of predictors IRSK^{2}and IRSK-G are both negative and insignificant. The quantile regression based coefficients show decreasing trend, are significant at the extreme quantiles of the conditional distribution of excess returns, and change from being positive to negative values, passing through a zero point around the median (Figures 1 and 2).

As the interest in this paper is to study the idiosyncratic risk-return relation, the rest of the paper is devoted to exploring only this relationship. Figures 3(a) and 3(b) graph quantile regression based coefficient values of pairs IRSK and sqrt_IRSK-G, and IRSK^2 and IRSK-G along with their corresponding t-ratios. The similarity in values is striking. It means that both measures of idiosyncratic risk produce the same pattern in relationship, and that extra effort in obtaining the GARCH model based estimates is not justified. In the work that follows, we shall base our discussion on IRSK estimates only.

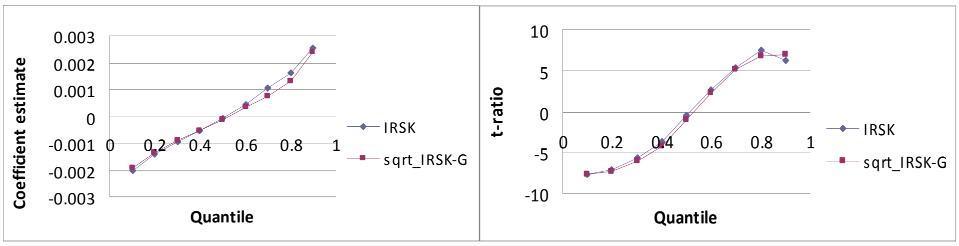

Figure 3 (a): IRSK and sqrt_IRSK-G coefficients & their corresponding t-ratios

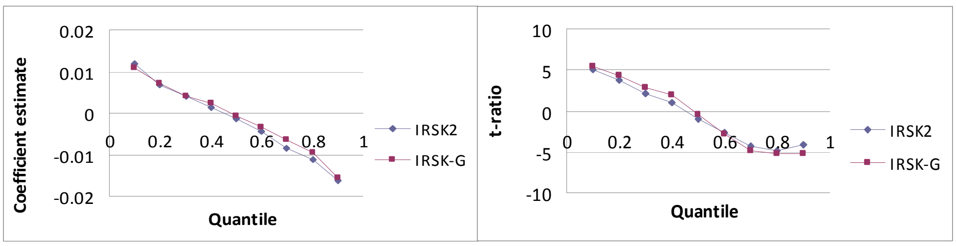

Figure 3(b): IRSK^2 and IRSK-G coefficients & their corresponding t-ratios

3.2 Patterns in predicted returns

In order to find out possible patterns in predicted returns from the quantile regression and OLS based estimates of idiosyncratic risk, we recorded mean IRSK values in each of the 91 panels. As contribution of explanatory variables other than idiosyncratic risk (Table 1 (b)) is almost negligible, we assume them to be zero. Using quantile regression and OLS estimates of coefficients of IRSK and $IRSK^2$ and the vector of mean IRSK values from the 91 panels, we predict stock returns. These returns are graphed in Figure 4.

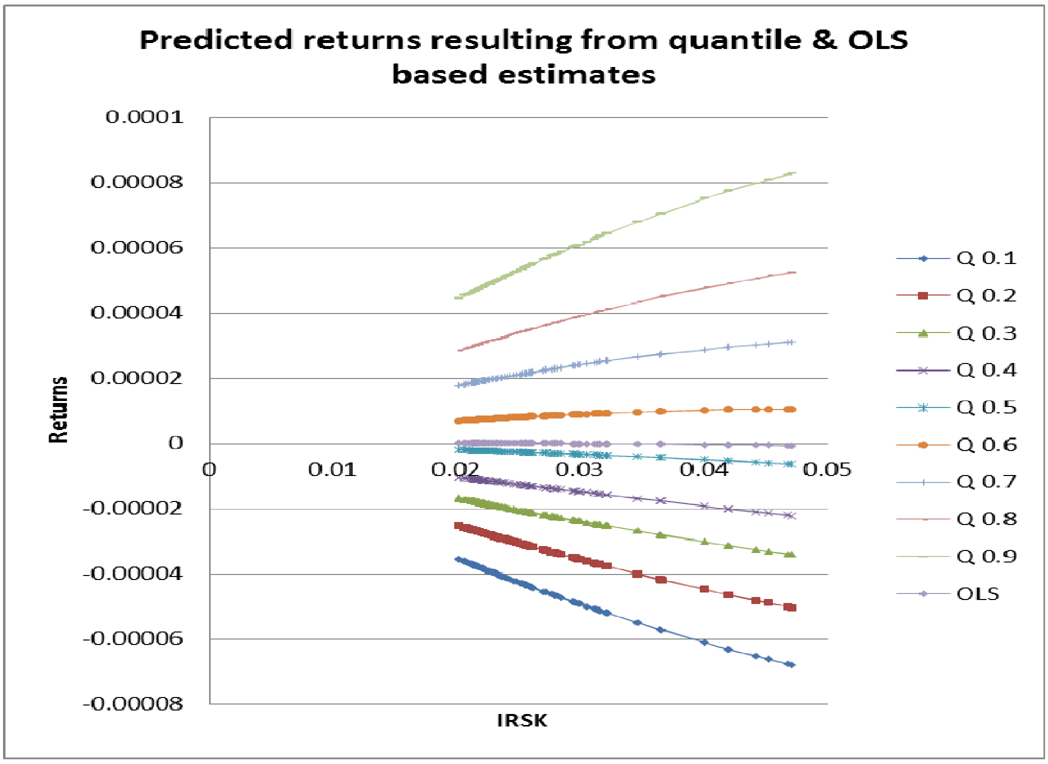

Figure 4: Predicted returns for a given vector of idiosyncratic risk.

We can make the following observations from prediction curves in Figure 4.

1. The predictions resulting from the risk-return relationship at various quantiles using the same vector of idiosyncratic risk values are quite different. For large IRSK values, the disparity in these predictions is much greater and would be harder to predict returns for high values of RISK. Since we corrected data for heteroskedasticity, it is unlikely that returns are influenced by heteroskedasticity. A likely scenario is that the shape of the conditional distribution of predicted returns changes with the quantiles; it is dynamic.

2. Return curves (lines) are not parallel, meaning that the prediction curves at various quantiles are quite different and that the shape of the conditional distribution of returns changes with quantiles. The relationship between idiosyncratic risk and returns is not linear.

3. Variation in spacing between quantile curves tells that the conditional distribution of returns is not symmetric.

3.3 Exploring the form of the idiosyncratic risk-return relation in the predictive model

Viewing patterns in pairs of IRSK and $IRSK^2$ coefficients as displayed in Tables 1(b) and 2(b), the relation between expected future excess stock returns R and lagged IRSK can be expressed as

$$R = a\,IRSK^2 + b\,IRSK + c \qquad (7)$$

In relation (7), *c* denotes the net effect of all explanatory variables in the model on returns when IRSK is zero, and can be calculated from the known values of average excess return *R1*, *lnsize*, *lnBM* etc. over the estimation window for a stock. The constant *c* may be positive or negative. The first two terms involving IRSK on the right hand side of equation (7) capture the effect of idiosyncratic risk over and above the contribution of other explanatory variables in the model.

Relation (7) represents a family of parabolic curves with vertex $[-b/2a, -(b^2-4ac)/4a]$. The shape and location of these parabolic curves in the *XY*- plane depends on the values of constants (parameters) *a, b* and *c*. While *c* is the Y-intercept of the parabolic curve resulting from IRSK (defined as standard deviation of residuals from FF three-factor model) being zero in equation (7), the location of the vertex, concavity or convexity of the curve depends on the values of *a* and b. There are four possibilities.

(i) *a* < 0 and *b* > 0: the curve is concave with vertex in first quadrant of the *XY*-plane. Entries in Table 1(b) corresponding to quantiles 0.6 to 0.9 and OLS represent this situation.

(ii) *a* < 0 and *b* < 0: the curve is concave with vertex in second quadrant of the *XY*-plane. This situation is possible only when IRSK is negative. Since IRSK cannot be negative, only part of the parabolic curve corresponding to positive values of IRSK passing through the first and fourth quadrant will be relevant and visible in drawings. Entries corresponding to quantile 0.5 in Table 1(b) and OLS in Table 2(b) OLS represent this situation.

(iii) *a* > 0 and *b* < 0: the curve is convex with vertex in fourth quadrant of the *XY*-plane. Entries in Table 1(b) corresponding to quantiles 0.1 to 0.4 represent this situation.

(iv) *a* > 0 and *b* > 0: the curve is convex with vertex lying in the third quadrant of *XY*-plane. Once again, as IRSK cannot be negative, only part of the curve corresponding to positive values of IRSK will be relevant. We do not observe this case for our data sample.

Figure 5 graphs are sketched generating estimates of predicted excess returns based on Ferson-Harvey estimates of IRSK and $IRSK^2$ coefficient pairs listed in Table 1(b) and using hypothetical IRSK values ranging from 0 to 0.3, in steps of 0.05. For the sake of convenience and without loss of generality, constant *c* is assumed to be zero. It is easy to see that estimated values of *a* and *b* listed in Table 2(b) will produce similar curves. Indeed, the relationship between expected future returns and lagged idiosyncratic risk is parabolic that changes with the quantiles of the conditional distribution of expected returns. Thus, owing to the underlying dynamic nature of this relationship, the future returns are not likely to be predicted accurately via the use of a simple linear model. Also, due to the parabolic nature of the idiosyncratic risk-return relationship, it is possible to realise the same amount of positive or negative returns for two different high and low values of IRSK. Besides, the returns may decrease or increase with increase or decrease in idiosyncratic risk level.

Graphs labelled Q 0.4 and Q 0.5 in Figures 5 indicate the possibility of parabolic shape changing from being convex to concave around the quantile 0.5, the median level. This means that there is a small range of values of IRSK for which an insignificant positive or negative relation with returns may result. Thus, observations of Ang et al. (2006, 2009), Fu (2009) and others about the idiosyncratic risk-return relationship being negative or positive are all plausible via the use of OLS

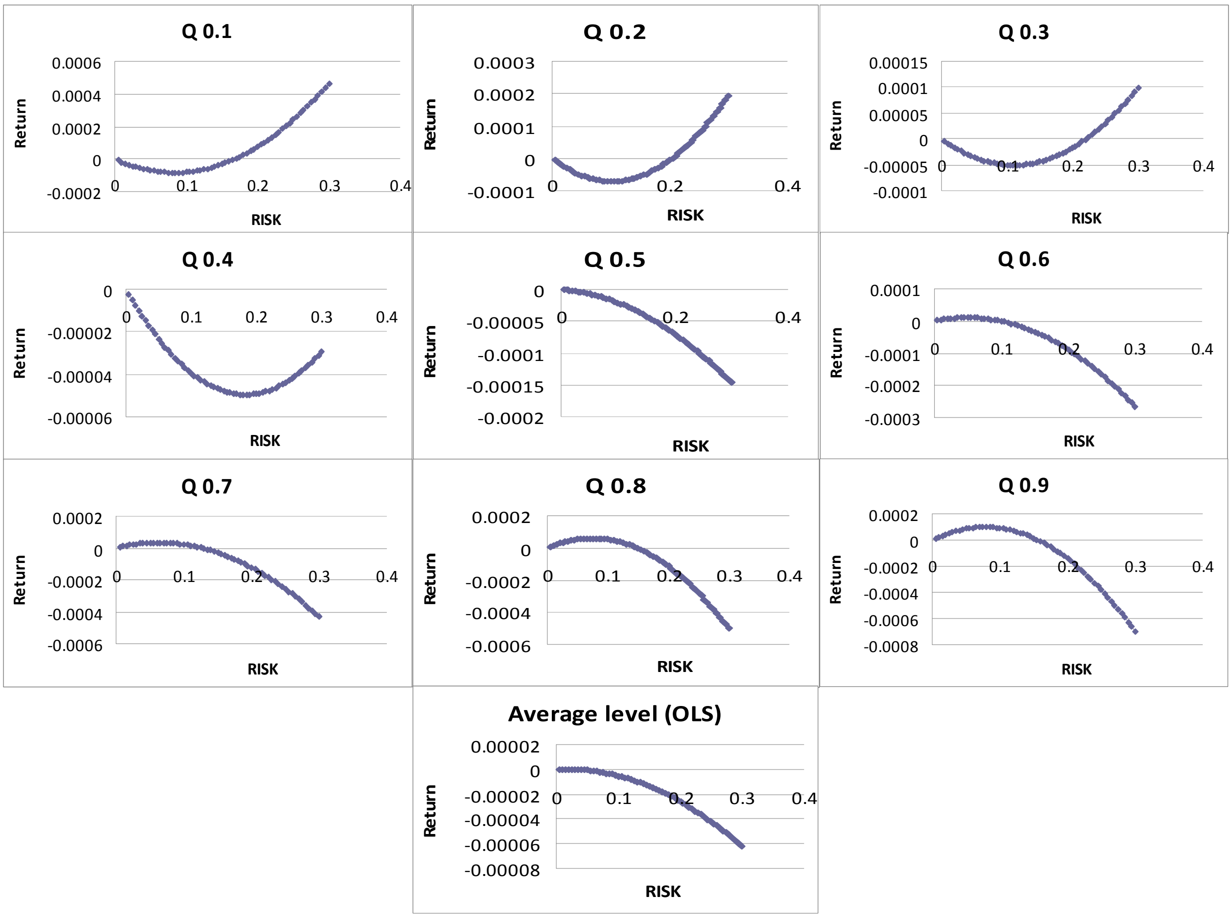

Figure 5: **Dynamic parabolic nature of the conditional distribution of returns.** The graphs show patterns in predicted returns conditional on the idiosyncratic risk values between 0 and 0.3, in steps of 0.05. The predicted returns are estimated from equation (7) using *c* =0, and coefficients *a* and *b* of $IRSK^2$ and IRSK as listed in Table 1 (b) for quantile 0.1 to 0.9 and the average level (ordinary least squares).

method. Besides, it is possible to observe this relationship not being significant as reported by Wei and Zhang (2005) and Bollen et al. (2009). The last curve labelled Average level (OLS) in Figure 5 may be mistaken to reflect a negative linear relation if IRSK values were recorded to fall between 0.1 and 0.2. Thus, the form of the idiosyncratic risk-return relation is far from being a simple predictive episode as mentioned by Goyal and Santa-Clara (2003). Moreover, the change in shape of the curves at various quantiles suggests that the conditional distribution of returns may not be symmetric, and thus the ordinary least squares based estimates that explain relationship at the mean level may not serve well at all.

The form of the parabolic curves at low and high quantiles of returns resemble the utility curves of risk-seeking and risk-aversion, respectively, rather than risk-neutral attitude. Thus, the idiosyncratic risk-return relationship pattern may be linked to risk-seeking and risk-aversion of investros towards trading of stocks.

3.4 Robustness of results: Idiosyncratic risk-return relation in CAPM one-factor model

In this section, we confirm the form of the idiosyncratic risk-relationship via the use of CAPM one-factor model. Idiosyncratic risk for stock *i* (i = 1, 2, 3, , 207) is measured as standard deviation of residuals resulting from OLS application of model

$$y_{id} = \alpha_i + \beta_i r_{md} + \varepsilon_{id} \ , \qquad (8)$$

various symbols in (8) are the same as explained in Section 2.3. Ferson-Harvey (1999) estimates from the application of OLS and quantile regression methods to panels are displayed in Table 3. Significant t-ratios for variables IRSK and $IRSK^2$ are presented in bold. It is clear that there is a quadratic relationship between idiosyncratic risk and stock returns for the CAPM one-factor model as well.

Table 3: Quadratic form of idiosyncratic risk and stock returns relation in a predictive CAPM one-factor model.

Quantile	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	OLS
Intercept	2.592E-06	3.594E-06	6.163E-06	6.639E-06	6.423E-06	6.439E-06	6.352E-06	1.061E-05	1.099E-05	8.699E-06
t_Intercept	0.7032836	1.0573121	1.7612825	2.0216015	1.8362146	1.6108916	1.5155622	2.4930488	2.4090298	2.7153869
R1	0.001429	0.00055	0.0002508	3.331E-05	-5.552E-05	4.597E-06	2.336E-05	0.0002596	-5.791E-05	0.0004062
t_R1	3.8006634	1.7605561	0.7770304	0.1124287	-0.2001705	0.0162727	0.0849387	0.7616362	-0.1031072	1.2878424
lnsize	-1.215E-07	-2.237E-07	-4.425E-07	-3.656E-07	-4.01E-07	-4.089E-07	-6.502E-07	-1.161E-06	-1.659E-06	-6.799E-07
t_lnsize	-0.2592396	-0.6158075	-1.2900895	-1.0920465	-1.1212603	-1.1138416	-1.601598	-2.9851428	-3.3109861	-2.1184883
lnbm	1.638E-06	1.415E-06	1.324E-06	1.484E-06	1.622E-06	1.741E-06	1.652E-06	1.327E-06	1.755E-06	1.431E-06
t_lnbm	1.93775	2.0439367	2.1368105	2.461108	2.6715871	2.6913367	2.3176558	1.7257751	1.7981075	2.5592431
beta	-9.209E-07	-6.837E-07	3.032E-07	2.676E-07	6.597E-07	9.695E-07	1.081E-06	1.89E-06	1.98E-06	7.815E-07
t_beta	-0.595365	-0.5637041	0.2767739	0.2638008	0.6702817	0.9122418	0.9903182	1.5667258	1.2671947	0.7400727
IRSK	-0.002179	-0.0013566	-0.0009174	-0.0005696	-9.116E-05	0.0004461	0.0009853	0.0014358	0.0024891	1.143E-05
t_IRSK	**-9.0494761**	**-7.1239762**	**-5.6993499**	**-4.0941877**	-0.6543601	**3.0091218**	**5.0264356**	**6.2766065**	**7.1321742**	0.0776437
$IRSK^2$	0.010285	0.0051272	0.0034498	0.0018191	-0.0004077	-0.0033554	-0.0055578	-0.0065673	-0.0123353	-0.0001249
t_IRSK^2	**6.2862574**	**3.2708577**	**2.6780281**	1.7260194	-0.3992161	**-3.0803068**	**-4.2148017**	**-4.1252476**	**-3.7994467**	-0.0899687
R^2	0.0891046	0.0704068	0.0640774	0.0614672	0.0601467	0.0618947	0.0643963	0.0711997	0.0840549	0.098642
Adj-R^2	0.0619136	0.0426577	0.0361394	0.0334513	0.0320913	0.0338915	0.0364677	0.0434743	0.0567132	0.0717358

We also applied *E/H/R* plan of 60/20/20 within CAPM one-factor model (results available on request) and noticed that quadratic form of relation between idiosyncratic risk and future stock returns holds. Thus, our findings about the form of idiosyncratic risk-return are not sensitive to the choice of CAPM one-factor and FF three-factor models, or the choice of estimation window *E*, the length *H* of the period of holding stocks to realise returns and *R* the rolling size of the window, or the measure of estimating idiosyncratic risk.

4 Conclusions and Implications

This paper investigates idiosyncratic risk and stock returns relation in the cross section in a heteroskedasticity corrected predictive model, where lagged idiosyncratic risk is used for explaining future stock returns. Idiosyncratic risk is measured with respect to FF three-factor model using two different measures. It is observed that the form of the relationship is not affected by the choice of measure used for estimating idiosyncratic risk; the GARCH based estimate does not have a superior performance.

The use of quantile regression method facilitates in capturing a more complete picture of the covariates effects on the conditional distribution of expected excess returns. It is established /shown that the relationship is dynamic, changes with quantiles of the conditional distribution of returns given a certain level of idiosyncratic risk, and has a parabolic form. The conclusions of past research are based on partial view of the relationship via the use OLS, and therefore the conflicting findings. The nature of the relationship as reported in this paper is invariant under the CAPM one-factor and FF three-factor models , and is robust to the choice of E/H/R plan values, i.e., to the choice of sample size used for estimation, period of holding stocks to realise returns and size of the rolling window. It is observed that the coefficients of FF factors SMB and HML are insignificant practically at all quantiles of the conditional distribution as well as at the mean level as captured by the ordinary least squares method. Given almost identical results based on FF three-factor and CAPM one-factor models, one wonders if the use of more involved framework of FF three-factor for asset pricing is justified.

The non-linear dynamic parabolic nature of the realtionship means it is much harder to predict direction of returns for a given level of idiosyncratic risk. Also high gains from holding high idiosyncratic risk may not be realized often as per the common belief. The idiosyncratic risk-return relationship is too dynamic to support idiosyncratic risk as a 'priced' item.

The major contributions of the paper are summarised below:

- A new perspective has been provided on the form of the much debated idiosyncratic volatility risk-return relationship puzzle in the finance literature via the use of quantile regression method. The form of the idiosyncratic risk-return relationship is not linear. It is dynamic and parabolic in nature. The parabolic shape changes from being convex at lower quantiles to being concave at higher quantiles of the conditional distribution of expected returns.
- The dynamic parabolic nature of relationship means the possibility of big losses from holding even small levels of idiosyncratic risk for some stocks, and big gains from holding the same amount of idiosyncratic risk for some other stocks. It is hard to defend idiosyncratic risk as a 'priced' item.
- It is shown that the form of the relationship is not sensitive to the choice of CAPM one-factor and FF three-factor models, and to the choice of measure used for estimating idiosyncratic risk.
- It is demonstrated that the idiosyncratic risk-return relationship is predictive in nature in the sense that future returns are linked to the recent past (lagged) idiosyncratic risk.

- The assessment of the relationship at various quantiles suggests higher impact of the idiosyncratic risk and other important stock specific explanatory variables at the extreme quantiles of the conditional distribution of returns than at the median level.

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