

Bank Stability and Market Discipline: Debt-for-Equity Swap versus Subordinated Notes

Alon Raviv*

Abstract

Policymakers are actively considering requiring banks to issue subordinated debt as a tool for monitoring banks by investors. However, subordinated debenture increases the probability of costly failure. We propose a novel instrument, ‘Debt-for-Equity Swap’ (DES), which pays a fixed income unless the value of the bank’s assets falls below a predetermined threshold. In such an event, the debt obligation is automatically converted to the bank’s common equities. We present closed-form solutions for the valuation of liabilities, deposit insurance and the value of bankruptcy costs of a bank that includes DES or subordinated debt in its capital structure. We compare quantitatively the effects of DES contract versus subordinated debt on regulatory and management goals as bank stability, depositor protection, incentives for risk taking, market discipline and the value of bankruptcy costs.

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The Hebrew University Business School, Mount Scopus, Jerusalem, 91905 Israel. Emails: araviv@stern.nyu.edu, Tel +1 972 37329883, Fax +1 972 588-1341. I am grateful to Dan Galai, Yoram Landskroner, Dan Volberg, Zvi Wiener, as well as seminar participants at the FMA meeting (New Orleans, 2004) for helpful comments.

Unlike firms in non-financial, unregulated industries, the primary creditors in a bank, the depositors, do not have sufficient incentives to monitor the bank, because of the implicit or explicit guarantees that are provided to commercial banks by national governments. As a result, governments, as the depositors' insurers, have a strong incentive to monitor banks in order to avoid insolvency. Moreover, the motivation of governments stemmed from the fact that bank insolvency may spillover to other sectors of the financial system and, through it, to the real economy (See Allen and Herring (2001) and Flannery (2003)).

Over the past decades, as the size and complexity of financial firms have enormously increased, government supervisors have found it more difficult and challenging to monitor and control banks in a timely manner by using traditional supervisory techniques such as minimum capital requirements and regular review of a bank's risk management procedures. Accordingly, considerable efforts have been made by banks and their supervisors to include capital instruments that enhance market discipline.¹

Recent studies have recommended subordinated notes and debentures (hereafter SND) as a preferred tool to discipline banks and policymakers are actively considering requiring banks to issue SND.² It is argued that the expected negative effect on subordinated debt prices to excessive risk-taking encourage their holders to monitor the bank closely on an on-going basis, in a way which is aligned with those of the deposits' insurers. Moreover, it is asserted that the SND can impose discipline indirectly by providing risk signals to other market participants and regulators who can then discipline the bank (See Evanoff and Wall (2001)).

While helping to monitor a bank's activities and increasing the size of the financial cushion for the deposit insurer, the subordinated debenture also increases its leverage and the probability of default by the issuing financial institution. According to

the classical structural approach, in the event of firm failure equity investors simply surrender the firm to the bond investors who proceed to operate the firm in the most efficient manner available.³ In reality, the costs of distress and bankruptcy are substantial, especially in the banking industry, in which a weak credit rating limits a bank's ability to trade foreign exchange and OTC derivatives and to extend lines of credit to borrowing customers.

To overcome the disadvantages of the SND mechanism, Flannery (2003) presents qualitatively a potential new capital instrument: 'Reverse Convertible Debentures' (RCD), which is converted into common stocks if the issuing firm's capital ratio falls below some prespecified level. The RCD conversion is triggered automatically: neither the issuer nor the investor will have an option regarding this conversion and the debentures convert at the *current* share price. When the firm's share price is depressed, part of the outstanding debt is forgiven and thus the incidence of costly failures is reduced.

Relying on the market price of the liabilities as the trigger for conversion may lead to price puzzling, since it is unclear under which conditions conversion would take place. The market price of each corporate liability is a function of the ex-post possible payoffs that are contingent on the value of the underlying asset of the corporation. However, by determining that the payoff of a claim could be replaced by another payoff at some future time, if its value ex-ante (market value) has touched some predetermined level, may lead to an undesired equilibrium in which conversion may not occur even if the corporation is in a bad standing and vice versa.

In this paper we elaborate the conversion mechanism by suggesting a capital instrument: Debt-for-Equity-Swap contract (hereafter DES), which has a fixed payoff upon maturity, unless the value of assets falls below some prespecified conversion

threshold at any time *until* debt's maturity. In such event, the debt contract is converted automatically according to a predetermined conversion ratio into the bank's common stocks. Under the presented mechanism, when the value of the bank's assets is depressed, the outstanding amount of the DES is forgiven and thus the incidence of costly failures is reduced.

For the evaluation and quantitative comparison of the effects of issuing DES contract versus subordinated debt, we adopt a contingent claim framework, *à la* Black and Cox (1976, hereafter BC) and Ericsson and Reneby (1998), where each of the bank's liabilities, under each of the two capital structures, is mimicked by a portfolio of barrier options. By using this modular approach we derive closed-form solutions for the liabilities of a bank, the cost of deposit insurance and the value of bankruptcy costs.

Relying on the derived closed-form solutions, we compare the effects of the DES and SND contracts on several crucial policy targets as bank stability, depositor protection, minimizing the value of bankruptcy costs and enhancement of market discipline. We analyze how the levels of volatility, leverage ratio, bankruptcy costs and the mandatory supervisory intervention affect these goals.

The valuation of a bank's liabilities requires setting a trigger point for mandatory supervisory intervention. Previous models for pricing SND contracts (See Levonian (2001) and Nivorozhkin (2001)) assume based on the Black-Scholes-Merton model that default can occur only at debt maturity if the value of assets falls below the debt's face value. We, like BC, assume that bank failure can occur at the first time when the value of assets falls below a predetermined threshold. In such event, the remaining assets of the bank are distributed among the claimholders according to their seniority. However, in our model, conversely to BC, liquidation may be costly. Moreover, the trigger point in our model is based on the assumption that the solvency of a bank is determined by its

regulator according to a minimum “adequate” capital level, which is expressed as a percentage of the book value of equity. Thus, a mandatory supervisory intervention occurs usually when the value of assets is well above the bank’s outstanding debt. Aware of this fact, the liquidation threshold in our model is set at least equal to the value of the bank’s outstanding debt.⁴

We prove that the difference between the default probabilities of a bank with SND and a bank with DES contract, holding everything else constant, is always positive. The power of DES as provider of depositors’ protection is more questionable. We prove that the ratio between the cost of insurance of a deposit in a bank that includes DES contract in its capital structure and a similar insurance of a deposit in a bank that includes instead SND may be lower, equal or larger than one. We show that the ratio between the costs of insurance of a deposit in a bank with SND contract and an identical deposit in a bank with DES contract is a decreasing function of the leverage ratio and the level of mandatory supervisory intervention, which is expressed as the ratio between the value of assets and the total face value of debts. We demonstrate that for a relatively low ratio of bankruptcy costs, the cost of deposit insurance is lower for a bank with SND contract, while the conversely occurs when bankruptcy costs are relatively high. However, as the level of supervisory intervention increases (in the form of relatively high liquidation threshold) the costs of deposits insurance are equalized at a higher ratio of bankruptcy costs.

It is argued that the SND contract can impose direct discipline on banks by charging high funding costs once excessive risk-taking activities are detected. The results of empirical studies that examine whether risky debt, issued by banks and bank holding companies, facilitates market monitoring and the control of risk taking have been mixed. Using our theoretical framework, we demonstrate that unlike the DES contract, the SND

contract could be almost insensitive to changes in the volatility of assets if the bank is highly regulated and the rate of bankruptcy costs is relatively low. We find that the effect of the regulatory intervention policy on the sensitivity of the SND's price to increase in assets volatility is not obvious, and depends on the leverage ratio. It is shown that while the sensitivity of the SND to changes in asset price is always positive, the sensitivity of the DES contract could be negative under relatively high conversion ratio, and its efficiency as a tool for market monitoring is therefore questionable.

Jensen and Meckling (1976) assert that the interests of bondholders and shareholders in a leverage firm strongly diverge regarding the risk that can accompany higher firm profits, since the increase in assets volatility results in a transfer of value from the debtholders of a firm to its equityholders. However, as demonstrated by Reisz and Perlich (2004), if the liquidation threshold is higher than the sum of the total liability, the probability of going bankrupt becomes too large and shareholders will shy away from any risky project. In this case we might observe a risk-avoidance problem *à la* John and Brito (2004). In consistency with this approach, Flannery (2003) asserts that the introduction of subordinated debt into a bank's capital structure increases the level of leverage and thus exacerbates the problem of risk-avoidance and might even hurt the competitive nature of a bank and its task as liquidity provider. He suggests that the inclusion of "Reverse convertible debentures" (RCD) instead of subordinated debt, can forestall financial distress without distorting the stockholders risk-taking incentives. In this paper, we show that the inclusion of DES contract in a bank's capital structure may not avoid the distortion of the shareholders' risk taking incentive. This effect depends among other factors on the level of the conversion threshold and on the conversion ratio. Moreover, the shareholders' risk-taking incentive in a bank with DES contract may be lower than the incentive of shareholders in bank that is identical in all other respects

except for replacing the DES contract with contract that does not include the conversion feature (i.e.: SND contract).

The rest of the paper is organized as follows. Section I presents the basic assumptions for the valuation of corporate liabilities of a bank with DES contract in its capital structure. Section II analyzes in a similar manner a bank with SND contract in its capital structure. Section III shows the valuation of a bank's claims under each of the two capital structures via options replication. Section IV compares quantitatively the effects of DES contract versus SND on bank's stability, depositor protection enhancement of market discipline and the value of bankruptcy costs. Section V concludes. All proofs are contained in the appendices.

I. Capital structure with DES contract

In this section we discuss the basic economic setting on which we base our model for pricing corporate liabilities of a bank with capital structure that consists of senior deposit, DES contract and equity. We show the valuation of these liabilities, and derive closed form solutions for the values of bankruptcy costs and for the cost of deposit insurance under the presented capital structure.

A. Setup and Key Assumptions

Consider a hypothetical bank with assets ω that are continuously traded in an arbitrage-free and complete market with riskless borrowing or lending at a constant rate of r . The

value of the bank's assets is independent of its capital structure, and is well described under the risk neutral probability by the following stochastic differential equation:

$$d\omega = (r - \delta)\omega dt + \sigma\omega dW \quad (1)$$

where W is a standard Brownian motion, δ is the institution's payout ratio and σ is the instantaneous constant standard deviation of the rate of return of the bank.⁵

To finance its assets, the bank issues three types of claims: a single zero-coupon deposit, a single DES contract and a residual equity claim with market value denoted by S . The zero-coupon deposit matures at time T , has principal value of F^B and market value of B . The depositor is the most senior security holder, and thus has priority over all classes of securities in a way that will be specified later on. It is assumed that the government, which supervises the bank through a specialized regulator, would force liquidation or reorganization at any time $t \in [0, T]$ if the value of assets has reaches an exogenous lower threshold K^l , where this threshold is defined as:

$$K^l = \lambda^l F^B \quad \text{where } \frac{\omega}{F^B} \geq \lambda^l \geq 1. \quad (2)$$

In contrast to the convention in most structural models in our model, the threshold level is at least equal to the face value of debt. Previous models assume, based on the nature of existing safety covenants or common practice and law, that the debtholders or regulators have the power to enforce liquidation or reorganization only if the value of assets has reached the debt's principal value.⁶ However, regulators usually determine the

solvency of a bank according to a minimum “adequate” capital level, which is expressed as a percentage of the book value of equity, and thus the reorganization of a bank occurs well above the bank’s debt face value. The time of default, where liquidation is declared, is denoted by τ_l and is defined formally by:

$$\tau_l = \inf \{ t > 0 \mid \omega_t \leq K^l \} \quad (3)$$

If default has occurred, so $T \geq \tau_l$, a fraction $0 \leq \gamma \leq 1$ of value will be lost due to bankruptcy costs.⁷ The costs of bankruptcy consist of, for example, losses due to suspended deliveries by cautious suppliers or the ex-post costs of over or under-investment incentives. In the banking industry, bankruptcy costs could also include the cutting down of trades in financial derivative, where counterparties aware of the bank’s condition would reduce derivative transactions in which they might have credit exposure to the distressed bank.

Similar to the deposit, the DES contract matures at time T and has a principal value of F^D and market value of D . The DES is converted automatically into α ($0 \leq \alpha \leq 1$) common stocks if the value of assets falls below some prespecified conversion threshold, denoted by K^c , at any time *prior* to debt’s maturity. While the level of the liquidation threshold is determined exogenously by the behavior of the regulator, the conversion threshold is set in the contract terms. Since the main goal of the DES issue is to reduce the incidence of costly failures, a natural choice is to set the conversion threshold, like the liquidation threshold, at a level which is at least equal to the sum of the principal value of the deposit and the DES, and thus $K^c \geq (F^B + F^D)$. This threshold level ensures that the event of bank insolvency may not occur before the

time of an enforced conversion, and thus the DES holder has no legal support to force early liquidation. However, the regulator may have incentive to liquidate the bank before the event of conversion if the DES principal amount is limited and the conversion threshold is relatively close to the level of the deposit face value. To ensure the efficiency of the conversion mechanism, the threshold level should be set at a sufficient level above the total sum of all claims' face value. The time of conversion is denoted by τ_c and is defined formally by:

$$\tau_c = \inf \{ t > 0 \mid \omega_t \leq K^c \} \quad (4)$$

B. Valuation of the Bank liabilities with DES contract

The market value of the bank, V , is equal to the sum of the value of its securities, i.e.: $V_t = S + B + D$. In the presence of bankruptcy costs and the deductibility of tax and interest payments, this value is not in general equal to the value of assets, ω .⁸ We assume neither tax nor interest deductibility, since we want to concentrate on the effects of the DES contract on the value of the bank's liabilities and on the value of bankruptcy costs. Under this assumption, the value of assets is at least equal to the bank's market value, such that $\omega \geq V$.

As presented by BC and similarly by Ericsson and Reneby (1998), the value of zero-coupon corporate security can be decomposed into two sources of value: first, its value at maturity, assuming the bank is not prematurely liquidated, and second, its value if the bank is liquidated before debt maturity, T .⁹ Although these two components are mutually exclusive, they are both possible outcomes and accordingly each contributes to

the present value of both equity and debt. Since the issuing of the DES contract involves the introduction of a conversion threshold, which is efficiently located above the liquidation threshold and below the value of the bank's assets, the values of the stock and the DES should be decomposed to three mutual exclusive sources of value, and can be expressed as:

$$S = E^Q \left\{ e^{-rT} \left[\begin{aligned} & (\omega_T - F^B - F^D) 1_{\{\tau_c > T\}} \\ & + (1 - \alpha) (\omega_T - F^B) 1_{\{\tau_l \geq T > \tau_c\}} \end{aligned} \right] + e^{-r\tau_l} \Phi^S 1_{\{\tau_l \leq T\}} \right\} \quad (5)$$

$$D = E^Q \left[\begin{aligned} & e^{-rT} F^D 1_{\{\tau_c > T\}} + \\ & \alpha e^{-rT} (\omega_T - F^B) 1_{\{\tau_l \geq T > \tau_c\}} + e^{-r\tau_l} \Phi^D 1_{\{\tau_l \leq T\}} \end{aligned} \right] \quad (6)$$

where E^Q denotes the conditional expectation under a risk neutral measure Q given all available information at time zero, and 1_ψ is the indicator function of the event ψ . The symbols Φ^S and Φ^D are the payoff functions upon liquidation of the stock and the DES contract respectively, which depend on the ratio of bankruptcy costs, and can be expressed as:

$$\Phi^S = \begin{cases} 0 & \gamma \geq 1 - \frac{F^B}{\omega} \\ (1 - \alpha) [K^l (1 - \gamma) - F^B] & \text{elsewhere} \end{cases} \quad (7)$$

$$\Phi^D = \begin{cases} 0 & \gamma \geq 1 - \frac{F^B}{\omega} \\ \alpha [K^l (1 - \gamma) - F^B] & \text{elsewhere} \end{cases} \quad (8)$$

The first term on the right hand side of each of equations (5) and (6) accounts for the cash flows that are generated if the value of assets has not touched the conversion threshold until maturity ($\tau_c > T$), as depicted in Figure 1.A. In such cases, the debtholder is fully paid and the equityholder receives the residual amount. The second term on the right hand side of each equation accounts for the cash flows that are generated if conversion has occurred but no liquidation process has happened until debt maturity, i.e. $\tau_l > T \geq \tau_c$, as depicted in Figure 1.B. The stockholder would deliver a portion of α common stock to the DES holder, while the DES holder would waive the debt obligation. The payoff to the DES holder at maturity in this state is equal to α units of the difference between the value of assets and the deposit face value, F^B . The initial stockholder would receive $(1-\alpha)$ units of the same payoff. Upon liquidation, where $T \geq \tau_l$, as depicted in Figure 1.C, the bank incurs costs that represent a portion γ of the value of its assets. If the proceeding assets are distributed according to absolute priority, then the depositor, as the most senior security holder, would receive the minimum between the debt face value and the remaining assets $(1-\gamma)K^l$. In the extreme case, when the senior depositor has been fully paid off, the initial equityholder and the DES holder would receive the residual as expressed through equations (7) and (8) respectively.

The current value of the senior deposit, provided neither conversion nor liquidation have occurred until the current time, is expressed by:

:

$$B = E^Q \left[e^{-rT} F^B 1_{\{\tau_l > T\}} + e^{-r\tau_l} \Phi^B 1_{\{T \geq \tau_l\}} \right] \quad (9)$$

where Φ^B is the payoff function upon liquidation, which depends on the ratio of bankruptcy costs, and can be written as:

$$\Phi^B = \begin{cases} K^l(1-\gamma) & \gamma \geq 1 - \frac{F^B}{K^l} \\ F^S & \text{elsewhere} \end{cases} \quad (10)$$

The current value of bankruptcy costs, denoted by BC , reflects the market value of a payoff of γK^l should liquidation occur. The value of the bank, V , is equal to the value of assets minus the value of bankruptcy costs. Therefore, we can write its current value as:

$$V = \omega - BC = \omega - E^Q \left[e^{-r\tau_l} K^l \gamma 1_{\{\tau_l \leq T\}} \right] = E^Q \left[e^{-rT} \omega_T 1_{\{T < \tau_l\}} + e^{-r\tau_l} K^l (1-\gamma) 1_{\{\tau_l \leq T\}} \right] \quad (11)$$

The payoffs to the claimholders for various realized asset values and the alternative ratio of bankruptcy cost are summarized in Table I.

C. The Introduction of Deposit Insurance in the Presence of DES Contract.

Two of the primary objectives of the regulator, who supervises the banking system, are to attain financial stability by avoiding systematic risk and to protect investors by ensuring that the financial services firm will be able to honor its liabilities to its depositors. While

the two measures capital adequacy and deposit insurance serve these two objectives, by imposing minimum capital adequacy the regulator tries to prevent ex-ante bank's run, where by introducing explicit or implicit deposit insurance an ex-post measure, which compensates the depositor upon default, is employed.¹⁰

In our model, the deposit insurance is assumed to absorb all risk for the senior debt and none for the DES contract and thus the deposit is fully insured or guaranteed.¹¹ In that case, the liability of the provider of the deposit insurance (the guarantor's liability) can be expressed as:

$$G = E^Q \left\{ e^{-r\tau_l} (F^B - K^l (1 - \gamma)) 1_{\{T \geq \tau_l\}} \right\} \quad (12)$$

For a bank of given size, this liability is affected by four factors: the degree of deposit leverage which is reflected in the relationship between ω and F^B ; the rate of bankruptcy costs, the riskiness of the bank's assets, which is reflected in the bank's choice of assets' volatility and the regulator mandatory intervention policy, which is reflected in the distance between the liquidation threshold and the deposit's principal amount. The higher the spread between the liquidation threshold and the deposit's principal amount the smaller is the loss of the depositor upon liquidation. Securing the face value of the deposit at any possible state can be achieved by both measures, via deposit insurance and/or by setting a relatively high level of minimum capital adequacy in the form of a relatively high liquidation threshold that causes the equity holder to carry all the burden of bankruptcy costs.

According to Equation (12), deposit insurance and minimum capital adequacy are substitutive measures for eliminating the potential losses of the depositor upon

liquidation. The regulator can increase the liquidation threshold, K^l , in a way which nullifies the payoff of the insurer upon liquidation. However, in our model, conversely to the minimum capital adequacy measure, the presence of deposit insurance has no influence on the bank's default probability.¹²

II. Capital structure with Subordinated Notes

In this section we present a model for valuation of bank liabilities with traditional capital structure that includes senior debt, SND contract and equity. We present the added assumptions to those presented in Section I.A and then derive valuation equations for each of the bank's securities under each possible state. In a similar manner, the cost of deposit insurance and the value of bankruptcy costs are derived.

Since we assume that the value of bank's assets is independent of its capital structure all the assumptions about the economy that were presented in Section I.A are holding. As in the previous presented capital structure, the bank is financed with equity and a single zero-coupon deposit with principal amount of F^{*B} and market value of B^* . However, instead of issuing a DES contract, the bank has issued a single zero-coupon subordinated debt that matures at time T , has a principal value of F^J and a market value of J .

As in Section I.A, the regulator policy is to force liquidation or reorganization if the value of assets falls below an exogenous threshold:

$$K^{*l} = \lambda^{*l} (F^{*B} + F^J) \quad \text{where} \quad \frac{\omega}{(F^{*B} + F^J)} \geq \lambda^{*l} \geq 1 \quad (13)$$

where the liquidation threshold is denoted by K^{*l} and the star superscript denotes a liquidation threshold of a bank with capital structure that includes SND. The liquidation threshold is at least equal to the sum of the principal values of the senior deposit and the SND. The time of default, where liquidation is declared, is denoted by τ_i^* and is defined formally by:

$$\tau_i^* = \inf \left\{ t > 0 \mid \omega_t \leq K^{*l} \right\} \quad (14)$$

If liquidation has not occurred until debt's maturity ($\tau_i^* > T$), the debtholders would receive its debt's face value, while the stockholder would receive the residual assets (See Figure 2.A). Upon liquidation, the residual assets of the bank, after bankruptcy costs have been incurred, would be distributed among the claimholders according to their seniority. The value of equity, provided that liquidation has not occurred by the current time can be expressed as:

$$S^* = E^Q [e^{-rT} (\omega_T - F^{*B} + F^J) 1_{\{\tau_i^* > T\}} + e^{-r\tau_i^*} \Phi^{*S} 1_{\{\tau_i^* \leq T\}}] \quad (15)$$

where S_i^* is the value of a stock issued by a bank with capital structure that includes SND, and Φ^{*S} is the stock's payoff function upon liquidation, which depends on the ratio of bankruptcy costs and can be written as:

$$\Phi^{*S} = \begin{cases} 0 & \gamma \geq \frac{K^{*l} - (F^{*B} + F^J)}{K^{*l}} \\ K^{*l}(1-\gamma) - (F^{*B} + F^J) & \text{elsewhere} \end{cases} \quad (16)$$

If the remaining assets after bankruptcy costs are not sufficient to completely pay off the deposit then the SND's holder receives nothing. Elsewhere, the holder of the SND receives the minimum between its debt's face value and the remaining assets. The SND can be valued as:

$$J = E^Q[e^{-rT} F^J 1_{\{\tau_l^* > T\}} + e^{-r\tau_l^*} \Phi^J 1_{\{\tau_l^* \leq T\}}] \quad (17)$$

where the function Φ^J is the SND's payoff function upon liquidation, which equals:

$$\Phi^J = \begin{cases} 0 & \gamma \geq \frac{K^{*l} - F^{*B}}{K^{*l}} \\ K^{*l}(1-\gamma) - F^{*B} & \frac{K^{*l} - F^{*B}}{K^{*l}} > \gamma > \frac{K^{*l} - (F^{*B} + F^J)}{K^{*l}} \\ F^J & \frac{K^{*l} - (F^{*B} + F^J)}{K^{*l}} \geq \gamma \end{cases} \quad (18)$$

The value of the deposit can be written as:

$$B^* = E^Q[e^{-rT} F^{*B} 1_{\{\tau_l^* > T\}} + e^{-r\tau_l^*} \Phi^{*B} 1_{\{T \geq \tau_l^*\}}] \quad (19)$$

where Φ^{*B} is the deposit's payoff function in the event of liquidation, which is equal to:

$$\Phi^{*B} = \begin{cases} K^{*l}(1-\gamma) & \gamma \geq \frac{K^{*l} - F^{*B}}{K^{*l}} \\ F^{*B} & \text{elsewhere} \end{cases} \quad (20)$$

The value of a bank with capital structure that includes SND, denoted by V^* , is equal to the value of assets minus the value of bankruptcy costs, denoted by BC^* . Therefore we can write the bank value as:

$$V^* = \omega - BC^* = \omega_t - E^Q \left[e^{-r\tau_t^*} K^{*l} \gamma 1_{\{\tau_t^* \leq T\}} \right] = E^Q \left[e^{-rT} \omega_T 1_{\{T < \tau_t^*\}} + e^{-r\tau_t^*} K^{*l} (1-\gamma) 1_{\{\tau_t^* \leq T\}} \right] \quad (21)$$

The need for deposit insurance under the presumed capital structure arises only in states in which the size of bankruptcy costs is larger than the difference between the level of the liquidation threshold and the deposit's face value. The cost of a deposit insurance that fully compensates the depositor upon liquidation is denoted by G^* and expressed by:

$$G^* = E^Q \left\{ e^{-r\tau_t^*} [F^B - K^{*l} (1-\gamma)] 1_{\{T \geq \tau_t^*\}} \right\} \quad (22)$$

The payoffs to the claimholders for various realized asset values and the alternative ratio of bankruptcy cost are summarized in Table II.

III. Pricing the Bank's Claims by Replicating Payoffs

The following valuation method utilizes the fact that each of the bank's securities can be expressed as a combination of four building blocks: down-and-out call and down-and-in call options, down-and-out and down-and-in Heaviside call options. Assuming no arbitrage, claims with identical payoff function must have equivalent value. Hence, in order to price the different liabilities we simply need to mimic each security by using combinations of these four basic claims. In this section we define the payoff function and the value of each option that serves as one of the building blocks for pricing the bank's liabilities, and then we show how to replicate the bank's different claims under each of the two capital structures.

A. Definitions of the Basic Claims

Definition 1: *If the value of assets has (has not) hit a lower barrier K until maturity then the holder of a down-and-in (out) call option receives at maturity, T , the maximum between zero and the difference between the value of assets, ω_T and an exercise price of F .*

Lemma 1: *The price of a down-and-in call option and down-and-out call option (with payoff given by definition 1) is:*

$$C^{di}(T, F, K) = e^{-rT} E^Q \left[(\omega_T - F)^+ 1_{\{\tau < T\}} \right]$$

$$C^{do}(T, F, K) = e^{-rT} E^Q \left[(\omega_T - F)^+ 1_{\{\tau \geq T\}} \right]$$

where $C^{di}(T, F, K)$ and $C^{do}(T, F, K)$ are the current values of down-and-in call and down-and-out call, with expiry at time T . The strike price of the options is equal to F

and the barrier is equal to K . The superscript ‘*di*’ indicates a down-and-in type contract and the superscript ‘*do*’ indicates a down-and-out type contract. τ is the first time that the value of assets has touched the lower barrier K .

Definition 2: *If the value of assets has hit a lower barrier K until maturity, T , the holder of a down-and-in Heaviside call (at hit) would receive a unit of 1\$ at the hitting time τ .*

Lemma 2: *The price of a down-and-in Heaviside call (with payoff given by definition 2) is:*

$$H^{di}(T, K) = E^Q \left[e^{-r\tau} 1_{\{T \geq \tau\}} \right]$$

where $H^{di}(T, K)$ is the current value of a down-and-in Heaviside call with expiry at time T and a barrier level of K .

Definition 3: *If the value of assets has not hit a lower barrier K until maturity, T , the holder of a down-and-out Heaviside call would receive a unit of 1\$ at maturity.*

Lemma 3: *The price of a down-and-out Heaviside call (with payoff given by definition 3) is:*

$$H^{do}(T, K) = e^{-rT} E_t^Q \left[1_{\{T < \tau\}} \right]$$

where $H^{do}(T, K)$ is the value of a down-and-out Heaviside call with expiry at time T and barrier level K . The reader will find in Appendix-A a reminder of the pricing formulas for all four basic barrier options that are needed.

B. Replicating Corporate Securities that Include DES Contract

The following options portfolios mimic the payoffs of the stock and the DES contract respectively:

$$S = C^{do}(T, F^B + F^D, K^c) + (1-\alpha)C^{di}(T, F^B, K^c) - (1-\alpha)C^{di}(T, F^B, K^l) + (1-\alpha)\Phi^S H^{di}(T, K^l) \quad (23)$$

$$D = F^D H^{do}(T, K^c) + \alpha C^{di}(T, F^B, K^c) - \alpha C^{di}(T, F^B, K^l) + \alpha \Phi^D H^{do}(T, K^c) \quad (24)$$

The long down-and-out call position represents the stock payoff if neither liquidation nor conversion have occurred. In this state, the holder of the DES contract has F^D units of down-and-out Heaviside call option. The long down-and-in call position account for the combined effects that occur upon reaching the conversion threshold: the dilution of the initial stockholder and the reduction of the level of debt to F^B .

To account for early liquidation and for the transfer of control to the senior depositor, a short down-and-in call position is introduced. This option, with barrier K^l , offset exactly upon liquidation the long down-and-in payoff with barrier level K^c , leaving the stockholder with zero payoffs. In the extreme case, in which the senior depositor has been fully paid, and thus $K^l(1-\gamma) - F^S > 0$, the initial equityholder and the DES holder would receive the residual assets.

The senior deposit is not influenced in any case by the conversion activity and thus its payoff should be mimicked only by options with barrier level that equal to the liquidation threshold:

$$B = F^B H^{do}(T, K^l) + \Phi^B H^{di}(T, K^l) \quad (25)$$

The current value of bankruptcy costs is equal to its magnitude times the present value of 1\$ conditional on future default. Thus its value is isomorphic to down-and-in Heaviside call option with payoff of γK^l . The value of a bank, V , reflects its assets value minus the value of bankruptcy costs and can be expressed as:

$$V = \omega - BC = \omega - \gamma K^l H^{di}(T, K^l) \quad (26)$$

The cost of deposit insurance can be expressed as a down-and-out Heaviside call option, with payoff upon liquidation which is equal to the difference between the face value of the deposit and the remaining assets of the bank:

$$G = [F^B - K^l(1-\gamma)]H^{do}(T, K^l) \quad (27)$$

C. Replicating Corporate Securities that Include SND

The value of each of the bank liabilities is identical to the value of a portfolio that consists of two types of options. The first are down-and-out options that mimic the payoff if early liquidation has not occurred, and the second are down-and-in options that mimic the payoff in the event of liquidation. The values of the stock, the SND contract and the senior deposit can be mimicked by the following options:

$$S^* = C^{do}(T, F^{*B} + F^J, K^{*l}) + \Phi^{*S} H^{di}(T, K^{*l}) \quad (28)$$

$$J = F^J H^{do}(T, K^{*l}) + \Phi^J H^{di}(T, K^{*l}) \quad (29)$$

$$B^* = F^{*B} H^{do}(T, K^{*l}) + \Phi^{*B} H^{di}(T, K^{*l}) \quad (30)$$

As for a bank with capital structure that includes DES contract, the value of the bank, V^* , is equal to the value of its assets minus a down-and-in Heaviside call option with payoff equal to the size of bankruptcy costs:

$$V^* = \omega - BC^* = \omega - \gamma K^{*l} H^{di}(T, K^{*l}) \quad (31)$$

where BC^* is the current value of bankruptcy costs of a bank that includes SND in its capital structure. The cost of deposit insurance, denoted by G^* , can be mimicked by a down-and-out call option with liquidation threshold K^{*l} , which can be expressed as:

$$G^* = [F^{*B} - K^{*l}(1 - \gamma)] H^{di}(T, K^{*l}) \quad (32)$$

IV. Debt-for-Equity Swap Contract versus Subordinated Notes

Equipped with closed form solutions for the valuation of a bank's liabilities, which includes a DES or alternatively an SND contract as part of its capital structure, we can compare these two capital structures with respect to the fundamental tasks of preventing

costly failures in banking, reducing the cost of deposits insurance, enhancing market discipline and reducing the current value of bankruptcy costs.

The effects of each contract on promoting the presented goals are compared in the following sections. As a base case for our analysis, we assume a bank with capital structure that is composed of a single zero-coupon deposit, with face value $F^B=95$, a single DES contract, with $F^D=95$, and a stock. Both liabilities mature in one year. The value of the bank's assets equals 130, the risk free interest rate is $r = 3\%$, the volatility of the bank's assets is 12% and no payout is expected ($\delta = 0$). Bankruptcy costs, as percentage of the value of the bank's assets upon liquidation, are equal to 20% and the parameter λ^c is 1.05, which means that conversion would occur if the value of assets is 105% of the total sum of the principal amounts (deposit plus DES). At the first time that the value of assets reaches the conversion threshold, the DES holder would receive automatically 0.25 of common stock issued by the bank in exchange for waiving the debt contractual obligation. The liquidation threshold is 96.9 and therefore liquidation occurs when the value of assets is 102% of the total outstanding debt, which consists by that time only of the senior deposit.

The compared bank is identical in all respects except for issuing SND instead of DES contract. The SND contract is otherwise identical to the DES contract except conversion. We assume that the regulator has a consistent mandatory intervention policy in the form of a liquidation threshold that equal to a constant parameter ($\lambda^l = \lambda^{*l}$) multiplied by the total debt's principal amount. As a result, the liquidation threshold of a bank with SND in its capital structure, K^{*l} , is equal to 102 in our example. Table III summarizes the input for the base case.

A. Reducing the Incidence of Costly failures.

In this section we compare the risk neutral probability of default of banks with the two capital structures and derive analytic expression for the difference between the two probabilities. The risk neutral probability that the value of assets would touch a lower threshold until debt maturity is equivalent to the probability that the running minimum of the log-asset value at maturity, T , would be below the adjusted default threshold $\ln(K/\omega)$. As presented in Giesecke (2003), employing the fact that the distribution of the minimum is inverse Gaussian and setting $m = (r - \delta) - \sigma^2/2$, we can write this probability as:

$$P(\tau < T) = N\left(\frac{\ln(K/\omega) - mT}{\sigma\sqrt{T}}\right) + e^{\frac{2m\ln(K/\omega)}{\sigma^2}} N\left(\frac{\ln(K/\omega) + mT}{\sigma\sqrt{T}}\right)$$

where N is the standard normal distribution function.

Lemma 4: *If the mandatory supervisory intervention policy is set as a constant fraction of the total debt's principal amount for all banks, then the difference between the risk neutral default probabilities of a bank with SND contract in its capital structure and an otherwise identical bank whose capital structure include DES contract can be calculated as:*

$$P(\tau^{*l} \leq T \leq \tau^l) = N\left(\frac{\ln(\lambda^l (F^B + F^D)/\omega) - mT}{\sigma\sqrt{T}}\right) + e^{\frac{2m\ln(\lambda^l (F^S + F^D)/\omega)}{\sigma^2}} N\left(\frac{\ln(\lambda^l (F^B + F^D)/\omega) + mT}{\sigma\sqrt{T}}\right) - N\left(\frac{\ln(\lambda^l F^B/\omega) - mT}{\sigma\sqrt{T}}\right) - e^{\frac{2m\ln(\lambda^l F^S/\omega)}{\sigma^2}} N\left(\frac{\ln(\lambda^l F^B/\omega) + mT}{\sigma\sqrt{T}}\right) \quad (33)$$

It is noteworthy that the closed-form expression reported in Lemma 4 for the difference between the default probabilities is an increasing function of the leverage ratio and the level of capital adequacy, which is measured in the form of the ratio between the liquidation threshold, and the total face value of debt. The relationship between the difference between the default probabilities and the volatility of assets is U-shape. To illustrate, Table IV presents the difference between the default probabilities for various levels of volatility, value of assets and liquidation threshold. When $\omega = 130$, $\lambda^l = \lambda^f = 1.02$ and $\sigma = 12\%$ the difference is equal to 2.02%. However, when the value of assets changes to 120 or to 140, the difference is 8.16% and 0.38% respectively. A decrease in the level of the minimum capital adequacy, in the form of lower liquidation threshold, where $\lambda^l = \lambda^f = 1.0$, decreases the difference between the default probabilities to 1.34%. However, a lower liquidation threshold could increase the cost of deposit insurance, as will be described at the following section.

B. Depositors Protection: The Cost of Deposits Insurance

While the DES contract definitely enhances financial stability under any market conditions compared to the SND contract, its advantage as a measure for reducing the cost of deposits insurance and enhancing depositors' protection is more ambiguous and depends on the ratio of bankruptcy costs, the value of assets (or the leverage ratio), the volatility of assets and the level of the liquidation threshold.

Lemma 5: *If the mandatory supervisory intervention policy is set as a constant fraction of the total debt's principal amount for all banks, then the difference between the costs of*

deposit insurance of a bank with SND contract and an otherwise identical bank with DES contract can be positive, negative or equal to zero.

- **Proof:** See Appendix B.

Table V illustrates that the ratio between the cost of insurance of a deposit in a bank with SND in its capital structure and the cost of insurance of a deposit in a bank with DES contract decreases with the level of liquidation threshold. When $\omega = 130$, $\lambda^l = \lambda^{*l} = 1.0$ and $\sigma = 18\%$, the difference between the risk neutral probabilities of default is 5.83% (See Table IV), and the ratio between the cost of deposits insurance is 143.9%. However, when the liquidation threshold increases, such that $\lambda^l = \lambda^{*l} = 1.1$, the difference between the risk neutral probabilities of default increases to 12.38% while the ratio between the costs of deposits insurance decreases to 98.8%, meaning that the cost of insurance of a deposit in a bank with DES contract becomes more expensive than an identical insurance of a deposit in a bank with SND contract in its capital structure.

As the value of the bank's assets decreases (the leverage is increased), issuing SND contract increases the probability of costly default and decreases the cost of deposit insurance compared to the DES contract. A decrease in the value of the bank's assets to $\omega = 120$ increases the difference between the default probabilities from 5.83% to 11.18% while decreasing the ratio between the costs of deposit insurance from 143.9% to 129.5%.

The behavior of the cost of deposit insurance with respect to the ratio of bankruptcy costs is shown in Figure 3. When the liquidation threshold is set at a relatively low level, such that $\lambda^l = \lambda^{*l} = 1.0$, we find that the cost of insurance of a

deposit in a bank with DES contract is higher than a deposit in a bank with SND contract until the ratios of bankruptcy costs are approximately equal to 7%. If the liquidation threshold is increased (in our example: $\lambda^l = \lambda^{*l} = 1.1$), the costs of deposits insurance are equalized when the ratios of bankruptcy costs are approximately 16%. Intuitively, we could think of it in the following way. Under a low ratio of bankruptcy costs the depositor in a bank with SND is fully paid in the event of liquidation if the level of liquidation threshold is relatively high, since the layers of equity and SND absorb all losses. An extra layer of debt does not protect the depositor in a bank with DES contract, since the event of conversion would always precede the liquidation event and thus the depositor begins to suffer losses at a lower ratio of bankruptcy costs. When the ratio of bankruptcy costs increases the effect of the extra financial cushion supplied by the SND contract to the depositor can not avoid losses and thus the depositors in both banks would not be fully paid upon liquidation and thus the cost of insurance of a deposit in a bank with SND contract is higher due to the higher probability of a liquidation event.

C. Efficiency Enhancement: The Value of Bankruptcy Costs

Although the deadweight value in the event of a bank failure is usually not limited to the collapsing bank, it is interesting to compare the value of bankruptcy costs of the two capital structures as a signal for efficiency.

Lemma 2.6: *If the mandatory supervisory intervention policy is set as a constant fraction of the total debt's principal amount for all banks, then the difference between the values of bankruptcy costs of a bank with SND contract and an otherwise identical bank with DES contract in its capital structure is always positive.*

- **Proof:** See Appendix 2.B

When the value of assets is relatively high (low leverage ratio), the difference between the values of bankruptcy costs (as % of assets' value) of a bank with SND in its capital structure and a bank with DES contract, that are otherwise identical, increases with assets volatility (See Figure 4). When the volatility of assets is relatively high liquidation would certainly occur under each of the two capital structures, and thus the difference between the two values of bankruptcy costs would converge to the difference between the two liquidation thresholds multiplied by the ratio of bankruptcy costs, i.e.: $\gamma(K^{*l} - K^l)/\omega$, which in the presented example is 0.93%. Conversely, when the value of assets is relatively low (high leverage ratio), the difference between the values of bankruptcy costs is humped shape with respect to assets' volatility, and as the value of assets increases this difference is maximized at a higher level of assets volatility.

D. Risk Control and Market Monitoring

In this section, we evaluate and compare the effects of the DES and the SND contracts on enhancing market monitoring and controlling the risk taking by the shareholders. A contract can enhance market monitoring due to its negative sensitivity to increased leverage or due to its negative sensitivity to an increase in assets' volatility. The influence of a contract on the shareholders risk taking is measured by the sensitivity of the stock to changes in assets' volatility.

1. Market Monitoring

The presence of most subordinated debt instruments within a bank's capital structure is justified by their protective nature and their ability to enhance market

monitoring by charging high funding costs once excessive risk-taking activities are detected. Levonian (2001) presents a theoretical model for pricing SND, which is based on the Black-Scholes-Merton contingent claims analysis, and finds that an increase in the risk of assets will decrease the value of subordinated debt for solvent banks

The results of empirical studies that examine whether risky debt issued by banks and bank holding companies enhances risk monitoring have been mixed. Studies done prior to 1992 failed to find a significant relationship between firm risk and yields on subordinated debt.¹³ More recent studies do indicate that risk is being appropriately priced (See Flannery and Sorescu, 1996, and Jagtiani, Kaufman and Lemieux, 2002).¹⁴ In a recent study, Krishnan, Ritchken, and Thomson (2003) examine whether changes in credit spreads reflect changes in firm specific risks. After controlling for changes in market-wide and liquidity factors, the authors do not find any consistent evidence for connection between the two. Moreover, the fact that banking firms are highly regulated could not explain the insensitivity of the subordinated debt spreads.

In order to explore and to compare the sensitivity of the SND and the DES prices to changes in assets volatility, we present in figures 5 and 6 the *vega* of the two contracts against the ratio of bankruptcy costs for different levels of liquidation threshold and leverage.¹⁵ When the ratio of liquidation costs is relatively low and banking firms are highly regulated, in the form of a relatively high liquidation threshold ($\lambda^* = 1.06$), the loss given default of the SND is zero. As a result, the SND price is insensitive to the level of volatility, and the SND contract is inefficient as a tool for providing market monitoring (See Figure 5). However, when the ratio of bankruptcy costs increases the SND holder would not be fully paid upon default, and therefore the contract price has negative vega. The effect of the mandatory regulatory intervention policy on the contract price is not

obvious and depends heavily on the leverage ratio. When the leverage ratio is relatively low $[(F^J + F^B)/\omega = 100/130]$, the SND's vega decreases with the level of the liquidation threshold and conversely, when the leverage is relatively high ($\omega = 110$) the SND's vega increases with the level of the liquidation threshold. In both of these cases, the vega of the DES contract receives negative values since a mandatory conversion would always precede the liquidation event (See Figure 6).

The efficiency of the SND over the DES contract as a tool for providing market monitoring exists in regions in which the ratio of bankruptcy costs is relatively high. A relatively high ratio of bankruptcy costs decreases the recovery rate of both contracts upon liquidation, however, the potential compensation of the DES holder upon conversion, in the form of common stocks, reduces its vega compared to the SND.

Figure 7 highlights the limitedness of the DES contract as a tool for providing market monitoring by plotting the value of the SND and the DES contract against the value of assets. While the value of the SND contract increases with assets value, the behavior of the DES contract depends on its conversion ratio (α). An increase in the value of assets produces two opposite effects: increasing the probability that the DES holder would be fully paid upon maturity and decreasing the probability of early mandatory conversion. As the conversion ratio increases (decreases) the later effect becomes more dominant (minor) and the value of the DES decreases (increases) with the value of assets.

2. Risk-Avoidance or Risk-Transfer?

In their seminal work, Black and Scholes (1973) and Merton (1974) offer the insight that equity value is identical to the price of a standard European call option on the total market value of the firm's assets with an exercise price equal to the promised payment of

corporate liabilities. However, as Galai and Masulis (1976) first pointed out, this option analogy suffers from the fact that the value of a standard call option is strictly increasing with assets' volatility of the underlying assets. Hence, a shareholder-aligned manager, who is faced with a choice between two different projects "would invest in the project of higher variance. Moreover, it is even possible that a more profitable investment project will be rejected in favor of a project with a higher variance of percentage returns", thereby transferring wealth from bondholders to shareholders. This asset-substitution problem was much developed in the agency literature, starting with Jensen and Meckling (1976). According to this approach, the interests of bondholders and shareholders in a leverage firm strongly diverge regarding the risk that can accompany higher firm profits since the increase in assets volatility results in a transfer of value from the debtholders of a firm to its equityholders.

The traditional methods employed by debtholders to deal with asset-substitution problems include increasing the required rate of return on their financial claims, design of safety covenants that limit a firm's ability to shift risk, and simple termination of the relationships with the companies. Although the market is able to constrain the behavior of non-bank firms, commercial banks' debtholders have weak incentives to protect the value of their claims due to the effects of implicit or explicit guarantees provided to commercial banks by national governments. However, by setting a minimum level of mandatory intervention, government's regulators have a strong impact on equityholders' ability to transfer risk. As analyzed by Reisz and Perlich (2004), if the liquidation threshold is higher than the sum of the total liability, the derivative of the stock price with respect to assets volatility (vega of the option) is negative for all σ : any risky investment makes the probability of going bankrupt too large and shareholders will shy away from any risky project. In this case, far from witnessing an asset substitution problem *à la* Jensen and

Meckling (1976), we might observe a risk-avoidance problem *à la* John and Brito (2004): a shareholder-aligned manager, afraid of losing growth options privy only to her, can shy away from risk and undertake projects with suboptimal risk levels. Reisz and Perlich (2004) support this finding numerically and find that the value of equity decreases for low leverage ratio.

Consistent with this attitude, Flannery (2003) asserts that the introduction of subordinated debt into a bank's capital structure increases the level of leverage and thus derives a higher liquidation threshold that exacerbates the problem of risk avoidance and might even hurt the competitive nature of bank and its task as liquidity provider. Moreover, it is suggested that the inclusion of "Reverse convertible debentures" (RCD), in a bank's capital structure instead of subordinated debt can forestall financial distress without distorting the stockholders risk-taking incentives. In the present paper it is shown that the effect of including DES contract in a bank's capital structure may not avoid the distortion of shareholders' risk-taking incentive. This effect depends among other factors on the level of the level of the conversion threshold and on the conversion ratio. Moreover, the shareholders' risk-taking incentive in a bank with DES contract can be lower than the incentive of the shareholders in an identical bank except for replacing the DES contract with contract that does not include the conversion feature (i.e.: SND contract).

To illustrate the effects of issuing SND versus DES contract on the stockholders' motivation to increase assets' volatility and to transfer risk, Figure 8 poses the value of the stock's vega versus the value of the bank's assets for different conversion threshold. In the extreme case, when the conversion ratio is equal to one (Figure 8.A), the stockholders lose all their shares when the value of the bank's assets reaches the

conversion threshold. Therefore, the value of the stock is equal to the value of a stock in an identical bank with SND contract in its capital structure in which the liquidation threshold is equal to the conversion threshold of the DES contract. For this case the stock's vega is always less or equal to zero and the risk-avoidance problem would not be forestalled.

When the conversion ratio decreases to 0.75 (Figure 8.B), the vega of the stock begins to receive insignificant value from a lower assets' value and thus the incentive for risk-avoidance is reduced. However, the DES contract superiority over SND as a mean for forestalling the risk-avoidance incentive depends on the difference between the contract's conversion threshold and the level of the mandatory intervention policy of the regulator in a bank with SND in its capital structure. As the difference increases, the DES contract becomes less effective as a means for reducing the risk-avoidance incentive compared to the SND contract.

When the conversion ratio further decreases to 0.25 (Figure 8.C), the vega of the stock can receive positive values for relatively low assets values. An increase in assets' volatility increases the probability of reaching the conversion threshold that results in early forced conversion in exchange for unlevering the debt notional amount. As the conversion ratio decreases, the influence of the debt removal on the stock price becomes major and positive vega, which encouraged risk transfer by the equityholders, can be observed.

V. Summary and Conclusions

The increasing size and complexity of banking organizations and the desire to lower the potential vulnerability of the banking and the financial systems to systemic risk, have lead to a continuous effort to shape innovative financial tools that would assist in monitoring and controlling banks and supplement the traditional supervisory methods. It is argued that subordinated debt can be a proper mechanism to enhance these tasks, since debtholders stand to suffer heavy losses in the event of insolvency and these losses motivate them to monitor the bank closely on an on-going basis. Yet, while removing part of the cost of deposits insurance from the insurer, the subordinated debenture increases bank leverage and thus the probability of default by the issuing financial institution.

In this paper we propose a novel financial instrument “Debt-for-Equity-Swap” contract (DES) that would automatically convert the debt obligation to a predetermined quantity of common equities if the value of assets falls below a predetermined threshold. Thus, when the assets of the bank perform poorly, the level of leverage is automatically reduced without involving the depositors, counterparties or supervisors.

By using a modular option pricing approach, we present closed form solutions for the valuation of liabilities, the cost of deposits insurance and the value of bankruptcy costs of a bank that includes DES or alternatively subordinated debt in its capital structure. We compare and evaluate quantitatively the effects of DES contract versus subordinated debt on major policy issues as banks' stability, depositor protection, value of bankruptcy costs, market monitoring and control of risk taking.

The policy implications of the paper highlight the fact that the DES contract has salient advantages over subordinated debt as an efficient tool for enhancing market

stability and firm efficiency by reducing the value of bankruptcy costs. The power of the DES as a provider of depositors' protection is more questionable. We find that the ratio between the cost of insurance of a deposit in a bank that includes DES contract in its capital structure, and a similar insurance of a deposit in a bank that includes SND instead may be lower equal or larger than one. While the difference between the default probabilities of SND and DES contract increases with the leverage ratio and the level of the liquidation threshold, the ratio between the costs of the deposit insurance is a decreasing function of the two mentioned factors

The efficiency of the DES contract on reducing the value of bankruptcy costs, in comparison with the SND contract, increases with the volatility of assets for relatively high leverage ratio. However, for relatively low leverage ratio the pattern is humped shape and not strictly inclining.

It is argued that including subordinated debt instruments within a bank's capital structure may enhance market discipline due to their negative sensitivity to changes in the level of volatility and leverage. However, the results of empirical studies have been mixed. We show the efficiency and the limitations of each of the two contracts as a tool for imposing market discipline. Conversely to the DES contract, the SND contract could be almost insensitive to changes in assets' volatility if the level of regulatory intervention, in terms of capital adequacy is relatively high and the rate of bankruptcy costs is relatively low. We show that the effect of the regulatory intervention policy on the sensitivity of the SND's price is not obvious and depends on the leverage ratio. When the leverage ratio is relatively low (relatively high) the sensitivity of the SND's price decreases (increases) with the level of regulatory intervention. It is shown that while the sensitivity of the SND to changes in asset price is always positive, the sensitivity of the DES contract could be positive under relatively high conversion ratio.

Appendix A

Below is a reminder of the pricing formulas of the four barrier options that serve as building blocks for the valuation of the bank's liabilities. For all the following options the barrier H is strictly larger than the strike price F and smaller than the starting value of assets ω , i.e. $\omega \geq H \geq F$. The presented formulas were derived by Merton (1973) and Rubinstien and Reiner (1991). The reader who is familiar with the formulas can skip this appendix.

The value of a down-and-in call and a down-and-out call at time zero is given respectively by:

$$C^{di} = \omega N(d_1) - Fe^{-rT} N(d_2) + \omega(H/\omega)^{\left(\frac{2r}{\sigma^2}+1\right)} N(d_5) - Fe^{-rT} (H/\omega)^{\left(\frac{2r}{\sigma^2}-1\right)} N(d_6) - \omega N(d_3) + Fe^{-rT} N(d_4)$$

$$C^{do} = \omega N(d_3) - Fe^{-rT} N(d_4) - \omega(H/\omega)^{\left(\frac{2r}{\sigma^2}+1\right)} N(d_5) + Fe^{-rT} (H/\omega)^{\left(\frac{2r}{\sigma^2}-1\right)} N(d_6)$$

where $N()$ denotes the standard normal cumulative probability function and:

$$d_1 = \frac{\ln(\omega/F) + T\left(r + \frac{\sigma^2}{2}\right)}{\sigma\sqrt{T}} \quad d_2 = \frac{\ln(\omega/F) + T\left(r - \frac{\sigma^2}{2}\right)}{\sigma\sqrt{T}}$$

$$d_3 = \frac{\ln(\omega/H) + T\left(r + \frac{\sigma^2}{2}\right)}{\sigma\sqrt{T}} \quad d_4 = \frac{\ln(\omega/H) + T\left(r - \frac{\sigma^2}{2}\right)}{\sigma\sqrt{T}}$$

$$d_5 = \frac{\ln(H/\omega) + T\left(r + \frac{\sigma^2}{2}\right)}{\sigma\sqrt{T}} \quad d_6 = \frac{\ln(H/\omega) + T\left(r - \frac{\sigma^2}{2}\right)}{\sigma\sqrt{T}}$$

The value of a down-and-in Heaviside call (with payoff at touch) and a down-and-out Heaviside call (with payoff at maturity) is given respectively by:

$$H^{di} = (H/\omega)\left(\frac{2r}{\sigma^2}\right)N(d_5) + (\omega/H)N(d_7)$$

$$H^{do} = e^{-rT} \left[N(d_4) - (H/\omega)\left(\frac{2r}{\sigma^2}-1\right)N(d_6) \right]$$

where:

$$d_7 = \frac{\ln(H/\omega) - T\left(r + \frac{\sigma^2}{2}\right)}{\sigma\sqrt{T}}$$

Appendix B

Lemma 5

The insurance cost of a deposit in a bank with capital structure that includes DES contract and the cost of insurance of a deposit in a bank with SND contract, which is identical in all other respects, are given by equations (27) and (32) respectively:

$$G = [F^B - K^l(1-\gamma)]H^{di}(T, K^l)$$

$$G^* = [F^{*B} - K^{*l}(1-\gamma)]H^{di}(T, K^{*l})$$

Since we assume that the two banks are identical in all other respects and the liquidation policy is set as a constant fraction of the total debt's principal amount for all banks, then we can write: $F^B = F^{*B}$, $F^D = F^J$ and $\lambda^l = \lambda^{*l}$. By substituting equations (2) and (13) into equations (27) and (32) respectively and dividing one by another we receive:

$$\frac{G}{G^*} = \frac{[F^B - \lambda^l F^B (1 - \gamma)] H^{di}(T, \lambda^l F^B)}{[F^B - \lambda^l (F^B + F^D)(1 - \gamma)] H^{di}[T, \lambda^l (F^B + F^D)]} \quad (2.A)$$

The right side of Equation (2.A) can be decomposed into two separate expressions. The first is always equal or greater than one, since by definition $0 \leq \gamma \leq 1$ and thus the expression $F^D(1 - \gamma)$ is always positive and as a result:

$$\frac{[F^B - \lambda^l F^B (1 - \gamma)]}{[F^B - \lambda^l (F^B + F^D)(1 - \gamma)]} \geq 1 \quad (2.B)$$

The second expression is the quotient of two down-and-in Heaviside call options (at hit).

The option derivative with respect to the barrier level is:

$$\frac{\partial H^{di}}{\partial K} = \frac{2rN'(d_5)}{\omega\sigma^3\sqrt{T}} + \frac{2rN(d_5)}{\omega\sigma^2} + \frac{N(d_7)}{\omega} + \frac{2rN'(d_7)}{\omega\sigma\sqrt{T}} \quad (2.C)$$

where:

$$N'(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

Since all the components of equation (2.C) are positive, the value of the option always increases with the level of the barrier, and consequently the following quotient is always smaller than one:

$$\frac{H^{di}(T, \lambda^l F^B)}{H^{di}(T, \lambda^l (F^B + F^D))} \leq 1 \quad (2.D)$$

Thus, the product of expression (2.B) with expression (2.D) can be lower, higher or equal to one.

Lemma 6

The values of bankruptcy costs of a bank with DES contract and of a bank with SND are given by equations (26) and (31) respectively:

$$BC = \gamma K^l H^{di}(T, K^l)$$

$$BC^* = \gamma K^{*l} H^{di}(T, K^{*l})$$

Since we assume that the two banks are identical in all other respects and the mandatory intervention policy is set as a constant fraction of the total outstanding debt for all banks, we can write: $F^B = F^{*B}$, and $\lambda^l = \lambda^{*l}$. By substituting equations (2) and (13) into equations (26) and (31) respectively and dividing one by another we receive:

$$\frac{BC_t}{BC_t^*} = \frac{\lambda^l F^B \gamma H_t^{di}(T, \lambda^l F^B)}{\lambda^l (F^B + F^D) \gamma H_t^{di}(T, \lambda^l (F^B + F^D))} \quad (2.E)$$

The right side of Equation (2.E) can be decomposed into two separates expressions. The first is always equal or smaller than one since by definition $0 \leq \gamma \leq 1$:

$$\frac{\lambda^l F^B \gamma}{\lambda^l (F^B + F^D) \gamma} = \frac{\lambda^l F^B \gamma}{\lambda^l F^B \gamma + \lambda^l F^D \gamma} \leq 1 \quad (2.F)$$

The second expression is always smaller than one, since by setting all other parameters constant, the value of a down-and-in Heaviside call always increases with the level of the barrier (See Lemma 5), and thus we have:

$$\frac{H^{di}(T, \lambda^l F^B)}{H^{di}[T, \lambda^l (F^B + F^D)]} \leq 1 \quad (2.G)$$

As a result the product of expressions (2.F) with (2.G) is always smaller or equal to one.

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Table I: Claimholders' Payoffs, in a bank with DES contract in its capital structure, for various realized asset values and ratio of bankruptcy costs

Type of liability	Case 1: $\tau_c > T$	Case 2: $\tau_l > T \geq \tau_c$	Case 3: $T \geq \tau_c$	
			$(K^l - F^B)/K^l \leq \gamma$	$\frac{(K^l - F^B)}{K^l} > \gamma$
Senior debt	F^B	F^B	$K^l(1-\gamma)$	F^B
DES contract	F^D	$\alpha(\omega_T - F^B)$	0	$\alpha(K^l(1-\gamma) - F^B)$
Equity	$\omega_T - (F^B + F^D)$	$(1-\alpha)(\omega_T - F^B)$	0	$(1-\alpha)(K^l(1-\gamma) - F^B)$

Table II: Claimholders' Payoffs, in a bank with SND contract in its capital structure, for various realized asset values and ratio of bankruptcy costs

Type of liability	Case 1: $\tau_l^* > T$	Case 2: $T \geq \tau_l^*$		
		$\gamma \geq \frac{K^{*l} - F^{*B}}{K^{*l}}$	$\frac{K^{*l} - F^{*B}}{K^{*l}} > \gamma > \frac{K^{*l} - (F^{*B} + F^J)}{K^{*l}}$	$\frac{K^{*l} - (F^{*B} + F^J)}{K^{*l}} \geq \gamma$
Senior debt	F^{*B}	$K^{*l}(1-\gamma)$	F^{*B}	F^{*B}
SND contract	F^J	0	$K^{*l}(1-\gamma) - F^{*B}$	F^J
Equity	$\omega_T - (F^{*B} + F^J)$	0	0	$K^{*l}(1-\gamma) - (F^{*B} + F^J)$

Table III: Market and contract data for the base case:

Bonds Maturity (T)	1 year	Conversion threshold	105
Deposit's principal amount (F^B/F^{*B})	95	Liquidation threshold with DES	96.9
DES/ SND principal amount (F^D/F^J)	5	Liquidation threshold with SND	102
Ratio of bankruptcy costs (γ)	0.20	Payout ratio (δ)	0%
Conversion ratio (α)	0.25	Risk free rate (r)	3%
Assets' volatility (σ)	12%	The value of assets (ω)	130

Capital structure with:	DES	SND
Stock price	33.06	32.88
DES price (credit spread)	4.73	4.71
Deposit price (credit spread)	92.05	91.93
Value of bankruptcy costs	0.17	0.58
Costs of deposit insurance	0.15	0.38

Table IV: The difference between the Default Probabilities of a bank with SND in its capital structure and a bank with DES contract for various levels of assets' volatility, value of assets and liquidation threshold (as percentage of the total outstanding debt)

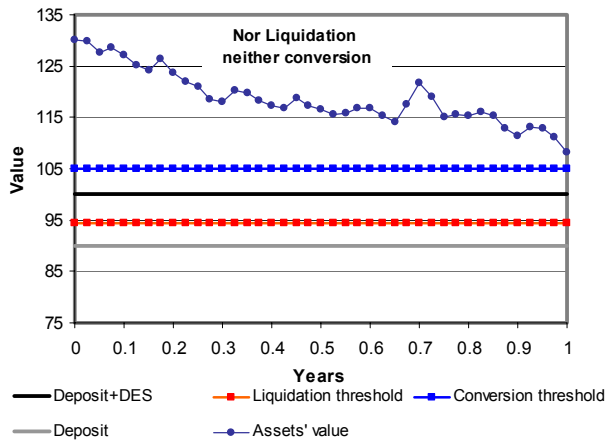
Value of assets	Liquidation threshold as percentage of the total outstanding debt	Assets' volatility		
		12.0%	18.0%	24.0%
$\omega=120$	100.0%	6.01%	11.18%	11.70%
	102.0%	8.16%	12.75%	12.53%
	104.0%	10.75%	14.34%	13.31%
	106.0%	13.74%	15.91%	14.04%
	108.0%	17.08%	17.44%	14.70%
	110.0%	20.69%	18.89%	15.28%
$\omega=130$	100.0%	1.34%	5.83%	8.28%
	102.0%	2.02%	6.97%	9.11%
	104.0%	2.95%	8.22%	9.95%
	106.0%	4.17%	9.55%	10.77%
	108.0%	5.72%	10.94%	11.57%
	110.0%	7.63%	12.38%	12.34%
$\omega=140$	100.0%	0.23%	2.68%	5.45%
	102.0%	0.38%	3.36%	6.15%
	104.0%	0.61%	4.13%	6.88%
	106.0%	0.94%	5.01%	7.63%
	108.0%	1.42%	5.99%	8.40%
	110.0%	2.08%	7.06%	9.17%

Parameters: See Table III.

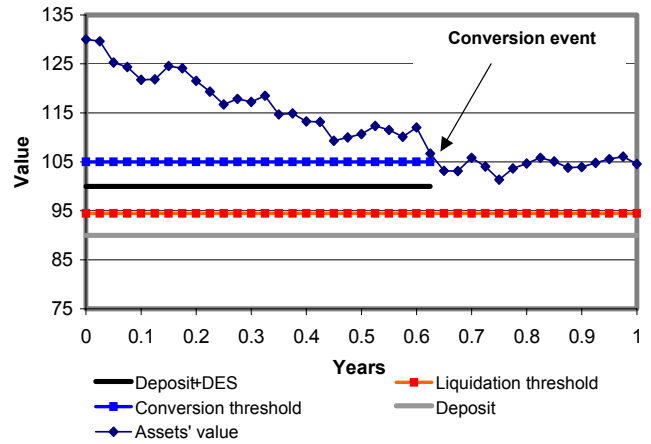
Table V: The ratio between the costs of insurance of a deposit in a bank with SND contract in its capital structure and a deposit in a bank with DES contract for various levels of assets' volatility, value of assets and liquidation threshold (as percentage of the total outstanding debt)

Value of assets	Liquidation threshold as percentage of the total outstanding debt	Assets' volatility		
		12.0%	18.0%	24.0%
$\omega=120$	100.0%	214.5%	129.5%	107.3%
	102.0%	196.1%	122.6%	102.8%
	104.0%	178.4%	115.4%	97.8%
	106.0%	161.2%	107.7%	92.3%
	108.0%	144.0%	99.3%	85.9%
	110.0%	126.1%	89.6%	78.3%
$\omega=130$	100.0%	275.9%	143.9%	113.6%
	102.0%	251.5%	136.1%	108.7%
	104.0%	228.1%	127.9%	103.4%
	106.0%	205.3%	119.2%	97.5%
	108.0%	182.6%	109.6%	90.7%
	110.0%	159.2%	98.8%	82.6%
$\omega=140$	100.0%	351.4%	159.4%	120.0%
	102.0%	319.6%	150.6%	114.8%
	104.0%	289.3%	141.4%	109.1%
	106.0%	259.8%	131.6%	102.8%
	108.0%	230.5%	120.9%	95.5%
	110.0%	200.5%	108.8%	87.0%

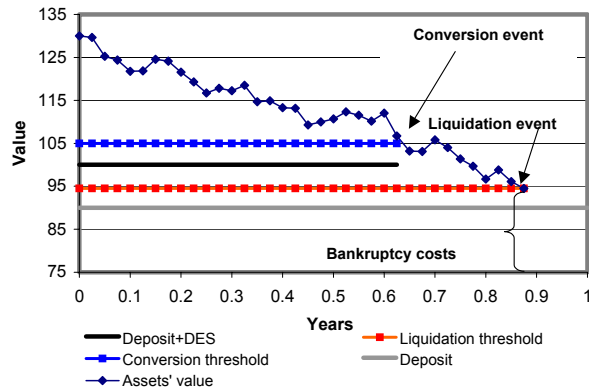
Parameters: See Table III.



(1.A)



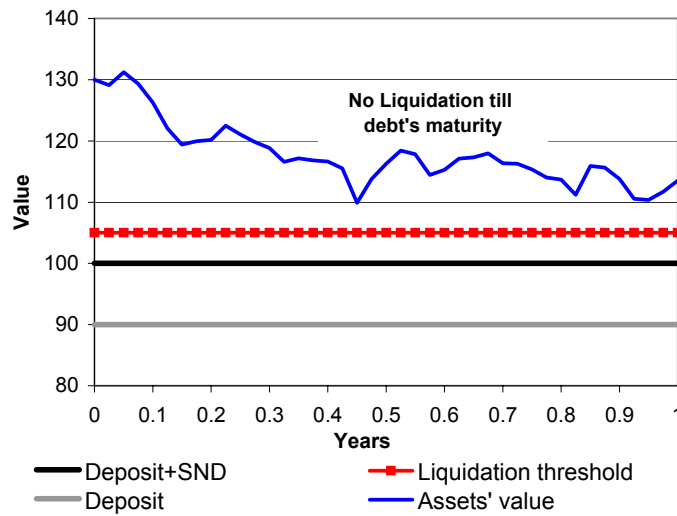
(1.B)



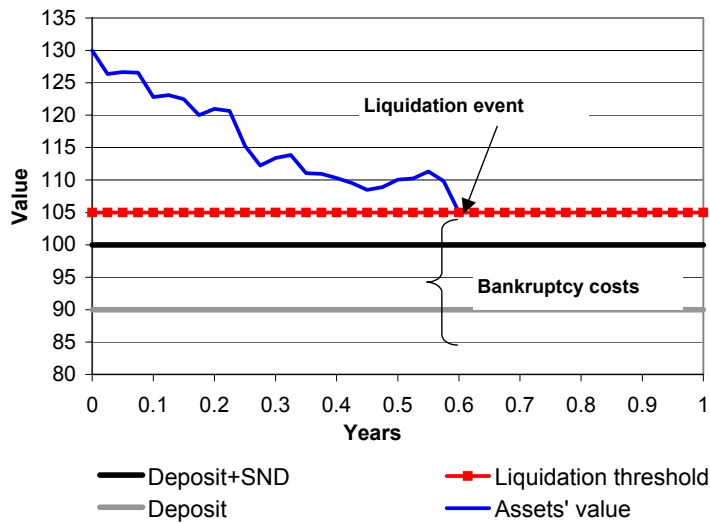
(1.C)

Figure :The Time Line of the Model for the DES contract

There are three possible states in the model: (1.A) The bank is not liquidated until debt maturity ($\tau_c > T$), the debtholders are fully paid, while the stockholders receive the residual. (1.B) The value of the bank's assets has reached the lower conversion threshold and as a result the DES holder receives a predetermined ratio of the bank's stocks in exchange for unwinding its debt obligation ($\tau_l > T \geq \tau_c$). (1.C) The value of the bank's assets has touched the liquidation threshold and liquidation (or reorganization) has occurred, bankruptcy costs of $K'(1-\gamma)$ are incurred. The firm is liquidated with all the proceeds being divided among claimholders according to their seniority ($\tau_c \leq T$). In the example, all the proceeds are paid to the depositor.



(2.A) No early liquidation

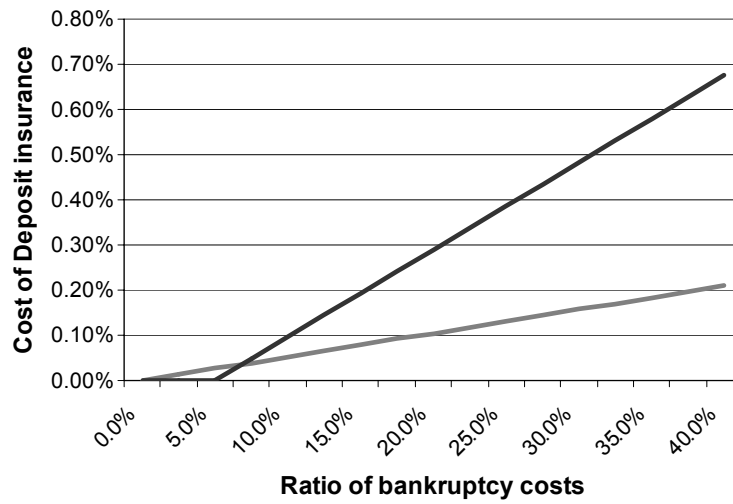


(2.B) Early liquidation

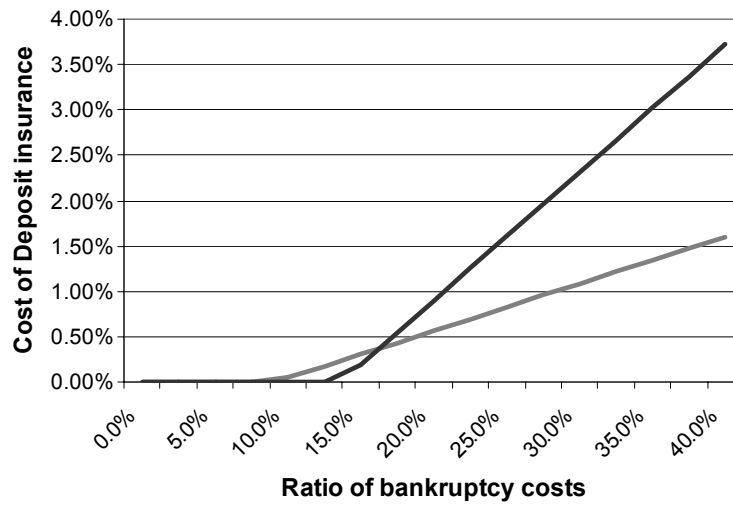
Figure 2: The Time Line of the Model for the SND contract

There are *two possible states* in the model: (2.A) since the bank is not liquidated until debt maturity ($\tau_t^* > T$), the debtholders are fully paid, while the stockholders receive the proceeds. (2.B) Liquidation occurs prior to debt maturity ($\tau_t^* \leq T$) and in such event, bankruptcy costs of $K^{nl}(1-\gamma)$ are incurred. The firm is liquidated with all the proceeds being divided among claimholders according to their seniority. In the example, all the proceeds are paid to the depositor.

Cost of Deposits Insurance vs. ratio of bankruptcy costs



$$\lambda^l = \lambda^{*l} = 1$$



$$\lambda^l = \lambda^{*l} = 1.1$$

Figure 3

The ratio of bankruptcy costs against the cost of deposit insurance as percentage of the deposit's face value for different levels of liquidation threshold

Gray line: Bank with DES Contract in its capital structure. Black line: Bank with SND in its capital structure. All other parameters are identical to the base case as appeared in Table III.

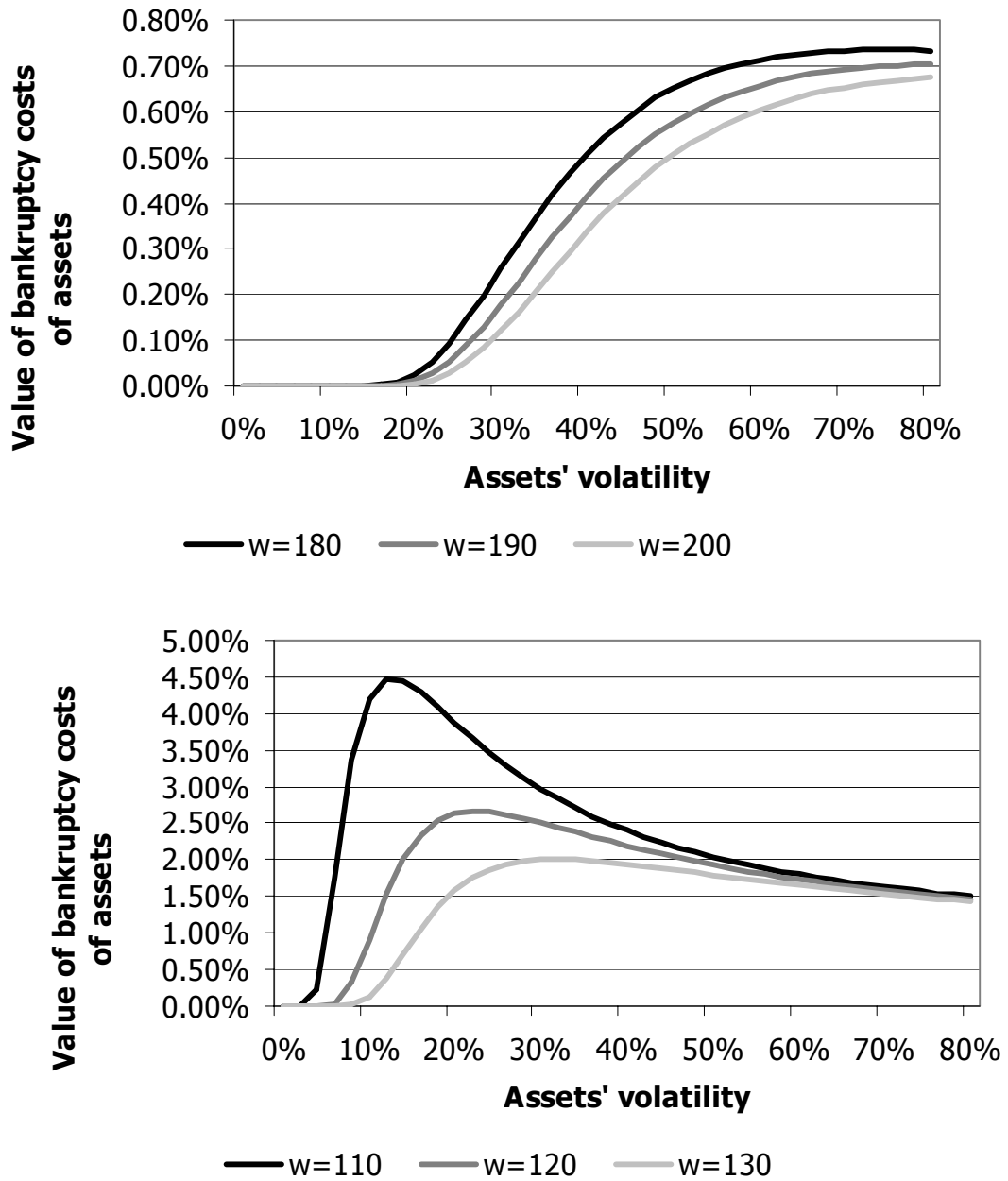


Figure 4

The volatility level against the difference between the values of bankruptcy costs (as percentage of assets' value) of a bank with SND in its capital structure and of a bank with DES contract in its capital structure that are all else identical, for different value of assets. All other parameters are identical to the base case as appeared in Table III.

SND's vega vs. ratio of bankruptcy costs

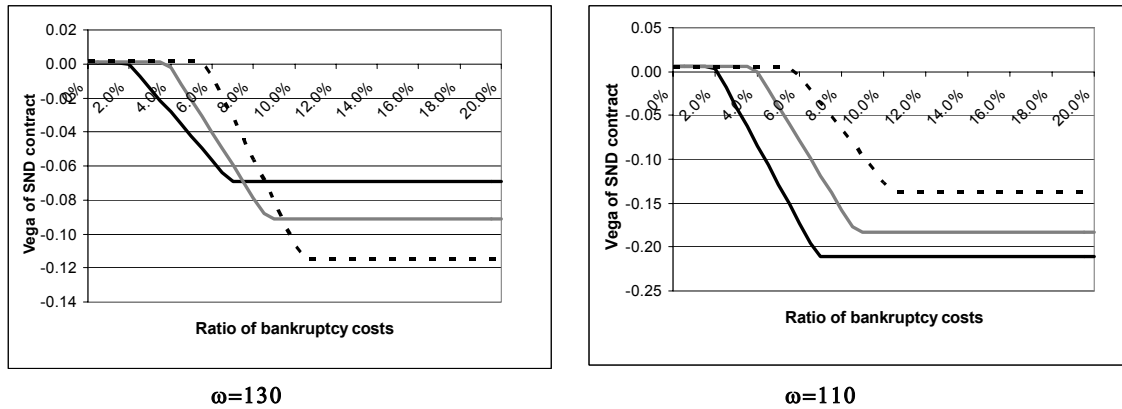


Figure 5

SND's vega versus the ratio of bankruptcy costs for different liquidation thresholds.

Black line: $\lambda^l=1.02$. Gray line: $\lambda^l=1.04$. Dashed line: $\lambda^l=1.06$. Assets' volatility is 24%. All other parameters are identical to the base case (See Table III).

DES's vega vs. ratio of bankruptcy costs

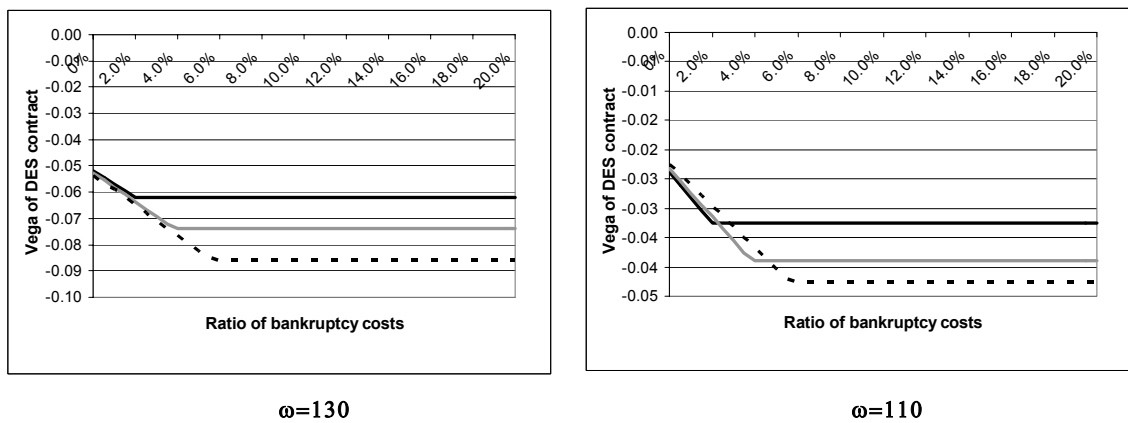


Figure 6

DES's vega versus the ratio of bankruptcy costs for different liquidation thresholds.

Black line: $\lambda^l=1.02$. Gray line: $\lambda^l=1.04$. Dashed line: $\lambda^l=1.06$. Assets' volatility is 24%. All other parameters are identical to the base case (See Table III).

DES and SND values vs. value of assets

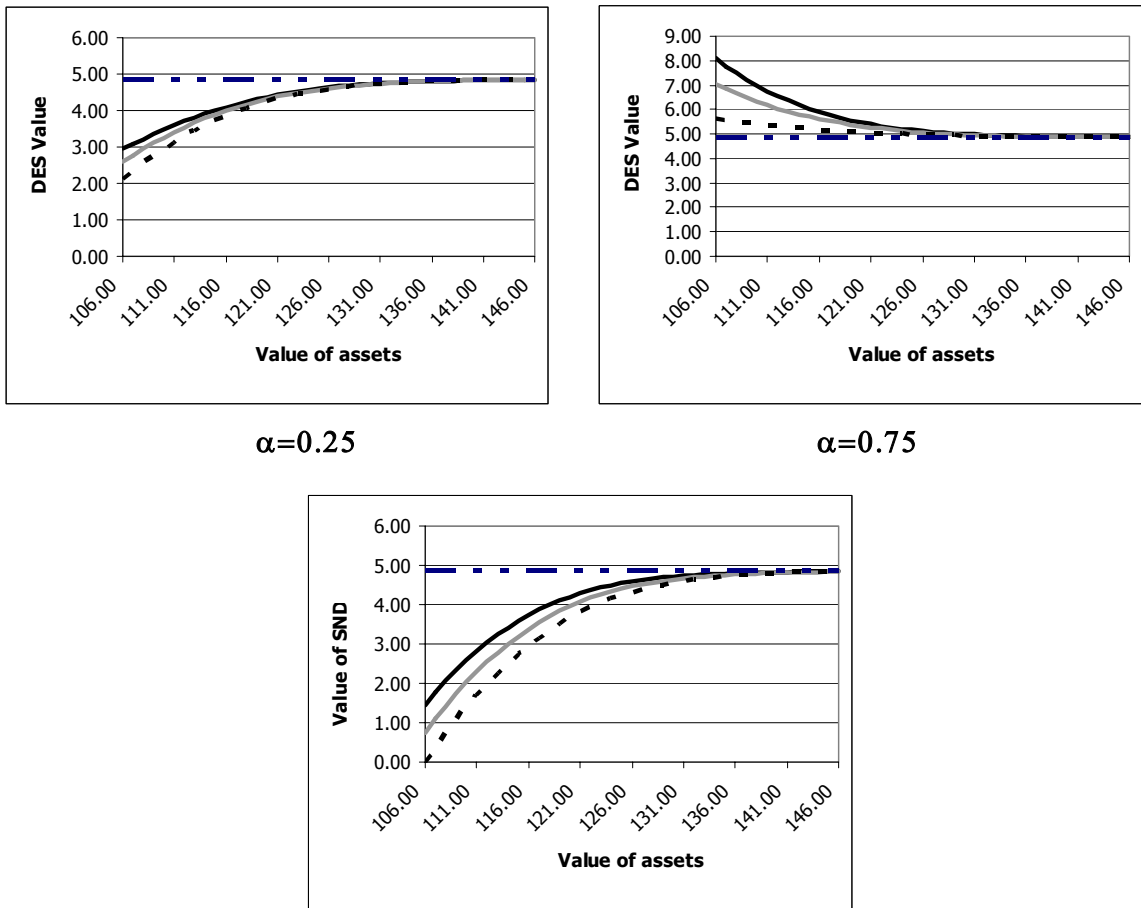
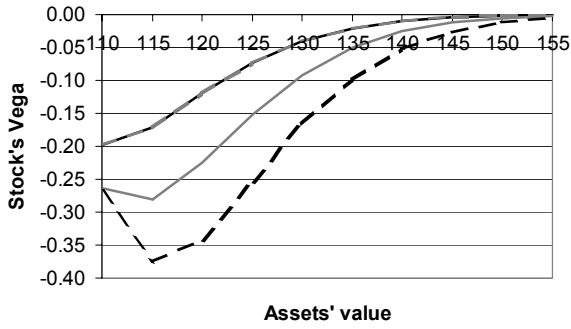


Figure 7

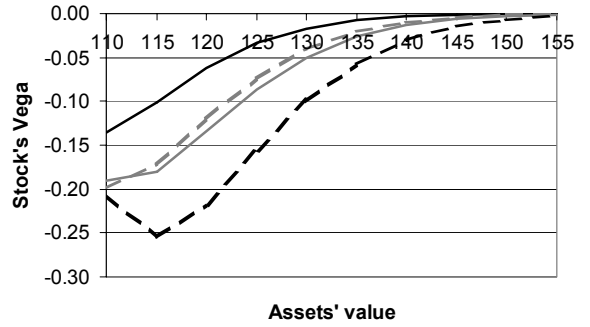
The values of SND and DES contracts versus the value of assets for different levels of liquidation and conversion thresholds.

Black line: $\lambda^s = \lambda^l = 1.02$. Gray line: $\lambda^s = \lambda^l = 1.04$. Dashed line: $\lambda^s = \lambda^l = 1.06$. Blue dashed line: risk-free price. All other parameters are identical to the base case (See Table III).

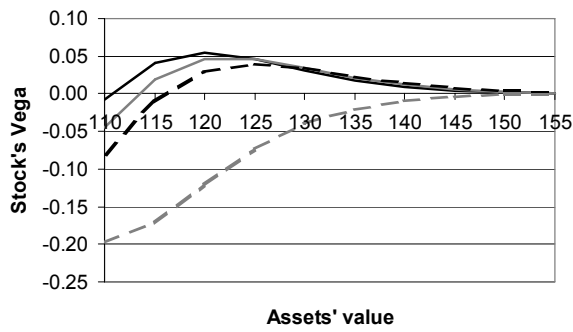
Stock's vega of vs. assets' value



(8.A) $\alpha = 1$



(8.B) $\alpha = 0.75$



(8.C) $\alpha = 0.25$

Figure 8

Stock's vega of a bank with DES contract in its capital structure versus assets' value for different conversion thresholds and conversions ratio (α). Black line: $\lambda^c = 1.02$. Gray line: $\lambda^c = 1.04$. Dashed black line: $\lambda^c = 1.06$, Dashed gray line: Stock's vega of a bank with SND with $\lambda^s = 1.02$. All other parameters are identical to the base case (See Table III).

Endnotes

¹ Market discipline has been explicitly recognized as one of the three pillars that allow banks and supervisors to evaluate properly the various risks that banks face. The additional two pillars are minimum capital requirements and regular review of a bank's risk management procedures (Basel Committee on Banking Supervision, 2001).

² For example, the U.S.A Federal Reserve task force (See Kwast et al. (2000)) has summarized 11 different forms for subordinated debt. Policymakers are actively considering requiring banks to issue SND. A mandatory SND requirement appears to be an important part of the market oriented reforms contained in the consultative paper issued by the Basel Committee on Banking Supervision (1999). The U.S Shadow Regulatory Committee (SFRC, 2000) has come out strongly in favor of mandatory SND as a mechanism for realizing enhanced market discipline of banks.

³ See Merton (1974 and 1977) and Black and Cox (1976).

⁴ As noted by Reisz and Perlich (2004), bank disclosure rules enforce liquidation when the capital of a bank falls below a threshold equal to 2% of assets.

⁵ To keep the notation as simple as possible, all variables without subscripts are present values.

⁶ In our model, liquidation may occur in or outside bankruptcy proceedings. We refer to liquidation and reorganization interchangeably.

⁷ As presented in Leland (1994) we focus on bankruptcy costs that are proportional to asset value when bankruptcy is declared. Alternatives such as constant bankruptcy costs, as suggested by Ericsson and Reneby (1998), or mixed costs, as suggested by Acharya et al. (1994), are explored readily within the framework developed. Deviations from absolute priority rule, in which bondholders do not receive all remaining value, as

showed by Franks and Torous (1989) and Eberhart, Moore and Roenfeldt (1990), can also be incorporated in the model.

⁸ In the literature, V is often referred to as the value of the levered firm and ω as the value of the unlevered firm.

⁹ Black and Cox (1976) decompose firm value into two additional components. The first is the value of the bond in the event that the value of the firms' assets has touched an upper boundary where reorganization is occurred. While we concentrate on the effects of bankruptcy on the bank's security, this upper threshold is not influential on the value of the bank's claims. The second is the value of intermediate payments in solvency. However, since we are dealing with zero coupon securities this component does not exist.

¹⁰ A summary of regulatory measures and regulatory objectives can be found in Allen and Herring (2001).

¹¹ A full coverage of loss by the insurer may generate a moral hazard problem, since the depositors' incentive to monitor bank risk is likely to become nil under full insurance. Insurance pricing based on individual risk is used in many insurance markets to reduce this form of moral hazard. However, our model can easily accommodate deviation from the full coverage assumption.

¹² According to Diamond and Dybvig (1983) deposit insurance also effect bank stability. Bank runs are self-fulfilling prophecies. If everyone believes that a banking panic is about to occur, it is optimal for each individual to try and withdraw her funds and since each bank has insufficient liquid assets to meet all of its commitments a systematic failing of banks would occur. However, in the presence of deposits insurance there are sufficient liquid assets to meet these genuine liquidity demands and there will be no panic.

¹³ Examples include Avery, Belton and Goldberg (1988) and Gorton and Santomero (1990). A review of this literature can be found in Flannery (1998) and Bliss (2000).

¹⁴ Other recent studies include De Young et al (2001), Morgan and Stiroh (2001) and Sironi (2002, 2003)

¹⁵ Vega is defined as the rate of change in the price of a contract with volatility.