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Dividend Taxation and DAX Futures Prices

- Working Paper -

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Abstract

Investors entering a DAX future contract intend to do this at the fair, arbitrage-free price. A pricing under perfect market assumptions ignoring dividend taxation can only give an approximation of this price. We examine the effect of dividend taxation on the future price of the total return index DAX. We analyse the historical tax regimes in Germany from 1990 until 2011, their implications for dividend taxation and thus DAX future prices. The regimes differ considerably in manner and magnitude of dividend taxation and hence allow to empirically derive a tax effect. We find a mispricing of the DAX future under the different tax regimes due to tax distortions. Previous literature has tried to find elaborated theoretical models incorporating tax effects on future prices but none of them has undertaken an extensive empirical evaluation of the tax effects over an extensive time period. Our analysis has implications for a correct DAX future valuation formula, the marginal investor in the futures market and its taxation as well as policy implications for the taxation of dividends.

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1 Introduction

In a frictionless world the prices of stock index futures are determined by the cost-of-carry relation and thus only depend on the current index level, the discount rate and the expected dividend yield. In the case of a total return index (which assumes that dividends are reinvested) the cost-of-carry relation simplifies even further because the expected dividend yield drops out of the pricing equation (e.g. Prigge and Schlag (1992), Bühler and Kempf (1995)). Consequently, in a frictionless world the level of a total return index should be equal to the discounted futures price. However, empirically this simple relation does not seem to hold. Several authors (e.g. Prigge and Schlag (1992), Loistl and Kobinger (1993), Bühler and Kempf (1995)) have documented that the prices of the DAX futures contract violate the simple cost of carry relation. They find that the futures contracts are systematically undervalued.

A potential explanation for the violation of the cost-of-carry relation is the taxation of dividends[1] in Germany. Investors are unable to perfectly replicate the DAX futures contract because the taxation of dividends is different in the spot market and the futures contract. The facts that (1) different investors may face different marginal tax rates and that (2) domestic investors are taxed differently than foreign investors (on this see McDonald (2001)) complicate the analysis. Several authors (Kempf and Spengel (1993),Röder and Bamberg (1994), Janssen and Rudolph (1995), Bamberg and Dorfleitner (2002), Weber (2004), Weber (2005) have tried to incorporate the tax treatment of dividends into the cost-of-carry relation. While some authors analyze whether there are arbitrage opportunities relative to a modified cost-of-carry model[2] (e.g. Röder and Bamberg (1994), Janssen and Rudolph (1995), Merz (1995)), no paper so far has provided reliable empirical evidence on the importance of dividend taxation for the pricing of DAX index futures.

Our paper closes this gap in the literature. We analyze the pricing of the DAX futures contracts in the period 1990-2011. The distinguishing feature of our empirical analysis is the fact that our sample period covers three different tax regimes. This allows us to analyze how changes in dividend taxation affect futures prices.

We find in our empirical analysis that there exists a mispricing between the fair futures price derived from the simple cost-of-carry formula and the empirically observed future contract. The mispricing is induced by dividend taxation and varies between the tax systems. Over the last 20 years, the daily mispricing and arbitrage opportunities in the DAX future contract gradually

[1] Cornell and French (1983a) document that US stock index futures were underpriced in the early 1980s. They offer an explanation which is based on the taxation of capital gains. While investors in the spot markets have the option to defer capital gains taxes, investors in the futures markets do not have this option. However, Cornell (1985) reconsidered the issue and concluded that "the timing option is not an important factor in picing stock index futures" (p. 89). We will therefore not pursue this issue.

[2] Several papers analyze the relation between spot and futures prices and try to infer the value of dividend tax shields or imputation tax credits, e.g. McDonald (2001), Cannavan et al. (2004) and Cummings and Frino (2008).

decreased. The empirical derivation of tax effects on future prices under the three tax regimes is not only of relevance for Germany but can give important general implications for an accurate valuation formula and policy implications for the design of a non-distorting tax systems. We find that with more generic taxation rules, future contracts are easier and more precise to price.

The paper is organized as follows. Section 2 describes the institutional background. In section 3 we derive our hypotheses. Section 4 describes our data set and presents descriptive statistics. Section 5 presents the methodology and results of our empirical analysis. Section 6 concludes.

2 Institutional Settings

The German DAX index future is one of the most liquid futures contracts in the world. In 2010, a total of 41 million contracts with a total capital volume of 3.5 trillion Euro were traded with a daily average of 160,000 contracts.

The underlying of the future contract, the German DAX index was introduced in 1987 and is in contrast to its international counterparts, e.g. S&P 500 or EuroStoxx 50, a total return (performance) index. Dividends, which German companies usually pay once a year, are re-invested into the dividend-paying stock. Once a year, the DAX index is adjusted and the re-invested dividends are re-distributed to all companies in the index proportionally to their market capitalization. This is done to prevent a bias in the index towards dividend-paying stocks. Theoretically the DAX assumes a re-investment of the gross-dividend (*Bruttobardividende*)[3] of every dividend-paying stock in the index. The re-investment at the gross-dividend is a critical assumption and has particular implications in asymmetric tax system as the German one.

In order to find a fair and arbitrage-free *future price* of the DAX index, several assumptions need to be made. These are the perfect market assumptions of the the cost-of-carry model first derived by Cornell and French (1983a,b)[4] Considering all these assumptions, one of the first derivations of the valuation of a future on an index was undertaken by Cornell and French (1983a,b) who find an arbitrage-free future price for a *price index*, e.g. like the S&P 500 as follows

$$F(t,T) = I_t e^{r(T-t)} - \sum_{i=1}^{N} D_i e^{r(T-t_i)} \tag{1}$$

Until delivery of the future the dividend payments are aggregated and deducted from the index

[3] Depending on the tax regime, it is roughly the gain after corporate tax.

[4] First, frictionless capital markets are assumed. There are no transaction costs, no short-sale restrictions, assets are perfectly divisible, there is an instantaneous order execution, non-stochastic interest rates, equal lending and borrowing interest rates. Furthermore, margin requirements are ignored. Second, dividend payments are assumed to be certain and known to the investor. Third, the re-investment risk, the risk that the dividends cannot invested at the ex-dividend stock price, is assumed away. During the lifetime of the future contract, the index composition does not change, e.g. due to capital expenditures or company exclusions from the index, and the index can be replicated by the investor. Lastly, in the most basic cost-of-carry model no taxes are assumed.

as they are not re-invested. Dividends are taxed at the investors marginal tax rate and will not create any tax distortions.

As for the *performance index* DAX, Bühler and Kempf (1995) find an adapted valuation formula. In this case, the index value should be unchanged by dividend payments as they are re-invested. Therefore, Bühler and Kempf (1995) come up with the following relation.

$$F(t,T) = DAX_t e^{r(T-t)} \tag{2}$$

In their argumentation, to derive the cost-of-carry price, the arbitrageur has to follow the index re-investment strategy and re-invest into dividend paying stocks and follow the yearly index rebalancing, to avoid unbalanced arbitrage positions. As a consequence, the no-arbitrage relation between the DAX index and future does not depend on dividend payments. What is more, they argue that taxes have no impact on the fair value of the future. It is assumed that the investor directly receives the gross-dividend (*Bruttobardividende*) on the dividend-ex day and is able to invest it at the reduced price ($Price_{ex-dividend}$). For all dividend payments to investor j the same effective tax rate s applies. Furthermore, Bühler and Kempf (1995) claim that the marginal investor, an institutional investor, has exactly the same tax rate as the one that is implicitly applied on dividends when the DAX is computed, and therefore no tax distortions will arise.

In contrast to this view, Kempf and Spengel (1993) were among the first to argue that investors' marginal tax rates have an influence on the fair futures price. They tax the gross-dividend at the marginal tax rate of the investor and therefore add the correct dividend payment to the index. [5]

Kempf and Spengel (1993) and Janssen and Rudolph (1995) as well as Bamberg and Dorfleitner (2002); Weber (2005) undertake theoretical price considerations and refinements of the future price valuation formula in different tax systems. However, the focus of this project is not to find the correct theoretical formula in the first place but to validate the DAX mispricing due to taxes in the data over and extended time period and different tax systems.
In a first step, we revisit and derive on the following pages the theoretical effects of taxes on futures prices. In a second step, we derive the particularities of the three tax systems and simulate the influence of taxes on futures prices in a sensitivity analysis. From these results and insights hypotheses are derived.

Derivation of the DAX cost-and-carry price with regard to taxes

In this section, we want to derive a fair futures price, valid in all tax regimes and for the main market participants. For this general approach, we define the following variables that we want to parametrize in the upcoming section. We define r as the interest expenditure of the time period

[5] Further refinements of the theoretical valuation formula, by relaxing perfect market assumptions can be found in Janssen and Rudolph (1995). They look at additional risks and costs to arbitrage. Janssen and Rudolph (1995) model transaction costs (short-sale costs, fees, liquidity costs) and interest rate taxation. Furthermore, they take into account capital gains taxation which could reduce profits from arbitrage and therefore the band in which arbitrage is possible. The focus of this project lies on taxes and therefore transaction costs are not considered.

$(T-t)$ and D as the dividend payments in the same time-period. We define s_k as tax on all capital gains and losses from stock and future transactions. The interest income and expenses are taxed with rate s_z and dividend payments are taxed with rate s_d. The following derivation of the fair futures price is based on Weber (2004) and Bamberg and Dorfleitner (2002) who show similar arbitrage tables and notations in their derivations.

In the presented strategy, we assume an arbitrageur who is selling the future and is long the stock index. The same no-arbitrage scenario is possible for an investor that is long in the future and short the underlying. This scenario gives the same no-arbitrage table, except that all positions are in the opposite direction.[6] A long position in the future involves a short position in the underlying which involves short-selling transaction costs. In our analysis we want to abstract from transaction costs and only analyze the influence of taxes on future prices.

[Insert Table 1 about here]

As can be seen from table 1[7] the arbitrageur initially invests into a portfolio of stocks S_0 that replicates the DAX index. This investment is financed with a credit in period $t = 0$. In the same period, the arbitrageur sells the equivalent amount of DAX Future contracts F short. In period zero the cash flows are exactly zero. Up to maturity the underlying index stocks pay dividends D. The dividends cause differing tax payments on the index level and on the personal portfolio level. The arbitrageur has to finance the tax differences with supplementary credits up to maturity. At maturity, the investor has to pay capital gains tax of $(1 - s_k)$ on the gains and losses of his DAX portfolio. Furthermore, he has to pay taxes at rate s_k on the profit of his future contract $F - S_T$. He repays the credit on the DAX portfolio and supplementary credit on dividend adjustment costs and subtracts from these payments his interest tax credit (the credit is at taxrate s_z). Eventually, he has to pay taxes on the reinvested dividends, at rate s_d, that he earns at maturity.

The cash-flows up to maturity add up to zero. To get the fair futures price, we add up the cash flows at maturity and solve for F. This results in a general futures price formula that corrects for taxes as in Bamberg and Dorfleitner (2002).

$$F = S_0 \left(1 + r\frac{1 - s_z}{1 - s_k}\right) - \left[s_k - s_d - r(T - \tau_j)s_d(1 - s_z)\right] \frac{1}{1 - s_k} D \tag{3}$$

The first part of the equation equals the simple cost-of-carry formula 2.

Tax System and the Marginal Investor

The German tax system is characterized by three reforms in the last twenty years. The first and oldest tax system is the so called **Vollanrechnungsverfahren (VOLL)**, an imputation

[6]This also holds for tax losses and tax gains. We assume a symmetric taxation of gains and losses due to simplification, as in Weber (2004).

[7]Bamberg and Dorfleitner (2002) uses the same no-arbitrage table to derive the futures price for the Halbeinkuenfteverfahren.

system that was in place until 2000. The main principle of the imputation system is to tax every individual's capital income at his personal marginal tax rate. In this system the taxes paid on a corporate level are credited to the investor in form of a tax credit. On the final tax assessment day, the investor will, depending on his personal income tax rate, either get a tax refund or has to pay additional taxes. In 2001, the half-income system **Halbeinkuenfteverfahren (HEV)** was introduced. In this system the imputation tax credits are abolished and dividends are taxed on a corporate and personal level. However, personal taxes are only levied on one half of the gross-dividend. In 2009, the half-income system was succeeded by the flat withholding tax system **Abgeltungssteuer (ABG)** is a flat tax of 25%, irrespective of the investor's tax bracket.[8]

In table 2, we present the tax rates of five exemplary marginal investors in the respective tax regimes. Among them, we show the cases of a private individual investor with low and high personal tax rates case (1) and (2), a joint stock company in case (3) and a financial institution in case (4). For completion, we present the taxation of a foreign investor in case (5).[9]

[Insert Table 2 about here]

In order to define the influence of taxes on the fair futures price, we need to know more about the marginal investor that drives the market. To determine the marginal investor that drives the futures price we follow the argumentation of Weber (2005). We agree that the arbitrage positions are mainly taken by institutional investors[10] taxed according to German tax laws, as shown in cases (3) and (4) in table 2.

These companies either hold long- term investments in the company or are trading stock and derivatives on their own or a client's account. The positions are kept in a trading book and taxed with the financial institutions corporate tax rate.[11]. Compared to institutional investors, private investors face some obstacles in trading futures contracts. Private investors have to fulfill the margin requirements of the exchanges and face considerable difficulties in the short-selling of stocks. The alternative of private investors to buy or short-sell certificates or ETFs, as proposed by Bamberg and Dorfleitner (2002) instead of the underlying index to hedge a position is a valid option. The advent of ETFs and certificates makes markets more complete and efficient, the no-arbitrage band smaller and provides more arbitrage signals and opportunities for private investors. However, even with these products private investors still face higher transaction costs as institutional investors, as they engage in ETF contracts provided by institutional investors who price-in their own transaction costs. Therefore, it is not unrealistic to assume that these institutional investors, in the form of financial institutions, drive the fair future value with their marginal tax rate. In the following analysis we take the financial institution as the relevant

[8]It can be lower than 25% if the investor has a lower personal tax rate.

[9]McDonald (2001) finds in the German tax imputation system arbitrage opportunities for foreign investors.

[10]Such as banks, financial institutions, pension funds, live insurances

[11]Financial institutions and corporations are classified according to §1 I, Ia and III Kreditwesengesetz (KWG)

marginal investor and base our main analysis on its tax rates and focus on cases (3) and (4) in table 2.

In the **Vollanrechnungsverfahren (VOLL)** tax regime, the taxation is best described by Röder and Bamberg (1994) and Kempf and Spengel (1993). In the case of a financial institution, the tax rate s is the same on interest, capital gains, and dividends ($s = s_z = s_k = s_d$). This tax rate equals to the coporate tax rate I and II (*Körperschaftssteuer + Gewerbesteuer*). In this scenario we assume a corporate tax rate $s = 0.583$ and therefore higher than the DAX assumed tax rate KST_{pout} of 0.3 (0.36 in later periods of the system). As we will see in the following sensitivity analysis, this tax rate differential causes a decrease in the theoretical fair futures price. In the case of a foreign institutional investor (case (5)), the described tax rate differential is only the lower bound as foreign investors not necessarily get the tax credit reimbursed as German investors get (see McDonald (2001)).

In the **Halbeinkuenfteverfahren (HEV)** tax regime, the taxation is best described in Weber (2004, 2005). In this tax regime, we have to differentiate between two cases for the institutional investors. In the first case (3), the institutional investor is considered a joint stock company holding its stock and future positions as long-term investment and not in a *tradingbook*.[12] In this case only 5 % of the dividends have to be taxed at the corporate tax (*dividend-privilege*).

In the second case (4), the institutional investor (financial institution, bank, live insurance, pension fund) holds its future and index positions as short-term positions. We consider case (4) the far more realistic scenario for institutional investors (see Weber (2004), pp. 134-137.). Therefore, companies according to §1I and Ia KWG that have to hold their short-term positions in the trading book, as well as financial institutions according to §1 I, Ia and III KWG are taxed according to §8b VII KStG. For all these investors the tax rate equals the corporate tax rate (again coporate tax rate I and II) $s = s_z = s_k = s_d$ as derived in table 2.

In the **Abgeltungssteuer (ABG)** tax regime the taxation of institutional investors is best described in Scheffler (2012). As in the Halbeinkuenfteverfahren, there are two relevant cases for the taxation of dividends. In the first scenario (3), institutional investors hold their position in the index-underlying stock as long-term investment. According to §8b Abs 1 KStG dividends and capital gains are not taxed on the investor's side. As in the Halbeinkuenfteverfahren, according to §8b Abs 5 KStG only 5% of dividends and capital gains are taxed at the corporate tax rate. In the more realistic scenario (4), we also face the same rules for institutional investors (§8b Abs 7,8 KStG) as under the Halbeinkuenfteverfahren. Here the institutional investors and their investments are classified according to §1a KWG. The investments are kept for short-term profit reasons and therefore the dividends and capital gains are taxed at the corporate tax rate. Therefore, we set the tax rate equal to $s = s_z = s_k = s_d$.

[12] One could also think of the positions as current assets and non-current assets instead of trading book position and long-term investment.

Sensitivity Analysis

In the following analysis we provide examples of a fair and tax-corrected futures price under the different tax regimes. To get these estimates, we use historical median values over the time period of the tax systems. For all the scenarios of the sensitivity analysis, we again assume that the marginal investor is a financial institution that holds its future and index positions for short-term profit reasons or as long-term investments. Therefore, we use in all following analyses the parameters and taxation rules of cases (3) and (4) in the previous analysis.

Vollanrechnungsverfahren

To calculate the futures price we use formula 3, a dividend yield of 1.78% and an interest rate of 3.96% p.a. (cont. compounded) and an index value of 2625 points. Furthermore, for simplifications we assume that the dividends are all paid halfway to maturity at $t = \frac{1}{2}$. Furthermore, we assume a time to maturity of $T = 1$ year:

$$\begin{aligned} F^* &= S_0\left[1 + r(\tfrac{1-s_z}{1-s_k})\right] - \left[s_k - s_d - r(T-\tau_j)s_d(1-s_z)\right]\frac{1}{1-s_k}D \\ &= 2625\,[1.0404] - [0.583 - 0.3484 - 0.0404 * 0.5 * 0.3484 * 0.417] * 2625 * 0.0178 * 2.4 \\ &= 2731 - (0.2346 - 0.00293) * 2625 * 0.0178 * 2.4 = 2731 - 25.98 = 2705.8 \end{aligned}$$

The fair futures price F* is about 26 index points (or 0.95 %) smaller than the theoretical futures price F from the simple cost-of-carry formula 2.

Halbeinkuenfteverfahren

To calculate the futures price we use formula 3 a dividend yield of 2.33% and an interest rate of 3.34% (cont. compounded) and an index value of 5030 points. Again, dividends are all paid halfway to maturity at $t = \frac{1}{2}$ and the maturity is $T = 1$. In Halbeinkuenfteverfahren, we have to differentiate between case (3) and (4).

$$\begin{aligned} F^* &= S_0\left[1 + r(\tfrac{1-s_z}{1-s_k})\right] - \left[s_k - s_d - r(T-\tau_j)s_d(1-s_z)\right]\frac{1}{1-s_k}D \\ &= S_0\,[1+r] - [-0.035 * (0.5)0.375(0.625)] * 5030 * 0.0233 * 1.6 \\ &= S_0\,[1.035] + 0.0041 * 5030 * 0.0233 * 1.6 \\ &= 5201 + 0.77 = 5202 \end{aligned}$$

The fair futures price F* is about 0.77 index points bigger than the theoretical futures price F from the simple cost-of-carry formula 2, if the investor is a financial institution and keeps its positions in the trading book.

If we assume that the financial firm holds long-term positions as in case (3), it will enjoy the dividend-privilege.

$$\begin{aligned} F^* &= S_0\left[1 + r(\tfrac{1-s_z}{1-s_k})\right] - \left[s_k - s_d - r(T-\tau_j)s_d(1-s_z)\right]\frac{1}{1-s_k}D \\ &= S_0\,[1+r] - [(0.375 - 0.05 * 0.375) - 0.035 * (0.5)0.375(0.625)] * 5030 * 0.0233 * 1.6 \\ &= S_0\,[1.035] - (0.35625 - 0.0041) * 5030 * 0.0233 * 1.6 \\ &= 5201 - 66.03 = 5134 \end{aligned}$$

The fair futures price F* is about 66 index points smaller than the theoretical futures price F from the simple cost-of-carry formula 2.

Abgeltungssteuer

To calculate the futures price we use the same specifications as above and a dividend yield of 3.4%, an interest rate of 1.31% (cont. compounded) and an index value of 5957 points. For case (4), we get the following price

$$F^* = S_0\left[1 + r(\tfrac{1-s_z}{1-s_k})\right] - \left[s_k - s_d - r(T-\tau_j)s_d(1-s_z)\right]\frac{1}{1-s_k}D$$
$$= S_0\left[1 + r\right] - \left[-0.0132(0.5)0.2544(0.7456)\right] * 5957 * 0.034 * 1/0.7456$$
$$= S_0\left[1.0132\right] + 0.0013 * 5957 * 0.034 * 1/0.7456$$
$$= 5957 + 0.35 = 5954$$

The fair futures price F* is about 0.35 index points bigger than the theoretical futures price F from the simple cost-of-carry formula 2.

If we consider case (3) and the dividend-privilege we find the following future price.

$$F^* = S_0\left[1 + r(\tfrac{1-s_z}{1-s_k})\right] - \left[s_k - s_d - r(T-\tau_j)s_d(1-s_z)\right]\frac{1}{1-s_k}D$$
$$= S_0\left[1 + r\right] - \left[(0.2544 - 0.05 * 0.2544) - 0.013 * (0.5)0.2544(0.7456)\right] * 5957 * 0.034 * 1/0.7456$$
$$= S_0\left[1.035\right] - (0.2417 - 0.0012) * 5957 * 0.034 * 1.34/$$
$$= 5201 - 65.27 = 5135.73$$

The fair futures price F* is about 65.27 index points bigger than the theoretical futures price F from the simple cost-of-carry formula 2.

For the Vollanrechnungsverfahren, a re-investment at the gross-dividend (Bruttobardividende) is only possible if the dividend has been taxed on the corporate level at the same rate as the investors marginal tax rate. In our case, the marginal investor, the financial institution, would need a taxrate of 30% / 36%. All marginal tax rates different to the corporate tax rate create systematic price distortions. As shown, these price distortions lead to an undervaluation of the future compared to formula 2. For the Halbeinkuenfteverfahren, the future price is either almost equal or less compared to the simple cost-of-carry formula 2. The size of the difference is smaller if we do not take into account the coporate tax II (Gewerbesteuer). The marginal investor, here the financial institution, additionally has to tax the dividends and capital gains at the corporate tax rate if we assume case (4). If we assume the dividend-privilege as in case (3), the potential future price is closer to the simple cost-of-carry formula 2. The same holds in the Abgeltungssteuer tax regime. It needs to be empirically tested if the marginal investor applies case (3) or case (4) in the taxation of dividends.

If we analyze the derivative of the futures price with respect to the dividend payment time τ_j, as shown in formula 4, we find that dividend payments closer to maturity have a negative influence on the futures price in all three tax regimes. However this effect is likely small, as it depends on the size of the interest rate.

$$\frac{\partial F}{\partial \tau_j} = -\left[s_d r \frac{1 - s_z}{1 - s_k} D \right] \tag{4}$$

3 Hypothesis Development

In all three tax regimes there seem to be systematic differences in the price of the DAX future and its theoretical value as in formula 2 due to tax distortions. Therefore, it needs to be analyzed empirically if these over- and undervaluation and a corresponding mispricing in the data set of DAX future contracts.

As discussed above, assuming that the marginal investor is a financial institution, the tax rate on dividends and capital gains systematically deviates from the tax rate that is implicitly assumed by the index. Therefore the following hypothesis 1 can be derived:

Hypothesis 1: The empirically observed DAX future price will be lower during dividend paying months in the Vollanrechnungsverfahren and equal or lower in the Halbeinkuenfteverfahren and Abgeltungssteuer tax regimes than the price derived from the simple cost-carry formula (2) without dividend adjustments.

German companies pay dividends usually once a year during summer months. Therefore, the June contract takes a special positioning compared to the other three contracts of the year. As empirically shown by Bühler and Kempf (1995) and Röder and Bamberg (1994) there are more arbitrage opportunities in June contracts than in other contracts. Investors try to capture as many dividends as possible in that month. The following hypotheses about the June contract phenomenon, as in Bamberg and Dorfleitner (2002), can be derived:

Hypothesis 2: The deviation of the DAX future from its theoretical value as in formula 2 is greater for the June contract than for the three other contracts. The cumulation of dividend payments usually seen between April and June leads results in a lower futures price in the Vollanrechungsverfahren and lower or equal futures prices in the Halbeinkuenfteverfahren and Abgeltungssteuer.

As we saw in the previous analysis, the dividend yield has an influence on the fair futures price and drives it away from its theoretical value as in formula 2. Therefore it can be hypothesized that:

Hypothesis 3: Higher and more frequent dividend payments will lead to a greater deviation of the DAX future from its theoretical value in formula 2 under all three regimes. Furthermore, higher dividend yields will lead to a more depressed futures price in the Vollanrechnungsverfahren and equal or lower futures prices in the Halbeinkuenfteverfahren and Abgeltungssteuer regime.

As we saw from the first derivatives of the futures price, dividend payments closer to maturity, i.e. with higher τ_t, reduce the fair futures price. Furthermore, as found by Bailey (1989), Cakici and Chatterjee (1991) and MacKinlay and Ramaswamy (1988), a higher time to maturity increases mispricing. Therefore, we can hypothesize:

Hypothesis 4: The closer the dividend payment to maturity, the more depressed futures price in the Vollanrechnungsverfahren and equal or lower futures prices in the Halbeinkuenfteverfahren and Abgeltungssteuer regime and the higher the mispricing. The effect is supposed to be small as it depends on the size of the interest rate. Also, the higher the time to maturity, the higher the mispricing.

4 Data and Summary Statistics

The dataset contains daily DAX futures price data, DAX index data, and dividend data of the DAX index constituents from Datastream over the time period from 1992 until 2011. The DAX future price data is on a per contract basis (for DAX futures with maturities March, June, September, and December). Every contract is split in a next-to-deliver 3-month trading period and a 6-month trading period before it becomes the next-to-deliver contract.[13] For the calculation of the mispricing per contract, the mispricing is once calculated only for next-to-deliver contracts and in a robustness check for contracts taking into account the whole trading period. The DAX dividend data consists of the annualized DAX dividend yield retrieved from Datastream. The Datastream dividend yield is based on historical dividend payments of the DAX companies. Furthermore, we construct monthly constituent lists of the DAX index and track the dividend payment reason, dividend payment dates, ex-dividend dates, as well as the dividend amount of the index constituents. From this we can derive a distribution of dividend

[13]The trading volume of inactive, non next-to-deliver contracts is very low and on average only about 3% of the active, next-to-deliver contract. The open interest of inactive contracts is only about 7% of the open interest of active contracts. An analysis of the trading volume and open interest over the whole time period and per contract can be found in figure 7, figure 8 and figure 9 in the appendix. .

payment frequency over the year. Additionally, we extrapolate the dividend payments from the difference of the DAX performance index and DAX price index. The risk-free interest rate data consists of the daily Frankfurt money market and interbank interest rates as well as the daily German interest swap rates for three month, six month and twelve month maturity. In general the rates will be interpolated linearly to match the maturity of the future contract. In a robustness check we also calculate the futures price by simply taking the 12 month maturity Frankfurt banks middle rate. In the early years of the DAX future, the DAX index is traded on Parkett whereas the future is traded on the Deutsche Termin Börse (DTB) / Eurex exchange. To exclude possible time shifts between the daily settlement prices of the DAX future on the Eurex / DTB and the closing price of the DAX index on the Parkett, we let the sample period start in mid 1992.

Within the time period of analysis from 1992 until 2011, the average dividend yield of the DAX companies within this around 2.34 per year. The yearly fluctuations of the dividend yield can be seen in table 3.

[**Insert Table 3 about here**]

There is a clear clustering of dividend payments in the second quarter of the year. This becomes evident in figure 1, the distribution of the number of annual dividend payments.

[**Insert Figure 1 about here**]

An overview of the average trading volume and open interest per future contract over the whole time period is given in table 4. The trading volume seems to be the same in all the contracts whereas the open interest is highest in the June contract. A more detailed trading volume and open interest analysis can be found in the appendix.

[**Insert Table 4 about here**]

5 Empirical Analysis

In the following empirical analysis, we want to first analyse the mispricing of the empirically observed DAX future price relative to the theoretical cost-of-carry as determined by the simple cost-of-carry formula 2. In a next step, we calculate the corrected mispricing relative to the dividend-adjusted tax formula as in equation 3. In a time-series analysis, the mispricing is explained by several factors derived in the hypothesis section 3. In a last step, several robustness checks are undertaken.

5.1 Mispricing per Contract

The daily mispricing relative to the simple cost-of-carry formula 2 is calculated as follows.[14]

$$\frac{DAX_t e^{r(T-t)} - FDAX_t^{empirical}}{FDAX_t^{empirical}} \tag{5}$$

We want to analyze the mispricing of the future contract on different observational levels. In a first step, we analyze the *mispricing per future contract.* Therefore, we aggregate the daily mispricing as *cumulative mispricing* per contract. In this analysis, the mispricing is only calculated for the 3 months trading period of the next-to-deliver future contract. The results for the enlarged period are analyzed in the robustness section.

Table 5 gives an overview of the *cumulative mispricing* per future contract. In this table one can clearly see that the cumulative mispricing in the Vollanrechnungsverfahren and Halbeinkuenfteverfahren is cleary biggest for the June contract. In the Vollanrechungsverfahren, the cumulative mispricing for all contracts, except for the June contract, is even negative. For the latest tax regime, the Abgeltungssteuer, this cannot be found. A more detailed analysis of the cumulative mispricing per contract is provided in figure 2. This figure gives a detailed picture of the cumulative mispricing per contract and supports the evidence found in table 5.

[**Insert Table 5 and Figure 2 about here**]

This first analysis partly supports hypothesis 1, as we find that the *cumulative mispricing* is highest for the June contract of the Vollanrechnungsverfahren, indicating that the empirical future price is as low as we postulated. The higher mispricing in the Abgeltungssteuer as compared to the Halbeinkuenfteverfahren remains unclear at first sight.

The clearest and most significant mispricing pattern around the June contract can be found in the Vollanrechnungsverfahren. From 2000 onwards the mispricing becomes smaller and less significant. In the Abgeltungssteuer regime the mispricing reaches a constant positive level.[15] Figure 3 illustrates the statistical significance of the mean mispricing per future contract.

[**Insert Figure 3 about here**]

About 41 % of all contracts have a mispricing different from zero, measured at a 1% significance level. A clear significance pattern for the positive mispricing in the Vollanrechungsverfahren can be observed.

Overall, in this first analysis of the mispricing, we find first evidence that the postulated hypotheses 1 and 2 are valid for the Vollanrechnungsverfahren. The mispricing seems positive,

[14]The mispricing calculation is analogous to MacKinlay and Ramaswamy (1988); Bailey (1989); Cakici and Chatterjee (1991); Bühler and Kempf (1995); Roll et al. (2007)

[15]The entirely positive mispricing in the Abgeltungssteuer tax regime might stem from different reasons. One possible explanation could be a constant negative price pressure of the future contract relative to the index due to negative expectations during the financial crisis.

significant and most pronounced in the Vollanrechnungsverfahren as postulated in hypothesis 1 and during dividend paying months, as postulated in hypothesis 2. In the Halbeinkuenfteverfahren as well as the Abgeltungsteuer tax regime the mispricing is constantly positive and not pronounced in the June contract.

5.2 Tax-corrected Mispricing

We re-calculate the mispricing with a tax-adjusted cost-of-carry formula as developed in equation 3. The *Tax Correction Factor* depends on the tax regime. It is derived from the institutional settings as explained in section 8.2 and based on the tax rates of the marginal investor in table 2. The detailed explanation of the correction factor is given in section 8.3 in the appendix. A crucial assumption for the tax correction factor is the assumed dividend tax rate of the marginal investor in the tax system. In the following calculations a marginal tax rate of the representative investor, a financial institution, is assumed as discussed and presented in table 2.

As can be seen in figure 4, the cumulative mispricing per contract is considerably reduced into the negative in the Vollanrechungsverfahren.

[**Insert Figure 4 about here**]

In the Halbeinkuenfteverfahren and Abgeltungssteuer system we have to differentiate two cases as described in section 2. In the first case (case (3) in section 2) , the financial institution has the dividend-privilege. In this scenario the futures price is depressed more heavily and therefore the mispricing becomes negative, as can be seen in figure 4. In case the financial institution keeps its positions in a trading book (case (4) in section 2) the tax-corrected future price is closer the empirical future price and almost no tax-related price corrections are necessary. Therefore, the mispricing is in the same magnitude as in the uncorrected case in figure 2.

[**Insert Figure 5 about here**]

Overall, we see that the tax corrections have the biggest influence in the Vollanrechnungsverfahren. In the Halbeinkuenfteverfahren and Abgeltungssteuer the tax-corrected future price depends on the marginal investors tax case and faces more corrections in the dividend privilege scenario.

5.3 Time-Series Analysis

After having examined the mispricing per contract, we want to analyze the evolvement of the mispricing in a more detailed time-series analysis. For this analysis we regress the daily mispricing on several explanatory factors to explain the derived hypotheses. In a first step, we test hypothesis 1 and 2 and have a more detailed look at how the different tax regimes influence the mispricing and in particular how the mispricing evolves during dividend paying months. In a second step, we analyze the June contract phenomenon and the influence of dividends on the mispricing as postulated in hypotheses 2 and 3. In a last analysis we test the influence of several explanatory factors as derived in the hypotheses as well as control variables on the mispricing in a time-series analysis

In table 6, panel (1) - (3), we test the influence of the tax system and June contract dummy on the mispricing. In panel (1), the daily mispricing with 5022 observations over the time period 1992 until 2011 of the next-to-deliver future contracts is regressed on a Vollanrechungsvefahren tax system dummy, June contract dummy and an interaction term. The Vollanrechungsverfahren dummy drives the mispricing to a negative daily average, indicating an overvaluation of the future contract with respect to formula 2. The June contract dummy has a positive but insignificant influence on the mispricing. The most interesting result provides the interaction term of tax system and June contract. It has a positive economically as well as statistically significant influence of 0.25% on the daily mispricing. This more than offsets the negative marginal effect of the Vollanrechnungsverfahren dummy alone. This result confirms hypothesis 1, namely that during dividend paying months the mispricing is positive in Vollanrechungsverfahren, indicating an undervaluation of the empirically observed futures price relative to formula 2 as was hypothesized. In panel (2), we find a positive and significant influence of the Halbeinkuenfteverfahren dummy and June contract dummy on the daily mispricing. The most interesting result in this panel is again the interaction term. The level of mispricing in the Halbeineinkuenfteverfahren during dividend paying months is reduced. The general June effect and June effect in the Halbeinkuenfteverfahren seem to cancel out. This supports the hypothesis 1 that the future contract is equal to or slightly undervalued relative to the simple cost of carry formula 2. The results in panel (3) for the Abgeltungssteuer are also in line with hypothesis 1 and indicate a slight undervalution of the future price as in the Halbeinkuenfteverfahren. Overall, we can confirm hypothesis 1, namely that during dividend paying months the DAX future is undervalued in the Vollanrechungsvefahren and almost equal in the Halbeinkuenfteverfahren and Abgeltungssteuer tax system relative to the simple cost of carry formula 2.

[Insert Table 6 about here]

In table 7 we analyze the June contract phenomenon and influence of dividends in more detail. Panel (1) - (3) provide regressions of the daily mispricing on a June contract dummy and the dividend yield in the Vollanrechnungsverfahren, Halbeinkuenfteverfahren and Abgeltungssteuer tax regime respectively. In Panel (1), we see that the marginal effect of dividend payment days has a significant positive influence on the mispricing. More dividend payment days increase the mispricing. This supports hypothesis 3. By looking at the the dividend yield, we find a marginal positive but insignificant influence of the dividend yield in the Vollanrechnungsverfahren. This evidence supports hypothesis 3. In Panel (2) and (3), we find a negative marginal effect of dividend payment days, which is not in line with the postulated hypothesis and a positive but insignificant influence of the dividend yield as expected.

[Insert Table 7 about here]

In a last step we want to verify hypotheses 1-3 and test hypothesis 4 in a detailed time series regression with more explanatory factors and control variables. We examine the mispricing in a complete sample over all tax regimes and in a subsample analysis. As in the previous analysis, the main explanatory factors are the tax regime dummies, the dividend yield and the June contract dummy. Additionally we have the time to maturity as explanatory factors. Several researchers, such as Bailey (1989); Cakici and Chatterjee (1991); MacKinlay and Ramaswamy (1988) have found that mispricing depends on the time to maturity of the futures contract. To verify hypothesis 4, we test the time to maturity as explanatory factor as well. As control variables we include the cost of carry, future contract dummies and various proxies to test the marginal influence of the dividend, such as the dividend payment days. Due to collinearity reasons, we cannot test all the different proxies in one regression specification. We set up additional specifications which are described in the robustness section 5.4. In the first regression specification in table 8, we analyze the full sample.

[Insert Table 8 about here]

In Panel (1) of table 8 we regress the daily mispricing on the tax system dummies, the June contract dummy as well as the interaction terms. Furthermore, we try to explain the mispricing with dividend payment days and the days to maturity of the next-to-deliver contract. The coefficients of the June contract and tax system dummies confirm the results found in table 6 and therefore hypothesis 1. The interaction term of the Vollanrechungsverfahren system and the June contract dummy has a positive economically as well as statistically significant influence of about 0.29% on the daily mispricing. The interaction term of the Halbeinkuenfteverfahren is also significant but has a weaker influence. The interaction term of the Abgeltungssteuer system is insignificant. The days to maturity of the next-to-deliver contract have an economically small but statistically significant positive influence on the daily mispricing. This means that the further the contract is away from its maturity the higher is its deviation from the theoretical value in formula 2. This result is in line with hypothesis 4. However, interaction term between dividend yield and time to maturity is positive but insignificant. The dividend payment days have a negative significant influence on the mispricing. In addition the dividend yield has a positive marginal effect on the mispricing as found in table 7.

In Panel (2) of the same table we analyze the mispricing on a contract basis and do not differentiate the daily mispricing on a tax system basis. We find that the only contract with a positive and significant influence on the mispricing throughout all the tax systems is the June contract. In this regression specification, we control for the marginal effect of the dividend yield and days to maturity. The time to maturity has a significant and positive influence whereas the dividend yield has an insignificant negative influence. Furthermore, the dividend payment days have a negative influence. The results found in panel (2) reinforce hypotheses 1 and 2.

After analyzing the full sample, we take a closer look at the three tax regime subsamples. Results for the Vollanrechnungsverfahren subsample can be found in table 9, panel (1)-(2).

[Insert Table 9 about here]

In Panel (1) of table 9 we again test for the influence of the dividend yield on the daily mispricing. In this regression, we can confirm the previous results. The June contract dummy has a positive significant influence. However, the interaction effect of the dividend yield during dividend paying months is negative but insignificant. Furthermore, we find that the interaction effect of the days to maturity and dividend yield of the next-to-deliver contract is insignificant. The days to maturity effect is positive and significant.

In Panel (2), we confirm the marginal effects of days to maturity as well as dividend yield and days to maturity interaction. Furthermore, we find that the June contract has the biggest positive influence on mispricing as previously discovered.

Overall, the subsample analysis with less observations of the Vollanrechungsverfahren confirms the results found in the full analysis. The mispricing, June phenomenon, dividend yield effect and time to maturity effect are as hypothesized under the Vollanrechnungsverfahren.

In Panel (3) of table 9 we regress the daily mispricing of the Halbeinkuenfteverfahren on the same factors as in Panel (1). As hypothesized and discovered in the previous analysis, we find that the June contract dummy has an overall positive influence on the mispricing. The interaction term between the dividend yield and June contract dummy has a positive but insignificant influence. Furthermore, the days to maturity have a positive influence on the mispricing. In Panel (4) of the subsample analysis, the June contract effect can be confirmed. However, the dividend yield has a significant negative influence on the mispricing.

Overall, the mispricing, June phenomenon and dividend yield effect are less pronounced than under the Vollanrechnungsverfahren. The June contract has its hypothesized effect but the dividend yield effect is much weaker and negative.

In Panel (5) - (6), we find the mispricing of the Abgeltungssteuer tax system in the same regressions as for the two previous systems. In panel (5) the dividend yield effect has opposite signs. Furthermore, the June contract dummy alone shows no significance. The interaction term of the June contract dummy with dividend yield is significant. Furthermore, the days to maturity have a positive significant influence. In Panel (6),the only significant coefficients are the days to maturity.

Overall, the mispricing, June phenomenon, dividend yield and trading volume effect are not as hypothesized and cannot be confirmed for the Abgeltungssteuer tax regime.

The detailed analysis of the mispricing in the different tax systems, the influence of dividends on the mispricing and the full- and subsample time-series regression analyses allow us to confirm most of the postulated hypotheses. Concerning hypothesis 1 and the level of mispricing we can confirm that the mispricing turns positive during dividend paying months in the Vollanrechungsverfahren. The mispricing in the Halbeinkuenfteverfahren and Abgeltungssteuer tax regime, on the contrary, turns negative during dividend paying months and is then on average zero. This result is, with somewhat weaker evidence, confirmed in the full- and subsample time-series analysis. The June contract phenomenon and hypothesis 2 is only robust for the Vollanrechnungsverfahren. The Halbeinkuenfteverfahren partially shows a June contract phenomenon in the subsample analysis which is not robust. The June contract phenomenon cannot be confirmed for the Abgeltungssteuer tax regime. Hypothesis 3 and the marginal effect of the dividend yield on the mispricing can be confirmed in the hypothesized direction for the Vollanrechnungsverfahren regime. Higher dividend yields have a positive effect on the mispricing. Furthermore, more dividend payment days increase the mispricing. We find a negative and sometimes insignificant effect of dividends in the Halbeinkuenfteverfahren and Abgeltungssteuer regime. Therefore, the influence of the dividend yield and dividend payment days on the mispricing seems to be mostly negative. The time to maturity effect as postulated in hypothesis 4 partially holds for all three tax regimes. Future contracts further away from their maturity show a higher mispricing.

5.4 Robustness Checks

In order to assert our results, we undertook various robustness checks. The most important factors determining the mispricing are the dividend yield, interest rate and the related compounding methods. Therefore, we tested for different ways to compound the dividend yield into the theoretical future price. In the base scenario, the dividend yield is added as an exponential factor. We also deduct the dividend payments as dividend sums from the index. The index dividend yield is based on a historical index dividend yield that incorporates all dividend payments of all index companies over the last year. As base case scenario for the future calculation, we take the historical dividend yield as investor expected future dividend yield. As a robustness check, we take the average dividend yield until maturity as dividend yield measure in our calculations. The results remain robust. In the time-series analysis we use several measures to proxy the dividend yield and simulate a robust marginal effect. Besides the previously described historical dividend yield and average dividend yield until maturity, we use the distribution of dividend payments per quarter. As further proxy for dividends, we use the dividend yield on a quarterly basis as well as the aggregated dividend payments per company per day.

As further robustness checks, we take different interest rates for the future calculation. As base scenario, we take the Frankfurt Money Market middle rate over the last 20 years with 3, 6, and 12 months to maturity. The interest rates are interpolated to fit the time to maturity. As robustness checks, we calculate the future price without term-structure interpolation and the 12 month Frankfurt Money Market middle rate for all maturities. Also, the results remain robust. Furthermore, we calculate the futures price using Eonia interest swap rates, to correct for the higher demand for risk-less government bonds in the last three years of our data sample. The results remain unchanged.

We calculate the mispricing for the next-to-deliver future contract trading period and use this as base case. We find that the trading volume and open interest is highest in the next-to-deliver contract months. The trading volume of the inactive, non next-to-deliver contracts is only about 3% of the active next-to-deliver contracts. The open interest of the inactive, non next-to-deliver contracts is only about 7% of the active next-to-deliver contracts. An analysis of the trading volume over the whole time period and per contract can be found in figure 7, figure 8 and figure 9 in the appendix. We also do the whole analysis for the full trading period of the future. The results are somewhat weaker but remain qualitatively the same.

To calculate the tax-rate corrected futures prices, we use a financial institution as marginal investor. We do the same calculations for a high net-wealth individual that should have similar arbitrage opportunities due to the high tax rates[16]. The results remain mostly unchanged for this scenario. We also checked for the systematic influence of differing trading hours between the index and future contracts in the last 20 years under the different tax systems. The influence seems unsystematic and cannot explain the mispricing.

6 Conclusion

We confirm in our empirical analysis the previously discovered result that there apparently exists a mispricing between the fair futures price derived from the simple cost-of-carry formula 2 and the empirically observed future contract. Besides its existence, this daily mispricing seems to be time varying throughout the year and depending on the taxation of the marginal investor / arbitrageur in the respective tax system. The three analyzed tax regimes considerably differ in manner and magnitude of dividend taxation. We postulate four hypotheses concerning the size and timing of the mispricing and eventually try to explain it with the taxation of dividends in the different German tax systems. We can confirm our hypothesis that the severest mispricing exists in the Vollanrechungsverfahren, where the gap between index-assumed dividend payments and after-tax dividend payments of the marginal investor is biggest. Furthermore, the mispricing correction has its biggest influence in the Vollanrechnungsverfahren. This mispricing can be explained with the described June contract phenomenon and the cumulation of dividend payments between April and June. An increase in the dividend yield during dividend paying months has a positive effect on the mispricing in the Vollanrechungsverfahren as hypothesized. Furthermore, the time to maturity effect is in the hypothesized direction. In the Halbeinkuenfteverfahren, the mispricing is reduced and the defined factors lose their explanatory power. The mispricing in the Abgeltungssteuer regime also cannot be explained with the hypothesized dividend taxation effects. If our assumptions hold and the marginal investors is the financial institution, it is most probably a financial institution that holds its stock and derivative positions in the trading book as this case has not effect on dividend taxation. Over the last 20 years, the daily mispricing and arbitrage opportunities in the DAX future contract have been reduced. In this empirical analy-

[16]However, one has to keep in mind that we neglect the higher transaction costs that the individuals face as compared to the financial institution

sis we were able to show that this reduction stems from the systematic change in the taxation of dividends. The abolishment of the tax-credit at the beginning of the 2000s has reduced the tax-induced mispricing of future contracts. With more generic taxation rules, future contracts on total return indices become easier and more precise to price.

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7 Figures and Tables

Figure 1: Quarterly Dividend Clustering

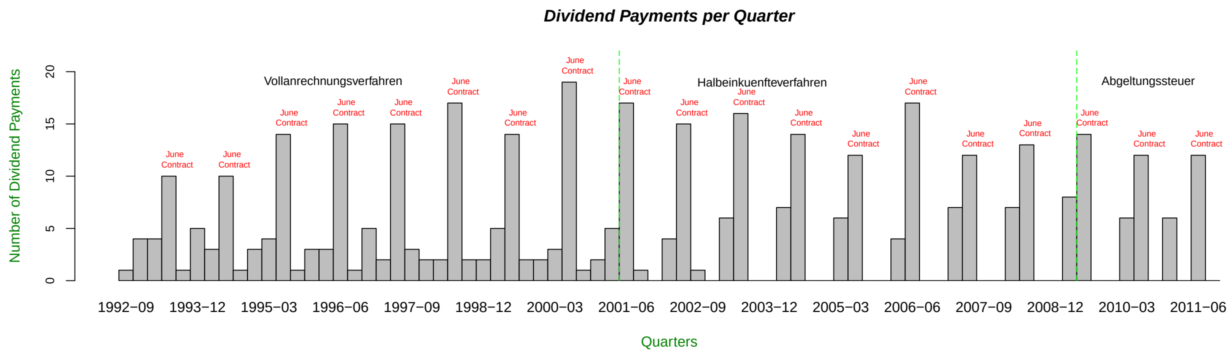


Figure 2: Cumulative Mispricing $\frac{DAX_t e^{r(T-t)} - FDAX_t^{empirical}}{FDAX_t^{empirical}}$ per contract under the three tax regimes: *Vollanrechnungsverfahren, Halbeinkuenfteverfahren, Abgeltungssteuer*

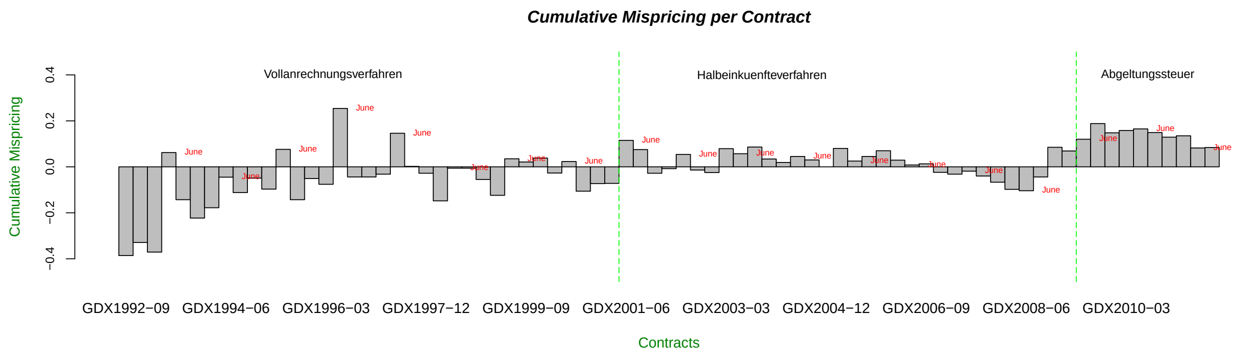


Figure 3: T-values of Mean Mispricing $\frac{FDAX_t^{theoretical\ tax\_corrected} - FDAX_t^{empirical}}{FDAX_t^{empirical}}$ per Future Contract Different from Zero, in the Three Tax Regimes: *Vollanrechnungsverfahren, Halbeinkuenfteverfahren, Abgeltungssteuer.*

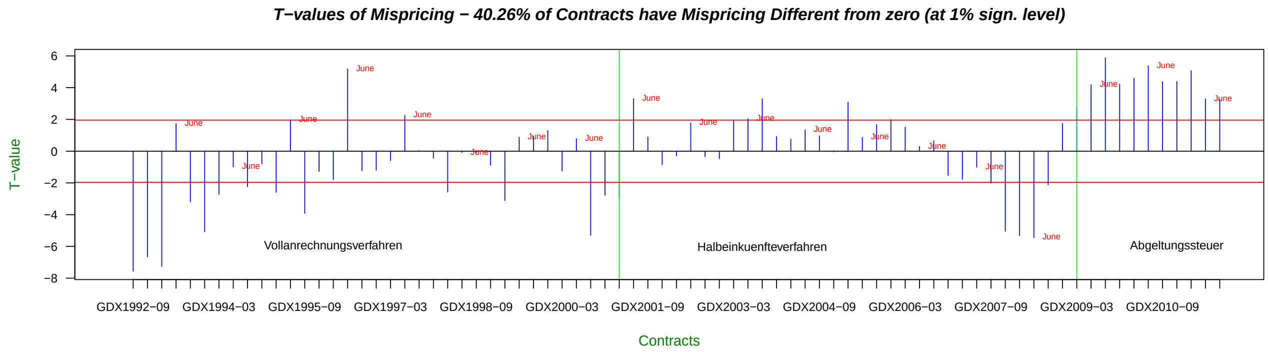


Figure 4: Corrected Cumulative Mispricing $\frac{FDAX_t^{theoretical\ tax\_corrected} - FDAX_t^{empirical}}{FDAX_t^{empirical}}$ per Contract in the Three Tax Regimes: *Vollanrechnungsverfahren (VOLL), Halbeinkuenfteverfahren (HEV), Abgeltungssteuer (ABG)* assuming a *dividend-privilege* of the marginal investor in the HEV and ABG regimes.

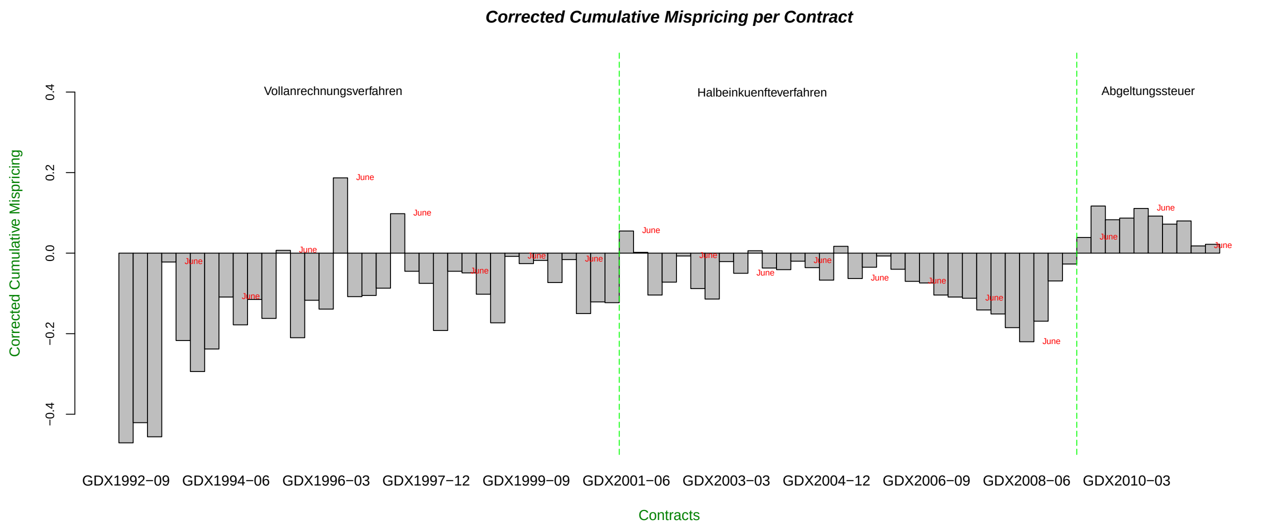


Figure 5: Corrected Cumulative Mispricing $\frac{FDAX_t^{theoretical\ tax\_corrected} - FDAX_t^{empirical}}{FDAX_t^{empirical}}$ per Contract in the Three Tax Regimes: *Vollanrechnungsverfahren (VOLL), Halbeinkuenfteverfahren (HEV), Abgeltungssteuer (ABG)* assuming *no dividend-privilege* of the marginal investor in the HEV and ABG regimes.

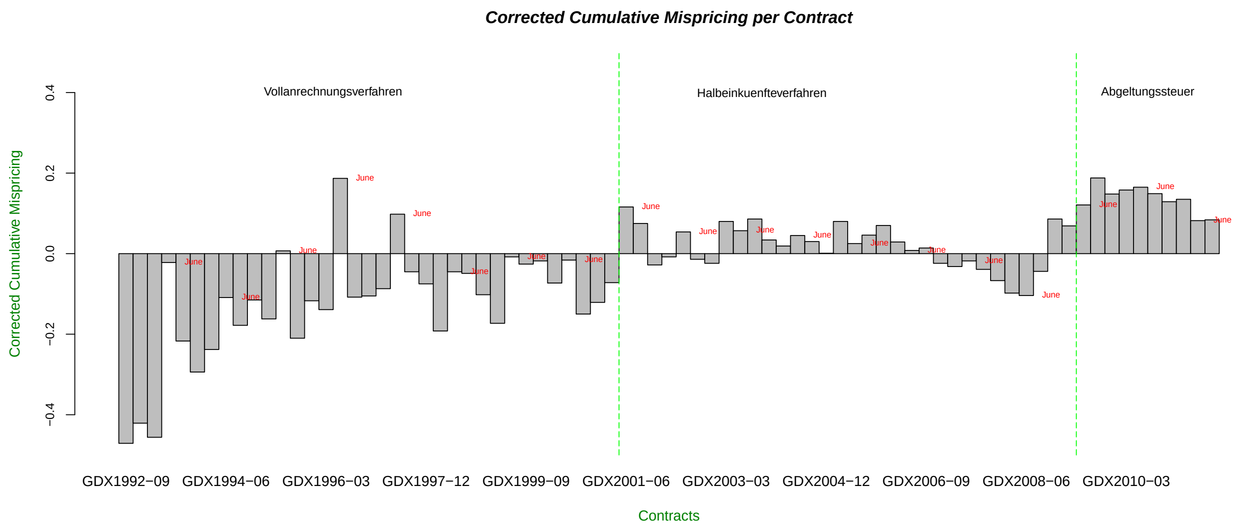


Figure 6: Cumulative Tax Correction per Future Contract in the Three Tax Regimes: *Vollanrechnungsverfahren (VOLL), Halbeinkuenfteverfahren (HEV), Abgeltungssteuer (ABG)* assuming a *dividend-privilege* of the marginal investor in the HEV and ABG regimes.

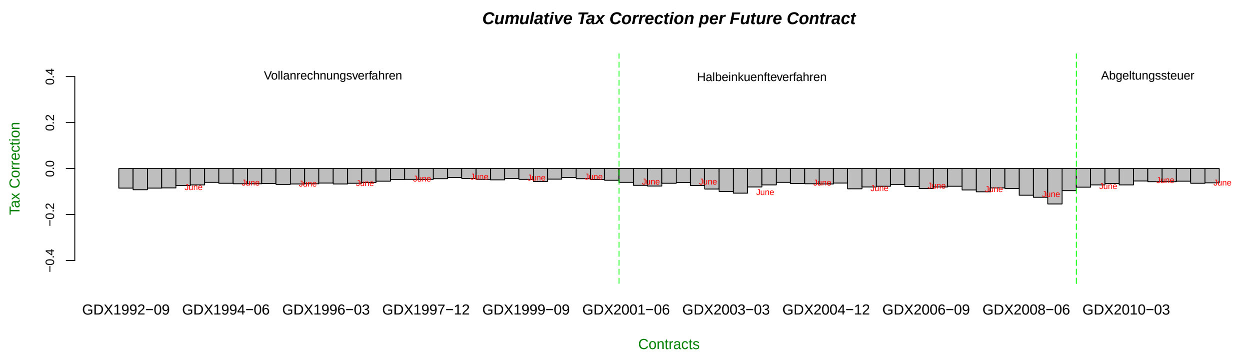


Table 1: No-Arbitrage to derive the fair future price

In this table, we derive the fair futures price $F*$ with no-arbitrage arguments. The no-arbitrage arguments are based on a cost-of-carry strategy that involves initial investment into a portfolio of stocks that replicates the DAX index. This investment is financed with a credit in period $t = 0$. In the same period, the arbitrageur sells a DAX Future contract short. Up to maturity the underlying index stocks pay dividends. The dividends cause different tax payments on the index level as on the personal portfolio level. The arbitrageur has to finance the tax differences with supplementary credits up to maturity.

Position	t=0	$t = \tau_j$	t=T
DAX portfolio long	$-\sum_{i=1}^{30} n(i)p_0(i)(:= -S_0)$	$(1 - s_d)\sum_j n(j)D(j)$	$\sum_{i=1}^{30} n(i)p_T(i) - \underbrace{s_k \sum_{i=1}^{30} n(i)[p_T(i) - p_0(i)]}_{\text{Capital gains tax}}$
FDAX short	± 0		$(F - S_T)(1 - s_k)$
credit	$+S_0$		$-S_0(1 + r(1 - s_z)(T - 0)$
supplementary credit		$s_d \sum_j n(j)D(j)$	$-s_d(1 + r(1 - s_z))(T - \tau_j)\sum_j n(j)D(j)$
dividend re-investment		$-\sum_j n(j)D(j)(= -\sum_j n(j)z(j)p_{ex}(j))$	$\sum_j n(j)z(j)p_T(j) - \underbrace{s_k \sum_j n(j)z(j)[p_T(j) - p_{ex}(j)]}_{\text{Capital gains tax}}$
Sum	0	0	0 →Solve for F* to get fair futures price.

s_k = captial gains tax, s_z = interest tax, s_d = dividend tax, r=interest rate not annualized but related to the time-interval $(T - t)$

Derivation of futures price F*:

$$0 = (1 - s_k)\underbrace{\left[\sum_{i=1}^{30} n(i)p_T(i) + \sum_j n(j)z(j)p_T(j)\right]}_{=:S_T}$$

$$+ (F - S_T)(1 - s_k) - S_0(1 + r(1 - s_z)(T - 0)) + s_k \underbrace{\sum_{i=1}^{30} n(i)p_0(i)}_{=:S_0}$$

$$+ [s_k - s_d[1 + r(1 - s_z)(T - \tau_j)]] \underbrace{\sum_j n(j)D(j)}_{=:\text{D}}$$

Table 2: Marginal investors and Tax Rates in the Three Tax Regimes Vollanrechungsverfahren (VOLL), Halbeinkuenfteverfahren (HEV), and Abgeltungssteuer (ABG)

In this table we show for five exemplary marginal investors the tax rates in the respective tax regime. Similar tables are used in Kempf and Spengel (1993) and Spengel and Zinn (2010). We follow Weber (2005) and incorporate the Corporate Tax II (GST) in the personal tax rate s. Furthermore, we ignore an additional tax on the Corporate Tax, a solidary surcharge tax which was introduced after the German reunification. The tax is only applicable in case the company has to pay corporate taxes and is obsolete in case of tax reimbursements.

VOLL	Case (1)	Case (2)	Case (3)	Case (4)	Case (5)
Marginal Investor	Private Individual	Private Individual	Company*	Company*	Foreign
Corporate Tax (KST_{pout}) in case of dividend payout	0.3/0.36	0.3/0.36	0.36/0.3	0.36/0.3	0.36/0.3
Corporate Tax (KST)	0.5/0.45	0.5/0.45	0.5/0.45	0.5/0.45	0.5/0.45
Corporate Tax II (GST)	-	-	GST=0.05*400%=0.2	0.2	-
Personal Tax (=s)	0.5	0.2	$s = KST + \frac{GST(1-KST)}{(1+GST)} = 0.583$ or s = 0.5417	0.583/0.5417	*
s_z	s	s	s	s	*
s_k	s	s	s	s	*
s_d	0.28571§/0.21875	-0.14286§/-0.25	0.3484 / 0.2839§	0.3484/0.2839§	*
HEV	Case (1)	Case (2)	Case (3)	Case (4)	Case (5)
Marginal Investor	Private Individual	Private Individual	Company*	Company*	Foreign
Corporate Tax (KST)	0.25	0.25	0.25	0.25	0.25
Corporate Tax II (GST)	-	-	GST=0.05*400%=0.2	0.2	-
Personal Tax (=s)	0.45	0.19	$s = KST + \frac{GST(1-KST)}{(1+GST)} = 0.375$; long-term**	$s = 0.375$; short-term**	‡‡
s_z	$\frac{s}{2}$	$\frac{s}{2}$	s	s	-
s_k	$\frac{s}{2}$	$\frac{s}{2}$	s	s	-
s_d	0.225	0.095	$s \times 0.05^{\circledast}$	s	*
ABG	Case (1)	Case (2)	Case (3)	Case (4)	Case (5)
Marginal Investor	Private Individual	Private Individual	Company*	Company*	Foreign
Corporate Tax (KST)	0.15	0.15	0.15	0.15	0.15
Corporate Tax II (GST)	-	-	GST=0.035*400%=0.14	0.14	-
Personal Tax (=s)	0.45	0.14	$s = KST + \frac{GST(1-KST)}{(1+GST)} = 0.2544$; long-term**	$s = 0.2544$; short-term**	‡‡
s_z	0.25	0.14	s	s	-
s_k	0.25	0.14	s	s	-
s_d	0.25	0.14	$s \times 0.05^{\circledast}$	s	*

§ the dividend tax rate s_d is calculated according to $D(1 - s_d) = D^{gross}(1 - s)$, where $D = (1 - KST_{pout})D^{gross}$

∗ financial institution according §1 I, Ia and III Kreditwesengesetz (KWG).

∗ ∗ tax rate depending if company sees investment as long-term or short-term trading-book asset.

‡ on foreign country but on top of KST and KST tax credit is not reimbursed assumed re-investment amount.

‡‡ depends on foreign country tax rate

⊛ Dividend-Privilege of Corporations. Not bound to minimum investment or holding period of investment. Only 5% of dividends are taxed. In case of losses, not deductible.

YEAR	Dividend Yield
1990	2.33
1991	2.52
1992	2.58
1993	2.25
1994	1.90
1995	2.02
1996	1.88
1997	1.46
1998	1.38
1999	1.40
2000	1.37
2001	1.96
2002	2.12
2003	2.57
2004	1.95
2005	2.22
2006	2.38
2007	2.44
2008	3.84
2009	4.16
2010	3.12
2011	3.56
(all)	2.34

Table 3: Annual Dividend Yield

Future Contract	Trading Volume Active Contract	Trading Volume Inactive Contract
March	80,280.26	2,389.43
June	81,956.61	3,209.67
September	81,503.76	2,806.28
December	87,766.86	2,956.17

Future Contract	Open Interest Active Contract	Open Interest Inactive Contract
March	156,832.35	9,220.19
June	205,569.18	18,016.98
September	146,055.24	8,112.05
December	152,668.65	8,717.13

Table 4: Future Contract Trading Data per active (next-to-deliver) and inactive (non next-to-deliver) contract.

	Abgeltungssteuer Mispricing	Halbeinkuenfteverfahren Mispricing	Vollanrechnungsverfahren Mispricing
March	0.3620	−0.0030	−1.0530
June	0.3670	0.1810	0.5450
Sept	0.4210	0.1510	−0.9170
Dec	0.2770	0.0450	−0.8140

Table 5: Cumulative Mispricing per Future Contract in the Three Tax Regimes

Table 6: Mispricing - Level Analysis

VARIABLES	(1) mispricing	(2) mispricing	(3) mispricing
VOLL	-0.00225***		
	(0.000174)		
d_JUNE	0.000170	0.00207***	0.00148***
	(0.000165)	(0.000239)	(0.000180)
VOLL_x_d_JUNE	0.00251***		
	(0.000344)		
HEV		0.000939***	
		(0.000163)	
HEV_x_d_JUNE		-0.00186***	
		(0.000306)	
ABG			0.00280***
			(0.000194)
ABG_x_d_JUNE			-0.00158***
			(0.000356)
Constant	0.000607***	-0.000794***	-0.000784***
	(9.19e-05)	(0.000124)	(9.33e-05)
Observations	5,022	5,022	5,022
R-squared	0.047	0.020	0.039

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 7: Dividend Analysis

VARIABLES	(1) mispricing	(2) mispricing	(3) mispricing
DIVIDEND_PAYMENT_DAYS	0.000777*	-0.000723***	-0.00157***
	(0.000454)	(0.000252)	(0.000350)
DIVIDEND_YIELD	5.77e-05	4.71e-05	0.000226
	(0.000130)	(0.000207)	(0.000149)
Constant	-0.00108***	0.000249**	0.00206***
	(0.000137)	(9.88e-05)	(0.000153)
Observations	2,227	2,088	707
R-squared	0.001	0.002	0.013

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 8: Time Series Regression - Full Sample

	(1)	(2)
VARIABLES	mispricing	mispricing
HEV	0.00182***	
	(0.000182)	
ABG	0.00371***	
	(0.000205)	
VOLL_June_dummy	0.00291***	
	(0.000294)	
HEV_June_dummy	0.000460**	
	(0.000201)	
ABG_June_dummy	0.000303	
	(0.000271)	
DIVIDEND_YIELD	0.000330**	-7.75e-05
	(0.000167)	(0.000114)
DIVIDEND_YIELD_x_d_JUNE	-0.00105***	
	(0.000344)	
DAYS_TO_MATURITY	5.42e-05***	5.30e-05***
	(4.02e-06)	(4.04e-06)
DAYS_TO_MATURITY_x_DIV_YIELD		
	(8.19e-06)	
DIVIDEND_PAYMENT_DAYS	-0.000416*	-0.000684***
	(0.000251)	(0.000252)
d_CONTRACT_MARCH		-0.000156
		(0.000205)
d_CONTRACT_JUNE		0.00145***
		(0.000211)
d_CONTRACT_SEPTEMBER		0.000124
		(0.000210)
Constant	-0.00347***	-0.00212***
	(0.000194)	(0.000177)
Observations	5,022	5,022
R-squared	0.102	0.051

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 9: Time Series Regression - Subsamples

	(1)	(2)	(3)	(4)	(5)	(6)
VARIABLES	mispricing	mispricing	mispricing	mispricing	mispricing	mispricing
d_CONTRACT_JUNE		0.00348***		0.000594**		0.000255
		(0.000388)		(0.000238)		(0.000262)
d_CONTRACT_SEPTEMBER		0.000507		0.000125		0.000310
		(0.000366)		(0.000253)		(0.000266)
d_CONTRACT_DECEMBER		0.000629*		-6.53e-06		0.000249
		(0.000368)		(0.000234)		(0.000276)
DIVIDEND_YIELD	0.000358*	0.000236	-0.00122	-0.000955**	-0.00334***	-0.000411
	(0.000184)	(0.000173)	(0.00128)	(0.000481)	(0.000962)	(0.000320)
DAYS_TO_MATURITY	2.38e-05***	2.47e-05***	5.94e-05***	5.92e-05***	0.000153***	0.000152***
	(7.03e-06)	(6.98e-06)	(5.32e-06)	(5.52e-06)	(5.82e-06)	(5.83e-06)
DAYS_TO_MATURITY_x_DIV_YIELD	-2.57e-05	-4.86e-05***	3.59e-06	5.14e-06	1.79e-06	3.78e-06
	(2.22e-05)	(1.34e-05)	(1.27e-05)	(1.46e-05)	(6.78e-06)	(7.36e-06)
d_JUNE	0.00316***		0.000550**		1.95e-05	
	(0.000314)		(0.000219)		(0.000219)	
DIVIDEND_YIELD_x_d_JUNE	-0.000990		0.000330		0.00304***	
	(0.000649)		(0.00122)		(0.000979)	
Constant	-0.00243***	-0.00280***	-0.00182***	0.000649	-0.00295***	-0.00313***
	(0.000258)	(0.000352)	(0.000218)	(0.000235)	(0.000212)	(0.000255)
Observations	2,227	2,227	2,088	2,088	707	707
R-squared	0.045	0.044	0.077	0.077	0.571	0.565

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

8 Appendix

8.1 Figures

Figure 7: Trading Volume and Open Interest of active (next-to-deliver) and inactive future contracts.

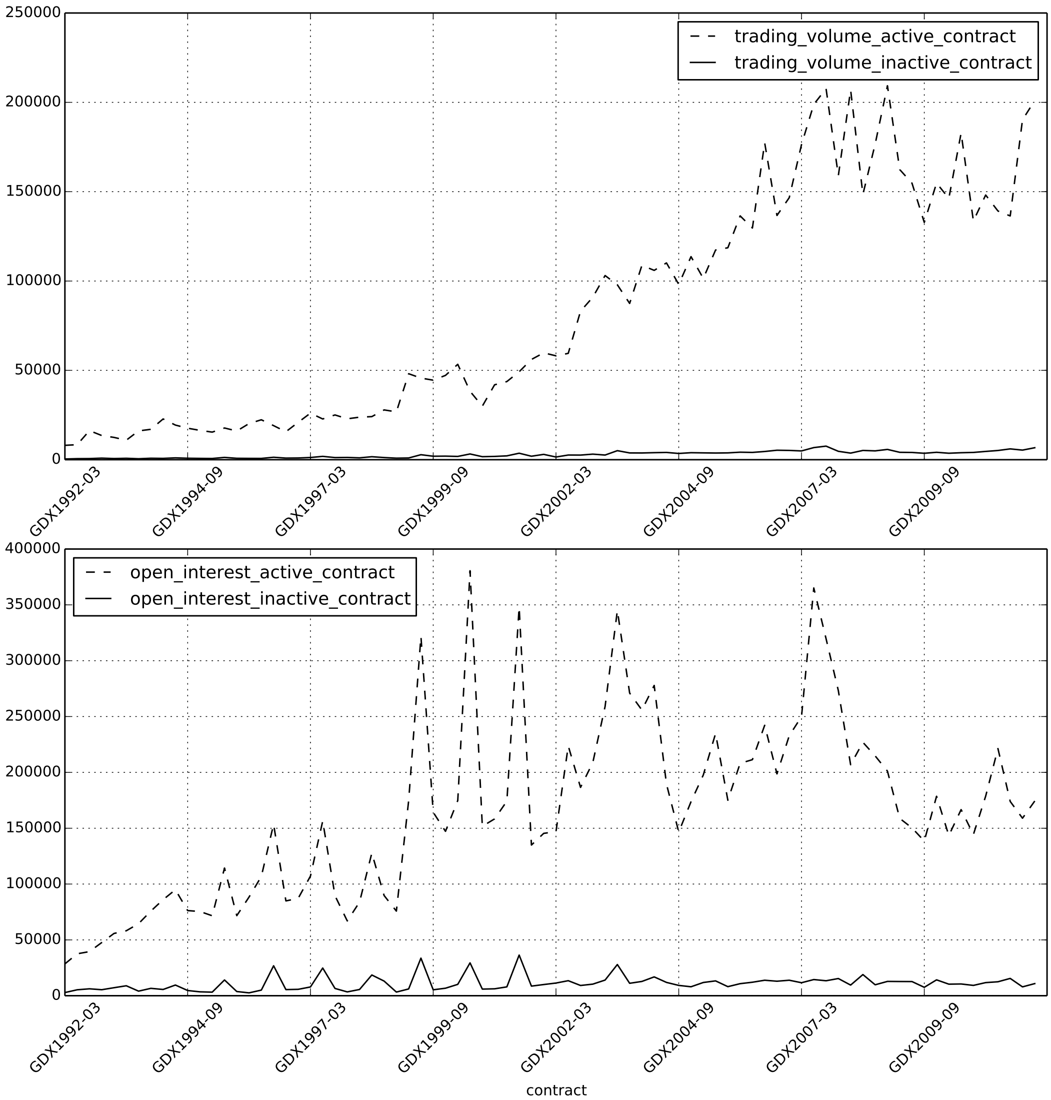


Figure 8: Ratio of trading volume in inactive versus active (next-to-deliver) contracts with March, June, September, December maturity.

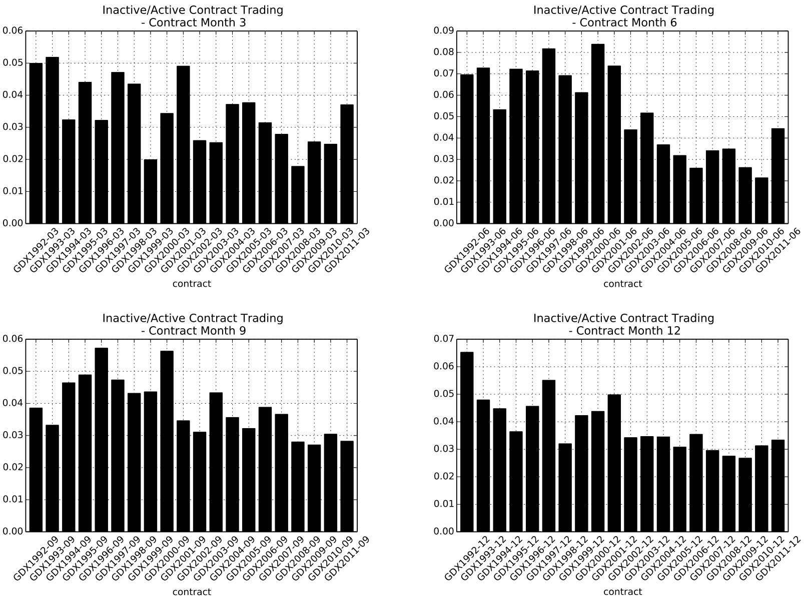
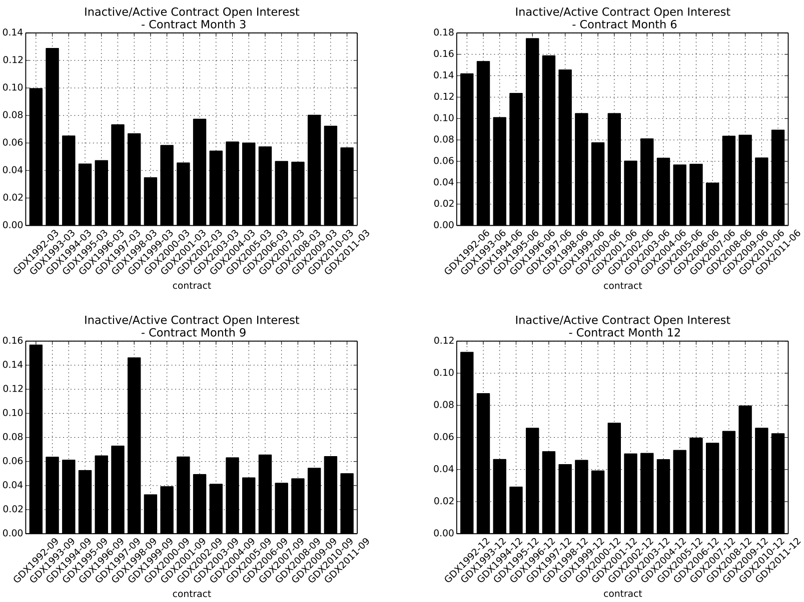


Figure 9: Ratio of open interest in inactive versus active (next-to-deliver) contracts with March, June, September, December maturity.



8.2 The German Tax Regimes (1990-2011)

The German tax system is characterized by three reforms in the last twenty years. Spengel and Zinn (2010) and Scheffler (2012) give a good overview of the German tax reforms and their implications for the different levels of corporate and personal income taxation. The first and oldest tax system is the so called Vollanrechnungsverfahren, an imputation system that was in place until 2001 when the Halbeinkuenfteverfahren (half-income system) was introduced. In 2009, the half-income system was succeeded by the Abgeltungsteuer (flat withholding tax system). Over the last twenty years German corporate taxes were lowered by a considerable amount. The taxes on the corporate level consist of two taxes. The first tax rate is the *Gewerbesteuer*, a regional tax that is levied by the German local authorities[17] which stayed constant at around 15%. The second corporate tax rate, *Körperschaftssteuer*, is a national tax that was 50% in 1990 and has been gradually reduced to 45% in 1998, 25% during half-income system in 2001, and eventually to 15% for the Abgeltungsteuer. The personal income tax rates for individuals vary over the three tax systems from as low as 14% under the Abgeltungssteuer for low income up to 53% under the Vollanrechnungsverfahren for the high income tax bracket. The focus of this project is on the taxation of dividends under the different regimes. For the determination of DAX future prices the amount that can be re-invested into the index, the dividend after corporate taxes or gross-dividend (Bruttobardividende), is of interest. The gross-dividend is not yet adjusted for the personal income tax level or corporate taxes in case of institutional investors. In the tax calculation several further particularities exist that can be considered. Under all three regimes there exists a withholding tax on dividend payments, the so called *Kapitalertragssteuer*. This tax is used under the Vollanrechnungsverfahren and Halbeinkuenfteverfahren as pre-payment to income tax. On individual assessment the pre-payment of the Kapitalertragssteuer is applied against the personal income tax. In case of the Abgeltungststeuer, the Kapitalertragsteuer is equal to the final withholding tax. The Kapitalertragssteuer is in all three systems a neutral tax with regard to futures pricing and can only change the fair future price by interest that accrues until final tax assessment day. A further tax that is levied under all three systems is the *Solidaritätszuschlag*. It is an additional tax of 5.5% to 7.5%, depending on the regime, on top of the tax liabilities. In addition, in all three tax regimes several tax allowances and rounding-off rules are possible but not considered in this first analysis. In the following passage certain particularities of the three systems are explained.

8.2.1 Vollanrechnungsverfahren

In 1977 the Vollanrechnungsverfahren tax system, an imputation system, was introduced. Its main principle is to tax every individual's capital income at his personal marginal tax rate. A good introduction into the basics of the Vollanrechnungsverfahren and the tax credit in

[17] Depending on the municipal area there is a different tax multiplier. The average multiplier is 400 which is multiplied by a tax rate of about 3.5%.

Germany is given in McDonald (2001). The system tries to reduce the total tax burden on corporate income. Shareholders should be compensated for tax already paid by the corporation. On the corporate level, capital gains are taxed at the Körperschaftssteuer. However, this Körperschaftssteuer is returned to the individual investor in form of a tax credit. The company pays a gross dividend to the investor of which the Körperschaftsteuer has been deducted. On the final assessment day of individual taxes the investor will, depending on his personal income tax rate, either get a tax refund or has to pay additional taxes. Only with a marginal income tax rate of 30% (or 36% for later years), the individual has been taxed correctly. The tax credit given to German investors is generally not recognized by tax laws of other countries. This induced an unequal treatment of domestic and foreign investors investing into German stocks and opened the possibility for arbitrage and created different fair prices for domestic and foreign investors of DAX futures. McDonald (2001) describes these arbitrage possibilities and their implications in detail.

8.2.2 Halbeinkuenfteverfahren

In 2001 the Vollanrechungsverfahren was replaced by the Halbeinkuenfteverfahren (half-income system). Bamberg and Dorfleitner (2002) give a good overview of the half-income system and its implication for dividend taxation and future prices. Under the half-income system the imputation tax credit is abolished. In this system the profits of the company are taxed on the corporate level as well as on the investor level. However, after corporate taxation the taxes are levied only on one half of the gross-dividend. In case the investor is a financial institution there are two cases. The dividend has to be taxed at the corporate tax rates of 25% if the financial institution keeps the positions in a trading book as described in section 2. In case the corporation keeps the positions as long-term investments, only 5% of the dividends are taxed at the corporate tax rate.

8.2.3 Abgeltungsteuer

In 2009 the half-income system was succeeded by the Abgeltungsteuer system. The Abgeltungsteuer is a flat tax of 25%. The tax rate applies irrespective of the investor's tax bracket. As in the half-income system the investor is taxed twice, on the corporate level with 15% Körperschaftssteuer and on an individual level with the flat tax, Abgeltungsteuer. The term flat tax needs to be understood as a flat withholding tax. The tax is withhold directly by the financial institution that holds the stocks for the individual investor of the dividend paying company. In case the investor is a financial institution there exist the same cases as in the Halbeinkuenfteverfahren with the only difference that dividends are taxed at the corporate tax rates of 15%.

8.3 The Tax System Correction Factors

In order to correct the theoretical futures prices the following tax correction factors for the different tax systems are calculated. This correction factors are multiplied by the reported dividend yield of the index and incorporated into the cost-of-carry pricing relation as in formula 3.

In the Vollanrechnungsverfahren, we correct with the following correction factor.

$$CF_{VOLL} = \left(-\left[s_k - s_d - r(T - \tau_j)s_d(1 - s_z)\right] \frac{1}{1 - s_k} D \right) \tag{6}$$

Here s_d represents the dividend tax rate that is in the Vollanrechnungsverfahren derived from:

$$D(1 - s_d) = D^{gross}(1 - s), \ where \ D = (1 - s_{div\_payout\_rate})D^{gross} \tag{7}$$

For the calculation of the DAX performance index it is assumed that $s_{div\_payout\_rate}$ is the only relevant tax rate that all market participants are subject to. Therefore, we correct the index implied tax rate with a tax rate of the marginal investor as explained in section 2. The correction factor follows the second part of equation 3 as explained in section 2.

In the Halbeinkuenfteverfahren and Abgeltungssteuer regime, we correct in two ways. In the first case, the tax rate s_d is the corporate tax rate of the marginal investor as explained in section and in the second case the tax rate is 5%* s_d as explained in 2.

$$CF_{HEV,ABG} = \left(-\left[s_k - s_d - r(T - \tau_j)s_d(1 - s_z)\right] \frac{1}{1 - s_k} D \right) \tag{8}$$