Effective and Cost-Efficient Volatility Hedging Capital Allocation: Evidence from the CBOE Volatility Derivatives

Yueh-Neng Lin[*]
Imperial College Business School and National Chung Hsing University
Email: *y-n.lin@imperial.ac.uk*, *ynlin@dragon.nchu.edu.tw*
Tel: +44 (0) 20 759 49168

Abstract

The challenge in "long volatility contracts" is to minimize the cost of carrying such insurance, as implied volatility continues to trade above realized levels. This study proposes a cost-efficient strategy for CBOE volatility contracts that is subject to substantial protection against severe downturns in a portfolio of S&P 500 stocks, while still participating upside preservation. The results show (i) timely hedging strategy removes the extreme negative tail risk and reduces the negative skewness in exchange for slightly fewer instances of large positive returns; (ii) dynamic volatility hedging capital allocation effectively solves the negative cost-of-carry problem; and (iii) using volatility contracts as extreme downside hedges can be a variable alternative to buying out-of-the-money S&P 500 index puts.

Keywords: Dynamic effective hedge; VIX calls; VIX futures; Variance futures; S&P 500 puts; Negative cost of carry

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[*] Corresponding author: Yueh-Neng Lin, Academic Visitor, Imperial College Business School, 53 Princes Gate, South Kensington Campus, London SW7 2AZ, United Kingdom, Phone: +44 (0) 20759-49168, Email: y-n.lin@imperial.ac.uk, ynlin@dragon.nchu.edu.tw. The author thanks Jun Yu, Walter Distaso and Jeremy Goh for valuable opinions that have helped to improve the exposition of this article in significant ways. The author also thanks Jin-Chuan Duan, Joseph Cherian and conference participants of the 2011 Risk Management Conference in Singapore for their insightful comments. Yueh-Neng Lin gratefully acknowledges research support from Taiwan National Science Council and Risk Management Institute, National University of Singapore.

1. Introduction

Using volatility as an asset class prior to the Q4 2008 financial crisis tended to capture historical excess returns by selling volatility as well as various strategies involving combinations of option positions. Hafner and Wallmeier (2008) and Egloff, Leippold and Wu (2010) analyze the implications of optimal investments in sizable short positions on variance swaps. Using data on S&P 500 index (SPX) options, Driessen and Maenhout (2007) show that with constant relative risk aversion, investors find it always optimal to short out-of-the-money (OTM) puts and at-the-money straddles. However, many shorting volatility strategies, following the spike in volatility in Q4 2008, have been susceptible to sudden large losses and were exposed to the high (positive) downside market beta, causing a re-evaluation of return requirements relative to risks. Similarly, relative-value strategies suffer from a lack of liquidity on the back of reduced supply and demand for exotic derivative structures. Long volatility strategies have gained popularity since 2008, primarily as a hedge against catastrophic scenarios, often referred to as "tail risk." Szado (2009) suggests that, while long volatility exposure may result in negative returns in the long term, it may provide significant protection in downturns.

Common examples of Chicago Board Options Exchange (CBOE) volatility instruments include the S&P 500 Index (SPX) Options, the Volatility Index (VIX) Futures, the VIX Options and the S&P 500 Three-Month Variance Futures (VT).[1] Figure 1 displays VIX, S&P 500 historical volatility and S&P 500 levels over the last two decades. The analysis reveals that the historical volatility provides information on the future realized volatility of the SPX market, and the VIX and SPX often mirror each other. The VIX appears to be an appropriate hedging tool against the potential downside of the broad equity market. While the spot VIX is difficult to replicate as a practical matter, investors trade futures and options on VIX as well as variance futures to express their view on the S&P 500's implied volatility.

[Figure 1 about here]

Since volatility often signifies financial turmoil, using volatility derivatives as extreme downside hedges is often referred to as portfolio diversification. For example, Kat (2003) proposes the purchase of OTM SPX puts to hedge risks of higher moments. Black (2006) finds that adding a small VIX position to an investment significantly reduces portfolio volatility. Moran and Dash (2007) discuss the benefits of a long exposure to VIX futures and VIX call options. Szado (2009) analyzes the diversification impacts of a long VIX exposure during the 2008 financial crisis. His

[1] The CBOE launched the SPX options in 1983, the VIX futures on March 26, 2004, the three-month variance futures (VT) on May 18, 2004, and the VIX options on February 24, 2006.

results suggest that, dollar for dollar, VIX calls provide a more efficient means of diversification compared to SPX puts. More recently, Alexander and Korovilas (2011) point out the hazards of volatility diversification if volatility trades are not carefully timed.

The challenge in holding such a volatility position is to minimize the cost of carrying such insurance, as implied volatility continues to trade above realized levels. In other words, any long positions on volatility contracts would have offered substantial returns during the financial crisis periods, but most long volatility positions also incurred devastating losses in the subsequent bull market. For example, Figure 2 shows VIX futures prices move downward for the majority of their recent history, except for periods of extreme stress and volatility of the 2008 financial crisis. As a result, long positions on VIX futures are expected to suffer losses incurred in futures rolls during normal volatility regimes. In contrast, backwardation in the VIX futures market during periods of stress such as in Q4 2008 presents a positive roll yield for investors with long positions on VIX futures.

Therefore, there exists a negative cost of carry in volatility futures that is possibly caused by the significant theta decay on the premia of underlying options used to replicate the volatility contracts. An investor often needs a dealer who is willing to take the other side of the trade on the exchange because of the lack of liquidity, while the dealers are simply replicating their volatility exposures with underlying option positions. It indicates that the volatility futures also have delta, gamma and theta. The last one is the most obvious in the marketplace — most of the price decay occurs closer to expiration. The amount of money the hedger loses in time decay must then be made back by additional volatility movement, and such is generally the case once the time has reached the financial crisis.

[Figure 2 about here]

Figures 3-6 demonstrate that cost of carry may be an extremely high financial cost if the volatility contracts are ineffectively traded. The sample period for trading naked volatility contracts, consisting of OTM SPX puts, VIX futures, variance futures and OTM VIX calls, spans from February 24, 2006 through September 9, 2009. Those figures present the cumulative dollar profit and loss (P&L) on the SPX ETF and the cumulative dollar P&Ls on volatility contracts plus the bank cash balance account of any receivable/payable required for monthly rolls. Note that the solid line of the lower right-hand-corner graph in each figure is the sum of the security asset and cash balance accounts represented by the solid lines in the upper half of each figure. Negative cost of carry[2] is indeed observed in the marketplace prior to the 2008 financial crisis. What the naïve hedger fails to realize is that in order for the volatility contract to be profitable the delta of the volatility contract must outpace its rate of decay.

[Figure 3 about here]

[Figure 4 about here]

[Figure 5 about here]

[Figure 6 about here]

In sum, this kind of downside or crash protection may be expensive because of its constantly negative cost of carry, and practically it might be impossible to time the market to pay for protection only during a significant market downturn. It is unclear how to allocate volatility capital in an equity portfolio efficiently. Traditional hedge ratio determination, usually involving either risk minimum or risk-adjusted return maximum, fails to take into account the unique features of volatility contracts. This study proposes a cost-efficient strategy to achieve the effectiveness of using CBOE volatility instruments as extreme downside hedges. After taking into account the costs of rolling contracts, this strategy provides meaningful protection against sudden and/or large market declines, while not imposing excessive costs under ordinary market

[2] The definition of negative carry is the cost of borrowing money to fund an investment that exceeds the profit earned.

conditions.

The study uses a long SPX portfolio and compares various hedging instruments including (i) VIX futures, (ii) VT futures, (iii) 10% OTM VIX calls, and (iv) 10% OTM SPX puts. In each case, out-of-sample hedging effectiveness is analyzed against a long position on a 100-lot unit of SPX ETF based on risk reduction and return improvement per unit cost of hedging. The cost of hedging is measured by the negative cost of carry on volatility contracts. The reason that the CBOE considers no cost of carry for VIX futures is that there is an absence of clearly defined way to replicate a VIX futures contract.[3] This study proposes replication schemes as upside volatility hedges for market dealers who short sell VIX or variance futures. The futures market price in excess of the dealer's replication cost is regarded as the implicit cost for futures long hedge; whereas, the upfront option premium is treated as a negative cost of carry for option long hedge. Empirical evidence suggests that (i) using volatility instruments as extreme downside hedges, especially when combined with an appropriate hedging technique, can be a viable alternative to buying a series of OTM SPX put options; (ii) using OTM VIX call options and VIX futures presents a cost-effective choice for extreme downside risk protection as well as for upside

[3] See http://cfe.cboe.com/education/vixprimer/features.aspx.

preservation; (iii) the pros and cons of using variance futures with benefits from boosted gains and discounted losses and costs reflected in a slightly higher strike than VIX futures more or less offset one another; and (iv) a rule-based strategy that dynamically allocates volatility hedging capital into an equity portfolio presents an effective and cost-efficient method as extreme downside hedges.

The primary contribution of this paper is a new methodology for solving two problems. The first, is to measure negative costs of carry implicit in the market prices of volatility derivatives. The second, is to propose an effective and cost-efficient strategy to allocate volatility hedging capital in an equity portfolio. The methodology is new in the hedging exercise using volatility derivatives because (i) it imposes a replication scheme directly on the volatility/variance futures; (ii) it does not require an "risk minimization or return maximization of the hedged portfolio" to estimate the hedge ratio; and (iii) it incorporates a rule-based dynamic strategy influencing the volatility hedging capital allocation in an equity portfolio.

The remainder of the paper is organized as follows. Section 2 describes the methodologies. Section 3 provides an analysis of the hedging results. Section 4 concludes the paper.

2. Methodology

This section provides an in-depth discussion of the methodologies used in this study: (i) the hedging strategy, (ii) the rolling methodology for volatility instruments, (iii) bid-ask spreads, and (iv) the negative costs of carry on volatility instruments.

2.1 Hedging Strategy

Traditional hedge ratio determination usually involves using either risk minimization, which minimizes variance, maximum drawdown and conditional value-at-risk of a hedged portfolio, or risk-adjusted return maximization. Those conventional hedging methods to determine hedge ratios for volatility products could incur substantial losses under normal market conditions, leaving frustrated investors wondering whether hedges are worth the expense. The inadequacy of traditional hedge ratios on volatility-based products is intuitive. Everyone knows that volatility is mean-reverting, so it stands to reason that a constant level of exposure will record sizable gains when volatility increases and will record equally large losses as volatility reverts to its long-term mean. Therefore, constant allocations to volatility hedge positions on monthly/quarterly rolling schemes are ineffective as hedges and inefficient as cost reduction technique. With dynamic volatility hedging capital allocation, investors can retain the effectiveness of the hedge when environments are abnormal and reduce costs during normal market conditions. Therefore, using dynamic hedging rules to allocation is a rational way to effectively exploit the unique features of volatility contracts.

This study proposes a variable sizing rule-based “Long VOLatility Hedging” (LVOLH) strategy to allocate hedging capital dynamically in response to changes in the prevailing volatility environment. The premise of the LVOLH strategy is that the allocation to volatility assets grows at an increasing percentage rate when the stock market slumps. The allocation pattern of the volatility component is governed by mathematical properties exhibited in the Fibonacci sequence, or appears as sums of oblique diagonals in Pascal’s triangle. Specifically, the LVOLH strategy consists of a long position in the SPX ETF, hedged with volatility positions that vary in accordance with how the LVOLH evaluates volatility risk. LVOLH largely uses the current level of realized volatility and the direction of the VIX trend to determine if a security risk is overvalued or undervalued. Generally, securities with a higher historical volatility carry more risk. Typically, VIX can be used as a trend-confirming indicator because it often trends in the opposite direction of the stock market. Despite a tendency to trend, the VIX can identify sentiment extremes that react to stock market movements (Whaley, 2000). Sharp stock market declines often produce exaggerated spikes in the VIX as panic grips the market, whereas a steady stock market advance produces a steady downtrend and relatively low levels for the VIX.

The allocations are evaluated on a daily basis, though changes in hedge ratios may occur less frequently. The volatility hedge of LVOLH is set to vary in a range of 0 −65% of the mark-to-market (MTM) value of the hedged portfolio. The LVOLH strategy is implemented by the following steps.

Step 1: Determine the realized volatility.

The annualized one-month historical volatility level, $RVOL_{t-1}$, of the SPX returns on the preceding business day is calculated as

$$RVOL_{t-1} = \sqrt{\frac{252 \cdot \sum_{n=1}^{22} \left[ln\left(\frac{SPX_{t-n}}{SPX_{t-n-1}} \right) \right]^2}{22}} \tag{1}$$

Step 2: Identify the short- and long-term implied volatility levels.

Calculate the 5-day and 20-day moving averages of one-month implied volatility represented by the VIX:

$$\overline{VIX}_{t-1,t-5} = \sum_{n=1}^{5} \frac{VIX_{t-n}}{5} \tag{2}$$

$$\overline{VIX}_{t-1,t-20} = \sum_{n=1}^{20} \frac{VIX_{t-n}}{20} \tag{3}$$

Step 3: Find out the daily implied volatility trend indicator.

$$1_{VIX,t-1} = \begin{cases} +1 \quad if \ \ \overline{VIX}_{t-1,t-5} \geq \overline{VIX}_{t-1,t-20} \\ -1 \quad if \ \ \overline{VIX}_{t-1,t-5} < \overline{VIX}_{t-1,t-20} \end{cases} \tag{4}$$

Step 4: Determine the implied volatility trend.

An implied volatility trend is constructed if the daily implied volatility trend indicators remain constant for at least 10 consecutive index business days. Therefore, on any index business day, t, the implied volatility trend ($VIXtrend_{t-1}$) is given by either an uptrend, a downtrend, or no trend:

$$VIXtrend_{t-1} = \begin{cases} +1 & if \;\; \sum_{n=1}^{10} 1_{VIX,t-n} = +10 \\ -1 & if \;\; \sum_{n=1}^{10} 1_{VIX,t-n} = -10 \\ 0 & if \; -10 < \sum_{n=1}^{10} 1_{VIX,t-n} < +10 \end{cases} \quad (5)$$

Step 5: Identify the target weighting of the volatility instrument.

The Fibonacci sequence is a recursive sequence, which one has to simply sum the preceding two numbers to calculate the next term:

$$0, 1, 1, 2, 3, 5, 8, 13, \cdots \quad (6)$$

Multiplying the Fibonacci sequence by the volatility basis 5%,[4] the target weight $\omega(t)$ of the volatility contract is determined as

$$0\%, 5\%, 5\%, 10\%, 15\%, 25\%, 40\%, 65\% \quad (7)$$

of the hedged portfolio.

Table 1 summarizes volatility hedging capital allocations of the LVOLH strategy. On each business day, the realized volatility is used in conjunction with the VIX trend for market timing. The resultant weighting of each volatility instrument in the hedged

[4] This study tried alternative volatility basis including 2.5%, 5%, 7.5% and 10%, and found that volatility basis of 5% on average provided the most satisfactory hedging results for the majority of volatility instruments.

portfolio will be allocated in accordance with the rule-based algorithm set forth above. The MTM value of the ETF portfolio times each of weightings ω divided by $(1-\omega)$ is the allocated hedging capital to the volatility instruments.

[Table 1 about here]

Graphical illustrations of the pre-defined volatility hedging capital allocations against a 100-lot unit of SPX ETF are presented in Figure 7. The graph illustrates the pre-defined weightings from February 24, 2006 to September 9, 2009. The volatility hedging capital allocation shows put option-like characteristics, because it tends to have little impact on the SPX portfolio during normal market conditions but gain profits during worst performing days of the S&P 500 equity markets. The LVOLH strategy makes volatility capital injection in market disruption and force majeure events, and withdrawal in regular trading days. Therefore, the proposed method to obtain exposure to volatility is thought to be a cost-efficient and effective choice as extreme downside risk hedge as well as upside preservation.

[Figure 7 about here]

2.2 Rolling Methodology for Volatility Instruments

The front-month series of volatility contracts are created by purchasing volatility contracts with at least five business days prior to their expiration to avoid liquidity problems in the last week of trading. Additional positions are purchased at their opening asks whenever a bullish volatility signal results in the volatility contracts becoming attractive; whereas, a portion of purchased positions are sold at their opening bids whenever a bearish volatility market results in the contracts turning unattractive. The study uses opening prices plus (minus) half of the bid-ask spreads as the synthetic opening ask (bid), because ask and bid prices at the opening of the market are not available. In addition, the study rolls any purchased futures five business days before the expiration date. In contrast, the study just lets any purchased VIX calls and SPX puts expire instead of trying to roll them forward, given good liquidity relative to the volatility futures markets and significant large bid-ask spreads in the options markets. This strategy is consistent with the real-world practice.

On each business day t, the MTM value of the hedged portfolio, consisting of one unit of 100-lot SPX ETF, h units of volatility instruments purchased on day t_{roll}, and a cash account that finances the positions and accumulates the trading profit and loss (P&L), is evaluated as

$$MTM(t) = ETF(t) + h(t_{roll}) \cdot cumP\&L_{INST}(t_{roll}, t) + cash(t) \qquad (8)$$

Any interest charges on a negative balance or interest accruals on a positive balance from the current period also become part of the P&Ls for the next period. The hedge ratio is calculated as

$$h(t_{roll}) = \frac{ETF^{open}(t_{roll}) \cdot \omega(t_{roll})}{(1-\omega(t_{roll})) \cdot INST_{t_{roll}}^{open\ ask}} \tag{9}$$

where $ETF^{open}(t_{roll}) = \$10 \cdot SPX^{open}(t_{roll})$ is the opening price of ETF on day t_{roll}; and $INST_{t_{roll}}^{open\ ask}$ indicates multiplier-adjusted opening asks of the futures instrument on the roll day t_{roll} or the option strikes. The contract multipliers are \$1,000 per VIX point for the VIX futures, \$50 per variance point for the VT contract, and \$100 per point of VIX options and SPX options, respectively.

The daily P&L should be computed based on a combination of the changes in market values of the assets and in the balance of cash account. For simplicity, the potential need to finance one's margin requirements is ignored. The day-t cumulative P&L of the volatility contract purchased on day t_{roll} is calculated using daily settlement prices of futures or midpoints of options; specifically, $cumP\&L_{vixcall}(t_{roll}, t) = \$100 \cdot C_t^{VIX}(T), cumP\&L_{spxput}(t_{roll}, t) = \$100 \cdot P_t^{SPX}(T)$, $cumP\&L_{vixfut}(t_{roll}, t) = \$1000 \cdot [F_t^{VIX}(T) - F_{t_{roll}}^{VIX}(T)]$, and $cumP\&L_{VT}(t_{roll}, t) = \$50 \cdot \left[F_t^{VT}(T) - F_{t_{roll}}^{VT}(T)\right]$.

This study expects that an effective hedging instrument will fluctuate like a crude mirror image of the P&L represented by the SPX ETF. This is roughly the case for both VIX futures and 10% OTM VIX calls as shown in Figures 3 and 5. In the case of VT futures, however, the study does not observe any “rough mirror image” resemblance between the solid line and the dotted line in the lower right-hand-corner graph in Figure 4 prior to the 2008 financial crisis, but such is generally the case after the crisis. In the case of 10% OTM SPX puts, the study observes a roughly straight line representing a steady increase in the negative carry as the time approaches the 2008 financial crisis. As in the case of volatility contracts, the study observes the “rough mirror image” resemblance once the time has reached the financial crisis.

2.3 Bid-Ask Spreads on Volatility Contracts

The bid-ask spreads have been taken into account when rolling forward and rebalancing the volatility positions. Table 2 provides summary statistics for bid-ask spreads of volatility contracts for monthly rolls. To reconcile the differences in multipliers across various volatility contracts, the unit of bid-ask spreads in Table 2 is expressed in US dollars. A spread of $s\%$, the degree at which the portion of daily trade prices could be explained by bid-ask spreads, is also reported in parentheses. Noticeably, VT futures consistently have larger bid-ask spreads in dollars than other volatility contracts. Significantly large bid-ask spreads in ratios for the 10% OTM SPX puts are observed, as indicative of a relative expensiveness in rolling costs when using those contracts and a relative cheapness when using other volatility contracts. Further, higher bid-ask spreads denominated in dollars are observed for all volatility contracts during the 2008 crisis period. In particular, the increased dollar percentages from bid-ask spreads in 10% OTM SPX puts are on average greater than other volatility contracts during the 2008 crisis periods. Based on the data compiled in Table 2, the VIX futures and 10% OTM VIX calls appear to be roughly comparable as extreme downside hedges in terms of dollar and ratio spreads, while 10% OTM SPX puts can be significantly expensive as extreme downside hedges.

[Table 2 about here]

2.4 Negative Costs of Carry for Volatility Instruments

There is a difference in hedging cost structure between options and futures. The cost for a long option hedge is the premium at open ask on each balance day, since money is paid up front. In contrast, hedging with futures is often considered to be "costless", since the hedger pays no explicit upfront premium. This study challenges this notion by identifying the existence of the implicit premium embedded in the futures price itself. If hedging with futures truly is "costless", then the futures market price should exactly equal the dealer's replication cost. The concept is justified by the fact that it may be practically impossible to time the market crashes and most short volatility positions incur devastating losses during the financial crisis periods. The market dealer who shorts the volatility contract can neutralize his exposure by replicating a long position on the volatility contract as it has sold. Under the assumption of deterministic interest rates,[5] the total profit for the dealer who writes the volatility futures and hedges it with replicated forwards is regarded as the implicit cost of carry for the hedger who takes a long volatility futures position.

2.4.1 Negative Costs of Carry for VIX Calls and SPX Puts

Suppose the hedger has $h(t_{roll}^{j})$ units of long option positions on day t_{roll}^{j} for day $j = 1, \cdots, N$ and the hedge requires $\left(h\left(t_{roll}^{j}\right) - h\left(t_{roll}^{j-1}\right)\right)^{+}$ units of the contracts to be additionally purchased at their opening ask prices on day t_{roll}^{j}. The total costs of the hedging after discounting would be equal to:

$$HC_{opt}(t_{roll}^{1}) = h(t_{roll}^{1}) \cdot \$100 \cdot Opt_{t_{roll}^{1}}^{open\ ask}$$

$$+ \sum_{j=2}^{N} \left(h\left(t_{roll}^{j}\right) - h\left(t_{roll}^{j-1}\right)\right)^{+} \cdot \$100 \cdot Opt_{t_{roll}^{j}}^{open\ ask} \cdot e^{-R\left(t_{roll}^{j}\right) \cdot \left(t_{roll}^{j} - t_{roll}^{1}\right)} \quad (10)$$

where $Opt_{t_{roll}^{j}}^{open\ ask}$ is the opening ask price of either a 10% OTM VIX call or 10% OTM SPX put purchased on the roll day t_{roll}^{j}. $R\left(t_{roll}^{j}\right)$ is a continuously-compounded interest rate that has a maturity equal to the option's expiration, and is obtained by linearly interpolating between the two closest US Treasury bill rates observed at day t_{roll}^{j}.

[5] Under deterministic interest rates, the futures contract can be usually treated as a forward contract.

2.4.2 Implicit Costs of Carry for VIX Futures

The VIX futures market price in excess of its replication cost is treated as the implicit cost for the hedger to take a long futures position. The VIX futures are not tied by the usual cost of carry relationship that connects other indexes and index futures, because the portfolio of SPX options used to replicate the VIX is ever changing which makes the index non-investable. This study points to the term structure of SPX implied volatilities to explain any perceived carry issues. The payoff of implied variance forwards, replicated from CBOE VIX Term Structure (denoted $VIXTerm$) or equivalently the portfolio of SPX options,[6] is convex in volatility. This means that an investor who is long implied variance forwards will benefit from boosted gains and discounted losses. This bias has a cost reflected in a slightly higher strike than the fair implied volatility, as documented by "Jensen's inequality", a phenomenon which is amplified when volatility skew is steep. Therefore, the cost of replicating a long VIX futures position using $VIXTerm$ is required to subtract convexity from implied variance forwards. This study points out that jointly using a strip of SPX options and VIX options would replicate VIX futures with regards to convexity.[7]

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[6] $VIXTerm$ is a representation of implied volatility of SPX options, and its calculation involves applying the VIX formula to specific SPX options to construct a term structure for fairly-valued variance. As a result, investors will be able to use $VIXTerm$ to track the movement of the SPX option implied volatility in the listed contract months.

[7] One alternative replication strategy to offset the dealer's short position on VIX futures is adopting

Using Martingale pricing theory and Jensen's inequality, the time-t fair price $Fwd_t^{VIX}(T)$ of VIX futures with maturity T is given by Lin (2007):

$$Fwd_t^{VIX}(T;\tau_0) = E_t^Q(VIX_T) \approx \sqrt{E_t^Q(VIX_T^2)} - \frac{var_t^Q(VIX_T^2)}{8\cdot\left[E_t^Q(VIX_T^2)\right]^{3/2}}$$

$$\approx \sqrt{Fwd_t^{VIX^2}(T;\tau_0)} - \frac{E_t^Q(VIX_T^4)-\left[Fwd_t^{VIX^2}(T;\tau_0)\right]^2}{8\cdot\left[Fwd_t^{VIX^2}(T;\tau_0)\right]^{3/2}} \quad (11)$$

where Q is the risk-neutral probability measure; $Fwd_t^{VIX^2}(T;\tau_0)$ is the time-t implied variance forwards starting at T and ending at $T+\tau_0$ with $\tau_0 = 30/365$, which can be replicated from $VIXTerm$ with a calendar spread:

$$Fwd_t^{VIX^2}(T;\tau_0) \approx \frac{1}{\tau_0}\left[VIXTerm_{t,T+\tau_0}^2 \cdot (T+\tau_0-t) - VIXTerm_{t,T}^2 \cdot (T-t)\right] \quad (12)$$

where $VIXTerm_{t,T}^2$ and $VIXTerm_{t,T+\tau_0}^2$ are the squares of VIX over $[t,T]$ and $[t,T+\tau_0]$, respectively. Using the generic spanning technique in Bakshi, Kapadia and Madan (2003), the risk-neutral fourth moment $E_t^Q(VIX_T^4)$ could be replicated by the quartic contract with final payoff of VIX_T^4 using a strip of calls and puts on VIX.

$H(VIX_T) = VIX_T^4$ is a twice-continuously differentiable function and it can be spanned algebraically (Bakshi, Kapadia and Madan, 2003), as in

put-call-futures parity. Using put-call-futures parity to synthesize a long VIX futures position, however, only utilizes the information in the VIX options market, whereas using convexity replication scheme exploits the trade information in both SPX options and VIX options markets.

$$H(VIX_T) = H\left(F_t^{VIX}(T)\right) + \left(VIX_T - F_t^{VIX}(T)\right)\left(\frac{\partial H(VIX_T)}{\partial VIX_T}\bigg|_{VIX_T = F_t^{VIX}(T)}\right)$$

$$+ \int_{F_t^{VIX}(T)}^{\infty} \left(\frac{\partial^2 H(VIX_T)}{\partial VIX_T^2}\bigg|_{VIX_T = K}\right)(VIX_T - K)^+ dK$$

$$+ \int_0^{F_t^{VIX}(T)} \left(\frac{\partial^2 H(VIX_T)}{\partial VIX_T^2}\bigg|_{VIX_T = K}\right)(K - VIX_T)^+ dK \quad (13)$$

or, equivalently,

$$VIX_T^4 = -3[F_t^{VIX}(T)]^4 + 4[F_t^{VIX}(T)]^3 VIX_T + \int_{F_t^{VIX}(T)}^{\infty} 12K^2 (VIX_T - K)^+ dK$$

$$+ \int_0^{F_t^{VIX}(T)} 12K^2 (K - VIX_T)^+ dK \quad (14)$$

Applying risk-neutral valuation to both sides of the above equation, one has the arbitrage-free price of the quartic contract as

$$E_t^Q\left[e^{-R(t)\cdot(T-t)} VIX_T^4\right] = [F_t^{VIX}(T)]^4 \cdot e^{-R(t)\cdot(T-t)} + \int_{F_t^{VIX}(T)}^{\infty} 12K^2 C_t^{VIX}(T,K) dK$$

$$+ \int_0^{F_t^{VIX}(T)} 12K^2 P_t^{VIX}(T,K) dK \quad (15)$$

which merely formalizes how VIX_T^4 can be synthesized from (i) a zero-coupon bond with positioning: $[F_t^{VIX}(T)]^4$, and (ii) a linear combination of calls and puts on VIX (indexed by K) with positioning: $12K^2$. Following the discretization methodology of CBOE VIX, the intrinsic values of the quartic contract can be statically constructed by observing the relevant market prices, and appealing to the following equation:

$$E_t^Q[VIX_T^4] \approx [F_t^{VIX}(T)]^4 + 12\sum_i {K_i}^2 \Delta K_i e^{R(t)\cdot(T-t)} Q_t^{VIX}(T, K_i) \quad (16)$$

where $Q_t^{VIX}(T, K_i)$ is the midpoint of the bid-ask spread for each VIX option with maturity T and strike K_i. K_0 is the first strike below the VIX futures price $F_t^{VIX}(T)$. K_i is the strike price of the ith OTM VIX option; a VIX call if $K_i > K_0$ and a VIX put if $K_i < K_0$; both VIX call and VIX put if $K_i = K_0$. $\Delta K_i = (K_{i+1} - K_{i-1})/2$ is the interval between strike prices, defined as the half difference between the strike on either side of K_i. ΔK for the lowest strike is simply the difference between the lowest strike and the next higher strike. Likewise, ΔK for the highest strike is the difference between the highest strike and the next lower strike. $R(t)$ is the time-t risk-free interest rate to expiration.

The implicit cost of carry for the hedger who takes a long VIX futures position initiated on day t_{roll}^j is calculated as

$$ICC_{t_{roll}^j}^{vixfut} = F_{t_{roll}^j}^{VIX,open\ ask}(T) - Fwd_{t_{roll}^j}^{VIX,open\ bid}(T) \quad (17)$$

where $F_{t_{roll}^j}^{VIX,open\ ask}$ is the synthetic opening ask price of VIX futures purchased on the roll day t_{roll}^j. Suppose the hedger has $h(t_{roll}^j)$ units of long futures positions on day t_{roll}^j for $j = 1, \cdots, N$ and the hedge requires $\left(h(t_{roll}^j) - h(t_{roll}^{j-1})\right)^+$ units of the contracts to be additionally purchased at their opening ask prices on day t_{roll}^j. The total costs of the hedging after discounting would be equal to:

$$HC_{vixfut}(t_{roll}^1) = h(t_{roll}^1) \cdot \$1000 \cdot ICC_{t_{roll}^1}^{vixfut} + \sum_{j=2}^{N} \left(h(t_{roll}^j) - h(t_{roll}^{j-1})\right)^+ \cdot \$1000 \cdot ICC_{t_{roll}^j}^{vixfut} \cdot e^{-R(t_{roll}^j)\cdot(t_{roll}^j - t_{roll}^1)} \quad (18)$$

2.4.3 Implicit Costs of Carry for VT Futures

CBOE variance futures offer an alternative to OTC variance swaps on the SPX. The distinction between variance futures and variance swaps is minimal, because the information contained in them is virtually identical. The value of forward-start VT futures is composed of 100% implied forward variance ($IUG_{T-\tau_1,T}$), as given by

$$F_t^{VT,fs}(T) = IUG_{T-\tau_1,T} \tag{19}$$

where $0 < t < T - \tau_1 < T$ and $\tau_1 = 0.25$ year. IUG represents the future variance of the SPX that is implied by the daily settlement price of the front-quarter VT futures. Once VT futures become the front-quarter contract, it enters the three-month window during which realized variance is calculated. The price of the front-quarter futures contract can be stated in two distinct components: the realized variance (RUG) and the implied forward variance (IUG). The value of front-quarter VT futures is given by

$$F_t^{VT,fm}(T) = \left(1 - \frac{T-t}{\tau_1}\right) \cdot RUG_{T-\tau_1,t} + \left(\frac{T-t}{\tau_1}\right) \cdot IUG_{t,T} \tag{20}$$

where $0 < T - \tau_1 < t < T$. The formula to calculate the annualized realized variance (RUG) is as follows[8]

$$RUG = 252 \cdot \sum_{i=1}^{N_a-1} R_i^2 / (N_e - 1) \tag{21}$$

where $R_i = \ln(P_{i+1}/P_i)$ is daily return of the S&P 500 from P_i to P_{i+1}; P_i is the

[8] See http://cfe.cboe.com/education/VT_info.aspx for the details. The RUG in Eqs. (20) and (21) multiplying 10,000 is the RUG data available in the Chicago Futures Exchange website.

initial value and P_{i+1} is the final value of the S&P500 used to calculate the daily return. This definition is identical to the settlement price of a variance swap with N prices mapping to $N-1$ returns. N_a is the actual number of days and N_e is the expected number of days in the observation period. The actual and expected number of days may differ if a market disruption event results to the closure of relevant exchanges, like what happened on September 11, 2001. For simplicity, N_e is approximated by N_a in this study.

From a theoretical viewpoint, the IUG portion of a variance futures can be seen as a representation of the volatility smile curve since the strike price of the VT futures is determined by the prices of SPX options of the same maturity and different strikes that make up a static portfolio replicating the payoff at maturity. The calculation methodology for the VIX represents the theoretical strike of a VT futures contract on the SPX with a maturity of one month. From a practical viewpoint, the VT futures and SPX option markets are closely linked through the hedging activity of market-makers. To a first approximation, a market-maker typically hedges a short position in the VT futures by creating a long VT forwards contract synthetically through buying the 10% OTM SPX puts.

Since the IUG portion of front-quarter VT forwards can be replicated by

$VIXTerm_{t,T}^2$ extracted from $VIXTerm$ with identical days to maturity, this study synthesizes the front-quarter VT forwards with the following equation:

$$Fwd_t^{VT,fm}(T) \approx \left(1 - \frac{T-t}{\tau_1}\right) \cdot RUG_{T-\tau_1,t} + \left(\frac{T-t}{\tau_1}\right) \cdot VIXTerm_{t,T}^2 \quad (22)$$

This study takes the initial forward VIX curve implicit in $VIXTerm$ to synthesize the forward-start VT forwards price, because forward-start VT forwards are completely attributable to IUG portion. That is, for $0 < t < T - \tau_1 < T$,

$$\begin{aligned} Fwd_t^{VT,fs}(T) &\approx Fwd_t^{VIX^2}(T - \tau_1; \tau_1) \\ &\approx \frac{1}{\tau_1}\left[VIXTerm_{t,T}^2 \cdot (T-t) - VIXTerm_{t,T-\tau_1}^2 \cdot (T - \tau_1 - t)\right] \end{aligned} \quad (23)$$

The implicit cost of carry for a long futures position initiated on day t_{roll}^j is defined as the futures market price in excess of its replicating forwards price:

$$ICC_{t_{roll}^j}^{VT} = F_{t_{roll}^j}^{VT,open\ ask}(T) - Fwd_{t_{roll}^j}^{VT,open\ bid}(T) \quad (24)$$

The total implicit costs of hedging for the hedger who has $h(t_{roll}^j)$ units of long VT futures positions on day t_{roll}^j for $j = 1,2,\cdots,N$ after discounting would be equal to

$$\begin{aligned} ICC_{t_{roll}^1}^{VT} &= h(t_{roll}^1) \cdot \$50 \cdot ICC_{t_{roll}^1}^{VT} \\ &+ \sum_{j=2}^{N}\left(h\left(t_{roll}^j\right) - h(t_{roll}^{j-1})\right) \cdot \$50 \cdot ICC_{t_{roll}^j}^{VT} \cdot e^{-R(t_j)\cdot(t_j - t_{roll}^1)} \end{aligned} \quad (25)$$

2.4.4 Results on Costs of Carry of Volatility Contracts

Table 3 reports descriptive statistics on both the explicit and implicit costs of carry denominated in dollars as applied to a unit of volatility contract purchased on each trading day. The table uses daily ask prices of options and futures prices and bid quotes of $VIXTerm$ from February 24, 2006 to September 9, 2009 for the monthly rolls. The costs of carry from the 2008 financial crisis are also separately tabulated in Table 3.

Only options have explicit upfront and easily quantifiable premia from the various forms of hedging instruments discussed above. These premia on SPX puts and VIX calls are found to be inexpensive when compared to the implicit costs of carry in dollars with VIX futures and VT futures. Interestingly, it is more costly to use VT (VIX) futures than to use 10% OTM SPX puts (VIX calls), but with substantial improvements to the upside, as shown in Figures 5 and 7 (Figures 4 and 6), despite the higher costs involved. This is less surprising considering that VT futures can be created from a series of SPX options in theory and VIX futures could be replicated from put-call-futures parity using comparable VIX options, and 10% VIX calls and 10% OTM SPX puts are among the cheapest liquid options available. This suggests a widely-accepted replication intuition among practitioners, since it seems rather usual that using the synthesized product is more expensive than using the raw materials.

Further, explicit and implicit costs of hedging would increase if one attempts to extend the hedge farther out into the 2008 crisis period. In particular, the costs of hedging with VT futures and SPX puts are diverse in very different market environments of falling stock prices panic and relatively stable stock prices, respectively. This is perhaps an indication that those contracts, without adopting any sophisticated hedging method, are more appropriate as hedging instruments in the absence of market crises. Conversely, the costs of carry reveal the viability of using VIX calls and VIX futures as extreme downside hedges when applied to a naïve hedging strategy. Furthermore, the findings of substantially relative low premia in the 10% OTM VIX call option market might represent unique properties of volatility to create trading opportunities, particularly to hedge equity volatility risk.

[Table 3 about here]

In next section, the study proposes an efficient and cost-effective way of using those volatility instruments to manage unwanted risks and preserve market returns.

3. Hedging Performance

The study focuses on a daily out-of-sample hedging horizon; that is, the rebalancing, checked every trading day, takes place on rebalance dates for monthly roll scheme of hedging instruments. Hedge effectiveness is measured based on the magnitude of risk reduction and/or adjusted-return enhancement per unit of effective hedging cost from before-the-hedge to after-the-hedge. The effective hedging cost are calculated as the explicit upfront premia for options or the implicit costs of carry for futures contracts that are measured by the market prices in excess of their replication costs.

3.1 Hedge Effectiveness Measures

Traditional risk/return measures such as Sharpe ratios and standard deviations are inadequate to measure risk for assets such as volatility with highly non-normal distributions and large tails. These are the three measures to gauge hedge performance when applied to a single hedging volatility instrument: (i) using maximum drawdown as a downside risk measure; (ii) using adjusted conditional Value-at-Risk as a measure of extreme tail risk; and (iii) using extended Sharpe ratio as a measure of the excess return relative to risk with highly non-normal distributions and large tails.

First measure is the magnitude of percentage maximum drawdown ($\%MaxDD$) reduction for monthly returns on the hedged portfolio from before-the-hedge to after-the-hedge per unit of effective hedging cost:

$$\frac{\%MaxDD(T;R_{before\ hedge}) - \%MaxDD(T;R_{after\ hedge})}{effective\ \ hedging\ \ cost} \qquad (26)$$

where $R(t)$ =MTM(t)/MTM(t-22)-1 is the monthly return. $\%MaxDD(T)$ is defined as the maximum sustained percentage decline (peak to trough) for period $[0, T]$, which provides an intuitive and well-understood empirical measure of the loss arising from potential extreme events (Magdon-Ismail et al., 2004; Magdon-Ismail and Atiya, 2004):

$$\%MaxDD(T;\{R\}_{t=0}^{T}) = \max_{0\le t\le T}\left[\frac{R_{0\le\tau\le t}^{peak}-R(t)}{R_{0\le\tau\le t}^{peak}}\right] \tag{27}$$

where $R_{0\le\tau\le t}^{peak} = \max_{0\le\tau<t}[R(\tau)]$ is the maximum dollar monthly return in the $[0,t]$ period.

Second measure is the magnitude of the expected shortfall or conditional Value-at-Risk ($CVaR$) (Rockafellar and Uryasev, 2002) reduction for monthly returns on the hedged portfolio at the confidence level $1-\alpha$ from before-the-hedge to after-the-hedge per unit of effective hedging cost:

$$\frac{CVaR_{1-\alpha}\left(T;\,R_{after\ hedge}^{CF}\right)-CVaR_{1-\alpha}\left(T;\,R_{before\ hedge}^{CF}\right)}{effective\ \ hedging\ \ cost} \tag{28}$$

where R^{CF} is the monthly returns on the hedged portfolio that uses the Cornish-Fisher expansion to incorporate skewness and kurtosis into the return distribution (Cornish and Fisher, 1938; Baillie and Bollerslev, 1992; Liang and Park, 2010):

$$CVaR_{1-\alpha}(R^{CF}) = \mu(R) + \sigma(R)\cdot E\left(z_{cf,1-\kappa}\middle|\kappa > 1-\alpha\right)$$

$$= \mu(R) + \sigma(R) \times E\left[\begin{array}{c|c} z_{1-\kappa} + \frac{1}{6}(z_{1-\kappa}^2 - 1)S(R) & \\ +\frac{1}{24}(z_{1-\kappa}^3 - 3z_{1-\kappa})K(R) & \kappa > 1-\alpha \\ -\frac{1}{36}(2z_{1-\kappa}^3 - 5z_{1-\kappa})S(R)^2 & \end{array}\right] \tag{29}$$

with $z_{1-\kappa}$ being the critical value for probability $1-\kappa$ with standard normal distribution (e.g. $z_{1-\kappa} = -1.64\ at\ \kappa = 95\%$), while μ, σ, S and K follow the standard definitions of mean, volatility, skewness and excess kurtosis, respectively, as computed from the monthly returns on the hedged portfolio.

Third measure is the magnitude of the extended Sharpe ratio (denoted ESR) enhancement for monthly returns on the hedged portfolio from before-the-hedge to after-the-hedge per unit of effective hedging cost:

$$\frac{ESR(T; R_{after\ hedge}) - ESR(T; R_{before\ hedge})}{effective\ \ hedging\ \ cost} \tag{30}$$

ESR is an omega-function-like measure. The numerator is a measure of upside cumulants while the standard deviation of returns in the denominator is replaced by a measure of downside cumulants (Karatzas and Shreve, 1998; Fernholz, 2002; Keating and Shadwick, 2002). This is a more balanced measure from the perspective of not only minimizing risk (which also tends to minimize returns) but also achieving a balance between upside and downside moments, and is generally consistent with the real-world practice in that traders tend to underhedge to preserve upside. The ESR is defined as

$$ESR = \frac{1}{z_\pi^- \sigma_\pi}\left(e_\pi + \frac{1}{2}(z_\pi^+ \sigma_\pi)^2 - \frac{1}{2}(z_\pi^- \sigma_\pi)^2\right) \tag{31}$$

where $e_\pi =$ excess monthly return rate of the hedged portfolio π; $\sigma_\pi =$ volatility of π; $z_\pi^+ = \frac{max(z_{cf,\kappa}(\pi),0)}{z_\kappa}$; $z_\pi^- = \frac{min(z_{cf,1-\kappa}(\pi),0)}{z_{1-\kappa}}$; for example, $z_\kappa = 2.33$ at $\kappa = 1\%$, $z_{1-\kappa} = -2.33$ at $1-\kappa = 99\%$.

3.2 Hedging Results

This section presents the empirical results of hedging a 100-lot unit of long SPX ETF with the LVOLH strategy as applied to: (i) the VIX futures; (ii) the variance futures; (iii) the 10% OTM VIX calls; and (iv) the 10% OTM SPX puts. Table 4 reports various statistics for monthly returns on the unhedged portfolio (ETF) and the LVOLH portfolio hedged with one of the four volatility contracts. In order to examine whether the LVOLH strategy provides economic benefits even in the absence of tail risks and abnormal market environments, the empirical analyses excluding the September to December 2008 panic period and the January to September 2009 relatively calm period are also separately tabulated.

[Table 4 about here]

3.2.1 VIX Futures

The graphical out-of-sample results of the unhedged ETF and the ETF portfolio hedged with the LVOLH strategy using VIX futures are plotted in Figure 8. Panel A looks at the MTM values of unhedged and hedged portfolios, and Panel B displays the histograms of their monthly returns. The hedged portfolio realizes outsized gains during the Q4 2008 panic period and also has considerable profits in the Q1-Q3 2009 relatively calm periods.

[Figure 8 about here]

The adoption of LVOLH with VIX futures has the following effects over the full sample period. First, the hedged portfolio removes monthly returns below -4%; for example, -14% and -27.32%. Second, the hedged portfolio adds returns greater than 10%; for example, the 20% and 126.39%. Third, the hedged portfolio increases the number of months with returns between -2% and 2%, that is, has a smoothing effect. In sum, the LVOLH strategy with VIX futures removes the extreme negative tail risk during the full sample period for slightly fewer instances of large positive returns. This results in a significant enhancement in skewness from -1.83 for the unhedged portfolio to 5.98 for the hedged portfolio. Further, the LVOLH-hedged portfolio with VIX futures produces an average of 3.23% per month, versus a -0.92% mean return for the unhedged SPX ETF alone, and the minimum monthly return is improved by seven times. Panel B of Table 4 shows the VIX futures portfolio is effective in reducing tail risk measured by percentage maximum drawdown, and Cornish-Fisher $CVaR$s at 95% and 99%, as well as produces impressive enhancement in extended Sharpe ratio from before-the-hedge (-3.08) to after-the-hedge (4.36).

While the spike in the MTM of the hedged portfolio during the late 2008 is dramatic, it is important to consider the performance of long volatility positions during normal periods. The graphical analyses excluding the Q4 2008 panic period and the Q1-Q3 2009 relatively calm periods are displayed in the lower graphs in each panel of Figure 8. With the financial crisis excluded, the hedged portfolio exhibits a mean monthly return of -0.19%, with a volatility of 2.47% versus -0.25% (mean) and 3.95% (volatility) for the unhedged SPX ETF. Noticeably, in contrast to ad hoc hedging results using conventional hedge ratios, the LVOLH strategy with VIX futures presents an upside preservation during the normal market environments.

The present value of total implicit costs of carry for the variable approach to allocate capitals to VIX futures positions is $9,447.73 during the full sample period. This consists of a mean cost of $106.84 when the 2008 financial crisis was excluded, and $330.71 when the Q4 2008 panic period and the Q1-Q3 2009 relatively calm periods were included. The LVOLH strategy with VIX futures is able to keep costs low under normal conditions in the form of higher minimum and mean monthly returns based on the volatility exhibited in and implied by the market. Further, the LVOLH allocation achieves large gains under crisis conditions, and retains nearly all of those gains once the market returns to normal. These results show that the LVOLH strategy with VIX futures provides economic benefits even in the absence of tails risks and abnormal market environments. In general, the results indicate the effectiveness of using the LVOLH strategy with the VIX futures. The technique is a cost-effective choice as hedging instruments for extreme downside risk protection and for upside preservation.

3.2.2 Variance Futures

As shown in Figure 9 and Table 4, the LVOLH-hedged portfolio with VT futures has not only gained substantial positive returns during extreme downside markets, but also incurred less devastating losses in the preceding bull market than a fixed or constant level of allocation. The LVOLH strategy with VT futures has removed monthly returns below -10%, reduced the frequency of poor monthly returns of -10% to -8%, and added a return greater than 120%. In sum, the VT futures portfolio removes the extreme negative tail risk during the full sample period and reduces the negative skewness in exchange for slightly fewer instances of large positive returns. The VT futures portfolio returns an average of 1.97% per month with a minimum of -10.11% and a maximum of 120.81%, versus a -0.92% mean monthly return with a minimum of -27.32% and a maximum of 9.91% for SPX ETF alone. The study observes a reduction in drawdown and an effective decline in Cornish-Fisher $CVaR$s with a significant improvement of the upside, resulting in a reliable proposed strategy during the financial crisis.

[Figure 9 about here]

With the financial crisis excluded, however, the performance of the VT futures portfolio has modest improvement, exhibiting a negative skewness of -0.62 versus -0.62 for the unhedged SPX ETF monthly returns. Since Fall 2008, the strategy costs approximately $1,110.18 via a lower position placed, versus a $4,960.66 cost for the VIX futures portfolio, to maintain the substantial returns. With the financial crisis again excluded, the VT futures portfolio exhibits a cost of just $1,676.89, versus $4,487.07 for the LVOLH strategy with VIX futures. The results suggest that the LVOLH strategy with VT futures provides cost benefits in the absence of tail risks and abnormal market environments at the expense of lacking some minimal level of portfolio protection, which is always in place for a VIX futures portfolio.

The practical issue with using the VT futures is that its market price considers the "look back" nature of maximum drawdown in SPX movements, and it is generally believed that the VT futures is recoiling in its P&L. The study observes a slight increase in losses toward the crisis representing the negative carry of any long VT futures strategy, while the upside for the VT futures portfolio is preserved once the financial crisis has reached. Consequently, the pros and cons of using VT futures with benefits from boosted gains and discounted costs via smaller hedge ratios reflected in a slightly higher strike, more or less offset one another. Such a negative carry would possibly deter any real-life traders from using such an instrument for hedging during normal market conditions.

3.2.3 10% Out-of-the-Money VIX Call Options

As shown in Table 4 and Figure 10, instead of attempting to hedge a portfolio of SPX ETF by buying an index put option, one may be able to accomplish it cheaper by purchasing VIX call options.

[Figure 10 about here]

The VIX call portfolio removes the extreme negative tail risk during the full sample period and reduces the negative skewness in exchange for slightly fewer instances of large positive returns. In particular, it removes monthly returns below -10% and adds a return greater than 102%. The adoption of the LVOLH strategy with 10% OTM VIX calls thus, has a positive effect on the overall mean; the ESR and the monthly return risk also have improved as measured by $\%MaxDD$ and $CVaR$s.

The mean monthly return and its risk measures, however, suggest the existence of a negative cost of carry for a long VIX call hedge during the normal market scenario, though it is much less severe than using 10% OTM SPX puts and comparable to the VT futures. As a result, using 10% OTM VIX calls is not as effective as VIX futures during the normal market episode, but it has produced more significant cost-effective upsides for the portfolio in the crisis period.

3.2.4 10% Out-of-the-Money SPX Puts

Comparing Figure 11 to Figure 7, it is noticeable that an adoption of the LVOLH strategy makes the 10% OTM SPX puts more responsive to shocks in the spot SPX ETF, making them more desirable as hedging instruments.

[Figure 11 about here]

The most conventional method for gaining long exposure to volatility has been the purchase of OTM SPX put options. Option buyers seem caught, however, between the rapid time decay afflicting short-dated contracts, and the rich premium and strike dependence plaguing longer-dated contracts. Many hedgers will typically either underhedge with a smaller than suitable notional amount, or use options further out-of-the-money, lowering the payoff when the options go into the money. Conversely, the study shows that investors who employ 10% OTM SPX puts will pay reduced premia by adopting the LVOLH strategy to determine the optimal hedge ratio, and will not forego substantial gains during strong bear markets. The LVOLH strategy with SPX puts, however, still contend with higher premia related to implied volatility skew and thus more expensive than the LVOLH strategy with VIX calls.

The adoption of the LVOLH strategy with 10% OTM SPX puts has the following effects. First, it removes monthly returns below -16%. Second, it reduces the frequency of poor (-8% to -16%). Third, it adds a return greater than 310%. In sum, the SPX put portfolio removes the extreme negative tail risk during the full sample period and reduces the negative skewness in exchange for slightly fewer instances of large positive returns.

With the 2008 financial crisis excluded, the SPX put portfolio shows a mean monthly return of -1.1296%, versus a -0.2547% mean return for the ETF alone, and ESR of -0.4623, versus -0.8246 ESR for the ETF alone. The results indicate the existence of a negative cost of carry for the SPX put portfolio. The strategy costs approximately $11,690.70, versus $4,126.17 for the VIX call portfolio during the full sample period. While the spike in SPX put portfolio returns during late 2008 is dramatic, they become costly positions with a cost of $7,827.30, versus $2,146.59 for the VIX call portfolio. Further, monthly return risks as measured by $\%MaxDD$ and VaRs improved versus the unhedged ETF.

3.3 Overall Comparison on Choices of CBOE Volatility Instruments

The study compares different hedging instruments based on their 1-month rolling time series, wherein liquidity can be found in real-life trading.[9] Table 5 reveals their hedging effectiveness normalized by their effective hedging costs when applied to hedging a 100-lot SPX ETF. The study highlights these results to show that the LVOLH strategy provides economic benefits to alternative volatility instruments even in the absence of tail risks and abnormal market environments.

[Table 5 about here]

Compared to the unhedged SPX ETF, the LVOLH strategy removes the extreme negative tail risk in exchange for slightly fewer instances of large positive returns, which generally exhibit a higher degree of positive skewness and kurtosis. The LVOLH strategy with VIX futures continues to be a reliable performer in preserving upside gains for portfolios during normal market periods. Monthly returns on the VT futures portfolio have the most cost-effective positive mean during the 2008 financial crisis period. The VIX call portfolio appears to be a more stable performer over time than the SPX put portfolio. What makes the VIX option different is that, VIX calls could rise in value much faster than a typical index put option during market downturns, because spikes in volatility tend to be relatively larger than the market

[9] In practice, quarterly rolling, for example, saves on transaction costs, but the longer-dated futures are also known to be less responsive to shocks in the spot VIX, making them less desirable as hedging instruments. Using longer-dated options is to help reduce the effects of time decay; however, keeping them 10% OTM may not be as cost-effective as buying OTM options each month and letting them expire.

movements that cause them. Potentially, this allows the hedger to offset some or all of the losses in his SPX ETF at a much lower cost. OTM VIX call purchases are, therefore, less expensive than OTM SPX puts.

The details of hedge effectiveness are given as follows. The ESR per unit of effective hedging cost is highest for the VT futures portfolio during the full sample period, followed by the VIX call portfolio, the SPX put portfolio, and the VIX futures portfolio. In particular, unit ESR of the VT futures portfolio is almost five times as large as that of the VIX futures portfolio. Still, significant reductions in unit $\%MaxDD$ and unit Cornish-Fisher $CVaR$s are observed for the VT portfolio. Interestingly, devastating gains from using VT futures only incur in the 2008 financial crisis period, but without substantial improvements to the upside despite the lower strategy costs involved during the normal market environments. In contrast, the VIX futures portfolio stands out in terms of the unit mean monthly return and unit ESR during the normal market period.

The LVOLH strategy typically requires position allocations that are conversely related to the magnitudes of negative carry costs and price movements of the hedging instruments. On the one hand, the VT futures are far less liquid than the VIX futures (Huang and Zhang, 2010), and their implied negative costs of carry could be very expensive, as shown in Table 3. The implied negative carry is caused by a theta decaying rate on the premia of SPX options used to replicate the VT futures. Both features of the VT futures results in fewer positions required for hedging a 100-lot SPX ETF. In contrast, 10% OTM VIX calls are among the cheapest liquid options available, which leads to more hedge positions reserved for hedging a 100-lot SPX ETF. Though the leverage on option premia can also magnify the effects of losses, a judicious use of the LVOLH strategy can help investors allocate hedging capital more efficiently.

In sum, the LVOLH strategy has produced reasonably consistent performance under almost all cases: the hedged portfolio using the LVOLH strategy has outsized gains during the Q4 2008 panic period and also participated in the Q1-Q3 2009 relatively calm periods. The LVOLH strategy is able to keep costs low under normal conditions in the form of lower effective hedging costs as well as higher minimum, mean and maximum monthly returns by allocating capital to hedging positions based on the volatility exhibited in and implied by the market. Therefore, the LVOLH strategy could be an acceptable hedging scheme among practitioners and academics.

4. Conclusion

Given the growing popularity of contracts deriving their values from the implied and realized volatilities of the SPX, it is important to develop effective and cost-efficient hedging strategies for these types of products. Previous studies have looked at a strategy of continuously buying SPX puts to protect a portfolio. While this is a viable method, the costs of the hedging would be expensive over time making the strategy a less-than-optimum deployment of funds.

This study explicitly identifies costs of carry for holding such volatility contracts. The effective hedging costs are calculated as the explicit upfront premia for options or the implicit costs of carry for volatility futures that are measured by the market prices in excess of their replication costs. By allocating capital to hedging positions based on the volatility exhibited in and implied by the market, one is able to keep costs low under normal conditions in the form of higher minimum and mean monthly returns. The strategy also provides significant benefits with reasonable transaction costs in the presence of tail risks and abnormal market environments. The allocation pattern of the volatility capital is governed by Fibonacci sequence.

The study examines using CBOE VIX futures, VT futures, 10% OTM VIX calls, and 10% OTM SPX puts as extreme downside equity hedges, and compares their effectiveness per unit of effective hedging costs. By replicating a dynamic allocation strategy with reasonable costs of rolling contracts, the empirical results show that using CBOE volatility contracts as extreme downside hedges, when combined with the

LVOLH strategy, can be a viable alternative to buying a series of OTM SPX puts. In particular, using 10% OTM VIX calls presents a cost-effective choice as hedging instruments to protect against market downside losses and to preserve upside gains. Further, the VIX term structure effects on any perceived carry issues are centered primarily on the negative roll yield caused by contango in the VIX futures market. The adoption of the LVOLH strategy, however, makes VIX futures substantially effective as a desirable hedging instrument with reasonable strategy costs even in the absence of tail risks and abnormal market environments. Finally, the implicit cost for the VT futures hedge depends on the term structure of volatility. This rollover P&Ls for the VT futures contracts will be negative in periods of low or decreasing volatility, and will be positive in periods of high or increasing volatility. In the long run, the rollover effect is a negative. There are, however, much better hedges found in the market when accompanied with the LVOLH strategy, which are cheaper if this strategy is well adopted.

In sum, this study reduces costs during normal market environments while retaining the effectiveness of the hedge when conditions are abnormal by adopting a rule-based approach to allocation. From September 15 to December 31, 2008 — a raging bear market by any definition — the passive SPX ETF underperforms every timely volatility hedge strategy by at least a whopping 11.14% in its monthly return. Volatility hedge strategies, however, perform differently with the 2008 financial crisis excluded. Timely hedge strategies on VT futures and 10% OTM VIX calls suffer slightly, but a timely hedge strategy on VIX futures performs superiorly, and also, a 10% OTM SPX put portfolio outperforms the SPX in risk-adjusted terms. Therefore, if the stock market outlook is bearish, CBOE volatility contracts should be an attractive asset class compared to buy-and-hold or long-only SPX ETF. Conversely, if the stock market outlook is bullish, the proposed timely strategy should reasonably give the hedger to expect a volatility contract to preserve the index.

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Table 1 Volatility Hedging Capital Allocations of the LVOLH Strategy

The weighting of the volatility component in a hedged portfolio is determined in accordance with the pre-defined weightings set forth below. The MTM value of the ETF portfolio times each of weightings w divided by $(1-w)$ is the allocated hedging capital to the volatility instruments.

Realized Volatility ($RVOL_{t-1}$)	Target Volatility Component Allocation: Implied Volatility Downtrend ($VIXtrend_{t-1}=-1$)	No Implied Volatility Trend ($VIXtrend_{t-1}=0$)	Implied Volatility Uptrend ($VIXtrend_{t-1}=+1$)
$RVOL_{t-1} < 10\%$	0%	5%	10%
$10\% \leq RVOL_{t-1} < 20\%$	5%	10%	15%
$20\% \leq RVOL_{t-1} < 35\%$	10%	15%	25%
$35\% \leq RVOL_{t-1} < 45\%$	15%	25%	40%
$45\% \leq RVOL_{t-1}$	25%	40%	65%

Table 2 Distribution of Bid-Ask Spreads

This table provides summary statistics for bid-ask spreads of volatility contracts. Monthly-rolling daily spreads are calculated covering the full sample period from February 24, 2006 to September 9, 2009, and the 2008 financial crisis period from September 15 to December 31, 2008. The unit of bid-ask spreads is the dollar premium quote. The multipliers for volatility contracts are illustrated as follows: the contract size of VIX futures is \$1,000 times the VIX; the contract multiplier for the VT futures is \$50 per variance point; and one point of SPX options and VIX options equals \$100. The figures in parentheses are spread ratios $s\%$, calculated as $s(t)\% = 100 \times BA(t)/p(t)$, where $p(t)$ indicates either day-t settlement prices of VIX and VT futures or the midpoints of SPX puts and VIX calls.

\$Bid-Ask Spread (Spread Ratio $s\%$)	VIX futures	VT futures	10% OTM VIX calls	10% OTM SPX puts
Panel A. Full Sample (February 24, 2006 – September 9, 2009)				
N	892	892	892	892
M	\$ 109.96 (0.50%)	\$ 2,677.24 (8.93%)	\$ 22.03 (9.26%)	\$ 97.03 (47.02%)
Mdn	90.00 (0.37)	1,000.00 (7.56)	15.00 (7.41)	50.00 (30.00)
Max	1,550.00 (6.09)	25,000.00 (43.27)	160.00 (66.67)	1,500.00 (200.00)
Min	10.00 (0.02)	100.00 (0.35)	5.00 (0.72)	5.00 (1.79)
SE	110.29 (0.42)	3,406.58 (5.64)	18.67 (7.14)	146.75 (44.03)
$Skewness$	4.70 (3.76)	2.16 (1.65)	2.67 (3.22)	4.25 (1.74)
$Kurtosis$	43.87 (38.63)	8.78 (6.38)	13.32 (18.93)	28.99 (5.77)
Panel B. Lehman Brothers Bankruptcy (September 15, 2008 – December 31, 2008)				
N	76	76	76	76
M	254.61 (0.52)	8,827.30 (5.13)	59.93 (6.44)	415.66 (18.15)
Mdn	230.00 (0.45)	8,125.00 (4.96)	60.00 (5.89)	380.00 (15.09)
Max	990.00 (1.75)	25,000.00 (11.61)	160.00 (17.14)	1,500.00 (52.63)
Min	10.00 (0.02)	250.00 (0.73)	5.00 (1.87)	30.00 (3.24)
SE	196.49 (0.38)	5,013.97 (2.16)	31.11 (3.00)	296.59 (10.11)
$Skewness$	1.10 (0.97)	0.62 (0.54)	0.90 (1.23)	1.46 (1.15)
$Kurtosis$	4.29 (3.53)	3.88 (3.20)	3.56 (5.06)	5.85 (4.17)

Table 3 Costs of Carry of Volatility Contracts

The table reports descriptive statistics on both the explicit and implicit costs of carry denominated in dollars as applied to a unit of volatility contract purchased on each trading day. The explicit cost of carry for a long option hedge is the upfront premium, while the implicit cost of carry for a long futures hedge is calculated as the price difference between the futures and its replicated forwards. Daily ask prices of options and futures, and daily bid prices of $VIXTerm$ are used over the period from February 24, 2006 to September 9, 2009 for the monthly rolls. The costs of carry with and without the Q4 2008 panic period and the Q1–Q3 2009 relatively calm periods are also separately tabulated in Panels B and C, respectively.

Volatility Contract	*Cost of carry*	*N*	*M*	*Mdn*	*Max*	*Min*	*SE*	*Skewness*	*Kurtosis*
Panel A. Full Sample (February 24, 2006 − September 9, 2009)									
VIX futures	*Ask price*	892	$24,076.2780	$22,145.0000	$66,400.0000	$10,570.0000	$12,123.1711	1.2011	3.7984
	Bid price of synthetic forwards	892	20,833.4896	19,180.3000	63,818.0000	7,350.5000	10,934.2709	1.3335	4.5900
	Cost of carry	892	3,242.7884	2,702.1000	18,119.8000	8.8749	2,314.0720	1.9514	9.3808
VT futures	*Ask price*	892	38,379.0191	20,437.5000	311,325.0000	3,075.0000	55,557.3162	2.9514	11.7788
	Bid price of synthetic forwards	892	35,240.2470	19,092.8400	280,578.2150	2,837.4800	51,553.2582	3.0007	12.1723
	Cost of carry	892	3,138.7721	1,264.1425	35,569.7750	3.8345	4,853.3350	3.4048	17.1838
10% *OTM VIX calls*	*Ask price*	892	142.3655	115.0000	790.0000	15.0000	100.0113	2.2095	10.3576
10% *OTM SPX puts*	*Ask price*	892	1,226.6312	890.0000	6,630.0000	20.0000	1,163.5844	1.4922	5.5230
Panel B. Exclding Fall 2008 and the 2009 Market Rally (February 24, 2006 − September 12, 2008)									
VIX futures	*Ask price*	643	17,985.5832	16,000.0000	28,850.0000	10,570.0000	5,341.1656	0.3330	1.5970
	Bid price of synthetic forwards	643	15,590.4942	14,203.9000	28,118.7000	7,350.5000	5,120.1521	0.4076	1.8624
	Cost of carry	643	2,395.0891	2,258.4000	15,212.1000	8.8749	1,433.2721	2.3969	17.4235
VT futures	*Ask price*	643	1,5008.5537	1,1750.0000	41,625.0000	3,075.0000	9,284.1879	0.5543	2.0745
	Bid price of synthetic forwards	643	1,3712.9724	11,115.0900	39,051.6750	2,837.4800	8,498.7778	0.5780	2.1134
	Cost of carry	643	1,295.5812	836.8700	6,174.9450	3.8345	1,108.6440	1.3362	4.3082
10% *OTM VIX calls*	*Ask price*	643	106.8196	100.0000	330.0000	15.0000	52.8410	0.9304	4.0035
10% *OTM SPX puts*	*Ask price*	643	923.0871	600.00	4,400.0000	20.0000	894.6941	1.3211	4.2780
Panel C. Lehman Brothers Bankruptcy (September 15, 2008 − September 9, 2009)									
VIX futures	*Ask price*	249	39,804.4578	40,090.0000	66,400.0000	24,780.0000	10,470.1526	0.3572	2.0373
	Bid price of synthetic forwards	249	34,372.6301	34,279.3000	63,818.0000	20,046.3000	10,312.9620	0.6827	2.7321
	Cost of carry	249	5,431.8273	5,102.0000	18,119.8000	245.1600	2,691.9566	1.3126	6.8404
VT futures	*Ask price*	249	98,729.2570	77,275.0000	311,325.0000	22,950.0000	76,113.9812	1.3529	3.5194
	Bid price of synthetic forwards	249	90,830.7591	70,723.3800	28,0578.2150	18,848.1950	71,114.5850	1.3859	3.6730
	Cost of carry	249	7,898.4979	5,755.9200	35,569.7750	13.1945	7,063.2836	1.7235	6.1725
10% *OTM VIX calls*	*Ask price*	249	234.1566	215.0000	790.0000	40.0000	130.2761	1.3679	5.5494
10% *OTM SPX puts*	*Ask price*	249	2,010.4819	1,660.0000	6,630.0000	95.0000	1,391.5173	1.0846	3.8537

Table 4 Monthly Returns for Unhedged SPX ETF and $LVOLH$-Hedged Portfolios

The table analyzes monthly returns on two portfolios, the unhedged SPX ETF and the SPX ETF hedged with the $LVOLH$ strategy using a CBOE volatility contract. Specifically, for each of hedged portfolios, the volatility contract used could be the VIX futures, VT futures, 10% OTM VIX call options, or 10% OTM SPX put options. The unhedged SPX ETF is a portfolio of holding one 100-lot unit of the S&P 500 in dollars. To mitigate the effect of non-normality, Panel B reports several risk-adjusted measures, including the percentage maximum drawdown, denoted $\%MaxDD$, the conditional Value-at-Risk computed using the Cornish-Fisher expansion at 95% and 99%, denoted $CVaR(95\%)$ and $CVaR(99\%)$ as well as the extended Sharpe ratio, denoted ESR. $\$PV(ICC)$ denotes the present value of total N_{ICC} explicit/implicit costs of carry, measured in million dollars. The full out-of-sample data period starts from February 24, 2006 to September 9, 2009. The hedging performance with/without the Q4 2008 panic period and the Q1-Q3 2009 relatively calm periods are separately tabulated.

Panel A. Monthly Return Distribution	N	Min	1%	5%	10%	Mdn	90%	95%	99%	Max
Full Sample (February 24, 2006 − September 9, 2009)										
Unhedged SPX ETF	43	-27.3176	-27.3176	-11.2350	-7.5050	1.0540	4.6114	5.8602	9.9129	9.9129
LVOLH-hedged VIX futures portfolio	43	-4.8916	-4.8916	-4.5936	-3.9143	0.5305	2.8805	9.6220	126.3934	126.3934
LVOLH-hedged VT futures portfolio	43	-10.1128	-10.1128	-7.3338	-6.0751	0.7727	3.2038	4.0131	120.8131	120.8131
LVOLH-hedged VIX call portfolio	43	-9.2271	-9.2271	-6.4649	-5.8131	0.8237	3.2879	4.0735	103.2279	103.2279
LVOLH-hedged SPX put portfolio	43	-15.8701	-15.8701	-7.1317	-6.0671	-0.6150	4.1299	6.3230	313.7220	313.7220
Excluding the 2008 Financial Crisis and the 2009 Market Rally (February 24, 2006 − September 12, 2008)										
Unhedged SPX ETF	32	-9.1103	-9.1103	-7.0120	-5.9235	1.3383	3.8277	4.4541	5.2913	5.2913
LVOLH-hedged VIX futures portfolio	32	-4.8916	-4.8916	-4.6641	-4.2021	0.3995	2.2621	3.9500	4.3951	4.3951
LVOLH-hedged VT futures portfolio	32	-8.5605	-8.5605	-6.6336	-5.7879	0.7737	3.2201	3.7786	4.3569	4.3569
LVOLH-hedged VIX call portfolio	32	-9.2271	-9.2271	-6.7270	-6.2441	0.6987	3.3121	3.8928	4.3241	4.3241
LVOLH-hedged SPX put portfolio	32	-7.3538	-7.3538	-6.9238	-6.0748	-0.6226	3.7223	4.4047	4.4940	4.4940
The Fall 2008 and the 2009 Market Rally (September 15, 2008 − September 9, 2009)										
Unhedged SPX ETF	12	-23.8836	-23.8836	-22.8582	-16.7056	-1.2387	7.8157	9.6133	9.9129	9.9129
LVOLH-hedged VIX futures portfolio	12	-3.8795	-3.8795	-3.6993	-2.6178	0.6791	53.8339	122.8433	134.3449	134.3449
LVOLH-hedged VT futures portfolio	12	-10.1128	-10.1128	-9.7040	-7.2512	0.6510	41.3045	119.0151	131.9669	131.9669
LVOLH-hedged VIX call portfolio	12	-5.0794	-5.0794	-5.0705	-5.0171	0.8024	34.7871	99.1149	109.8363	109.8363
LVOLH-hedged SPX put portfolio	12	-15.8701	-15.8701	-14.6643	-7.4295	-0.2205	106.5264	300.1395	332.4083	332.4083

Panel B. Risk Characteristics	M	SE	$Skewness$	$Kurtosis$	$\%MaxDD$	$CVaR(95\%)$	$CVaR(99\%)$	ESR	N_{ICC}	$\$PV(ICC)$
Full Sample (February 24, 2006 − September 9, 2009)										
Unhedged SPX ETF	-0.9157	6.3350	-1.8330	8.3519	616.2748	-19.6657	-28.7716	-3.0847	57	NA

LVOLH-hedged VIX futures portfolio	3.2343	19.5929	5.9839	38.0733	299.0536	97.9652	209.9187	4.3561	57	9,447.7270
LVOLH-hedged VT futures portfolio	1.9717	18.9111	5.9532	38.0291	326.6880	91.6402	197.1330	4.1363	57	1,676.8887
LVOLH-hedged VIX call portfolio	1.6505	16.2191	5.8892	37.5164	313.3851	76.2824	164.0296	3.6352	56	4,126.1659
LVOLH-hedged SPX put portfolio	6.1256	48.2188	6.2463	40.3695	301.2663	266.8934	575.0133	9.3288	57	11,690.7046
Excluding the 2008 Financial Crisis and the 2009 Market Rally (February 24, 2006 − September 12, 2008)										
Unhedged SPX ETF	-0.2547	3.9521	-0.6204	2.1433	322.0689	-8.8991	-10.5164	-0.8246	42	NA
LVOLH-hedged VIX futures portfolio	-0.1870	2.4747	-0.3339	2.3992	299.0536	-5.4441	-6.5940	-0.3425	42	4,487.0707
LVOLH-hedged VT futures portfolio	-0.6485	3.5803	-0.6179	2.1095	326.6880	-8.4621	-9.8957	-0.8549	41	566.7057
LVOLH-hedged VIX call portfolio	-0.8723	3.6271	-0.5603	2.2136	313.3851	-8.7765	-10.3371	-0.8598	41	1,979.5756
LVOLH-hedged SPX put portfolio	-1.1296	3.6846	-0.1334	1.7204	301.2663	-8.3189	-9.2040	-0.4623	42	3,863.4023
The Fall 2008 and the 2009 Market Rally (September 15, 2008 − September 9, 2009)										
Unhedged SPX ETF	-2.2985	9.5164	-0.9576	3.2745	1049.9530	-25.0769	-31.2249	-2.9069	15	NA
LVOLH-hedged VIX futures portfolio	12.7733	38.7267	2.9014	9.6448	102.8877	35.5745	111.0506	100.5345	15	4,960.6563
LVOLH-hedged VT futures portfolio	9.7474	38.6783	2.9613	9.9017	107.6631	35.5771	114.2406	107.1868	16	1,110.1831
LVOLH-hedged VIX call portfolio	8.8389	31.9150	2.9778	9.9580	104.6245	30.9119	96.7011	90.3497	15	2,146.5903
LVOLH-hedged SPX put portfolio	26.5485	96.4931	2.9956	10.0213	104.7743	95.7566	297.5098	276.9426	15	7,827.3023

Table 5 Hedging Performance per Unit of Effective Hedging Costs

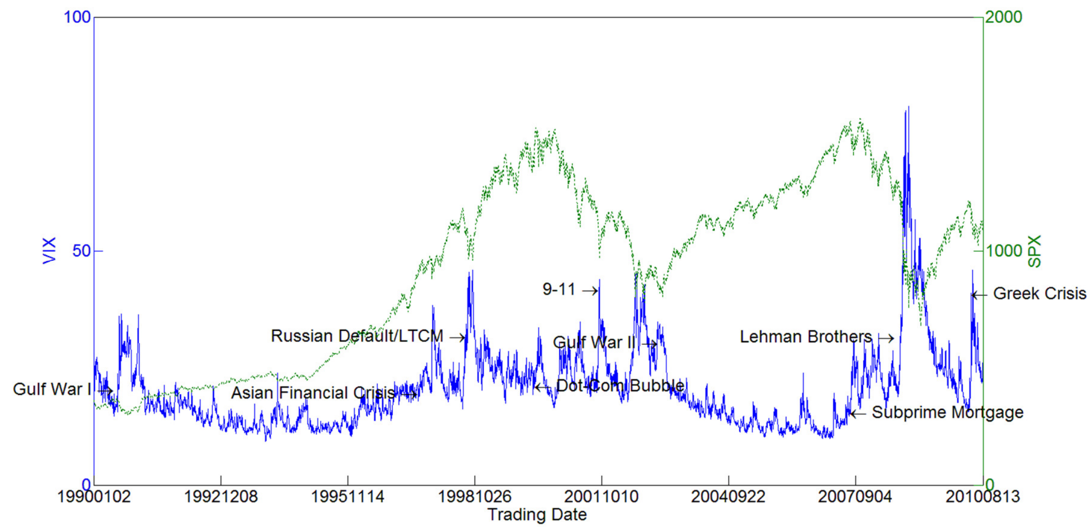
This table reports hedging effectiveness of alternative $LVOLH$-hedged portfolios that is analyzed based on monthly mean returns, enhanced extended Sharpe ratio (ESR) and risk reduction in adequate risk measures per unit of effective hedging costs, denoted HE_{unit}. Effective hedging costs are measured by the negative costs of carry on volatility instruments. The explicit cost of carry for a long option hedge is the premium at open ask on each rebalance and roll day. Since the hedger pays no explicit upfront premium, the implicit cost of carry for a long futures hedge is the futures price in excess of its replicating cost calculated from a synthetic forwards contract. The risk measures include percentage maximum drawdown ($\%MaxDD$) and Cornish-Fisher conditional Value-at-Risk ($CVaR$) computed at 95% and 99%. $PV(ICC)$ denotes the present value of total explicit/implicit costs of carry, measured in ten thousand dollars ($1,000). The first row in each Panel gives the statistics on the unhedged SPX ETF. The remaining rows give the statistics on the hedged portfolio under the five performance measures used, ranked by the symbol $Rank$ starting from the most effective hedging instrument. The sample period is from February 24, 2006 to September 9, 2009. The hedging performance with/without the Q4 2008 panic period and the Q1-Q3 2009 relatively calm periods are also separately tabulated.

	N	$Measure$	$Rank_{Measure}$	$\$PV(ICC)$ ($unit$=$1,000)	$Rank_{ICC}$	HE_{unit}	$Rank_{HE_{unit}}$
$Full\ Sample\ (February\ 24, 2006 - September\ 9, 2009)$							
$Panel\ A.\ Measure = Monthly\ Return\ Mean$ & $HE_{unit} = \Delta Mean/\$PV(ICC)$							
$Unhedged\ SPX\ ETF$	43	-0.9157	5	NA	NA	NA	NA
$LVOLH$-$hedged\ VIX\ futures\ portfolio$	43	3.2343	2	9.4477	3	0.4393	4
$LVOLH$-$hedged\ VT\ futures\ portfolio$	43	1.9717	3	1.6769	1	1.7219	1
$LVOLH$-$hedged\ VIX\ call\ portfolio$	43	1.6505	4	4.1262	2	0.6219	2
$LVOLH$-$hedged\ SPX\ put\ portfolio$	43	6.1256	1	11.6907	4	0.6023	3
$Panel\ B.\ Measure = Extended\ Sharpe\ Ratio$ & $HE_{unit} = \Delta ESR/\$PV(ICC)$							
$Unhedged\ SPX\ ETF$	43	-3.0847	5	NA	NA	NA	NA
$LVOLH$-$hedged\ VIX\ futures\ portfolio$	43	4.3561	2	9.4477	3	0.7876	4
$LVOLH$-$hedged\ VT\ futures\ portfolio$	43	4.1363	3	1.6769	1	4.3062	1
$LVOLH$-$hedged\ VIX\ call\ portfolio$	43	3.6352	4	4.1262	2	1.6286	2
$LVOLH$-$hedged\ SPX\ put\ portfolio$	43	9.3288	1	11.6907	4	1.0618	3
$Panel\ C.\ Measure = Percentage\ Maximum\ Drawdown$ & $HE_{unit} = \nabla\%MaxDD/\$PV(ICC)$							
$Unhedged\ SPX\ ETF$	43	616.2748	5	NA	NA	NA	NA
$LVOLH$-$hedged\ VIX\ futures\ portfolio$	43	299.0536	1	9.4477	3	33.5765	3
$LVOLH$-$hedged\ VT\ futures\ portfolio$	43	326.6880	4	1.6769	1	172.6929	1
$LVOLH$-$hedged\ VIX\ call\ portfolio$	43	313.3851	3	4.1262	2	73.4071	2
$LVOLH$-$hedged\ SPX\ put\ portfolio$	43	301.2663	2	11.6907	4	26.9452	4
$Panel\ D.\ Measure = Cornish-Fisher\ CVaR(95\%)$ & $HE_{unit} = \nabla CVaR(95\%)/\$PV(ICC)$							
$Unhedged\ SPX\ ETF$	43	-19.6657	5	NA	NA	NA	NA
$LVOLH$-$hedged\ VIX\ futures\ portfolio$	43	97.9652	3	9.4477	3	12.4507	4
$LVOLH$-$hedged\ VT\ futures\ portfolio$	43	91.6402	1	1.6769	1	66.3764	1

LVOLH-hedged VIX call portfolio	43	76.2824	4	4.1262	2	23.2536	3
LVOLH-hedged SPX put portfolio	43	266.8934	2	11.6907	4	24.5117	2
Panel E. $\boldsymbol{Measure = Cornish - Fisher\ CVaR(99\%)}$ & $HE_{unit} = \nabla CVaR(99\%)/\$PV(ICC)$							
Unhedged SPX ETF	43	-28.7716	5	NA	NA	NA	NA
LVOLH-hedged VIX futures portfolio	43	209.9187	3	9.4477	3	25.2643	4
LVOLH-hedged VT futures portfolio	43	197.1330	1	1.6769	1	134.7165	1
LVOLH-hedged VIX call portfolio	43	164.0296	4	4.1262	2	46.7265	3
LVOLH-hedged SPX put portfolio	43	575.0133	2	11.6907	4	51.6466	2
Excluding the 2008 Financial Crisis and the 2009 Market Rally (February 24, 2006 − September 12, 2008)							
Panel A. $\boldsymbol{Measure = Monthly\ Return\ Mean}$ & $HE_{unit} = \Delta Mean/\$PV(ICC)$							
Unhedged SPX ETF	32	-0.2547	2	NA	NA	NA	NA
LVOLH-hedged VIX futures portfolio	32	-0.1870	1	4.4871	4	0.0151	1
LVOLH-hedged VT futures portfolio	32	-0.6485	3	0.5667	1	-0.6950	4
LVOLH-hedged VIX call portfolio	32	-0.8723	4	1.9796	2	-0.3120	3
LVOLH-hedged SPX put portfolio	32	-1.1296	5	3.8634	3	-0.2265	2
Panel B. $\boldsymbol{Measure = Extended\ Sharpe\ Ratio}$ & $HE_{unit} = \Delta ESR/\$PV(ICC)$							
Unhedged SPX ETF	32	-0.8246	3	NA	NA	NA	NA
LVOLH-hedged VIX futures portfolio	32	-0.3425	1	4.4871	4	0.1074	1
LVOLH-hedged VT futures portfolio	32	-0.8549	4	0.5667	1	-0.0536	4
LVOLH-hedged VIX call portfolio	32	-0.8598	5	1.9796	2	-0.0178	3
LVOLH-hedged SPX put portfolio	32	-0.4623	2	3.8634	3	0.0938	2
Panel C. $\boldsymbol{Measure = Percentage\ Maximum\ Drawdown}$ & $HE_{unit} = \nabla\%MaxDD/\$PV(ICC)$							
Unhedged SPX ETF	32	322.0689	4	NA	NA	NA	NA
LVOLH-hedged VIX futures portfolio	32	299.0536	1	4.4871	4	5.1293	2
LVOLH-hedged VT futures portfolio	32	326.6880	5	0.5667	1	-8.1508	4
LVOLH-hedged VIX call portfolio	32	313.3851	3	1.9796	2	4.3867	3
LVOLH-hedged SPX put portfolio	32	301.2663	2	3.8634	3	5.3845	1
Panel D. $\boldsymbol{Measure = Cornish - Fisher\ CVaR(95\%)}$ & $HE_{unit} = \nabla CVaR(95\%)/\$PV(ICC)$							
Unhedged SPX ETF	32	-8.8991	4	NA	NA	NA	NA
LVOLH-hedged VIX futures portfolio	32	-5.4441	1	4.4871	4	0.7700	2
LVOLH-hedged VT futures portfolio	32	-8.4621	5	0.5667	1	0.7711	1
LVOLH-hedged VIX call portfolio	32	-8.7765	3	1.9796	2	0.0619	4
LVOLH-hedged SPX put portfolio	32	-8.3189	2	3.8634	3	0.1502	3
Panel E. $\boldsymbol{Measure = Cornish - Fisher\ CVaR(99\%)}$ & $HE_{unit} = \nabla CVaR(99\%)/\$PV(ICC)$							
Unhedged SPX ETF	32	-10.5164	4	NA	NA	NA	NA

LVOLH-hedged VIX futures portfolio	32	-6.5940	1	4.4871	4	0.8742	2
LVOLH-hedged VT futures portfolio	32	-9.8957	5	0.5667	1	1.0953	1
LVOLH-hedged VIX call portfolio	32	-10.3371	3	1.9796	2	0.0906	4
LVOLH-hedged SPX put portfolio	32	-9.2040	2	3.8634	3	0.3397	3
The 2008 Financial Crisis and the 2009 Market Rally (September 15, 2008 − September 9, 2009)							
Panel A. Measure = Monthly Return Mean & $HE_{unit} = \Delta Mean/\$PV(ICC)$							
Unhedged SPX ETF	12	-2.2985	5	NA	NA	NA	NA
LVOLH-hedged VIX futures portfolio	12	12.7733	2	4.9607	3	3.0383	4
LVOLH-hedged VT futures portfolio	12	9.7474	3	1.1102	1	10.8503	1
LVOLH-hedged VIX call portfolio	12	8.8389	4	2.1466	2	5.1884	2
LVOLH-hedged SPX put portfolio	12	26.5485	1	7.8273	4	3.6854	3
Panel B. Measure = Extended Sharpe Ratio & $HE_{unit} = \Delta ESR/\$PV(ICC)$							
Unhedged SPX ETF	12	-2.9069	5	NA	NA	NA	NA
LVOLH-hedged VIX futures portfolio	12	100.5345	3	4.9607	3	20.8524	4
LVOLH-hedged VT futures portfolio	12	107.1868	2	1.1102	1	99.1672	1
LVOLH-hedged VIX call portfolio	12	90.3497	4	2.1466	2	43.4441	2
LVOLH-hedged SPX put portfolio	12	276.9426	1	7.8273	4	35.7530	3
Panel C. Measure = Percentage Maximum Drawdown & $HE_{unit} = \nabla\%MaxDD/\$PV(ICC)$							
Unhedged SPX ETF	12	1,049.9530	5	NA	NA	NA	NA
LVOLH-hedged VIX futures portfolio	12	102.8877	1	4.9607	3	190.9153	3
LVOLH-hedged VT futures portfolio	12	107.6631	4	1.1102	1	848.7698	1
LVOLH-hedged VIX call portfolio	12	104.6245	2	2.1466	2	440.3861	2
LVOLH-hedged SPX put portfolio	12	104.7743	3	7.8273	4	120.7541	4
Panel D. Measure = Cornish − Fisher CVaR(95%) & $HE_{unit} = \nabla CVaR(95\%)/\$PV(ICC)$							
Unhedged SPX ETF	12	-25.0769	5	NA	NA	NA	NA
LVOLH-hedged VIX futures portfolio	12	35.5745	3	4.9607	3	12.2265	4
LVOLH-hedged VT futures portfolio	12	35.5771	2	1.1102	1	54.6342	1
LVOLH-hedged VIX call portfolio	12	30.9119	4	2.1466	2	26.0826	2
LVOLH-hedged SPX put portfolio	12	95.7566	1	7.8273	4	15.4374	3
Panel E. Measure = Cornish − Fisher CVaR(99%) & $HE_{unit} = \nabla CVaR(99\%)/\$PV(ICC)$							
Unhedged SPX ETF	12	-31.2249	5	NA	NA	NA	NA
LVOLH-hedged VIX futures portfolio	12	111.0506	3	4.9607	3	28.6808	4
LVOLH-hedged VT futures portfolio	12	114.2406	2	1.1102	1	131.0285	1
LVOLH-hedged VIX call portfolio	12	96.7011	4	2.1466	2	59.5950	2
LVOLH-hedged SPX put portfolio	12	297.5098	1	7.8273	4	41.9985	3

Panel A. Time series of VIX and the S&P 500 index



Panel B. Time series of one-month historical volatility and the S&P 500 index

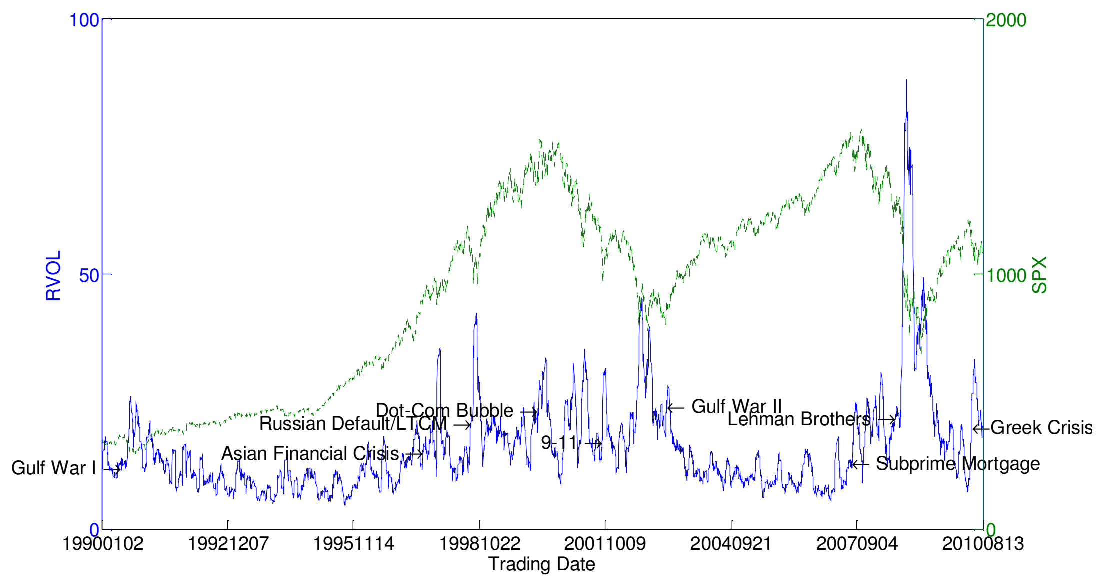


Figure 1. Financial turmoil (SPX market downturn) against investor sentiment (VIX) and market volatility ($\boldsymbol{RVOL}$). $RVOL$ at time t refers to one-month historical volatility, calculated as 100 multiplied by the square root of annualized mean-square SPX returns over t-22 trading days to t-1 trading days. Panel A (B) plots trading dates versus VIX ($RVOL$) with y-axis labeling on the left (solid line) and versus SPX with y-axis labeling on the right (dotted line).

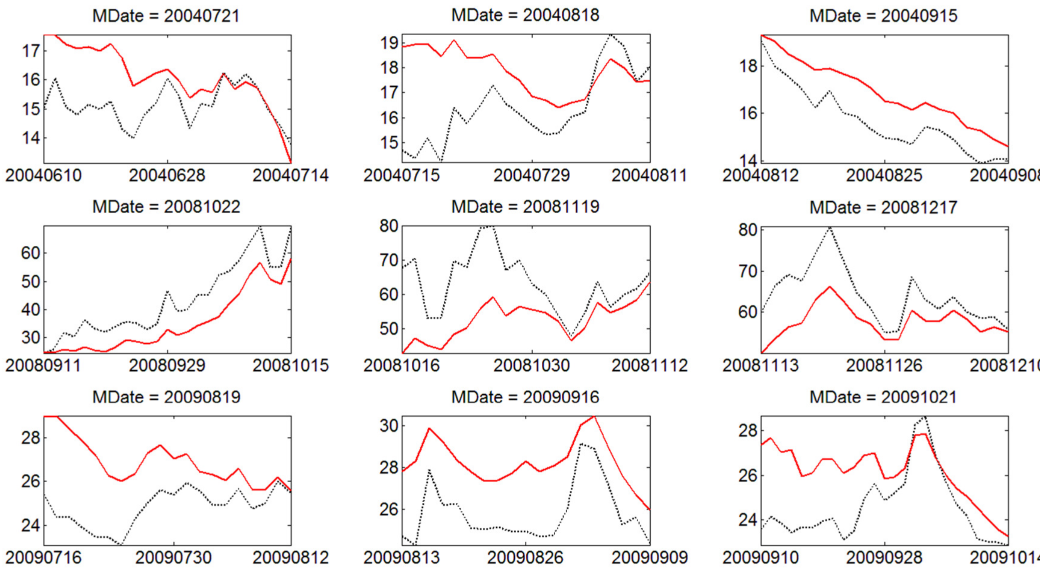


Figure 2. Spot VIX versus VIX futures across maturity months. The subplot of each contract month for VIX futures plots trading dates versus VIX futures prices with a solid line and versus spot VIX with a dotted line.

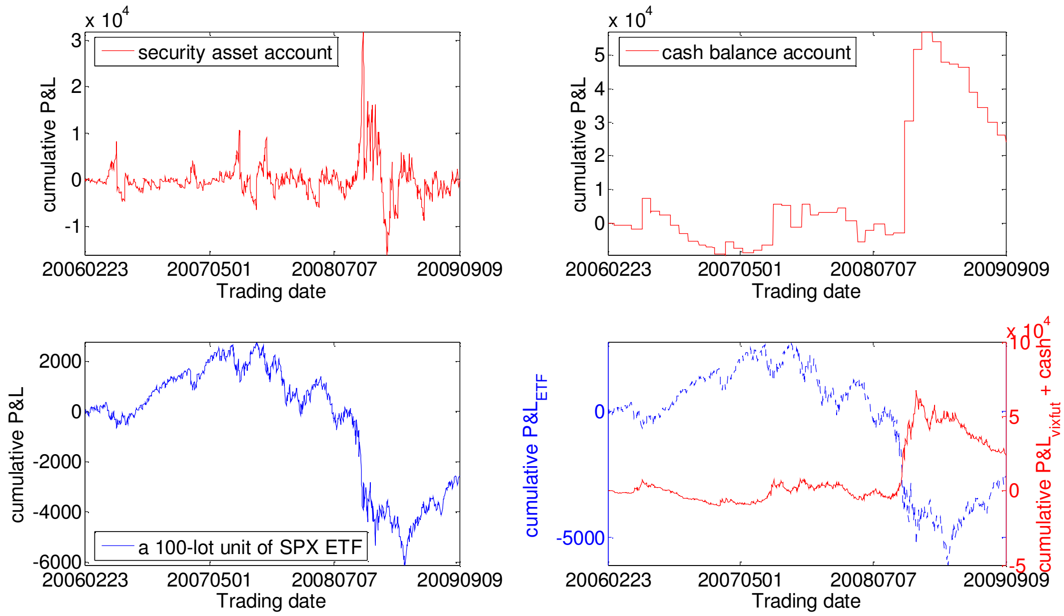


Figure 3. Cumulative dollar P&Ls of the ETF and VIX futures. The rolling strategy covers the period from February 24, 2006 to September 9, 2009. SPX ETF is valued at approximately 1/10th the value of the SPX and typically tend to be transacted in 100-lot (or round-lot) increments. The contract size of VIX futures is $1,000 times the VIX. $cumP\&L_{vixfut} + cash$ is the accumulation of the security asset and cash balance accounts of VIX futures. The fourth subplot plots trading dates versus cumulative P&L of a 100-lot unit of SPX ETF (dotted line) with y-axis labeling on the left, and versus cumulative P&Ls of VIX futures and the cash account (solid line) with y-axis labeling on the right.

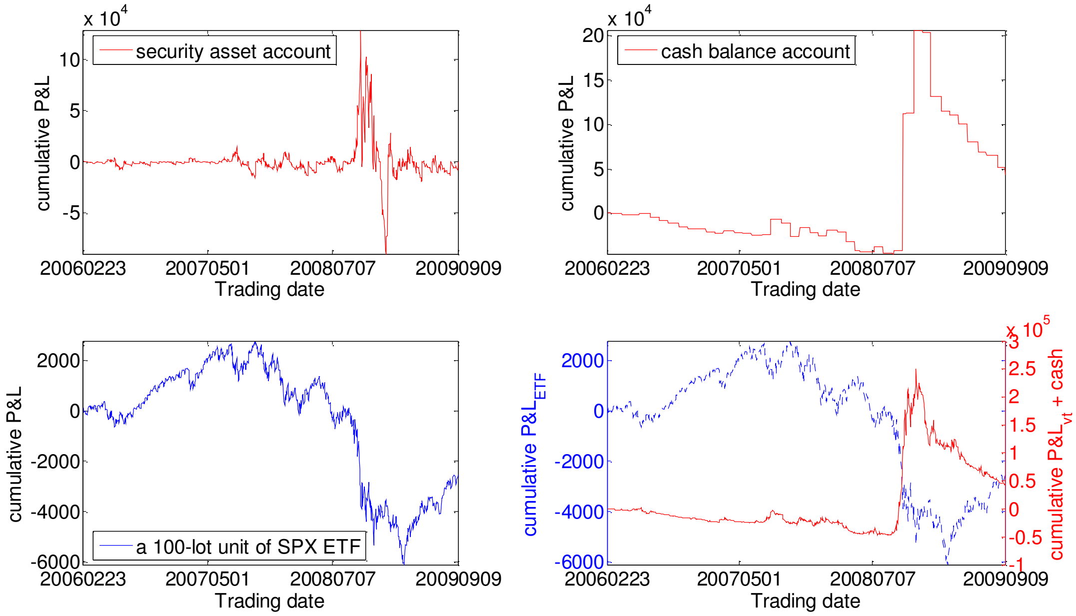


Figure 4. Cumulative dollar P&Ls of the ETF and VT futures. The rolling strategy covers the period from February 24, 2006 to September 9, 2009. SPX ETF is valued at approximately 1/10th the value of the SPX and typically tend to be transacted in 100-lot (or round-lot) increments. The contract multiplier for the VT futures is $50 per variance point. $cumP\&L_{VT} + cash$ is the accumulation of the security asset and cash accounts of VT futures. The fourth subplot plots trading dates versus cumulative P&L of a 100-lot unit of SPX ETF (dotted line) with y-axis labeling on the left, and versus cumulative P&L of VT futures and the cash account (solid line) with y-axis labeling on the right.

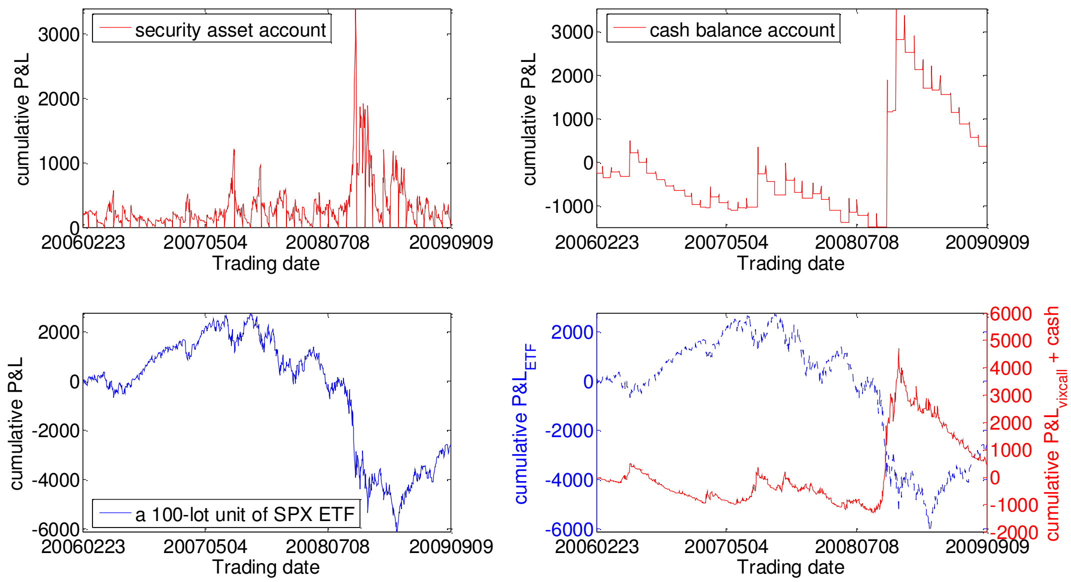


Figure 5. Cumulative dollar P&Ls of the ETF and 10% out-of-the-money VIX calls. The monthly rolling strategy covers the period from February 24, 2006 to September 9, 2009. SPX ETF is valued at approximately 1/10th the value of the SPX and typically tend to be transacted in 100-lot (or round-lot) increments. One point of VIX options equals $100. $cumP\&L_{vixcall} + cash$ is the accumulation of the security asset and cash accounts of SPX puts. The fourth subplot plots trading dates versus cumulative P&L of a 100-lot unit of SPX ETF (dotted line) with y-axis labeling on the left, and versus cumulative P&Ls of VIX calls plus the cash account (solid line) with y-axis labeling on the right.

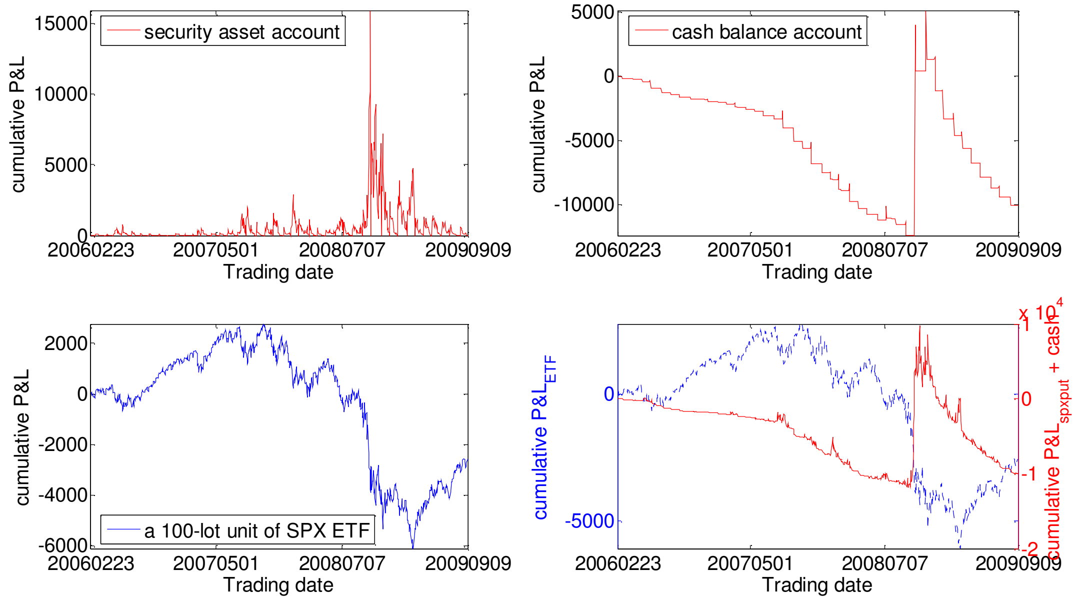


Figure 6. Cumulative dollar P&Ls of the ETF and 10% out-of-the-money SPX puts. The monthly rolling strategy covers the period from February 24, 2006 to September 9, 2009. SPX ETF is valued at approximately 1/10th the value of the SPX and typically tend to be transacted in 100-lot (or round-lot) increments. One point of SPX options equals $100. $cumP\&L_{spxput} + cash$ is the accumulation of the security asset and cash accounts of SPX puts. The fourth subplot plots trading dates versus cumulative P&L of a 100-lot unit of SPX ETF (dotted line) with y-axis labeling on the left, and versus cumulative P&Ls of SPX puts plus the cash account (solid line) with y-axis labeling on the right.

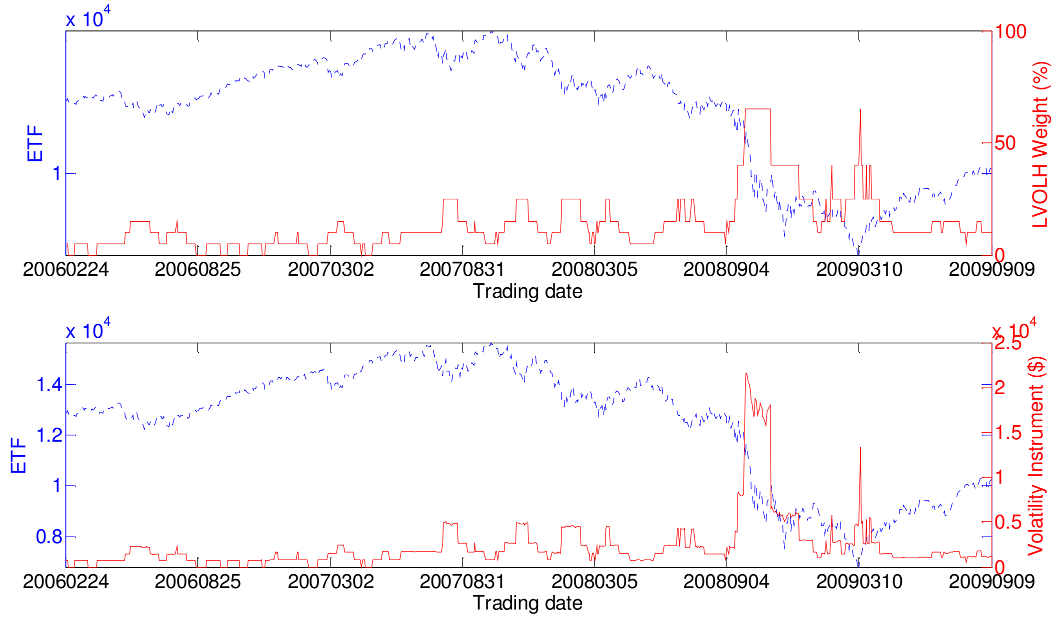
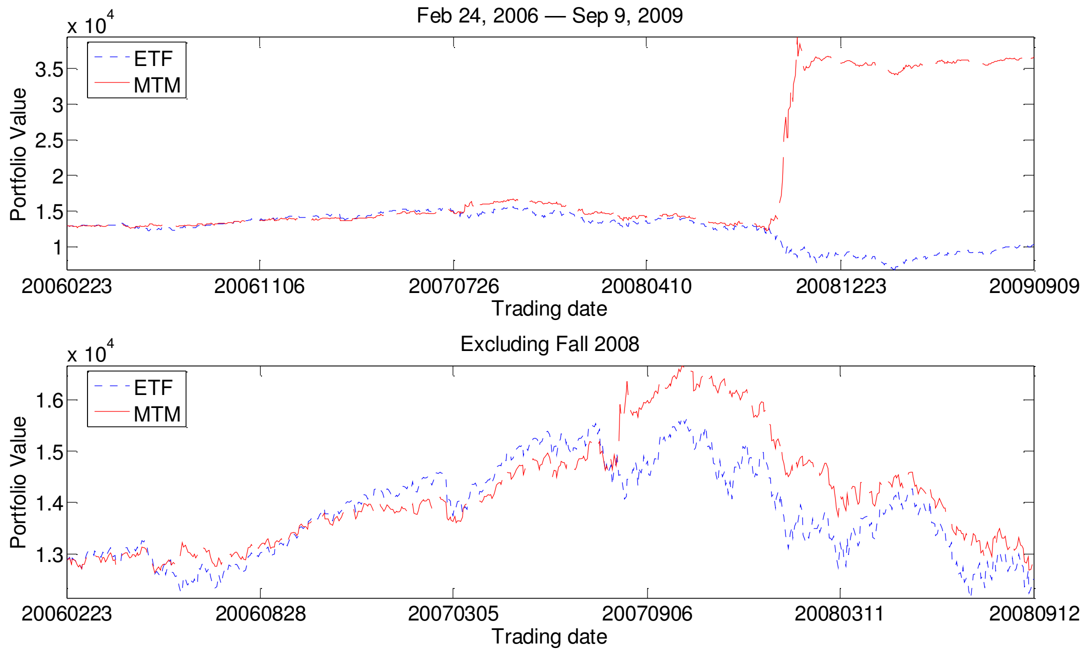


Figure 7. The MTM value of ETF and the pre-defined volatility hedging capital allocations for the LVOLH strategy. The hedging exercise covers the period from February 24, 2006 to September 9, 2009. SPX ETF is valued at approximately 1/10th the value of the SPX and typically tend to be transacted in 100-lot (or round-lot) increments. The two subplots plot trading dates versus the MTM value of a 100-lot unit of SPX ETF (dotted line) with y-axis labeling on the left, and versus pre-defined weightings and the volatility hedging capital allocations in dollars set forth in Table 1 for the LVOLH strategy with y-axis (solid line) labeling on the right. The MTM value of the ETF portfolio times each of weights w divided by $(1-w)$ is the allocated hedging capital to the volatility instruments.

Panel A. Mark-to-market of unhedged vs. hedged portfolios



Panel B. Monthly return distribution of unhedged vs. hedged portfolios

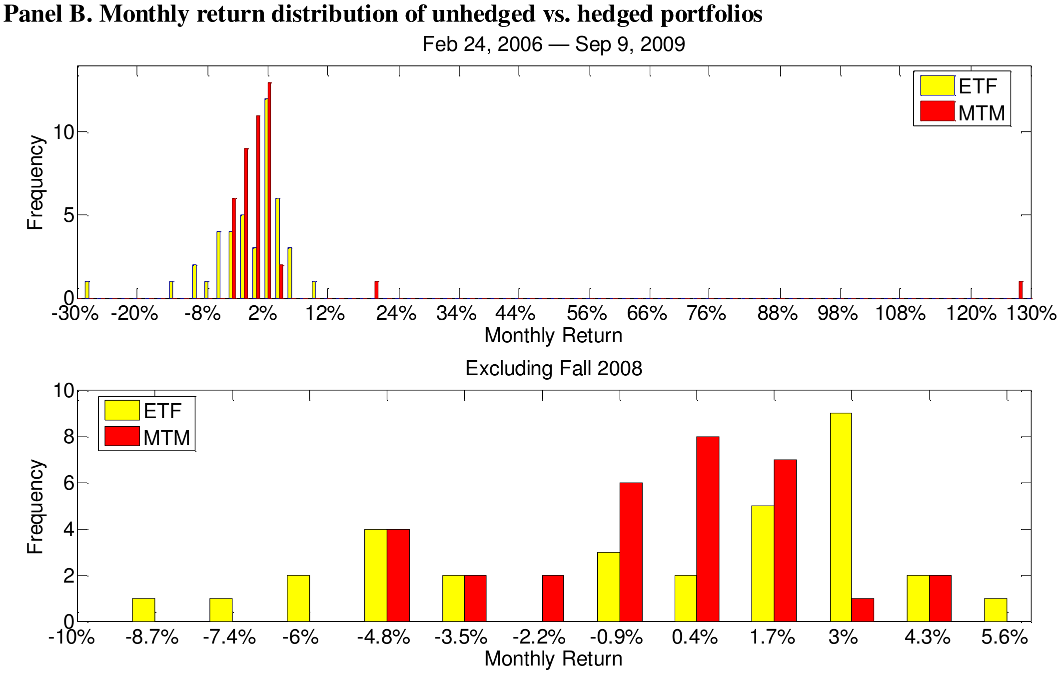
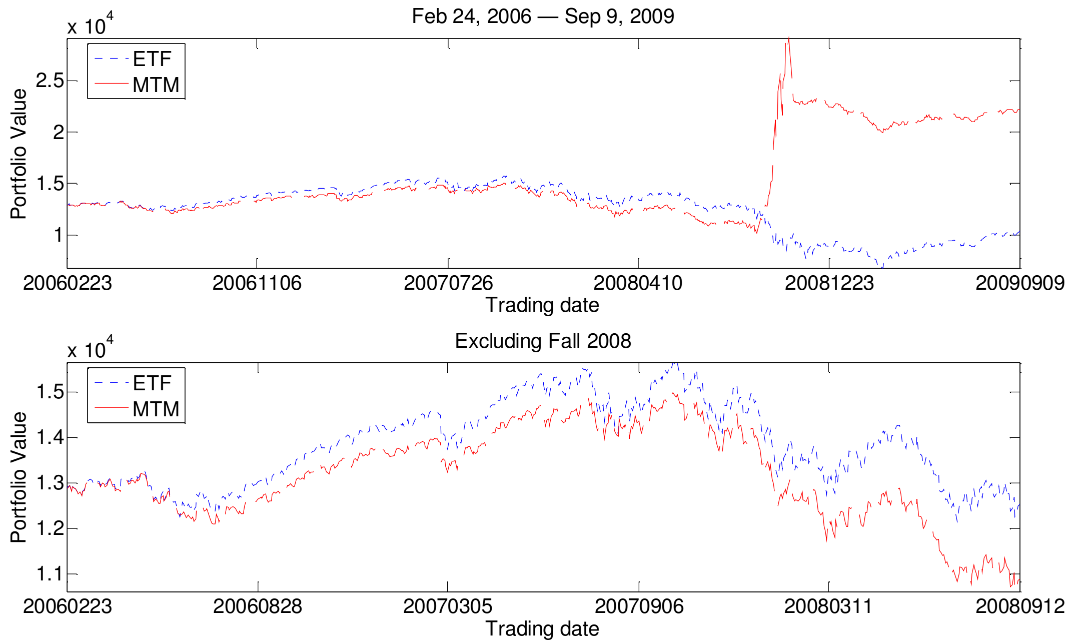


Figure 8. Out-of-sample VIX futures portfolio using the LVOLH strategy. The ETF lines in Panel A are the unhedged MTM of holding one 100-lot unit of the S&P 500 in dollars. The MTM lines are the hedged MTM by adopting LVOLH strategy to dynamically allocate hedging capitals to VIX futures. The rolling strategy covers the full sample period from February 24, 2006 to September 9, 2009, and the period with the Q4 2008 panic period and the Q1-Q3 2009 relatively calm periods excluded (denoted *Excluding Fall* 2008). The histograms of monthly returns on the unhedged vs. hedged portfolios are presented in Panel B. Each of the graphs is plotted across the full sample period and the period excluding Fall 2008.

Panel A. Mark-to-market of unhedged vs. hedged portfolios



Panel B. Monthly return distribution of unhedged vs. hedged portfolios

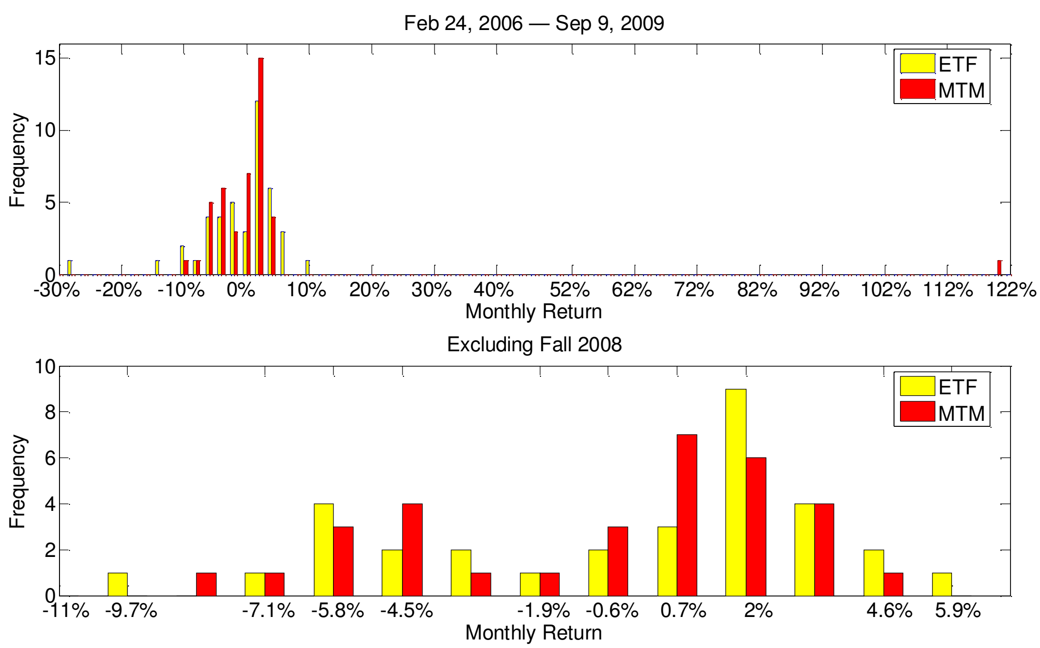
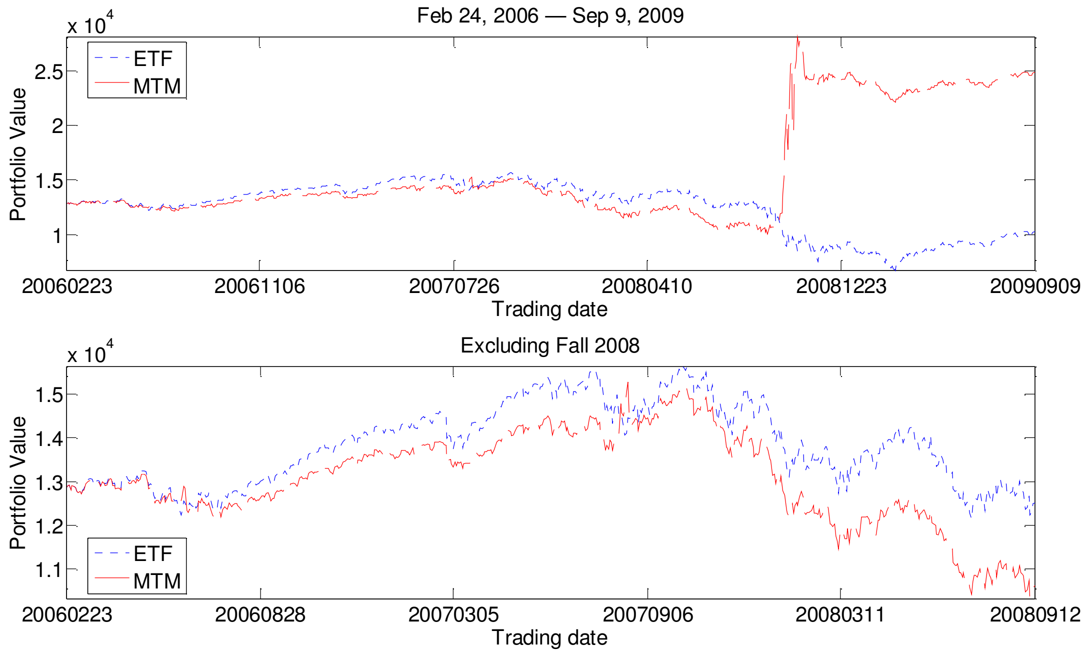


Figure 9. Out-of-sample VT futures portfolio using the LVOLH strategy. The ETF lines in Panel A are the unhedged MTM of holding one 100-lot unit of the S&P 500 in dollars. The MTM lines are the hedged MTM by adopting LVOLH strategy to dynamically allocate hedging capitals to VT futures. The rolling strategy covers the full sample period from February 24, 2006 to September 9, 2009, and the period with the Q4 2008 panic period and the Q1-Q3 2009 relatively calm periods excluded (denoted *Excluding Fall* 2008). The histograms of monthly returns on the unhedged vs. hedged portfolios are presented in Panel B. Each of the graphs is plotted across the full sample period and the period excluding Fall 2008.

Panel A. Mark-to-market of unhedged vs. hedged portfolios



Panel B. Monthly return distribution of unhedged vs. hedged portfolios

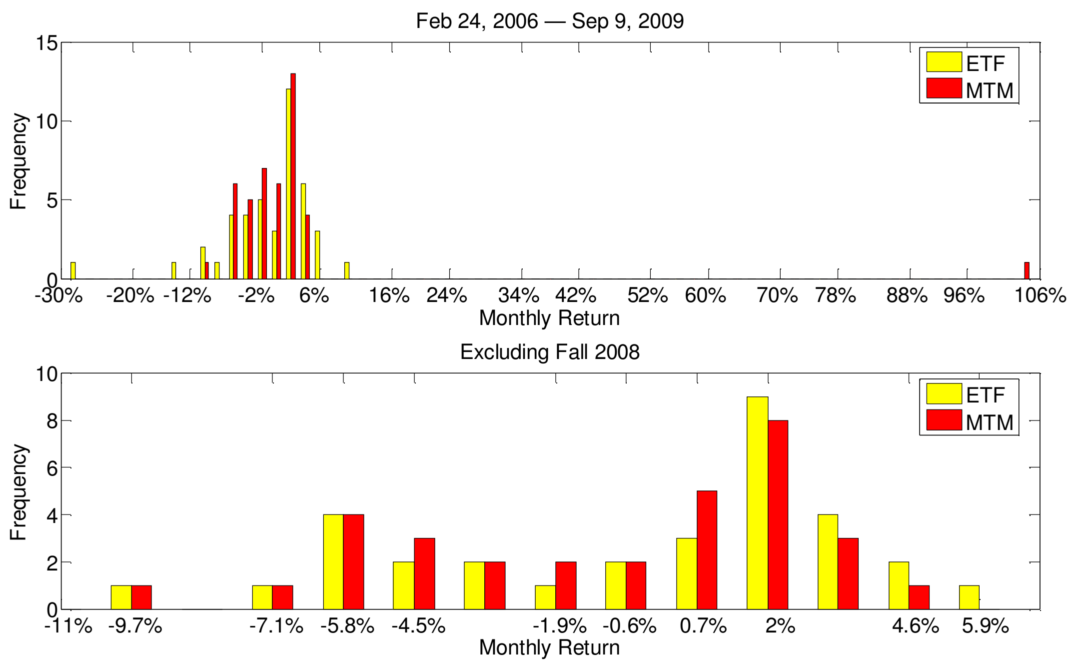
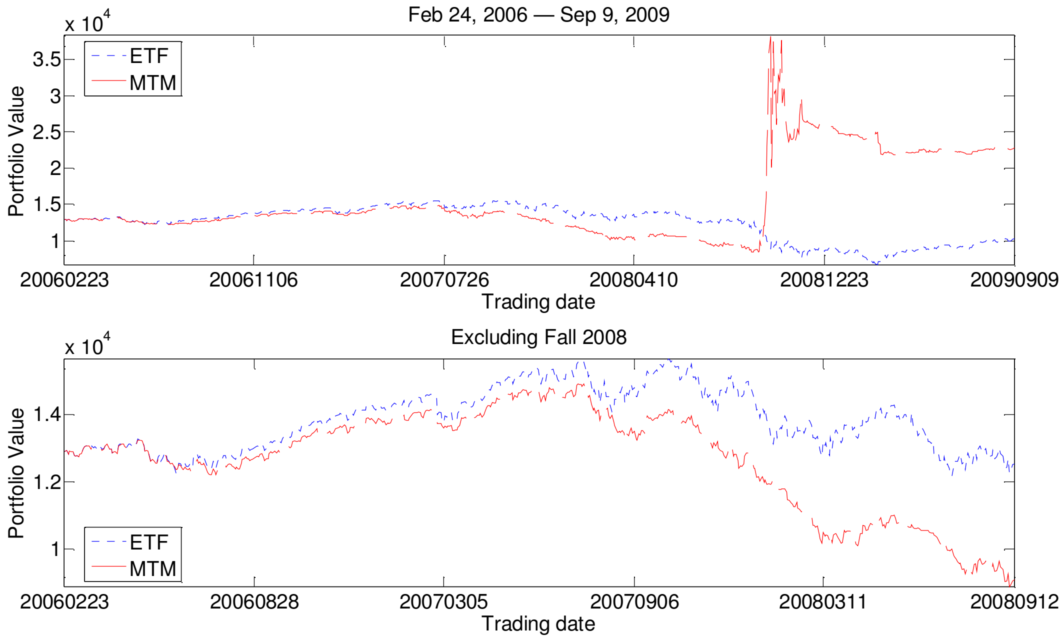


Figure 10. Out-of-sample OTM VIX call portfolio using the LVOLH strategy. The ETF lines in Panel A are the unhedged MTM of holding one 100-lot unit of the S&P 500 in dollars. The MTM lines are the hedged MTM by adopting LVOLH strategy to dynamically allocate hedging capitals to 10% out-of-the-money VIX call options. The rolling strategy covers the full sample period from February 24, 2006 to September 9, 2009, and the period with the Q4 2008 panic period and the Q1-Q3 2009 relatively calm periods excluded (denoted *Excluding Fall* 2008). The histograms of monthly returns on the unhedged vs. hedged portfolios are presented in Panel B. Each of the graphs is plotted across the full sample period and the period excluding Fall 2008.

Panel A. Mark-to-market of unhedged vs. hedged portfolios



Panel B. Monthly return distribution of unhedged vs. hedged portfolios

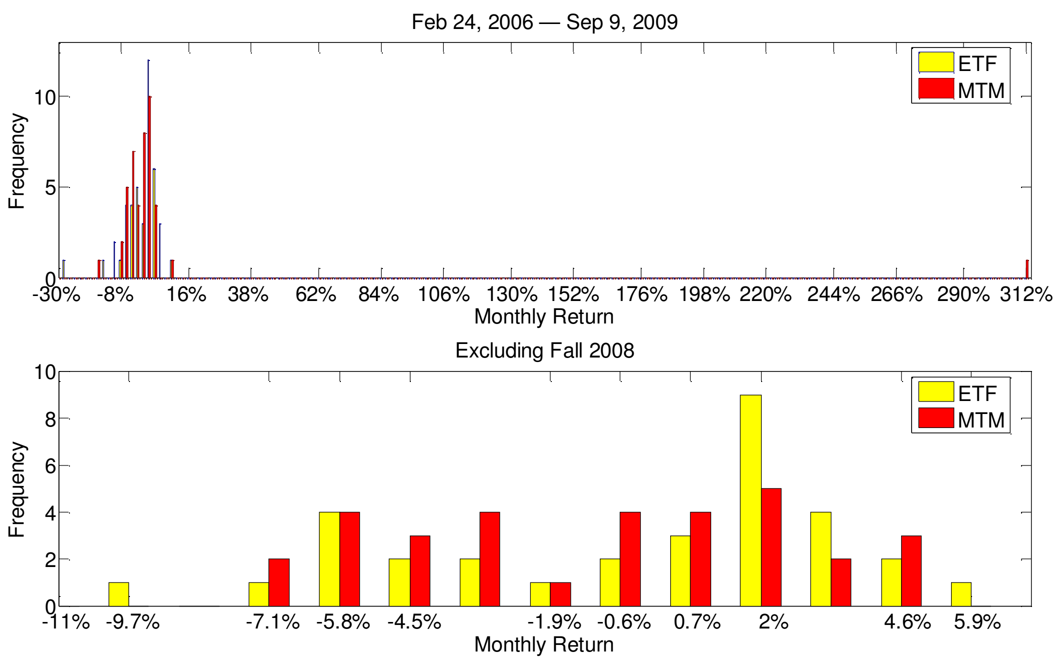


Figure 11. Out-of-sample OTM SPX put portfolio using the LVOLH strategy. The ETF lines in Panel A are the unhedged MTM of holding one 100-lot unit of the S&P 500 in dollars. The MTM lines are the hedged MTM by adopting LVOLH strategy to dynamically allocate hedging capitals to 10% out-of-the-money SPX put options. The rolling strategy covers the full sample period from February 24, 2006 to September 9, 2009, and the period with the Q4 2008 panic period and the Q1-Q3 2009 relatively calm periods excluded (denoted *Excluding Fall* 2008). The histograms of monthly returns on the unhedged vs. hedged portfolios are presented in Panel B. Each of the graphs is plotted across the full sample period and the period excluding Fall 2008.