Liquidity ‘life cycle’ in US Treasury bonds[◊]

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Abstract

This paper examines the predictable behavior of the liquidity premia involved in the prices of the U.S. government bonds. We use different measures of expected future liquidity and illiquidity, in addition to current liquidity and illiquidity. Proxied by a trading activity measure, liquidity of these fixed income securities goes over different stages throughout their life. A bond is actively traded after issued. It is the on-the-run for its time to maturity. After other new issues burst into the market, it becomes an off-the-run, and its trading activity loses intensity. A high portion of the issue is kept in investors’ inactive portfolios and its trading fades out. Through the liquidity ‘life cycle’ function, we are able to estimate the current liquidity and expected future liquidity. Using the GovPx dataset, we analyze the influence of both variables in the observed yield spreads of U.S. Treasury bonds. We find that expected future liquidity affects bond prices more than current liquidity.

Keywords: liquidity, fixed income, pricing, life cycle, government bonds.

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1. Introduction.

Liquidity is a key factor in the pricing of fixed income securities. A number of papers emphasize their role. Since the Amihud and Medelson (1991)'s seminal work, there have been many studies showing that security's liquidity is priced in Treasury markets[1]. The observed differences in prices imply that market participants price liquidity. Investors are willing to pay a higher price for the most liquid assets. Otherwise, the most liquid securities are traded with a liquidity premium that implies higher price and therefore lower yield to maturity.

The traditional static liquidity analysis examines differences in liquidity between assets, i.e. they are due to different bond characteristics as well as bond's fundamentals, such as bond age, time of maturity, amount outstanding, and coupon rate. Recent papers propose liquidity measures focus on the bid-ask spread behavior, such as different adaptations of the Roll measure (1984) to the fixed income market, or on the price impact of a trade per unit traded, i.e. Amihud (2002) illiquidity measure.

The recent availability of transaction prices in the secondary U.S. corporate bond markets, i.e. the TRACE data set, has allowed the development of this new branch of literature. This literature often translates stock market liquidity measures to the new potentially analyzable data set. Beside a number of idiosyncratic aspects of bond markets, some critical characteristics of these markets prevent a blind adaptation of measures. On the one hand, Treasury bond markets and especially corporate bond markets are much less liquid than stock exchange markets. In their original expression, some proxies are not able to be accurately computed and some modifications are needed. Other ones lose their essence. On the other hand, stocks have infinite maturity. Current liquidity can be a good proxy of future liquidity for a stock. Most of the popular liquidity measures take a static picture of liquidity in a point of time. In most cases this sentence is completely wrong for a bond. Bonds have finite maturity. In the case of US Treasury debt, they mature in two to thirty years. As we analyze, there is a bond liquidity life cycle. Bond aging reduces and even fades away the bond liquidity.[2]

We emphasize that market participants take into account that a bond has a finite life and its liquidity goes through differences stages. The trading activity of two government bonds, all characteristics equal except time to maturity, can be equally intense during a day, but liquidity premium involved in their prices should probably be different. The reason is that market participants consider the potential future liquidity of each bond. The buyer of the oldest bond is wishing to pay a lower price that he would

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[1] Kamara (1994), Fleming (2003), Chen, Lesmond, and Wei (2007), Pasquariello and Vega (2009), Favero, Pagano, and Bon Thadden (2010), Jankowitsch, Nashikkar and Subrahmanyam (2010), Goyenko, Subrahmanyam and Ukhov (2011), Lin, Wang, and Wu (2011), Bao, Pan, and Wang (2011), Dick-Nielsen, Feldhütter, and Lando (2011) study different aspects about liquidity in debt markets.

2 The previous empirical literature has assumed that a bond's current liquidity remains at the same level over the time, with a few exceptions include Goldreich, Hanke and Nath (2005) who show that yield spreads in U.S. Treasury notes depends primarily on future liquidity, and Diaz, Merrick and Navarro (2006) who study the importance of expected future liquidity in Spanish bond liquidity premiums.

pay for the youngest bond since its expected future liquidity is lower. Investors price the costs of illiquidity that they would incur whether unwind positions before maturity. In this sense, Goldreich, Hanke and Nath (2005) observe the relevance of the future liquidity, and Díaz, Merrick and Navarro (2006) analyze its impact on prices of Spanish government bonds. Thus, we consider both the current liquidity and the expected future liquidity.

The main objective of this paper is to examine the yield spreads impact of the whole liquidity life cycle in the U.S. Treasury bond market. First, we propose the individual market share of each bond as a metric of the current liquidity and modelize the link between bond liquidity and bond age. Sarig and Warga (1989) observe that bond liquidity depends inversely on age. The on-the-run bond of a certain maturity, i.e. the just-issued bond or the bond issued at the most recent auction, is by far the more liquid bond. This issue focuses the trading activity of the market. All institutional investors are wishing to include this bond in their portfolios. The higher the liquidity, the higher price to pay for the bond, so the bond is more expensive. It has a higher price and a lower yield-to-maturity.[3] But in the next future, the bonds will become an off-the-run bond when a new on-the-run bond is issued. Even the market distinguishes between the first off-the-run, the second off-the-run, and so on. This means that our measurement of liquidity, the individual bond market share, changes predictably over time. Because of this pattern, we can see that liquidity covaries with bond's age in a regular and predictable way over the time. If bond status goes through a 'life cycle', we can say that also bonds liquidity goes through a similar 'life cycle'.[4]

Second, we also analyze whether the liquidity life cycle is also observed in the liquidity measures proposed in the recent literature.[5] As mentioned, our liquidity measure is proxied by a measure of trading activity. Thus, we also examine the bond age dependence of other popular proxies: the Roll (1984) measure, the Amivest Liquidity ratio, the Amihud (2002) measure, the Bao, Pan and Wang (2011) measure, the bid-ask spread, and the numbers of "runs" as in Sarig and Warga (1989) or zero-trading as in Chen, Lesmond, and Wei, (2007). Roll (1984) finds that, under certain assumptions, consecutive returns can be interpreted as a bid-ask bounce. Thus, the covariance in price changes provides a measure of the effective bid-ask spread. The Amivest Liquidity ratio (1985) is used, among others, by Cooper, Groth and Avera (1985), as a measure of price impact, i.e. larger liquidity implies lower price impact. It is computed as the average between the volume traded and the absolute return. The Amihud (2002) measure relates the price impact of a trade to the trade volume. It is defined as the price impact of a trade per unit traded. The Bao, Pan and Wang (2011)

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[3] Krishnamurthy (2002) observes that variations in the bond/old-bond spread is driven by the Treasury supply of bonds as well as aggregate factors affecting investors' preference for liquid assets.

[4] Expression used by Diaz, Merrick and Navarro (2006) to reflect the pattern of Spanish fixed income securities liquidity as a function of bond age.

[5] Many studies have focused on identifying the most appropriate proxy for liquidity in Treasury markets (e.g. Fleming, 2003) and in corporate debt markets (Amihud, 2002, and Jankowitsch, Nashikkar and Subrahmanyam, 2010).

measure is the negative covariance between the price change from a time and the price change from the previous period. It is a measure of illiquidity that is applied for corporate bonds. According to Sarig and Warga (1989) terminology, a price "run" appears when two consecutive daily prices are identical. This is a measure of the trading activity.

Third, we model the liquidity life cycle that will let to measure current liquidity and hence let us to estimate expected future liquidity. With these two measures of liquidity, we try to quantify the effect of the whole liquidity life cycle in the prices of the U.S. Treasury bonds. We consider the expected future liquidity to control for the liquidity life cycle. This should be a key input in the investors' making decision process in case they consider the possibility of unwinding positions before maturity.

Fourth, we estimate yield spreads from the observed yield-to-maturity at which these assets are traded in the market and the yield-to-maturity of theoretical Treasury bonds[6]. The prices of these fictitious bonds are obtained from discounting the original cash flows by the spot rates[7]. For each bond and day we obtain the price of a theoretical bond with the same cash flow structure using spot rates. These zero coupon interest rates are estimating from the Svensson (1994) methodology. Svensson model is a parametric and parsimonious model that specifies a functional form for the instantaneous risk free spot rate which is a function of the term to maturity. The functional form of the model allows for a wide range of potential shapes of the term structure not covered by simple linear or log-based estimation procedures. We test empirically which measure, current and expected future liquidity, further influences the observed yield spreads.

Our paper contributes to the existing literature in different ways. We use a liquidity 'life cycle' function to quantity the observed yield spreads in U.S. Treasury fixed income securities. Also, we link bond liquidity with bond age, and distinguish bonds by term to maturity, so we examine both Treasury notes and Treasury bonds. We are able to investigate which liquidity measure drives the observed yield spreads on each term to maturity.

This paper is related to Goldreich, Hanke and Nath (2005), who show that expected future liquidity is the main component of the liquidity premium observed between on-the-run and off-the-run bonds. Their results are only for U.S. two-year

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[6] Many papers estimate spread yields as the difference between the yield to maturity on a bond with an original time to maturity and the yield to maturity on another government bond with different original time to maturity. I.e., spreads are measured by the difference between yield-to-maturity on a short term bonds and yield-to-maturity on long term bonds with same left time to maturity. See Campbel (1991) and Kamara (1994) among others. In other cases, spread yields are measured as differences between corporate yield and Treasury yield. See Duffee (1998), Huang and Huang (2002) and Longstaff et al (2005).

[7] We fit an standard discounting and equation to data on bond prices and cash flows:

$$P = \frac{C}{(1+R_1)} + \frac{C}{(1+R_2)^2} + \cdots + \frac{C+F}{(1+R_n)^n}$$

where P is the bond price, F is the face value, and R_i is the spot rate. See McCulloch (1971) and (1975), and Schaefer (1981). Once we have traded price, we estimate yield-to-maturity. At the same time, we estimate a theoretical price for a bond with same features about coupon and time to maturity. From this theoretical price bond, we will obtain a theoretical yield-to-maturity, notes, while ours are extensive to the rest of notes and bonds. Also we measure expected future liquidity using a liquidity ‘life cycle’ function. Our work is also related to Diaz, Merrick and Navarro (2006) who analyze the market liquidity of Spanish Treasuries and the impacts of the changes before the entry into the European Economic and Monetary through the role of a bond liquidity ‘life cycle' function. We extend our analysis to U.S. debt market. Another work related to our paper is Goyenko, Holden and Trzinka (2009) that analyses different liquidity proxies in order to answer the question Do liquidity measures measure liquidity? in stock markets. They use different liquidity measures with different liquidity benchmarks widely used in previous literature. We also use different liquidity proxies, although our market is a fixed income public debt market, U.S. Treasury Market.

Our paper is organized as follows. Section 2 deals with liquidity in debt markets, and different measures used to quantify liquidity. Section 3 describes the specific characteristics of U.S. debt market, and data and sample period. In section 4 we show methodology together with the estimated liquidity and illiquidity variables and in section 5 we present the empirical analysis. Section 6 is about a further analysis for different terms to maturity. Finally, section 7 concludes.

2. Liquidity in debt markets.

Liquidity is a key aspect in determining the price and the return offered by fixed income assets. A basic definition is that which defines liquidity as the ability of an asset to be turned into money. We say that an asset is liquid if it can be traded on the market in a short period of time without causing significant losses in value. Fleming (2003) includes a definition of liquidity from O'Hara (1995) and Engle and Lange (1997): “a liquid market is defined as one in which transactions can be done without cost”.

In practice, a market with low transaction costs is known as a liquid market, while one in which there are high transaction costs is called illiquid one. Measuring these costs is not simple, since they depend on numerous factors like the size of the negotiation, time, place of negotiation, and partners.

In particular, high liquidity would indicate that an asset can be negotiated quickly and without significant loss of value. In this case investors would expect higher asset prices, and lower yield to maturity. In contrast, lower liquidity means that the cost to trade an asset will be high, so investors would expect lower prices, and in contrast a higher profit.

Liquidity depends on several factors that influence the liquidity of fixed income assets, such as, among others, amount outstanding, age, term to maturity, issue status, economic activity cycle, interest rates volatility, investor risk aversion, etc.[8]

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[8] See for example Fisher (1959), the larger the size of the issue, the easier the bond trading.

The issue status is also a liquidity determinant. The newest issue of Treasury securities for each different original term to maturity focuses the interest of investors. Most financial institutions and mutual and pension fund managers try to incorporate these just-issued assets in their portfolios. The "*on-the-run*" bond attracts the market liquidity and trading activity[9]. In this sense, several studies find evidence of the phenomenon called the '*on-the-run liquidity phenomenon'* in U.S. Treasury securities[10]. The most recently issued (*on-the-run*) government securities of a certain term to maturity have generally higher prices and higher bond-level liquidity than previously issues (*off-the-run*) maturing on similar dates.

The measurement and monitoring of liquidity are relevant for making investment decisions in fixed income markets, in particular in government bond markets. In times of financial turmoil, there is the phenomenon known as 'flight to quality',[11] where some market participants abruptly decrease their portfolio exposure to securities bearing credit risk. They prefer safer securities, the default risk-free issues.[12] Another phenomenon observed in financial markets is known as 'flight to liquidity'. It means that investors put their interest in highly-liquid securities such as government fixed income securities. They prefer higher liquid securities rather than less-liquid securities[13].

In previous literature, there are a number of different bond liquidity measures. Measures such as trading volume, trading frequency, bid-ask spreads, quote sizes, trade sizes, price impact coefficients, and on-the-run/off-the-run yield spreads have been traditionally used to measure liquidity in an effective way[14]. Recently, the availability of high-frequency data, especially the case of the TRACE data set from US corporate bond market, has allowed incorporating and adapting from stock exchange markets new liquidity measures in the analysis of fixed income liquidity.

Díaz and Navarro (2002) use measures such as trading frequency and turnover to measure liquidity in the Spanish debt market. Goldreich, Hanke and Nath (2005) use the average spread quoted bid-ask, the average effective spread bid-ask, the average size

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[9] Buy and holder investors ascribe to holding these more liquid securities, because they can sell more quickly and without high losses. Moreover, higher liquidity of on-the-run Treasuries also makes them ideal securities for market intermediaries who wish to create short positions. They can easily be borrowed and sold when initiating a short position, and just as easily repurchased when closing one out. Althought off-the-run bonds are cheaper than on-the-run bonds, investors think that those are hard to find and scarce in markets. See Vayanos and Weill (2007).

[10] See, for example, Brandt, Kavaiecz and Underwood (2007), Mizrach and Neely (2008), and Pasquariello and Vega (2009).

[11] See, for example, Bernanke and Gertler (1995), Longstaff (2002), Vayanos (2004) and Beber, Brandt and Kavajecz (2008).

[12] There have recently been several episodes of negative interest rates resulting from Treasury bill auctions in United States, Switzeland and Germany, i.e. investor asked to pay more than the nominal amount for the promise of receiving the nominal amount in the maturity date.

[13] See Fleming and Remolona (1999).

[14] Fleming (2003) examines some measures used in the literature to quantify the liquidity in order to determine which one assess and track liquidity better. His analysis reveals that the bid-ask spread, one of the most widely used in the literature as a proxy for liquidity, is a useful tool for assessing and tracking Treasury market liquidity.

quoted, the number of quotes per day, the number of trades per day, or the daily volume among others to measure liquidity in the U.S Treasury market. They find evidence that the quoted spread and measures of market trading activity adds the greatest explanatory power and the other measures, depth measures, add little explanatory power to explain the yield difference between off-the-run and on-the-run notes. Díaz, Merrick and Navarro (2006) use the individual market share of each type of issue and the status of the issue in the Spanish debt market. Another measure used by Goyenko, Subrahmanyam and Ukhov (2008), is the quoted bid-ask spread, which relates the price range with the average effective spread. Ejsing and Sihvonen (2009) use trading volume, quoted depth and the quoted bid-ask spread, besides the "liquidity ratio" proposed by Bollen and Whaley (1998).

The new set of liquidity measures imported from the stock exchange markets focuses the attention in the Amihud (2002)'s illiquidity measure, which may be applicable to fixed income assets. Originally, this measure based on Kyle (1985) was proposed for equity market, but it has been widely used on fixed income[15]. It relates price impact to trade volume. Johnson (2008) uses the bid-ask spread and price impact illiquidity measure in government bonds. Bao, Pan and Wang (2011) propose a measure of illiquidity but for the case of corporate bonds, as is the covariance between changes in prices. This Bao measure is related to Roll measure (1984) which allows to estimate the effective bid-ask spread as twice the square root of the negative covariance between price changes for stocks. Amivest (Cooper, Groth and Avera (1985), and, Amihud, Medelson and Lauterback (1997) among others) measure is a liquidity ratio that is a good indicator of market liquidity or depth. All these new set of measures can be applicable to fixed income, because of the availability of new databases.

3. U.S. debt market: Description and Data.

3.1. U.S. debt market.

U.S. Treasury securities are default risk-free debt instruments issued by the U.S. government. These securities play an important, even unique, role in international financial markets because of their safety, liquidity and low transaction costs.

U.S. debt market is the largest debt market in the world, both by trading volume and by number of investors and trades. From April 2001, the amount outstanding of U.S. government debt was more than \$5 billion, and of this quantity more than \$3.2 billion was on public holds, and \$2.8 billion was traded on financial markets. In August 2007, it was more than \$9 billion. To December 2011, the amount outstanding of U.S. government debt has been more than \$15 billion. This increase suppose near 200% in ten years. The U.S. Treasury sells securities through auctions on a regular schedule to finance the national debt. Government bonds offer the security and safety of the U.S.

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[15] Jankowitsch et al (2010) and Friewald et al (2011).

federal government. These bonds, as they provide greater security, offer less interest than other bonds with similar characteristics in term and / or maturity.

There are three types of government securities in the U.S. Treasury market[16]:

1) *Treasury Bills*: These securities have the shortest maturity, a year or less. The Federal Reserve Bank of U.S. sells these bills at discount in denominations of $10.000 to $1 million. 21% of the debt traded in the market in April 2001 is composed by bills with a maturity of one year or less, and in December 2011, 15% of the marketable U.S. debt is in bills.

2) *Treasury Notes*: These securities have intermediate maturities: 2-, 3-, 5-, 7- and 10-year notes. Notes pay coupons every six months. Bonds with an original maturity of 2- and 5-year are auctioned monthly the last day of each month. So that at any time there are 24 issues outstanding. The sale is also done through auctions, and is subsequently traded in secondary markets. Notes with an original maturity of 3- and 10-year are auctioned quarterly (February, May, August and November), on 15th February, May, August and November. 7-year notes are auctioned quarterly, and its maturity date is on 15th January, April, July and October. In April 2001, 52% of the debt traded on financial markets is for bonds with intermediate maturities, and made up 66.5% of the debt in December 2011.

3) *Treasury Bonds*: These bonds have the longest maturity term, 20 and 30 years and pay coupon every six months. They are issued through auctions conducted by the Federal Reserve Bank, and the negotiation of these bonds in the secondary market is quick and easy. 20- and 30-year bonds are issued quarterly; 30-year bonds are issue are due 15th February, May, August and November, and 20-year bonds due first working day on January, April, July and October. 21% of negotiated debt in April 2001 on markets corresponds to longer-term bonds, and in December 2011 it was near 11%.

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[16] Mizrach and Neely (2008) analyze the microestructure el the U.S. Treasury Market, and describe the types of debt instruments: Treasury Bills, Treasury Notes, Treasury Bonds and STRIPS.

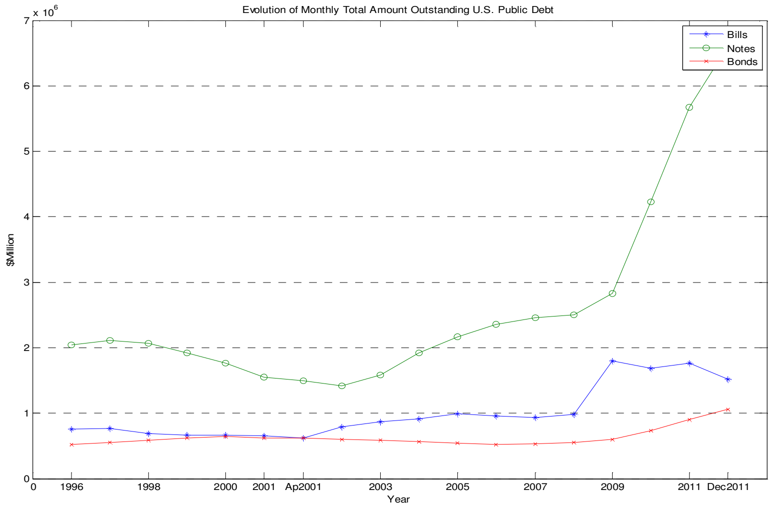

Figure 1 Evolution of Monthly Total Amount Outstanding US Public Debt. It shows the evolution of January's total amount outstanding of US public debt, from January 31, 1996 to January 31, 2011. Includes monthly level from April 2001, and December 2011.

Most U.S. debt securities consist of medium-term and long-term maturity, being about 50% of the total debt issued. In terms of trading activity, the U.S. Treasury debt is one of the largest sectors of the bond market. The total volume of debt and size of any individual issue is higher compared to the other bond market sectors [17].

Since April 2001 to December 2011, the amount outstanding held by the public has changed: from 21.6% for bills, 52.7% for notes and 21.7% for bonds on 2001, to 15.3% for bills, 66.5% for notes and 10.7% for bonds on 2011.

Issuance of each different type of U.S. Treasury depends on funding needs and monetary policy objectives. From 1993 U.S. Treasury does not issue notes with 7-year maturities any more. In 1998 U.S. Treasury suspends 3-year notes issuance, and resumes its issuance in 2003. The 30-year bond issuance restarts on 2006 after had been suspended in 2001. From 1986 any new 20-year bond is issued until 2004. Thus, issuance cycles are different across securities. Table 1 shows schematically the calendar with the issue and maturity dates of each type of U.S. Treasury securities.

Treasury securities go through different phases: *when issued, on-the-run* and *off-the-run*. Each of these stages presents different market structures.

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[17] The major issues from the U.S. Treasury market imply that the secondary market is very liquid, with large trading volumes and bid-ask spreads narrow, as shown by Fleming and Sarkar (1998).

Table I. Release Schedule

	2-y notes	3-y notes	5-y notes	7-y notes	10-y notes	20-y bonds	30-y bonds
January	Issue (l.d.) Maturity (l.d.)		Issue (l.d.) Maturity (l.d.)	Issue (15) Maturity (15)		Issue (f.d.)	
February	Issue (l.d.) Maturity (l.d.)	Issue (15) Maturity (15)	Issue (l.d.) Maturity (l.d.)		Issue (15) Maturity (15)	Maturity (15)	Issue (15) Maturity (15)
March	Issue (l.d.) Maturity (l.d.)		Issue (l.d.) Maturity (l.d.)				
April	Issue (l.d.) Maturity (l.d.)		Issue (l.d.) Maturity (l.d.)	Issue (15) Maturity (15)		Issue (f.d.)	
May	Issue (l.d.) Maturity (l.d.)	Issue (15) Maturity (15)	Issue (l.d.) Maturity (l.d.)		Issue (15) Maturity (15)	Maturity (15)	Issue (15) Maturity (15)
June	Issue (l.d.) Maturity (l.d.)		Issue (l.d.) Maturity (l.d.)				
July	Issue (l.d.) Maturity (l.d.)		Issue (l.d.) Maturity (l.d.)	Issue (15) Maturity (15)		Issue (f.d.)	
August	Issue (l.d.) Maturity (l.d.)	Issue (15) Maturity (15)	Issue (l.d.) Maturity (l.d.)		Issue (15) Maturity (15)	Maturity (15)	Issue (15) Maturity (15)
Septembre	Issue (l.d.) Maturity (l.d.)		Issue (l.d.) Maturity (l.d.)				
October	Issue (l.d.) Maturity (l.d.)		Issue (l.d.) Maturity (l.d.)	Issue (15) Maturity (15)		Issue (f.d.)	
November	Issue (l.d.) Maturity (l.d.)	Issue (15) Maturity (15)	Issue (l.d.) Maturity (l.d.)		Issue (15) Maturity (15)	Maturity (15)	Issue (15) Maturity (15)
December	Issue (l.d.) Maturity (l.d.)		Issue (l.d.) Maturity (l.d.)				

This table shows in parentheses the issue and / or maturity day. f.d. refers to the first working day of the month, and l.d. refers to the last working day. Following this calendar, at any time there should be 24 issues outstanding for 2- and 5-year notes, and there should be 12 issues outstanding for 3-year note; 28 issues outstanding for 7-year notes; 40 issues outstanding for 10-year notes; and 120 issues outstanding for 30-year bonds.

The primary market is where the debt is sold through auctions to investors. In the first instance, the U.S. Treasury publishes a calendar with the next upcoming auction dates on the first Wednesday of February, May, August and November. The vast of bids are submitted several days before the auction because U.S. Treasury doesn't announce auction information until few days before. Short-term bills are auctioned weekly; 2- and 5-year notes are auctioned monthly; instead 3-, 7-, and 10-year notes and 30-year bonds are auctioned four times a year.

The secondary market is an over-the-counter market where takes place trading between dealers, brokers, institutional and private investors, including foreign ones. It is composed of the *when-issued*, and the on-the-run and off-the-run issues. In the *when-issued* market, securities are traded several days before the auction. The settlement date of these transactions coincides with the auction settlement date. The just-issued security is the *on-the-run* among that those who have the same original term to maturity. The on-the-run issues concentrate most of the trading volume in this secondary market.[18] After a new issue is auctioned, the new bond is the on-the-run and the former on-the-run becomes the first off-the-run, the former first off-the-run becomes the second off-the-run, and so on.

Each U.S. Treasury issue is identified by a sole identification number referred as CUSIP (Committee on Procedures Uniform Securities Identification). In some cases, a new tranche of an outstanding issue is auctioned. The outstanding tranche and the new tranche are completely fungible. They share all the characteristics, i.e. CUSIP, coupon rate, maturity date.

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[18] Fabozy and Fleming (2005) argue that about 70% of total trading volume is concentrated in the section on-the-run.

3.2. Data and Sample Period.

The dataset used in the analysis of the U.S. Treasury liquidity has been obtained from the database GovPx (Government (securities) Pricing Information System). This database collects trading information from five of the six larger majority brokers trading in the interdealer market. Its creation in 1991 was in order to demands to provide greater transparency of U.S. Treasury market. Brokers report quote and trade information from their trading activity to GovPx system that take place through participating interdealer brokers. The dataset includes only trades and quotes registered among them. The trading activity among dealers, and between dealers and their customers is beyond the computation of the data. The posted data includes the best bid and ask quotes, the quote sizes, and the price and size of each trade. For 1996, GovPX daily trading volume averaged was \$77.1 billion. Average daily volume in 1997 was \$79.7 billion, more than the 1998 average of \$71.5 billion. On 1999 was \$52.5 billion, and during the first half of 2000, GovPX average daily volume was \$39.6 billion, 25% less than the daily average for 1999. From 2001, Treasury volume has been fallen off. GovPX does not provide a reliable indicator of transactions after March 2001, when started electronic trading.[19]

Our initial sample includes every trade between January 1996 and December 2006. We analyze 2-, 3-, 5-, and 10-year Treasury notes and 30-year Treasury bonds[20]. Although U.S. Treasury suspended 3-year notes and 30-year bonds issuances on a few times, we include this securities in the analysis. 7-year Treasury notes and 20-year Treasury bonds also have been taken into account to let us estimate individual market share for each issue.

New electronic trading platform emerges from the beginning of our century. The trading activity of the traditional interdealer brokers drops and GovPx leaves to report volume information since May 2001. As for we compute trading measures that consider the trading volume, we can only use data from January 1996 to April 2001. Furthermore, during 2001 information is limited causing distortion in used measures, so we reduce our data sample from January 1996 to December 2000. To complement the dataset, we use information about amount outstanding and auction details obtained from the official website of U.S. Treasury.

For the study period, there are 1272 trading days and 251680 observations. We compute daily data of all outstanding Treasury notes and bonds, even whether they are not traded and the trading volume is zero. We control for the issuance of new tranches

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[19] Mizrach and Neely (2006) analyses the transtition from GovPx to electronic trading in the secondary Treasury market.

[20] Most of the studies on U.S. Treasury securities are focused on bonds with maturities of 2, 5 and 10 years, as those are issues that have never been interrupted, with regular broadcast dates, and are available greater number of observations and information. Goldreich, Hanke and Nath (2005) use data from 2-year bonds, Pasquariello and Vega (2009) use data from bonds to 2, 5 and 10 years, as Fleming (2003), Strebulaev (2002) uses data from bonds 2, 3, 5 and 10 years, which are those with more regular releases.

of an outstanding issue. For instance, this is the case of some 5-year notes. Three years after its issuance, a new tranche is issued as a 2-year note. As any new auctioned asset, the trading activity rises dramatically. Even GovPx changes the way the issue is denominated. The original CUSIP is now reported as “2-year note”. We consider the issue as a 5-year note during the period from the original issuance until the new tranche is issued. From this time, we consider the issue as a different “2-year note”.

Table II shows the total number of outstanding issues and their average trading volume. Several patterns can be observed. By far, the most actively traded issues are 2-year and 10-year notes. By contrast, 7-year notes and 20-year bonds can be considered illiquid securities, they are rarely traded. During our sample period, no new issuances of these assets take place. 2- and 5-year notes have a regular number of simultaneous outstanding issues during period of analysis because U.S. Treasury always has issued those type of notes, without interruptions. Also, 2- and 5-year notes have the largest number of different traded issues in secondary market. Average volume per issue is very low in case of 7-year notes and 20-year bonds, because the trading is the lowest. In contrast, for 2- and 3-year notes is the highest for our sample.

Table II. Summary of Data

	2-year	***3-year***	***5-year***	***7-year***	***10-year***	***20-year***	***30-year***
Number of Simultaneous Outstanding Issues	*24*	*12 or less*	*24*	*18 or less*	*40 or less*	*16 or less*	*120 or less*
Number of Observations	*31167*	*12327*	*67768*	*10090*	*44309*	*20832*	*65187*
Number of Observations with non-zero Volume Traded	*27684*	*10150*	*33871*	*5662*	*11778*	*345*	*13986*
Number of Trading days	*1302*	*1302*	*1302*	*1302*	*1302*	*1302*	*1302*
Number of Different Traded Issues	*83*	*22*	*95*	*18*	*49*	*16*	*56*
Total Aggregate Volume (thousand $)	*9007475*	*2203020*	*6167517*	*128718*	*4370513*	*20832*	*1409381*
Average Trading Volume per Day (thousand $)	*325’79*	*217’05*	*182’09*	*22’73*	*371’07*	*8’35*	*100’77*
Average Trading Volume per Issue	*108523’79*	*100137’27*	*64921’23*	*7151*	*89194’14*	*180*	*25167’52*
% of Aggregate Volume on status on-the-run	*72%*	*74%*	*67%*	*0%*	*67%*	*0%*	*57%*
% of Aggregate Volume on status off-the-run	*28%*	*26%*	*33%*	*100%*	*33%*	*100%*	*43%*

Table II shows information for subsample data, from January 1996 to December 2000, for all outstanding issues. 7-year notes and 20-year bonds don’t present new issues during the period analyzed, so all issues are on the off-the-run status.

4. Methodology.

Our approach consists into panel data regressions to analyze the observed bond yield spreads changes. We have time-series cross-sectional observations, including some quantitative and qualitative measures. We analyze spread yields between U.S Treasury securities yields and theoretical bonds yields, i.e., securities with the same original term to maturity that were the most liquid in the market and with equal coupon rate. We do this distinguishing by kind of issue, i.e. 2-year note, 3-year note, … to avoid any potentially cross-sectional differences between notes and bonds, like differences in coupon, different tax treatments[21] ... Also markets in which are traded may have different characteristics that can influence the determinants of liquidity. Therefore it is

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[21] Tax differences may influence when measuring the effect of liquidity, (Strevulaev (2002)). The analysis of each issue separately, can avoid the tax differentials between short-term securities and securities in the medium and long term.

desirable to separate and analyze liquidity of each type of issue separately. So we will analyze spread yields between securities with the same features. Therefore the analysis of yield spreads over the bond 'life cycle' is not affected.

In this section firstly we present our measures of current and expected future liquidity and illiquidity. As we have seen above, there are many liquidity measures in previous literature to quantify liquidity securities and its use depends on the analysis and on the availability data.

In the specific case of 2-year notes, we estimate different liquidity and illiquidity proxies, and from these proxies we distinguish between current and future liquidity and illiquidity. We will use these measures to try to explain observed yield spreads, in addition to other control measures.

For other maturities, 3-, 5-, and 10-year notes and 30-year bonds, we realize the same analysis but using only one liquidity proxy, market share liquidity measure, to explain our liquidity benchmark, yield spreads.

4.1. Market Share.

Individual market share measures bond-level current liquidity. It is an indirect but widely measure of market liquidity. It is the ratio between a bond trading volume and the total market trading volume, for all outstanding issues, during a time period. We determine the market share of each issue, for the period analyzed, using the following expression:

$$MS_{i,t} = \frac{TV_{i,t}}{TTV_t} \qquad for\ j = 0,1, \dots 1272 \qquad (1)$$

where:

$MS_{i,t}$ is the market share for security i on day *t*; $TV_{i,t}$ is the total volume traded for security i on t, and TTV_t is the total traded volume of all securities traded on the market on day t. Market share is estimated individually for each security on each day, and after then we distinguish by term to maturity, i.e., by type of issue.

The average market share over the bond's life cycle reflects the bond status, i.e. if it is a newly issued bond or if some time has gone since its issuance. The most recent issue for an original maturity, i.e. the on-the-run, is considered the benchmark, and it is the most traded security for this maturity. The rest of issues at the same maturity are off-the-run and they are less liquid.

To determine individual securities market share, we have included all observations of all outstanding issues (2-, 3-, 5-, 7- and 10-year notes and 20- and 30-year bonds) in the period analyzed. Securities data from phase *when-issued* have not been taken into account. In this issues are set out all the trades after the auction announcement and prior to auction purchases. Also we have not included data from older bonds because it has not been possible to obtain information, particularly 30-year bond issues with issuance date before 1980. This is a small number of titles (approximately 2% of total bonds sample) and they are extremely illiquid.

To represent the behavior of notes and bonds, in first instance we estimate age in weeks for security i at any moment t as the time elapsed from its issuance date to the day considered:

$$Age_{i,t} = \frac{D.issue_{i,j} - D.traded_{i,j}}{5} \tag{2}$$

where $D.issue_{i,t} - D.traded_{i,t}$ are the number of laborer days between trading date and issuance date. We take into account the day of the week in which the bond has been issued, i.e., if it's on Monday, Tuesday… to control for working days and for holidays. So we use weekly sections to establish age in weeks.

To determine an average market share measure according to age, we summarize all individual market share measures sorted by age ranges, and average them for number of securities at the same age range:

$$AvMS_t = \frac{\sum_{t=1}^{n} MS_{i,t}}{N_t} \tag{3}$$

where N_t is the number of bonds for each age range. Thus we have an average market share for bonds of 1 week, another average measure for bonds of 2 weeks, and so on.

Figure 2 shows the specific behavior of the 2-year notes average market share. We observe that the newer issues for 2-year notes are the mostly traded on market and are the most liquid. However the older issues, from week 4 to maturity date, have a lower trading and are much more illiquid. When a bond is 4 weeks old, a newer issue is coming, and the oldest on-the-run became the newest off-the-run. If any security has a security-level liquidity at any moment through its life, this security-level liquidity will go through a 'life cycle' too. We can follow liquidity over time, through the bond's age, in what is referred as liquidity 'life cycle'. This cycle is the same over time for older and newer issues, and this pattern is what we want to model; it is the first objective of this paper.

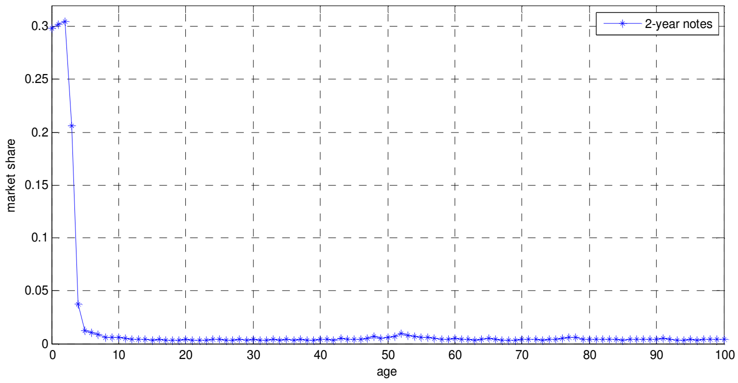

Figure 2. It represents the average market share in %, in terms of the age in weeks, for 2-year notes.

Furthermore, the same behavior over the time is observed for the rest of original maturities, and the time when they become off-the-run depends on the different issuance cycles across securities. 5-year notes are issued monthly, thus since week 4 older notes become first off-the-run notes. 3- and 10-year notes are issued four times a year, so from week 12 go on status off-the-run, and the same goes for 30-year bonds. 5-year notes has a spike around week 156 from issue, which corresponds with new issues on the same reference but with a term to maturity of 2 years. It's a new issue, which attracts investor's interest again, which makes large market share measure at that time as shown in figure (A1).

If we set the average market share of all issues by term to maturity, for the first 100 weeks of the security's life, we can observe that 2- and 5-year notes go on status off-the-run faster than the other issues (see figure 3). 3- and 10-year notes stay on status on-the-run during more time, until a new issue is realized that is, until week 12 from its issuance.

What we can see is that there is a similar behavior of the average market share for all types of issues. The newly issued bonds, the on-the-run bonds, are the ones that attract market trading, and are therefore those with greater liquidity. When there are new issues with the same term to maturity, these bonds don't trade actively and go to the status off-the-run, which remain relatively illiquid until maturity date. This phenomenon referred as 'on-the-run phenomenon' has been observed in the U.S. Treasury bonds, and widely studied in previous literature, Brandt, Kavaiecz and Underwood (2007), Mizrach and Neely (2008), and Pasquariello and Vega (2009), among others.

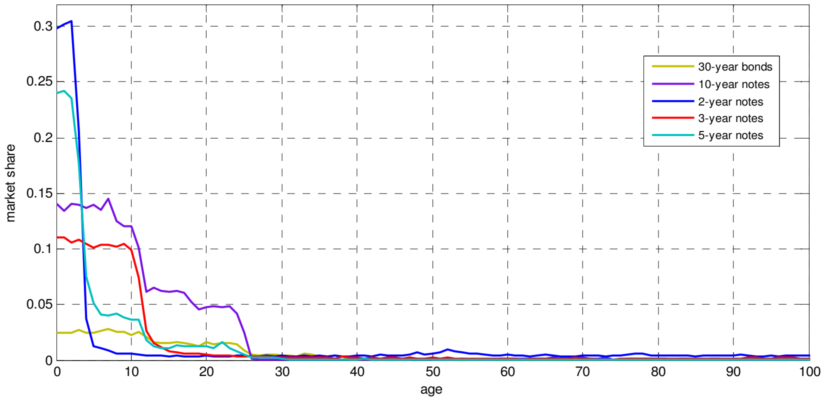

Figure 3. 2-, 3-, 5- and 10-year notes and 30-year bond market share. It represents the average market share in %, in terms of security's age measures in weeks, for securities with original maturities of 2, 3, 5, 10, and 30 years.

It is also possible to see in figure (3) that the securities which represent the vast of market share and that are the most widely traded are 2- and 5-year notes, and are also the largest number of securities issuances.

4.2. Other liquidity and illiquidity proxy measures.

4.2.1. Roll.

Roll (1984) finds that, under certain assumptions, consecutive returns can be interpreted as a bid-ask bounce. Thus, the covariance in price changes provides a measure of the effective bid-ask spread. The estimator for the effective bid-ask spread of bond *i* on day *t* is defined by

$$Roll_{i,t} = 2\sqrt{-Cov(\Delta p_t, \Delta p_{t-1})} \tag{4}$$

where Δp_t is the change in prices or absolute return from *t-1* to *t*. In those cases where the covariance between price changes is positive, the Roll measure is undefined, thus we give Roll measure value zero. To compute this measure, we use a rolling window of at least 21 days. We use daily data from 5 weeks, where there are maximum of 25 trading days. We estimate Roll measure for each individual bond, and to represent by age, we determine an average Roll measure according to age in weeks. We summarize all individual Roll measures sorted by age ranges, and average them for number of securities at the same age range:

$$AvRoll_t = \frac{\sum_{t=1}^{n} Roll_{i,t}}{N_t} \tag{5}$$

Where N_t is the number of observations for each age range in weeks.

4.2.2. Amivest.

The Amivest Liquidity ratio (1985)[22] is the average between the volume traded and the absolute return. We compute an average for each bond as:

$$Amivest_{i,t} = \frac{1}{D_i} \cdot \sum_{t=1}^{D_i} \frac{TV_{i,t}}{|r_{i,t}|} \tag{6}$$

where D_i is the number of days for which data are available for bond i into the subsample; $TV_{i,t}$ is the trading volume for bond *i* on day *t*; and $|r_{i,t}|$ is the absolute return of bond i on day t. We do not consider zero-return day data because the measure is undefined in those cases; it will be calculated over all non-zero-return days.

Likewise, we compute an average Amivest measure sorted by age ranges in weeks to represent in figure (4):

$$AvAmivest_t = \frac{\sum_{t=1}^{n} Amivest_{i,t}}{N_t} \tag{7}$$

Where N_t is the number of observations for each age range in weeks.

4.2.3. Amihud.

The Amihud (2002) measure relates the price impact of a trade to the trade volume. It is related to the Amivest measure, and is a more intuitive measure for price impact than that. Also we estimate an average by bond to see the relationship between volume and price changes as:

$$Amihud_{i,t} = \frac{1}{D_i} \cdot \sum_{t=1}^{D_i} \frac{|r_{i,t}|}{TV_{i,t}} \tag{8}$$

where D_i is the number of days for which data are available for bond i into the subsample; $TV_{i,t}$ is the trading volume for bond *i* on day *t*; and $|r_{i,t}|$ is the absolute return of bond i on day t. This measure is undefined for zero-volume days, so we do not consider those days; it will be calculated over all positive-volume days. Also, we compute an average Amihud measure by age in weeks as

$$AvAmihud_t = \frac{\sum_{t=1}^{n} Amihud_{i,t}}{N_t} \tag{9}$$

Where N_t is the number of observations for each age range in weeks.

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[22] The Amivest ratio has been used, among others, by Cooper et al. (1985), Amihud et al. (1997), Berkman and Eleswarapu (1998), and by Goyenko et al. (2009).

4.2.4. Bao.

The Bao, Pan and Wang (2011) measure is the negative covariance between the price change from a time and the price change from the previous period. It is defined by

$$\gamma_{i,t} = -Cov(\Delta p_t, \Delta p_{t+1}) \tag{10}$$

As Roll measure, is an illiquidity measure. We compute it for individual bonds, and we also use a rolling window of at least 21 days, i.e. the number of laborer days in 5 weeks. In contrast to Roll measure, it is defined for all trading days.

We determine an average Bao measure according to age in weeks, as

$$Av\gamma_t = \frac{\sum_{t=1}^{n} \gamma_{i,t}}{N_t} \tag{11}$$

Where N_t is the number of observations for each age range in weeks.

Table III provides some summary descriptive statistics for liquidity and illiquidity proxies, from 1996 to 2000. Market share has an average of 0.017696, similar to Roll mean, and half of Amivest mean that is over 0.039. Original estimated series for Amivest measure has changed scale for comparison with other series, dividing result by 100. Amihud and Bao measures present similar maximum values, while minimum are quite different.

Table III. Principal descriptive statistics.

	Market Share	**Amihud**	**Roll**	**Bao**	**Amivest**
Average	0,0176959	0,000237	0,024822	0,001174	0,039358
Std dev	0,0613697	0,005790	0,064865	0,015968	0,287116
Min	0,0000236	0,000000	0,000000	-0,011737	0,000000
Max	0,0034992	0,000007	0,007764	0,000012	0,005469
Median	0,6955222	0,819320	1,170894	0,343449	27,87504

This table shows principal statistics for estimated liquidity and illiquidity proxies measures, for data subsample from January 1996 to December 2000, and for a total number of observations equal to 21617.

Figure (4) shows these liquidity and illiquidity measures sorted by age for 2-year notes, from 1996 to 2000. Behavior of Market Share and Amivest are quite regular, while Amihud, Roll and Bao measures present a regular pattern for first 4 weeks on bonds life and for week 60 until the end. Between week 4 and week 60, Amihud, Roll and Bao measures present values very different and with many fluctuations.

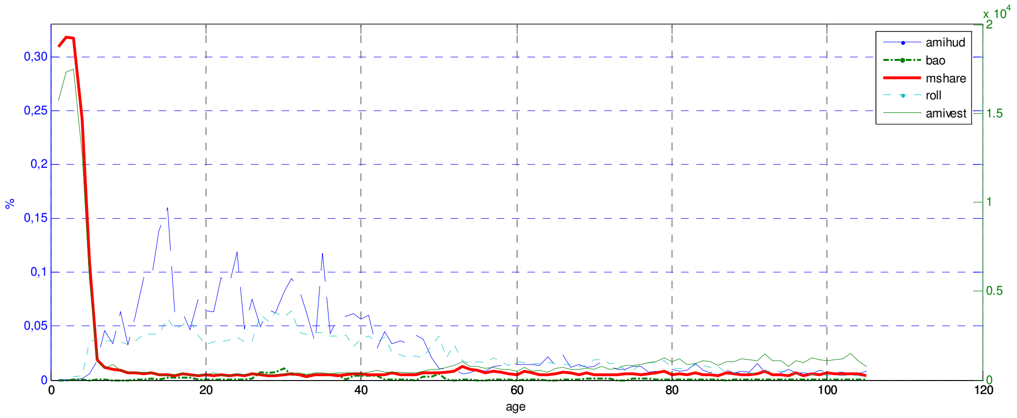

Figure 4. 2-year notes liquidity and illiquidity proxies. It represents the average market share, Amivest ratio, Amihud, Roll and Bao measures in terms of security's age measured in weeks, for securities with original maturities of 2 years.

For first 4 weeks, illiquidity measures have the lowest values, as we expected. On first life notes weeks, securities are very liquid. It is easy to buy and sell those securities, and costs for trading are low. So illiquidity must be close to zero. In contrast, liquidity must have the highest values, and that is what we can observe in figure (4).

4.3. Current Liquidity.

4.3.1. 2-year notes current liquidity and illiquidity.

We continue this section analyzing liquidity and illiquidity proxies measures, from which we will obtain our current and future liquidity measures.

Although all maturities, 2-, 3- 5- and 10-year notes, and 30-year bonds have the same regular pattern, we will analyze more detailed specific issuance of 2-year notes.

Liquidity measured by market share or by Amivest Ratio becomes predictable over time. As shown in figure (4), liquidity follows a pattern quite regular. For the first four weeks since issuance date of notes these measures take the highest values. Bao measure present a value near zero in many cases, and so a pattern regular too.

Illiquidity Amihud and Roll measures present a different pattern. For first four weeks since issuance date, both measures take value zero, so that means that illiquidity is zero and notes are highly liquid, as we expected. In following weeks, measures become irregular with successive jumps, until last 30-40 weeks to maturity, when values are near zero. Those measures are more irregular but we will also try to model them.

These results suggest that we may model security's liquidity as a function of security's age. A valid functional form to reflect this behavior would be from an exponential function[23], in the case of Amivest and Market Share liquidity proxies. The following exponential expression measures the average market share depending on the bond age:

$$MS_{i,t} = c + \beta_1 \exp\left(-\beta_2 \cdot age_{i,t}^2\right) + \beta_3 \cdot \beta_4^{age} + u_{i,t} \tag{12}$$

To model the Amivest Ratio, we use another exponential function, similar to the equation used in Díaz, Merrick and Navarro (2006). Amivest ratio presents a more pronounced hump for first weeks, and next equation takes it into account:

$$Amivest_{i,t} = \beta_1 \exp\left[-\beta_2 \cdot (age_{i,t} - \beta_3)^2\right] + \beta_4 \cdot \beta_5^{age_{i,t}} + u_{i,t} \tag{13}$$

To model Amihud, Bao and Roll measures, previously we have smooth initial obtained series. After smoothing, Amihud illiquidity measure, can be model as

$$Amihud_{i,t} = \exp(\beta_1 + \beta_2/age + \beta_3 \ln(age)) + u_{i,t} \tag{14}$$

Roll measure, can be model following next exponential equation:

$$Roll_{i,t} = c + \beta_1 \exp\left(-\frac{age}{\beta_2}\right) + \beta_3 \exp\left(-\frac{age}{\beta_4}\right) + u_{i,t} \tag{15}$$

And equation proposed for Bao measure is defined by:

$$Bao_{i,t} = \exp(\beta_1 + \beta_2 \cdot age + \beta_3 \cdot age^2) + u_{i,t} \tag{16}$$

The estimation results of equations (12) to (16) presented in Table III shows parameter values, and the equations applied in each case. Figures (5) and (6) show actual and estimated values for all those liquidity measures.

Table III. Principal results from estimated equations (12) to (16).

	Market Share	**Amivest**	**Amihud**	**Bao**	**Roll**
c	0,0048	-	-	-	-0,0002
$\boldsymbol{\beta_1}$	-2,4380	1,3480	7,6132	-20,0873	-27,1131
$\boldsymbol{\beta_2}$	0,3202	0,3255	-53,5453	0,9272	22,4061
$\boldsymbol{\beta_3}$	4,4020	2,9980	-2,6620	-0,0143	27,1023
$\boldsymbol{\beta_4}$	0,4713	2,0840	-	-	22,5146
$\boldsymbol{\beta_5}$	-	0,6266	-	-	-
Standard error	0,000023	0,000127	0,004564	0,000966	0,002920
R^2	0,9567	0,9966	0,8358	0,8411	0,9574

Estimated equations as follows:

$$MS_{i,t} = c + \beta_1 \exp\left(-\beta_2 \cdot age_{i,t}^2\right) + \beta_3 \cdot \beta_4^{age} + u_{i,t}$$
$$Amivest_{i,t} = \beta_1 \exp\left[-\beta_2 \cdot (age_{i,t} - \beta_3)^2\right] + \beta_4 \cdot \beta_5^{age_{i,t}} + u_{i,t}$$
$$Amihud_{i,t} = \exp(\beta_1 + \beta_2/age + \beta_3 \ln(age)) + u_{i,t}$$
$$Bao_{i,t} = exp(\beta_1 + \beta_2 \cdot age + \beta_3 \cdot age^2) + u_{i,t}$$
$$Roll_{i,t} = c + \beta_1 \exp(-age/\beta_2) + \beta_3 \exp(-age/\beta_4) + u_{i,t}$$

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[23] The original exponential form was proposed by Heligman and Pollard (1980) on their seminal work about human mortality, where they establish a relationship between mortality and age. Hence the name ‘life cycle’. Díaz, Merrick and Navarro (2006) use an equation inspired in their actuarial research, and here we use a version of this exponential form for the U.S. Treasury securities.

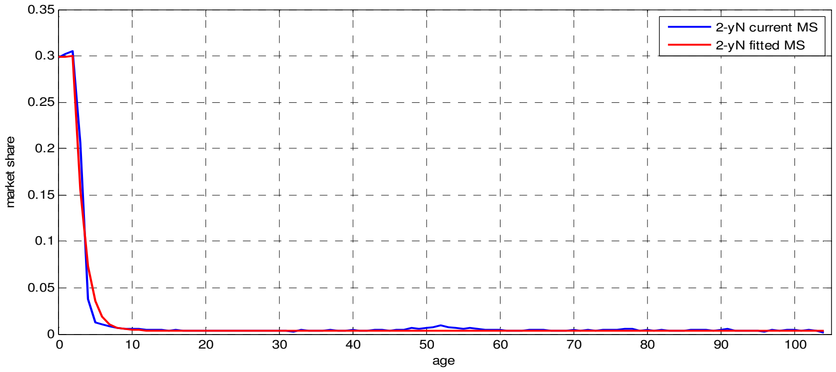

Figure 5. 2-year notes current and estimated average market share. It represents the current and estimated average market share from the exponential form described in the table (III), depending on note age. The note age is measured in weeks elapsed from its issue.

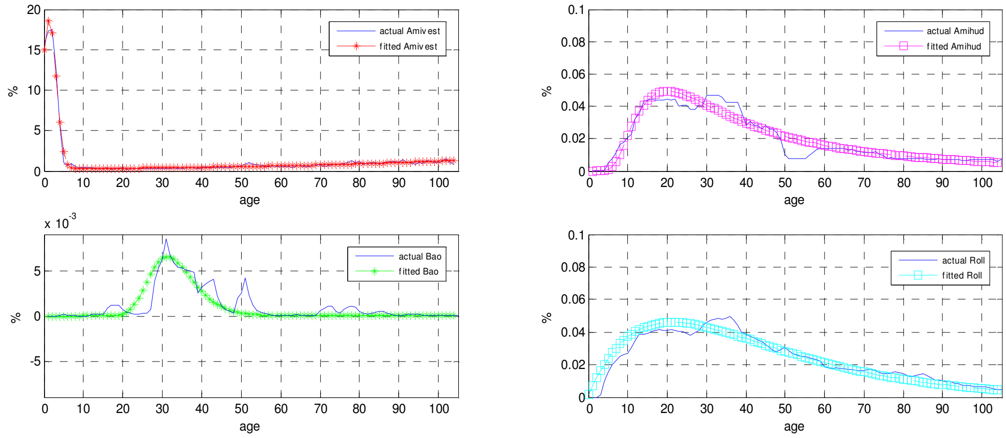

Figure 6. 2-year notes current and estimated average Amivest, Amihud, Bao and Roll measures. It represents the current and estimated average measures after smoothing series, from the exponential forms described in table (III), depending on note age. The note age is measured in weeks elapsed from its issue.

Estimated adjusted equations present high R^2, with values close to 85% and 99%. Firstly we have subjected to smoothing the series regarding measures Amihud, Roll and Bao, since that those measures present different peaks along them. It has not been necessary in case of market share and Amivest measures. The adjustment let us estimate the future path of liquidity, as a function of bond's age, as we see next.

4.3. Future Liquidity.

4.3.1. 2-year notes future liquidity and illiquidity.

We have established a link between liquidity and bond's age, a deterministic variable. At any time during liquidity life cycle, there is a bond-level liquidity that is its current-level liquidity and there is a future-level liquidity remaining until maturity date[24]. Both current and future expected liquidity are important in price changes. So we want to estimate bond expected future liquidity at any time for its remaining time to maturity.

Once we have measured current liquidity and illiquidity with our different exponential forms, we are able to estimate future liquidity and future illiquidity through our liquidity bond 'life cycle' function. We will use estimated functions on previous section to estimate those future measures.

But as we have seen before, liquidity goes over a cycle and expected future liquidity for one week bonds is much more different than for fifty weeks bonds. Liquidity behavior leads us to estimate expected future liquidity at any time, based on the remaining age of the bond, and not on the average liquidity realized until that time[25]. As we have fitted different functions for different liquidity measures, we can project the future path of liquidity using those estimated functions.

With estimated coefficients of each form set according to table (III), we estimate the expected value of market share for bond *i* at time $t + j$:

$$E_t[MS_{i,t+j}] = \hat{c} + \hat{\beta}_1\, exp(-\hat{\beta}_2 \cdot age_{i,t+j}^2) + \hat{\beta}_3 \cdot \hat{\beta}_4^{age_{i,t+j}} \tag{17}$$

This is the 2-year notes conditional expected market share for bond *i* during any future day $t+j$. For each note we have a day *t* conditional expectation for market share liquidity measure. For each term to maturity we estimate an expected value of future liquidity.

The expected average market share for each bond at time *t*, would be the average over m_{it} of the total expected future market share:

$$\overline{MS}_{i,t,t+m_{it}} = \frac{1}{m_{it}} \sum_{j=1}^{m_{it}} E[MS_{i,t+j}] \tag{18}$$

where m_{it} is the number of laborer days of bond *i* until it's maturity. Thus we estimate from day $t+1$ to $t+m_{it}$ an average expected market share. This average expected future market share will be the expected future liquidity for each security by issue, since we argue that agents have rational expectations. Figure (8) shows how expected future liquidity is lower at any time than current liquidity, as this measure is the average of liquidity remaining to maturity.

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[24] Goldreich, Hanke and Nath (2005) showed that lifetime liquidity includes both, security's current liquidity, and expected future path liquidity.

[25] Elton (1999) shows that the use of average realized returns as a proxy for expected returns is misplaced. He analyzes realized returns from U.S. government bonds and concludes that there are much better estimates of expected returns from realized return. In this case, we consider realized liquidity instead of realized returns, and the same goes for expected liquidity instead of expected returns.

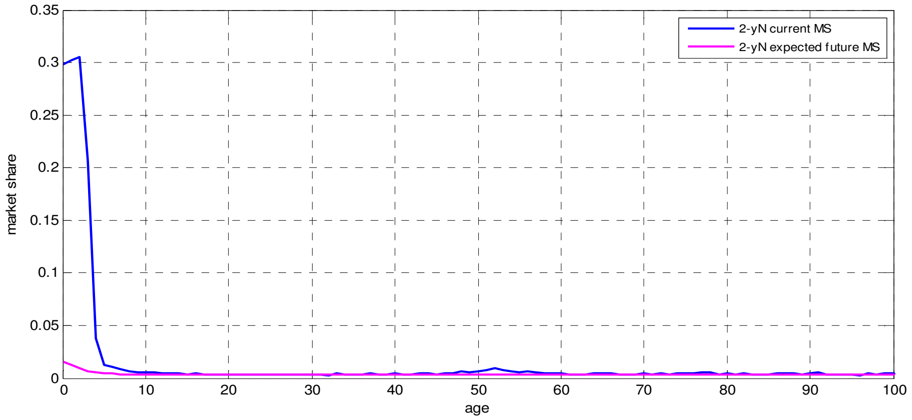

Figure 8. 2-years notes average current market share and expected future market share. The average is calculated by age in weekly ranges, controlling for laborer days.

We can also estimate future liquidity and illiquidity measures for Amivest, Amihud, Roll and Bao estimated measures, as follows:

$$E\left[Amivest_{i,t+j}\right] = \hat{\beta}_1 \exp\left[-\hat{\beta}_2 \cdot \left(age_{i,t} - \hat{\beta}_3\right)^2\right] + \hat{\beta}_4 \cdot \hat{\beta}_5^{age_{i,t}} \tag{19}$$

$$E\left[Amihud_{i,t+j}\right] = \exp\left(\hat{\beta}_1 + \hat{\beta}_2/age + \hat{\beta}_3 \ln\left(age\right)\right) \tag{20}$$

$$E\left[Roll_{i,t+j}\right] = \hat{c} + \hat{\beta}_1 \exp\left(-age/\hat{\beta}_2\right) + \hat{\beta}_3 \exp\left(-age/\hat{\beta}_4\right) \tag{21}$$

$$E\left[Bao_{i,t+j}\right] = \exp\left(\hat{\beta}_1 + \hat{\beta}_2 \cdot age + \hat{\beta}_3 \cdot age^2\right) \tag{22}$$

The expected average measure to each bond at time *t*, would be the average over m_{it} of the total expected future measure for each note:

$$\overline{Amivest}_{i,t,t+m_{it}} = \frac{1}{m_{it}} \sum_{j=1}^{m_{it}} E\left[Amivest_{i,t+j}\right] \tag{23}$$

$$\overline{Amihud}_{i,t,t+m_{it}} = \frac{1}{m_{it}} \sum_{j=1}^{m_{it}} E\left[Amihud_{i,t+j}\right] \tag{24}$$

$$\overline{Roll}_{i,t,t+m_{it}} = \frac{1}{m_{it}} \sum_{j=1}^{m_{it}} E\left[Roll_{i,t+j}\right] \tag{25}$$

$$\overline{Bao}_{i,t,t+m_{it}} = \frac{1}{m_{it}} \sum_{j=1}^{m_{it}} E\left[Bao_{i,t+j}\right] \tag{26}$$

where m_{it} is the number of laborer days remaining until maturity for bond *i* on day *t*. Those averages expected future measures will be the measures of future liquidity and illiquidity in each case.

Figure (9) shows average expected future liquidity sorted by note's age in weeks:

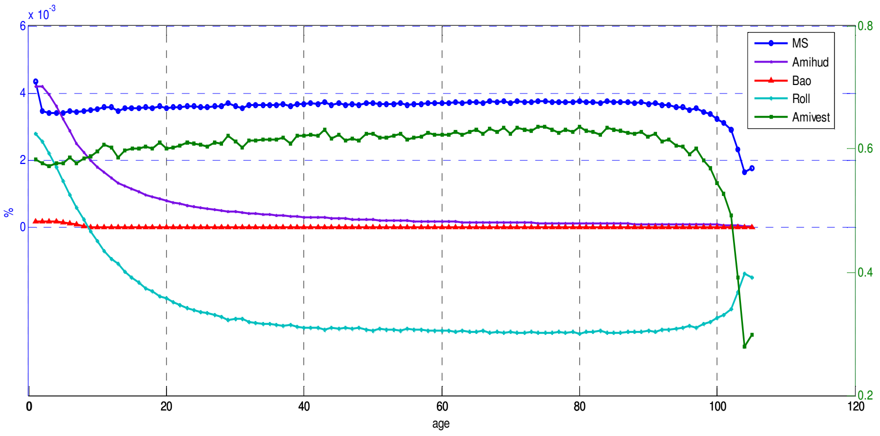

Figure 9. 2-years bond average expected future Market Share, Amihud, Roll, Bao and Amivest measures. The average is calculated for all measures by age in weekly ranges.

Illiquidity measures follow the same pattern. On first weeks, illiquidity is close to zero, the lowest values, so expected future illiquidity at that moment is high because is an average of expected future illiquidity. For remaining weeks, future illiquidity is going down because the average is made over expected future values that become smaller. In case of liquidity measures, for 4 first weeks liquidity presents the highest values. Thus expected future liquidity at that moment is quite large than expected future liquidity for the rest of weeks until maturity.

5. Empirical Results. Analysis of yield spreads.

In this section we analyze yield spreads between securities[26]. We calculate yield spreads as differences between an outstanding security yield and another most liquid and theoretical security yield with the same term to maturity. We want to see the differences in yields between assets with the same characteristics in terms of maturity and coupon payments.

Yield spread between two Treasury bonds with similar characteristics should be interpreted as a 'liquidity premium'. The so-called liquidity premium bond measures the yield spread between less liquid and more liquid bonds. Investors demand a higher return or a lower price as the lower the liquidity of a title. Besides, the liquidity premium measures the price spread between less liquid and more liquid bonds that also could be described as 'price premium'. Therefore liquidity premium can be measured by yield spreads or by price spreads.

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[26] The empirical literature propose different measures to yield spreads. As example Amihud and Medelson (1991), Strebulaev (2001), Goldreich, Hanke and Nath (2005) or Díaz, Navarro and Merrick (2006).

Differences in yield may be explained by differences in liquidity. To that purpose we include both current liquidity and expected future liquidity that we have obtained using a bond liquidity 'life cycle' function, as well as current illiquidity and expected future illiquidity. Our initial hypothesis is that newer securities have higher security-level liquidity and the older ones have lower security-level liquidity. So we expect that newer notes would have lower yield spreads because prices are high, and that the older have larger yield spreads because those notes are cheaper due to investors don't want these notes in them portfolios.

We will relate these liquidity measures with yield spreads between securities and other securities considered the most liquid in markets, in time-series regressions, besides other bond characteristic, trading activity and status variables.

To do this, we first calculate, at a time *t*, the yield that has a security maturating at *T* and that pays *c* coupons at interest rates prevailing in *t*. Additionally, we calculate the yield that would have another theoretical security that were issued on *t*, with maturity date at *T* and with the same number of coupons equal to *c*, discounted at spot interest rates. The difference between these yields is equal to our yield spreads.

We get the spot interest rates using Svensson (1994) procedure that estimates the term structure through a parametric and parsimonious model. Its functional form is a function of term to maturity. The expression is as follows

$$r_f(T,\beta) = \beta_0 + \beta_1\left(\frac{1-exp\left(-\frac{T}{\tau_1}\right)}{\frac{T}{\tau_1}}\right) + \beta_2\left(\frac{1-exp\left(-\frac{T}{\tau_1}\right)}{\frac{T}{\tau_1}} - exp\left(-\frac{T}{\tau_1}\right)\right) + \beta_3\left(\frac{1-exp\left(-\frac{T}{\tau_2}\right)}{\frac{T}{\tau_2}} - exp\left(-\frac{T}{\tau_2}\right)\right) \tag{27}$$

where $r_f(T,\beta)$ is the risk free spot rate over time to maturity T as a function of the Beta-factors. $\beta_0, \beta_1, \beta_2, \beta_3, \tau_1$ and τ_2 are the parameters to be estimated; they are computed by a non-linear optimization program aiming to minimize the squared deviation between estimated and true interest rates. The Svensson method is more precise and has the benefit to "smooth out" a potential market mispricing due to illiquidity as for example in the case of very long-term bonds. Now, to calculate the theoretical price, and therefore the theoretical yield, we discount each of the remaining security cash flows until its maturity, to the interest rate on each data for each term.

Once we estimate current security yield at time *t* and theoretical security yield also at *t*, we calculate, for each of securities, the yield spreads as:

$$YS_{i,t} = y_{curr_{i,t}} - y_{theo_{i,t}} \tag{28}$$

where $y_{curr_{i,t}}$ is the current yield for security *i* on *t*, and $y_{theo_{i,t}}$ is the theoretical yield for security *i* on *t*, whose price we have get discounting bond cash flows to spot interest rates.

Yield spreads for older notes and bonds are greater than those observed for the newer notes and bonds. For new securities, as they are the most liquid on-the-run securities, we are comparing the yield from a new bond with the theoretical yield for another bond that would be too new. So yield spreads are quite small. In the case of older securities, less liquid off-the-run securities, they have a higher yield respect to a security with similar characteristics in term to maturity and coupon payments. Thus yield spread is greater than on-the-run yield spreads.

5.1. 2-year notes yield spreads.

Liquidity premium are higher during on-the-run periods and lower for the rest of bonds. It would indicate that investors are willing to pay a premium on price in order to buy the newest assets, i.e. the most liquid assets, and so the yield for those assets is going to be lower. And as we have seen that liquidity is a function of bonds age, we can say that the lower age is the lower liquidity premium is, because liquidity is an inverse function of bond's age.

Figure (10) shows yield spreads of 2-year notes. We can see that the larger the age is, the higher the liquidity premium is.

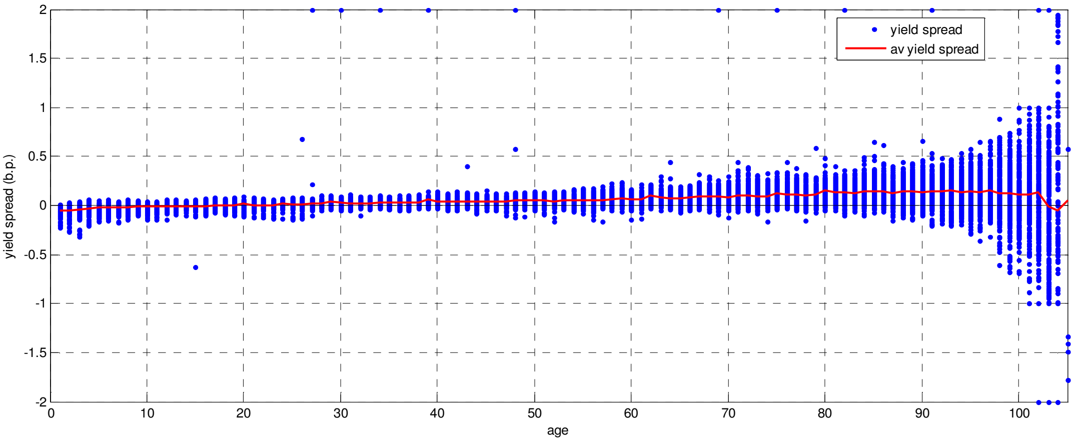


Figure 10. 2-year notes yield spreads. Yield spread measured as the difference between current yield and theoretical yield, in basic points, by bond's age in weeks.

As we can see in figure (10), newer notes have in most of the cases negative yield spreads. As bond's age increases, yield spreads begin to be positive indicating that current and theoretical prices are quite different. Specifically theoretical prices begin to be higher than current prices, leading to higher current yields and lower theoretical yields, and therefore higher yield spreads.

We have estimate different current and future liquidity and illiquidity measures to try to explain these differences in yield observed in 2-year U.S. Treasury securities.

Table IV. Liquidity and Illiquidity measures correlation matrix.

	MS	**Amihud**	**Roll**	**Bao**	**Amivest**	$\overline{\text{MS}}$	$\overline{\text{Amıhud}}$	$\overline{\text{Roll}}$	$\overline{\text{Bao}}$	$\overline{\text{Amıvest}}$
MS	1,0000									
Amihud	-0,0109	1,0000								
Roll	-0,0022	-0,0051	1,0000							
Bao	-0,0038	-0,0013	0,7786	1,0000						
Amivest	0,4244	-0,0054	-0,0033	-0,0024	1,0000					
$\overline{\text{MS}}$	-0,0535	0,0048	-0,0246	-0,0051	-0,0817	1,0000				
$\overline{\text{Amıhud}}$	0,7040	-0,0035	-0,0122	-0,0096	0,2483	0,0996	1,0000			
$\overline{\text{Roll}}$	0,6328	-0,0067	0,0050	-0,0041	0,2579	-0,4995	0,8071	1,0000		
$\overline{\text{Bao}}$	0,7829	-0,0093	-0,0029	-0,0042	0,2972	0,0063	0,8873	0,7547	1,0000	
$\overline{\text{Amıvest}}$	-0,0960	0,0052	-0,0244	-0,0049	-0,0958	0,9942	0,0651	-0,5327	-0,0296	1,0000

Table IV shows correlation between current and expected future liquidity and illiquidity measures.

Firstly, attending to correlation matrix, we can see how high correlation is between measures, so we estimate an ortogonalized version for current and future Amivest, Roll and Bao measures, as follows:

$$ort\_Bao_{i,t} = c + \gamma_1 Roll_{i,t} + v_{i,t} \tag{29}$$

$$ort\_\overline{Roll}_{i,t} = c + \gamma_1 \overline{Am\imath hud}_{i,t} + v_{i,t} \tag{30}$$

$$ort\_\overline{Bao}_{i,t} = c + \gamma_1 \overline{Am\imath hud}_{i,t} + v_{i,t} \tag{31}$$

$$ort\_\overline{Am\imath vest}_{i,t} = c + \gamma_1 \overline{MS}_{i,t} + v_{i,t} \tag{32}$$

In addition, to analyze yield spreads, we will consider current liquidity and illiquidity, and expected future liquidity and illiquidity, besides other variables to explain yield spreads, such as duration, that would be reflecting observed differences in term to maturity, amount outstanding by issue, bonds age, and other control variables to control for interest rates, such as level, slope and curvature. Coupon is important because different coupons securities have different taxes. High coupon bonds are subject to higher taxation in U.S. Age is also important in yield spreads, like we have seen in previous sections.

Table (V) shows the correlation matrix of these variables, with the ortogonalized versions of liquidity and illiquidity measures. Yield spreads correlation is high with variables as age, duration and expected future Amihud measure. The sign between liquidity premiums and current and expected future measures, is positive whit liquidity measures and negative with future measures as we expected. This is indicating that the higher liquidity is the lower yield spread is, and in contrast that the higher illiquidity is the higher liquidity premiums are.

Table V. Correlation Matrix

	Agvolume	Amihud	$\overline{\mathrm{Amihud}}$	Amivest	Coupon	Curvat	YS	DSONTR	Dur	Age	Level	MS	$\overline{\mathrm{MS}}$	$\overline{\mathrm{RAmivest}}$	RBao	$\overline{\mathrm{RBao}}$	Roll	$\overline{\mathrm{RRoll}}$	AmOuts	Slope	SBidAsk
Agvolume	1,0000																				
Amihud	-0,0099	1,0000																			
$\overline{\mathrm{Amihud}}$	0,7064	-0,0035	1,0000																		
Amivest	0,3262	-0,0054	0,2483	1,0000																	
Coupon	-0,0410	0,0075	-0,0594	0,0124	1,0000																
Curvat	0,0011	-0,0007	0,0404	-0,0130	-0,1559	1,0000															
YS	0,0905	0,0081	0,1715	-0,0359	0,0119	-0,0150	1,0000														
DSONTR	0,7848	-0,0082	0,7671	0,3405	-0,0167	-0,0075	0,0741	1,0000													
Dur	0,3170	0,0162	0,6927	0,1202	-0,1004	0,0125	0,2171	0,3547	1,0000												
Age	-0,3158	-0,0165	-0,6946	-0,1202	0,0947	-0,0118	-0,2175	-0,3543	-0,9997	1,0000											
Level	-0,0255	0,0076	0,0019	0,0103	0,0592	0,5984	0,1522	-0,0638	0,0209	-0,0220	1,0000										
MS	0,8963	-0,0109	0,7040	-0,4244	-0,0299	0,0094	-0,0966	0,8169	0,3315	-0,3305	-0,0007	1,0000									
$\overline{\mathrm{MS}}$	0,0669	0,0048	0,0996	-0,0817	-0,0273	0,1444	-0,0726	0,0113	0,0324	-0,0300	-0,1134	-0,0535	1,0000								
$\overline{\mathrm{RAmivest}}$	-0,4015	0,0040	-0,3168	-0,1362	0,0025	0,0184	-0,0494	-0,3563	-0,1432	0,1424	-0,0138	-0,3987	0,0000	1,0000							
RBAO	-0,0006	0,0043	-0,0001	0,0004	0,0248	-0,0279	0,0015	-0,0005	-0,0079	0,0074	-0,0522	-0,0033	0,0224	0,0019	1,0000						
$\overline{\mathrm{RBao}}$	0,2818	-0,0134	0,0000	0,1667	0,0813	-0,0374	-0,0726	0,4307	-0,3716	0,3757	0,0189	0,3430	-0,1780	-0,1166	-0,0066	1,0000					
Roll	-0,0047	-0,0051	-0,0122	-0,0033	-0,0016	0,0030	0,0131	-0,0049	-0,0251	0,0254	-0,0050	-0,0022	-0,0246	0,0007	0,0000	0,0172	1,0000				
$\overline{\mathrm{RRoll}}$	-0,0052	-0,0065	0,0000	0,0974	0,0326	-0,1412	0,0810	0,0467	0,0005	-0,0037	0,1133	0,1094	-0,9823	-0,1374	-0,0213	0,1414	0,0252	1,0000			
AmOutst	-0,0002	0,0224	0,0032	-0,0240	-0,0232	0,0825	-0,0435	-0,0464	0,0074	-0,0080	0,2186	-0,0028	-0,0391	-0,0016	-0,0002	-0,0160	-0,0012	0,0358	1,0000		
Slope	0,0211	0,0096	0,0574	-0,0423	0,2638	-0,0012	0,0335	-0,0044	-0,0074	0,0071	-0,0187	-0,0476	0,6415	0,0602	0,0294	-0,1022	-0,0252	-0,6346	0,0540	1,0000	
SBidAsk	-0,0025	0,0007	-0,0025	-0,0013	0,0035	-0,0007	-0,0023	-0,0020	-0,0085	0,0087	-0,0001	-0,0035	0,0037	0,0013	-0,0009	0,0014	0,0005	-0,0026	-0,0022	0,0002	1,0000

Table V shows correlation between other liquidity and control variables.

The regression estimated includes variables to control for interest rates: level, slope and curvature. Also includes other explanatory variables, as duration, to take into account the influence term to maturity and age, and a *dummy* variable to control for the status impact in yield spreads. This *dummy* variable takes value 1 if bond is on-the-run and 0 otherwise. Estimated equations and results are shown on in table (VI):

Table VI. Current liquidity/illiquidity and expected future liquidity/illiquidity impacts on yield spreads controlling for interest rates, status, duration and other price impact variables.

Estimated Equation	Variables	Coefficients	t-Statistics	Prob	R^2
(1)	C	-0,0006	-19,4846	0,0000	0,0096
	MS	0,0033	19,6325	0,0000	
	Amihud	-0,0025	-0,8845	0,3764	
	Roll	-0,0008	-2,6138	0,0090	
	RBao	0,0008	0,3077	0,7583	
	Amivest	0,0000	-1,4476	0,1477	
(2)	C	0,0088	11,3056	0,0000	0,0605
	MS	-0,0027	-8,4404	0,0000	
	Amihud	-0,0038	-1,1492	0,2505	
	Roll	-0,0006	-1,8201	0,0688	
	RBao	0,0020	0,7539	0,4509	
	Amivest	0,0000	1,0709	0,2842	
	$\overline{\text{MS}}$	-2,6913	-11,8738	0,0000	
	$\overline{\text{Amihud}}$	0,6680	17,7488	0,0000	
	$\overline{\text{RRoll}}$	-2,7476	-10,5894	0,0000	
	$\overline{\text{RBao}}$	-14,4255	-13,9413	0,0000	
	$\overline{\text{RAmivest}}$	-0,0187	-11,5823	0,0000	
(3)	C	0,0030	1,9761	0,0482	0,1753
	MS	0,0011	1,5510	0,1209	
	Amihud	-0,0040	-1,2477	0,2121	
	Roll	-0,0002	-0,6551	0,5124	
	RBao	0,0037	1,3581	0,1744	
	Amivest	0,0000	-0,0094	0,9925	
	$\overline{\text{MS}}$	-1,8139	-4,9524	0,0000	
	$\overline{\text{Amihud}}$	0,6311	5,0247	0,0000	
	$\overline{\text{RRoll}}$	-1,9112	-5,0173	0,0000	
	$\overline{\text{RBao}}$	-0,9605	-0,3939	0,6936	
	$\overline{\text{RAmivest}}$	-0,0130	-5,0044	0,0000	
	Level	0,0617	5,8388	0,0000	
	Slope	0,0413	3,2653	0,0011	
	Curvat	-0,3447	-6,2293	0,0000	
	DSONTR	-0,0022	-4,3310	0,0000	
	Dur	0,0005	3,2517	0,0011	
	AmOutst	0,0000	-5,4123	0,0000	
	SBidAsk	0,0000	0,1231	0,9020	
	Agvolume	0,0000	1,2653	0,2058	
(4)	C	0,0033	2,3264	0,0200	0,1747
	$\overline{\text{MS}}$	-1,8995	-5,3450	0,0000	
	$\overline{\text{Amihud}}$	0,6541	5,1270	0,0000	
	$\overline{\text{RRoll}}$	1,9985	-5,3403	0,0000	
	$\overline{\text{RBao}}$	-1,1659	-0,4925	0,6224	
	$\overline{\text{RAmivest}}$	-0,0136	-5,4053	0,0000	
	Level	0,0616	5,7848	0,0000	
	Slope	0,0415	3,2929	0,0010	
	Curvat	-0,3430	-6,2506	0,0000	
	DSONTR	-0,0021	-4,3945	0,0000	
	Dur	0,0005	3,1998	0,0014	
	AmOutst	0,0000	-5,3932	0,0000	
	Agvolume	0,0000	2,0989	0,0358	
(5)	C	0,0071	9,4454	0,0000	0,1596
	$\overline{\text{MS}}$	-3,0968	-13,9912	0,0000	
	$\overline{\text{Amihud}}$	1,0043	10,2299	0,0000	
	$\overline{\text{RRoll}}$	-3,3228	-12,5515	0,0000	
	$\overline{\text{RBao}}$	-8,2163	-3,9756	0,0001	
	$\overline{\text{RAmivest}}$	-0,0219	-12,4634	0,0000	
	Level	0,0556	5,2704	0,0000	
	Slope	0,0341	2,6053	0,0092	
	Curvat	-0,3423	-5,9462	0,0000	
	DSONTR	-0,0022	-4,9031	0,0000	

Regression estimated for a number of observations equal to 27.158. In regression we have applied Newey-West correction for heteroscedasticity estimators.

Estimated equations are as follows:

$$(1)\ YS_{i,t} = c + \beta_1 MS_{i,t} + \beta_2 Amihud_{i,t} + \beta_3 Roll_{i,t} + \beta_4 RBao_{i,t} + \beta_5 Amivest_{i,t} + u_{i,t} \tag{33}$$

$$\begin{aligned}(2)\ YS_{i,t} = {} & c + \beta_1 MS_{i,t} + \beta_2 Amihud_{i,t} + \beta_3 Roll_{i,t} + \beta_4 RBao_{i,t} + \beta_5 Amivest_{i,t} + \\ & + \beta_6 \overline{MS}_{i,t} + \beta_7 \overline{Amihud}_{i,t} + \beta_8 \overline{RRoll}_{i,t} + \beta_9 \overline{RBao}_{i,t} + \beta_{10} \overline{RAmivest}_{i,t} + u_{i,t}\end{aligned} \tag{34}$$

$$\begin{aligned}(3)\ YS_{i,t} = {} & c + \beta_1 MS_{i,t} + \beta_2 Amihud_{i,t} + \beta_3 Roll_{i,t} + \beta_4 RBao_{i,t} + \beta_5 Amivest_{i,t} + \beta_6 \overline{MS}_{i,t} + \beta_7 \overline{Amihud}_{i,t} + \\ & + \beta_8 \overline{RRoll}_{i,t} + \beta_9 \overline{RBao}_{i,t} + \beta_{10} \overline{RAmivest}_{i,t} + \beta_{11} level_{i,t} + \beta_{12} slope_{i,t} + + \beta_{13} curvat_{i,t} + \\ & + \beta_{14} DSONTR_{i,t} + \beta_{15} Dur_{i,t} + \beta_{16} AmOutst_{i,t} + \beta_{17} SBidAsk_{i,t} + \beta_{18} agvolume_{i,t} + u_{i,t}\end{aligned} \tag{35}$$

$$\begin{aligned}(4)\ YS_{i,t} = {} & c + \beta_1 \overline{MS}_{i,t} + \beta_2 \overline{Amihud}_{i,t} + \beta_3 \overline{RRoll}_{i,t} + \beta_4 \overline{RBao}_{i,t} + \beta_5 \overline{RAmivest}_{i,t} + \beta_6 level_{i,t} + \beta_7 slope_{i,t} + \\ & + \beta_8 curvat_{i,t} + \beta_9 DSONTR_{i,t} + \beta_{10} Dur_{i,t} + \beta_{11} AmOutst_{i,t} + \beta_{12} Agvolume_{i,t} + u_{i,t}\end{aligned} \tag{36}$$

$$\begin{aligned}(5)\ YS_{i,t} = {} & c + \beta_1 \overline{MS}_{i,t} + \beta_2 \overline{Amihud}_{i,t} + \beta_3 \overline{RRoll}_{i,t} + \beta_4 \overline{RBao}_{i,t} + \beta_5 \overline{RAmivest}_{i,t} + \\ & + \beta_6 level_{i,t} + \beta_7 slope_{i,t} + \beta_8 curvat_{i,t} + \beta_9 DSONTR_{i,t} + u_{i,t}\end{aligned} \tag{37}$$

where $YS_{i,t}$ measures yield spread for each security *i* on day *t*; $MS_{i,t}$ is the current market share of security *i* on day *t* ; $Amihud_{i,t}$ is the current Amihud measure for note *i* on day *t*; $Roll_{i,t}$ is the current Roll measure for note *i* on day *t*; $RBAO_{i,t}$ are the residuals from an ortogonalized version of Bao current measure; $Amivest_{i,t}$ is the current Amivest measure for security *i* on day *t*; $\overline{MS}_{i,t}$ is the average expected future liquidity of the note *i* at time *t*; $\overline{Amihud}_{i,t}$ is the future Amihud measure for note *i* on day *t*; $\overline{RRoll}_{i,t}$ are the residuals from ortogonalized version of Roll future measure; $\overline{RBao}_{i,t}$ are the residuals from ortogonalized version of Bao future measure; $\overline{RAmivest}_{i,t}$ are the residuals from ortogonalized version of Amivest future measure; $level_{i,t}, slope_{i,t}\ and\ curvat_{i,t}$ are control variables for interest rates; $DSOTR_{i,t}$ is a *dummy* variable that reflects bond status. It takes value 1 if is an on-the-run bond, and 0 otherwise; $dur_{i,t}$ is the Macaulay duration for note *i* on day *t*; $AmOutst_{i,t}$ is the amount outstanding for bond i on day t; and $AgVolume_{i,t}$ is the total aggregate volume traded for note *i* on day *t*.

The different regressions results showed in table (VI) reflects that both, current liquidity and future liquidity, explain the observed yield spreads on U.S. Treasury notes, including other control variables. Although only market share and Roll current liquidity measures, add significant explanatory power to explain liquidity premiums. If we distinguish between current and future measures, the last ones are more decisive in explaining yield spreads. The R-squared for estimated equation (4) is about 17%, and all the variables include add explanatory power for observed yield spreads to a 99% confidence level, except Bao future liquidity measure. Coefficients signs for liquidity-related variables are negative, as we expected. In case of illiquidity-related variables, sign has result to be either positive or negative, but in case of Bao measure it has result not to be significant. This result implies that the lower liquidity is, both current and expected future, the larger U.S. debt securities yield spread is. In terms of magnitude, expected future liquidity seems to be most important rather than current liquidity, as coefficients accompanying explanatory variables are higher. Status notes variable coefficient present negative sign; it would indicate that new notes on status on-the-run have lower yield spread, than bonds in the off-the-run status. In the rest of equations, both the significance and magnitude of the coefficients accompanying future liquidity variables has proved to be higher. These results confirm the importance of future liquidity in explaining yield spreads in U.S. Treasury 2-year notes.

6. Further Analysis of yield spreads. 3-, 5- and 10-year notes, and 30-year bonds.

In this section we do the same analysis to explain yield spreads for 3-, 5- and 10-year notes and for 30-year bonds but only with one liquidity proxy measure, market share. We don't take into account illiquidity measures. Our current liquidity measure is the individual daily market share. On first instance we estimate liquidity life cycle functions taken into account the different type of issue by term to maturity. After that we can estimate the future path of liquidity, and thus the average expected future liquidity that will be our future liquidity. Once we have liquidity measures in time series data, we estimate yield spreads regression over our current and expected future liquidity besides other control variables.

6.1. Current Liquidity.

3- and 5-year notes market share measures have a very similar behavior, while 10-year notes and 30-year bonds is different (see figure (A1)). So we will apply different functions for different issues. For example, for 3- and 5-year notes we apply equation (12), similar to 2-year notes. In the case of 10-year notes, the liquidity behavior is different, and the equation that we will estimate is as follows:

$$MS_{i,t} = c + \beta_1 dummy_1 + \beta_2 dummy_2 + \beta_3 dummy_3 + \beta_4 dummy_4 + u_{i,t} \qquad (38)$$

where $dummy_i$ are dummy variables that reflected this effect: $dummy_1$ takes value 1 for 10-year notes aged between 0 and 7 weeks, and 0 otherwise; $dummy_2$ takes value 1 for bonds with aged between 8 and 11 weeks, and 0 otherwise; $dummy_3$ takes value 1 for bonds aged between 12 and 25 weeks, and 0 otherwise; and $dummy_4$ takes value 1 for bonds aged 26 weeks and 1560 weeks, and 0 otherwise.

And in the case of 30-year bonds, which behave differently from other bonds, the estimated equation is given by the following expression:

$$MS_{i,t} = c + \beta_1 \, exp(-\beta_2 \cdot age_{i,t}) + u_{i,t} \qquad (39)$$

The estimation of equations (38) and (39), presented in Table (AI) shows the results for each issue, and the equations applied in each case, and in figure (A2) we can see the weekly average or each term to maturity where we can see the forms obtained.

6.2. Future liquidity.

For 3-, 5- and 10-year notes and 30-year bonds, expected future liquidity is obtained equal than in 2-year notes.

With estimated coefficients of each form set according to table (AI), we estimate the expected value of market share for bond *i* at time *t + j*:

$$E[MS_{i,t+j}] = \hat{c} + \hat{\beta}_1 \, exp(-\hat{\beta}_2 \cdot age_{i,t+j}^2) + \hat{\beta}_3 \cdot \hat{\beta}_4^{age_{i,t+j}} \quad (40)$$

The expected average market share for each bond at time *t*, would be the average over m_{it} of the total expected future market share:

$$\overline{MS}_{i,t,t+m_{it}} = \frac{1}{m_{it}} \sum_{j=1}^{m_{it}} E[MS_{i,t+j}] \quad (41)$$

where m_{it} is the number of laborer days remaining until maturity for bond *i* on day *t*.

6.3. Yield spreads.

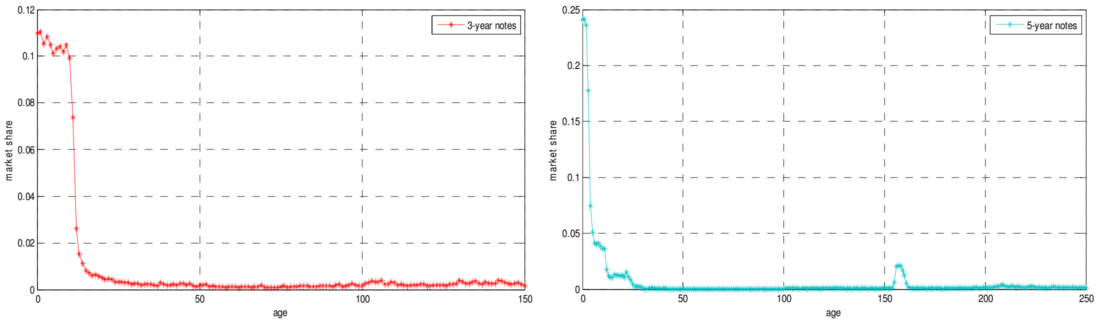
The same way as with 2-year notes, yield spreads are obtained as the difference between actual bonds yields and theoretical bonds yields with similar coupon and term to maturity. Spot rates to calculate bond prices and yields are estimate with Svensson procedure also. If we separate them by security term to maturity, estimated regressions and results are showed in table (AII), where $YS_{i,t}^{ma}$ are the yield spreads for each '*m*' term to maturity. We observe that in all regressions current and future market share variables coefficients have negative sign, as we expected. This result indicates the inverse relationship that exists between yield spreads and security-level liquidity. The lower liquidity is the higher yield spread is. In terms of magnitude, every coefficient accompanying future liquidity measures are higher, indicating that future liquidity is more important and significant in explaining liquidity premiums Also in all regressions duration coefficient has negative sign that would indicate the inverse relationship between yield spreads and bond age: the longer duration is the lower yield spread is. Moreover, control variables for interest rates coefficients are statistically significant in almost all cases.

7. Conclusion.

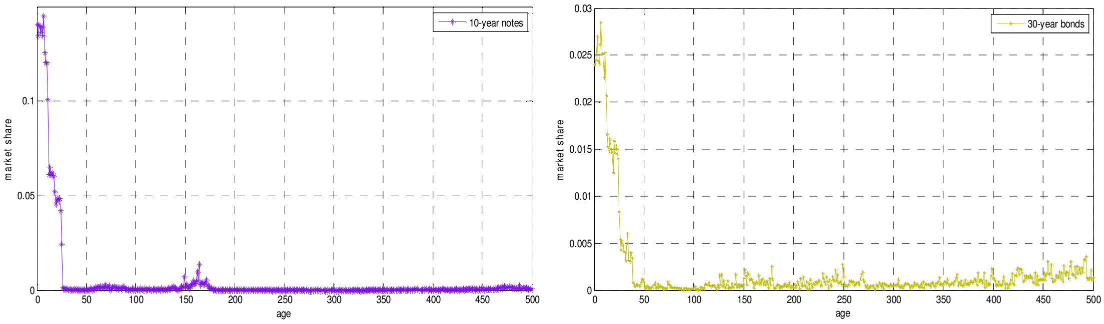
In this paper we have analyzed the observed yield spreads U.S. Treasury notes and U.S. Treasury bonds for the period from January 1996 to December 2000. Changes in yield are interpreted like changes in current and expected future liquidity and illiquidity. It is possible to model liquidity measured by market share and by other liquidity proxies since they have a quite regular pattern. We use bond liquidity 'life cycle’ function that allows us to express changes in yield spreads through changes in liquidity 'life cycle’.

Our results for the yield spreads analysis for bonds and notes with maturities of 2, 3, 5, 10 and 30 years, reveal that the whole liquidity life cycle explain the yield spreads of U.S. Treasuries studied. The role of the liquidity 'life cycle' has allowed us to collect liquidity over the entire life of the bond, not only the current liquidity. In terms of size, expected future liquidity has proved to be more important in all cases than current liquidity. Distinguishing by liquidity proxy, market share and Bao measures seem to be those that best reflected the behavior of yield spreads. Taking each issue individually distinguishing by term to maturity, the results show that both current and future liquidity and illiquidity, in addition to other variables, explain the observed yield spread for the period considered. In these cases, also expected future liquidity is more important in magnitude than current liquidity.

A. Additional Tables and Figures.



a) 3-year notes *b) 5-year notes*



c) 10-year notes *d) 30-year bonds*

Figure A1. It represents the average market share in %, in terms of the age in weeks, for (a) 3-year notes, (b) 5-year notes, (c) 10-year notes and (d) 30-year bonds.

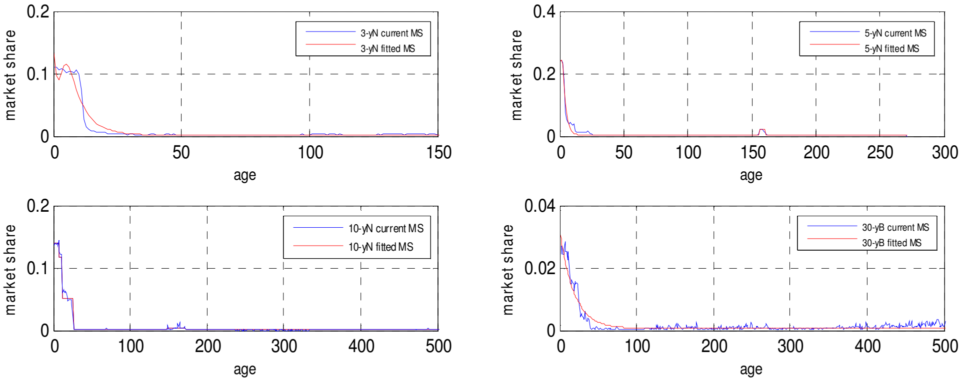


Figure A2. 3-, 5- and 10-year notes and 30-year bonds current and estimated average market share measures. It represents the current and estimated average market share from the exponential forms described in table (AI), depending on notes and bonds age. Age is measured in weeks elapsed from its issue.

Table AI. Principal results from estimated equations (38) and (39).

3-year note	$MS_{i,t} = c + \beta_1 \, exp\left(-\beta_2 \cdot age_{i,t}^2\right) + \beta_3 \cdot \beta_4^{age_{i,t}} + u_{i,t}$	
Estimated coefficients		
c	0.00122	(4.354e-005, 0.002396)
$\boldsymbol{\beta_1}$	-0.2035	(-0.2515, -0.1554)
$\boldsymbol{\beta_2}$	0.07664	(0.06504, 0.08824)
$\boldsymbol{\beta_3}$	0.3351	(0.2813, 0.3889)
$\boldsymbol{\beta_4}$	0.8452	(0.8306, 0.8598)
R^2	0.9378	
5-year note	$MS_{i,t} = c + \beta_1 \, exp\left(-\beta_2 \cdot age_{i,t}^2\right) + \beta_3 \cdot \beta_4^{age_{i,t}} + \beta_5 dummy\_re + u_{i,t}$	
Estimated coefficients		
c	0.002131	(0.001486, 0.002777)
$\boldsymbol{\beta_1}$	−0.2794	(-0.3298, -0.229)
$\boldsymbol{\beta_2}$	0.9636	(0.835, 1.092)
$\boldsymbol{\beta_3}$	0.5176	(0.4678, 0.5673)
$\boldsymbol{\beta_4}$	0.672	(0.6524, 0.6916)
$\boldsymbol{\beta_5}$	0.0184	(0.0144, 0.0225)
R^2	0.9724	
10-year note	$MS_{i,t} = c + \beta_1 dummy_1 + \beta_2 dummy_2 + \beta_3 dummy_3 + \beta_4 dummy_4 + u_{i,t}$	
Estimated coefficients		
c	0.00044672	(0.0002557, 0.0006378)
$\boldsymbol{\beta_1}$	0.13838003	(0.1369002, 0.1398599)
$\boldsymbol{\beta_2}$	0.11639718	(0.1143131, 0.118481,)
$\boldsymbol{\beta_3}$	0.05157218	(0.504466, 0.0526978)
$\boldsymbol{\beta_4}$	0.00368476	(0.0028163, 0.0045533)
R^2	0.99023729	
30-year bond	$MS_{i,t} = c + \beta_1 \, exp\left(-\beta_2 \cdot age_{i,t}\right) + u_{i,t}$	
Estimated coefficients		
c	0.0005666	(0.0005107, 0.0006224)
$\boldsymbol{\beta_1}$	0.03185	(0.03092, 0.03277)
$\boldsymbol{\beta_2}$	0.05491	(0.05257, 0.05726)
R^2	0.8536	

In parenthesis, the confidence intervals for the 95% level.

Table AII. Current market share and expected future market share impact yield spreads and other control variables and liquidity measures.

	Estimated coefficients	***t-statistics***	***Probability***	R^2	R^2 ***Adjusted***
3-years note					
C	0.535563	33.46066	0.0000	0.281156	0.280667
MS	-0.073513	-4.829590	0.0000		
$\overline{MS}$	-3.176693	5.989212	0.0000		
LEVEL	-8.832227	-30.14547	0.0000		
SLOPE	0.121382	0.239067	0.8111		
CURVAT	65.08687	23.95450	0.0000		
DUR	-0.067360	-24.04294	0.0000		
DSONTR	-0.001171	-0.418282	0.6757		
5-years note					
C	0.555315	44.87578	0.0000	0.194359	0.194267
MS	-0.024473	-2.123135	0.0337		
$\overline{MS}$	-85.17954	-55.99831	0.0000		
LEVEL	-6.442159	-32.04623	0.0000		
SLOPE	-1.079891	-7.164619	0.0000		
CURVAT	27.11034	36.57081	0.0000		
DUR	-0.009989	-16.29780	0.0000		
DSONTR	0.229246	34.94764	0.0000		
10-years note					
C	0.425409	68.82237	0.0000	0.266142	0.266008
MS	-0.157194	23.24127	0.0000		
$\overline{MS}$	-6.275210	-5.316544	0.0000		
LEVEL	-7.524968	-81.70184	0.0000		
SLOPE	1.657129	13.54118	0.0000		
CURVAT	24.10872	36.30173	0.0000		
DUR	-0.009557	-27.78697	0.0000		
DSONTR	-0.019461	-4.659376	0.0000		
30-years bond					
C	2.833040	50.41777	0.0000	0.557693	0.557620
MS	-1.374734	-13.84843	0.0000		
$\overline{MS}$	-1034.779	-19.96021	0.0000		
LEVEL	-14.26114	-48.26992	0.0000		
SLOPE	13.74632	28.58096	0.0000		
CURVAT	0.553261	0.569196	0.5692		
DUR	-0.127385	-49.81802	0.0000		
DSONTR	0.297650	17.91993	0.0000		

All regressions adjusted by Newey-West correction for heteroscedasticity estimators. Equations estimated are as follows:

$$YS_{i,t}^{3a} = c + \beta_1 MS_{i,t}^{3a} + \beta_2 \overline{MS}_{i,t}^{3a} + \beta_3 Level_{i,t}^{3a} + \beta_4 Slope_{i,t}^{3a} + \beta_5 Curvat_{i,t}^{3a} + \beta_6 Dur_{i,t}^{3a} + \beta_7 DSONTR_{i,t}^{3a} + u_{i,t} \quad (42)$$

$$YS_{i,t}^{5a} = c + \beta_1 MS_{i,t}^{5a} + \beta_2 \overline{MS}_{i,t}^{5a} + \beta_3 Level_{i,t}^{5a} + \beta_4 Slope_{i,t}^{5a} + \beta_5 Curvat_{i,t}^{5a} + \beta_6 Dur_{i,t}^{5a} + \beta_7 DSOTR_{i,t}^{5a} + u_{i,t} \quad (43)$$

$$YS_{i,t}^{10a} = c + \beta_1 MS_{i,t}^{10a} + \beta_2 \overline{MS}_{i,t}^{10a} + \beta_3 Level_{i,t}^{10a} + \beta_4 Slope_{i,t}^{10a} + \beta_5 Curvat_{i,t}^{10a} + \beta_6 Dur_{i,t}^{10a} + \beta_7 DSONTR_{i,t}^{10a} + u_{i,t} \quad (44)$$

$$YS_{i,t}^{30a} = c + \beta_1 MS_{i,t}^{30a} + \beta_2 \overline{MS}_{i,t}^{30a} + \beta_3 Level_{i,t}^{30a} + \beta_4 Slope_{i,t}^{30a} + \beta_5 Curvat_{i,t}^{30a} + \beta_6 Dur_{i,t}^{30a} + \beta_7 DSONTR_{i,t}^{30} + u_{i,t} \quad (45)$$

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