On Credit Spreads, Credit Spread Options and Implied Probabilities of Default

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Abstract

This study uses the two-factor valuation framework of Longstaff and Schwartz (1992a) to model the stochastic evolution of credit spreads and price European-type credit spread options. The level of the credit spread is the first stochastic factor and its volatility is the second factor. The advantage of this setup is that it allows the fitting of complex credit curves. Calibration of credit spread options prices is carried out using a replicating strategy. The estimated credit spread curves are then used to imply default probabilities under the Jarrow and Turnbull (1995) and Jarrow, Lando and Turnbull (1997) credit risk models.

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1. Introduction

Credit spread options are contracts, which “bet” on the potential movement of corporate bond yields relative to the movement of government bond yields. Credit spreads can be thought of as the compensation investors receive to accept all the incremental risks inherent in holding a particular bond instead of some “riskless” benchmark. Spread options are often used in the market for speculation and hedging of credit risk; ever since credit risk was commoditised, they constitute one of the better OTC instruments to replicate spread movements with controlled downside risk depending on the employed hedging strategy.

In theory, the pricing of a European credit spread option resembles the valuation of an interest rate option. Assuming a complete market / no-arbitrage framework, the standard approach is to take the expectation under the “risk-neutral” probability measure of the contingent claim’s terminal payoff function. Hence, deriving the risk – neutral distribution of the dynamics of the spread is the major step in obtaining a closed-form solution for credit spread options. So far, both the structural and reduced-form credit risk modelling approaches failed to provide a closed-form expression for such contingent claims. For that reason, most research into the pricing of credit spread options has concentrated in numerical solutions.

In this paper, we propose a “spread-based” framework for pricing credit spread options in which the short term credit spread rate (i.e., the rate at which corporations arrange their short term financing) and its volatility are the two stochastic variables that drive the underlying uncertainty within the Longstaff and Schwartz (1992a) – hereafter LS- general equilibrium two-factor interest rate model (see Rebonato (1996) for a survey of interest rate models). An obvious advantage of our approach is a unified pricing framework for both interest rate sensitive and credit sensitive securities. Furthermore, the estimation of the credit spread curves using the LS model facilitates in implying probabilities of default using the Jarrow and Turnbull (1995) framework -hereafter JT- and the transition rating matrix using the Jarrow, Lando and Turnbull (1997) model –hereafter JLT. In that sense, we differ from Arvanitis, Gregory and Laurent (1999) in that implied probabilities of default are obtained directly from credit spreads rather than bond prices.

Interest rate models have been used before to model the dynamics of credit spreads (e.g., Duffie and Singleton (1994)). Our view is that the choice of the modelling framework has to be congruent with the observed credit spread characteristics. Pringent, Renault and Scaillet (2001) report that credit spread series show strong mean reversion. Furthermore, Duffie (1999) found that credit spread volatilities display GARCH-type effects.

The LS framework has several advantages that make it attractive as a candidate for modelling credit spreads and pricing credit spread options, notably it is an affine model, thus enabling closed-form solutions for contingent claims, it can accommodate mean reversion and stochastic volatility and may give rise to complex shapes of credit curves. Longstaff & Schwartz (1995), following an empirical investigation into credit spreads, proposed a mean reverting model for the logarithm of the credit spread. The specification of their latter model is such that it provides closed form valuation expressions for risky bonds as well as risky floating rate debt. It is a two-factor model where one factor is the default free rate and the second factor (possibly correlated with the first) is the (default) risky rate. In essence, two yield curves can be estimated simultaneously based on observed default-free and defaultable bond prices. Whilst appealing, the Longstaff and Schwartz (1995) framework has the main drawback that it only allows for monotonically increasing or hump shaped curves to be estimated, types that are clearly at odds with the observed characteristics of credit spreads.

The remainder of the paper is organized as follows. Section 2 describes the LS (1992) two-factor equilibrium interest rate model, our preferred estimation method and way of inferring the risk-neutral density for option pricing. Section 3 outlines our adaptation of the LS framework in modelling the stochastic evolution of credit spreads and pricing credit spread options. Section 4 presents the data set and empirical results. Section 5 extents our credit spread approach in extracting implied probabilities of default and transition matrices. Finally, Section 6 concludes the paper.

2. The Theoretical Model

2.1 The Longstaff and Schwartz Model

The LS model considers a stylised version of the economy in which interest rates are obtained endogenously in an equilibrium set up. In their model, agents (investors) are faced at each point in time with the choice between investing or consuming the single good produced in the economy. If C(t) represents consumption at time t, the goal of the representative investor is to maximise, subject to budget constraints, his additive preferences of the form:

$$E_t[\exp(^{(-\rho s)\ln C(s)ds})] \quad (1)$$

Consumption at a future time s is ‘discounted’ to the present time t by a logarithmic utility discounting rate ρ. Consumption or reinvestment decisions have to be made subject to budget constraints of the form:

$$dW = W\,\frac{dQ}{Q} - Cdt \quad (2)$$

i.e., the infinitesimal change in wealth W over time dt is due to consumption (-Cdt) and returns from the production process (dQ / Q), scaled by the wealth invested in it (hence the constant-return-to-scales technology assumption). The returns on the physical investment (the only good produced by the economy) are in turn described by a stochastic differential equation of the form

$$\frac{dQ}{Q} = (\mu X + \theta Y)dt + \sigma X dz_1 \quad (3)$$

, where dz_1 is the Brownian motion increment, μ, θ and σ are constants, and X and Y are two state variables (economic factors) chosen in such a way that X is the component of the expected returns unrelated to production uncertainty (i.e., to dz_1), and Y is the factor correlated with dQ. X and Y are described by the following stochastic differential equations

$$dX = (a - \beta X)dt + \gamma X dz_2 \quad (4)$$

$$d\mathrm{Y} = (\delta - \varepsilon Y)dt + \phi Y dz_3 \quad (5)$$

Given the assumptions made, there is no correlation between the processes dz_1 and dz_2, on the one hand, and between dz_2 and dz_3 on the other. If one accepts that the optimal consumption is ρW (see Cox Ingersol and Ross (1985) –CIR henceforth), direct substitution of (3) and of the optimal consumption in the budget constraint equation (2) gives the stochastic differential equation for the dynamics of wealth:

$$dW = (\mu X + \theta Y \text{ - } \rho)Wdt + \sigma WY dz_1 \quad (6)$$

Having obtained the stochastic differential equation obeyed by the wealth process of the representative investor, following CIR (1985), the partial differential equation that any contingent claim, H, satisfies is given by

$$\frac{\partial^2 H}{\partial x^2}\frac{x}{2} + \frac{\partial^2 H}{\partial y^2}\frac{y}{2} + (\gamma - \delta x)\frac{\partial H}{\partial x} + (\eta - (\xi + \lambda)\gamma)\frac{\partial H}{\partial y} - rH = \frac{\partial H}{\partial \tau} \qquad (7)$$

, where x = X / c^2 , y = Y / f^2 , g = a / c^2 , e = ξ , δ = b, η = d / f^2, r is the instantaneous riskless rate, and the market price of risk is endogenously derived to be proportional to y, rather than exogenously assumed to have a certain functional form.

The set of equations and assumptions described above provide a general equilibrium model for the economy as a whole. Contingent claims are priced in this framework as endogenous components of the economy, and their prices are therefore equilibrium prices.

In particular, for the case of a zero-coupon bond, F, with terminal condition F(r, V, 0) = 1, following separation of variables, equation (7) yields:

$$F(r, V, \tau) = A^{2\gamma}(\tau)B^{2\eta}(\tau)\exp(\kappa\tau + C(\tau)r + D(\tau)V) \quad (8)$$

The expression for the continuously compounded yield of a zero-coupon bond, Y, can be directly obtained as the negative of log(F(T)) / T

$$Y(T) = \frac{-\kappa T + 2\gamma \log A(T) + 2\eta \log B(T) + C(T)r + D(T)V}{T} \quad (9)$$

For practical option pricing applications, achieving a good fit to the term structure of volatilities can be as important as fitting the yield curve correctly. The volatility of rates of different maturities can be obtained by deriving the volatility of zero-coupon bond prices for different maturities, and then applying Ito's lemma to convert the price volatility to yield volatility. Hence, the instantaneous volatility of bond returns is

$$\begin{aligned} Var[dF(T)] = \sigma^2_{F(T)} &= r\left[\frac{\alpha\beta\psi^2(e^{\varphi T}-1)A^2(T) - \alpha\beta\varphi^2(e^{\psi T}-1)B^2(T)}{\varphi^2\psi^2(\beta-\alpha)}\right] + \\ &\quad V\left[\frac{-\alpha\psi^2(e^{\varphi T}-1)A^2(T) - \beta\varphi^2(e^{\psi T}-1)B^2(T)}{\varphi^2\psi^2(\beta-\alpha)}\right] \quad (10) \end{aligned}$$

2.2 Estimation of the Longstaff-Schwartz Model

Earlier studies by Hordahl (2000) and Rebonato (1996) propose that a mixed (historical/implied) parametrisation procedure should be used for the calibration of the LS model. A purely implied approach is the one which regards the two state variables and the six parameters as fitting quantities, whereas a historical/implied approach involves the estimation of the short-rate volatility using time series and applying it to the model having only the six parameters as fitting quantities.

In practice, the LS model is frequently estimated using cross-sectional data on T-bills and bonds/swaps for some specific point in time. This results in a new set of parameters each time the model is estimated. Using cross-sectional data rather than a time series approach to estimate the parameters of the model could possibly capture changes in the dynamics of the term structure in a much more timely manner. Whilst this approach is not entirely compatible with the equilibrium set up of the model, it is nevertheless used in order to fit the model to observed bond prices as closely as possible.

The estimation procedure relies on (8), which provides a closed form solution for the discount function. Using this expression, the six parameters of the LS model can be estimated with cross-sectional bond price/swap rate data, given initial values of the two state variables r and V. As a first step, the initial values of r and V are determined as follows: The short rate r is represented by the average of the yield of the most liquid short term instrument, i.e., a T-bill over the examined period, whereas the initial value of the variance of interest rate changes is estimated using a simple GARCH(1,1) model, assuming a constant conditional mean:

$$r_t - r_{t-1} = \mu + \varepsilon_t, \qquad \varepsilon_t / \Omega_{t-1} \sim N(0, h_t) \quad (11)$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1}, \qquad (12)$$

, where Ω_{t-1} denotes the information set at time t-1.

Once the initial values of r and V are estimated, the next step is to estimate the six model parameters using cross-sectional data on T-Bills and Bonds across the Euro area for a reference date. Assuming that the observed market prices of these instruments differ from the prices obtained by the LS model (the “true” specification) by an error term with expected zero value, the estimates of the parameters of the LS model are obtained by minimizing the distance between the observed market prices and the model’s theoretical prices of bills and bonds:

$$\Theta = \arg\min_{\Theta} \sum_{i=1}^{n} [P_i - P_i(r, V, \Theta)]^2 \quad (13)$$

, where P_i denotes the observed price of bill/bond(i) among the n different securities and P_i (r, V, Θ) is the corresponding LS price given the current values of r, V and the parameter vector.

2.3 Option pricing using the Longstaff-Schwartz Density

Given the well documented problems (see Rebonato (1996)) of the closed form approach suggested by LS (1992a,b) for estimating the risk neutral density, we have employed the Monte Carlo methodology of Hordahl (2000). Discretised versions of the processes for the short rate and its variance are used to simulate possible future realisations of r and V. Specifically, by using an Euler approximation and assuming weekly time steps, discrete versions of the continuous-time dynamics are obtained as follows:

$$r_{t+\Delta t} - r_t = \left(\alpha\gamma + \beta\eta - \frac{\beta\delta - \alpha\xi}{\beta - \alpha} r_t - \frac{\xi - \delta}{\beta - \alpha} V_t \right) \Delta t + \alpha \sqrt{\frac{\beta r_t - V_t}{\alpha(\beta - \alpha)}} \sqrt{\Delta t \varepsilon_{1,t+\Delta t}}$$

$$V_{t+\Delta t} - V_t = \left(\alpha^2\gamma + \beta^2\eta - \frac{\alpha\beta(\delta - \xi)}{\beta - \alpha} r_t - \frac{\beta\xi - \alpha\delta}{\beta - \alpha} V_t \right) \Delta t + \alpha^2 \sqrt{\frac{\beta r_t - V_t}{\alpha(\beta - \alpha)}} \sqrt{\Delta t \varepsilon_{1,t+\Delta t}}$$
$$+ \beta^2 \sqrt{\frac{V_t - \alpha r_t}{\beta(\beta - \alpha)}} \sqrt{\Delta t \varepsilon_{2,t+\Delta t}} \qquad (14)$$

, where $\Delta t = 1/52$ (weekly interval) and $\varepsilon_{1,t+\Delta t}$, $\varepsilon_{2,t+\Delta t}$ are drawn from two independent standard normal distributions. Note that since we have assumed that the local expectations hypothesis holds, the above processes are approximations of the risk neutral dynamics of r and V.

The above equations were used to simulate future values of r and V, in a recursive manner starting form the initial values r_0 and V_0, as outlined in the previous section. In this study, a time horizon of up to half a year is chosen, which means that 26 future values of r and V are simulated, given the choice of $\Delta t = 1/52$. This process is then repeated 20,000 times with the same parameter values. Hence, for each of the 26 future weeks following the date of estimation, the described procedure produces a simulated sample consisting of 20,000 r's and 20,000 V's.

The next step is to obtain an estimate of the risk neutral distribution (RND) of the future short-term interest rate at each time, which was done by using a simple histogram. In turn, under the RND of the short rate we can price European call, C(t,T), and put options, P(t,T), on bonds using:

$$C(t,T) = E_t^Q[\exp(-\int_t^T r_s ds)\max(0, r(T) - X)], \quad (15)$$

under the risk neutral measure Q and

$$P(t,T) = E_t^Q[\exp(-\int_t^T r_s ds)\max(0, X - r(T)] \quad (16)$$

Hence, using the Monte Carlo simulation we can use the discrete approximations of (15) and (16) to calculate option prices:

$$C(t,T) = \frac{1}{N}\sum_{i=1}^{N}\exp\left(\sum_{s=t}^{T-1} r_{i,s}\Delta t\right)\max(0, r_{i,T} - X), \quad (17)$$

$$P(t,T) = \frac{1}{N}\sum_{i=1}^{N}\exp\left(\sum_{s=t}^{T-1} r_{i,s}\Delta t\right)\max(0, X - r_{i,T}) \quad (18)$$

, where Δt is the time step in the Monte Carlo simulation.

3. The Methodology

3.1 Credit Spread Curves

There is sufficient empirical evidence to indicate that credit spreads and credit spread indices exhibit characteristics found in default free interest rates. Recent studies by Pedrosa and Roll (1998) and Koutmos (2002) point out that credit spreads show *mean reversion* in levels, *term structure, time variation and jump characteristics* in their respective *volatilities.*

Following the lead of Ramaswamy and Sundaresan (1996) who used a direct assumption about the stochastic process followed by the credit spread, we adapt the two-factor framework of LS to model the dynamics of the level of the short term credit spread and its instantaneous volatility:

$$\begin{aligned} dspread_t &= (\alpha - \beta spread_t) + \gamma \sigma_t dZ_{1,t} \\ d\sigma_t &= (\delta - \varepsilon \sigma_t) + \eta \sigma_t dZ_{2,t} \end{aligned} \qquad (19)$$

This two-factor model is a very flexible tool to capture the stochastic evolution of credit spreads as it can accommodate both the mean reversion and the time varying volatility features of credit spreads (and credit spread indices).

The estimation of credit spread curves is carried out using bond prices of various European corporates and EUR denominated government benchmark bonds (see next Section). We apply a standard bootstrapping methodology to the benchmark and corporate bonds in order to obtain market zero coupon discount factors. Once the discount factors of all the bonds were obtained, the spread discount factors were calculated by:

$$spread \text{ zero Disount Factor} = P(0,T) - P^i(0,T) \quad \text{should be} > 0 \qquad (20)$$

$P(0,T)$ is a risk free zero coupon bond paying 1 currency unit at maturity

$P^i(0,T)$ is a risky zero coupon bond of rating i under the assumption of paying 1 currency unit at maturity if there is no default or δ (recovery rate) in case of default.

Note that the spread zero discount factor should always be positive. This is mainly insured by the fact that the benchmark discount curve is usually higher than the risky discount curves. In cases where this difference is less than zero then a potential mispricing has occurred.

Finally, we estimate the volatilities of the risk free short rate and of the respective short term credit spreads using the GARCH(1,1) model in (11) and (12).

3.2 Pricing Credit Spread Options

We adopt an engineering approach and assume that the price of an at-the-money credit spread option is equivalent to two vanilla options written on two assets, a government bond and a corporate bond. Hence, for a call on a credit spread we assume that the following condition holds when at the money:

$$Spread(i) = yield(i) - yield_{Benchmark}$$

$$Call\ on\ spread(i) = Call\ on\ yield(i) + Put\ on\ yield_{Benchmark} \quad (21)$$

Note that this approximation is carried out for calibration purposes only and it holds theoretically if both the risk free and the defaultable bonds are discounted by the same risk free curve

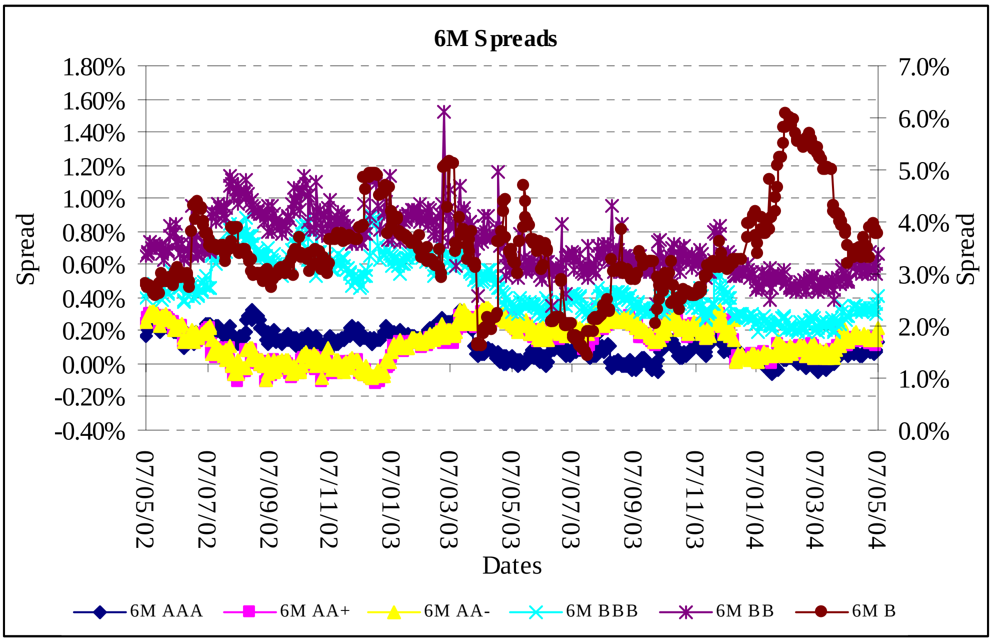
To illustrate, consider a call on the AA+ spread. This call option could be replicated by going long a call on a AA+ corporate yield and long on a put on the respective government benchmark yield. This just replicates the position on the underlying which is long the AA+ credit spread or just long AA+ calls.

4. Empirical Results

4.1 Credit Spread Data

Two years of daily data were collected from Bloomberg between 07/05/02 – 07/05/04 for the 6M Euro-LIBOR rate, and 6M AAA, AA+, AA-, BBB, BB, B credit spreads. All the corporate bonds were from the industrial sector apart from the AAA which was from the financial sector. The date, which we fitted the 6 different credit spread curves was the 7th of May 2004. The bonds were stripped to create the zeroes using a bootstrapping procedure. All the bonds used to fit the credit spread curves are listed in Appendix 1. Figure 1 plots the 6M credit spreads over time.

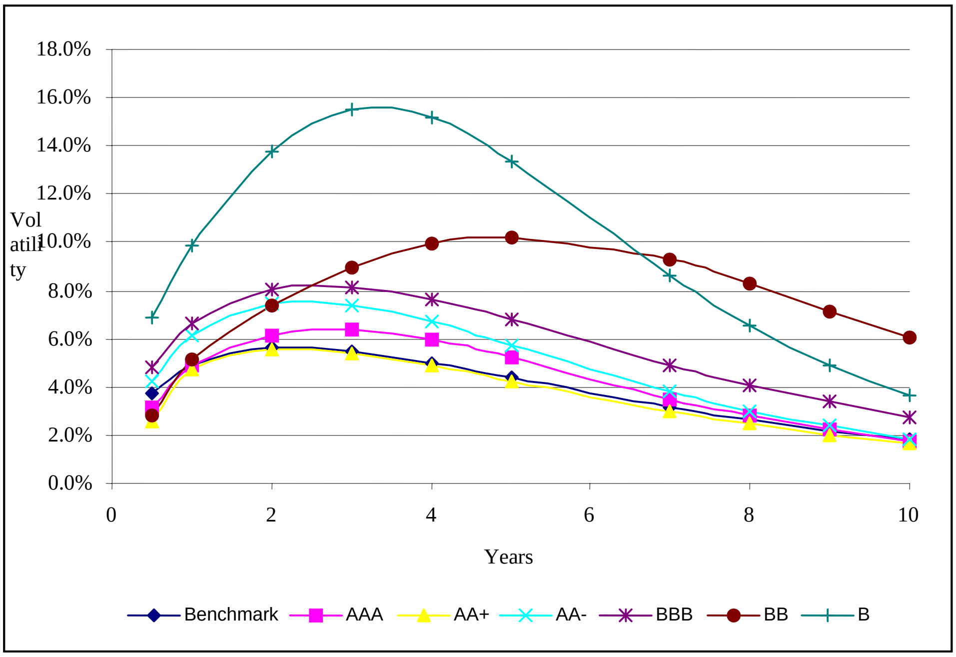
Figure 1: 6M credit spreads (May-02 – May-04)



It is quite clear from the Figure above that the B spread is the highest of all. The higher rated spreads such as the 6M AAA and 6M AA+, 6M AA- are often quite low, as low as zero. This usually occurs when there is a huge amount of liquidity in the corporate market, which makes the short term financing among highly rated institutions "almost" risk free. This increased liquidity is related to the monetary policy by the European Central Bank, which reduced the level of interest rates during the period examined. Arguable, in periods of low interest rates there is an increased risk appetite since the cost of borrowing is quite low.

Figure 2 plots the volatilities of the credit spreads.

Figure 2: Credit Spread Volatility Curves (07/05/04)



4.2 Estimating Credit Spread Curves

As before, the date of our analysis is the 7th of May 2004. After a long time of low credit spreads and continuous spread tightening we see the first signs of a reversal in the credit markets. The first 5 months of 2004 have been quite interesting since the increased liquidity and the global deflation theme has taken its toll. Investment in 2004 is not a "sure thing" anymore across all asset classes and especially in credit where careful selection of credit –based investments superseded the theme of going long in all types of corporate bonds. During the period of examination, credit spread curves showed the first signs of spread widening.

Table 1 shows the estimated parameters of the spread-based model in (19) using the discretized Monte Carlo approach in (14) and the minimization procedure in (13). The first two rows report the initial average spread levels and their volatilities. The last four rows report parameter values that indicate that the spread reverts to its long term mean and so its volatility.

Table 1: Estimation of the Parameters of the Spread-Based Model.

	AAA Finance	**AA+ Industrials**	**AA- Industrials**	**BBB Industrials**	**BB Industrials**	**B Industrials**
Mean Spread	0.11%	0.12%	0.14%	0.46%	0.72%	3.44%
V	27.72%	12.87%	12.21%	8.33%	19.55%	5.53%
α	0.114	0.085	0.123	0.089	0.042	0.097
β	18.515	11.973	14.995	21.995	22.000	15.759
γ	0.070	0.184	0.100	0.167	0.052	0.112
δ	-0.179	0.109	-0.086	-0.001	-0.307	-0.432
ε	0.001	-0.001	-0.001	-0.002	0.000	-0.001
η	3.459	1.691	2.346	0.768	0.741	-21.009

Table 2 shows the estimated spread discount factors and credit spreads based on the estimated parameters for the AA+, following the approach outlined in (20). The optimisation has worked quite well since reported differences between observed and estimated spread discount factors are quite small.

Table 2: Spread Discount Factors for EUR AA+ spread curve (07/05/04)

Maturity /years	**Observed spread discount factors**	**Estimated spread discount factors**	**Estimated credit spreads in %**
0.25	0.00051	0.00124	0.510%
0.5	0.00093	0.00163	0.364%
1	0.00105	0.00170	0.211%
2	0.00306	0.00272	0.152%
3	0.00514	0.00429	0.164%
4	0.00632	0.00708	0.189%
5	0.01009	0.00983	0.214%
7	0.01352	0.01282	0.254%
8	0.01449	0.01436	0.268%
9	0.01362	0.01690	0.281%
10	0.02210	0.01976	0.291%

In Figure 3 we observe the difference between the observed B-rated Industrial credit spread curve and spread-based model estimated credit spread curve. The fit is surprisingly good even for a complex curve like this, enforcing our choice of model. Liquidity effects on the examined corporate bonds could be responsible for the actual shape or simply it is a reflection of expectations.

Figure 3: EUR B Industrial Credit Spread Curve (07/05/04)

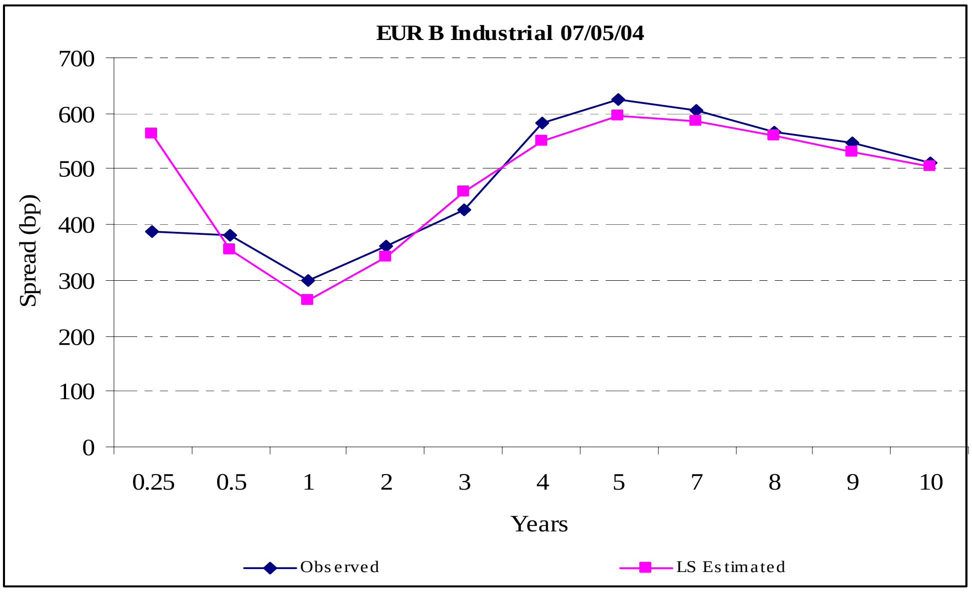


Figure 4 shows the estimated discount factors per credit spread curve including the estimated benchmark discount factor curve. The natural observation is that the spread zero discount factors are upward sloping curves as opposed to the downward slope of the benchmark discount factor curve. The absence of monotonic patterns in the discount functions of the credit spread curves is striking. It is clear that the shape of the credit curves is quite complex. Possible reasons are liquidity, which is always a major factor in the cash market of fixed income securities and potential arbitrage opportunities due to market mispricing.

Figure 4: Zero Discount Factors (07/05/04)

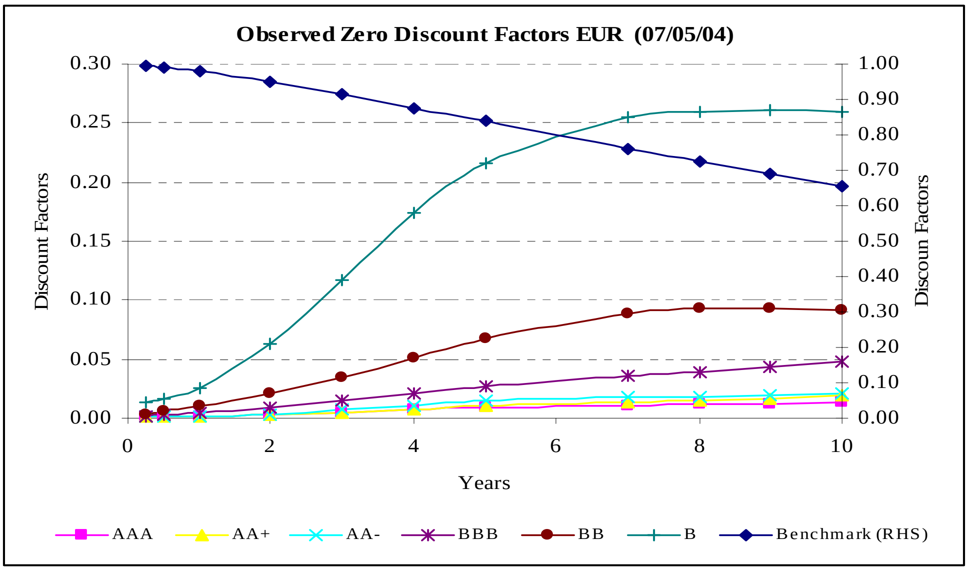
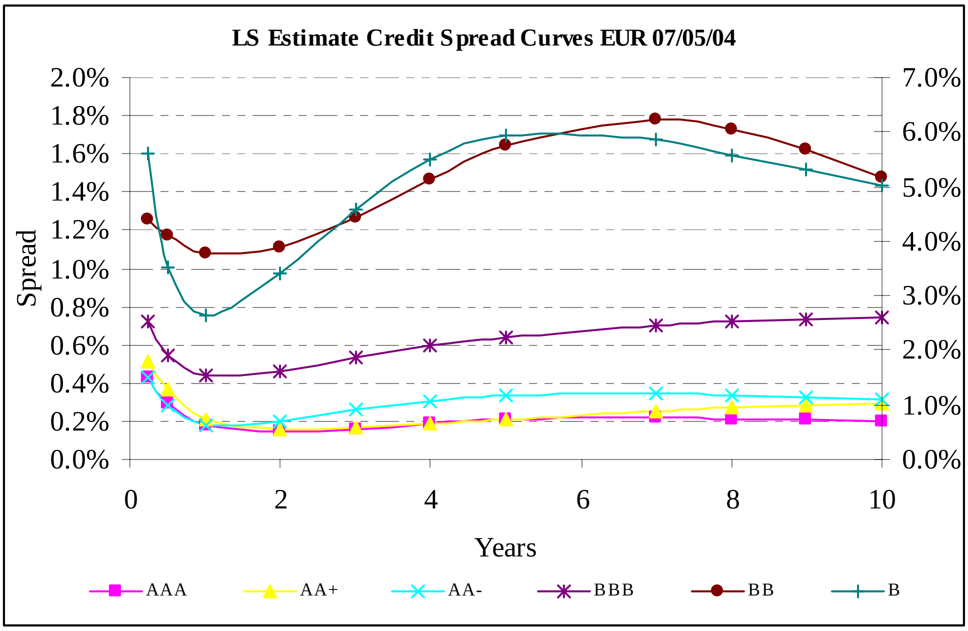


Figure 5 shows the spread-based model estimated credit spreads. Comparing it to the actual credit spreads shown in Figure 1, one observes that the estimated credit spreads are very close to the recorded ones.

Figure 5: Estimated Credit Spread Curves (07/05/04)



The only serious mispricing occurs for the AAA curve. During the 2 – 7 year period the spread is higher than AA-. This is something that does not occur in the observed credit curves since the level of risk of holding AAA credit compared to AA- asset is always lower. A possible explanation is that the volatility of the AA+ and AA- curves is quite low relative to the AAA volatility for the period examined (see Figure 2). Another reason could be that the 2 to 7 year slice of the AA+ and AA- curves has not moved for some time, i.e. lack of liquidity or even lack of activity.

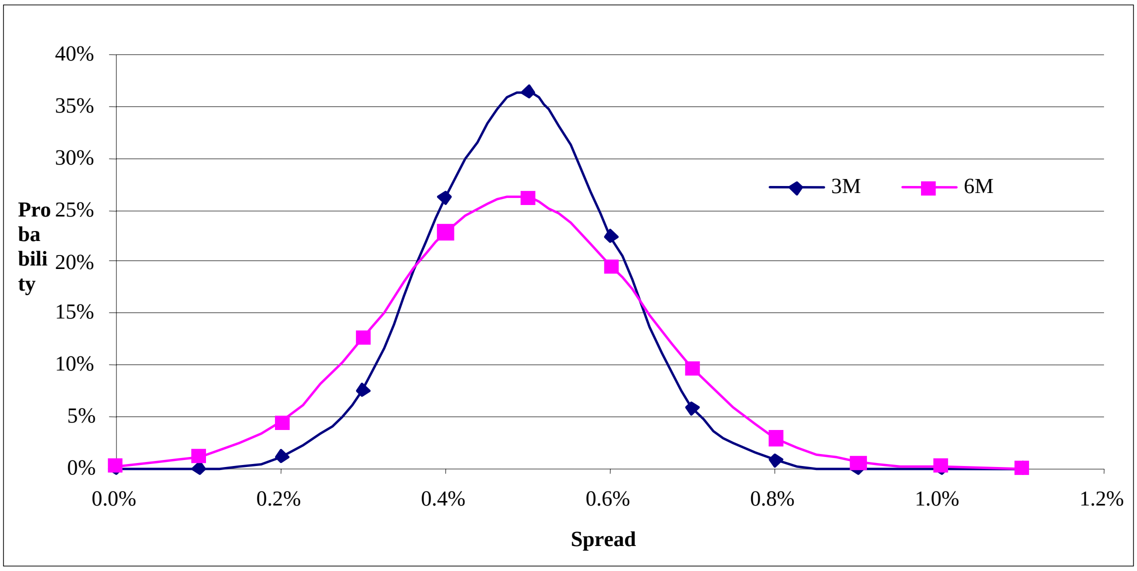
The fact that the part of the AAA curve yields higher than the AA+ and AA- shows that although the spread-based LS model can fit complicated curve shapes and takes into account the volatility of credit spreads, nevertheless there is no guarantee that (i) the credit spread discount curves are strictly positive and (ii) the premium of holding a higher rated security is lower than the premium of holding a lower rated security.

The main reason for this shortcoming is that the spread-based model is only concerned with fitting an observed credit spread term and its volatilities. However, the previously documented ability of the model to fit complex spread curves can be used to one's advantage since, as we shall see later, there is quite a lot of information that can be extracted out of the estimated credit spread curves. Furthermore, the estimated credit spread curves can give us good insight in where the "equilibrium" of the short credit spread might be and also where the level of the forward credit spreads are.

4.3 Pricing Credit Spread Options

The estimated credit spread curves are in turn used to provide the forward structure of the credit spread with the view to pricing credit spread options; in line with our adopted approach, we assume that the spread is a stochastic variable which follows the diffusion equations in (16). Figure 6 shows the risk-neutral implied probability density function (RND) for 3M and 6M AA+ industrials using the methodology outlined in Section 2.3.

Figure 6: RND of Future 3M and 6M Credit spreads for AA+ Industrials (07/05/04).



Using the pricing methodology of Section 3.2, we price options on different credit spreads. The "calibration" of the credit spread options was performed via "Greeks" replication. The at-the-money spread options were replicated by buying 1 unit of an at-the-money corporate bond of rating (i) and buying ½ a unit of a government bond. For example the combination of the VMG bond and the OBL government bond options(see Appendix 1) will yield a Total Delta = (1x Duration(i) x Delta of option) + (0.5 x Duration x Delta of Option), i.e., Total Delta of strategy = 1 x 2.574 x 0.5009 + 0.5 x 2.276 x (-0.499) = 0.50254, which is matched but the ATM spread option. Naturally, for different option moneyness, appropriate weights should be used to account for delta (and the rest of the Greeks) matching and for the payoff of credit spread options.

As a general rule, the two bonds are chosen to be of similar duration (see Table 3) and the options are struck at the at-the-money implied forward level, as extracted from the observed discount curve of the 07/05/04. The respective deltas are 0.5009 for the calls and -0.4999 for the puts.

The maturity of the bonds is approximately 3Y and 2Y and the option maturities are 3M, in other words, the credit spread options priced are 3M options on 3Y and 2Y underlyings, respectively. The spread options were struck at the same spread strike as the replicating strategy and the option implied volatility of the strategy was matched as well.

Table 3: Underlying Government and Corporate Bonds

Strike Call	**Strike Put**	**Government Bond**	**Coupon**	**Corporate Bond**	**Coupon**	**Duration Government**	**Duration Corporate**
2.61	2.52	OBL	4	VMG	4	2.574	2.276
2.77	2.57	BKO	2	Total	3.875	1.728	1.864
2.96	2.57	BKO	2	Bosch	5.25	1.728	1.964
3.27	2.57	BKO	2	Renault	5.125	1.728	1.972

The total cost of the strategy (see Table 4) was expected to be higher than the respective option prices out of the spread-based model. The reason is that the fractional difference in the time value between the combination of the two options compared to the single spread option would increase the overall premium by almost the same amount. This small difference between the model-induced spread option values and the replicating strategy is shown in Table 4. The advantage of the spread-based model over the industry's standard Black's (1976) model is mainly that our two-factor framework accounts for the stochastic nature of the volatility function.

Table 4: Credit Spread Option Prices

Rating	Option Maturity/ Spread Maturity	Price Difference	LS Price (decimals)	Cost of Strategy (decimals)	Government Bond Put Option	Corporate Bond Call Option	Spread Strike (bp)
AA	3M_3Y	0.008	0.553	0.545	0.499	0.295	9
AA-	3M_2Y	-0.006	0.367	0.374	0.315	0.216	20
BBB	3M_2Y	-0.003	0.403	0.406	0.315	0.248	39
BBB	3M_2Y	-0.013	0.541	0.555	0.315	0.397	70

Longstaff and Schwartz (1995) arrive to an interesting result about credit spread options. Based on their proposed model, which assumes that credit spreads are conditionally log-normally distributed, they conclude that the value of call credit spread options can be less than the their intrinsic value. Based on our results, this finding is questionable. Table 4 reports that the value of credit spread options is higher than its intrinsic value. The reason is that, in our framework, the pricing of credit spread options is no different than the pricing of interest rate option, a clear advantage over the LS (1995) specification.

5. Implied Probabilities of Default and Transition Matrices

Arguably, one of the end results of credit risk modelling is to infer the survival probabilities and, possibly, transition matrices in order to price credit sensitive contingent claims. Credit spreads have been closely linked to the survival probabilities by many researchers, for example, JT (1995), JLTurnbull (1997), Madan and Unal (1994). Prior to using our estimated credit spreads in order to derive the survival probabilities[2] we need to formally outline our choice of the theoretical credit risk model.

[2] Since in these models first the implied probabilities are derived and then the credit spreads.

5.1`An Iterative Procedure to Extract Default Probabilities from Credit Spreads

Within the JT (1995) equivalent recovery model in which the recovery rate δ is taken to be an exogenous constant, we assume that both the riskless interest rate r(t) and the spread s(t) evolve under the LS (1992) framework. Let B(t,T) be the time t price of a default-free zero-coupon bond paying 1 currency unit at time T. The money market account accumulates returns at the spot rate as:

$$B(t,T) = \exp\int_0^t r(s)ds \qquad \text{in the continuous case}^{3} \quad (22)$$

Let D(t,T) be the time t price of a risky zero-coupon bond promising to pay 1 currency unit at time T if there is no default, and if default occurs it pays the recovery rate δ < 1. Following the assumption that the stochastic processes for default-free spot rate and bankruptcy are statistically independent under the risk neutral probability measure Q, we arrive at the standard equation:

$$\begin{aligned} D(t,T) &= E_t\left(\frac{B(t)}{B(T)}\right)E_t(\delta 1_{\{\tau^* \le T\}} + 1_{\{\tau^* > T\}}) \\ &= B(t,T)(1 - Q_t(t,T) + Q_t(t,T)\delta) \end{aligned} \quad (23)$$

Assuming that default hasn't already occurred, the survival probability is:

$$Q_t(t,T) = \frac{1 - D(t,T)/B(t,T)}{(1-\delta)} \quad (24)$$

Since both D(t,T) and B(t,T) are zero-coupon bonds, we can express the difference between the two zeroes in terms of their instantaneous spread s(t,T):

$$Q_t(t,T) = \frac{s(t,T)}{(1-\delta)B(t,T)} \quad (25)$$

---

[3] In the discrete time case $B(t) = \exp(\sum_{i=o}^{t-1} r(i))$

Minor re-arrangement of the above expression relates the default probabilities with the forward credit spreads:

$$Q_t(t,T)(1-\delta) = 1 - e^{-\int_t^T s(t,\tau)d\tau} \qquad (26)$$

Furthermore, for a small time dt, the short-term credit spreads are directly related to local default probabilities q(t,t+dt). Local default probabilities are the probability of default between t and t + dt, conditional on no default prior to time t. In addition, we can relate the probability of default to the intensity of the default process as in Arvanitis, Gregory and Laurent (1999):

$$\lambda(t) = \frac{q(t,t+dt)}{dt} \qquad (27)$$

The next step is to use the spread discount factors obtained under our spread-based model to derive the local default probabilities and, subsequently, the conditional default probabilities. The following iterative procedure will be used in order to extract the survival probabilities out of the term structure of credit spreads.

For $t < \tau < T$

$$q(t,\tau) = \frac{B(t,\tau) - D(t,\tau)}{(1-\delta)B(t,\tau)} = \frac{s(t,\tau)}{(1-\delta)B(t,\tau)}$$

$$q(t,\tau+1) = \frac{\{B(t,\tau+1) - D(t,\tau+1)\} - \{B(t,\tau) - D(t,\tau)\}}{(1-q(t,\tau)(1-\delta)B(t,\tau+1)}$$

$$= \frac{s(t,\tau+1) - s(t,\tau)}{(1-q(t,\tau)(1-\delta)B(t,\tau+1)}$$

$$\bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet$$

$$\bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet$$

$$q(t,T) = \frac{\{B(t,T) - D(t,T)\} - \{\sum_{\tau=\tau+1}^{T} B(t,\tau) - D(t,\tau)\}}{\{\prod_{\tau=\tau+1}^{T}(1-q(t,\tau)\}(1-\delta)B(t,T)} \qquad (28)$$

$$= \frac{s(t,T) - \{\sum_{\tau=\tau+1}^{T} s(t,\tau)\}}{\{\prod_{\tau=\tau+1}^{T}(1-q(t,\tau)\}(1-\delta)B(t,T)}$$

Following the determination of the local default probabilities we can now determine the cumulative survival probabilities taking into account again the two possibilities at each time point: default and no-default.

$$Q(t,\tau) = q(t,\tau)$$

$$Q(t,\tau+1) = q(t,\tau) + (1-q(t,\tau))q(t,(\tau+1) \qquad (29)$$

$$Q(t,\tau+2) = q(t,\tau) + (1-q(t,\tau))q(t,(\tau+1) + (1-q(t,\tau))(1-q(t,\tau+1))q(t,\tau+2)$$

In the following subsection, using the ratings model of Jarrow, Lando and Turnbull (1997) – hereafter JLT-, we will demonstrate how the risk premia as estimated using the LS (1992) model can be used to derive the implied ratings transition matrix.

5.2 Inferring Transition Probabilities from Credit Spreads

In JLT (1997), the dynamics of K- possible credit ratings are represented by a Markov chain. The first state of the Markov chain corresponds to the best credit quality and the (K-1) state the worst before default. The Kth state represents default and is an absorbing state, which pays the recovery rate δ times the security's par value.

The dynamics of credit ratings are characterised by a set of transition matrices Q'(t,T) (k x k matrices) for any period between time t and T with each of their elements $q_{ij}(t,T)$ representing the probability of migrating from rating i at time t to rating j at time T. The last column of Q'(t,T) ($q_{ik}(t,T)$) gives the default probabilities.

In JLT (1997) and Arvanitis, Gregory and Laurent (1999), the transition probability matrix is expressed exponentially:

$$Q'(t,T) = \exp[\Lambda(T-t)] \quad (30)$$

The matrix Λ (k x k) is called the generator of transition matrices Q'(t,T) and is assumed to be diagonalisable[4]. JLT (1997) postulated that the generator matrix, under the equivalent martingale measure, may be expressed as:

$$\overline{\Lambda}(t) = U(t)\Lambda(t) \quad (31)$$

, where U(t) is the vector of the risk premia which transform the historical generator matrix to the risk neutral. The elements of the generator matrix are directly related to probabilities: The probability of staying to the same rating i from time t to t + dt is 1 + λ_{ii} dt. The probability of going from rating i to rating j (i≠j) is λ_{ij} dt and the probability from rating i to default is λ_{ik} dt (j≠k). The transition probabilities are

[4] For example $\Lambda = \Sigma \, D \, \Sigma^{-1}$ where D is a diagonal matrix

constrained by further assumptions is order to ensure the proper evolution of credit spreads[5].

Since the generator matrix is diagonalisable, using (30) we get:

$$Q'(t.T) = \Sigma \exp[D(T-t)]\Sigma^{-1} \quad (32)$$

, where D represent the eigenvalues of the generator matrix and Σ represent its' eigenvectors.

Using (27) the probabilities of default are being expressed:

$$q_{iK}(t,T) = \sum_{j=1}^{K-1} \sigma_{ij}\sigma_{ij}^{-1}[\exp[d_j(T-t)-1] \qquad (33)$$

where σ_{ij} are the elements of Σ and σ_{ij}^{-1} are the elements of Σ^{-1} for $1 \leq \mathrm{i} \leq \mathrm{K} - 1$

Hence, using (28) we can write:

$$D(t,T,\Lambda) = B(t,T)(1 - q_{iK}(t,T) + q_{iK}(t,T)\delta) \quad (34)$$

Using (28) and (29), we can solve for the credit spread of rating i:

$$s^i(t,T,\Lambda) = (\delta - 1)B(t,T)\sum_{j=1}^{K-1}\sigma_{ij}\sigma_{ij}^{-1}[\exp[d_j(T-t)-1]$$

or (35)

$$= (\delta - 1)B(t,T)q_{iK}$$

Equation (35) shows how the credit spreads are related to the eigenvectors and eigenvalues of the generator matrix. Furthermore, it provides the term of credit spreads based on a given generator matrix. JLT (1997) did outline a procedure that could be used to estimate the above using as inputs market prices of default free zero-coupon bonds, risky zero-coupon bonds and the historical generator matrix. The procedure involves estimating the risk premia using the observed market prices and

[5] λ_{ij} >= 0 always. The sum of transition probabilities $(1+\lambda_{ij}) + \sum_{\substack{i=1 \\ i \neq j}}^{K} \lambda_{ij} = 1, \quad \mathrm{j} = 1,.....,\mathrm{K}$, is equal to one. The k-th state is absorbing, i.e., $\lambda_{kj} = 0$. Finally, state (i+1) is always more risky than state I, $\sum_{j \geq k} \lambda_{ij} \leq \sum_{j \geq k} \lambda_{i+1,j}$.

then multiplying the risk premia diagonal matrix ($diag(\pi_1,.....\pi_{K-1},1)$) by the historical generator matrix. In this way the Q'(0,t) matrix is calculated and subsequently the Q'(0,t+1) matrix can be calculated using:

$$Q'(t,T) = \exp(diag(\pi_1,....\pi_{k-1},1)\Lambda(T-t)) \quad (36)$$

This iterative procedure produces the risk neutral transition matrix based on current risk premia. Their next step is to minimise the risk premia by minimising the difference between the theoretical risky zero-coupon bond prices estimated using the risk-neutral transition matrix and the observed risky zero-coupon bond prices.

Along similar lines, following the estimation of the credit spreads we minimise the difference between our estimated spreads and the spreads derived (using (27)) under the historical transition matrix[6] as published in JLT (1997). Essentially, we use the risk premia as derived from our estimated spread curves in order to derive the risk neutral generator matrix.

5.3. Implied Probabilities of Default: The Results

By estimating zero-coupon spreads and given a historical recovery rate, we extract default probabilities in the same way as one would use risky zero-coupon bonds and default free zero-coupon bonds. Using the estimated spread curves (see Section 4.2), the relevant Bloomberg data[7] and the weighted recovery rate (0.3265) as used by JLT (1997), we are in a position to infer the cumulative probabilities of default.

The pattern of the cumulative default probabilities follows the risk premia one as obtained from the estimated spread discount factors. The probability of default is higher for lower ratings and increases with time. If we take a closer look at Figures 7

[6] This is the historical transition matrix as published by Moody's. Other rating agencies such as S&P also publish similar matrices on a regular basis.

[7] See appendix 2 as the default probability curves were obtained for the 7th of May 2004.

and 8 for example, we will realise that the probabilities are not straightforward exponential curves as one would expect, instead there is a higher level of convexity. A potential explanation may be the inclusion of the volatility of the spread when the curves were estimated.

Figure 7: Cumulative Default Probabilities

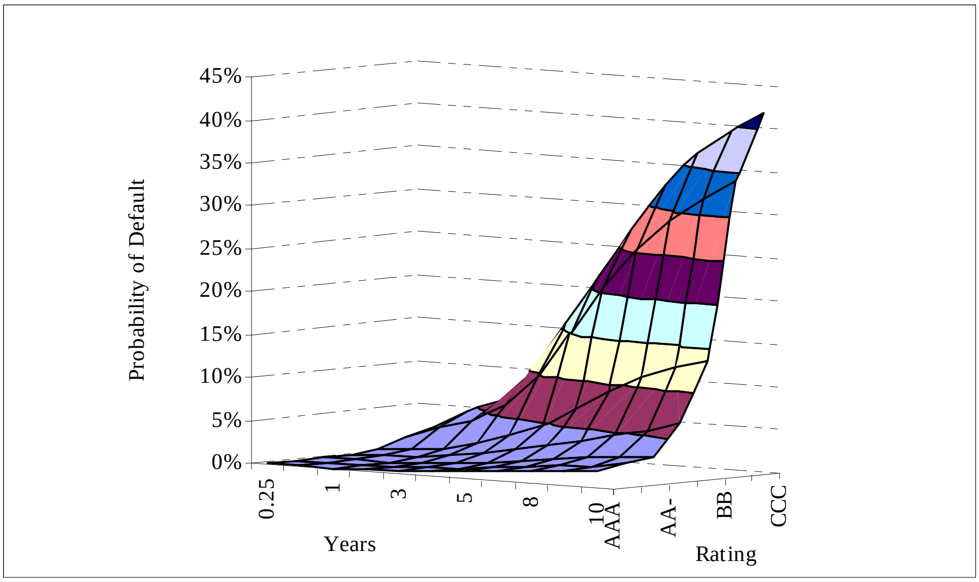
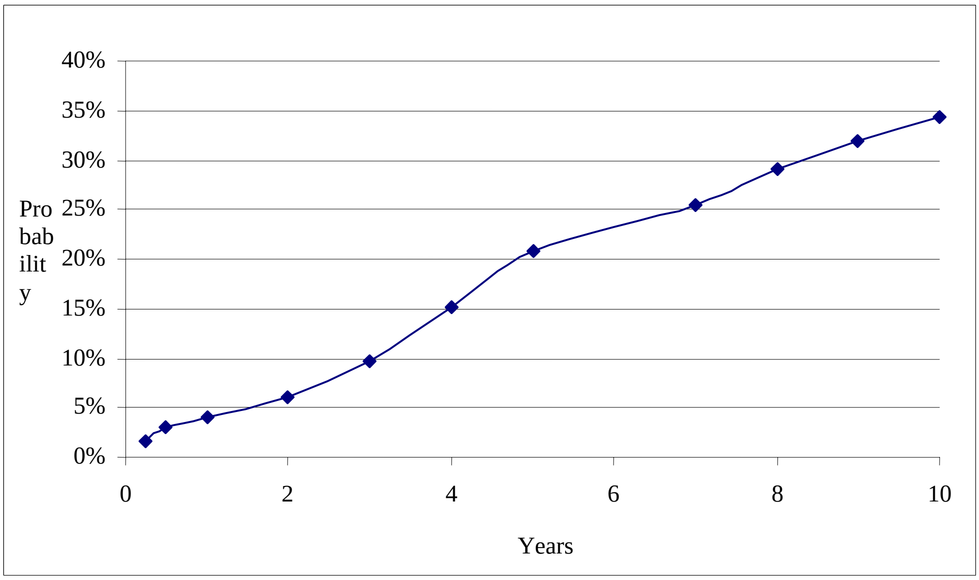


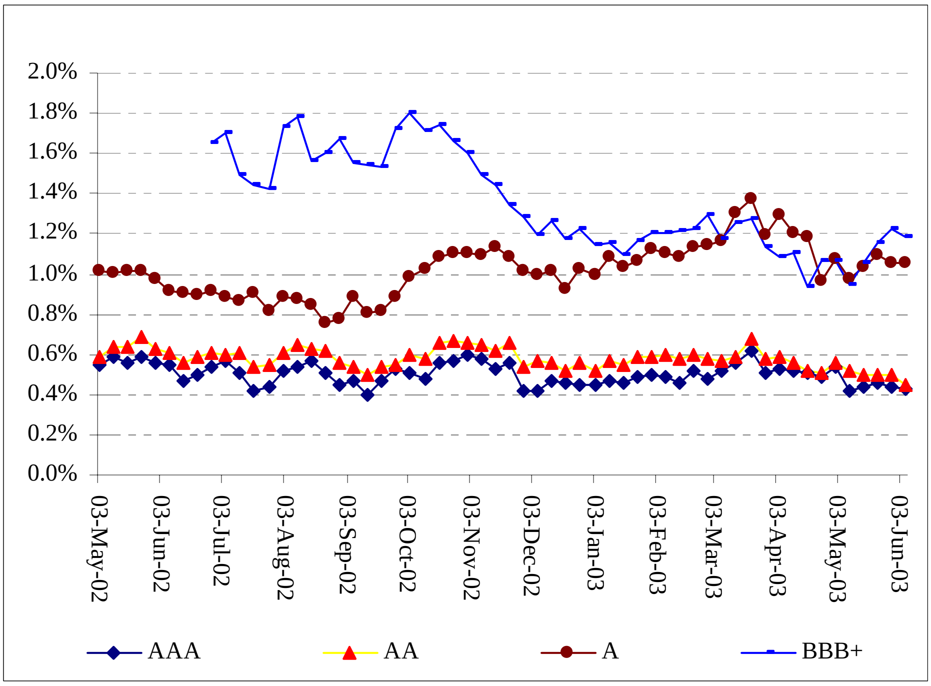
Figure 8: Implied Cumulative Default Probability Curve of Rating B



Hence the shape of the implied probability of default, which shows that the term structure of default is directly related to the term structure of the credit spread curves, both statically and dynamically. This is easily deduced since our inputs are the credit spreads themselves.

Figure 9 shows plots of weekly time series of the implied default probabilities. It is evident that default risk is relatively low during the period examined. This is in accordance with the environment of low interest rates (US and Euro area), which allows corporations to borrow money at historically low interest rates, thus making their debt servicing cost very low.

Figure 9: Evolution of 2Y implied default probabilities



An interesting observation is that for a brief period of time the probability of default of rating BBB+, as implied from the estimated credit spreads, was lower than the probability of default of rating A. This is clearly in violation of no-arbitrage. However, in our case this could be due to a mis-pricing that, thankfully, did not last long. Alternatively it could be the result of market pricing in a rating upgrade event for the sample of bonds examined. This could make sense since a number of corporations (again fuelled by a low interest rate environment) were placed on a positive outlook by major rating agencies.

5.4. Implied Transition Ratings Matrix: The Results

Table 5 shows the 1-year historical transition matrix reported in JLT [1997], whereas Table 6 reports our results for the implied transition ratings matrix using the spread risk premia as obtained from the spread-based model.

Table 5 (JLT (1997): 1 Year Historical Transition Matrix)

	AAA	**AA**	**A**	**BBB**	**BB**	**B**	**CCC**	**D**
AAA	-0.1154	0.1020	0.0083	0.0020	0.0032	0.0000	0.0000	**0.0000**
AA	0.0091	-0.1043	0.0787	0.0104	0.0031	0.0031	0.0000	**0.0000**
A	0.0009	0.0308	-0.1172	0.0688	0.0107	0.0048	0.0000	**0.0010**
BBB	0.0006	0.0046	0.0714	-0.1711	0.0701	0.0174	0.0020	**0.0049**
BB	0.0004	0.0023	0.0086	0.0814	-0.2531	0.1181	0.0144	**0.0273**
B	0.0000	0.0020	0.0034	0.0075	0.0568	-0.1929	0.0478	**0.0753**
CCC	0.0000	0.0000	0.0126	0.0131	0.0223	0.0928	-0.4319	**0.2856**
D	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**

Table 6: Implied Transition Rating Matrix (07/05/04)

	AAA	**AA**	**A**	**BBB**	**BB**	**B**	**CCC**	**D**
AAA	-0.1302	0.1148	0.0092	0.0022	0.0035	0.0001	0.0001	**0.0001**
AA	0.0158	-0.1815	0.1370	0.0180	0.0052	0.0053	0.0001	**0.0002**
A	0.0017	0.0584	-0.2214	0.1298	0.0203	0.0090	0.0002	**0.0018**
BBB	0.0007	0.0057	0.0954	-0.2292	0.0938	0.0233	0.0027	**0.0065**
BB	-0.0001	0.0013	0.0055	0.0495	-0.1540	0.0718	0.0087	**0.0165**
B	-0.0001	0.0018	0.0025	0.0056	0.0425	-0.1445	0.0356	**0.0563**
CCC	-0.0001	0.0003	0.0043	0.0041	0.0066	0.0275	-0.1278	**0.0845**
D	0.0000	-0.0001	-0.0001	0.0000	0.0001	0.0001	0.0000	**0.0000**

There are striking differences between the two generator matrices especially in the last column, which shows the implied cumulative probability of default. The historical matrix seems to produce much higher probabilities of default in comparison to the implied generator matrix. This is easily explained by the fact that the levels of credit spreads at 07/05/04 are very low. One could argue that default risk, as being viewed by the market (risk premia), is quite low with a tendency to increase, since the probabilities of being at the same rating after 1 year are lower than the historical transition probabilities.

Table 7 shows the model-based input spreads in comparison to the spreads implied using the risk neutral generator matrix (Table 6).

Table 7: Credit Spreads used for Calibration to Infer the Transition Ratings Matrix

Rating	Spread as estimated from historical matrix in bp	Spread as estimated from risk-neutral matrix in bp	Model-based Spread in bp	Difference in bp
AAA	21	1	7	6
AA	29	1	11	10
A	46	12	12	0
BBB	50	44	41	-3
BB	89	112	107	-5
B	472	386	375	-11
CCC	1698	586	578	-8

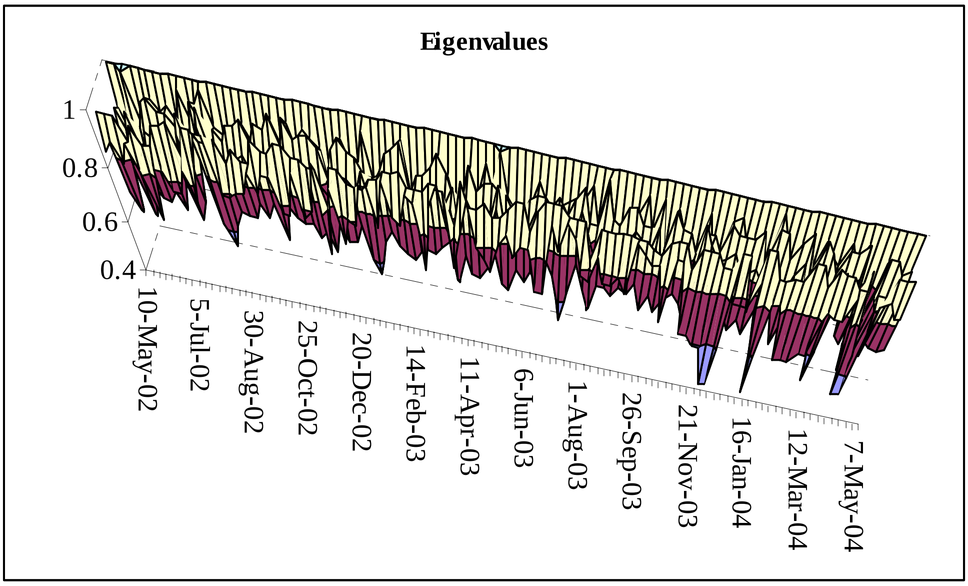
In absolute terms, the differences seem quite small. An interesting point however, is that the 1Y AAA and AA spreads as implied from the rating matrix are 1bp whereas the model-based are six to ten times higher.

On the other hand, the implied spreads using the historical generator matrix are very much different as it can be seen by looking at the default probabilities (last column in Table 5). This shows again that the current risk premia are much different than the historical averages, hence the big difference between the spreads.

This difference between the spreads needs to be thought carefully when one is measuring risk and even when one is marking to market. As we've just seen, historical default data show that the "fair value" of credit spreads is much higher than they are at the point of our evaluation. However, one cannot measure risk based on that "fair value" because market rates fluctuate according to market expectations.

What is mainly important when measuring risk is the time horizon, the volatility of the underlying rates over the specified horizon and the details of the risky position (maturity, size, direction). An examination of the dynamic evolution of the eigenvalues of the generator matrix (see Figure 10), derived from the weekly credit spreads as obtained from our spread-based model, helps in understanding how the risk neutral transition probability matrix shapes up over time.

Figure 10: Weekly Evolutions of Eigenvalues



6. Conclusion

In this paper, using the flexible framework of LS (1992), we propose a two-factor model for the dynamic evolution of credit spreads. The credit spread level and its instantaneous volatility are the two stochastic factors. It was shown that, as a theoretical model, it is capable of providing a fairly accurate pricing framework, where complex credit curve shapes can be accommodated, thus reflecting observed market rates, as well as the volatility term structure making the model quite appealing for option pricing applications.

We fitted the model to benchmark and corporate data and estimated its parameters using a cross-section / Monte Carlo simulation approach. Using a replication strategy, we carried on pricing credit spread options. Furthermore, the credit spread curves were used to infer default probabilities and transition matrices, showing an alternative way for their extraction based directly on spread data rather than bond prices.

The reported results are quite encouraging for the ability of our framework to deal with real issues in credit markets and form the basis of a valuation and credit risk management system. What remains for future research is to explore as to whether similar reassuring results could be found in different epochs of the credit markets.

Appendices

Appendix 1: List of Bonds at 07/05/2004

Coupon	Coupon Frequency	Maturity	Time to Maturity	Name	Price	Moody's Rating
2	1	20-Oct-04	1.48	GERMAN TREASURY BILL	99.066	Benchmark
2.5	1	18-Mar-05	1.89	BUNDESSCHATZANWEISUNGEN	100.225	Benchmark
2	1	10-Mar-06	2.88	BUNDESSCHATZANWEISUNGEN	98.971	Benchmark
4	1	16-Feb-07	3.84	BUNDESOBLIGATION	102.754	Benchmark
4.25	1	15-Feb-08	4.85	BUNDESOBLIGATION	103.42	Benchmark
3.25	1	17-Apr-09	6.03	BUNDESOBLIGATION	98.596	Benchmark
5.375	1	4-Jan-10	6.76	BUNDESREPUB. DEUTSCHLAND	108.367	Benchmark
5.25	1	4-Jan-11	7.78	BUNDESREPUB. DEUTSCHLAND	107.77	Benchmark
5	1	4-Jan-12	8.79	BUNDESREPUB. DEUTSCHLAND	105.994	Benchmark
4.5	1	4-Jan-13	9.81	BUNDESREPUB. DEUTSCHLAND	102.044	Benchmark
4.25	1	4-Jan-14	10.82	BUNDESREPUB. DEUTSCHLAND	99.555	Benchmark
6.25	1	4-Jan-24	20.96	BUNDESREPUB. DEUTSCHLAND	117.212	Benchmark
4.75	1	4-Jul-34	31.61	BUNDESREPUB. DEUTSCHLAND	96.144	Benchmark
4.5	1	11-Aug-04	0.27	LEASE ASSET BACKED SECS	100.69	AAA
5	1	28-Jan-05	0.74	VAUBAN MOBILISATION GAR	101.94	AAA
7.4	1	13-Apr-05	0.95	CSSE DE REF DE L'HABITAT	104.54	AAA
3.625	1	19-Sep-05	1.39	CDC IXIS	101.47	AAA
2.75	1	6-Mar-06	1.86	CIF EUROMORTGAGE	100.07	AAA
6	1	6-Jun-06	2.11	CSSE DE REF DE L'HABITAT	106.21	AAA
6.75	1	24-Jul-06	2.24	FI MORTGAGE SECURITIES	108.10	AAA
4	1	30-Oct-06	2.52	VAUBAN MOBILISATION GAR	102.18	AAA
4	1	30-Jul-07	3.28	VAUBAN MOBILISATION GAR	101.98	AAA
3.5	1	12-Nov-07	3.57	CIF EUROMORTGAGE	100.74	AAA
5.375	1	28-Jan-08	3.78	VAUBAN MOBILISATION GAR	106.43	AAA
5	1	25-Apr-08	4.03	CSSE DE REF DE L'HABITAT	105.67	AAA
2.75	1	26-Jun-08	4.20	CDC IXIS	97.22	AAA
5	1	15-Jul-08	4.25	COLONNADE SECURITIES BV	105.49	AAA
4.4	1	9-Oct-08	4.49	CDC FINANCE - CDC IXIS	102.98	AAA
4.5	1	28-Oct-08	4.54	VAUBAN MOBILISATION GAR	103.41	AAA
4.75	1	29-Oct-08	4.54	CIF EUROMORTGAGE	104.76	AAA
4.5	1	12-Nov-08	4.58	SAGESS	103.44	AAA
4.375	1	28-Apr-09	5.05	VAUBAN MOBILISATION GAR	102.27	AAA
4.25	1	15-Jul-09	5.26	COLONNADE SECURITIES BV	101.91	AAA
5.8	1	21-Jul-09	5.28	CDC FINANCE - CDC IXIS	109.16	AAA
4	1	25-Oct-09	5.55	CAISSE REFINANCE HYPOTHE	100.67	AAA
5.875	1	15-Apr-10	6.03	COLONNADE SECURITIES BV	109.96	AAA
5.75	1	25-Apr-10	6.05	CSSE DE REF DE L'HABITAT	108.76	AAA
3.625	1	16-Jul-10	6.28	CIF EUROMORTGAGE	98.21	AAA
6.125	1	2-Aug-10	6.33	CDC IXIS CAPITAL MARKETS	110.48	AAA
4.375	1	25-Apr-11	7.07	CREDIT D'EQUIPEMENT PME	102.04	AAA
4.2	1	25-Apr-11	7.07	CSSE DE REF DE L'HABITAT	100.59	AAA
5.25	1	27-Apr-11	7.07	SAGESS	106.46	AAA
5.375	1	6-Jul-11	7.27	CDC IXIS	107.37	AAA

6	1	28-Oct-11	7.58	VAUBAN MOBILISATION GAR	110.79	AAA
9	1	4-Jun-12	8.19	CDC FINANCE - CDC IXIS	131.62	AAA
5.25	1	30-Jul-12	8.35	VAUBAN MOBILISATION GAR	105.69	AAA
4.625	1	11-Oct-12	8.55	CIF EUROMORTGAGE	102.22	AAA
4.25	1	25-Feb-13	8.93	SAGESS	98.46	AAA
3.75	1	29-Jul-13	9.36	VAUBAN MOBILISATION GAR	94.07	AAA
5	1	25-Oct-13	9.61	CSSE DE REF DE L'HABITAT	104.11	AAA
4.625	1	29-Oct-13	9.62	GE CAPITAL EURO FUNDING	100.35	AAA
4.5	1	10-Dec-13	9.73	CIF EUROMORTGAGE	100.30	AAA
4.25	1	25-Oct-14	10.62	CSSE DE REF DE L'HABITAT	97.54	AAA
5.625	1	5-Oct-04	0.42	TOTAL S.A.	101.27	AA+
5.375	1	2-Jun-05	1.09	TOTAL S.A.	102.9797	AA+
5.75	1	29-Sep-05	1.42	TOTAL S.A.	104.1994	AA+
3.875	1	5-May-06	2.02	TOTAL S.A.	102.025646	AA+
3.5	1	28-Jan-08	3.78	TOTAL CAPITAL SA	100.3972	AA+
6.75	1	25-Oct-08	4.53	TOTAL S.A.	112.359751	AA+
4.5	1	23-Mar-09	4.95	ELF AQUITAINE	103.1601115	AA+
5.125	1	21-Jul-09	5.28	TOTAL S.A.	105.8525402	AA+
3.75	1	11-Feb-10	5.85	TOTAL CAPITAL SA	98.84618	AA+
6	1	15-Jun-10	6.19	DEUTSCHE BAHN FINANCE BV	110.095819	AA+
5.375	1	31-Jul-12	8.35	DEUTSCHE BAHN FINANCE BV	106.290683	AA+
4	1	15-Jul-13	9.32	CORES	96.07047325	AA+
5.125	1	28-Nov-13	9.70	DEUTSCHE BAHN FINANCE BV	104.194642	AA+
4.25	1	8-Jul-15	11.33	DEUTSCHE BAHN FINANCE BV	95.409185	AA+
4.75	1	14-Mar-18	14.05	DEUTSCHE BAHN FINANCE BV	99.0625	AA+
5.75	1	25-Jul-05	1.23	BASF AG	103.7633	AA-
5	1	4-Jul-06	2.19	SIEMENS FINANCIERINGSMAT	104.1403	AA-
5.25	1	19-Jul-06	2.23	ROBERT BOSCH GMBH	104.6612	AA-
5.5	1	12-Mar-07	2.89	SIEMENS FINANCIERINGSMAT	107.501	AA-
6	1	12-Nov-09	5.60	AGBAR INTERNATIONAL BV	109.5807945	AA-
6.125	1	9-Jun-10	6.18	ENI SPA	111.2364045	AA-
3.5	1	8-Jul-10	6.26	BASF AG	97.08931	AA-
5.75	1	4-Jul-11	7.26	SIEMENS FINANCIERINGSMAT	106.0491085	AA-
5.25	1	3-Jul-12	8.28	POSTE ITALIANE SPA	106.1753285	AA-
4.625	1	30-Apr-13	9.11	ENI SPA	100.43	AA-
4.375	1	8-Jul-13	9.30	SCHIPHOL NEDERLAND B.V.	98.3458685	AA-
5	1	12-Jul-04	0.18	METRO FINANCE BV	100.459494	BBB
4.375	1	15-Jul-04	0.19	LAFARGE	100.1325	BBB
5.1	1	3-Feb-05	0.76	LAFARGE	101.7728837	BBB
4.625	1	4-Mar-05	0.84	LAFARGE SA	101.662287	BBB
5.875	1	14-Apr-05	0.95	CASINO GUICHARD PERRACH	102.823	BBB
5.875	1	4-Jul-05	1.18	COCA COLA ERFRISCHUNGETR	104.325139	BBB
4.75	1	8-Nov-05	1.53	VEOLIA ENVIRONNEMENT	102.883294	BBB
5.125	1	16-Dec-05	1.63	WOLTERS KLUWER NV	102.755013	BBB
5.8	1	20-Dec-05	1.64	RENAULT S.A.	104.32669	BBB
5.75	1	9-Mar-06	1.86	METRO FINANCE BV	104.4988325	BBB
5.125	1	26-Jun-06	2.17	LAFARGE SA	104.002809	BBB
5.75	1	5-Jul-06	2.19	ACCOR	104.8090935	BBB
4.75	1	6-Jul-06	2.19	CASINO GUICHARD PERRACH	103.253698	BBB
5.25	1	14-Jul-06	2.22	THYSSENKRUPP FINANCE BV	103.273288	BBB
5.125	1	21-Jul-06	2.24	RENAULT S.A.	104.0950895	BBB
5.75	1	4-Dec-06	2.61	REPSOL INTL FINANCE	105.934727	BBB

5	1	20-Dec-06	2.66	ACCOR	103.521971	BBB
6	1	7-May-07	3.04	IMERYS SA	106.9570215	BBB
6.25	1	11-Jun-07	3.14	WOLTERS KLUWER NV	105.646174	BBB
6.375	1	26-Jul-07	3.26	LAFARGE SA	108.0797	BBB
6.375	1	19-Oct-07	3.50	RENAULT S.A.	108.650034	BBB
5.875	1	23-Nov-07	3.60	CASINO GUICHARD PERRACH	106.73	BBB
5.4	1	3-Feb-08	3.80	LAFARGE SA	105.213752	BBB
5.125	1	13-Feb-08	3.83	METRO AG	104.23	BBB
6	1	6-Mar-08	3.89	CASINO GUICHARD PERRACH	107.75	BBB
6.125	1	10-Apr-08	3.98	ARCELOR FINANCE	107.31	BBB
5.875	1	27-Jun-08	4.20	VEOLIA ENVIRONNEMENT	107.6119	BBB
5.875	1	6-Nov-08	4.57	LAFARGE SA	107.2164	BBB
4.5	1	19-Nov-08	4.60	SES GLOBAL SA	102.1875	BBB
4.8	1	22-Dec-08	4.69	LOTTOMATICA SPA	100.2632495	BBB
6.125	1	26-Jun-09	5.21	RENAULT S.A.	108.7227	BBB
5.875	1	10-Jul-09	5.25	SOCIETE DES CIMENTS FRAN	106.76686	BBB
6.35	1	1-Oct-09	5.48	UPM-KYMMENE CORP	109.443937	BBB
5.25	1	28-Apr-10	6.06	CASINO GUICHARD PERRACH	103.8952	BBB
6	1	5-May-10	6.08	REPSOL INTL FINANCE B.V.	108.08	BBB
4.625	1	28-May-10	6.14	RENAULT S.A.	101.1394	BBB
5.125	1	24-Sep-10	6.48	ARCELOR FINANCE	102.1968	BBB
6.125	1	23-Jan-12	7.83	UPM-KYMMENE CORP	108.165776	BBB
5.875	1	1-Feb-12	7.85	VEOLIA ENVIRONNEMENT	107.2922	BBB
6	1	27-Feb-12	7.92	CASINO GUICHARD PERRACH	106.6245	BBB
4.875	1	28-May-13	9.19	VEOLIA ENVIRONNEMENT	99.44154	BBB
5	1	22-Jul-13	9.34	REPSOL INTL FINANCE	99.99	BBB
5.448	1	4-Dec-13	9.72	LAFARGE SA	102.56	BBB
5.375	1	28-May-18	14.26	VEOLIA ENVIRONNEMENT	98.75	BBB
6.125	1	25-Nov-33	29.98	VEOLIA ENVIRONNEMENT	99.76	BBB

Appendix 2: FMC Definition (Bloomberg)

FMC curves are created using prices from new issue calendars, trading/portfolio systems, dealers, brokers, and evaluation services which are fed directly into the specified bond sector databases on an overnight basis. All prices are used.

All bonds for each sector are then subject to option adjusted spread (OAS) analysis and the option-free yields are then plotted from the fair market yield curve without any yields being distorted by embedded calls, puts, or sinks. This allows bonds with very different structures to be compared on an equivalent basis. A best fit curve is then drawn from the option-free yields, resulting in a specific yield curve for each bond category.

Debt issues are divided into hundreds of sectors that are grouped by several variables such as rating or industry type. The sectors are numbered, and an option-free yield curve is constructed daily for each sector. The ratings categories for each sector are expressed as Bloomberg Composite Ratings, which are blends of Moody's Investor Service and Standard & Poor's ratings.

COMP	MOODY'S	**S&P**
AAA	Aaa	AAA
AA1	Aa1	AA+
AA2	Aa2	AA
AA3	Aa3	AA-
A1	A1	A+
A2	A2	A
A3	A3	A-
BBB1	Baa1	BBB+
BBB2	Baa2	BBB
BBB3	Baa3	BBB-
BB1	Ba1	BB+
BB2	Ba3	BB
BB3	Ba3	BB-

B1	B1	B+
B2	B2	B
B3	B3	B-
CCC1	Caa1	CCC+
CCC2	Caa2	CCC
CCC3	Caa3	CCC-

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