A Jackknife-Type Estimator for Portfolio Revision*

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Abstract

This article proposes a novel approach to optimal portfolio revision. The current literature on portfolio optimization uses a somewhat naive approach, where portfolio weights are always completely revised after a predefined fixed period. However, one shortcoming of this procedure is that it ignores parameter uncertainty in the estimated portfolio weights, as well as the biasedness of the in-sample portfolio mean and variance as estimates of the expected portfolio return and out-of-sample variance. To rectify this problem, we propose a Jackknife procedure to determine the optimal revision intensity, i.e., the percent of wealth that should be shifted to the new, in-sample optimal portfolio. We find that our approach leads to highly stable portfolio allocations over time, and can significantly reduce the turnover of several well established portfolio strategies. Moreover, the observed turnover reductions lead to statistically and economically significant performance gains in the presence of transaction costs.

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*Comments are welcome.

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1 Introduction

Despite the theoretical appeal of Markowitz's (1952) mean-variance paradigm, it has three prominent criticisms. First, the mean-variance algorithm is believed to yield highly unbalanced allocations (see, e.g., Green and Hollifield (1992) as well as Black and Litterman (1992)). Second, the impact of even small changes in the input parameters can cause dramatic shifts in the derived allocations (see, e.g., Best and Grauer (1991), Chopra (1993), and Chopra and Ziemba (1993)). Third, its repeated use over time leads to comparatively unstable allocations, resulting in higher turnovers (see Chopra et al. (1993)). The literature contains several potential remedies for the first two issues; relatively little attention, however, has focused on the immense turnover of mean-variance portfolios and the associated question of portfolio revisions.

In this paper, we tackle the question of optimal portfolio revision. Specifically, we explore when and how much investors should consider shifting from their current portfolios to new in-sample optimal portfolios. We assess this question based on the established rolling sample approach to portfolio optimization, in which investors typically revise their entire portfolios after a predefined but arbitrarily chosen time period. The investor then holds the new optimal portfolio, which is optimized over the corresponding new in-sample period. This revision policy appears intuitive, because the new portfolio dominates the old one in terms of in-sample mean-variance efficiency. The fixed portfolio revision frequency is also in line with the myopic nature of the Markowitz framework.

However, a significant shortcoming of this approach is that it is based on in-sample mean-variance efficiency. As Kan and Zhou (2007), Siegel and Woodgate (2007), and Basak et al. (2009) have shown, in-sample portfolio mean and variance are not consistent estimators of expected out-of-sample return and variance because of the potential for estimation errors. Specifically, these authors note that in-sample portfolio returns that are optimized in the presence of estimation errors tend to overestimate expected out-of-sample returns. In-sample variance, on the other hand, tends to underestimate expected out-of-sample variance.[1]

As a result of this in-sample overoptimism, investors may be reluctant to revise their portfolios without a concurrent increase in expected out-of-sample performance. To deal with this issue, we base our revision policy on the expected out-of-sample performance of a portfolio, which we consider as the natural investor objective for portfolio revisions.

We use Basak et al.'s (2009) Jackknife procedure to obtain consistent estimates of the expected out-of-sample portfolio performance of the investor's current portfolio versus the new, in-sample optimal portfolio. We then need to determine what percentage to allocate to the new portfolio. We provide an analytic expression for the optimal revision intensity, and evaluate our revision policy by applying it to various minimum-variance portfolio strategies across five datasets. We could also apply our portfolio revision framework to mean-variance portfolios, but our focus is solely on minimum-variance portfolios because broad empirical evidence has shown that they perform better out-of-sample (see Jobson and Korkie (1981) as well as Jagannathan and Ma (2003), among others).

We assess the performance of our revision policy on simulated datasets as well as on five empirical datasets. For both simulation and empirical analysis, we achieve comparable portfolio performance in terms of the Sharpe ratio and the certainty equivalent return, suggesting our revision policy can lead to substantially lower turnovers than policies with fixed revision frequencies. Empirically, we observe that our revision policy reduces average monthly turnovers by up to 77.6%, with average reductions ranging from 23.3% for the Ledoit and Wolf (2004a) to 38.9% for the shortsale constrained minimum-variance portfolio.

If we include trading costs in our analysis, we observe that, because of the sizable turnover reduction, our portfolio revision policy yields economically and statistically significant performance gains. Assuming proportional round-trip transaction costs of 1%,

[1]Kan and Zhou (2007) as well as Siegel and Woodgate (2007) demonstrate that this in-sample overoptimism is inversely related to the number of observations available to estimate the input parameters for the optimization parameters. It also increases with the number of assets.

we find that, compared to monthly or annual portfolio revision, investors with a risk aversion of five would theoretically be willing to pay an annual management fee of between 101 and 168 basis points (bps). Our results are especially strong for datasets with larger asset universes, where the higher ratio of number of assets to number of time periods (N/T) increases the estimation error of the covariance matrix estimates. This creates more unbalanced allocations, along with higher turnover.

There are two main strands of literature on portfolio revisions of mean-variance efficient portfolios: dynamic revision strategies (e.g. Chen et al. (1971), Kamin (1975), and Leland (1999)), and retrospectively determined optimal revision frequencies or ranges (e.g. Arnott and Lovell (1993), Plaxco and Arnott (2002), and Tokat and Wicas (2007)).

Dynamic revision strategies are generally complex, computationally intensive, and thus somewhat limited to small investment universes (see Donhue and Yip (2003)). Notably, the Markowitz and van Dijk (2003) as well as Kritzman et al. (2009) approximations represent scalable alternatives to dynamic revision strategies. However, they remain a heuristic approximation to the underlying dynamic program. The heuristic nature of these approximations is also reflected in the retrospectively determined optimal revision frequencies or ranges. The revision frequencies do not follow any objective optimality criteria, and may thus be regarded as heuristic.

In addition to the two major approaches to portfolio revisions, the literature contains two minor approaches: the imposition of turnover constraints (Schreiner (1980)), and the use of statistical process control charts for determining optimal portfolio revisions (Golosnoy (2009)). Both approaches require threshold values to trigger the revision process, however, which renders the results highly sensitive to the chosen threshold. Furthermore, all of the approaches mentioned thus far neglect the impact of estimation errors and any corresponding in-sample overoptimism on the revision decision.

Our paper contributes to the existing literature in three ways. First, our approach represents a new method to overcoming the problem of optimal portfolio revision by focusing on the (consistently estimated) expected out-of-sample performance of the revised portfolio, which we regard as the natural objective for revision. Accordingly, our approach takes into account the problem of estimation errors and in-sample overoptimism. Second, we derive our estimator by optimizing expected out-of-sample portfolio performance. This represents an exact solution to the revision problem, rather than a heuristic one. By focusing on the expected out-of-sample performance of the revised portfolio, our approach is easily scalable and computationally inexpensive, even for large investment universes. Third, our estimator gives a purely data-driven solution to the portfolio revision problem. It requires neither calibration nor a threshold value. It is thus easily implementable, and of interest for practical applications. The remainder of this paper is organized as follows. Section 2 describes our framework for optimal portfolio revision, reviews the various minimum-variance portfolio strategies that we use to evaluate the profitability of our policy, and outlines our performance evaluation procedure. Section 3 discusses our proposed revision policy using simulated data, while section 4 explores its performance on empirical datasets. Section 5 gives our conclusions.

2 Methodology

2.1 Optimal Portfolio Revision Policy

Throughout this paper, we consider the typical myopic minimum-variance portfolio optimization problem. The investor's objective is to minimize $w'\Sigma w$, where w denotes a column vector of optimal portfolio weights, and Σ represents the population covariance matrix. Because Σ is not observable, we use an estimate based on sample information, denoted by S. The sample estimate of the portfolio variance is thus $\hat{w}'S\hat{w}$, where $\hat{w}$ represents the sample estimate of w.

In the traditional rolling sample approach to portfolio optimization, investors determine $\hat{w}_t$ at the end of each period t using the sample information from the previous using τ months. The portfolio weights are held constant over the consecutive period, $t + 1$. At the end of the period, the investor receives the corresponding out-of-sample return $\hat{w}_t' R_{t+1}$, where R_{t+1} denotes the cross-section of excess returns in period $t + 1$. The investor then determines the new optimal portfolio weights, $\hat{w}_{t+1}$, conditional on the sample information of the new estimation period, which ranges from $t + 2 - \tau$ to $t + 1$. Finally, investors completely revise their portfolio weights to the new portfolio weights, $\hat{w}_{t+1}$, since the old portfolio weights, $\hat{w}_t$, are no longer efficient conditional on the new in-sample information. This procedure is then repeated over the entire sample period.

Note that Kan and Zhou (2007), Siegel and Woodgate (2007), and Basak et al. (2009) show that the in-sample variance of a portfolio is not a consistent estimator of expected out-of-sample variance. This intuitively appears to be the quantity of interest when investors revise their minimum-variance portfolios.

Basak et al. (2009) particularly demonstrate that the in-sample variance of a portfolio can suffer from overoptimism and systematically underestimate out-of-sample variance. Hence, because the new in-sample optimal portfolio weights do not necessarily yield lower expected out-of-sample variance, investors may not be better off revising their weights from the previous period to the current period. Rather, it may be more advantageous to hold a combination of both weight estimates, because this could yield diversification benefits if the expected out-of-sample variances are not perfectly correlated. Moreover, holding the weights of the previous period constant may even yield a lower expected out-of-sample variance than shifting them toward the new ones.

Aside from the expected out-of-sample variance, managing the turnover resulting from a portfolio revision is crucial for the practical implementation of a portfolio strategy. Typically, higher turnover hampers implementation, and can adversely affect portfolio performance in the presence of trading costs. Hence, investors will generally prefer less frequent revisions of portfolio weights due to a decrease in turnover.

Aside from the expected out-of-sample variance, the turnover resulting from revising a portfolio is crucial when it comes to the practical implementation of a portfolio strategy. Typically, a higher turnover hampers practical implementation and adversely affects portfolio performance in the presence of trading costs. Hence, associated with a decrease in turnover, less frequent revision of portfolio weights will generally be preferred by investors.

2.1.1 Portfolio revision in a two-period setting

For the sake of simplicity, we first describe our portfolio revision policy in a two-period setting. Our objective is to minimize the expected out-of-sample portfolio variance, $Var\left(\hat{w}_t^{Rev'} R_{t+1}\right)$, where $\hat{w}_t^{Rev}$ is the convex combination of the estimated portfolio weights of the current and previous period, denoted by $\hat{w}_t$ and $\hat{w}_{t-1}$, respectively. The portfolio weights of our proposed revision policy in period t are obtained by $\hat{w}_t^{Rev} = \alpha\hat{w}_{t-1} + (1-\alpha)\hat{w}_t$, where α is the fraction of wealth that the investor keeps in the portfolio weights from the previous period, $\hat{w}_{t-1}$. We can express the quantity of interest (the expected out-of-sample portfolio variance), as follows:

$$\min_{\alpha} Var\left(\hat{w}_t^{Rev'} R_{t+1}\right) = Var\left(\alpha\hat{w}'_{t-1}R_{t+1} + (1-\alpha)\hat{w}'_t R_{t+1}\right). \tag{1}$$

The first and second derivatives of $Var\left(\hat{w}_t^{Rev'} R_{t+1}\right)$ yield:

$$\begin{aligned} Var'\left(\hat{w}_t^{Rev'} R_{t+1}\right) &= \alpha Var(\hat{w}'_{t-1}R_{t+1} - \hat{w}'_t R_{t+1}) - Var(\hat{w}'_t R_{t+1}) \\ &\quad + Cov(\hat{w}'_{t-1}R_{t+1}, \hat{w}'_t R_{t+1}), \qquad (2) \\ Var''\left(\hat{w}_t^{Rev'} R_{t+1}\right) &= Var(\hat{w}'_{t-1}R_{t+1} - \hat{w}'_t R_{t+1}). \qquad (3) \end{aligned}$$

Setting $Var'\left(\hat{w}_t^{Rev'} R_{t+1}\right) = 0$ and solving for α yields the optimal combination intensity, α^*, which minimizes the expected out-of-sample portfolio variance as follows:

$$\alpha^* = \frac{Var(\hat{w}'_t R_{t+1}) - Cov(\hat{w}'_{t-1}R_{t+1}, \hat{w}'_t R_{t+1})}{Var(\hat{w}'_{t-1}R_{t+1} - \hat{w}'_t R_{t+1})}. \tag{4}$$

Because (3) is positive everywhere, the expression in (4) is verified as a global minimum for the expected out-of-sample variance. However, note that α^* is not a bona fide estimator, since it depends on the unobservable quantities $Var(\hat{w}_t' R_{t+1})$, $Cov(\hat{w}_{t-1}' R_{t+1}, \hat{w}_t' R_{t+1})$, and $Var(\hat{w}_{t-1}' R_{t+1} - \hat{w}_t' R_{t+1})$. Hence, we need to estimate the quantities in (4) consistently.

To do this, we use Basak et al.'s (2009) Jackknife-type estimator of the expected out-of-sample portfolio return moments. Following this methodology, we first drop the i-th observation from the in-sample period and compute the resulting covariance matrix based on the remaining $\tau - 1$ returns, denoted by $S_{t,i}$. Next, we determine the corresponding optimal portfolio weights, denoted by $\hat{w}_{t,i}$. Finally, we compute the Jackknife return, $R_{t,i}^{JK}$, that would have been achieved from holding $\hat{w}_{t,i}$ in period i, given by $\hat{w}_{t,i}' R_i$. Repeating the aforementioned process for all $i = \{1, .., \tau\}$ leaves us with τ Jackknife returns $\{R_{t,1}^{JK}, R_{t,2}^{JK}, \cdots, R_{t,\tau}^{JK}\}$.

Based on the underlying Jackknife assumption that returns on risky assets are identically and independently distributed over time, Basak et al. (2009) show that the distribution of $R_{t,i}^{JK}$ follows the same distribution as $\hat{w}_t' R_{t+1}$. The sample variance based on the τ Jackknife returns thus represents a natural estimator of the out-of-sample variance, $Var(\hat{w}_t' R_{t+1})$, denoted by:[2]

$$\widehat{VAR}^{JK}(\hat{w}_t' R_{t+1}) = \frac{1}{\tau} \sum_{i=1}^{\tau} \left(R_{t,i}^{JK} - \mu_{\hat{w}_t}\right)^2, \text{ with} \tag{5}$$

$$\mu_{\hat{w}_t} = \frac{1}{\tau} \sum_{i=1}^{\tau} R_{t,i}^{JK}. \tag{6}$$

Note that when we estimate the remaining quantities in Equation (4), $Cov(\hat{w}_{t-1}' R_{t+1}, \hat{w}_t' R_{t+1})$ and $Var(\hat{w}_{t-1}' R_{t+1} - \hat{w}_t' R_{t+1})$, we find only $\tau - 1$ overlapping Jackknife returns available for computation.

This is attributable to the use of the rolling sample procedure. The intuition of this procedure is that only the last τ observations are relevant for determining the quantity of

[2] See Basak et al. (2009) for further discussion of this point.

interest. We thus need to compute $Cov(\hat{w}'_{t-1}R_{t+1}, \hat{w}'_t R_{t+1})$ and $Var(\hat{w}'_{t-1}R_{t+1} - \hat{w}'_t R_{t+1})$ based on the same observations (the most recent τ returns).

In period t the Jackknife return for $\hat{w}_{t-1}$ is missing in the last period of the sample used to estimate $\hat{w}_t$. To overcome this problem, we compute the out-of-sample return for $\hat{w}_{t-1}$ for the missing observation, which is $\hat{w}'_{t-1}R_t$. From this, we can obtain a vector of approximate Jackknife returns[3] for $\hat{w}_{t-1}$, consisting of $\left\{\hat{w}'_{t-1}R_t, R^{JK}_{t-1,1}, R^{JK}_{t-1,2}, \cdots, R^{JK}_{t-1,\tau-1}\right\}$. Because the sequence of all Jackknife returns for $\hat{w}_{t-1}$ has the same distribution as $\hat{w}'_{t-1}R_t$ using it does not alter the distribution. It further enables us to obtain consistent estimates of $Cov(\hat{w}'_{t-1}R_{t+1}, \hat{w}'_t R_{t+1})$ and $Var(\hat{w}'_{t-1}R_{t+1} - \hat{w}'_t R_{t+1})$ based on the last τ observations. We can thus estimate $Cov(\hat{w}'_{t-1}R_{t+1}, \hat{w}'_t R_{t+1})$ as:

$$\begin{aligned}\widehat{Cov}^{JK}\left(\hat{w}'_{t-1}R_{t+1}, \hat{w}'_t R_{t+1}\right) &= \frac{1}{\tau}\left(\left(\hat{w}'_{t-1}R_1 - \mu_{\hat{w}_{t-1}}\right)\left(R^{JK}_{t,1} - \mu_{\hat{w}_t}\right)\right) + \\ &\tfrac{1}{\tau}\textstyle\sum_{i=2}^{\tau}\left(R^{JK}_{t-1,i} - \mu_{\hat{w}_{t-1}}\right)\left(R^{JK}_{t,i} - \mu_{\hat{w}_t}\right), \text{ with} \\ \mu_{\hat{w}_{t-1}} = \tfrac{1}{\tau}&\left(\hat{w}'_{t-1}R_1 + \textstyle\sum_{i=2}^{\tau} R^{JK}_{t-1,i}\right) \text{ and } \mu_{\hat{w}_t} \text{ as defined in (6),}\end{aligned} \tag{7}$$

and $Var(\hat{w}'_{t-1}R_{t+1} - \hat{w}'_t R_{t+1})$ as:

$$\begin{aligned}\widehat{Var}^{JK}\left(\hat{w}'_{t-1}R_{t+1} - \hat{w}'_t R_{t+1}\right) &= \frac{1}{\tau}\left(\left(\hat{w}'_{t-1}R_1 - R^{JK}_{t,1}\right) - \mu_{\hat{w}_{t-1}-\hat{w}_t}\right)^2 + \\ &\tfrac{1}{\tau}\textstyle\sum_{i=2}^{\tau}\left(\left(R^{JK}_{t-1,i} - R^{JK}_{t,i}\right) - \mu_{\hat{w}_{t-1}-\hat{w}_t}\right)^2, \text{ with} \\ \mu_{\hat{w}_{t-1}-\hat{w}_t} = \tfrac{1}{\tau}&\left(\left(\hat{w}'_{t-1}R_1 - R^{JK}_{t,1}\right) + \textstyle\sum_{i=2}^{\tau} R^{JK}_{t-1,i} - R^{JK}_{t,i},\right),\end{aligned} \tag{8}$$

which finally allows us to estimate α^* as:

$$\hat{a}^* = \frac{\widehat{Var}^{JK}(\hat{w}'_t R_{t+1}) - \widehat{Cov}^{JK}(\hat{w}'_{t-1}R_{t+1}, \hat{w}'_t R_{t+1})}{\widehat{Var}^{JK}(\hat{w}'_{t-1}R_{t+1} - \hat{w}'_t R_{t+1})}. \tag{9}$$

Naturally, $\hat{a}^*$ may lie outside the desired range between 0 (the portfolio is totally revised

[3]The term "approximate Jackknife returns" accounts for the mixture of out-of-sample returns and Jackknife returns in the vector.

to the new portfolio weights) and 1 (the portfolio is not revised at all). To ensure that $\hat{a}^*$ lies within the desired interval, we cut off values that lie outside the $[0,1]$ interval, so that:

$$\hat{a} = max\left[0, min\left(1, \hat{a}^*\right)\right]. \tag{10}$$

2.1.2 Portfolio revision in a multi-period setting

It is straightforward to extend the revision scheme of the two-period case to a multi-period setting. In a multi-period setting, the weights of the previous period are already a convex combination of those from the preceding $t-1$ periods. However, two minor adjustments are necessary. First, we must change the notation of the old portfolio weights from the previous period from $\hat{w}_{t-1}$ to $\hat{w}_{t-1}^{Rev}$ to account for the fact that $\hat{w}_{t-1}^{Rev}$ is not fully determined by $\hat{w}_{t-1}$. In other words, the revised portfolio weights of the last period have not been or have only been partially revised to the portfolio weight estimate $\hat{w}_{t-1}$.

Second, we must track the solution to the out-of-sample variance minimization problem in (1) over time, to determine both the share of less recent portfolio weight estimates of $\hat{w}_{t-1}^{Rev}$ and the corresponding determination of the expected out-of-sample variance. We can thus describe investors' revision decisions in each period t by the following minimization problem:

$$\min_{\alpha_t} Var\left(\hat{w}_t^{Rev'} R_{t+1}\right) \quad = \quad Var\left(\alpha_t \hat{w}_{t-1}^{Rev'} R_{t+1} + (1-\alpha_t)\hat{w}_t' R_{t+1}\right). \tag{11}$$

Similarly to the two-period case, this solves as:

$$\alpha_t^* = \frac{Var(\hat{w}_t' R_{t+1}) - Cov(\hat{w}_{t-1}^{Rev'} R_{t+1}, \hat{w}_t' R_{t+1})}{Var(\hat{w}_{t-1}^{Rev'} R_{t+1} - \hat{w}_t' R_{t+1})}. \tag{12}$$

Again, we face the problem that α_t^* is not a bona fide estimator. Accordingly, we need to estimate $Var(\hat{w}_t' R_{t+1})$, $Cov(\hat{w}_{t-1}^{Rev'} R_{t+1}, \hat{w}_t' R_{t+1})$, and $Var(\hat{w}_{t-1}^{Rev'} R_{t+1} - \hat{w}_t' R_{t+1})$. Because

$Var(\hat{w}_t' R_{t+1})$ does not differ from the two-period case, we can estimate it by using the estimator in (5).

Estimating $Cov(\hat{w}_{t-1}^{Rev\prime} R_{t+1}, \hat{w}_t' R_{t+1})$ and $Var(\hat{w}_{t-1}^{Rev\prime} R_{t+1} - \hat{w}_t' R_{t+1})$, however, is more complex. Because both quantities depend on $\hat{w}_{t-1}^{Rev}$ it may be a convex combination of $\{\hat{w}_{t-1}, \hat{w}_{t-2}, \ldots, \hat{w}_{t-n}\}$. We thus need to compute the expected out-of-sample variance for $\hat{w}_{t-1}^{Rev\prime}$ by taking all n portfolio weights into account.[4]

To illustrate this in practice, let $r_{j,t}$ denote the vector of approximate Jackknife returns corresponding to $\hat{w}_t$, where j denotes the period for which the portfolio weights and the corresponding Jackknife returns $\left\{R_{j,1}^{JK}, R_{j,2}^{JK}, \cdots, R_{j,\tau}^{JK}\right\}$ were initially calculated. Whenever α_t takes a value of 0, the following procedure applies to the consecutive periods: we obtain return vector $r_{t,t}$, consisting of τ Jackknife returns $\left\{R_{t,1}^{JK}, R_{t,2}^{JK}, \cdots, R_{t,\tau}^{JK}\right\}$ in the first period, t. In the following period, $t+1$, we obtain the new return vector, $r_{t+1,t+1}$, which again consists of τ Jackknife returns $\left\{R_{t+1,1}^{JK}, R_{t+1,2}^{JK}, \cdots, R_{t+1,\tau}^{JK}\right\}$. The previous return vector, $r_{t,t}$, then becomes $r_{t,t+1}$ in $t+1$, indicating that it comprises $\tau - 1$ Jackknife returns corresponding to portfolio $\hat{w}_t$ and one out-of sample return $\left\{\hat{w}_t' R_{t+1}, R_{t,1}^{JK}, R_{t,2}^{JK}, \cdots, R_{t,\tau-1}^{JK}\right\}$. After the revision decision in $t+1$, we determine the approximate Jackknife return vector of the revised portfolio, r_{t+1}^{Rev}, as follows:

$$r_{t+1}^{Rev} = \alpha_1 r_{t,t+1} + (1-\alpha_1) r_{t+1,t+1}.$$

Consequently, the entries in the vector of approximate Jackknife returns r_{t+1}^{Rev} are given by $\left\{\alpha_1 \hat{w}_t' R_{t+1} + (1-\alpha_1) R_{t+1,1}^{JK}, \alpha_1 R_{t,1}^{JK} + (1-\alpha_1) R_{t+1,2}^{JK}, \cdots, \alpha_1 R_{t,\tau-1}^{JK} + (1-\alpha_1) R_{t+1,\tau}^{JK}\right\}$. In

[4]Note that the number of portfolio weights, n, contained in $\hat{w}_{t-1}^{Rev}$ depends on the historical values of $\{\alpha_{t-1}, \alpha_{t-2}, \cdots \alpha_{t-n}\}$. As long as none of the n alphas is equal to zero, $\hat{w}_{t-1}^{Rev}$ will be a convex combination of all portfolio weights determined over the n periods. In contrast, as soon as any alpha equals zero, the portfolio weights are completely revised to the new, in-sample optimal weights, which render the old weights and their corresponding approximate Jackknife returns obsolete.

the next period, the same procedure applies as follows:

$$\begin{aligned} r_{t+2}^{Rev} &= \alpha_2[\alpha_1 r_{t,t+1} + (1-\alpha_1) r_{t+1,t+1}] + (1-\alpha_2) r_{t+2,t+2} \\ &= \alpha_2 r_{t+1}^{Rev} + (1-\alpha_2) r_{t+2,t+2}. \end{aligned}$$

It is obvious that, unless the n-th α_t takes a value of zero, the vector of approximate Jackknife returns corresponding to w_t is determined by:

$$r_{t+n}^{Rev} = \alpha_n r_{t+n-1}^{Rev} + (1-\alpha_n) r_{t+n,t+n}.$$

Let $r_{t,i}^{Rev}$ and $r_{t,t,i}$ denote the i-th element of the vector r_t^{Rev} and $r_{t,t}$, respectively. Following the intuition of the revision scheme described above, we can then estimate $Cov(\hat{w}_{t-1}^{Rev'} R_{t+1}, \hat{w}_t' R_{t+1})$ as:

$$\widehat{Cov}^{JK}\left(\hat{w}_{t-1}^{Rev'} R_{t+1}, \hat{w}_t' R_{t+1}\right) = \frac{1}{\tau}\sum_{i=1}^{\tau}\left(r_{t-1,i}^{Rev} - \frac{1}{\tau}\sum_{i=1}^{\tau} r_{t-1,i}^{Rev}\right)\left(r_{t,t,i} - \frac{1}{\tau}\sum_{i=1}^{\tau} r_{t,t,i}\right), \quad (13)$$

and $Var(\hat{w}_{t-1}^{Rev'} R_{t+1} - \hat{w}_t' R_{t+1})$ as:

$$\widehat{Var}^{JK}\left(\hat{w}_{t-1}^{Rev'} R_{t+1} - \hat{w}_t' R_{t+1}\right) = \frac{1}{\tau}\sum_{i=1}^{\tau}\left(\left(r_{t-1,i}^{Rev} - r_{t,t,i}\right) - \frac{1}{\tau}\sum_{i=1}^{\tau}\left(r_{t-1,i}^{Rev} - r_{t,t,i}\right)\right)^2, \quad (14)$$

which finally allows us to estimate α^* as:

$$\hat{a}^* = \frac{\widehat{Var}^{JK}(\hat{w}_t' R_{t+1}) - \widehat{Cov}^{JK}(\hat{w}_{t-1}^{Rev'} R_{t+1}, \hat{w}_t' R_{t+1})}{\widehat{Var}^{JK}(\hat{w}_{t-1}^{Rev'} R_{t+1} - \hat{w}_t' R_{t+1})}. \quad (15)$$

Analogously to the two-period case, we trim the estimator $\hat{a}^*$ according to Equation (10) in order to ensure that the values lie inside the $[0,1]$ interval.

2.2 Description of Evaluated Portfolios

In this section, we briefly describe the minimum-variance portfolio strategies we use to evaluate our proposed revision policy. The difference between the various portfolio strategies is attributable to the different estimators of the variance-covariance matrix. All portfolio weights are computed by solving the following optimization problem:

$$\begin{aligned} &\min_{\hat{w}} \hat{w}' S \hat{w} \\ &\text{s.t. } \hat{w}' 1_N = 1, \end{aligned} \tag{16}$$

where 1_N denotes a vector of ones of appropriate size. The well-known solution to this minimization problem is given by:

$$\hat{w} = S^{-1} 1_N (1'_N S^{-1} 1_N)^{-1}, \tag{17}$$

which is the sample estimate of the global minimum-variance portfolio weights. For our analysis we focus on the most prominent covariance matrix estimators and corresponding minimum-variance portfolios in the portfolio selection literature. An overview of all portfolios is given in Table 1.

[INSERT TABLE 1 HERE]

Our selection of minimum-variance portfolios may be divided into two groups: the first is based on a single covariance matrix estimate whereas the second is based on a combination of two covariance matrix estimators. Within the first group of minimum-variance portfolios, we consider three covariance matrix estimators. First, we evaluate the minimum-variance portfolio based on the sample covariance matrix (SAMPLE) as given by:

$$S_{Sample} = \frac{1}{\tau - 1} X X', \tag{18}$$

where X denotes a $\tau \times N$ matrix of de-meaned returns. Second, we consider the suitability of our revision policy for the short sale constrained minimum-variance portfolio. In order to compute the optimal weights of the short sale constrained minimum-variance portfolio (MIN-C), the optimization problem in (16)-(17) is extended by the following additional weight constraint:

$$\hat{w}_i \geq 0, i = 1, 2, ..., N. \tag{19}$$

Jagannathan and Ma (2003) show that the portfolio weights of the short sale constrained minimum-variance portfolio correspond to those of the global minimum-variance portfolio with the following covariance matrix estimator:

$$S_{SC} = S_{Sample} - (\lambda 1' + 1\lambda'), \tag{20}$$

where $\lambda = \{\lambda_1, \lambda_2, ..., \lambda_N\}$ denotes a vector of Lagrangian multipliers for the non-negativity constraints in (19). Third, we assess our revision policy using a minimum-variance portfolio based on the 1-factor covariance matrix (1F). For our analysis, we employ Sharpe's (1967) single-index model which assumes that the return of stock i at time t, $r_{i,t}$, can be described by:

$$r_{i,t} = \alpha_i + \beta_i r_{M,t} + \delta_{i,t}, \tag{21}$$

where α_i is the mispricing of stock i, β_i is the beta factor of stock i, $r_{M,t}$ is the excess return of the market over the risk-free asset at time t, and $\delta_{i,t}$ is the residual variance of stock i at time t. The covariance matrix estimate implied by this model is then given by:

$$S_{1F} = \hat{\sigma}^2_M \hat{\beta}\hat{\beta}' + \hat{\Delta}, \tag{22}$$

where $\hat{\sigma}_M$, $\hat{\beta}$, and $\hat{\Delta}$ are the estimates of market variance, beta factors and a diagonal matrix with the residual variances $\hat{\delta}_{i,t}$, respectively.[5]

The second group of minimum-variance portfolios comprises the three shrinkage estimators proposed by Ledoit and Wolf (2003, 2004a, 2004b). All considered shrinkage estimators of the covariance matrix take the following form:

$$S_{LW} = \phi \hat{F} + (1 - \phi) S_{Sample}. \tag{23}$$

Obviously, S_{LW} is a convex combination of the sample covariance matrix and a shrinkage target $\hat{F}$, with the shrinkage intensity, ϕ, taking values between 0 and 1. The optimal shrinkage intensity is determined by minimizing the quadratic loss of S_{LW}.[6] Following the authors, we consider three different candidates for $\hat{F}$: the single-factor model covariance matrix (LW1F) (Ledoit and Wolf (2003)), a multiple of the identity matrix (LWID) (Ledoit and Wolf (2004a)), and the constant correlation model (LWCC) (Ledoit and Wolf (2004b)).

2.3 Performance Evaluation Method

In order to evaluate the profitability of our proposed revision policy, we benchmark the performance of each portfolio strategy based on our revision policy relative to the performance of the same strategy based on other revision frequencies. In particular, we choose revision frequencies of one, six, and twelve months as benchmarks.[7] We assess the

[5]Following the results of Chan et al. (1999), we do not include other factor models beside the single index model in our analysis, as additional factors to the market factor do not improve portfolio performance.

[6]We refer the interested reader to the papers by Ledoit and Wolf (2003, 2004a, 2004b) for a derivation of the optimal shrinkage intensity and an in-depth discussion of the estimators.

[7]Our choice of benchmark revision frequencies is driven by the commonly employed revision frequencies in the contemporaneous portfolio choice literature. While more recent studies (e.g., DeMiguel et al. (2009a, 2009b) and Tu and Zhou (2011)) employ a monthly revision scheme, the contributions by Chan et al. (1999), Jagannathan and Ma (2003), as well as Ledoit and Wolf (2003), are based on a twelve month revision frequency. Besides the considered one, six, and twelve months revision frequencies, we compute all results for a quarterly revision frequency. Given the qualitative and quantitative similarity of the three and six months revision frequency, we do not report the results for the three months revision frequency. However, the results are available from the authors upon request.

profitability of the revision policies for each portfolio strategy i using the following four performance measures: the portfolio variance ($\hat{\sigma}^2$), the Sharpe ratio ($\widehat{SR}$), the certainty equivalent ($\widehat{CEQ}$) return, and the average portfolio turnover (TRN):

$$\hat{\sigma}_i^2 = \frac{1}{T-\tau-1}\sum_{t=\tau}^{T-1}\left(w_{i,t}' r_{t+1} - \hat{\mu}_i\right)^2 \tag{24}$$

$$\text{with } \hat{\mu}_i = \frac{1}{T-\tau}\sum_{t=\tau}^{T-1} w_{i,t}' r_{t+1},$$

$$\widehat{SR}_i = \frac{\hat{\mu}_i - r_f}{\hat{\sigma}_i}, \tag{25}$$

$$\widehat{CEQ}_i = \hat{\mu}_i - \frac{\gamma}{2}\hat{\sigma}_i^2, \tag{26}$$

$$TRN_i = \frac{1}{T-\tau-1}\sum_{t=\tau}^{T-1}\sum_{j=1}^{N}\left|w_{i,j,t+1} - w_{i,j,t+}\right|, \tag{27}$$

where $w_{i,j,t}$ denotes the portfolio weight of strategy i in asset j at time t, $w_{i,j,t+}$ the corresponding weight before rebalancing at time $t+1$, and $w_{i,j,t+1}$ the congruent weight after rebalancing at time $t+1$.

In order to measure the statistical significance of the differences in the aforementioned performance metrics between our proposed revision policy and the considered benchmark revision strategies, we resort to bootstrapping methods. In our application of bootstrap methods to performance hypothesis testing, we explicitly control for the impact of time series effects (e.g., autocorrelation, volatility clustering, and non-normality) in portfolio returns and turnover on the aforementioned performance metrics. Accordingly, we test for each portfolio strategy whether the performance metric difference between our revision policy and a fixed revision frequency is statistically different from zero.

In the following, we describe the procedure for conducting hypothesis testing with the turnover.[8] In order to capture the time series effects of monthly turnovers[9], we

[8] The remaining approaches to bootstrap hypothesis testing follow the described procedure closely and differ only with respect to the resampling of the test statistic. The procedure for the p-value computation is equivalent across all approaches.

[9] We found on average a first-order autocorrelation in monthly turnovers of 0.2.

employ the circular block bootstrap proposed by Politis and Romano (1992). Hence, we randomly draw a fixed number, b, of consecutive observations, i.e. blocks of turnovers from the original time series of monthly turnovers, which is wrapped into a circle. We repeat the random draw of turnover blocks m times such that $mb = T - \tau$, which equals the original size of the time series of monthly turnovers. Repeating this K times leaves us with K bootstrap samples, each comprising a time series of T monthly turnovers. We may now assess whether the empirical difference in the average monthly turnover between a particular portfolio strategy based on our revision policy, Opt, is significantly different from the same portfolio strategy using a fixed portfolio revision frequency n. That is, $H_0 : trn_{Opt} - trn_n = 0$. We compute the corresponding two-sided p-value for the stated null hypothesis using remark 3.2 by Ledoit and Wolf (2008) as follows: let d denote the absolute value of the empirical average turnover difference between our revision policy, Opt, and the considered benchmark portfolio revision frequencies n. Further, let d_k^B denote the absolute value of the same difference based on the k-th bootstrap sample. We may then compute the p-value as:

$$PV = \frac{\#\left\{d_k^B > d\right\} + 1}{K + 1}.$$

For the purpose of performance hypothesis testing using the variance, we follow Ledoit and Wolf (2011) and employ a studentized circular block bootstrap. Accordingly, we compute the p-value for the null hypothesis that the variance difference between a particular portfolio strategy based on our revision policy, Opt, and a benchmark revision frequency, n, is significantly different from zero: $H_0 = log(\hat{\sigma}^2_{Opt}) - log(\hat{\sigma}^2_n) = 0$.[10]

For hypothesis testing with the Sharpe ratio, we use Ledoit and Wolf's (2008) studentized circular block bootstrap. Again, we report the two-sided p-value for the null hypothesis that the Sharpe ratio for a portfolio strategy using our revision policy, Opt, is equal to that of the same portfolio strategy revised according to a fixed revision frequency

[10] The log-transformation follows Ledoit and Wolf (2011) in order to obtain better finite-sample properties.

n: $H_0 = \widehat{SR}_{Opt} - \widehat{SR}_n = 0$. Similar to the inference with the Sharpe ratio, we apply a modified version of Ledoit and Wolf's (2008) studentized circular block bootstrap for hypothesis tests with the certainty equivalent (CEQ) return.[11] Correspondingly, we report the two-sided p-value for the following null hypothesis: $H_0 = \widehat{CEQ}_{Opt} - \widehat{CEQ}_n = 0$. For all bootstraps, we use a block length of $b = 5$ and base our reported p-values on $K = 1,000$ bootstrap iterations.

3 Simulation Study

In this section we use simulated data to evaluate the performance of our portfolio revision policy under clinical conditions, i.e. *i.i.d.* data, where results are not influenced by time series characteristics of asset returns. Our simulation setup follows the approach and parameterization of Tu and Zhou (2011). Specifically, we simulate returns according to a three factor model with mispricing, which takes the following form:

$$r_{i,t} = \alpha_i + \beta_{i,Market} r_{t,Market} + \beta_{i,Size} r_{t,Size} + \beta_{i,B/M} r_{t,B/M} + \epsilon_{i,t} \tag{28}$$

We model the factor returns to follow a multivariate normal distribution with the means and covariance matrix matching the empirical values of monthly returns on the three Fama and French (1993) factors (market, size, and book-to-market) from July 1963 to August 2007.[12] The factor loadings $\beta_{i,Market}$, $\beta_{i,Size}$, and $\beta_{i,B/M}$, as well as the mispricing factor, α_i, are constant over the simulation and randomly paired. The market factor loadings are evenly spread between 0.9 and 1.2, the size factor loadings between -0.3 and 1.4, the book-to-market factor loadings between -0.5 and 0.9, and the mispricing factor between -0.02 and 0.02 in the base case. The variance-covariance matrix of noise is set to be diagonal, with the elements on the main diagonal drawn from a uniform distri-

[11] The adjustments to the studentized block bootstrap by Ledoit and Wolf (2008) are outlined in Appendix A.1.

[12] The choice of the time horizon corresponds to the one chosen by Tu and Zhou (2011).

bution with support $[0.1, 0.3]$. This results in a cross-sectional average annual idiosyncratic volatility of 20%. Using the described simulation setup, we generate $T = 11,000$ monthly returns for an investment universe of $N = 25$ assets and an estimation period of $\tau = \{60, 120, 240, 480, 960\}$.

[INSERT TABLE 2 HERE]

Table 2 shows that our proposed revision policy leads to significantly lower out-of-sample variances than the benchmark revision frequencies of one, six, and twelve months for all portfolio strategies. Furthermore, we observe that the decrease in the out-of-sample variances is highly significant across all estimation periods. While the reductions are sizable for the shorter estimation periods of 60 and 120 months, the reductions become comparatively small for larger estimation periods comprising 240, 480, and 960 months. Yet, we find that the decrease in the out-of-sample variance remains statistically significant even for the largest estimation period of 960 months. From this, we conclude that concentration on the expected out-of-sample variance as a revision criterion helps to achieve a lower realized out-of-sample variance.

[INSERT TABLE 3 HERE]

The risk-adjusted performance, as measured by the Sharpe ratio and the CEQ return, is reported in Tables 3 and 4. Concerning the shorter estimation periods, we mostly do not observe significant differences between our revision policy and the considered benchmark revision frequencies. Exceptions are the SAMPLE and 1F minimum-variance portfolio strategies. We find for both portfolio strategies significant increases in the CEQ returns compared to all benchmark revision frequencies when the estimation period comprises 60 months.

For the estimation period of 240 months, we observe again significantly higher CEQ returns for the SAMPLE, 1F, and LWID strategies in comparison to all benchmark revision frequencies. Additionally, we also note that our revision policy yields significantly higher Sharpe ratios for the 1F portfolio strategy vis-à-vis all considered benchmark revision frequencies. For the estimation periods of sample sizes 480 and 960 months, we observe significant increases in the Sharpe ratio and CEQ return across most portfolio strategies.

[INSERT TABLE 4 HERE]

The average monthly turnovers of the various portfolio strategies are reported in Table 5. The results demonstrate that our proposed revision policy yields a dramatic decrease in turnover compared to the considered benchmark revision frequencies. The observed turnover reductions are statistically significant for all portfolio strategies and estimation period sizes. In particular, we observe for the estimation period comprising 60 months a decrease in turnover between 48% (1F) and 66% (SAMPLE) vis-à-vis the commonly applied monthly revision frequency. Compared to the six and twelve months revision frequencies, we find similar figures, though the reductions become unsurprisingly smaller due to the lower turnover of these benchmarks. Specifically, we observe that the turnover of the portfolios revised according to our policy is reduced between 33% (1F and LWCC) and 49% (SAMPLE) compared to a six months revision frequency. For revision frequencies of twelve months, we observe reductions between 24% (LWCC and Min-C) and 39%. The extension of the estimation period from 60 to 120 months is accompanied by a decrease in the turnover reduction. Correspondingly, we find that our revision policy reduces the turnover between 32% (1F) and 52% (SAMPLE) compared to the monthly revision frequency; between 24% (1F) and 38% (SAMPLE) compared to the six month revision frequency; and between 20% (1F) and 31% (SAMPLE) compared to the twelve months revision. We ascribe the lower turnover reduction of our revision policy in comparison to the results from the shorter estimation period to the generally higher stability of portfolio weights over time when the estimation period is increased. Accordingly, we find that the turnover reduction of our revision policy decreases proportionately with the estimation period length.

For the 960 months estimation period, the decrease in turnover attributable to our revision policy compared to all considered benchmark revision frequencies amounts to roughly 5% across all portfolio strategies. However, we find that, similar to the out-of-sample variances, even these small differences are significant.

[INSERT TABLE 5 HERE]

In summary, we find that our portfolio revision policy reduces the out-of-sample variances as well as the turnover of the considered minimum-variance portfolio strategies significantly. The observed turnover reductions do not come at the cost of a lower risk-adjusted performance as measured by the Sharpe ratio or CEQ return. On the contrary, we even find small increases in the aforementioned performance metrics, especially if the estimation period is large. In the following, we step away from the clinical conditions entertained in our simulation study and evaluate our revision policy on real datasets.

4 Empirical Results

4.1 Data

Table 6 provides an overview of the empirical data sets we consider for the empirical evaluation of our portfolio revision policy. The ten and forty-eight industry portfolios, as well as the six and twenty-five portfolios sorted by size and book-to-market ratios represent different cuts of the U.S. stock market and are available from Kenneth French's website. The aforementioned data sets span from July 1963 to December 2008.

[INSERT TABLE 6 HERE]

Our single stock data set comprises U.S. single stocks and spans from July 1965 to December 2008. The construction of the single stock data set is similar to the construction in Chan et al. (1999), DeMiguel et al. (2009b), and Jagannathan and Ma (2003).

Specifically, we select all NYSE, AMEX, and NASDAQ listed stocks from the CRSP data base that have no missing returns over the entire sample period. From this filtered subset, we select the 100 largest stocks according to their average market capitalization over the sample period.

For the risk free rate, we use the 30-day T-bill rate, which we also obtained from Kenneth French's website.

4.2 Evaluation of the Optimal Revision Policy

Table 7 presents the out-of-sample variances for the analyzed portfolio strategies revised according to our optimal portfolio revision policy (Opt), and the benchmark revision frequencies of one, six, and twelve months. Even though the portfolio variances of the optimal revision policy are in general comparable to those with fixed revision frequencies, we observe several significant deviations for all data sets and portfolio strategies. Regarding these deviations, two general observations can be made:

First, the larger the asset universes the more significant differences occur. For the STOCK data set, the SAMPLE and 1F strategies have significantly lower out-of-sample variances if they are revised according to our revision policy instead of one of the three fixed revision frequencies. We attribute the significant variance reduction in comparison to the fixed revision frequencies to a shrinkage-like effect of our revision policy. Unless alpha takes the value of 1, the revised portfolio weights from our revision policy are always a weighted average of the new and old portfolio weights (which may themselves be a combination of less recent weights). The averaging over the new, unbiased portfolio weights and old, biased[13] portfolio weights introduces a bias on the resulting revised portfolio weights. However, the introduction of the aforementioned bias allows a reduction in estimation variance and hence a lower mean-squared-error of the revised portfolio

[13]Even though the old portfolio weights represent unbiased estimates conditional on recent samples, they represent a biased estimate of optimal portfolio weights conditional on the current sample.

weights.

[INSERT TABLE 7 HERE]

Second, more significant differences in the out-of-sample variance between our revision policy and the fixed revision frequencies are observed for the extended revision frequencies, especially for the annual revision frequency. For most data sets and portfolio strategies, the shorter fixed revision frequencies yield lower portfolio variances compared to longer frequencies. This observation is not surprising since the shorter the revision frequency, the faster are the weights adjusted to changing market dynamics, which have an effect on the optimal weights, e.g. changes in the correlation structure of individual assets or changes in volatility.

[INSERT TABLE 8 HERE]

The Sharpe ratio and the CEQ return are reported in Tables 8 and 9, respectively. The CEQ returns are calculated for a risk aversion coefficient of five. We observe only a few significant performance differences between the optimal revision policy and the fixed revision frequencies. Specifically, we find significantly higher Sharpe ratios on the 6FF data set for LWCC, on the 48IND data set for SAMPLE, and on the STOCK data set for 1F and SAMPLE. Except from the observed deviations, we find that the Sharpe ratios and CEQ returns for the fixed revision frequencies are highly comparable to those of the optimal revision policy.

[INSERT TABLE 9 HERE]

Finally, Table 10 reports the portfolio turnovers. In almost all cases the turnover of our optimal revision policy is significantly lower than the turnover of the corresponding portfolio strategies revised according to a fixed revision frequency. The largest turnover differences are observed when comparing our revision policy to the monthly revision frequency. In particular, the turnover of SAMPLE is reduced by 71.3% for the STOCK data set, while the turnover of the Min-C strategy is reduced by 77.6% for the 25FF data set. However, even when compared to the annual revision frequency, we find considerable reductions in turnover. For instance, the turnover of the 1F and SAMPLE portfolio strategies for the STOCK data set is reduced by 36.3% and 41.7%, repectively. The average reductions according to the portfolio strategies considering all revision frequencies add up to 36.7% (SAMPLE), 27.7% (1F), 27.3% (LW1F), 23.3% (LWID), 28.0% (LWCC), and 38.9% (Min-C). These results indicate that the average reductions in turnover are highly comparable across portfolio strategies, which underpins the robustness of our revision policy in the sense that the turnover reductions are independent of the choice of the portfolio strategy.

[INSERT TABLE 10 HERE]

4.3 Optimal Revision Policy in Presence of Transaction Costs

So far we have discussed the results in the absence of transaction costs. However, transaction costs play a vital role in the practical implementation of portfolio strategies. Accordingly, from a practical perspective it is most interesting to look at the performance of the portfolio strategies after transaction costs. Furthermore, the portfolio performance evaluation in the presence of transaction costs allows us to evaluate the economic value, i.e. the increase in portfolio performance attributable to the turnover reduction of our portfolio revision policy.

Critically, our portfolio revision policy does not take trading costs into account when determining the optimal fraction of wealth to be kept in the old portfolio weights. While our portfolio revision policy could easily be modified to take transaction costs into account by altering the objective function to a maximization of the expected out-of-sample CEQ return minus trading costs, we find that this does not yield a favorable out-of-sample performance. We attribute this to the trade-off between an estimation error prone quantity (the expected out-of-sample performance) and a constant, estimation error free quantity (trading costs).[14]

In order to assess the performance of our proposed optimal revision policy in the presence of transaction costs, we report the differences of the Sharpe ratios and CEQ returns between our optimal revision policy and the fixed revision frequencies for all data sets and portfolio strategies as a function of transaction costs. Specifically, we apply one way transaction costs between 0 and 100 bps. We have chosen this interval for one-way transaction costs because the mid-point of 50 bps results in round-trip transaction costs of 1%. This is in line with recent literature on transaction costs and portfolio optimization.[15] Hence, we regard it as a positive result when our optimal revision policy achieves a better performance compared to fixed revision frequencies with transaction costs of 50 bps or lower. The lower the transaction costs that are required to outperfrom the fixed revision frequencies, the better is the performance of our proposed optimal revision policy.

[INSERT FIGURE 1 HERE]

Figure 1 presents the Sharpe ratio differences for the STOCK data set.[16] All curves of the Sharpe ratio differences have a positive slope due to our proposed revision policy generating consistently lower turnover compared to the fixed revision frequencies. Accordingly, the performance differences increase with higher transaction costs. In almost all cases the curve of the Sharpe ratio differences is above the zero line (where the Sharpe ratio difference is zero). This indicates a higher performance even without transaction

[14]Our observations are in line with DeMiguel et al. (2010), who consider trading costs directly in the objective function of a myopic mean-variance portfolio optimization problem.

[15]See for example DeMiguel et al. (2009b), Kirby and Ostdiek (2011), and Brandt et al. (2009). All aforementioned papers employ proportional one-way transaction costs of 0.5% over a similar time period as the one in this paper. Brandt et al. (2009) further model proportional transaction costs in a more sophisticated way by treating them as a function of the market value and time so as to account for the fact that transaction costs are typically lower for stocks with a large market capitalization and that transaction costs have declined over time. In doing so, they also include the empirical observations of other authors regarding transaction costs (e.g., Keim and Madhavan (1997) and Hasbrouck (2009)). The average proportional one-way transaction costs of the function are again 0.5%, which further supports this number to be an appropriate assumption.

[16]We present the figures for the Sharpe ratio and CEQ return differences only for the STOCK data set, since it represents tradable capital market data. The figures for the remaining data sets are omitted due to space considerations, but are available upon request from the authors.

costs. This is the case for SAMPLE, Min-C and 1F. For LW1F with a revision frequency of twelve months the Sharpe ratio differences curve intersects the zero line before transaction costs of 10 bps, which means that for transaction costs larger than 10 bps our revision policy attains a higher Sharpe ratio. For LWID with revision frequencies of one and six months we observe the intersections at similar low transaction costs. Only for LWID and LWCC with a revision frequency of twelve months do we find higher Sharpe ratios for our revision policy at considerably higher transaction costs of above 90 and 50 bps, respectively.

In addition to these strong empirical results, we also note significant Sharpe ratio differences (at the 10% level) at or below transaction costs of 50 bps for many portfolio strategies and revision frequencies. Significant results are indicated by the intersection of the lower confidence band and the zero line. Not surprisingly, we find the strongest results for the highest revision frequency and portfolio strategies with higher portfolio turnovers. In these cases we observe a steeper slope of the Sharpe ratio differences curves and the confidence bands, as well as an intersection of these curves with the zero line at relatively low transaction costs.

Figure 2 depicts the CEQ return differences for the STOCK data set. Overall, the results are highly comparable to those for the Sharpe ratio differences. All of the curves representing CEQ differences have a positive slope. The slope, again, increases with higher revision frequencies. For SAMPLE, Min-C, 1F, and LW1F, the CEQ return difference curves always lie above the zero line, resulting in larger CEQ returns for our proposed revision policy already at zero transaction costs. The highest transaction costs - where the zero line is intersected - are observed for LWID with a revision frequency of twelve months. For the remaining three cases (LWID and LWCC) where the CEQ difference is negative at zero transaction costs the intersections of the CEQ return differences curve and the zero line are at 40bps or below.

As for the Sharpe ratio differences, the CEQ return differences are in many cases significantly different from zero, with the strongest results achieved for those portfolio strategies and revision frequencies with the largest turnover, i.e. SAMPLE and 1F.

[INSERT FIGURE 2 HERE]

In addition to the evaluation of statistical significance, we also aim to highlight the economic significance of our results. Comparing the differences of CEQ returns between the portfolios revised according to our optimal revision policy and portfolios revised according to a fixed revision frequency, we obtain the maximum management fee an investor would be willing to pay for our revision policy. At transaction costs of 50 bps an investor would be willing to pay an annual management of 1,716 basis points (SAMPLE), 358 basis points (1F), 60 basis points (LW1F), 48 basis points (LWID), 44 basis points (LWCC), and 52 basis points (Min-C). These results show that our proposed revision policy delivers consistent and economically meaningful performance gains when transaction costs are applied.

For the remaining data sets (6FF, 25FF, 10Ind, and 30 Ind), we observe a similar pattern for the Sharpe ratio and CEQ return differences.[17] We find a positive slope for the Sharpe ratio and CEQ return difference curves across all data sets and for all portfolio strategies. Furthermore, in most cases, our proposed revision policy either has a positive Sharpe ratio or the CEQ return differences already at transaction costs of zero, or the difference curves intersect the zero line at reasonable transaction costs of 50 bps or lower. However, with regard to the statistical significance of performance differences we find more statistically significant results at transaction costs of 50 bps or below for the data sets with larger asset universes (25FF, 48IND and STOCK) and find less strong results for the smaller data sets (6FF and 10IND).

If we consider the economic significance across all data sets and portfolio strategies, for transaction costs of 50 bps, an investor with a risk aversion of five would be willing

[17]Due to space considerations, we do not show the results here. However, the results are available from the authors upon request.

to pay an annual management fee of 101 basis points for the optimal revision strategy compared to a fixed annual revision frequency. Compared to the semi-annual and monthly revision frequency, the corresponding management fees amount to 112 and 168 basis points, respectively. Focusing on the considered portfolio strategies, we find that an investor would be willing to pay an annual management fee of 435 basis points (SAMPLE), 120 basis points (1F), 53 basis points (LW1F), 48 basis points (LWID), 93 basis points (LWCC), and 14 basis points (Min-C), respecively. All of the aforementioned numbers can be considered economically meaningful.

5 Conclusion

In this paper, we developed a novel approach to portfolio revision based on the expected out-of-sample performance of the underlying portfolio strategy. Contrary to the commonly applied fixed revision frequencies to portfolio revision, which suggest revision of a portfolio after a predefined time period, our policy determines if and how much of the previous portfolio weights need to be revised. Our approach is thereby purely data driven, requiring no calibration or threshold to trigger the revision of the portfolio. In particular, our revision policy builds on the Jackknife procedure proposed by Basak et al. (2009) to estimate the expected out-of-sample moments of portfolios. Based on the estimate of the expected out-of-sample performance, we derive a closed form solution for the optimal fraction of wealth to be allocated to the new, in-sample optimal portfolio weights.

Using both simulations and empirical data sets, we demonstrate that our proposed revision policy leads to a substantial reduction in portfolio turnover compared to all considered fixed revision frequencies (one, six, and twelve months). Besides a substantially reduced turnover, the results from our simulation study show that portfolios revised according to our revision policy achieve a significantly reduced out-of-sample variance. At the same time, we find that the risk-adjusted performance, as measured by the Sharpe ratio and CEQ return, is equal to or even slightly above that of the portfolios revised according to fixed revision frequencies.

Our empirical results confirm the findings from our simulation study to a great extent. Again, we note that our revision policy yields sizable turnover reductions compared to fixed revision frequencies. In particular, we find that our revision policy reduces the turnover by up to 77.6% for individual portfolio strategies. On average, we observe turnover reductions of 44.9%, 27.4%, and 18.9% compared to fixed revision frequencies of one, six, and twelve months, respectively. At the same time, we find that the out-of-sample portfolio performance in terms of Sharpe ratios or CEQ returns is similar or even slightly better than the performance that is obtained by revising portfolios according to fixed revision frequencies. Accounting for transaction costs, we note that the substantial turnover reductions attributable to our revision policy translate in many cases into economically and statistically significant gains in terms of Sharpe ratios or CEQ returns. On average, an investor with a risk aversion of five would be willing to pay 101 basis points for the optimal revision strategy compared to a fixed annual revision frequency. Compared to the semi-annual and monthly revision frequency, an investor would by willing to pay 112 and 168 basis points, respectively. We observe that the results are especially strong for data sets with larger asset universes, where the turnover is usually higher.

A Appendix

A.1 The studentized circular block bootstrap for hypothesis testing with the certainty equivalent return

The difference between two Sharpe ratios is given by $\Delta_{SR} = f(\mu_1, \mu_2, \sigma_1, \sigma_2) = \frac{\mu_1}{\sigma_1} - \frac{\mu_2}{\sigma_2}$. Using uncentered moments as in Ledoit and Wolf (2008), the difference between two Sharpe ratios may be denoted by $\Delta_{SR} = f(u) = f(a, b, c, d) = \frac{a}{\sqrt{c-a^2}} - \frac{b}{\sqrt{d-b^2}}$. Equivalently, the difference between two certainty equivalent returns is given by:

$$\Delta_{CEQ} = f(v) = f(a, b, c, d, \gamma) = a - \frac{1}{2}\gamma(c - a^2) - b + \frac{1}{2}\gamma(d - b^2) \tag{29}$$

where γ denotes the risk aversion coefficient. Let the sample counterpart of $\Delta_{CEQ} = f(v) = f(a, b, c, d, \gamma)$ be denoted by $\hat{\Delta}_{CEQ} = f(\hat{v}) = f(\hat{a}, \hat{b}, \hat{c}, \hat{d}, \gamma)$. Following equation (4) of Ledoit and Wolf (2008) the standard error of $\hat{\Delta}_{CEQ}$ is defined as:

$$s(\hat{\Delta}_{CEQ}) = \sqrt{\frac{\nabla' f(\hat{v})\hat{\Psi}\nabla f(\hat{v})}{T}} \tag{30}$$

where $\nabla' f(\hat{v})$ denotes the gradient of the certainty equivalent difference based on sample data and $\hat{\Psi}$ is a consistent estimator of Ψ, which is an unknown symmetric positive semi-definite matrix. In particular, the gradient of the certainty equivalent return difference is given by:

$$\nabla' f(\hat{a}, \hat{b}, \hat{c}, \hat{d}, \gamma) = \left(1 + \gamma\hat{a}, -1 - \gamma\hat{b}, -\frac{1}{2}\gamma, \frac{1}{2}\gamma, \frac{1}{2}(\hat{a}^2 - \hat{c} + \hat{d} - \hat{b}^2)\right) \tag{31}$$

whereas $\hat{\boldsymbol{\Psi}}$ is obtained via kernel estimation using the the prewhitened QS kernel of Andrews and Monahan (1992) as proposed by Ledoit and Wolf (2008). In contrast to the gradient of the Sharpe ratio difference (see Ledoit and Wolf (2008) p.852), the gradient of the certainty equivalent difference has five entries since the difference depends not only on the sample moments $\hat{a}, \hat{b}, \hat{c}$ and $\hat{d}$ but also on the risk aversion coefficient γ.

For the inference concerning the sample certainty equivalent return difference, the standard error for each bootstrap sample of the proposed studentized circle block bootstrap by Ledoit and Wolf (2008) is given by:

$$s(\hat{\Delta}^*_{CEQ}) = \sqrt{\frac{\nabla' f(\hat{v}^*)\hat{\Psi}^* \nabla f(\hat{v})^*}{T}} \tag{32}$$

where $s(\hat{\Delta}^*_{CEQ}), f(\hat{v}^*)$ and $\hat{\Psi}^*$ denote the bootstrap sample counterparts of $s(\hat{\Delta}_{CEQ}), f(\hat{v})$ and $\hat{\Psi}$. Following Ledoit and Wolf (2008) p.854, we may then define:

$$y^*_t = \left(r^*_{ti} - \hat{\mu}^*_i, r^*_{tn} - \hat{\mu}^*_n, r^{*2}_{ti} - \hat{\Gamma}^*_i, r^{*2}_{tn} - \hat{\Gamma}^*_n, 0 \right) \;\; t = 1, .., T \tag{33}$$

where the last entry in y^*_t is zero since the risk aversion coefficient is a constant. Having the new y^*_t and letting l denote the necessary number of blocks to construct a bootstrap sample, the definitions for ζ_j and $\hat{\Psi}^*$ are those given by Ledoit and Wolf (2008) p.854:

$$\zeta_j = \frac{1}{\sqrt{b}} \sum_{t=1}^{b} y^*_{(j-1)b+t} \;\; t = 1, .., T \text{ and } \qquad \hat{\Psi}^* = \frac{1}{l} \sum_{j=1}^{l} \zeta_j \zeta j'.$$

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Table 1: List of considered portfolio strategies

The table lists the minimum-variance portfolio strategies we use to evaluate our portfolio revision policy.

#	Description	Abbreviation
1	Minimum-variance portfolio based on the sample covariance matrix	SAMPLE
2	Minimum-variance portfolio based on the sample covariance matrix with short-sale constraints	Min-C
3	Minimum-variance portfolio based on covariance matrix implied by the single-index model	1F
4	Minimum-variance portfolio as a weighted average of the sample covariance matrix and the covariance matrix implied by the single-index model	LW1F
5	Minimum-variance portfolio as a weighted average of the sample covariance matrix and a scalar multiple of the identity matrix	LWID
6	Minimum-variance portfolio as a weighted average between the sample covariance matrix and the covariance matrix implied by the constant correlation model	LWCC

Table 2: Simulation results for variances

The table reports the variances of the various portfolio strategies revised according to the considered revision policies in the simulation study. The considered estimation periods comprise $\tau = \{60, 120, 240, 480, 960\}$ observations. The p-values are computed using the studentized circular block bootstrap by Ledoit and Wolf (2011) for the null hypothesis that the variance of our optimal revision policy, Opt, is equal to the variance of a particular benchmark revision policy, n, i.e. $H_0 : \hat{\sigma}^2_{Opt} - \hat{\sigma}^2_n = 0$. ***,**, and * denote significant differences at the 1, 5, and 10 percent level, respectively.

Portfolio	Revision	Estimation period length (τ)				
Strategy	Policy	60	120	240	480	960
SAMPLE	Opt	0.00267	0.00212	0.00194	0.00185	0.00182
	1M	0.00310***	0.00226***	0.00200***	0.00189***	0.00183***
	6M	0.00311***	0.00226***	0.00200***	0.00189***	0.00183***
	12M	0.00312***	0.00226***	0.00200***	0.00189***	0.00183***
Min-C	Opt	0.00219	0.00209	0.00202	0.00198	0.00196
	1M	0.00224***	0.00211***	0.00203***	0.00199***	0.00196**
	6M	0.00224***	0.00211***	0.00203***	0.00199***	0.00196**
	12M	0.00224***	0.00211***	0.00203***	0.00199***	0.00196**
1F	Opt	0.00215	0.00209	0.00207	0.00204	0.00207
	1M	0.00223***	0.00218***	0.00216***	0.00215***	0.00213***
	6M	0.00224***	0.00218***	0.00216***	0.00215***	0.00213***
	12M	0.00224***	0.00218***	0.00216***	0.00215***	0.00213***
LW1F	Opt	0.00214	0.00201	0.00192	0.00184	0.00182
	1M	0.00220***	0.00206***	0.00195***	0.00188***	0.00183***
	6M	0.00221***	0.00206***	0.00195***	0.00188***	0.00183***
	12M	0.00221***	0.00206***	0.00195***	0.00188***	0.00183***
LWID	Opt	0.00227	0.00205	0.00193	0.00185	0.00182
	1M	0.00237***	0.00213***	0.00197***	0.00189***	0.00183***
	6M	0.00237***	0.00213***	0.00197***	0.00189***	0.00183***
	12M	0.00238***	0.00213***	0.00197***	0.00189***	0.00183***
LWCC	Opt	0.00225	0.00205	0.00194	0.00185	0.00182
	1M	0.00232***	0.00212***	0.00198***	0.00189***	0.00183***
	6M	0.00232***	0.00212***	0.00197***	0.00189***	0.00183***
	12M	0.00232***	0.00212***	0.00197***	0.00189***	0.00183***

Table 3: Simulation results for Sharpe ratios

The table reports the Sharpe ratios of the various portfolio strategies revised according to the considered revision policies in the simulation study. The considered estimation periods comprise $\tau = \{60, 120, 240, 480, 960\}$ observations. The p-values are computed using the studentized circular block bootstrap by Ledoit and Wolf (2008) for the null hypothesis that the Sharpe ratio of our optimal revision policy, *Opt*, is equal to the Sharpe ratio of a particular benchmark revision policy, n, i.e. $H_0 : SR_{Opt} - SR_n = 0$. ***,**, and * denote significant differences at the 1, 5, and 10 percent level, respectively.

Portfolio	Revision	Estimation period length (τ)				
Strategy	Policy	60	120	240	480	960
SAMPLE	Opt	0.0854	0.0865	0.0951	0.0999	0.1002
	1M	0.0800	0.0869	0.0922	0.0964**	0.0984*
	6M	0.0807	0.0867	0.0921	0.0968**	0.0985*
	12M	0.0791	0.0863	0.0918*	0.0965**	0.0985*
Min-C	Opt	0.1190	0.1163	0.1186	0.1219	0.1228
	1M	0.1210	0.1187	0.1197	0.1210	0.1222
	6M	0.1199	0.1183	0.1195	0.1212	0.1222
	12M	0.1182	0.1178	0.1193	0.1209	0.1222
1F	Opt	0.1109	0.1032	0.1066	0.1092	0.1100
	1M	0.1095	0.1038	0.1022**	0.1019***	0.1010***
	6M	0.1090	0.1032	0.1020**	0.1021***	0.1010***
	12M	0.1087	0.1029	0.1015***	0.1018***	0.1010***
LW1F	Opt	0.1091	0.1015	0.1030	0.1035	0.1024
	1M	0.1097	0.1035	0.1019	0.1012*	0.1008*
	6M	0.1094	0.1030	0.1017	0.1016	0.1009
	12M	0.1086	0.1026	0.1011	0.1013*	0.1009
LWID	Opt	0.1057	0.0975	0.0998	0.1020	0.1014
	1M	0.1051	0.0985	0.0981	0.0991**	0.0996*
	6M	0.1055	0.0983	0.0980	0.0995*	0.0998*
	12M	0.1039	0.0979	0.0975	0.0992*	0.0998
LWCC	Opt	0.1181	0.1089	0.1065	0.1056	0.1036
	1M	0.1220	0.1108	0.1061	0.1037	0.1021
	6M	0.1219	0.1102	0.1059	0.1040	0.1022
	12M	0.1204	0.1095	0.1055	0.1037	0.1023

Table 4: Simulation results for certainty equivalent returns

The table reports the certainty equivalent returns of the various portfolio strategies revised according to the considered revision policies in the simulation study. The considered estimation periods comprise $\tau = \{60, 120, 240, 480, 960\}$ observations. The p-values are computed using the studentized circular block bootstrap described in the appendix (A.1) for the null hypothesis that the certainty equivalent (CEQ) return of our optimal revision policy, *Opt*, is equal to the certainty equivalent of a particular benchmark revision policy, n, i.e. $H_0 : CEQ_{Opt} - CEQ_n = 0$. ***,**, and * denote significant differences at the 1, 5, and 10 percent level, respectively.

Portfolio	Revision	Estimation period length (τ)				
Strategy	Policy	60	120	240	480	960
SAMPLE	Opt	-0.0022	-0.0013	-0.0006	-0.0003	-0.0002
	1M	-0.0032***	-0.0015	-0.0008**	-0.0005***	-0.0003**
	6M	-0.0032***	-0.0015	-0.0008**	-0.0005***	-0.0003**
	12M	-0.0033***	-0.0015*	-0.0008***	-0.0005***	-0.0003**
Min-C	Opt	0.0001	0.0001	0.0002	0.0004	0.0005
	1M	0.0001	0.0001	0.0003	0.0004	0.0005
	6M	0.0000	0.0001	0.0003	0.0004	0.0005
	12M	-0.0001	0.0001	0.0002	0.0004	0.0005
1F	Opt	-0.0002	-0.0005	-0.0003	-0.0001	-0.0001
	1M	-0.0004	-0.0006	-0.0006***	-0.0006***	-0.0006***
	6M	-0.0004**	-0.0006	-0.0006***	-0.0006***	-0.0006***
	12M	-0.0004**	-0.0006	-0.0006***	-0.0006***	-0.0006***
LW1F	Opt	-0.0003	-0.0004	-0.0002	-0.0001	-0.0001
	1M	-0.0003	-0.0004	-0.0003	-0.0003**	-0.0002**
	6M	-0.0003	-0.0004	-0.0003	-0.0003**	-0.0002**
	12M	-0.0004	-0.0004	-0.0004*	-0.0003***	-0.0002**
LWID	Opt	-0.0006	-0.0007	-0.0004	-0.0002	-0.0002
	1M	-0.0008	-0.0007	-0.0005*	-0.0004***	-0.0003**
	6M	-0.0007	-0.0007	-0.0005*	-0.0003***	-0.0003**
	12M	-0.0008*	-0.0008	-0.0006**	-0.0004***	-0.0003**
LWCC	Opt	0.0000	-0.0002	-0.0001	-0.0001	-0.0001
	1M	0.0001	-0.0002	-0.0002	-0.0002**	-0.0002**
	6M	0.0001	-0.0002	-0.0002	-0.0002**	-0.0002*
	12M	-0.0001	-0.0002	-0.0002	-0.0002**	-0.0002*

Table 5: Simulation results for turnovers

The table reports the turnovers of the various portfolio strategies revised according to the considered revision policies in the simulation study. The considered estimation periods comprise $\tau = \{60, 120, 240, 480, 960\}$ observations. The p-values are computed using the circular block bootstrap proposed by Politis and Romano (1992) for the null hypothesis that the turnover of our optimal revision policy, Opt, is equal to the turnover of a particular benchmark revision policy, n, i.e. $H_0 : trn_{Opt} - trn_n = 0$. ***,**, and * denote significant differences at the 1, 5, and 10 percent level, respectively.

Portfolio	Revision	Estimation period length (τ)				
Strategy	Policy	60	120	240	480	960
SAMPLE	Opt	0.2111	0.1234	0.0984	0.0886	0.0869
	1M	0.6344***	0.2576***	0.1446***	0.1054***	0.0920***
	6M	0.4153***	0.2002***	0.1302***	0.1028***	0.0917***
	12M	0.3478***	0.1804***	0.1236***	0.1009***	0.0913***
Min-C	Opt	0.0617	0.0473	0.0406	0.0386	0.0381
	1M	0.1504***	0.0869***	0.0562***	0.0441***	0.0399***
	6M	0.0971***	0.0677***	0.0512***	0.0433***	0.0399***
	12M	0.0819***	0.0606***	0.0485***	0.0424***	0.0397***
1F	Opt	0.1202	0.1059	0.1011	0.1012	0.1052
	1M	0.2306***	0.1558***	0.1250***	0.1143***	0.1110***
	6M	0.1799***	0.1409***	0.1216***	0.1138***	0.1110***
	12M	0.1615***	0.1335***	0.1193***	0.1132***	0.1109***
LW1F	Opt	0.1265	0.1026	0.0915	0.0860	0.0859
	1M	0.2613***	0.1692***	0.1222***	0.0994***	0.0901***
	6M	0.1917***	0.1426***	0.1135***	0.0975***	0.0899***
	12M	0.1688***	0.1320***	0.1091***	0.0960***	0.0895***
LWID	Opt	0.1417	0.1077	0.0928	0.0859	0.0856
	1M	0.3293***	0.2015***	0.1313***	0.1013***	0.0903***
	6M	0.2267***	0.1605***	0.1191***	0.0989***	0.0901***
	12M	0.1949***	0.1457***	0.1133***	0.0971***	0.0897***
LWCC	Opt	0.1160	0.0929	0.0851	0.0819	0.0833
	1M	0.2446***	0.1669***	0.1186***	0.0956***	0.0875***
	6M	0.1755***	0.1355***	0.1082***	0.0935***	0.0873***
	12M	0.1537***	0.1238***	0.1031***	0.0919***	0.0869***

Table 6: List of data sets

The table lists the various data sets used for the evaluation of the portfolio performance, their abbreviations, the number of assets that each data set comprises, the time period over which we use data from each particular data set, and the data sources. All data sets comprise monthly data and apply in case of portfolio data the value weighting scheme to the respective constituents. The data sets from Kenneth French are taken from his website `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html` and represent different cuts of the U.S. stock market. The STOCK data set comprises single stocks from the Center for Research in Security Prices (CRSP).

#	Data set	Abbreviation	N	Time period	Source
1	6 Fama and French portfolios of firms sorted by size and book-to-market ratio	6FF	6	07/1963-12/2008	K. French
2	25 Fama and French portfolios of firms sorted by size and book-to-market ratio	25FF	25	07/1963-12/2008	K. French
3	10 industry portfolios representing the U.S. stock market	10IND	10	07/1963-12/2008	K. French
4	48 industry portfolios representing the U.S. stock market	48IND	48	07/1963-12/2008	K. French
5	100 Stocks with the largest average market capitalization	STOCK	100	07/1963-12/2008	CRSP

Table 7: Empirical results for variances

The table reports the variances of the various portfolio strategies revised according to the considered revision policies on the empirical data sets. The p-values are computed using the studentized circular block bootstrap by Ledoit and Wolf (2011) for the null hypothesis that the variance of our optimal revision policy, Opt, is equal to the variance of a particular benchmark revision policy, n, i.e. $H_0 : \hat{\sigma}^2_{Opt} - \hat{\sigma}^2_n = 0$. ***,** and * denote statistical significance at the 1, 5, and 10 percent level, respectively.

Portfolio Strategy	Revision Policy	Data set 6FF	25FF	10IND	48IND	STOCK
SAMPLE	Opt	0.00167	0.00152	0.00138	0.00194	0.00503
	1M	0.00162**	0.00149	0.00136	0.00189	0.00679***
	6M	0.00170	0.00156	0.00138	0.00193	0.00632***
	12M	0.00174*	0.00157	0.00135	0.00203	0.00675***
Min-C	Opt	0.00198	0.00186	0.00138	0.00144	0.00153
	1M	0.00201	0.00185	0.00138	0.00140**	0.00150
	6M	0.00200	0.00187	0.00138	0.00143	0.00152
	12M	0.00201	0.00188	0.00136	0.00143	0.00156
1F	Opt	0.00214	0.00299	0.00144	0.00154	0.00177
	1M	0.00222	0.00315**	0.00142	0.00158	0.00245***
	6M	0.00233**	0.00336**	0.00144	0.00159	0.00253***
	12M	0.00243*	0.00351**	0.00142	0.00161	0.00247***
LW1F	Opt	0.00169	0.00143	0.00136	0.00145	0.00157
	1M	0.00164**	0.00139	0.00134	0.00138**	0.00149*
	6M	0.00172	0.00145	0.00135	0.00141	0.00155
	12M	0.00176*	0.00146*	0.00133**	0.00145	0.00158
LWID	Opt	0.00167	0.00136	0.00133	0.00154	0.00166
	1M	0.00163	0.00133	0.00131*	0.00145***	0.00158*
	6M	0.00168	0.00139	0.00132	0.00148**	0.00165
	12M	0.00171	0.00141**	0.00130***	0.00153	0.00172
LWCC	Opt	0.00179	0.00153	0.00136	0.00148	0.00158
	1M	0.00193***	0.00200***	0.00133	0.00141**	0.00152*
	6M	0.00205***	0.00215***	0.00136	0.00146	0.00158
	12M	0.00214**	0.00224***	0.00135	0.00148	0.00159

Table 8: Empirical results for Sharpe ratios

The table reports the Sharpe ratio of the various portfolio strategies revised according to the considered revision policies on the empirical data sets. The p-values are computed using the studentized circular block bootstrap by Ledoit and Wolf (2008) for the null hypothesis that the Sharpe ratio of our optimal revision policy, *Opt*, is equal to the Sharpe ratio of a particular benchmark revision policy, n, i.e. $H_0 : SR_{Opt} - SR_n = 0$. ***,** and * denote statistical significance at the 1, 5, and 10 percent level, respectively.

Portfolio	Revision	Data set				
Strategy	Policy	6FF	25FF	10IND	48IND	STOCK
SAMPLE	Opt	0.2070	0.2345	0.1393	0.0814	0.0162
	1M	0.2084	0.2438	0.1451	0.0568*	-0.010
	6M	0.2064	0.2378	0.1434	0.0485**	-0.017
	12M	0.2069	0.2496	0.1378	0.0513**	-0.032*
Min-C	Opt	0.1228	0.1214	0.1377	0.1259	0.1043
	1M	0.1202	0.1230	0.1415	0.1347	0.1018
	6M	0.1204	0.1233	0.1412	0.1306	0.0982
	12M	0.1202	0.1275	0.1404	0.1333	0.0991
1F	Opt	0.1080	0.1184	0.1524	0.1179	0.0943
	1M	0.0987	0.1216	0.1651	0.1015	0.0614
	6M	0.0985	0.1216	0.1631	0.1026	0.0608*
	12M	0.1008	0.1198	0.1586	0.1057	0.0623
LW1F	Opt	0.1947	0.2201	0.1422	0.0932	0.0673
	1M	0.1955	0.2300	0.1473	0.0798	0.0629
	6M	0.1931	0.2250	0.1458	0.0809	0.0619
	12M	0.1936	0.2316	0.1398	0.0836	0.0677
LWID	Opt	0.1669	0.2116	0.1388	0.1051	0.0524
	1M	0.1617	0.2107	0.1464	0.0938	0.0507
	6M	0.1587	0.2066	0.1458	0.0945	0.0553
	12M	0.1603	0.2095	0.1410	0.0961	0.0633
LWCC	Opt	0.1413	0.1926	0.1362	0.0984	0.0519
	1M	0.1200**	0.1672	0.1400	0.0895	0.0458
	6M	0.1208*	0.1716	0.1423	0.0900	0.0455
	12M	0.1235	0.1752	0.1356	0.0940	0.0545

Table 9: Empirical results for certainty equivalent returns

The table reports the certainty equivalent returns of the various portfolio strategies revised according to the considered revision policies on the empirical data sets. The p-values are computed using the studentized circular block bootstrap described in the appendix A.1 for the null hypothesis that the certainty equivalent (CEQ) return of our optimal revision policy, Opt, is equal to the certainty equivalent of a particular benchmark revision policy, n, i.e. $H_0 : CEQ_{Opt} - CEQ_n = 0$. ***,**, and * denote significant differences at the 1, 5, and 10 percent level, respectively.

Portfolio	Revision	Data set				
Strategy	Policy	6FF	25FF	10IND	48IND	STOCK
SAMPLE	Opt	0.0042	0.0053	0.0017	-0.0012	-0.0114
	1M	0.0043	0.0056	0.0019	-0.0022	-0.0178**
	6M	0.0042	0.0054	0.0018	-0.0027**	-0.0172**
	12M	0.0042	0.0059	0.0016	-0.0027**	-0.0196***
Min-C	Opt	0.0005	0.0005	0.0016	0.0011	0.0002
	1M	0.0003	0.0006	0.0018	0.0015	0.0001
	6M	0.0003	0.0006	0.0017	0.0013	0.0000
	12M	0.0003	0.0008	0.0017	0.0014	0.0000
1F	Opt	-0.0003	-0.0010	0.0021	0.0007	-0.0004
	1M	-0.0009	-0.0010	0.0026	0.0001	-0.0030**
	6M	-0.0010	-0.0013	0.0025	0.0001	-0.0032**
	12M	-0.0011	-0.0016	0.0024	0.0002	-0.0030**
LW1F	Opt	0.0037	0.0047	0.0018	0.0000	-0.0012
	1M	0.0038	0.0051	0.0020	-0.0004	-0.0012
	6M	0.0037	0.0049	0.0019	-0.0004	-0.0014
	12M	0.0037	0.0052	0.0017	-0.0004	-0.0012
LWID	Opt	0.0026	0.0044	0.0017	0.0002	-0.0020
	1M	0.0024	0.0043	0.0020	-0.0001	-0.0019
	6M	0.0023	0.0042	0.0019	-0.0001	-0.0018
	12M	0.0023	0.0043	0.0018	-0.0001	-0.0016
LWCC	Opt	0.0015	0.0037	0.0016	-0.0000	-0.0018
	1M	0.0004**	0.0024	0.0017	-0.0001	-0.0020
	6M	0.0003**	0.0025	0.0018	-0.0002	-0.0021
	12M	0.0003	0.0026	0.0016	-0.0001	-0.0018

Table 10: Empirical results for turnovers

The table reports the turnover of the various portfolio strategies revised according to the considered revision policies on the empirical data sets. The p-values are computed using the circular block bootstrap proposed by Politis and Romano (1992) for the null hypothesis that the turnover of our optimal revision policy, *Opt*, is equal to the turnover of a particular benchmark revision policy, n, i.e. $H_0 : trn_{Opt} - trn_n = 0$. ***,**, and * denote significant differences at the 1, 5, and 10 percent level, respectively.

Portfolio	Revision	Data set				
Strategy	Policy	6FF	25FF	10IND	48IND	STOCK
SAMPLE	Opt	0.1408	0.3539	0.0881	0.3359	0.9925
	1M	0.2215***	0.7868***	0.1604***	0.8066***	3.4585***
	6M	0.1828***	0.5518***	0.1207***	0.5389***	2.0778***
	12M	0.1715***	0.4764***	0.1023**	0.4624***	1.7010***
Min-C	Opt	0.0143	0.0226	0.0283	0.0434	0.0641
	1M	0.0290**	0.1007***	0.0773***	0.1172***	0.1598***
	6M	0.0206***	0.0416***	0.0397***	0.0558***	0.0892***
	12M	0.0201**	0.0334***	0.0365***	0.0472	0.0755*
1F	Opt	0.0844	0.1470	0.0680	0.1352	0.1158
	1M	0.1218***	0.2379***	0.0997***	0.1874***	0.1730***
	6M	0.1168***	0.2328***	0.0885***	0.1733***	0.1624***
	12M	0.1155***	0.2309***	0.0821***	0.1655***	0.1539***
LW1F	Opt	0.1262	0.2850	0.0834	0.1968	0.2174
	1M	0.1990***	0.5600***	0.1416***	0.3643***	0.3615***
	6M	0.1638***	0.4115***	0.1096***	0.2694***	0.2781***
	12M	0.1535***	0.3613***	0.0934*	0.2391***	0.2504**
LWID	Opt	0.0689	0.1985	0.0670	0.2133	0.2640
	1M	0.0899***	0.3138***	0.1097***	0.3957***	0.4857***
	6M	0.0804**	0.2430***	0.0869***	0.2848***	0.3586***
	12M	0.0757	0.2178	0.0744	0.2517**	0.3203***
LWCC	Opt	0.0635	0.1618	0.0627	0.1851	0.1885
	1M	0.1067***	0.2895***	0.1005***	0.3292***	0.2872***
	6M	0.0969***	0.2483***	0.0815***	0.2461***	0.2313***
	12M	0.0925***	0.2313***	0.0701	0.2184**	0.2105*

Figure 1: Sharpe ratio differences for the STOCK data set

The eighteen panels in this figure show the Sharpe ratio differences (Δ SR) between our proposed revision policy and fixed revision frequencies of one, six, and twelve months as a function of proportional one-way transaction costs in basis points (bps). The Sharpe ratio differences are represented by dashed lines while dotted lines represent the 90% confidence interval. For the confidence interval, we use bootstrapped standard errors, which we obtain from the studentized circular block bootstrap from Ledoit and Wolf (2008).

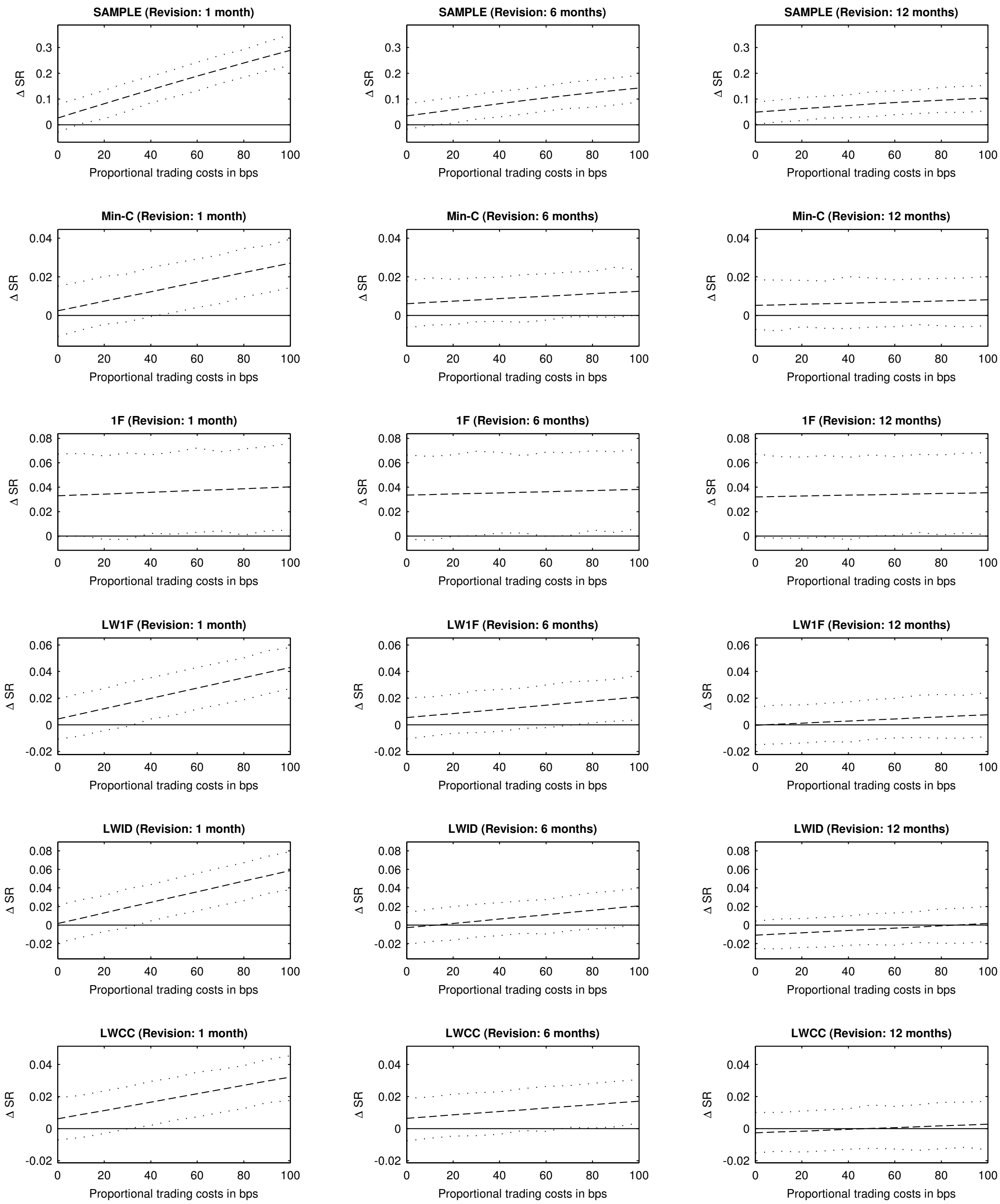

Figure 2: Certainty equivalent return differences for the STOCK data set

The eighteen panels in this figure show the Sharpe ratio differences (Δ SR) between our proposed revision policy and fixed revision frequencies of one, six, and twelve months as a function of proportional one-way transaction costs in basis points (bps). The Sharpe ratio differences are represented by dashed lines while dotted lines represent the 90% confidence interval. For the confidence interval, we use bootstrapped standard errors, which we obtain from the studentized circular block bootstrap described in the appendix (A.1).