On the role of behavioral finance in the pricing of financial derivatives: the case of the S&P 500 options

ABSTRACT

The object of this study was to investigate some implications of the tenets of behavioral finance on the pricing of financial derivatives. In particular, based on the work by Wolff et al (2009) we have investigated how prospect theory (Kahneman and Tversky, 1979) can be intregrated into the Black and Scholes (1973) option pricing framework. We have then used the resulting "behavioral version" of the Black-Scholes equation to price market quoted options. As an empirical test we have calibrated three-month market-quoted call options on the Standard & Poor's 500 index (SPX) at the Chicago Board of Options Exchange (CBOE) during the period January to December 2007. As a comparison, we have also calibrated the Heston (1993) stochastic volatility option pricing model to the same contracts. Our results show that during the period of study the market option prices are captured better by the behavioral version of the Black-Scholes equation than by the Heston stochastic volatility model. Further work is required to investigate if this is the case for other option types and under different market conditions.

Keywords: Black-Scholes Equation, Prospect Theory, Heston Stochastic Volatility Model, S&P500 Index, Option Pricing.

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1. INTRODUCTION

Option pricing is one of the most studied and fascinating topics in finance. The Black and Scholes (BS) formula, derived in 1973, it is still the benchmark of plain vanilla European option pricing for traders, market makers, sales people that use and price options. Even if widely used the BS formula is based on the existence of a set of "ideal conditions" in the market: short term interest rates are known and constant through time; the distribution of possible stock price is lognormal; the variance is constant; there are no transaction costs in buying and selling stocks and options; it is possible to borrow any fraction of the price of a security at the short term interest rate; there are no limitations on short selling. Furthermore in the framework of traditional finance investors are assumed to correctly update their beliefs when they receive new information and, conditional on those beliefs, make choices that maximize their expected utility.

BS is widely used because it is very easy to implement, it provides a closed solution to option pricing and it also allows risk coefficients and sensitivities calculation.

However the "ideal conditions" of the model are far away from reality. The observed returns exhibit fatter tails and higher peaks that what should be consistent with BS hypothesis of normally distributed returns. The volatility is not constant through time and strikes and is often negatively correlated with stock price level. Empirical evidence has also proved the existence of a substantial numbers of anomalies with respect to the expected utility theory.

In these circumstances, it was natural that the finance literature started to re-explore option pricing trying to develop new models, able to give a more accurate description of the reality, and in the same time to keep the simplicity of Black and Scholes Model. Different approaches have been proposed. A first alternative to BS is the constant elasticity of variance model (CEV) proposed by Cox and Ross (1976). This model allow to incorporate the idea that volatility may increase as stock prices decreases (distribution with heavy fat tails as observed empirically). The Jarrow and Rudd (1982), Corrado and Su (1996) models adjust BS model taking into consideration skewness and Kurtosis in underlying returns. Jump diffusion models (Merton 1976) incorporate the fact the observed prices do not move continuously but they jump from time to time. Discrete-time Garch option pricing models (Heynen et al. (1994), Duan (1996) Heston and Nandi (2000), Duan (2001)) that incorporate heteroskedasticity and negative correlation between volatility an spot returns have been proved to be successfully on explaining option prices. Finally stochastic volatility models (Hull and White 1988) introduce the idea of not constant observed volatility. In these models the volatility itself is described as a stochastic process. One of the most widely used stochastic volatility model is today the Henston (1993) Model. In this model not only the stock prices but the volatility as well follow a geometric Brownian Motion (BM). The two BM, the one of the stock and the one of the volatility, are correlated and the volatility is assumed to be mean reverting to a long term level. The Heston model is consistent with different return distributions and it allows to incorporate skewness and Kurtosis. The model is also computationally convenient since it provide a closed form solution for European Options. In general the Heston model is the most famous and widespread of the models that try to improve Black and Scholes relaxing the assumptions upon which it is based. However the rationality assumption is not proved by market observation and experiments.

In recent years another avenue of research to improve BS has been explored: incorporating the idea of non rational and non risk neutral investors. The BS derivation is built on the possibility to construct a risk-free replicating portfolio. If this is not true, then a possible explanation to the deviation of observed option prices respect to what implied BS could be related to the behavioral finance aspects.

With the pioneer works of two psychologists Kahneman and Tversky (1979, 1992), finance literature started to reconsider the role of attitudes, emotions and in general behavioral biases in investors' decisions and actions. Kahneman and Tversky Prospect Theory draws a framework of investors' preferences deviating significantly from those postulated into the Von Neumann and Morgenstern (1947) Expected Utility Theory. Four main features of prospect theory appear relevant for behavioral-based asset pricing model. Firstly, investors are risk averse in the domain of gains but they tend to transform their attitude into risk seeking while facing losses. This relates to a second condition, that value is assigned to losses and gains rather than final wealth. There is no aggregation of positions into total wealth. To state it differently, investors' value function (prospect theory correspondent to utility function) is defined on deviations from a reference point and is normally concave for gains (implying risk aversion) while commonly convex for losses (risk seeking). A third relevant aspect refers to a further feature of the value function named loss aversion: the function is steeper for loss than for gains. In other words losses are looming higher than gains. Finally, prospect theory suggests that individuals' subjective perception of probability, so called decision weights, differs from objective ones. Decision weights are generally lower than the corresponding probabilities, except in the range of low probabilities. Behavioral literature is also providing evidences on the way agents are presenting and interpreting problems. According to Kahneman and Tversky (1979), agents' preferences are influenced by the way prospects are presented (framing). Since, decisions process consists of an editing stage, when prospects are coded and categorized and complex problems are broken down into simpler sub problems, and an evaluation stage, when prospects with the highest value is chosen. As the editing can lead to different representations, the decision can change accordingly. Framing is at the basis of mental accounting, being the way a problem is subjectively interpreted. According to theory (Thaler, 1980) people group their assets into a number of nun-fungible metal accounts and these accounts are influenced by the way relevant information is displayed. Considering options, one might view naked options versus options in combination with its underlying asset: segregating in terms of entities; or one might aggregate or segregate current cash flows versus future cash flaw: segregating in terms of time.

Most of the behavioral finance literature has been focused on stock markets and the empirical evidence on behavioral stock prices anomalies is huge. Behavioral finance has been applied to derivatives pricing in a lesser degree.

To the best of our knowledge, the first "behavioral" option pricing model is Shefrin and Statman's (1993). Concentrating on the features of value function, they compare a binomial value function modified option price of covered calls with prices based upon the CRR model (Cox, Ross, Rubenstein, 1979). Precisely, they conclude that writers of such options are influenced by their value function. The prospect theory expected value of the covered call position exceeds the prospect theory expected value of the stock-only position for investors who are sufficiently risk-averse in the domain of gains. Breuer and Perst (2004) extend the analysis by including the weighting function to a prospect theory model of discount reverse convertible (DRCs), a combination of a risk free asset with put options writing. By comparing behavioral pricing with Black Scholes in a multi period continuous time setting they conclude that investments in risk free assets are preferred in low drift stock markets; while stocks are chosen in high drift markets and finally DCRs are preferred in a medium drift markets. Several other researchers have been drawing their attentions to the possible behavioral explanations of peculiarities in index-options prices, precisely to the steepness of the volatility smiles. Raisel (2003), Hodges, Tompkins and Ziemba (2003) and Gemmill and Schackleton (2005) they all account for option-pricing anomalies through the use of prospect theory. Mental accounting is used by Abbink and

Rockenbach (2006) to explain difference in pricing behaviour between students and professional traders.

In a very recent study Wolff et al (2009) studied option prices in an economy where investors are valuing call options according to the cumulative prospect theory of Kahneman and Tversky. They considered two prospect option pricing models, based on whether cash flows are either considered to be segregated or aggregated over time. They compared these models with the Black-Scholes model and the stochastic volatility model of Heston on an empirical analysis of European call options on the S&P 500 index. Their results show that prospect option pricing models significantly improve the fitting performance, as compared with the Black-Scholes model and that especially the aggregated version's performance is at least equivalent to the Heston model.

In this paper we investigate the performance of a BS behavioral modified model and of the Heston model in terms of fitting actual option prices. Our paper differ from previous literature on various ways. First, in contrast to the above-mentioned work of Wolff et al (2009), we concentrate on the year 2007 and strictly on options with three-months to maturity. Second, in agreement with Wolff et al (2009) we use a behavioral, prospect theory modified Black-Sholes option pricing model. Third, our aim is not to show that option prices incorporate behavioral aspect as in previous literature but to analyze if behavioral BS model perform better than Heston model on pricing. Finally, we intend to show that a fast and easy-to-compute behavioral BS option pricing formula can be used successfully to describe actually-traded option contracts.

In the following sections we describe the metyhodology and data used, our results and conclude with a brief discussion of our findings.

2.
METHODS

In our paper we compare option pricing obtained through Heston model and BS behavioral model to verify wich of the two perform better in terms of fitting real market data. To reach this objective we apply Heston model, we develop and apply a BS behavioral model and apply it and finally we build an error function to calculate the pricing performances. In this section we introduce both models, the market data used and the calibration procedure employed.

2.1
Market Data

We have collected daily option prices for calls and puts on the S&P 500 index from the Chicago Board Options Exchange (CBOE) during the period January to December 2007. We used the Market Data Express service (www.marketdataexpress.com). To reduce the amount of data to process, we concentrated only on call contracts with a expiry date of approximately three months (range 90 to 100 days included). We further eliminated the data from contracts with a negligible daily trading volume (less than 10 contracts) and those which were far-from-the-money, above 2000 and below 600. The above choices brought the total number of data points to 1502 contracts. In the resultant sample the range of strikes varied between 600 and 1900, while the spot price varied roughly between 1400 and 1600. Figure 1 shows a graphical depiction of the data used. The red line indicates the spot value of the index for each day. The blue dots indicate the call contracts for the various strikes for each day.

Apart from the option prices, the daily yield curves were downloaded and processed from the British Bankers Association (www.bbalibor.com). Figure 2 shows the USD LIBOR 3M in percent for the period of the study.

2.2
Prospect Theory

In our empirical analysis we also propose a behavioral, prospect theory modified Black-Sholes option pricing model based on Wolff et al (2007). The value of a prospect depends on both a value function (v), defining the value of an outcome (x) and a weighting function (w), which insists on the probability of that same outcome (q). In a prospect with non negative outcome (x_1, q_1) and a negative one (x_2, q_2), we get:

$$V = w^+(q_1)v^+(x_1) + w^-(q_2)v^-(x_2) \quad [1]$$

Both value function and weighting function can be modelled in different ways, see Fox and Poldrack (2009) for an extensive review, but we decide to maintain the original formulations by Kahneman and Tversky (1982) being respectively:

$$v(x) = \begin{cases} x^a, x \geq 0 \\ -\lambda(-x^b), x < 0 \end{cases} \quad [2]$$

$$w(q) = \frac{q^\gamma}{\left[q^\gamma + (1-q^\gamma)\right]^{/\gamma}} \quad [3]$$

Parameters a, b, λ, in the value functions are respectively constants determining the curvature of the function in the domains of gains and losses and the degree of loss aversion in the case of losses. In the weighting function, γ controls for perceived over and underweighting of small and large probabilities. More precisely in Kahneman and Tversky cumulative prospect theory γ assumes different values for losses and gains prospects, but we decided to mantain it invariant in our simulation as in Prelec (1998).

As for the option, we assume that the marginal investor prices it according to the valuation of the prospect theory. In addition we assume that the marginal investor is “affected” by both entity and time segregated mental accounting. When a marginal investor is writing a European style call option on a stock index she will face two possible state at the time of expiration (t=T). First, if the price of the underlying asset is higher than the exercise price

(S_T>K), the option will be exercised. The probability of being exercised (q) is given by:

$$q = \int_K^{\infty} f(S_t)dS_t \qquad [4]$$

with S_T being the price of the underlying asset at expiration, K being the exercise price and f(S_T) being the probability density function of S_T. The expected value (conditional on exercising the option), denoted by x, is then equal to:

$$x = -\int_k^{\infty} \frac{(S_T - K)f(S_T)dS_t}{} / q \qquad [5]$$

However, in a second state, when the price of the underlying asset is lower than the exercise price (S_T ≤K), the option will not be exercised. In such a case the probability that the option is not exercised is equal to (1-q) and its pay-off is zero.

Combining the two state and keeping in mind that when the option is exercised, from the writer’s point of view the expected value of the option is then a loss, the value function presented in Equation 1 can be simplified as follows:

$$V = w^-(q)v^-(x) \qquad [6]$$

However, equation [6] does not take into account that in addition to the potential loss at t=T, the writer of the option receives at t=0 a premium c. Assuming to invest *c* at a risk free rate *(r_f)* at t=T its value is equal to *cexp(r_f T)*. In equilibrium, the prospect value of *c* should equal the prospect value of *x*

$$v^+(c\exp(r_f T)) + w^-(q)v^-(x) = 0 \qquad [7]$$

By substituting [2] into [7] we achieve the following option value:

$$c = \exp(-r_f T)\left(w^-(q)\lambda(-x)^b\right)^{/a} \qquad [8]$$

Assuming a geometric Brownian price process with drift α and volatility σ, the future density function of the price of the underlying asset is

$$f(S_T) = \exp(-\left[\ln(S_T / S_0) - (\alpha - \sigma^2 / 2)T\right]^2 / 2\sigma^2 T) / S_T \sigma \sqrt{2\pi T} \qquad [9]$$

With the equation [4], [5], and [9], q and x can be determined

$$q = \Phi(\delta_{-1}) \qquad [10]$$

And

$$x = K - S_0 \exp(\alpha T)\Phi(\delta_1) / \Phi(\delta_{-1}) \qquad [11]$$

with S_0 being the current price of the underlying asset and $\Phi(\delta_m)$ the cumulative standard normal distribution of δ_m, defined by

$$\delta_m = (\ln(S_o / K) + (\alpha + m\sigma^2 / 2)T) / \sigma\sqrt{T} \qquad [12]$$

Substituting the equation [10] and [11] into equation [8] gives the value of the call option from the writer's point of view

$$Call^{PROSPECT}(K, T; a, b, \lambda, \gamma, \sigma) = \exp(-r_f T)(\lambda w^{-}\left[\Phi(\delta_{-1})\right]\left[S_0 \exp(\alpha T)\Phi(\delta_1) / \Phi(\delta_{-1}) - K\right]^{b})^{1/a} \qquad [13]$$

2.3
Heston Stochastic Volatility Model

Heston (1993) developed a model in which stock prices and volatility follow a geometric Brownian motion with negative correlation. More specifically:

$$dS_t = \mu S_t dt + \sqrt{V_t} S_t W_t^1 \quad [14]$$

$$dV_t = k(\vartheta - V_t)dt + \sigma \sqrt{V_t} S_t W_t^2 \quad [15]$$

$$dW_t^1 W_t^2 = \rho dt \quad [16]$$

Where S_t is the underlying process, V_t is the volatility process, W_t^1 and W_t^2 are the Brownian motion for stocks and volatility respectively and ρ is their correlation coefficient. ϑ is the long run mean of volatility and k is the speed or rate of reversion of the volatility to the long run mean. σ is the volatility of the volatility and V_t is a square root mean reverting process.

In BS the randomness in the option value is due to the randomness of the underlying. If the underlying (as in the case of stocks) is tradable, the option can be hedged by continuously trading the underlying. This makes the market complete.

In Heston model option value depends on the randomness of the underlying and of the volatility of the asset's return. Only the asset is tradable. Volatility is not traded. This makes the market incomplete with lot of implications in term of pricing. Heston model is consistent with different returns distribution hypothesis. Instead if ρ >0 then the volatility will increase when asset prices increase. This will lead to fat right tail. Viceversa if ρ <0 then volatility will increase when prices decrease creating a fat left-tail distribution. σ affect the kurtosis. When σ =0 the volatility is deterministic and log returns will be normally distributed. When σ increases the kurtosis will increase creating fat tails on both sides of the distribution.

According to Heston the price of a European call option is

$$Call^{HESTON}(K,T;\theta,\kappa,\xi,\rho,V_0) = S_0 P_1 - K \exp\left(-r\tau\right) P_2 \quad [17]$$

with S_0 being the current price of the underlying asset, K being the strike price, r being the risk-free interate rate and τ being the time till expiration. P_1 and P_2 are 'probabilities' determined by

$$P_j = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \mathrm{Re}\left[\frac{f_j \exp\left(-i\phi Log(K)\right)}{i\phi} \right] d\phi \quad [18]$$

With

$$f_j = \exp\left(C_j + D_j V_0 + i\phi Log(S)\right) \quad [19]$$

And

$$C_j = ri\phi\tau + \frac{\kappa\theta}{\xi^2}\left\{\left(b_j - \rho\xi i\phi + d_j\right)\tau - 2Log\left[\frac{1-g_j\exp\left(d_j\tau\right)}{1-g_j}\right]\right\} \quad ([20]$$

$$D_j = \frac{b_j - \rho\xi i\phi + d_j}{\xi^2}\left[\frac{1-\exp\left(d_j\tau\right)}{1-g_j\exp\left(d_j\tau\right)}\right] \quad ([21]$$

$$g_j = \frac{b_j - \rho\xi i\phi + d_j}{b_j - \rho\xi i\phi - d_j} \quad ([22]$$

$$d_j = \sqrt{\left(\rho\xi i\phi - b_j\right)^2 - \xi^2\left(2u_j i\phi - \phi^2\right)} \quad ([23]$$

$$b_1 = \kappa + \lambda - \rho\xi,\ b_2 = \kappa + \lambda \quad ([24]$$

$$u_1 = 0.5,\ u_2 = -0.5 \quad ([25]$$

2.4
Calibration

For each day in the analysis period (113 total), equations [13] and [17]] were calibrated to the market-quoted options prices for each combination of *K* and *T* available. The parameter vector Ξ to calibrate for the Heston model was:

$$\Xi^{HESTON} = \left\{\theta, \kappa, \xi, \rho, V_0\right\}$$

And for the BS Prospect model was:

$$\Xi^{PROSPECT} = \left\{a, b, \lambda, \gamma, \sigma\right\}$$

The constrained Levenberg Marquard method in Mathematica 7.0 was used, assuming constant values for the calibrated parameters. The minimization problem solved for each day in the period was then:

$$\min_{\Xi}\left(\left\|\vec{C}^{MODEL}(K,T) - \vec{C}^{MARKET}(K,T)\right\|_2\right)$$

Following calibration, as a measurement of the quality of the calibrations, the norm of the vector of price differences between market and model was also computed for each day in the analysis period. Thus:

$$L_2^{HESTON} = \left\| \vec{C}^{HESTON}(K,T) - \vec{C}^{MARKET}(K,T) \right\|_2$$

$$L_2^{PROSPECT} = \left\| \vec{C}^{PROSPECT}(K,T) - \vec{C}^{MARKET}(K,T) \right\|_2$$

Where L_2^{HESTON} is the scalar which represents the norm of the price differences in the case of Heston. Where $L_2^{PROSPECT}$ is the scalar which represents the norm of the price differences in the case of Prospect Theory. $\| \|_2$ represents the Euclidean norm.

Where $\vec{C}^{PROSPECT}(K,T)$ is the vector of option prices as calculated with the Prospect Theory pricing model, $\vec{C}^{HESTON}(K,T)$ is the vector of option prices as calculated with the Heston Stochastic Volatility model pricing model, $\vec{C}^{MARKET}(K,T)$ is the vector of option prices as quoted in the market. Note that these three variables are function of strike and maturity, K and T, respectively.

3. RESULTS

Both the Prospect Theory and the Heston Stochastic Volatimilty Model were implemented in the scientific software Wolfram Mathematica 7. Each was separately calibrated to the market data for each day of the period of analysis (Jan-Dec 2007), so as to obtain a best fit between the mathematical model and the market data.

Figure 3 shows an example of the market data and the calibrated of the models for the 25th January 2007. The x-axis represents strike, while y-axis represents option price. The black dots represent individual market-quoted call options on the S&P500 index for that day. All contracts have a maturity of 86 days. The USD LIBOR3M was 5.36%, while the spot value of the index was 1423.90. The red line represents the calibrated Heston model, the green line the calibrated Prospect Theory Black Scholes and the blue line represents the classic Black Scholes equation as reference. On that particular day the error norms were 12.3023 for Prospect and 14.7980 for Heston. As can be seen all models can be calibrated to closely follow the market data.

Figure 5 shows the calibrated parameters for the Prospect Theory Black Scholes model. Each day the model is calibrated to market data and these parameters calculated. The x-axis represents dates in 2007. The graphs correspond from top to bottom, to parameters: σ, λ and γ. The *a* and *b* parameters remained in the range [0.5, 0.9]. The γ parameter remained in the range [0.8, 1.0]. The λ parameter remained in the range [1.0, 1.86]. The σ parameter remained in the range [0.0670, 0.41]. The mean and standard deviation of the results are shown in Table 1. All the Prospect Theory parameters σ, λ and γ are consistent with the literature

Figure 6 shows the calibrated parameters for the Heston Stochastic Volatility model. Each day the model is calibrated to market data and these parameters calculated. The x-axis represents dates in 2007. From top to bottom: initial volatility (V0), mean reversion speed parameter (κ), mean reversion level (θ), and the volatility of volatility (ξ). The V_0 parameter remained in the range [0.0335, 0.1944]. The κ parameter remained in the range [0.5000, 1.0000]. The θ parameter remained in the range [0.0700, 0.6150]. The ξ parameter remained in the range [2.5, 3.5]. The ρ parameter remained in the range [-0.9500, -0.8500]. The mean and standard deviation of the results are shown in Table 2.

Figure 4 shows the calibration quality of the Prospect Theory Black Scholes and the Heston stochastic volatility model . The quality is measured in terms of the Euclidean norm of the error vector (i.e. the difference between model price and market price) after calibration, for each day in the period of study. From top to bottom: the norm of the error for Prospect Theory $L_2^{PROSPECT}$, the norm of the error for the Heston Model L_2^{HESTON} and the difference between the norm of the error for Prospect Theory and the norm of the error fo the Heston Model ($L_2^{HESTON} - L_2^{PROSPECT}$). The mean of the differences was 17.8182, with a standard deviation of 15.0826. With a maximum difference of +70.1449 and a minimum of -25.7982.

Figure 7 shows indicates which model is better in the sense of having a smaller error norm for each day in the study. The x-axis represents dates in 2007. In the top graph a value of 1 indicates the days in which the Prospect Theory Black Scholes model was better than the Heston Stochastic Volatility model. In the bottom graph the value of 1 indicates the days in which the Heston Stochastic Volatility model was better than the Prospect Theory Black Scholes model. From a total of 113 business days in the study, Prospect Theory Black Scholes was better 103 days, while the Heston Stochastic Volatility model was better 10 days. This represents a proportion of 91% and is illustrated in Figure 8.

Finally, a statistical test was conducted in order to confirm the significance between the difference in the quality of the calibrations, as measured in terms of the means in the error norms. A one-way ANOVA test performed in MATLAB, which resulted in an F-value of 40.48 and p-value of 1.1038×10^{-9}, which confirm the hypothesis that the means are significantly different. Figure 9 shows this graphically, there Heston Stochastic Volatility model (column 1) and Prospect Theory Black Scholes (column 2).

4. CONCLUSIONS

In this contribution we have investigated how a modified version of the Black Scholes equation based on Prospect Theory is able to describe market-quoted option prices. Our results show that this "Behavioral Black Scholes" formula captures very well three-month call options on the S&P 500 index. Even when comparing with an advanced model, such as the Heston Stochastic Volatility model, the Prospect Theory formula shows a better performance – measured in terms of the difference between model and market data – at least for the type of contracts (three-month to maturity, not far-from-the-money, calls) and the period investigated (2006). In addition, the Prospect Theory Black Scholes formula is easier to compute and calibrate, as it does not involve the complex integral (equation [18]) known to present numerical difficulties in the literature.

The type of investigation that we have conducted, as is in the nature of empirical research, has a number of limitations. First, we have used constant parameters. The use of time-dependent parameters in the Heston model would certainly improve its performance, but at the expense of a greater mathematical complexity in its computation. Second, we have only investigated three-month call options. Further work is required to investigate put options, other maturities and very out-of-the-money prices. Third, we have analyzed a single year. It would be advisable to study other periods including the financial crisis of 2008/2009. Fourth, we have relied on the in-built calibration methodology available in Mathematica. It is possible that more advanced calibration methods (hybrid, local-global, genetic algorithms) would lead different results. Further work on these issues is under way.

In conclusion, our empirical analysis suggests that a behavioral-based BS model can perform better than Heston model on pricing. This alternative fast and easy-to-compute behavioral BS option pricing formula can be used successfully to describe actually-traded option contracts.

The authors would like to thank Dr Michelle Baddeley, Department of Economics, University of Cambridge, UK, for her important contributions during the preparation of this paper.

TABLES

Table 1

Calibration results for Prospect Theory

Parameter	Mean ± Std dev
a	0.7929 ± 0.0495
b	0.7929 ± 0.0495
γ	0.8035 ± 0.0265
λ	1.1758 ± 0.2008
σ	0.1504 ± 0.0574

Table 2

Calibration results for the Heston Stochastic Volatility model

Parameter	Mean ± Std dev
κ	0.5133 ± 0.0807
V_0	0.0800 ± 0.0358
θ	0.2334 ± 0.1042
ξ	3.0012 ± 0.0421
ρ	–0.9004 ± 0.0082

FIGURES

Figure 1

Market data

Market data used for the study. The x-axis represents dates in 2007. The y-axis strike. The blue dots represent individual European call option contracts, each with a maturity of approximately three months. There are 1502 contracts (dots) in total. The red line indicates the spot value of the SPX index as a function of time. Data source: Chicago Board Options Exchange (CBOE).

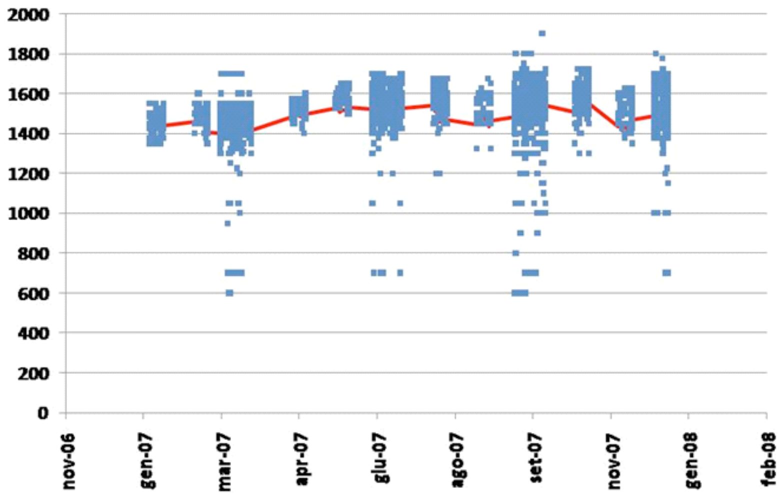


Figure 2

Interest rates

Interest rates during the period of study. The x-axis represents dates in 2007. The y-axis represents USD LIBOR 3M in percent. Source: British Bankers Association.

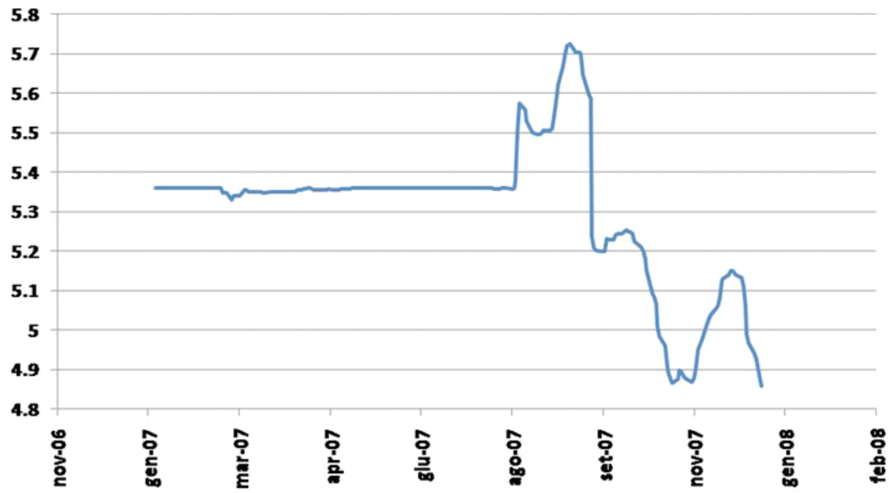


Figure 3

Calibration Example for 25th January 2007

Market data and calibrated models for the 25th January 2007. The x-axis represents strike. The y-axis represents option price. Black dots represent individual market-quoted call options on the S&P500 index for that day. All contracts have a maturity of three months (86 days). USD LIBOR3M = 5.36%. Spot = 1423.90. The red line represents the calibrated Heston model, the green line the calibrated Prospect Theory Black Scholes and the blue line represents the classic Black Scholes equation. Market data source: Chicago Board Options Exchange (CBOE).

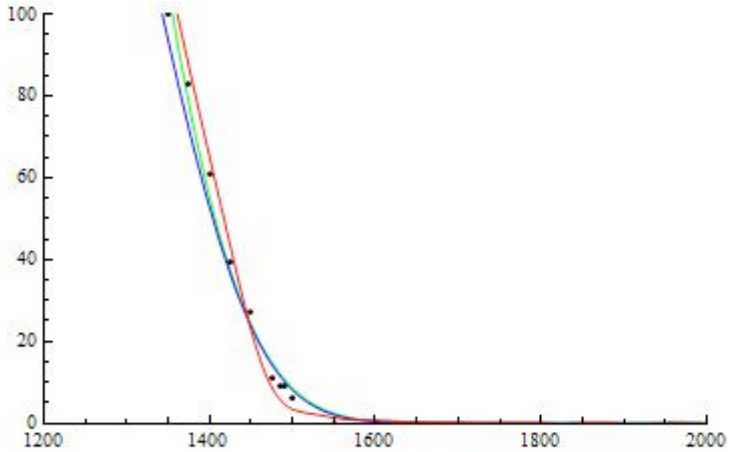


Figure 4

Calibration quality

Calibration quality of the Prospect Theory Black Scholes and the Heston stochastic volatility model . The quality is measured in terms of the Euclidean norm of the error vector (i.e. the difference between model price and market price) after calibration, for each day in the period of study. Above: the norm of the error for Prospect Theory. Middle: the norm of the error for the Heston Model. Below: difference between the norm of the error between Heston and the norm of the error for Prospect Theory.

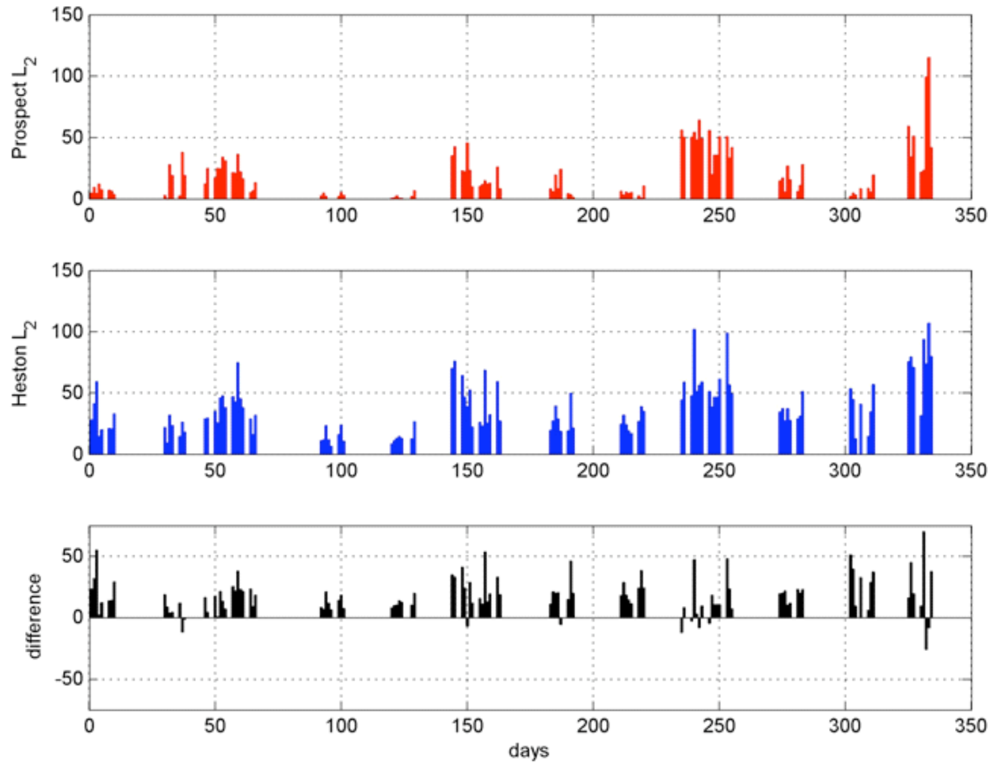


Figure 5

Calibrated parameters: Prospect Theory model

Calibrated parameters for the Prospect Theory model. Each day the model is calibrated to market data and these parameters calculated. The x-axis represents dates in 2007. Above: volatility parameter (σ). Middle: lambda parameter (λ). Below: gamma parameter (γ).

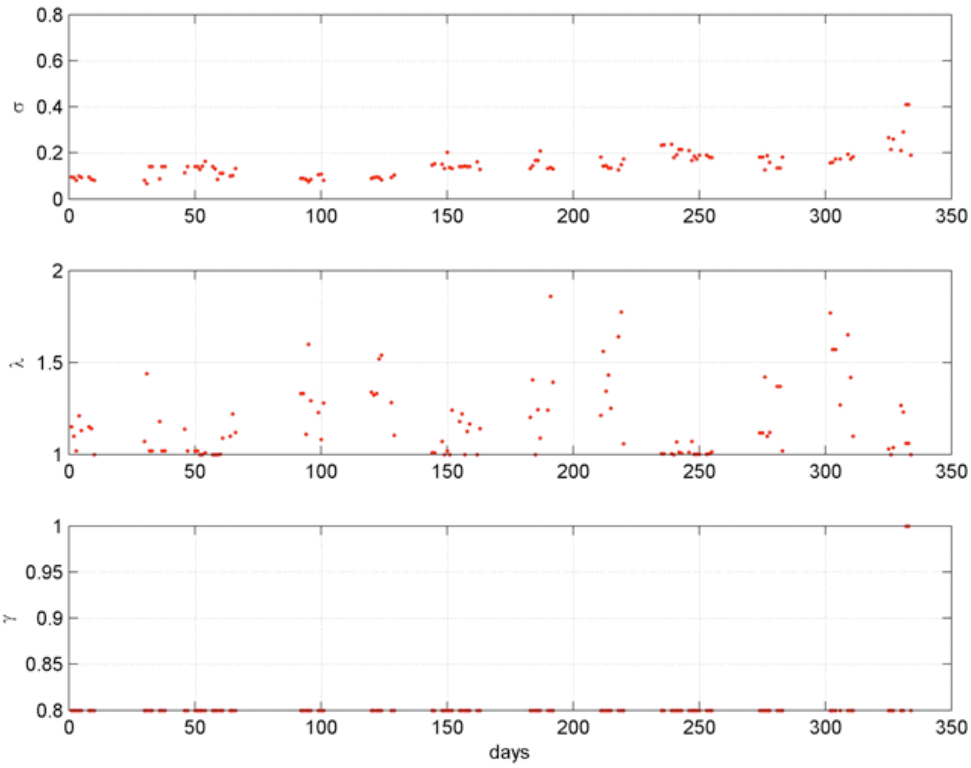


Figure 6

Calibrated parameter: Heston Stochastic Volatility model.

Calibrated parameters for the Heston Stochastic Volatility model. Each day the model is calibrated to market data and these parameters calculated. The x-axis represents dates in 2007. Above: initial volatility (V0). Middle above: mean reversion speed parameter (κ). Middle below: mean reversion level parameter (θ). Below: volatility of volatility parameter (ξ).

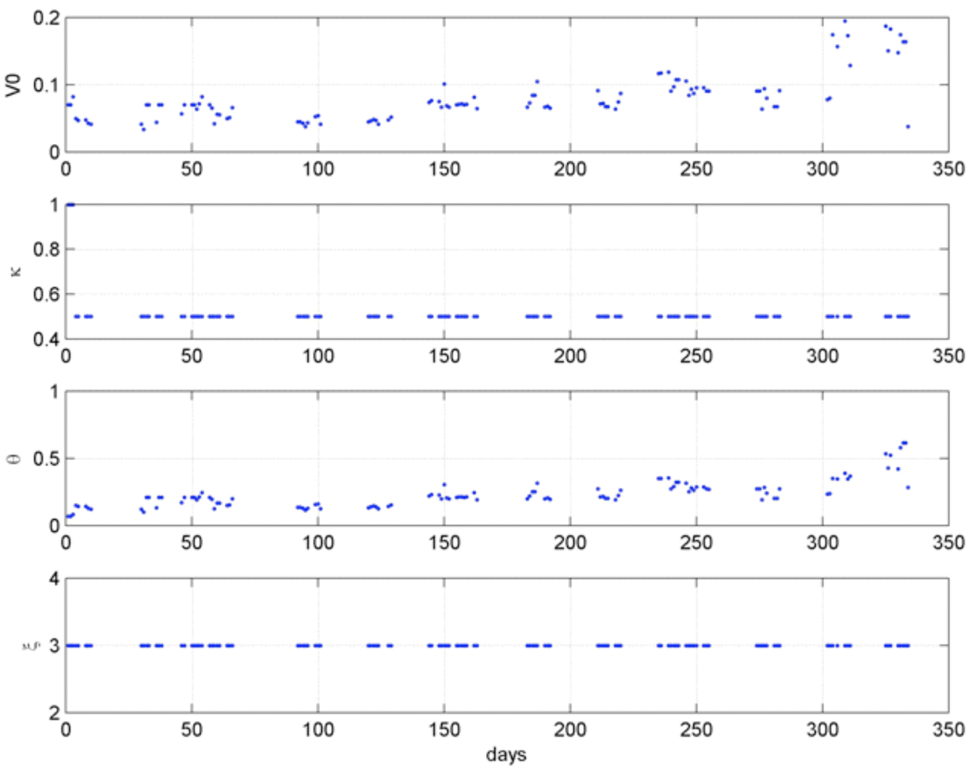


Figure 7

Which model is better?

The graphs below indicate which model had a smaller error norm for each day in the study. The x-axis represents dates in 2007. Above: shown in red, the value of +1 indicates the days in which the Prospect Theory Black Scholes model was better than the Heston Stochastic Volatility model. Below: shown in blue, the value of +1 indicates the days in which the Heston Stochastic Volatility model was better than the Prospect Theory Black Scholes model.

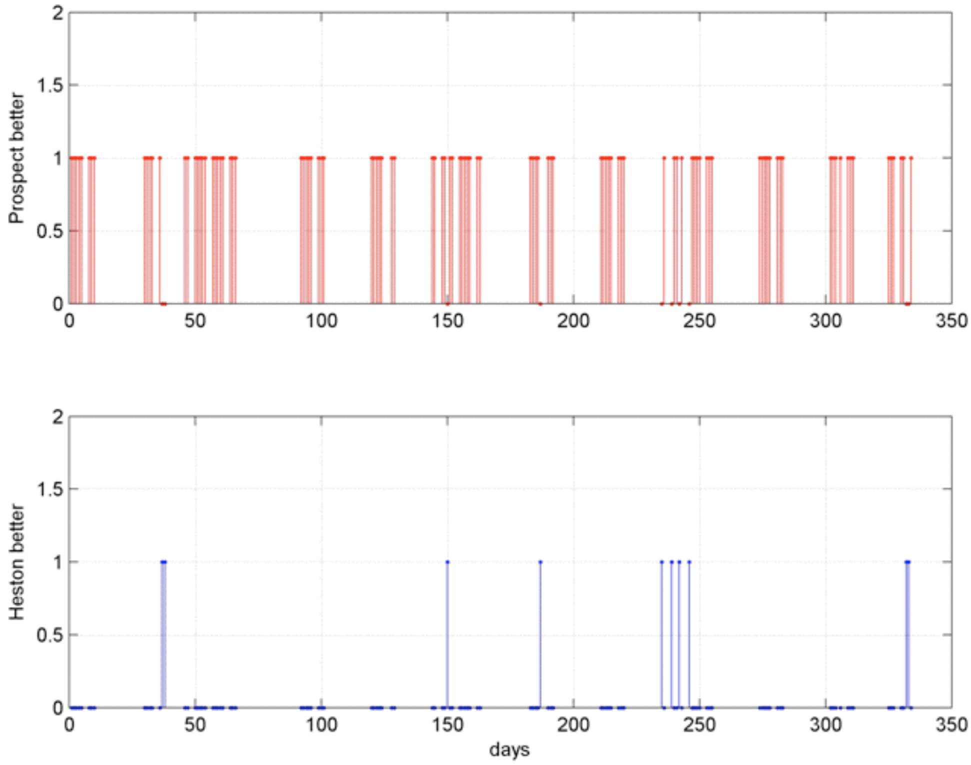


Figure 8

Which model is better?

Pie chart to indicate the proportion of days in which one model outperforms the other. Of the 113 business days in the study, Prospect Theory Black Scholes was better 103 days, while the Heston Stochastic Volatility mode was better 10 days. A proportion of 91% .

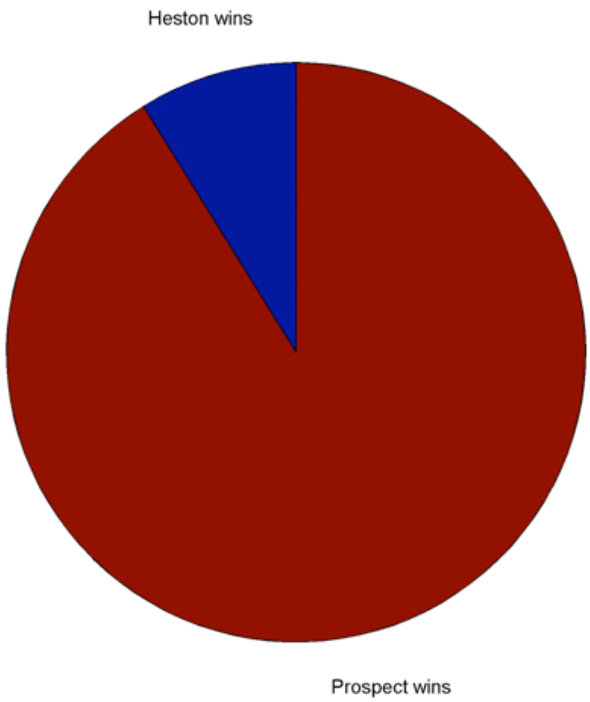
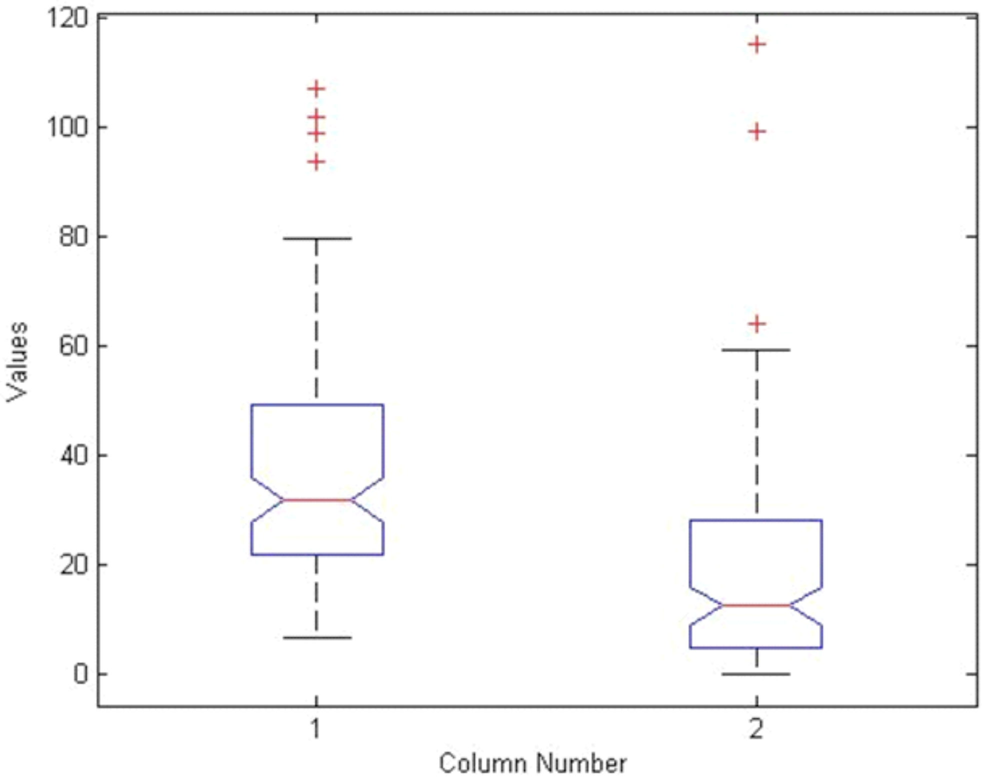


Figure 9

Statistical test

Statistical test for the significance in the difference between the error norms of Heston Stochastic Volatility model (column 1) and Prospect Theory Black Scholes (column 2). One-way

ANOVA test performed. The results were: F-value is 40.48 and p-value is 1.1038×10^{-9}, which confirm the hypothesis that the means are significantly different.



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