What do frictions mean for $Q-$theory testing?

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Abstract

This paper develops and empirically tests a model designed to distinguish the role of real and financing frictions on firms' investment, debt financing and equity financing policies. Real frictions include fixed costs of investment and adjustment costs. Financing frictions include taxes, collateral constraints, flotation costs of equity and dividend constraints. Because of financing frictions, all corporate policies are interrelated and depend on average Q. Due to fixed costs of investment and binding financing constraints, the sensitivity of corporate policies to Q is non-linear. The empirical tests demonstrate that both the endogeneity and non-linearities created by real and financing frictions are economically significant. The model then relates the effects of real and financing frictions on corporate policies to stock returns. The paper provides a rationale for the documented poor performance of Q-theory in explaining investment, and for the differential performance of the neoclassical investment model in explaining investment and stock returns. The paper extends Q-theory to explain debt and equity issues, and shows that market to book sorts control for non-linearities in investment policies.

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1 Introduction

The neoclassical investment model usually referred to as the Q-theory of investment constitutes a benchmark in financial economics to explain firm behavior. Its empirical performance, however, is controversial. Caballero (1997) documents poor performance of Q-theory to explain investment both in the aggregate and in the cross section. Fazzari, Hubbard and Petersen (1988) and Hennessy (2004) show that average Q is not a sufficient statistic by showing that cash flows and debt overhang effects are also significant in explaining investment. Meanwhile, Cochrane (1991, 1996) shows that a factor pricing model for stock returns based on the same theory of investment is not rejected both in the cross section and over time.[1] More recently, Philippon (2007) shows that Q-theory also performs well in explaining bond yields. Several questions arise from these facts and motivate this paper. First, why does the same model perform poorly in explaining investment but quite successfully in explaining stock returns? Second, what is the relation between investment, financing and stock returns in the context of Q-theory? Finally, why does the neoclassical model of investment explain bond behavior in the time series? This paper provides a unified framework to answer these questions by modelling and testing empirically the effects of both real and financing frictions on firms' optimal investment, debt financing and equity financing policies.

The core of the paper builds on two main observations concerning the effect of real and financing frictions on firms' corporate policies. Real frictions include fixed costs of investment and adjustment costs. Financing frictions include taxes, flotation costs of equity, collateral constraints and dividend constraints. The first observation is that in the presence of financing frictions the optimal investment and financing policies of the firm are interrelated and depend on average Q. The second observation is that both fixed costs of investment and binding financing frictions create optimal inaction regions in corporate policies. The main contribution is to estimate all corporate policies and to show that both the endogeneity and non-linearities caused by real and financing frictions are economically significant. The paper then introduces an exogenous pricing kernel and relates the findings on corporate policies to stock returns. The differences between the empirical approach in this paper and that of the empirical literature on Q-theory provide a potential answer as to why the same model has been successful in explaining stock returns but not investment policies.

The paper highlights that the estimation of the sensitivity of investment to Q is biased

[1] Liu, Whited, and Zhang (2007) also provide supporting empirical evidence on the goodness of fit of the neoclassical investment model for stock returns.

if the empirical approach fails to account for the effect of real and financing frictions on investment. The fact that coefficients on average Q have been typically low or insignificant surveyed by Caballero (1997) implies that Q-theory has been tested *on average*, neglecting the role of financing and the lumpiness in investment policies. In a frictionless environment with quadratic adjustment costs, Hayashi (1982) posits average Q as a sufficient statistic for investment. Due to real and financing frictions, this paper shows that investment is sensitive to Q and additional financing variables in the active region of investment, and insensitive to Q in the inertia region. The interaction between optimal investment and financing policies requires the use of instruments to control for endogeneity. The existence of inertia regions requires the use of sorts to test the sensitivity of investment to Q *only* when firms are actively investing. Since firms optimally invest conditional on marginal q, market to book equity ratios sort for inertia regions of investment in all equity financed, unconstrained firms.

Given the link between stock returns and investment returns observed by Cochrane (1991) and Restoy and Rockinger (1994), the main observations concerning the effect of real and financing frictions on corporate policies then translate into stock returns. In the presence of financing frictions, the returns to investment are endogenously related to the returns to financing. Furthermore, the lumpiness in investment policies due to fixed costs of investment results in non-linearity between stock returns and market to book ratios. In the empirical estimation, the first observation requires the use of instruments to control for the changing financing costs of funding investment opportunities. The second observation requires the use of sorts to control for inertia regions of investment in stock returns.

The paper then suggests one reason why Q-theory has historically performed better for stock returns is that the empirical approach in the investment based asset pricing literature has used both instruments and sorts on market to book, *naturally* controlling for the impact of real and financing frictions on firm behavior. The standard conditional estimation applied by Cochrane (1996) and Liu, Whited and Zhang (2007) uses default premia, dividend yields and term premia as instruments for the estimation of stock returns. I argue that these instruments control for changes in firms' costs of funding investment. The sorts by market to book equity introduced by Fama and French (1992, 1993) have been used in the empirical asset pricing literature to control for sample heterogeneity and ease the dimensionality problem of portfolio analysis. This paper shows that sorts by market to book control for non linearities in investment policies.

The model predicts multiple regimes in all corporate policies depending on whether financing constraints are binding and firms are actively investing. Concerning optimal investment policies, the empirical estimation focuses on three main results. First, controlling for financing frictions, fixed costs of investment create an optimal inaction region in which investment is insensitive to Q. Second, in the active region of investment, investment is relatively insensitive to Q when firms are subject to collateral constraints, and/or jointly subject to binding dividend and share repurchase requirements. Abel and Eberly (1994) and Hennessy, Levy and Whited (2006) obtain similar results in frameworks that respectively confine attention to real and financing frictions. The novelty of the model presented here is to incorporate both. This leads to a third novel prediction: financing frictions induce a larger inertia region for positive investment than that predicted by Abel and Eberly's neoclassical framework. This offers a complementary rationale for the insensitivity of investment to average Q in empirical tests.

Financing frictions thus induce underinvestment in two alternative ways. In the active region of investment, investment is relatively insensitive to Q when firms are subject to binding financing constraints. In the inactive region of investment, this paper shows that financing frictions may *also* induce underinvestment by enlarging the inertia region of optimal investment policies. In particular, firms subject to binding collateral constraints or dividend constraints require a higher marginal product of capital to reinitiate investment. Given reversibility in investment policies, the model further shows that financing frictions affect the incentives of firms to postpone disinvestment.

The model also provides testable implications for the optimal equity and debt financing policies as a function of Q. The empirical estimation focuses on three main results for financing policies. First, controlling for real and binding financing frictions, both equity and financing policies are positively and significantly related to average Q. Given that both investment and financing policies contribute to firm value, all corporate policies are interrelated and depend on average Q. Second, equity issues are sensitive to Q when firms are not financially constrained and are in their active region of investment. Firms with no equity issuance (due to high costs of equity funding) and no payout to shareholders (due to lack of internal resources) are financially constrained and should have low coefficients on Q. Finally, a novel prediction is that lumpy real investment may induce lumpy debt issuance due to binding collateral constraints. In the model, a firm lacking sufficient collateral to increase its debt financing is more severely constrained to refinance when it is optimal not to invest. This causal mechanism is distinct from that modeled by Tserlukevich (2008), whose theory of debt lumpiness hinges upon the debt tax shield. My paper hinges upon credit rationing cum lumpy real policies.

The empirical tests control for both the endogeneity and non linearities in corporate policies using the linear GMM generalized instrumental approach. To assess the endogeneity between investment and financing, I estimate corporate policies using linear GMM and use all lagged regressors as instrumental variables. This is in line with Hansen (1982) and Hansen and Singleton (1982). I also cluster data by firm history and include lagged changes in retained earnings, lagged changes in working capital, year dummies and industry dummies. To control for non-linearities, I sort the sample into quantiles to identify alternative regimes in corporate policies. In the case of investment, the sorting criterion builds on the prediction that marginal q equals market to book ratios plus additional variables controlling for the shadow costs of financing. The empirical section considers a double sort on market to book ratios and the net resources of the firm before investing; the working assumption is that these two variables jointly proxy for the levered marginal q determining inaction regions. The sorting criteria applied to debt and equity financing policies also build on the optimality conditions of the model to identify responsiveness to marginal q, binding collateral constraints and binding dividend constraints.

The empirical evidence for a sample of US industrial firms between 1980 and 2005 suggests that the both endogeneity and non-linearities described in the model are economically significant. Concerning endogeneity, the coefficient on Q estimated under linear GMM is usually higher than the one obtained under OLS; the sign and significance of the remaining coefficients in all corporate policies also changes. Consistent with the model, the endogeneity between investment and financing is further demonstrated by the fact that both debt and equity issues are significantly related to average Q in the sample.

Concerning non-linearities, the empirical evidence complements Barnett and Sakellaris (1998) and suggests the existence of inaction regions in investment policies. Investment is usually insensitive to Q when market to book equity ratios are sufficiently low; this matches the prediction that positive investment optimally occurs once the marginal product of capital exceeds a lower bound.[2] The empirical evidence also suggests that financially constrained firms postpone investment to a higher marginal product of capital. When firms are financially constrained, the coefficient on Q is only significantly related to investment for higher market to book ratios. Furthermore, firms with higher net funding resources have higher coefficients on Q in their active region of investment. The estimation results for debt and equity financing policies also support the prediction that financing depends positively on average Q. Consistent with the model, the responsiveness of Q to positive debt and equity issues is non-linear and depends on the inaction regions of investment and binding financing constraints.

[2] See Tables $3-7$ for further details.

The last section of the paper relates the findings on corporate policies to stock returns by incorporating an exogenous pricing kernel to the basic model. I show that binding financing constraints reduce firm value and increase market betas. This provides an alternative testable implication on the role of financing frictions on firm value. Stock returns depend on price-earnings effects, equity issues, market to book, changes in leverage, idiosyncratic risk, and a market premium for aggregate risk. These findings are consistent with both empirical studies on the cross section of stock returns as well as with theoretical models discussing these regularities.[3] The model then contributes in showing that the lumpiness in investment policies affects the sensitivity of stock returns to marginal q. The paper predicts that the usual market to book sorts applied by the empirical asset pricing literature have economic content; they control for the non-linear sensitivity of stock returns with respect to q.

The paper thus contributes to the current literature of Q-theory and financing frictions in several dimensions. First, it provides a benchmark model of investment and financing with testable implications on all corporate policies as a function of Q. The model elaborates on the link between real and financing frictions: binding collateral constraints propagate lumpiness between investment and debt financing, and the inertia region of investment shifts due to binding financing constraints. Second, the paper provides an empirical approach to test optimal corporate policies in the presence of both endogeneity and non linearities between corporate policies and Q. Finally, the paper suggests an economic rationale for the use of sorts in the empirical investment based asset pricing literature, and proposes an explanation to the differential performance of the neoclassical investment model for stock returns and corporate policies.

The paper is related to several strands of the financial economics literature. First, the paper relates to the macro papers on Q-theory for all equity financed firms. Hayashi (1982) derives the frictionless case and first tests the model empirically; Abel and Eberly (1994, 1996) consider alternative adjustment cost functions and discuss the role of fixed costs; Barnett and Sakellaris (1998) test empirically the non-linearity of investment for all equity financed firms; Caballero, Engel and Haltiwanger (1997), and Cooper, Haltiwanger and Power (1999) argue that non-convexities and irreversibility play a central role in the investment process. Second, the paper relates to the literature of financing frictions. Fazzari, Hubbard and Petersen (1988) provide empirical evidence on the investment-cash flow sensitivity; Whited (1992) and Bond and Meghir (1994) test the Euler condition for investment for constrained firms; Hennessy, Levy and Whited (2006) consider endogenous

[3] See Cochrane (1996), Berk, Green and Naik (1999) and Liu, Whited and Zhang (2007).

financing without fixed investment costs. This paper merges both strands of literature and highlights the role of fixed costs of investment policies in all corporate policies of the firm; it further proposes an alternative empirical approach to that of Barnett and Sakellaris (1998) and Hennessy, Levy and Whited (2006) to estimate all corporate policies in the presence of real and financing frictions.

The predictions of the model also relate to other papers in the investments and real options literature. Lamont (2000) suggests that time to build affects the link between investment and marginal q; this paper also highlights that the observed investment rates are not necessarily matched by the optimality condition of investment. De Marzo et al (2007) predict in a model of agency that investment is relatively insensitive to average Q when the firm is financially constrained; this paper yields a similar prediction in an alternative framework. Philippon (2007) suggests that Q-theory performs well in explaining bond yields in the time series, this paper highlights the interaction between investment and financing. Tserlukevich (2008) explores the effect of tax yields and fixed costs of investment for optimal debt policies; this paper shows that binding collateral constraints propagate lumpiness from real to financing policies. Carlson, Fisher and Giammarino (2005a) and Novy-Marx (2007) discuss the role of operating leverage for stock returns; this paper also elaborates on the role of fixed costs, primarily for corporate policies.

Finally, the paper relates to the investment based asset pricing literature. Cochrane (1991) and Restoy and Rockinger (1994) provide the link between investment returns and stock returns. Yaron, Gomes and Zhang (2002), Obreja (2006) and Gomes and Schmid (2007) further assess the link between investment and financing decisions for stock returns. This paper highlights the impact of fixed costs of investment on the sensitivity of stock returns to marginal q. The model also relates to Berk, Green and Naik (1999) and Carlson, Fisher and Giammarino (2005a) who derive implications for the cross section in a partial equilibrium set-up.

The paper is divided in five sections. Section 2 describes the main set-up. Section 3 discusses both the testable implications for corporate policies and the asset pricing implications of the model. Section 4 includes the empirical estimation of the testable implications for corporate policies in Section 3. Section 5 relates the results in Section 4 to the investment based asset pricing literature. Section 6 concludes.

2 The Model

2.1 The Problem of the Firm

Consider a firm run by a manager who decides on its optimal investment, external equity and debt financing policies. The manager maximizes the market value of the existing shares. The model distinguishes between the initial shareholders and the new shareholders incorporated through subsequent external equity financing issues. Investors are risk neutral and discount cash flows at a constant risk free rate $r > 0$.[4]

The gross operating profits of the firm denoted by $F(K_t, \epsilon_t)$ are a function of the current capital stock K_t and some diversifiable shocks ϵ_t. The firm is a price-taker and its production function exhibits constant returns to scale, such that its gross profits F are linear in capital. The function F is also twice continuously differentiable and strictly increasing in all its arguments. The state variable ϵ_t captures innovations in both input and output prices, and evolves according to a diffusion process

$$d\epsilon_t = \mu\left(\epsilon_t\right) dt + \sigma\left(\epsilon_t\right) dW_t \tag{1}$$

where W_t is a standard Wiener process.

Capital is acquired by undertaking gross investment at a rate I_t, and the capital stock depreciates as a fixed proportional rate δ_k. The capital stock K_t then evolves according to

$$dK_t = \left[I_t - \delta K_t\right] dt, \quad K_0 > 0 \tag{2}$$

When the firm undertakes gross investment, it incurs different types of costs. First, the firm incurs a direct cost of purchase of capital. I set the price of capital equal to 1 such that this cost is equal to I_t. Second, the firm incurs both adjustment costs and fixed costs of investment given by

$$G(I_t, K_t) = \frac{\alpha}{2} K_t \left[\frac{I_t}{K_t} - \delta_k\right]^2 + \Phi_t^I l\left(K_t\right)$$

The first term of G represents the costs of adjusting plant and equipment and is quadratic and homogeneous of degree one in both I_t and K_t. The second term of G represents fixed costs of investment, where Φ_t^I is an indicator function for non-zero investment. The fixed costs of investment $l\left(K_t\right)$ are such that $G(I_t, K_t)$ are an homogeneous function of I and K.[5]

[4] More precisely, r is the constant yield of a tax exempt risk free bond.

[5] In line with Abel and Eberly (1994), fixed costs do not affect the rescaling property of S. Davis and Norman (1990) and Kurshev and Strebulaev (2007) consider alternative set-ups where fixed costs break down the homogeneity of the value function.

The firm has multiple sources of external financing, and each of these sources is subject to different financing frictions. Concerning debt financing, the firm has access to a credit line

$$dB_t = [b_t - rB_t]\, dt, \quad B_0 > 0 \tag{3}$$

where b_t measures new bank borrowing or reductions in the debt buffer stock, the endogenous state variable B_t denotes the credit line balance. A debt covenant given by $dB_t \leq \eta dK_t$ where $\eta = \frac{K_0}{B_0}$ ensures that the credit line is risk-free. The debt covenant ensures that new debt financing is backed by new investment projects.[6]

Firms are subject to corporate taxes that yield a tax benefit of debt. I consider a tax benefit function $J(b_t, B_t)$ showing decreasing marginal tax benefits of debt as documented empirically by Graham (2000). Graham (2000) documents that the tax function is generally flat for small interest deductions but, because tax rates fall as interest expense increases, eventually becomes downward sloping.[7] A reduced functional form for the tax benefits of debt that is in line with these observations is given by

$$J(b_t, B_t) = \tau_c B_t \left[r - \frac{\gamma}{2} \left(r - \frac{b_t}{B_t} \right)^2 \right] \tag{4}$$

where τ_c is the average marginal tax rate on corporate income and γ reflects the curvature of the tax benefits of debt. Alternatively, Bond and Meghir (1994) and Gomes, Yaron and Zhang (2006) consider a convex interest schedule on the debt obligations that is increasing in book leverage. This alternative approach yields similar results as those obtained with (4).[8]

The firm may also fund its investment by means of new external equity funding x_t. Following Hayashi (1985), the firm cannot execute share repurchases such that $x_t \geqslant 0$. Equity issues are subject to convex flotation costs of underwriting as documented by Altinkilic and Hansen (2000). The function for flotation costs of equity issues H_t is assumed to be

$$H(x_t) = \frac{\vartheta}{2} x_t^2 \tag{5}$$

The costs of raising external equity thus cause the firm to retain funds in order to reduce reliance on external equity financing.

[6] This covenant is stronger than the one implied by $B_t \leq \eta K_t$. The covenant in this paper facilitates the analysis to obtain testable implications in Section 4.

[7] Interest deductions reduce taxable income, which decreases the probability that a firm will be fully taxable in all current and future states, which in turn reduces the tax benefit from incremental deductions.

[8] Note that a convex interest rate increasing in the credit line balance combined with a linear tax schedule yields concave tax benefits of debt.

The budget constraint of then firm is then given by

$$D_t = (1 - \tau_c) F (K_t, \epsilon_t) + \tau_c \delta K_t - (I_t + G_t) + b_t - rB_t + J_t + x_t - H_t \tag{6}$$

where D_t are the dividends to shareholders. The first two terms on the right-hand side of (6) represent the gross operating profits of the firm net of depreciation and related investment costs. The last two terms on the right hand side of (6) represent external sources of funds due to both debt financing and equity issues net of flotation costs.

Dividends to shareholders D_t are subject to both personal taxes and taxes on capital gains upon realization. I denote $m < 1$ the index of the stock market's preference for capital gains income over dividend income (Hayashi, 1985).[9] The initial shareholders of the firm further require $D_t \geqslant \overline{D}$ such that there is a lower bound to the dividends they receive (Gomes, Yaron and Zhang, 2006). Without loss of generality, I consider $\overline{D} = 0$ throughout the paper.

The vector (K_t, B_t, ϵ_t) captures all the relevant information at each instant t. At each point in time, the manager chooses the optimal investment I_t and the financing policies b_t and x_t that maximize the value of existing shares S_t. In sum, the manager maximizes the value of current equity holdings such that

$$S(K_t, B_t, \epsilon_t) = \max_{b_t, I_t, x_t} E_t \left[\int_0^\infty e^{-rs} (mD_s - x_s) \, ds \right] \tag{7}$$

subject to (1), (2), (3), (6) and the constraints $dB_t \leq \eta dK_t, D_t \geqslant 0$ and $x_t \geqslant 0$. The term $-x_t$ in (7) reflects the dilution of the market value of existing shares at time t. The corresponding Bellman Equation for the optimization problem of the manager in (7) is then given by

$$rS = mD - x + (I - \delta K) S_K + (b - rB) S_B + \mu(\epsilon) S_\epsilon + \frac{\sigma(\epsilon)^2}{2} S_{\epsilon\epsilon} \tag{8}$$

subject to the collateral constraints on the credit line $\varphi [dB - \eta dK] = 0$ and the non-negativity constraints $\lambda D = 0$ and $\nu x = 0$, where φ, λ and ν are the corresponding Lagrange multipliers of these constraints.

2.2 Optimal Investment Policies

Denote the marginal product of capital by $q \equiv S_K$. Using the approach in Abel and Eberly (1994), the investment policy that maximizes (8) in the presence of fixed costs of investment

[9] At the personal level, dividends are taxed at a constant rate τ_p. Capital gains are taxed at rate τ_g upon realization. Then $m < 1$ denotes the stock market's preference for capital gains income over dividend income such that $m = \frac{1-\tau_p}{1-\tau_g}$.

and financing constraints is given by

$$\frac{I}{K} = \begin{cases} \delta - \frac{1}{\alpha} + \frac{q-\varphi\eta}{\alpha(m+\lambda)} & \text{if } q \ \notin [q_1, q_2] \\ 0 & \text{if } q \ \in [q_1, q_2] \end{cases} \tag{9}$$

where the cut-off values $q_1 < 0$ and $q_2 > 0$ on the marginal product of capital determine an optimal inaction region for investment.

The optimal policy in (9) encompasses optimal investment rules discussed elsewhere in the literature and provides new insights on the role of financing frictions on investment. When firms are not subject to any type of fixed costs ($[q_1, q_2] = \varnothing$) or financing constraints ($\varphi = 0, \lambda = 0$ and $\nu = 0$), the optimal investment policy is a continuous function of the marginal product of capital and q is a sufficient statistic for investment (Hayashi, 1982). When all equity financed firms are subject to fixed costs of adjustment ($[q_1, q_2] \neq \varnothing$), Abel and Eberly (1994) show that the optimal investment policy is non-linear in q since there is a non-degenerate inaction region where firms find it optimal not to invest.[10]

Binding financing constraints affect the responsiveness of investment to marginal q in two alternative ways. All else equal, in the active region investment is relatively insensitive to q when the firm is financially constrained. Conversely, investment is sensitive to q when the firm is not subject to binding financial constraints. This is illustrated in Figure 1. Binding dividend constraints ($\lambda > 0$) dampen the sensitivity of investment to q; Hennessy, Levy and Whited (2006) provide a similar result for flotation costs of equity.[11] The optimal policy in (9) also shows that binding collateral constraints ($\varphi > 0$) induce underinvestment. In the extreme case where firms are *always* constrained to binding collateral constraints ($\varphi > 0 \ \forall t$), I show in the Appendix that the optimal investment policy is given by

$$\frac{I}{K} = \begin{cases} \delta - \frac{1-\eta}{(\alpha+\eta\gamma\tau_c)} + \frac{q}{(m+\lambda)(\alpha+\eta\gamma\tau_c)} & \text{if } q \ \notin [q_1, q_2] \\ 0 & \text{if } q \ \in [q_1, q_2] \end{cases} \tag{10}$$

such that both binding collateral and dividend constraints dampen the sensitivity of investment to q.

Binding financing frictions *also* induce underinvestment by shifting the optimal inaction region for positive investment to higher values of q_2. This is illustrated in Figure 2. Binding financing constraints affect the cut-off values that determine the optimal inaction region of

[10] See Appendix. I derive the cut-off values for the optimal inaction region when firms are also subject to financing constraints; the case described by Abel and Eberly (2004) is obtained when $\varphi = 0, \lambda = 0$ and $m = 1$.

[11] The shadow cost of dividend constraints in the model is the shadow cost of equity financing when $v = 0$. The first order condition for positive equity issues implies $m + \lambda \simeq 1 + H_x$. See Section 2.3.

investment. I show in the Appendix that the relevant cut-off values of the optimal inertia region satisfy

$$q_i = (m + \lambda + \varphi\eta) - \delta\alpha\,(m + \lambda) + \sqrt{[\delta\alpha\,(m + \lambda)]^2 + 2f\alpha\,(m + \lambda)} \qquad (11)$$

for $i = 1, 2$ where $q_1 < 0$ is the upper bound for asset sales and $q_2 > 0$ is the lower bound for investment.

Equation (11) shows that financing frictions affect the optimal inertia region of investment in alternative ways. Firms subject to binding collateral constraints postpone investment to higher values of $q_2 > 0$ and initiate disinvestment at a higher $q_1 < 0$ relative to the financing frictionless case by Abel and Eberly (1994). Binding collateral constraints increase the price of purchasing and selling capital; this induces firms to postpone investment to a higher positive marginal q and to initiate disinvestment at a higher negative marginal q. Meanwhile, firms subject to binding dividend constraints, personal and capital taxes postpone investment to higher values of $q_2 > 0$ if the shadow cost of collateral constraints is sufficiently low, and initiate asset sales at a lower cut-off value $q_1 < 0$ of the marginal product of capital.[12] Figure 2 illustrates that these frictions widen the optimal inertia region in a similar fashion than an increase in the real fixed costs of investment.

The empirical literature on Q-theory has typically interpreted the inverse of the coefficient on Q as the curvature of adjustment costs of investment α. Expressions (9)-(11) predict that such inference is biased if the estimation fails to account for the effect of real and financing frictions on investment. All else equal, financing frictions *increase* the implied curvature of adjustment costs and dampen the sensitivity of marginal q to investment. Furthermore, in the presence of fixed costs, the sensitivity of investment to marginal q reduces to zero. Expressions (9)-(11) thus rationalize the high variability of point estimates for the curvature of adjustment costs in the macro literature on Q-theory. Using the Q-theoretic approach, estimates for the curvature of adjustment costs range from over 20 (Hayashi, 1982) to as low as 3 (Gilchrist and Himmelberg, 1995).

2.3 Optimal Financing Policies

Consider the optimal debt financing policy. The shadow cost of debt financing $S_B < 0$ equates the marginal financing benefit of issuing debt to the corresponding marginal costs.

[12] The optimality condition for equity issues explained below relates these predictions to the impact of higher flotation costs of equity on inertia regions.

The optimal debt financing policy then satisfies

$$\frac{b}{K} = \begin{cases} r\frac{B}{K} - \frac{1}{\gamma\tau} + \frac{1}{(m+\lambda)\gamma\tau_c} S_B \frac{B}{K} & \text{if } \varphi = 0 \text{ and } q \notin [q_1, q_2] \\ r\frac{B}{K} + \eta\frac{I}{K} - \eta\delta & \text{if } \varphi > 0 \text{ and } q \notin [q_1, q_2] \\ r\frac{B}{K} - \eta\delta & \text{if } \varphi > 0 \text{ and } q \in [q_1, q_2] \end{cases} \tag{12}$$

Binding dividend constraints dampen the sensitivity of debt issues to the shadow cost of debt financing. Fixed costs of investment jointly with binding collateral constraints create lumpiness in the optimal debt policy.

The optimal policy in (12) shows that binding collateral constraints and fixed costs of investment jointly create lumpiness in debt financing policies. When the firm is in its inaction region of investment and is also subject to binding collateral constraints, the optimal financing policy is lumpy even in the absence of fixed costs of debt financing. Binding debt covenants may subordinate the optimal debt financing policy to investment; in particular, the binding constraint $dB = \eta dK$ implies $b = \eta I + rB - \eta\delta K$. A firm subject to binding collateral constraints may find it optimal not to increase its book leverage by more than $\frac{rB}{K} - \eta\delta$ when the firm is in its inaction region of investment. This rate ensures that the credit line balance is kept constant after both repaying interest expenses and repurchasing debt to control for the depreciation of collateral.[13]

Finally, consider equity issuing policies. The optimal external equity issuance policy of the firm is given by

$$\frac{x}{K} = \begin{cases} \frac{1}{\vartheta}\left(\frac{1-\nu}{m+\lambda} - 1\right)\frac{1}{K} & \text{if } \lambda = \frac{m}{1-H_x} \text{ and } \nu = 0 \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

at any point in time. While the model does not incorporate fixed costs of equity financing explicitly for the sake of tractability, the optimal equity issuance policy is still lumpy and depends primarily on the payout policy of the firm and the constraint on share repurchases (Hayashi, 1985; Bond and Meghir, 1994; Hennessy and Whited, 2005).

The optimal policy in (13) shows that binding financing frictions induce multiple regimes in equity issues. When dividends constraints are binding ($\lambda > 0$) and share repurchases are not binding ($v = 0$), firms optimally issue equity. This corresponds to the case where firms exhaust their net revenue to finance all the investment, and optimally issue shares to finance a higher level of investment. The optimal policy may also be not to issue any equity ($v > 0$).

[13] Notice that the empirical evidence suggests that $r < 0.1$, $\frac{B}{K} \approx 0.3$, $\delta_k \approx 0.3$ and $\eta < 1$. This implies $b < 0$ when $\varphi > 0$ and $q \in [q_1, q_2]$. When firms are subject to binding collateral constraints and they are in their inaction region of investment, they optimally repurchase debt to compensate for the depreciation of collateral.

This may correspond to two alternative situations. If $\lambda = 0$, firms have sufficient net internal resources to finance investment and distribute dividends to shareholders. If $\lambda > 0$, firms generate insufficient revenue to finance all investment opportunities, but face high flotation costs of equity that prevent them from issuing new shares. Firms subject to simultaneously binding dividend and share repurchasing constraints are financially constrained.

3 Testing Q-theory on Corporate Policies

3.1 Empirical Approach

Section 2 characterizes the optimal investment, debt financing and equity financing policies of the firm as a function of firm characteristics. In a nutshell, the model predicts that binding financing constraints reduce the responsiveness of investment to q. Binding financing constraints also decrease the responsiveness of the optimal debt and equity financing to their corresponding shadow costs. Fixed costs of investment create optimal inaction regions in investment, which shift with binding collateral and dividend constraints. The optimal financing policies are also lumpy due to both the lumpiness in investment policies and binding financing constraints.

The goal of this Section is to estimate the investment and financing policies of firms as derived in Section 2 and assess whether real and financing frictions are economically significant. The empirical approach to estimate all corporate policies is based on the generalized instrumental variable estimators proposed by Hansen (1982) and Hansen and Singleton (1982). An attractive feature of this method is that the parameters of the first order condition on investment are estimated without explicitly solving for the structural relation between investment and financing.

The empirical approach builds on two main observations. First, financing frictions create endogeneity between investment and financing. To control for the endogeneity between investment and financing, I use all lagged explanatory variables in each of the first order conditions for investment and financing in line with Hansen (1982) and Hansen and Singleton (1982). Second, fixed costs of investment and binding financing frictions create non-linearity of all corporate policies with respect to Q. To control for these non-linearities, I sort observations in quantiles to identify the alternative regimes in all corporate policies derived in Section 2. I provide further details on the estimation below.

There have been other approaches different from the one in this paper to control for the impact of either real or financing frictions on investment. Barnett and Sakellaris (1998) consider an all equity financed firm and find that investment has a nonlinear relation with average Q. The estimation approach applies the technique developed by Hansen (1996) to test models where there are nuisance parameters that are not identified under the null hypothesis; the nuisance parameters are the thresholds $[q_1, q_2]$ discussed in Section 2. This paper contributes to Barnett and Sakellaris (1998) in two dimensions. First, I consider both fixed costs *and* financing frictions in the estimation of optimal investment policies. Second, I propose an estimation approach using sorts that controls for inaction regions without pre-setting a specific number of regimes.

Arellano and Bond (1991) propose a single equation approach that estimates dynamic models consistently for short and unbalanced panels. The technique uses GMM and expresses investment either in differences or by transforming all variables in terms of orthogonal deviations (Arellano and Bover, 1995). Bond and Meghir (1994) use this approach to test the Euler condition for investment in a dynamic model of investment and financing. While the empirical approach in this paper also uses linear GMM, I hereby test for optimal investment and financing policies in levels as a function of Q, and correct both for endogeneity and non-linearities in the data.

Finally, an alternative estimation technique is that proposed by Erickson and Whited (2000) who use non-linear GMM to control for measurement error in Q using higher order moments. While Erickson and Whited (2000) focus on measurement error as a potential explanation of the bad empirical performance of Q-theory, the model and the tests in this paper focus on the lack of consistency of empirical tests due to the endogeneity between investment and financing.[14]

The sample has been drawn from the merged CRSP-COMPUSTAT database, considering only the 1985-2005 period for US manufacturing firms in SICs 2000-3999. The sample has been filtered for missing data, or for observations where total assets, the gross capital stock or sales are either zero or negative. All firms with less than five consecutive years of accounting data have been deleted from the sample. All variables have been winsorized to eliminate the effect of extreme values in the estimation. Table 1 provides further details on database construction and the correspondence of the main variables in the database to those described by the model in Section 2. Table 2 provides the relevant sample statistics.

[14] The technique to control for measurement error in Erickson and Whited (2000) requires that there is no endogeneity between investment and financing. Conversely, the empirical approach in this paper does not control for potential measurement error.

3.2 Testing Q-theory on Investment Policies

The optimal investment policy in (9) is a function of the marginal product of capital q and the shadow cost of debt financing S_B which are unobservable. Denote average $Q \equiv \frac{S+B}{K}$ as the ratio of the market value of the firm to its fixed assets. Using a similar approach as in Hennessy (2004) and Hennessy, Levy and Whited (2006), average Q overstates marginal q by incorporating the costs of binding financing frictions on equity such that

$$q = Q - (S_B + 1)\frac{B}{K} - \frac{\Xi}{K} - \frac{\Lambda}{K} - \frac{R}{K} \tag{14}$$

where Ξ is a function of the net present value of the flotation costs of equity issues, Λ reflects the net present value of the cost of dividend constraints, and R is the net present value of the shadow cost of collateral constraints on equity.

Using both the optimal investment policy in (9) and the alternative expression for marginal q given in (14) provides a testable implication for investment rates in the active region of investment, namely

$$\frac{I}{K} \equiv \phi_{10} + \underset{(+)}{\phi_{11}} Q + \underset{(+)}{\phi_{12}} \delta + \underset{(+)}{\phi_{13}} \frac{rB}{K} + \underset{(-)}{\phi_{14}} \frac{b}{K} + \underset{(-)}{\phi_{15}} \frac{D}{K} + \underset{(-)}{\phi_{16}} \frac{x}{K} + \phi_{17} \frac{c}{K} \tag{15}$$

where all empirical tests are done for positive investment $I > 0$.

The testable implication in (15) characterizes the wedge between marginal q and average Q derived in (14) differently. Equation (14) characterizes the wedge between marginal q and average Q by means of the shadow cost of future dividend constraints, the net present value of flotation costs, and the shadow cost of future binding debt covenants. The testable implication in (15) incorporates current equity issues and current dividend payments to re-express the first two terms in terms of observables. I assess the shadow cost of binding constraints on firms' capital structure later on by constraining the estimation to either low leveraged and high leveraged firms.[15] Given the persistence of dividend payments, current dividend payments are taken as a proxy of future dividend payments to shareholders. Current equity issues are used a proxy of flotation costs of equity both today and in the future (i.e. listing fees, etc.). The ratio cash holdings to capital at the time of investment controls for the cash flow policy of the firm, which is overlooked in Section 2 and may shed light on firms' financing prospects (Fazzari, Hubbard and Petersen, 1988; Almeida, Campello and Weisbach, 2004).

[15] An earlier version of the model considered a dummy for highly levered firms in the regression. However, the method by Hansen (1982) requires the use of lagged regressors and instruments; the use of a dummy for lagged high leverage though affects the robustness of the J-tests. I therefore abstract from the term $\frac{R}{K}$ in the benchmark case and then group firms according to book leverage in Table 7.

I apply the GMM generalized instrumental approach to control for the endogeneity between investment and financing. I consider one lagged regressors as instruments in line with Hansen (1982) and Hansen and Singleton (1982). I consider one lag for all regressors. I further incorporate lagged changes in working capital to capital ratios and lagged changes in retained earnings to capital ratios as additional instruments for the past financing decisions of the firm. Both the lagged change in retained earnings and the lagged change in working capital contribute to the estimation of investment policies as suggested by C-tests in untabulated results.[16] I also include year dummies and industry dummies using the 17-industry groups by Fama and French, and cluster data by firm history.[17]

I further control for non-linearities between investment and Q by estimating Equation (15) in quantiles. The sorting criterion stems directly from the model and is intended to identify the inaction region $[q_1, q_2]$ by sampling the data in quantiles according to their implied marginal q. The unobservable marginal q is restated in terms of average Q and other financing variables using (14). To isolate the impact of financing on marginal q, the working assumption is that marginal q can be re-expressed as a function of both the market to book equity ratio of the firm and an additional variable θ that reflects net funding resources of the firm before investment. I therefore apply a double sort on market to book and θ to control for non-linearities in investment policies. All tests consider a maximum 5 quantiles by sort; a higher number of quantiles does not affect results as demonstrated in the robustness checks below.

The sorting variable θ is constructed using the budget constraint such that

$$\theta = \frac{\pi}{K} + \frac{b}{K} + \frac{x}{K} - \frac{rB}{K} - \frac{D}{K}$$

where π denotes cash flows. The budget constraint systematically links investment to financing decisions irrespective of whether the firm is actively investing or refinancing. Whenever $\theta > 0$, the firm has available resources to invest; conversely, $\theta \leq 0$ implies that the firm may be constrained to invest. Table 1 suggests that $\theta \leq 0$ identifies firms that are constrained to invest. When $\theta \leq 0$ (Panel B), firms are smaller in size, have a lower Q, and issue both less equity and less debt than all firms on average (Panel A).

[16] I have checked the marginal contribution of each of these instruments to all GMM estimations in the paper using C-tests or GMM distance tests. The null that these additional orthogonality conditions are actually satisfied holds at the 5% level for almost all quantiles and estimations in the paper. The exception is the GMM estimation in Table 5 with firms with negative net funding resources; then the lagged change in retained earnings is rejected by C-tests and therefore not included as an instrument.

[17] The 17-industry group definitions are provided in Kenneth French's website. Controlling for heterogeneity without clustering by firm yields similar results to those reported in Tables 3-14.

Tables $3-7$ describe the estimation results for (15) using generalized linear GMM with sorts. The model in Equation (15) is identified and check if both the order condition and the rank condition of the system hold in all quantiles. J-tests of overidentying restrictions test relate to the order condition that requires a higher number of instruments than endogenous regressors. The joint null hypothesis of J-tests is that all instruments are valid instruments; a rejection of this hypothesis casts doubt on the validity of the model.[18] A-tests relate to the rank condition of the system and test whether instruments are effectively correlated with the endogenous regressors. The null hypothesis of A-tests is that the correlations between the instruments and the endogenous regressors are not significantly different from zero; a rejection of this hypothesis ensures that the model is identified.[19] The model is identified if the estimation results simultaneously reject A-tests and fail to reject J-tests.

Consider first the testable predictions related to fixed costs of investment. Controlling for financing frictions, the model predicts that fixed costs of investment create an optimal inaction region in which the coefficient ϕ_{11} is zero. Outside this region, the coefficient ϕ_{11} should be positive and significant. Table 3 shows the estimation results of using linear GMM and sorting the working sample into quantiles with market to book equity as the single sorting criterion. Results complement the empirical evidence by Barnett and Sakellaris (1998) and support the observation that fixed costs create non-linearities in investment.[20] In particular, the coefficient on Q is only significant when market to book equity ratios are larger than 2.[21] This suggests the existence of optimal inertia regions of investment for low market to book equity ratios. Interest expenses, debt issues, equity issues and cash holdings are also significant in explaining investment.

Consider now the implications for financing frictions. When firms are actively investing, the model predicts that financing frictions dampen the sensitivity of investment to Q. Firms with binding financing constraints or higher costs of financing (i.e. $\theta < 0$) should have lower coefficients on Q when actively investing. The model also predicts that binding collateral constraints and dividend constraints enlarge the inertia region for positive investment, such that $q_2 > 0$ increases for financially constrained firms. Table 4 sorts observations into quantiles of market to book equity and constrains the sample to firms with positive net

[18] Under the null, the J-test statistic is distributed as chi-squared in the number of overidentifying restrictions. See Hayashi (2000, pp. 227-228, 407, and 417).

[19] The A-test is a generalization of the Anderson canonical correlations rank statistic to the non-i.i.d. case. A failure to reject the null hypothesis of the A-test suggests the model is unidentified. See Baum et al (2007) for further clarifications on this test.

[20] See also Cooper and Haltiwanger (2006).

[21] This is the upper bound of the quantile whose market to book average equals 2.5.

funding resources. Results provide strong evidence that unconstrained firms have higher coefficients on Q than all firms on average as reported in Table 3. Table 5 then constrains the estimation in Table 3 to firms with negative net funding resources. Consistent with the model, the inertia region expands to higher market to book ratios, and the coefficients on average Q in those quantiles are lower than those observed in Tables 3 and 4. J-tests and A-tests suggest that the model is identified when firms are actively investing.

Table 6 complements Tables $3-5$ and shows the estimation results of applying the linear GMM and using firms' net funding resources θ as the single sorting criterion. This sorting criterion does not identify an inaction region in investment; the coefficients on Q are significant in all quantiles. These findings suggest that the sort on market to book equity ratios controls for inaction regions in investment; meanwhile, the sort on θ controls for the impact of financing on marginal q. Results also suggest that the model is identified only if the estimation controls for non-linearities in investment with market to book sorts; J-tests and A-tests confirm the goodness of fit of the model only for firms with positive net funding resources. The empirical evidence shows a quadratic relation between the coefficient on Q and θ; this supports the assumption of convex costs of financing in Section 2. Furthermore, cash holdings are negatively related to investment when $\theta < 0$ and positively related to investment when $\theta > 0$. This supports the prediction that firms anticipating future financing constraints may have incentives to hoard cash and short term investments today.[22]

Concerning equity related constraints, the model predicts that firms with no dividend distributions and no issuance activity have a lower coefficient in Q. Panel A of Table 7 sorts the sample by market to book equity and constrains observations to non-dividend paying and (simultaneously) non-equity-issuing firms in all quantiles. Results for both sorted and unsorted observations show that average Q is not significantly different from zero; A-tests reject that the rank condition holds in the model for most market to book quantiles and the model is only identified in the unsorted estimation. Concerning collateral constraints on debt financing, the model predicts that firms with high book leverage that are constrained in their leverage policy have coefficient on Q that is either low or not significantly different from zero. Panel B in Table 7 sorts the sample by market to book equity and constrains observations to highly levered firms (i.e. $\frac{B}{K} > 0.6$). The coefficients on Q are not significant and model is rejected by either J-tests and/or A-tests.

Overall, Tables $3-7$ provide an explanation for the high variability of point estimates for the curvature of adjustment costs in the empirical literature on Q-theory. All significant coefficients other than ϕ_{11} in Tables $3-7$ are in line with the predictions of the model except

[22] See Fazzari, Hubbard and Petersen (1988) and Almeida, Campello and Weisbach (2004).

for net debt issues, which are significantly and positively related to investment. The sorting criterion on market to book commonly used in the investment asset pricing literature hereby controls for non-linearities in investment policies.

3.3 Testing Q-theory on Financing Policies

Consider first the case of debt financing policies. Using both Equation (14) and (12), the testable implication for the optimal debt policy if firms are in there active region of investment and are not subject to collateral constraints is given by

$$\frac{b_t}{K_t} \equiv \phi_{10} + \underset{(+)}{\phi_{11}} Q + \underset{(+)}{\phi_{12}} \delta + \underset{(+)}{\phi_{13}} \frac{rB}{K} + \underset{(-)}{\phi_{14}} \frac{I}{K} + \underset{(-)}{\phi_{15}} \frac{D}{K} + \underset{(-)}{\phi_{16}} \frac{x}{K} + \phi_{17} \frac{c}{K} \tag{16}$$

where all empirical tests are done for net increases in long term debt $b > 0$.[23] When firms are not subject to collateral constraints and are actively investing, the optimal debt financing policy is positively related to average Q such that $\phi_{11} > 0$. The optimal debt financing policy is insensitive to Q otherwise such that ϕ_{11} is not significantly different from zero. The coefficient ϕ_{14} is significant only if the firm is actively investing irrespective of whether collateral constraints are binding. The reminder of the coefficients are significant when the firm is actively investing and is not constrained in its debt policy.

I estimate (16) using a similar approach as that of investment policies. I apply a double sort on $\frac{S}{K-B}$ and $\frac{B}{K}$ to control for non-linearities in positive debt issues. The sort on market to book equity controls for inaction regions in investment policies. The underlying assumption is that average Q is a good proxy for marginal q when controlling for the impact of real frictions on investment.[24] The sort on book leverage $\frac{B}{K}$ controls whether the firm is subject to binding collateral constraints as predicted by the model. The underlying assumption is that highly levered firms are more likely to be constrained in increasing their book leverage ratio.

Tables $8-9$ provide the empirical estimations of Equation (16). Consistent with the predictions of the model, Table 8 shows that optimal debt policies are positively and significantly related to average Q in the region where firms are actively investing according to Tables $3-5$. Both J-tests and A-tests confirm that the model is identified for all quantiles; this is also the case for the unsorted estimation. Results in Table 9 further suggest that the testable implication in Equation (16) only holds when firms are not constrained in their capital structure decisions. In Panel A (lower leverage), the model is accepted when market

[23] Reductions in long term leverage are less likely influenced by an investment motive, and most probably influenced by distress concerns or agency costs not explicitly addressed in Section 2.

[24] See Tables $3-6$.

to book ratios are sufficiently high and firms are actively investing. As an exception, the model is also accepted by J-tests and A-tests for the lowest quantile of market to book ratios where firms are not investing. In Panel B (higher leverage), the coefficients on Q are insignificant and A-tests suggest that the model is not identified with the exception of the highest quantile on market to book. The coefficient on average Q is also insignificant in the unsorted estimation.

Consider now the case of equity financing policies. While (13) does not provide an explicit expression for x as a function of Q, the model requires that the firm pays no dividends when issuing equity. If marginal investment is financed with new equity funding, then x is positively related to Q. Using (8) and considering $D_t = 0$, equity issues are positively related to investment and negatively related to debt issues. Reordering terms in Equation (15), a testable empirical implication for net equity issues is given by

$$\frac{x_t}{K_t} \equiv \phi_{10} + \underset{(+)}{\phi_{11}} Q + \underset{(+)}{\phi_{12}} \delta + \underset{(+)}{\phi_{13}} \frac{rB}{K} + \underset{(-)}{\phi_{14}} \frac{b}{K} \underset{(-)}{\phi_{15}} \frac{D}{K} + \underset{(-)}{\phi_{16}} \frac{I}{K} + \phi_{17} \frac{c}{K} \tag{17}$$

where all empirical tests are done for positive net equity issues $x > 0$.[25]

The optimal equity financing policy is also lumpy and non-linear in Q. Given that all tests are done for positive equity issues, the observed non-linearities in Q should come from the lumpiness in investment policies. Nonetheless, the working assumption in the model that positive dividend distributions and equity issues are perfectly negatively correlated should not necessarily hold in the data. I thus estimate (17) using a double sort on market to book equity and dividends to control for non-linearities in equity issues. The sort on market to book controls for inaction regions in investment policies. Discriminating among firms with positive payouts or no payouts further controls for the interaction between payout policies and equity issues.

Controlling for the payout policy of the firm and real frictions on investment, the model predicts that net equity issues are positively and significantly related to average Q such that $\phi_{11} > 0$. However, if firms are in their optimal inertia region of investment, the lumpiness in optimal investment policies might affect coefficients ϕ_{11} and ϕ_{16}. Tables $10 - 11$ provide the empirical results of estimating Equation (17). In Table 10, the coefficients on Q are significant for the higher market to book equity ratios where firms are actively investing according to Tables 3-5. This suggests that fixed costs of investment induce lumpiness in equity issues, even if the cut-off of value for the inaction region in equity is higher than the one for investment observed in Table 3. Interestingly, market to book ratios in equity

[25] Reductions in long term leverage are less likely influenced by an investment motive, and most probably influenced by distress concerns or agency costs not explicitly addressed in Section 2.

issuing firms are higher than those observed for investment and debt financing; the range of average market to book ratios in equity issues goes from 0.86 to 5.08 (compared to 0.73 to 4.65 in debt issues).

Consistent with the assumptions of the model, Table 11 further demonstrates that the relation between equity issues and average Q holds when equity issues are negatively related to dividend distributions. When firms are not distributing dividends (Panel A) and issuing shares, the model is accepted for the same quantiles that are accepted in Table 10. Conversely, when firms show positive dividend distributions (Panel B), the model is identified according to A-tests and the coefficients on Q are not significant in all quantiles. Results also show that the sensitivity of equity issues to Q is higher for firms that are not distributing dividends (Panel A of Table 11) than for all firms on average (Table 10).

3.4 Robustness checks

The estimation approach throughout the paper is based in two fundamental observations. First, financing frictions create endogeneity between investment and financing policies. Second, both real and financing frictions create lumpiness in corporate policies. Tables 12 and 13 consider two different robustness checks that suggest that both instruments and sorts are economically significant for corporate policies.

Table 12 compares the OLS estimation results to the corresponding GMM estimation for all corporate policies without using sorts. This is a robustness check on the existence of endogeneity between investment and financing. Average Q is significant for all cases; the use of instruments may change the sign and increases the magnitude of significant coefficients on average Q in all cases. The Hausman test (H-pval) on unclustered data shows that the GMM estimation is consistent, assuming that the model is correctly specified. The endogeneity between investment and financing is further demonstrated by the fact that both debt and equity issues are significantly related to average Q in the sample irrespective of the empirical approach.

Table 13 estimates investment policies using OLS and market to book sorts only. The comparison with Table 3 constitutes a robustness check to assess the relative magnitude of endogeneity vis a vis non-linearities in investment policies. In Panel A, inaction regions in investment also show when applying OLS to sorted data; still, the coefficients on Q are significantly lower. The coefficient on Q is also significant for the lowest quantile on market to book, where the GMM tests suggest insignificancy. When the sample in Panel A is constrained to firms with positive net funding resources, the coefficients on Q in the active region are either equal or lower than those for all firms on average; this also holds for unsorted data and is at odds with the predictions of the model.

As a complementary robustness check, Table 14 considers the use of a higher number of quantiles in the estimation of investment policies. Panel A considers GMM estimation with market to book sorts. Results are comparable to those in Table 3; the implied cut-off value for the inertia region for all firms on average is a market to book equity ratio of 2. J-tests and A-tests suggest that the model is identified in the active region of investment. Panel B constrains the estimation to firms with positive net funding resources. As in Table 4, the inertia region shrinks to higher market to book ratios. As a caveat, some A-tests fail to reject the null hypothesis at the 5% level.

4 Asset pricing implications

4.1 An extension of the basic model

The neoclassical investment model described in Hayashi (1982) and Abel and Eberly (1994) for all equity financed firms links stock returns to firm characteristics. This was noted by Restoy and Rockinger (1990) and Cochrane (1991, 1996) in a production based set-up. I hereby introduce an exogenous stochastic discount factor to assess the impact of both real and financing frictions on stock returns.[26]

Assume that the gross operating profits of the firm are both affected by both idiosyncratic shocks ϵ_t and undiversifiable aggregate shocks z_t such that $F(K_t, z_t, \epsilon_t)$. Furthermore, assume that the firm is subject to a stochastic discount factor such that

$$dz_t = -rz_t dt - \sigma_z z_t dW_t^z \tag{18}$$

where r stands the for short run risk free rate of the market. Since the firm is subject to a single source of aggregate risk by Equation (18), the asset pricing implications of the model predict that conditional CAPM holds once controlling for the firm characteristics.

Denote the market beta of equity by $\beta = \frac{zS_z}{S}$. Using equation (14), it is then possible to derive expressions for equity market betas, namely

$$\beta = \frac{zK}{S} q_z + \frac{zB}{S} S_{Bz} + \frac{z\left[\Lambda_z + \Xi_z + R_z\right]}{S} \tag{19}$$

Equation (19) reflects the underlying determinants of market betas. In particular, market betas are jointly affected by the investment and financing policies undertaken by the firm.

[26] For the sake of brevity, the set-up in Section 2.3 considers a risk-neutral framework. The same optimal corporate policies would hold for an exogenous pricing kernel as the one described in this subsection.

The contribution of each of these policies to the market beta depends on whether financing constraints are binding or not. Overall, financing constraints reduce firm value and increase market betas. Equation (19) thus provides an alternative testable implication for the impact of financing frictions on firm value.

While (19) provides an insight as to the fundamentals affecting equity betas, it does not provide a testable implication for stock returns. Consider now the reduced form expression for stock equity returns such that $R_S^e \equiv r_t + \xi(S)$, where the risk premium on equity $\xi(S)$ is given by $-cov(dS_t, dz_t)$. An alternative expression of stock returns can be obtained directly from the Bellman equation of (7), namely

$$R_S^e = \left(\frac{mD - x}{S}\right) + \frac{dK}{S}q + \frac{dB}{S}S_B + \left(-r + \frac{\sigma_z^2}{2}\frac{zS_{zz}}{S_z}\right)\beta + \frac{\sigma(\epsilon)^2}{2}\frac{S_{\epsilon\epsilon}}{S} \tag{20}$$

Equation (20) predicts that stock returns are positively related to earnings to price ratios; negatively related to net equity issues due to dilution; positively related to market to book for positive net increases in capital; negatively related to changes in leverage; positively related to a market premium for aggregate risk; and positively related to a premium on idiosyncratic risk. These predictions are consistent with most empirical studies on the cross section of stock returns (Fama and French (1992, 1993, 1997)), as well as with theoretical models discussing these regularities (Cochrane (1991), Berk et al (1999)). The negative relation between changes current leverage and stock returns is in line with Welch (2004) and Zhang et al (2005).

Most importantly, Equation (20) highlights that in the presence of fixed costs of investment and other frictions, the responsiveness of stock returns to market to book is non-linear. Consider first the frictionless case by Abel and Eberly (1994) where marginal q equals market to book. Equation (20) can then be re-expressed as a function of the rewards to investment $\Psi(q, K) = (q-1)I - G(I, K)$ such that

$$R_S^e = \frac{F(K, z, \epsilon)}{S} - \delta + \Psi(q, K) + \left(-r + \frac{\sigma_z^2}{2}\frac{zS_{zz}}{S_z}\right)\beta + \frac{\sigma(\epsilon)^2}{2}\frac{S_{\epsilon\epsilon}}{S} \tag{21}$$

where the optimal investment policy ensures that $\Psi(q, K)$ is positive when the firm is actively investing and zero in the inertia region. Consider now the case with financing frictions. While it is not longer true that marginal q equals market to book (see (14)), the same intuition applies. The optimal investment policy with fixed costs implies that stock returns are non-linear in market to book; furthermore, this non-linearity is time-varying since financing frictions affect the optimal inertia regions in investment policies as observed in Figure 2.

The main observations concerning the effect of real and financing frictions on corporate policies then translate into stock returns. In the presence of financing frictions, the returns to investment are endogenously related to the returns to financing. Furthermore, the lumpiness in investment policies due to fixed costs of investment results in non-linearity between stock returns and market to book ratios. In the empirical estimation of stock returns, the first observation requires the use of instruments to control for the changing financing costs of funding investment opportunities. The second observation requires the use of sorts to control for the impact of inertia regions of investment on stock returns.

4.2 Why has the model worked for stock returns?

Both Cochrane (1991) and Restoy and Rockinger (1994) show that stock returns and investment returns are analogous in a frictionless environment with quadratic adjustment costs of investment. Using this identity, Cochrane (1991, 1996) and more recently Liu, Whited and Zhang (2007) have documented that investment based asset pricing performs well in explaining stock returns. As a key feature, these models give no substantive role for financing; still, the conditional estimation usually controls for term premia, default premia and dividend yields. Recent models by Gomes Yaron and Zhang (2002), Obreja (2006) and Gomes and Schmid (2007) provide more refined dynamic investment models to assess the role of financing frictions on stock returns. Nonetheless, the standard estimation of investment asset pricing models with no distinct role for capital structure has been successful in explaining stock returns. A natural question that arises then is why the neoclassical frictionless investment model by Hayashi (1982) is successful in explaining stock returns and still has done a poor job in explaining investment data at both the aggregate and firm level.

Cochrane (1991) provides potential answers to this question. First, he suggests that the potential measurement error in Q arising in investment policies may be attenuated when working with stock returns at a higher frequency. Second, he suggests that models expressed as relations between returns can better capture firm's responses to time varying risk premia. This paper provides an alternative reason why Q-theory has historically performed better for stock returns, based on the impact of real and financing frictions on firms' corporate policies. This paper suggests that the estimated sensitivity of investment rates to Q is biased if the empirical approach fails to account for the effect of real and financing frictions on investment. The fact that coefficients on average Q have been typically low or insignificant surveyed by Caballero (1997) could be attributed to the biased estimation approach in empirical Q-theory, which has neglected the effect of frictions on the sensitivity of investment to Q. Meanwhile, the standard empirical approach in the investment based asset pricing literature has used both instruments and sorts, *naturally* controlling for the impact of real and financing frictions on stock returns.

The usual conditional estimation for stock returns has used default premia, dividend yields, term premia and the short term interest rate as instruments (see Fama and French, 1989; Ferson and Harvey, 1991). I consider these variables as natural instruments to control for the interaction between investment returns and the related costs of funding investment opportunities.

The empirical approach in investment based asset pricing also applies market to book and size sorts to ease the dimensionality problem of portfolio analysis (see Cochrane, 1991; Liu, Whited and Zhang, 2007). This paper demonstrates that fixed costs of investment create non-linearities between investment and marginal q, and that sorts by market to book control for this effect. Equation (21) also shows that the lumpiness in investment policies induces non-linearity between stock returns and market to book. This suggests market to book sorts as a useful tool to control for non-linearities in both investment and stock returns data. Market to book sorts are proposed by Fama and French (1992, 1993) in the empirical asset pricing literature. The paper then suggests that the usual sorts considered in the empirical asset pricing literature have economic content.

5 Conclusions

This paper provides an empirical approach to test the role of real and financing frictions on the investment, debt financing and equity financing policies of the firm. Due to the existence of financing frictions, the model provides testable implications for investment policies and also for financing policies as a function of average Q. Due to real and financing frictions, the sensitivity of optimal policies to Q is highly non-linear. The model contributes in showing that binding collateral constraints propagate lumpiness between investment and debt financing, and predicts that inertia regions of investment shift due to binding financing constraints.

The paper provides a simple empirical approach to control for both the endogeneity and non linearities created by real and financing frictions. The generalized linear approach by Hansen (1982) and Hansen and Singleton (1982) controls for endogeneity in all corporate policies. The use of sorts on market to book *a la* Fama and French (1992, 1993) controls for non-linearities in investment. The budget constraint of the firm controls for the impact of financing resources on investment. The paper then suggests that the use of sorts in the investment based asset pricing literature has economic content.

Finally, there has been little questioning as to why the Q-theory of investment model is successful in explaining stock returns but not investment policies. The current paper derives a stylized dynamic set-up to answer this question and provides supporting empirical evidence that financing frictions (i) create endogeneity between investment and financing and (ii) generate non linearity between investment rates and Q. While the standard conditional approach in investment asset pricing models naturally controls for these features using both instruments and sorts, the empirical tests of Q-theory in levels have overlooked the joint effect of real and financing frictions on corporate policies.

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6 Appendix

6.1 Average Q and marginal q

Consider first the general case where the firm alternates between periods of non-binding collateral constraints and periods of binding collateral constraints. The resulting the Bellman equation of S is given by

$$rS = (m+\lambda) D + (\nu - 1) x + dKS_K + dBS_B + \mu(\epsilon) S_\epsilon + \frac{\sigma(\epsilon)^2}{2} S_{\epsilon\epsilon} + \varphi [b - (\eta I + rB - \eta\delta K)]$$

Define the Dynkin's operator on a twice differentiable function, say f, as

$$A(f) = dKf_K + dBf_B + \mu(\epsilon) f_\epsilon + \frac{\sigma(\epsilon)^2}{2} f_{\epsilon\epsilon}$$

Consider the solution for the marginal cost of capital q. Differentiating with respect to K and using the operator A then yields

$$\begin{aligned} rq &= (m+\lambda) D_K + \lambda_K D + (\nu - 1) x_K + \nu_K x + I_K (q - \eta\varphi) \\ &\quad + b_K (S_B + \varphi) + \varphi_K [b - (\eta I + rB - \eta\delta K)] + A(q) - \delta_k q \end{aligned} \tag{22}$$

Using (6), D_K can be rewritten as

$$D_K = (1 - \tau_c) F_K + \tau_c \delta - (1 + G_I) I_K - G_K + b_K (1 + J_b) + x_K (1 - H_x)$$

The optimality conditions on I, b and x are given by

$$\begin{aligned} q &= (m+\lambda)(1 + G_I) + \varphi\eta \\ -S_B &= (m+\lambda)(1 + J_b) + \varphi \\ 0 &= (m+\lambda)(1 - H_x) + (\nu - 1) \end{aligned}$$

Replacing by the optimality conditions in equation (22) and using D_K, it then holds that

$$\begin{aligned} rq &= (m+\lambda) [(1 - \tau_c) F_K + \tau_c \delta - G_K] + \lambda_K D + \nu_K x \\ &\quad + \varphi_K [b - (\eta I + rB - \eta\delta_k K)] + A(q) - \delta_k q \end{aligned} \tag{23}$$

Multiplying by K, using the fact $A(q)K - \delta_k q \equiv A(qK) - Iq$ and invoking homogeneity of F and G yields

$$\begin{aligned} rqK &= (m+\lambda) [D - [b - rB + J] - [x - H]] - \lambda D_K K \\ &\quad - \nu x_K K - \varphi K [b_K - (\eta I_K + rB_K - \eta\delta)] + A(qK) \end{aligned}$$

where the complementarity slackness conditions on λN and vD hold pointwise. The Bellman equation for S can be written in a symmetric form as

$$rS = (m+\lambda)\, D + (\nu - 1)\, x + A(S) + \varphi\, [b - (\eta I + rB - \eta\delta K)]$$

Substracting and rearranging terms,

$$\begin{aligned} A(S-qK) - r(S-qK) &= -\,(m+\lambda)\,[(b - rB + J) + [x - H]] - (\nu - 1)\, x \\ &\quad -\lambda D_K K - v x_K K - \varphi K\, [b_K - (\eta I_K + rB_K - \eta\delta)] \end{aligned} \tag{24}$$

Using the properties of H, it holds that $H'(x)\, x = 2H(x)$. Then given the first order condition for equity issues, equation (24) can be re-expressed as

$$\begin{aligned} A(S-qK) - r(S-qK) &= -\,(m+\lambda)\,(b - rB + J) - (m+\lambda)\, H(x) \\ &\quad -\lambda D_K K - \nu x_K K - \varphi K\, [b_K - (\eta I_K + rB_K - \eta\delta)] \end{aligned}$$

Consider now the marginal cost of debt financing S_B. Applying the same approach as for q, it holds that

$$\begin{aligned} A(S_B B) - r(S_B B) &= -\,(m+\lambda)\,[b - rB + J] + \lambda D_B B \\ &\quad +\nu x_B B + \varphi B\, [b_B - (\eta I_B + r)] \end{aligned} \tag{25}$$

Finally, adding (24) and (25) yields the Euler equation for S such that

$$\begin{aligned} A(S - qK - S_B B) - r(S - qK - S_B B) &= -\,(m+\lambda)\, H(x) - \lambda\,(D_K K + D_B B) \\ &\quad -\nu\,(x_K K + x_B B) \\ &\quad -\varphi K\, [b_K - (\eta I_K - \eta\delta)] \\ &\quad -\varphi B\, [b_B - (\eta I_B + r)] \end{aligned}$$

Using Dynkin's formula and assuming the no bubble condition on firm value, the expression above yields

$$q_t K_t + S_{B_t} B_t = S_t - \Xi_t - \Lambda_t - R_t$$

where Λ_t reflects the shadow cost of the non-negativity constraints on equity value

$$\Lambda_t = E\left[\int_t^{\infty} e^{-rs} \lambda_s\, [D_{K_s} K_s + D_{B_s} B_s] + \nu_s\, [x_{K_s} K_s + x_{B_s} B_s]\, ds\right]$$

and Ξ_t is the net present value of equity flotation costs, namely

$$\Xi_t = E\left[\int_t^{\infty} e^{-rs}\,(m + \lambda_s)\, H_s ds\right]$$

and R_t reflects the shadow cost of collateral constraints on equity value, namely

$$R_t = E\left[\int_t^{\infty} e^{-rs} \varphi_s K\, [b_{Ks} - (\eta I_{Ks} - \eta\delta)] + \varphi_s B\, [b_{Bs} - (\eta I_{Bs} + r)]\, ds\right]$$

6.2 Binding collateral constraints

Consider now the extreme case of $\varphi > 0 \; \forall t$ where the collateral constraint is always binding such that $dB = \eta dK$ for all periods. The binding collateral constraint on debt implies $b = \eta \left(I + (r - \delta) K \right)$ and, due to the initial condition, it also holds that $B = \eta K$. The stock of debt of the firm is now proportional to the stock of capital and thus value function of the manager $S(K, \epsilon)$ does not depend on B. The optimality conditions with respect to K is given by

$$q = (m + \lambda) \left[(1 + G_I) - \eta (1 + J_b) \right] \tag{26}$$

such that the binding debt covenants extend the adjustment costs of investment to which the firm is subject to. Reordering terms in (26), the optimality condition of investment is

$$\frac{I}{K} = \delta - \frac{1 - \eta}{\alpha + \eta\gamma\tau_c} + \frac{q}{(m + \lambda)(\alpha + \eta\gamma\tau_c)} \tag{27}$$

such that both non-negativity constraints and debt covenants dampen the sensitivity of the investment capital ratio to marginal q.

Using the same approach as for the cases of non-binding collateral constraints, the expression for firm value is given by

$$qK = S - \Lambda - \Xi \tag{28}$$

Average Q overstates marginal q by the shadow cost of share repurchase constraints and flotation costs. The additional term related to the shadow cost of debt issuance disappears because the firm has lost its ability to decide on its optimal debt financing policy; all variables related to debt financing can be restated in terms of capital and investment. Note that equation (28) would be the same for an all-equity financed firm subject to non-negativity constraints and flotation costs of equity.

6.3 The inaction region of investment

In line with Abel and Eberly (1994), firms only invest when the maximand $\Psi(q, K) = (q - (m + \lambda + \varphi\eta)) I - (m + \lambda) G(I, K)$ is greater than zero. The inaction region of investment is thus given by $q \in [q_1, q_2]$ where $q_i \;\; i = 1, 2$ are the roots of the maximand $\Psi(q, K)$. The roots of the maximand depend on the shadow costs of financing and the firm's fixed costs of investment.

The optimality condition in the active region of investment is given by (9) and is such that it maximizes $\Psi(q, K)$. Replacing by (9) in $\Psi(q, K)$, dividing by K, and equating to zero yields

$$\frac{1}{2\alpha\left(m+\lambda\right)}\left(q-\left(m+\lambda+\varphi\eta\right)\right)^{2}+\delta\left(q-\left(m+\lambda+\varphi\eta\right)\right)-f=0 \tag{29}$$

where f is the fixed cost of investment per unit of capital. The roots of (29) are then cut-off values q_1 and q_2 of the optimal inaction region, and have the form

$$q_i=\left(m+\lambda+\varphi\eta\right)-\delta\alpha\left(m+\lambda\right)+\sqrt{\left[\delta\alpha\left(m+\lambda\right)\right]^{2}+2f\alpha\left(m+\lambda\right)}$$

The case of Abel and Eberly (1994) with no wedge on the price of capital obtains for $\lambda=0,\varphi=0$ and $m=1$, such that

$$q_i=1-\delta\alpha+\sqrt{\delta^{2}\alpha^{2}+2f\alpha}$$

for q_i $i=1,2$.

The statement that firms subject to binding collateral constraints $\varphi>0$ postpone positive investment to higher values of q implies

$$\frac{\partial q_2}{\partial\varphi}=\eta>0$$

Note that this is also true for q_1 such that firms disinvest at a higher marginal product of capital.

Consider now the proof that firms subject to flotation costs of equity may postpone positive investment to higher values of q. The optimality condition for equity issues when $\nu=0$ implies

$$1+H_x\simeq\left(m+\lambda\right)$$

where I have used a first order approximation of the term $\left(1-H_x\right)^{-1}$. Then an increase in the flotation costs of equity also induces firms to postpone investment since

$$\frac{\partial q_2}{\partial\left(m+\lambda\right)}=1-\delta\alpha+\left[\left(\delta\alpha\left(m+\lambda\right)\right)^{2}+2f\alpha\left(m+\lambda\right)\right]^{-\frac{1}{2}}\left[\left(\delta\alpha\right)^{2}\left(m+\lambda\right)+f\alpha\right]>0$$

The converse is true for the disinvestment cut-off such that firms with higher flotation costs postpone investment to a lower marginal product of capital.

Finally, if the fixed costs of investment per unit of capital f are larger, the firm also postpones investment to higher values since

$$\frac{\partial q_2}{\partial f}=\left[\left(\delta\alpha\left(m+\lambda\right)\right)^{2}+2f\alpha\left(m+\lambda\right)\right]^{-\frac{1}{2}}\alpha\left(m+\lambda\right)>0$$

The converse is true for q_1 such that inertia region increases for both investment and disinvestment.

Figure 1: The sensitivity of investment to q in the active region of investment

This table illustrates the impact of financing frictions on the derivative of optimal investment rates to marginal q. The solid line corresponds to the frictionless case (Hayashi, 1982), where the curvature of adjustment costs does not change with financing. The dotted dashed line applies to firms are subject to dividend constraints. Keeping b constant, the implied sensitivity of investment to q decreases when x increases. Given the optimality conditions for equity issues, these results also apply to the impact of flotation costs of equity on investment policies. The dashed line corresponds to the case where firms are affected by binding collateral constraints; keeping x constant, the sensitivity of investment rates is lower than the frictionless case. I chose the case where collateral constraints are permanently binding (Equation 10) for illustrative purposes.

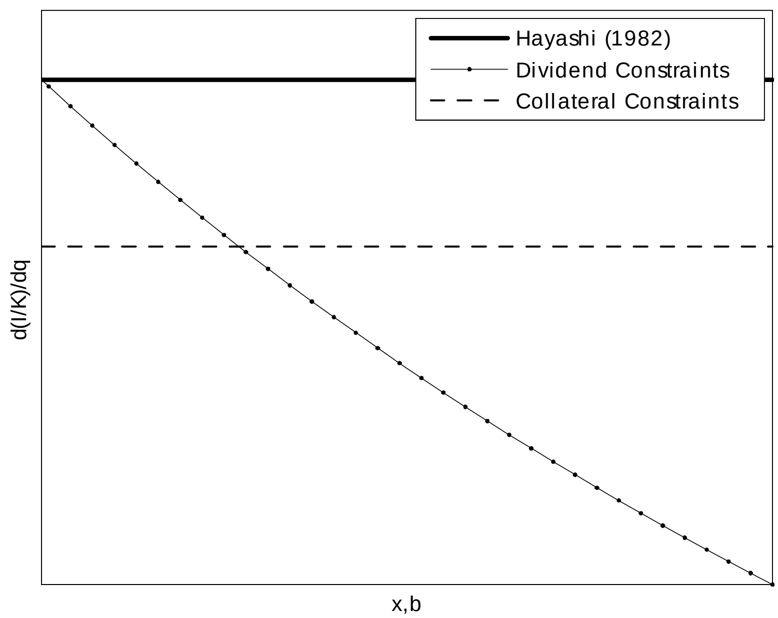

Figure 2: The inaction region of investment $q \in [q_1, q_2]$ and financing frictions

This figure illustrates an alternative channel through which financing frictions induce underinvestment. Firms only invest when the maximand $\Psi(q, K) = [q - (m + \lambda + \varphi\eta)]I - (m + \lambda)G(I, K)$ is greater than zero. The roots of the maximand q_1 and q_2 determine the inaction region of investment and depend both on the fixed costs of investment and the shadow costs of financing. The solid line corresponds to the financing frictionless case by Abel and Eberly (1994). The dotted dashed line corresponds to firms subject to dividend constraints; the inaction region becomes larger when collateral constraints are not binding. Given the optimality conditions for equity issues, these results also apply to the impact of flotation costs of equity on investment policies. The dashed line corresponds to firms subject to binding collateral constraints. Firms subject to binding collateral constraints postpone positive investment to higher values of q and also postpone sales of capital to higher values of q with respect to the financing frictionless case. The dashed dotted line corresponds to a financing frictionless case where firms have higher fixed costs than the solid line. The figure suggests that both an increase in fixed costs or binding financing frictions enlarge the inertia region of positive investment.

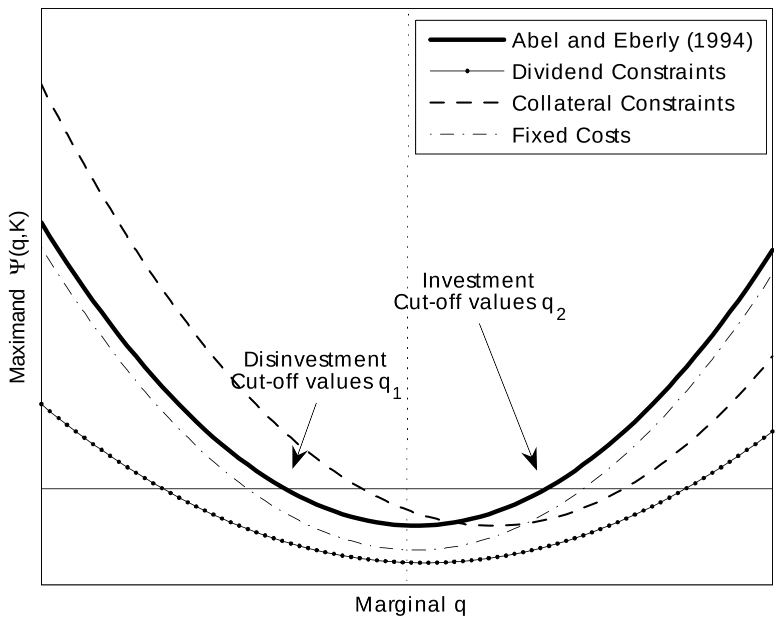

Table 1: Variable Definitions

This table lists the main variables of the model in Section 2. and provides their corresponding COMPUSTAT item in the working database used in Section 3. The working sample is drawn from the merged CRSP-COMPUSTAT annual database, considering the 1985-2005 period for US manufacturing firms in SICs 2000-3999. The sample is filtered for missing data, or for observations where total assets, the gross capital stock or sales are either zero or negative. All observations that fail to obey standard accounting identities are deleted. A firm is included in the sample if and only if it has at least five consecutive years of complete accounting data.

Variable	Description.	CRSP-COMPUSTAT Correspondence
S	market value of equity	product of item 199 times item 25
I	investment	difference between items 30 and 107
K	gross capital stock	item 7
δ	depreciation rate	item 14 divided by item 7
Q	average Q	$S + B$ divided by K
b	net debt issuance	difference between item 111 and item 114
B	total long term debt	sum of item 9 and item 34
rB	interest expenses	item 15
x	net equity issuance	difference between item 108 and item 115
D	total dividends paid	sum of item 21 and item 19
c	cash holdings	item 1
π	cash flows	sum of item 18 and item 14

Table 2: Summary Statistics

The database consists of public US industrial firms between 1985 and 2005. Panel A provides the average sample statistics. Panel B suggests that $\theta < 0$ identifies firms that are constrained to invest. When $\theta < 0$, firms are smaller in size, have lower Qs, lower investment to capital ratios and issue both less equity and less debt than firms in Panel A. Panel C shows that firms paying no dividends are smaller in size, have higher Qs and higher cash holdings than those in Panel A. Highly levered firms (Panel D) have lower Q, lower cash holdings and lower investment to capital ratios. Highly levered firms are those in the highest decile of book leverage.

Variable	**p25**	**p50**	**p75**	$\bar{x}$	σ	**N**
Panel A: All observations						
Q	0.983	1.367	2.058	1.751	1.244	21562
$\frac{I}{K}$	0.031	0.056	0.092	0.070	0.058	21562
$\frac{b}{K}$	-0.027	-0.002	0.012	0.000	0.090	21562
$\frac{x}{K}$	-0.002	0.000	0.008	0.017	0.105	21562
$\frac{c}{K}$	0.026	0.093	0.253	0.172	0.196	21562
$\frac{B}{K}$	0.076	0.242	0.408	0.268	0.218	21562
ln(S)	3.204	4.728	6.524	4.935	2.289	21562
Panel B: $\theta > 0$						
Q	0.879	1.155	1.613	1.428	0.964	10375
$\frac{I}{K}$	0.026	0.047	0.077	0.058	0.046	10375
$\frac{b}{K}$	-0.052	-0.014	0.000	-0.031	0.080	10375
$\frac{x}{K}$	-0.004	0.000	0.003	-0.002	0.066	10375
$\frac{c}{K}$	0.022	0.071	0.205	0.151	0.190	10375
$\frac{B}{K}$	0.112	0.288	0.460	0.307	0.231	10375
ln(S)	2.827	4.232	5.932	4.478	2.183	10375
Panel C: $D = 0$ and $x = 0$						
Q	0.789	1.038	1.449	1.302	0.940	2561
$\frac{I}{K}$	0.022	0.041	0.076	0.061	0.062	2561
$\frac{b}{K}$	-0.044	-0.007	0.011	-0.009	0.101	2561
$\frac{x}{K}$	0.000	0.000	0.000	0.000	0.000	2561
$\frac{c}{K}$	0.018	0.058	0.179	0.136	0.181	2561
$\frac{B}{K}$	0.165	0.378	0.573	0.381	0.259	2561
ln(S)	1.625	2.470	3.563	2.733	1.592	2561
Panel D: $\frac{B}{K} > 0.6$						
Q	1.040	1.293	1.651	1.488	0.740	1821
$\frac{I}{K}$	0.027	0.051	0.088	0.070	0.065	1821
$\frac{b}{K}$	-0.048	-0.001	0.086	0.025	0.147	1821
$\frac{x}{K}$	0.000	0.000	0.003	0.009	0.061	1821
$\frac{c}{K}$	0.011	0.027	0.066	0.058	0.091	1821
$\frac{B}{K}$	0.645	0.701	0.788	0.726	0.100	1821
ln(S)	2.193	3.339	4.639	3.466	1.719	1821

Table 3: Investment Policies sorted on $\frac{S}{K-B}$ only

This table shows the estimation results for investment using generalized linear GMM in quantiles sorted by market to book equity ratios. Instruments include all lagged regressors, the lagged changes in retained earnings, the lagged changes in working capital, year dummies and industry dummies. Observations are clustered by firm history. Each quantile is identified by its average market to book equity ratio. Results support the prediction that the relation between investment and Q is non-linear. Both J-tests and A-tests ensure that the model is identified in all quantiles. The coefficient on Q is only significant when market to book equity ratios larger than 2, where 2 is the lower bound of the fourth quantile. This suggests potential inaction regions in investment policies. Interest expenses, debt issues, equity issues and cash holdings prove all to be significant in explaining investment.

Variables	$\frac{S}{K-B}$ **quantiles**					**No**
	0.69	**1.20**	**1.72**	**2.50**	**4.88**	**Sort**
Q	0.002	0.035	0.027	0.051	0.015	0.010
	0.15	1.06	1.35	3.37	4.74	5.03
δ_k	0.647	0.800	1.031	1.085	1.007	0.899
	10.19	6.13	6.12	7.15	6.78	14.90
$\frac{rB}{K}$	0.539	1.827	2.205	1.489	1.292	1.325
	2.93	2.90	2.34	3.28	2.56	6.03
$\frac{b}{K}$	0.957	2.015	2.094	1.471	1.400	1.673
	5.80	3.91	2.97	3.33	3.30	8.22
$\frac{D}{K}$	0.034	0.045	0.001	-0.058	-0.020	0.018
	1.57	1.27	0.02	-1.33	-0.40	1.12
$\frac{x}{K}$	-0.157	-0.503	-0.486	-0.469	-0.271	-0.330
	-0.64	-1.69	-2.24	-3.56	-3.43	-6.02
$\frac{c}{K}$	0.027	0.111	0.130	0.055	0.070	0.081
	1.93	2.71	2.25	1.64	1.83	4.99
N	4313	4312	4313	4312	4312	21562
Clusters	1276	1621	1673	1633	1342	2468
RMSE	0.082	0.169	0.184	0.147	0.145	0.152
J_2**-stat**	11.9	2.7	0.5	1.2	5.5	3.7
J_2**-pval**	0.003	0.266	0.766	0.542	0.064	0.056
A_3**-stat**	58.1	20.2	16.0	20.7	18.8	69.2
A_3**-pval**	0.000	0.000	0.001	0.000	0.000	0.000
$\frac{\bar{I}}{K}$	0.059	0.070	0.074	0.075	0.075	0.070
$\bar{\theta}$	-0.088	-0.043	-0.005	0.023	0.053	-0.012
$\ln(\bar{S})$	3.229	4.484	5.250	5.638	6.075	4.935

Table 4: Investment Policies sorted on $\frac{S}{K-B}$ for $\theta > 0$

This estimation constrains the sample in Table 3 to firms with positive net funding resources. Results show that firms which are unconstrained to invest have higher coefficients on Q. Furthermore, the inaction region observed in Table 3 shrinks to lower market to book equity ratios. J-tests and A-tests ensure the goodness of fit of the model; as an exception, the model is not identified for the first quantile of market to book. Interest expenses, debt issues, equity issues and cash holdings are significant in explaining investment. The average investment rates and firm size in all quantiles are higher than those observed in Table 3.

Variables	$\frac{S}{K-B}$ **quantiles**					**No**
	0.69	**1.20**	**1.72**	**2.50**	**4.88**	**Sort**
Q	0.028	0.052	0.039	0.052	0.018	0.019
	0.58	1.03	2.43	3.47	3.89	6.90
δ_k	0.815	0.790	1.229	1.326	1.491	1.184
	6.55	2.90	6.84	8.11	5.66	11.75
$\frac{rB}{K}$	-0.674	0.108	0.579	0.499	0.537	0.035
	-2.12	0.15	1.36	1.20	1.07	0.16
$\frac{b}{K}$	0.996	1.866	1.594	1.135	1.923	1.767
	3.81	2.05	4.02	3.05	2.77	6.27
$\frac{D}{K}$	-0.218	-1.355	-0.167	-0.203	-0.099	-0.331
	-0.75	-1.49	-0.71	-1.01	-0.66	-2.50
$\frac{x}{K}$	-0.560	-1.066	-0.279	-0.304	-0.248	-0.350
	-0.92	-1.26	-2.44	-2.96	-2.54	-4.92
$\frac{c}{K}$	0.084	0.170	0.146	0.057	0.106	0.133
	2.42	1.62	3.10	1.61	1.79	4.55
N	1323	1924	2349	2709	2882	11187
Clusters	690	1055	1215	1275	1106	2321
RMSE	0.090	0.186	0.137	0.117	0.182	0.162
J_2**-stat**	6.3	1.2	0.7	1.5	1.9	0.9
J_2**-pval**	0.042	0.559	0.707	0.473	0.386	0.352
A_3**-stat**	6.7	10.2	25.4	22.0	12.6	46.2
A_3**-pval**	0.082	0.017	0.000	0.000	0.006	0.000
$\frac{\bar{I}}{K}$	0.075	0.084	0.085	0.083	0.082	0.082
$\bar{\theta}$	0.078	0.088	0.102	0.119	0.167	0.118
$\ln(\bar{S})$	3.416	4.559	5.369	5.769	6.392	5.359

Table 5: Investment Policies sorted on $\frac{S}{K-B}$ for $\theta < 0$

This estimation constrains the sample in Table 3 to firms with negative net funding resources. Instruments include all lagged regressors, year dummies, industry dummies, and lagged changes in working capital; the lagged changes in retained earnings do not contribute to the estimation and are not included. Results show that firms which are constrained to invest have lower coefficients on Q than firms in Tables 3 and 4. Furthermore, the inaction region is larger than that observed in Table 4 for firms with positive net funding. J-tests and A-tests ensure the goodness of fit of the model in the higher quantiles where firms are actively investing. The model is rejected on average in the absence of market to book sorts. Interest expenses, debt issues, equity issues and cash holdings are significant in explaining investment. The average investment rates and firm size in all quantiles are lower than those observed in Table 3.

Variables	**$\frac{S}{K-B}$ quantiles**					**No**
	0.69	**1.20**	**1.72**	**2.50**	**4.88**	**Sort**
Q	0.027	0.013	0.024	0.023	0.006	0.008
	2.69	1.10	2.70	2.26	1.99	7.48
δ_k	0.497	0.551	0.518	0.473	0.344	0.494
	9.81	10.20	9.99	4.02	3.34	16.99
$\frac{rB}{K}$	0.109	-0.180	0.157	0.682	-0.957	0.204
	0.58	-0.41	0.65	1.47	-1.99	2.01
$\frac{b}{K}$	0.462	0.059	0.300	0.809	-0.566	0.316
	2.16	0.14	1.60	2.02	-1.52	2.92
$\frac{D}{K}$	0.017	0.020	-0.003	-0.038	-0.011	0.030
	0.96	1.83	-0.19	-1.44	-0.27	3.22
$\frac{x}{K}$	-0.254	-0.016	-0.192	-0.222	0.077	-0.193
	-1.06	-0.14	-2.24	-1.46	0.69	-4.74
$\frac{c}{K}$	-0.021	-0.018	-0.018	-0.037	-0.052	-0.021
	-2.28	-2.15	-2.02	-2.16	-2.19	-4.71
N	2990	2387	1964	1603	1430	10374
Clusters	1113	1192	1100	939	777	2284
RMSE	0.049	0.040	0.044	0.076	0.077	0.048
J_1**-stat**	31.4	26.6	8.7	1.3	0.6	121.3
J_1**-pval**	0.000	0.000	0.003	0.247	0.456	0.000
A_2**-stat**	9.4	2.4	8.2	10.2	6.1	32.5
A_2**-pval**	0.009	0.304	0.017	0.006	0.047	0.000
$\frac{\bar{I}}{K}$	0.051	0.059	0.061	0.061	0.061	0.058
$\bar{\theta}$	-0.161	-0.149	-0.133	-0.140	-0.176	-0.152
$\ln(\bar{S})$	3.146	4.424	5.108	5.415	5.435	4.478

Table 6: Investment Policies sorted on θ only

This table shows the estimation results for investment using generalized linear GMM in quantiles sorted by θ, where a lower θ suggests that firms are constrained to invest. Instruments include all lagged regressors, lagged changes in retained earnings, lagged changes in working capital, year dummies and industry dummies. Each quantile is identified by its corresponding average θ. Results suggest a quadratic relation between the coefficient on Q and θ; the empirical evidence supports the assumption of convex costs of financing. The model fails to reject J-tests only when firms have positive net funding resources. Cash holdings are negatively related to Q when $\theta<0$ and positively related to Q when $\theta>0$.

Variables	$\frac{S}{K-B}$ **quantiles**					**No**
	-0.29	**-0.06**	**0.01**	**0.07**	**0.21**	**Sort**
Q	0.014	0.007	0.010	0.012	0.022	0.010
	5.19	4.00	8.23	7.85	4.59	5.03
δ_k	0.400	0.709	0.723	1.076	1.341	0.899
	8.00	14.12	12.44	15.47	6.78	14.90
$\frac{rB}{K}$	-0.722	0.286	-0.007	0.104	-0.145	1.325
	-2.66	2.17	-0.08	0.64	-0.39	6.03
$\frac{b}{K}$	-0.410	0.893	-0.091	0.863	1.628	1.673
	-2.08	2.93	-0.60	4.91	4.17	8.22
$\frac{D}{K}$	0.033	-0.029	0.195	-0.180	-0.795	0.018
	2.46	-0.66	3.33	-1.59	-1.91	1.12
$\frac{x}{K}$	-0.085	-0.328	-0.055	-0.289	-0.321	-0.330
	-1.24	-4.50	-1.27	-4.30	-3.36	-6.02
$\frac{c}{K}$	-0.028	-0.022	-0.026	0.051	0.220	0.081
	-3.42	-2.27	-3.94	3.58	3.07	4.99
N	4315	4312	4313	4312	4310	21562
Clusters	1711	1774	1737	1757	1754	2468
RMSE	0.063	0.065	0.041	0.068	0.197	0.152
J_2**-stat**	26.8	59.1	89.7	51.1	1.8	3.7
J_2**-pval**	0.000	0.000	0.000	0.000	0.407	0.056
A_3**-stat**	15.0	6.4	55.6	88.6	26.1	69.2
A_3**-pval**	0.002	0.096	0.000	0.000	0.000	0.000
$\frac{\bar{I}}{K}$	0.056	0.060	0.067	0.076	0.093	0.070
$\frac{\bar{S}}{K-B}$	2.212	2.154	2.154	2.193	2.270	2.196
$\ln(\bar{S})$	4.063	5.222	5.584	5.285	4.521	4.935

Table 7: Investment Policies sorted by $\frac{S}{K-B}$ for $D = 0$ and $x = 0$, and $\frac{B}{K} > 0.6$

Panel A constrains the estimation of investment policies in Table 3 to firms that both issue no equity and pay no dividends. Results supports the prediction that firms that are not paying dividends and not issuing equity show lower (insignificant) coefficients on Q with respect to those in Table 3. J-tests and A-tests further show that the model is not accurate to explain the investment rates of constrained firms. Panel B supports the prediction that firms with high book leverage may be constrained in their investment policies. The model is rejected by either J-tests and/or A-tests.

Panel A: $D = 0$ and $x = 0$						
Variables	$\frac{S}{K-B}$ **quantiles**					**No**
	0.69	**1.20**	**1.72**	**2.50**	**4.88**	**Sort**
Q	0.012	0.007	0.000	-0.019	0.005	-0.003
	0.72	0.29	0.00	-0.90	0.66	-0.39
N	1199	521	314	274	253	2561
Clusters	537	371	244	216	180	927
RMSE	0.065	0.062	0.064	0.088	0.058	0.121
J_2**-stat**	15.2	18.1	8.9	9.4	8.3	12.5
J_2**-pval**	0.004	0.001	0.063	0.052	0.081	0.014
A_3**-stat**	13.4	2.6	9.5	3.7	4.3	11.1
A_3**-pval**	0.020	0.761	0.092	0.599	0.503	0.049
$\frac{\bar{I}}{K}$	0.05	0.06	0.07	0.07	0.07	0.06
$\bar{\theta}$	-0.09	-0.06	-0.04	-0.06	-0.09	-0.07
$\ln(\bar{S})$	2.37	2.88	3.15	3.08	3.20	2.73
Panel B: $\frac{B}{K} > 0.6$						
Variables	$\frac{S}{K-B}$ **quantiles**					**No**
	0.69	**1.20**	**1.72**	**2.50**	**4.88**	**Sort**
Q	-0.040	-0.007	0.003	0.044	0.016	0.004
	-0.42	-0.29	0.07	1.38	1.37	0.30
N	520	388	290	294	329	1821
Clusters	279	238	204	214	213	623
RMSE	0.222	0.068	0.075	0.077	0.072	0.115
J_2**-stat**	0.7	8.6	0.7	3.1	6.9	0.9
J_2**-pval**	0.699	0.014	0.688	0.214	0.031	0.634
A_3**-stat**	2.0	10.9	3.4	9.9	15.0	18.0
A_3**-pval**	0.580	0.013	0.328	0.020	0.002	0.000
$\frac{\bar{I}}{K}$	0.06	0.07	0.08	0.07	0.08	0.07
$\bar{\theta}$	-0.11	-0.07	-0.04	-0.04	-0.08	-0.07
$\ln(\bar{S})$	2.87	3.36	3.77	3.89	3.89	3.47

Table 8: Debt Policies sorted on $\frac{S}{K-B}$ only

This table shows the result of estimating positive debt issues through generalized linear GMM in quantiles sorted by market to book ratios. Instruments include all lagged regressors, lagged changes in retained earnings, lagged changes in working capital, year dummies and industry dummies. All observations are clustered by firm. Each quantile is denoted by its corresponding average market to book equity ratio. Results suggest that optimal debt policies are significantly related to average Q in the region where firms are actively investing; otherwise the model is rejected and does not explain positive debt issues. R-tests suggest that instruments are valid for all quantiles.

Variables	$\frac{S}{K-B}$ **quantiles**					**No**
	0.73	**1.25**	**1.73**	**2.44**	**4.65**	**Sort**
Q	0.093	0.077	0.055	0.025	0.002	0.010
	2.11	1.93	2.03	1.96	2.02	2.56
δ_k	-0.022	0.256	-0.466	-0.128	0.301	-0.079
	-0.09	0.72	-2.04	-0.73	0.80	-0.73
$\frac{rB}{K}$	-0.446	0.011	-0.270	0.518	0.004	0.045
	-1.87	0.03	-1.07	1.48	0.01	0.30
$\frac{I}{K}$	0.098	0.133	0.319	0.369	-0.045	0.185
	0.69	0.71	2.43	3.07	-0.19	2.45
$\frac{D}{K}$	-0.068	0.200	-0.001	-0.086	0.088	0.062
	-1.00	1.23	-0.01	-1.13	0.60	1.11
$\frac{x}{K}$	0.467	0.834	0.721	0.252	0.614	0.760
	0.78	1.27	1.69	0.87	2.15	3.82
$\frac{c}{K}$	-0.132	-0.080	-0.231	-0.064	-0.051	-0.118
	-3.09	-1.32	-4.76	-1.47	-0.67	-3.75
N	754	751	752	751	751	3759
Clusters	306	371	390	367	293	623
RMSE	0.072	0.087	0.080	0.075	0.106	0.091
J_2**-stat**	25.3	1.8	0.2	3.4	2.8	1.2
J_2**-pval**	0.000	0.413	0.916	0.184	0.253	0.538
A_3**-stat**	22.4	19.2	75.5	26.0	54.0	232.2
A_3**-pval**	0.000	0.000	0.000	0.000	0.000	0.000
$\frac{\bar{I}}{K}$	0.089	0.100	0.101	0.099	0.096	0.097
$\bar{\theta}$	0.014	0.050	0.063	0.078	0.115	0.064
$\ln(\bar{S})$	4.019	5.361	5.991	6.480	7.145	5.799

Table 9: Debt Policies sorted on $\frac{S}{K-B}$ and $\frac{B}{K}$

This table considers the sorted estimation of positive debt issues in Table 8 conditional on book leverage. For the sake of brevity, only coefficients on Q and tests on goodness of fit are reported. Highly levered firms are defined as those in the highest decile of the book leverage within the working sample. In Panel A (lower leverage), positive debt issues are significantly related to average Q when firms are actively investing. As an exception, the model is also accepted for the lowest quantile of market to book, where firms are not actively investing according to Tables 3-5. In Panel B (higher leverage), the model is usually identified except for the highest quantile on market to book, and the coefficient on Q is always insignificant.

Panel A: $\frac{B}{K} < 0.6$						
Variables	$\frac{S}{K-B}$ **quantiles**					**No**
	0.73	**1.25**	**1.73**	**2.44**	**4.65**	**Sort**
Q	0.083	0.026	0.051	0.031	0.004	0.009
	2.63	0.67	1.84	2.04	1.95	2.39
N	628	654	670	683	668	3303
Clusters	270	337	359	335	262	610
RMSE	0.066	0.085	0.069	0.067	0.093	0.082
J_2**-stat**	2.0	3.4	0.2	2.8	0.3	2.9
J_2**-pval**	0.377	0.183	0.914	0.246	0.840	0.229
R_3**-stat**	22.8	17.9	76.8	28.0	26.3	171.3
R_3**-pval**	0.000	0.000	0.000	0.000	0.000	0.000
$\frac{\bar{I}}{K}$	0.090	0.102	0.099	0.099	0.097	0.097
$\bar{\theta}$	0.018	0.051	0.058	0.080	0.122	0.067
$\ln(\bar{S})$	4.174	5.557	6.222	6.695	7.486	6.056
Panel B: $\frac{B}{K} > 0.6$						
Q	0.145	0.092	0.346	0.066	0.028	0.013
	1.11	1.17	0.64	1.04	1.01	0.67
N	126	97	82	68	83	456
Clusters	78	64	62	57	61	182
RMSE	0.095	0.094	0.100	0.140	0.135	0.104
J_2**-stat**	4.9	1.1	0.2	3.2	2.9	1.7
J_2**-pval**	0.086	0.566	0.897	0.201	0.234	0.429
A_3**-stat**	4.7	1.7	0.2	1.7	31.9	56.4
A_3**-pval**	0.192	0.634	0.971	0.639	0.000	0.000
$\frac{\bar{I}}{K}$	0.088	0.083	0.116	0.097	0.096	0.095
$\bar{\theta}$	-0.002	0.042	0.101	0.060	0.056	0.046
$\ln(\bar{S})$	3.249	4.051	4.099	4.305	4.397	3.939

Table 10: Equity Policies sorted on $\frac{S}{K-B}$ only

This table shows the result of estimating net equity issues through generalized linear GMM sorted by market to book equity ratios. Instruments include all lagged regressors, lagged changes in retained earnings, lagged changes in working capital, year dummies and industry dummies. Each quantile is denoted by its average market to book equity ratio. J-tests reject the model in the lowest quantile of market to book equity ratios. Equity issues are positively and significantly related to average Q in the action region of investment according to Table 3. Investment to capital ratios, dividend payments to shareholders, debt issues and interest expenses are also significant in explaining equity issues. The range of average market to book ratios of all quantiles suggest that equity issuing firms have a higher market to book ratio than all firms on average.

Panel A: Sort by $\frac{S}{K}$

Variables	$\frac{S}{K-B}$ **quantiles**					**No**
	0.86	**1.41**	**1.95**	**2.78**	**5.08**	**Sort**
Q	0.015	0.063	0.156	0.163	0.047	0.041
	0.05	0.56	1.76	2.24	2.82	7.18
δ_k	0.190	1.495	2.119	3.136	2.560	1.853
	1.79	2.52	2.59	2.30	3.11	6.24
$\frac{rB}{K}$	0.282	3.540	3.860	5.699	5.246	3.606
	1.62	3.02	3.12	2.40	4.24	6.87
$\frac{b}{K}$	0.245	2.727	3.067	5.990	3.655	3.309
	1.37	2.49	2.16	2.33	3.51	5.92
$\frac{D}{K}$	-0.130	-1.651	-1.951	-3.487	-1.622	-1.873
	-1.05	-2.43	-2.44	-2.49	-2.78	-6.02
$\frac{I}{K}$	-0.029	0.052	0.151	0.293	-0.090	0.126
	-1.15	0.37	0.82	0.55	-0.36	1.24
$\frac{c}{K}$	0.029	0.250	0.186	0.149	0.337	0.218
	2.00	2.76	2.68	1.25	4.53	6.45
N	1775	1769	1774	1775	1773	8866
Clusters	696	823	863	858	698	1332
RMSE	0.054	0.277	0.298	0.540	0.426	0.341
J_2**-stat**	8.4	1.2	2.7	0.5	1.1	4.3
J_2**-pval**	0.015	0.558	0.266	0.796	0.589	0.117
A_3**-stat**	17.4	9.2	10.2	10.2	33.2	72.7
A_3**-pval**	0.001	0.027	0.017	0.017	0.000	0.000
$\frac{\bar{I}}{K}$	0.067	0.073	0.076	0.078	0.074	0.074
$\bar{\theta}$	-0.054	0.011	0.033	0.068	0.090	0.029
$\ln(\bar{S})$	3.798	4.850	5.144	5.383	5.590	4.953

Table 11: Equity Policies sorted on $\frac{S}{K-B}$ and D

This table shows the estimation of positive net equity issues through generalized linear GMM in quantiles conditional on payout policies. For the sake of brevity, only coefficients on Q and tests on goodness of fit are reported. In Panel A, ($D = 0$) the model is accepted when firms are actively investing according to the cut-off values implied by Table 4. The coefficients on Q are higher than those in Table 10 for the same quantiles, suggesting that firms that issue equity and do not distribute dividends have a lower cost of capital than the average sample. J-tests fail to reject the goodness of fit of the model in the higher quantiles; meanwhile, A-tests suggest that the model is identified only at the 6In Panel B ($D > 0$), the model is rejected by A-tests in all quantiles but the first, and the coefficient on Q is not significant. The investment based explanation for equity issues holds when dividends are negatively correlated to equity issues.

Panel A: $D = 0$

Variables	$\frac{S}{K-B}$ **quantiles**					**No**
	0.86	**1.41**	**1.95**	**2.78**	**5.08**	**Sort**
Q	0.026	-0.001	0.224	0.198	0.052	0.044
	1.56	-0.01	2.72	1.98	3.01	6.75
N	1222	1035	1071	1190	1415	5933
Clusters	530	573	603	632	565	1033
RMSE	0.056	0.356	0.292	0.443	0.398	0.353
J_2**-stat**	16.3	3.4	6.1	1.7	2.4	7.7
J_2**-pval**	0.001	0.338	0.109	0.646	0.493	0.054
A_3**-stat**	16.9	8.9	9.3	9.0	34.3	64.8
A_3**-pval**	0.002	0.063	0.055	0.060	0.000	0.000
$\frac{\bar{I}}{K}$	0.064	0.070	0.073	0.077	0.073	0.071
$\bar{\theta}$	-0.061	0.018	0.042	0.073	0.096	0.036
$\ln(\bar{S})$	3.369	4.040	4.422	4.716	5.177	4.377

Panel B: $D > 0$

Variables	$\frac{S}{K-B}$ **quantiles**					**No**
	0.86	**1.41**	**1.95**	**2.78**	**5.08**	**Sort**
Q	0.030	-0.018	0.021	0.005	0.017	0.008
	1.45	-0.44	0.34	0.55	1.58	1.91
N	553	734	703	585	358	2933
Clusters	247	306	309	262	157	534
RMSE	0.034	0.082	0.249	0.065	0.081	0.049
J_2**-stat**	0.1	0.0	0.1	0.5	1.3	2.6
J_2**-pval**	0.963	0.996	0.975	0.791	0.526	0.273
A_3**-stat**	16.1	0.6	0.6	2.7	5.7	12.1
A_3**-pval**	0.001	0.905	0.905	0.442	0.130	0.007
$\frac{\bar{I}}{K}$	0.073	0.077	0.082	0.079	0.080	0.078
$\bar{\theta}$	-0.039	0.000	0.018	0.055	0.067	0.016
$\ln(\bar{S})$	4.748	5.992	6.246	6.741	7.225	6.118

Table 12: OLS vs GMM without Sorts

This table compares the OLS estimation results of corporate policies to the GMM estimation described in Section 3. Results support the prediction that financing frictions create endogeneity between investment and financing policies. Average Q is significant for all cases and relates positively to all corporate policies. The use of instruments may change the sign and increases the magnitude of significant coefficients in all cases. The Hausman test (H-pval) on unclustered data shows that the GMM estimation is consistent if the model is correctly specified.

Panel A: $\frac{I}{K}$			Panel B: $\frac{b}{K}$			Panel C: $\frac{x}{K}$		
	OLS	**GMM**		**OLS**	**GMM**		**OLS**	**GMM**
Q	0.006	0.010	**Q**	0.005	0.010	**Q**	0.018	0.041
	13.31	5.03		3.02	2.56		9.95	7.18
δ_k	0.540	0.899	δ_k	-0.505	-0.079	δ_k	-0.082	1.853
	21.10	14.90		-7.79	-0.73		-1.51	6.24
$\frac{rB}{K}$	-0.434	1.325	$\frac{rB}{K}$	0.826	0.045	$\frac{rB}{K}$	0.857	3.606
	-17.26	6.03		9.57	0.30		9.08	6.87
$\frac{b}{K}$	0.157	1.673	$\frac{I}{K}$	0.398	0.185	$\frac{b}{K}$	-0.163	3.309
	22.26	8.22		14.23	2.45		-9.76	5.92
$\frac{D}{K}$	-0.016	0.018	$\frac{D}{K}$	-0.038	0.062	$\frac{D}{K}$	0.167	-1.873
	-2.64	1.12		-1.39	1.11		5.39	-6.02
$\frac{x}{K}$	0.040	-0.33	$\frac{x}{K}$	-0.141	0.760	$\frac{I}{K}$	-0.131	0.126
	7.72	-6.02		-5.12	3.82		-3.89	1.24
$\frac{c}{K}$	-0.06	0.081	$\frac{c}{K}$	0.05	-0.118	$\frac{c}{K}$	0.23	0.22
	-23.25	4.99		2.23	-3.75		16.40	6.45
R^2	0.263		R^2	0.133		R^2	0.205	
N	21562	21562	**N**	3759	3759	**N**	8866	8866
Clusters	2468	2468	**Clusters**	623	623	**Clusters**	1332	1332
RMSE	0.050	0.152	**RMSE**	0.075	0.091	**RMSE**	0.116	0.341
J_2**-stat**		3.7	J_2**-stat**		1.2	J_2**-stat**		4.3
J_2**-pval**		0.056	J_2**-pval**		0.538	J_2**-pval**		0.117
A_3**-stat**		69.2	A_3**-stat**		232.2	A_3**-stat**		72.7
A_3**-pval**		0.000	A_3**-pval**		0.000	A_3**-pval**		0.000
$\frac{\bar{I}}{K}$		0.07	$\frac{\bar{I}}{K}$		0.10	$\frac{\bar{I}}{K}$		0.07
$\bar{\theta}$		-0.01	$\bar{\theta}$		0.06	$\bar{\theta}$		0.03
$\ln(\bar{S})$		4.94	$\ln(\bar{S})$		5.80	$\ln(\bar{S})$		4.95
H-stat		110.81	**H-stat**		63.64	**H-stat**		96.94
H-pval		0.000	**H-pval**		0.008	**H-pval**		0.000

Table 13: Applying OLS and Market to Book Sorts to Investment Policies

This table provides an additional test of the magnitude of endogeneity vis a vis non-linearities in corporate policies. In Panel A, inaction regions in investment still appear when applying OLS to sorted data; still, the coefficients on Q are significantly lower and the lowest quantile on market to book is also significant. Panel B constrains the estimation to firms with positive net funding resources; the implied inaction region shrinks, yet the coefficients on Q are significantly lower than those in Table 4.

Panel A: All observations

Variables	$\frac{S}{K-B}$ **quantiles**					**No**
	0.69	**1.20**	**1.72**	**2.50**	**4.88**	**Sort**
Q	0.009	0.003	0.008	0.006	0.004	0.006
	2.18	1.06	3.65	3.78	5.41	13.31
N	4313	4312	4313	4312	4312	21562
Clusters	1276	1621	1673	1633	1342	2468
RMSE	0.045	0.050	0.047	0.051	0.053	0.050
R^2	0.241	0.260	0.311	0.255	0.262	0.262
$\frac{\bar{I}}{K}$	0.059	0.070	0.074	0.075	0.075	0.070
$\bar{\theta}$	-0.088	-0.043	-0.005	0.023	0.053	-0.012
$\ln(\bar{S})$	3.229	4.484	5.250	5.638	6.075	4.935

Panel B: $\theta > 0$

Variables	$\frac{S}{K-B}$ **quantiles**					**No**
	0.69	**1.20**	**1.72**	**2.50**	**4.88**	**Sort**
Q	-0.002	-0.003	0.006	0.006	0.004	0.005
	-0.32	-0.68	2.63	3.13	5.03	9.53
N	1323	1924	2349	2709	2882	11187
Clusters	690	1055	1215	1275	1106	2321
RMSE	0.054	0.058	0.052	0.053	0.054	0.055
R^2	0.270	0.275	0.339	0.283	0.294	0.285
$\frac{\bar{I}}{K}$	0.075	0.084	0.085	0.083	0.082	0.082
$\bar{\theta}$	0.078	0.088	0.102	0.119	0.167	0.118
$\ln(\bar{S})$	3.416	4.559	5.369	5.769	6.392	5.359

Table 14: Sorting Investment into More Quantiles of Market to Book

This table reports the coefficients on Q and goodness of fit indicators for all quantiles when sorting investment rates into 10 quantiles of market to book. Panel A considers all observations and shows similar results to those in Table 3 using 5 quantiles. The implied active region of investment starts on the seventh quantile, whose lower bound is a market to book equity ratio of 2. Both J-tests and A-tests confirm the goodness of fit of the model in the active region; as an exception, the J-test in the last quantile of market to book is rejected at the 5 per cent level. Panel B constrains the estimation to firms with positive net funding resources. In line with results in Table 4, the inaction region shrinks for firms with positive net funding resources. J-tests ensure the goodness of fit of the model in the active region; A-tests however are sometimes rejected at the 5 per cent threshold.

Panel A: All Observations

Variables	$\frac{S}{K-B}$ **quantiles**										**No**
	0.53	**0.85**	**1.08**	**1.32**	**1.57**	**1.86**	**2.23**	**2.77**	**3.64**	**6.11**	**Sort**
Q	-0.021	0.000	-0.001	-0.041	0.016	0.046	0.062	0.050	0.008	0.016	0.010
t-stat	-0.50	0.00	-0.03	-0.93	0.58	1.09	2.29	3.31	0.47	5.38	5.03
N	2157	2156	2156	2156	2156	2157	2156	2156	2156	2156	21562
Clusters	818	1040	1147	1181	1214	1215	1207	1143	1076	863	2468
RMSE	0.099	0.071	0.088	0.133	0.124	0.299	0.159	0.108	0.190	0.077	0.152
J_2**-stat**	4.8	10.5	23.8	0.2	0.4	0.3	0.4	4.4	0.3	7.2	3.7
J_2**-pval**	0.089	0.005	0.000	0.921	0.822	0.878	0.821	0.110	0.858	0.028	0.056
A_3**-stat**	20.5	29.5	9.0	17.9	12.8	3.5	14.2	8.1	7.7	24.6	69.2
A_3**-pval**	0.000	0.000	0.029	0.000	0.005	0.320	0.003	0.045	0.053	0.000	0.000
$\frac{\bar{I}}{K}$	0.055	0.062	0.069	0.071	0.071	0.076	0.076	0.074	0.075	0.075	0.070
$\bar{\theta}$	-0.097	-0.079	-0.062	-0.024	-0.012	0.003	0.018	0.028	0.046	0.060	-0.012
$\ln(\bar{S})$	2.832	3.626	4.289	4.680	5.125	5.375	5.488	5.787	6.013	6.136	4.935

Panel B: $\theta > 0$

Variables	$\frac{S}{K-B}$ **quantiles**										**No**
	0.53	**0.85**	**1.08**	**1.32**	**1.57**	**1.86**	**2.23**	**2.77**	**3.64**	**6.11**	**Sort**
Q	-0.161	-0.046	0.071	-0.004	0.032	0.033	0.070	0.045	0.022	0.018	0.019
t-stat	-0.50	-0.66	2.29	-0.10	1.53	1.95	2.49	1.83	1.65	4.63	6.90
N	548	775	857	1067	1123	1226	1331	1378	1433	1449	11187
Clusters	345	525	616	715	766	824	866	842	811	677	2321
RMSE	0.204	0.071	0.066	0.135	0.092	0.168	0.135	0.137	0.137	0.100	0.162
J_2**-stat**	1.1	2.4	20.2	1.3	0.1	3.6	0.6	2.2	0.8	3.3	0.9
J_2**-pval**	0.585	0.306	0.000	0.527	0.957	0.166	0.727	0.340	0.680	0.192	0.352
A_3**-stat**	3.0	5.9	4.4	14.9	26.1	6.9	15.5	6.8	5.2	16.8	46.2
A_3**-pval**	0.386	0.117	0.225	0.002	0.000	0.074	0.001	0.080	0.159	0.001	0.000
$\frac{\bar{I}}{K}$	0.073	0.077	0.085	0.084	0.082	0.087	0.084	0.081	0.082	0.082	0.082
$\bar{\theta}$	0.070	0.083	0.084	0.090	0.100	0.103	0.116	0.122	0.150	0.183	0.118
$\ln(\bar{S})$	2.996	3.712	4.333	4.741	5.299	5.434	5.573	5.958	6.285	6.498	5.359