The Dynamics of Going Public

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Abstract

This paper develops a signalling game in which the decision to raise public equity is a real option of the firm. Firms signal their type through the timing to go public, the fraction of shares issued and the underpricing of shares. The model provides a tractable approach to solve for signalling games in a real options framework and relates signalling to IPO waves. In hot markets, younger firms go public and there is overvaluation of low quality issuers. In cold markets, better firms accelerate their decision to go public and optimally underprice their shares to avoid imitation. The model solves for underpricing endogenously and provides a rational approach to analyze underpricing and market timing. The empirical section tests the model and highlights firm age as a significant firm characteristic in equity issues. The paper provides supporting evidence that the age of issuing firms decreases during hot IPO markets.

Keywords: IPOs, SEOs, real options, signalling games, asymmetric information.
JEL Classification Numbers: G14, G31, G32

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1 Introduction

Ever since Ibbotson (1975) has first documented the large underpricing in initial public offerings (IPOs), there has been a growing literature on IPOs. The study of IPOs gained further attention during the 1990s with the seminar work by Ritter (1991) on long run underperformance and the increased IPO activity until the burst of the NASDAQ bubble. Most of the theory on IPOs, though, is static in nature: it posits multi period set-ups where the time at which firms become public is taken as given. For instance, the papers by Rock (1986) and Welch (1989) provide alternative explanations to underpricing in IPOs where the time at which the firm goes public is deterministic. This paper thus adds to the current literature in IPOs by analyzing the decision to go public as a real option of the firm. As a distinct feature, the framework emphasizes that the timing to raise public equity is also relevant to explain the decision to go public.

In line with Welch (1989), Grimblatt and Hwang (1989) and Allen and Faulhaber (1989), there is asymmetric information between managers and market players. Since the single managing shareholder of the firm has better information about his firm’s future cash flows, he may provide information about the true value of the firm through the going public strategy. This paper explores a dynamic signalling game of the decision to raise public equity, where firms reveal their type through both dynamic signals such as the timing to do a public offering and static signals such as the fraction of shares issued and the underpricing of shares. While the static signals have been studied elsewhere (see Vandemaele, 1997, for a survey), this paper suggests that the time at which the firm raises public equity may also convey information to market players. The model is thus strongly related to the literature on real options and asymmetric information initiated by Grenadier (1999).

Several contributions follow from analyzing the decision to go public in the current framework. First, this paper adds to the study of real options and asymmetric information in considering the impact of signalling on the option exercise strategies of firms. When firms reveal their type in equilibrium, the model shows that more productive firms go public earlier to make imitation more costly. When there is no revelation in equilibrium, less productive firms accelerate the decision to go public to enhance the pricing of their shares. The model extends to show that firms optimally pool in hot markets and reveal their type otherwise, such that on average younger firms go public during periods of high IPO activity.

Second, the paper provides a tractable approach to model underpricing in a real options set-up. While there has been a growing literature on option exercise strategies and run-
up effects,\textsuperscript{1} this literature solves for the timing to exercise the option assuming that is always informative and then defines underpricing as the difference between the actual value of the firm and its prior expected value by market players. By contrast, this model considers underpricing as a signal and demonstrates that underpricing and the timing to go public must be jointly determined in equilibrium. The usual approach in the literature to solve for optimal timing and underpricing need not be an equilibrium; this paper suggests that the optimal timing to go public in separating equilibria must solve for underpricing endogenously, ensuring that market beliefs are self-fulfilling.

The implications on underpricing are directly translated into abnormal returns. The model predicts non negative abnormal returns for issuing stocks when firms reveal their type, and negative abnormal returns for issuing stocks when the lack of information by market players leads to overvaluation of low quality firms. This result differs significantly from the usual approach in the real options literature that rules out pooling equilibria and instead predicts negative announcement effects for bad types when good firms reveal their type (Carlson et al, 2006a).

Finally, the model reviews the seminal work by Welch (1989) and Grimblatt and Hwang (1989), and provides a dynamic framework to analyze underpricing in IPOs. The model thus provides a rational approach to analyze underpricing and market timing. When more productive firms reveal their type, younger firms provide higher underpricing to outside investors to separate from less productive firms. During hot markets, instead, less productive firms go public earlier to profit from overvaluation. The model extends to show that firms with several investment projects optimally time their equity issues such that underpricing is decreasing across offerings (Baron, 1982).

The basic IPO model thus characterizes each of the signals conveyed by issuers when firms reveal their type. First, the time to raise public equity is informative as long as firms with more productive growth options choose to go public earlier. This result is opposite to the conclusion by Lucas and McDonald (1990) in a dynamic set-up with adverse selection. Second, firms issue lower amounts of equity in the presence of higher underpricing costs. The fraction of shares issued is decreasing in asymmetric information (Vandemaele, 1997). Finally, underpricing is the only signal where issuers re-allocate their signalling costs to outside investors and thus ensure a market for their stocks. This paper thus provides a positive role for underpricing in contrast with current literature on IPOs (Grimblatt and Carlson et al (2006a) provide a real options model of SEOs with announcement effects. Other papers on real options and announcement effects are Carlson et al (2006b), Morellec and Zdhanov (2005) and Hackbarth and Morellec (2007).
The dynamic set-up of the model also explains the strategy to raise public equity in subsequent public offerings (SEOs). When the firm has more than one growth option, multiple offering strategies enable managers to allocate the costs of asymmetric information across time. Managers then face a trade-off between higher underpricing costs in fewer offerings or higher underwriting fees with more SEOs. The dynamic implications of the strategy to go public are strongly related to the empirical findings by Denis (1991) and Derrien and Kecskés (2007), who show that multi-stage offering strategies are intended to lower the costs of going public.

The empirical section of the paper complements the available literature on IPOs by considering firm age as a significant moment of the going public decision. First, I calibrate the model with real data and assess quantitatively the main predictions concerning firm age, proceeds, underpricing and gross spreads at public offerings. Second, using a sample of US public offerings between 1980 and 2005, I estimate the main moments of public offerings simultaneously and confirm that firm age, underpricing, underwriting spreads and the fraction of shares issued jointly determine the decision to raise public equity. Finally, I provide supporting empirical evidence for the prediction that the average age of issuing firms decreases during periods of high IPO activity.

The paper is divided in five sections. Section 2 describes the basic IPO model and its main results. Section 3 discusses further extensions of the model to explain SEOs and its asset pricing implications. Section 4 tests the model empirically by means of calibration and estimation. Section 5 concludes. All proofs are provided in the Appendix.

1.1 Related literature

This paper is related to several strands of literature in financial economics. In line with the papers by Welch (1989), Grinblatt and Hwang (1989) and Allen and Faulhaber (1989), I explain IPO underpricing using a model of signalling. The current paper adds to these in deriving a game where the timing of the offering also conveys information about the firms’ investment prospects.

The model also relates to recent papers examining the relation between real options, asymmetric information and learning. Grenadier (1999) presents a model with imperfect information where agents infer the private signals through option exercise strategies. Lambrecht and Peraudin (2003) consider the impact of incomplete information and preemption into an equilibrium model of firms facing real investment decisions. Grenadier and Wang (2005) solve for a screening game in a real options framework where better firms have in-
centives to delay their investment. I hereby provide model a signalling game in a real options set-up where bad firms accelerate the option to go public in hot markets and delay the option to go public otherwise.

The model is also extremely related to Carlson, Fisher and Giammarino (2006a) who provide a model for SEOs where the decision to raise public equity is a real option of the firm. The results by Carlson et al (2006a), though, assume that the timing of the SEO is always informative and determine announcement effects as the difference between the actual value of the firm at the SEO and its prior expected firm value. In the model, good firms have positive announcement effects at the time of their issue, while bad firms have negative announcement effects when good firms reveal their type. I hereby show that both the timing to go public and the underpricing at public offerings are jointly determined in equilibrium. By contrast, announcement effects for issuing firms are solely driven by market beliefs when firms do not reveal their type in equilibrium.

Lucas and Mcdonald (1990) also present a of equity issues under adverse selection. In their set-up, undervalued firms optimally delay their investment until undervaluation is corrected and overvalued firms issue immediately to prevent a downward assessment in the future. By contrast, this model suggests that in hot markets high quality firms have incentives to pool (at the expense of undervaluation) and optimally choose to accelerate the decision to raise public equity otherwise.

Pastor and Veronesi (2003) provide a model of rational IPO waves using a discount rate argument, such that low required returns cause investment-driven issuance and low average returns follow. In this paper, the implications of IPO waves are fully associated to the distribution of high and low quality firms in the market. This approach adds to the literature on IPO waves by showing that the average age of firms going public decreases during hot markets.

Several implications of the paper match the empirical evidence by Chemmanur, He and Nandy (2007). First, the authors explore the relationship between ex ante product market characteristics and the going public decision; this paper derives the going public strategy of the firm as a function of its cash flows, its long term investment plan and its issuance costs of public capital. Second, the authors show that firms with larger sales growth and productivity are more likely to go public. The optimal strategy of firms in separating equilibria is a function of the productivity of their growth options. Third, the authors estimate a probit model of the decision to go public as a function of firm characteristics; this paper provides a closed form solution for such probability. Finally, the authors study the dynamics of a firm’s product market performance before and after the
IPO; they conclude that sales growth exhibit an inverted U-shaped pattern with peak in the year of the offering. The dynamics of firm cash flows around the IPO in this model are in line with this observation.

Finally, the model relates to recent findings by Lowry and Schwert (2002), Gomes and Phillips (2007) and the papers by Chen and Ritter (2000), Ritter and Welch (2002) and Ritter (2003). Lowry and Schwert (2002) provide empirical evidence that the underpricing of current IPOs contains information about the valuation of future IPOs; in this model, underpricing is a tool for managers of good firms to prevent bad firms from issuing earlier. Gomes and Phillips (2007) show that asymmetric information affects underpricing in public offerings; the current paper provides a rationale for this observation. Chen and Ritter (2000), Ritter and Welch (2002) and Ritter (2003) provide general empirical facts on IPOs; I consider these in the calibrated version of the model.

## 2 Basic IPO model

### 2.1 Main assumptions

Consider a private firm with both assets in place and a growth option to invest. The firm is all equity financed and it is run by a manager who is the single shareholder. I thus explicitly rule out conflicts of interest between managers and shareholders. Assets in place generate a continuous stream of cash flows \((X_t)_{t \geq 0}\) governed by the diffusion

\[
dX_t = (r - \delta)X_t dt + \sigma X_t dZ_t, \quad X_0 > 0
\]

where \(W_t\) is a standard Brownian motion under a risk-neutral measure \(Q\) and \(\delta\) is the difference between the required return on risk and the actual growth rate of \(X_t\).

The growth option to invest enables the manager to expand firm size and cash flows by a factor \(\theta > 1\) at a cost \(I\). There are two different types of firms \(j = L, H\) according to the quality of their growth options \(\theta_j\), such that \(\theta_L < \theta_H\). While managers know the true value of their growth options, market players only know that high types occur with probability \(p\) and low types occur with probability \((1 - p)\).

The option to invest can be funded by either private or public capital. The manager of firm \(j\) can decide to stay private and fund investment with internal funds. Alternatively, he can decide to go public and fund investment with public capital. The decision to raise public equity and the decision to invest are assumed to be taken simultaneously at the IPO. I denote by \(\alpha_j\) the percentage share of firm \(j\) traded publicly at the moment of investment \(I\).
I assume further that firm $j$ finances a share $\alpha_j$ of the investment cost $I$ with the proceeds from the public offering.

There are thus three stages in firm life cycle. First, the manager of firm $j$ decides to go public and perform shelf registration (Stage 0). Second, the listed firm has the option to perform an IPO and increase its scale using public funding (Stage 1). Finally, the firm becomes mature (Stage 2) and is worth the net present value of its upgraded stream of cash flows $\theta_j(X_t)_{t \geq 0}$. Figures 1–2 illustrate the timeline of the going public procedure.

The cost of equity depends on whether the firm is private or public. Private firms are assumed to have a higher cost of equity than public firms. One potential rationale for this assumption is that in the presence of liquidity risk, liquidity shocks are costly for private shareholders only; this is case in this paper.\(^2\) At any point in time, private shareholders can face liquidity shocks with probability $\lambda_j dt$ over the interval $[t, t + dt]$. Liquidity shocks constrain private shareholders to sell their stake in the firm at a value $V_t$ where $\eta < 1$. Once the firm becomes public, liquidity shocks are not costly anymore ($\eta = 1$). Market players have no information on both the probability and the size of the liquidity shocks affecting private shareholders.

While going public eliminates the cost associated with liquidity shocks, it entails costs related to underpricing, disclosure and underwriting activities. First, the firm announces its decision to become public and does its shelf registration. At Stage 0, the firm faces a fixed cost due to disclosure expenses $D$. Second, the firm performs the public offering and announces the percentage amount of shares $\alpha_j$ to be issued. At Stage 1, the firm is subject to both underpricing costs and underwriting fees.

Underwriting fees are composed of a fixed cost $f$ and a variable cost which depends on the percentage amount of shares issued $\alpha_j$, such that

$$F(\alpha_j) = f + c \alpha_j^\gamma$$

Following Altinkiliç and Hansen (2000), marginal underwriting fees are an increasing function of $\alpha_j$ so that $\gamma > 1$. The functional form of underwriting fees is in line with $U$-shaped average spreads acknowledged both theoretically and empirically.\(^3\) Fixed costs $f$ initially cause scale economies, but as issue size increases diseconomies of scale emerge in the spread due to rising placement costs.

Underpricing costs at the offering result from the asymmetric information between the manager and market players. I consider two different types of underpricing. First, ex-


\(^3\)The spread is the percentage difference between underwriting fees and gross proceeds at the public offering. For a discussion on $U$-shaped spreads, see Altinkiliç and Hansen (2000).
ante underpricing arises if the manager optimally chooses to sell the shares of the firm at a percentage discount \( \epsilon_j \leq 1 \). Managers optimally choose to underprice their shares to convey information to market players (Grimblatt and Hwang, 1989). Second, the shares sold by firm \( j \) can be mispriced at the public offering due to the inability of market players to infer firm type out of the signals provided by the manager. Ex-post underpricing \( U_j \) (either positive or negative) is thus the difference between the value of the firm under perfect information \( V_j \) and the market price of the firm \( P_j \) once the offering has been announced.

In separating equilibria, the beliefs by market players are self-fulfilling and then the ex-ante underpricing by managers coincides with the ex-post underpricing of market players. In pooling equilibria, the actions taken by firms do not convey information to outside investors and ex-post underpricing arises.

### 2.2 Benchmark under perfect information

Consider first Stage 1. The manager’s problem is to maximize firm value \( V_{ij} \) for initial shareholders by choosing the optimal timing to IPO as described by \( x_{1j} \), the optimal fraction of shares to be issued \( \alpha_{1j} \) and the optimal discount on prices \( \epsilon_{1j} \). Under perfect information, managers have no incentives to underprice their shares and hence underpricing costs are equal to zero (\( \epsilon_{1j} = 1 \)).

Using standard arguments, the manager chooses the optimal timing to IPO \( x_{1j} \) that solves

\[
 rV_{1j} = (r - \delta) X \frac{\partial V_{1j}}{\partial X} + \frac{\sigma^2}{2} X^2 \frac{\partial^2 V_{1j}}{\partial X^2} + X
\]

subject to the boundary conditions

\[
 V_{ij} \bigg|_{X_t=x^*_{ij}} = \frac{\theta_{ij}}{\delta} x_{1j} - I_1 - F(\alpha_{1j})
\]

\[
 \frac{\partial V_{1j}}{\partial X_t} \bigg|_{X_t=x^*_{ij}} = \frac{\theta_{ij}}{\delta} - \alpha_{1j} \frac{\partial U_{1j}}{\partial X_t}
\]

The ordinary differential equation (2) imposes an equality between the required rate of return of the firm and the expected return on the option to do the public offering and the assets in place of the firm. The value matching condition in equation (3) imposes an equality between the value of the firm on shelf and the pay-off of the option to do the public offering. Thus, the value of firm on shelf equals, at the time of the public offering, the surplus that the manager extracts from the IPO net of underwriting fees and investment costs. The smooth pasting condition in equation (4) ensures that the option to IPO is exercised along the optimal path by requiring continuity of the slopes at the trigger threshold. Firm value
$V_{1j}^*$ under perfect information is thus given by

$$V_{1j}^* = \frac{X_t}{\delta} + \left( \frac{\theta_{1j} - 1}{\delta} x_{1j}^* - I - F(\alpha_{1j}) \right) \left( \frac{X_t}{x_{1j}^*} \right)^v$$

where the optimal threshold at which to invest and IPO $x_{1j}^*$ under perfect information results from solving equations (3) and (4) and thus depends on firm characteristics.

Finally, the manager chooses the percentage amount of shares $\alpha_{1j}$ to finance a fraction $\alpha_{1j}$ of total investment costs $I_1$ at the time of the IPO. The optimal fraction $\alpha_{1j}$ is such that the marginal costs of issuing public equity at the time of the IPO are equal to the average cost of financing a fraction $\alpha_{1j}$ of the investment costs $I_1$.

**Proposition 1** The optimal strategy to perform an IPO under perfect information is such that

$$\{ x_{1j}^*; \alpha_{1j}^*; \epsilon_{1j}^* \} = \left\{ \left( \frac{I_1 + F_j(\alpha_{1j}^*)}{\theta_{1j} - 1} \right) \left( \frac{v\delta}{v - 1} \right); \left( \frac{I_1}{\gamma c} \right)^{\frac{1}{v - 1}}; 1 \right\}$$

where $v > 1$ denotes the positive root of the firm’s ordinary differential equation (2).

Consider now Stage 0. Before shelf registration, the firm is worth the net present value of its original stream of cash flows $(X_t)_{t \geq 0}$ and the option to go on shelf. Since the firm is still private, it is subject to liquidity shocks. Using standard arguments, the manager’s problem in Stage 0 solves:

$$r V_{0j} = (r - \delta) X \frac{\partial V_{0j}}{\partial X} + \frac{\sigma^2}{2} X^2 \frac{\partial^2 V_{0j}}{\partial X^2} + X - \lambda_j(1 - \eta)V_{0j}$$

The left hand side of the equation represents the required rate of return on firm value. The right hand side is equal to the expected return on the option to go on shelf, cash flows earned by the firm before exercising the option and liquidity shocks. Alternatively, the term related to liquidity shocks in the left-hand side could be passed onto the right hand side of the equation. In this case, the right-hand side would show the total required rate of return on firm value once adjusting for liquidity shocks.

The ordinary differential equation is solved subject to the following boundary conditions

$$V_{0j} \big|_{X_t=x_{0j}} = V_{1j} - D$$

$$\frac{\partial V_{0j}}{\partial X_t} \bigg|_{X_t=x_{0j}} = \frac{\partial V_{1j}}{\partial X_t}$$

The value matching condition in (7) imposes an equality between the value of the private firm and the pay-off of the option to do the shelf registration. Thus, the value of the private firm
firm on shelf equals, at the time of shelf registration, the surplus that the manager extracts from the possibility of doing a public offering net of disclosure costs. The smooth pasting condition in (8) ensures that the option to go on shelf is exercised along the optimal path by requiring continuity of the slopes at the trigger threshold. The function for firm value $V_{0j}$ that solves the manager’s problem in Stage 0 is given by

$$V_{0j} = \frac{X_t}{\delta + \lambda(1-\eta)} + \left( V_{1j}(x_{0j}) - \frac{x_{0j}}{\delta + \lambda(1-\eta)} - D \right) \left( \frac{X_t}{x_{0j}} \right)^u$$

where the optimal timing to go on shelf $x_{0j}$ results from solving (7) and (8) simultaneously.

**Proposition 2** The optimal timing for firm $j$ to perform shelf registration $x_{0j}$ is such that it solves

$$x_{0j} = \frac{D - \left( \frac{u-v}{u} \right) V_{1j}}{\left( \frac{u-1}{u-v} \right) \frac{\lambda_j(1-\eta)}{\delta+\lambda_j(1-\eta)} - 1} \left( \frac{u\delta}{u-v} \right)$$

(9)

where $u > v > 1$ denotes the positive root of the firm’s ordinary differential equation at Stage 0.

The optimal timing to go on shelf $x_{0j}$ reflects the change in the cost of capital of the firm once it is listed and depends on the threshold to do the IPO $x_{1j}$ and the cost of liquidity shocks $\lambda_j(1-\eta)$. Equation (9) suggests that the present value of assets in place in Stage 0 equals the net costs of shelf registration (i.e. $D - \left( \frac{u-v}{u} \right) V_{1j}$) divided by a weighted average of the cost of capital before and after Stage 1 (i.e. $\left( \frac{u-1}{u-v} \right) \frac{\lambda_j(1-\eta)}{\delta+\lambda_j(1-\eta)} - 1$), multiplied by a mark-up reflecting the added value of waiting to go on shelf (i.e. $\frac{u\delta}{u-v}$).

### 2.3 Equilibria under asymmetric information

Consider now the case where managers have private information about the type of growth options of the firm. The strategies derived in Propositions 1 – 2 under perfect information do not necessarily hold in equilibrium, since low types can find it profitable to mimic the strategy of high types. The IPO game at Stage 1 is thus a signalling game with multiple signals $\{x_{1j}, \alpha_{1j}, \epsilon_{1j}\}$ and three players: outside investors (the market) with no private information, and two firms of different types. In the current set-up, the decisions of the game are the IPO strategy followed by each firm and the transfers are the share price $P_j$ paid by outside investors at each offering. The equilibrium strategy of firm $j$ in Stage 1 is to maximize firm value given the strategy of the other firms and the beliefs by market players.
In the model, uncertainty on firm types is resolved at the IPO (Stage 1) by either separating or pooling equilibria depending on firms’ incentives to reveal their type to market players. At Stage 0, the timing to for shelf registration is both a function of IPO strategies revealing firm type and firm specific parameters on liquidity shocks. The signalling game is only related to Stage 1 as long as in Stage 0 market players have no information on liquidity shocks affecting private shareholders.

Finally, a necessary condition such that there is a signalling game is that lower types have incentives to imitate higher types. This condition implies that

$$\left[ \frac{I_1 + F(\alpha^*_1)}{\nu - 1} + \frac{(1 - \alpha^*_1)(\theta_{1H} - \theta_{1L})}{\delta} x_{1H}^* \right] (\frac{X_t}{x_{1H}^*})^\nu > \left[ \frac{I_1 + F(\alpha^*_1)}{\nu - 1} \right] (\frac{X_t}{x_{1L}^*})^\nu$$

the value of the option to go public for low types under pooling is more valuable than the their option to go public under perfect information. Since higher types optimally issue earlier, this condition must hold for any value of $X_t \leq x_{1H}^*$. I assume that $\nu (1 - \alpha^*_1) \geq 1$ such that the inequality above holds for all $\theta_L < \theta_H$ throughout the paper.

### 2.3.1 Separating Equilibria

As long as low types have incentives to imitate high types, there may be both separating and pooling equilibria of the IPO game. Since firms can reveal their type $\theta_j$ through alternative signals, there are multiple separating equilibria of the IPO game in which firms attain the same expected firm value in equilibrium.

A separating equilibrium exists as long as the value function of the managers complies with single crossing conditions, and the equilibrium strategies comply both with individual rationality (IR) and incentive compatibility constraints (ICC). The single crossing conditions refer to how the value of firm $j$ changes as a function of the signals given to market players. The relevant single crossing conditions for firm $j$ in the IPO game are given by

$$\frac{\partial}{\partial \theta} \left[ \frac{\partial V_{1j}}{\partial x_{1j}} \right] > 0; \quad \frac{\partial}{\partial \theta} \left[ \frac{\partial V_{1j}}{\partial \theta_{1j}} \right] < 0; \quad \frac{\partial}{\partial \theta} \left[ \frac{\partial V_{1j}}{\partial \theta_{Lj}} \right] > 0$$

where the derivation is provided in the Appendix.

Individual rationality (IR) constraints ensure that agents participate in the principal’s mechanism, and are usually defined as the reservation value of the agent if he does not participate in the game. In a real options framework, though, IR constraints are implied

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4 Note though that if the probability of liquidity shocks is known to be the same for all firms, Stage S may convey information to market players. I rule out this case for simplicity.
by the value matching conditions on firm value: agents optimally decide to participate once their growth option is in the money.

Finally, incentive compatibility constraints (ICC) ensure that the manager of firm $j$ has no incentives to imitate other types. The ICCs of the IPO game are then given by

$$V_{1L} \geq \bar{V}_{1L} \quad (10)$$

$$V_{1H} \geq \bar{V}_{1H} \quad (11)$$

for any $X_t \leq x_{1H}$ where $V_{1j}$ is the value of firm $j$ when it performs its own optimal strategy and $\bar{V}_{1j}$ is the value of firm $j$ when it deviates. Nonetheless, the relevant ICC for the signalling game is that of low types (10) due to single crossing.

In equilibrium, the low type performs its optimal strategy under perfect information and high types deviate from their first best to separate from low types. The problem of firm $H$ under asymmetric information is different from the one under perfect information in two aspects. First, high types are constrained by the ICC of low types (10) such that

$$\chi_{1H} \left[ \bar{V}_{1L} - V_{1L} \right]_{X_t = x_{1H}} = 0 \quad (12)$$

where $\chi_{1H}$ is the Lagrange multiplier of high types on the ICC of low types. The Lagrange multiplier $\chi_{1H} > 0$ thus reflects the marginal cost for higher types of signalling its true type when the ICC of low types is binding. The complementary slackness condition in (12) is such that either the constraint is binding and multiplier is positive $\chi_{1H} > 0$, or the constraint is slack and $\chi_{1H} = 0$. Second, the firm is subject to an additional constraint ensuring that beliefs by market players are self fulfilled in equilibrium. This additional condition thus equates ex-ante to ex-post underpricing.

The problem of firm $H$ is thus given by

$$rV_{1H} = (r - \delta) X \frac{\partial V_{1H}}{\partial X} + \frac{\sigma^2}{2} X^2 \frac{\partial^2 V_{1H}}{\partial X^2} + X$$

subject to

$$V_{1H}|_{X_t = x_{1H}} = \frac{\theta_{1H}}{\delta} x_{1H} - I_1 - F(\alpha_{1H}) - \alpha_H U_{1H} - \chi_{1j} \left[ \bar{V}_{1L} - V_{1L} \right] \quad (13)$$

$$\frac{\partial V_{1H}}{\partial X_t}|_{X_t = x_{1H}} = \frac{\theta_{1H}}{\delta} - \alpha_j \frac{\partial U_{1H}}{\partial X_t} - \chi_{1j} \left[ \frac{\partial \bar{V}_{1L}}{\partial X_t} - \frac{\partial V_{1L}}{\partial X_t} \right] \quad (14)$$

$$U_{1H}|_{X_t = x_{1H}} = (1 - \epsilon_{1H}) \frac{\theta_{1H}}{\delta} x_{1j} \quad (15)$$

where the smooth pasting and value matching conditions (13)-(14) are constrained by the ICC of the low types (10) and condition (15) ensures there is no mispricing in equilibrium.
The approach to solve for the problem of firm $H$ in this paper is slightly different to the optimization subject to ICCs proposed by Grenadier and Wang (2005); I show in the Appendix that both approaches yield the same result.\footnote{Grenadier and Wang (2005) solve the problem sequentially. First, they compute firm value using the value matching and smooth pasting conditions. Second, they define a Lagrangean where they maximize firm value subject to additional constraints. I hereby solve firm value and account for ICCs at the same time. See Appendix for derivation.}

Given conditions (13)-(15), the value for firm $H$ in Stage 1 is thus given by

$$V_{1H} = \frac{X_1}{\delta} + \left(\frac{(1 - \alpha_{1H} + \alpha_{1H} \epsilon_{1H}) \theta_{1H} - 1}{\delta} x_{1H} - I_1 - F(\alpha_{1H})\right) \left(\frac{X_1}{x_{1H}}\right)^v$$

Finally, the manager of firm $H$ also chooses the optimal percentage amount of shares to finance a fraction $\alpha_{1H}$ of the investment costs $I_1$. Equating marginal costs of issuance to average costs of investment, this yields

$$\alpha_{1H}^{-1} \gamma c (1 - \chi_{1H}) + [(1 - \epsilon_{1H}) \theta_{1H} + \chi_{1H} (\epsilon_{1H} \theta_{1H} - \theta_{1L})] \frac{x_{1H}}{\delta} = I_1$$

such that optimal percentage amount of shares is a function of signalling costs.

**Proposition 3** There exists an optimal separating equilibrium for the IPO game in Stage 1 where the low type firm performs its optimal IPO strategy under perfect information, and high types follow the strategy \{\(x_{1H}, \alpha_{1H}, \epsilon_{1H}\)\} such that

\[
x_{1H} = \frac{(I_1 + F(\alpha_{1H}))}{\delta v (1 - \chi_{1H})} (1 - \alpha_{1H} + \alpha_{1H} \epsilon_{1H}) \theta_{1H} - 1 - \chi_{1H} (1 - \alpha_{1H}) \theta_{1L} + \alpha_{1H} \epsilon_{1H} \theta_{1H} - 1)
\]

\[
\alpha_{1H} = \frac{(I_1 - [(1 - \epsilon_{1H}) \theta_{1H} + \chi_{1H} (\epsilon_{1H} \theta_{1H} - \theta_{1L})] \frac{x_{1H}}{\delta})}{\gamma c (1 - \chi_{1H})}
\]

\[
\epsilon_{1H} = I_1 + F (\alpha_{1H}) + \frac{[I_1 + F(\alpha_{1H})]}{\alpha_{1H} \theta_{1H}} \left(\frac{X_1}{x_{1H}}\right)^v - \frac{(1 - \alpha_{1H}) \theta_{1L} - 1}{\alpha_{1H} \theta_{1H}}\]

Proposition 3 summarizes the optimal strategy to do an IPO when firms reveal their private information in equilibrium. The strategies of higher types in Proposition 3 differ from those of Proposition 1 as long as the ICCs are binding for high types and thus $\chi_{1H} > 0$. When $\chi_{1H} = 0$ and $\epsilon_{1H} = 1$, all strategies converge to the case of perfect information stated in Proposition 1.

There are multiple separating equilibria that are incentive compatible, depending on the combination between $\chi_{1H}$ and all the signals issued by the firm. Two extreme cases are relevant. First, the manager of the firm can reveal private information to market players...
solely through the timing of the IPO and the fraction of shares issued; in this case, the complementary slackness condition in (12) is attained with $\chi_{1H} > 0$ and $\epsilon_{1H} = 1$. This result is in line with the literature initiated by Grenadier (1999) where the timing of the exercise of growth options conveys information to market players. Second, the manager can also reveal firm type by means of underpricing; in this case, it holds that $\chi_{1H} > 0$ and $\epsilon_{1H} < 1$. In particular, when $\chi_{1H} \to 0^+$ and $\epsilon_{1H} < 1$, firms provide the highest underpricing, such that the difference between proceeds under perfect information and proceeds under asymmetric information is mainly driven by underpricing. Proposition 3 thus shows that it can also be optimal to underprice shares. This is in line with Ibbotson’s (1975) conjecture that new issues can be underpriced in order to "leave a good taste in investors’ mouth".

Table 3 provides the calibrated version of the basic IPO model; it illustrates the difference between the strategies of firms under perfect information and those under asymmetric information. In separating equilibria, high types optimally choose to go public earlier such that $x_{1H} < x_{1L}$. Panel B considers the case where $\chi_{1H} > 0$ and $\epsilon_{1H} = 1$; high and medium firms reveal their type only through the timing to go public and the fraction of shares issued. Panel C illustrates the case where $\chi_{1H} \to 0^+$ and $\epsilon_{1H} < 1$; the timing to go public for higher types is the closest to their first-best, and instead managers provide underprice such that $\epsilon_{1H} < \epsilon_{1L}$.

The multiplicity of equilibria implied by Proposition 3 also provides a functional relation between the different signals in equilibrium. In particular, the model suggests that the static signals commonly reviewed in the IPO literature optimally depend on the time to raise public equity. Figure 3 illustrates the comparative statics of the different signals used by firms in separating equilibria. All else equal, younger firms must provide higher underpricing to ensure incentive compatibility. This provides a rational explanation for the link between underpricing and market timing when firms reveal their type. Also, older (and larger) firms issue less percentages of equity in equilibrium. Finally, the multiplier $\chi_{1H}$ for firm $H$ is increasing in $\epsilon_{1H}$; this illustrates the duality between underpricing and the shadow cost of signalling in the model.

### 2.3.2 Pooling equilibria

In pooling equilibria, IPO strategies are not informative, and market players price new shares given their beliefs. In the current set-up, there exists a pooling equilibrium in which both high and low firms follow the strategy of high types under perfect information and outside investors price their shares according to the expected value of firm types. There is ex-post underpricing for higher types and ex-post overvaluation for low types in equilibrium.
Proposition 4  There exists a pooling equilibrium of the IPO game such that

\[
\{x_{1j}; \alpha_{1j}; \epsilon_{1j}\} = \left\{ \left( \frac{I_1 + F_j(\alpha_1^*)}{\theta_{1H} - 1} \right) \left( \frac{v\delta}{v - 1} \right); \left( \frac{I_1}{\gamma c} \right)^{\frac{1}{\gamma - 1}}; 1 \right\}
\]

(17)

all firms imitate the strategy of high types under perfect information.

Propositions 3 – 4 thus show that the impact of asymmetric information on the strategy to go public depends on the incentives of firms to reveal their type. Consider first the implications for the timing to go public. When firms separate, the time to go public is a signal and better firms go public earlier to make imitation more costly. When firms pool, the timing to go public is not informative and worse firms go public earlier to enhance the value of their shares. Consider now the percentage amount of shares issued. When firms reveal their type, high firms issue a lower fraction of shares due to higher underpricing costs. In pooling equilibria, all types issue the optimal fraction of shares under perfect information. Finally, consider underpricing. High types are underpriced ex-ante in separating equilibria and ex-post when firms pool. Low types are fairly priced when there is revelation in equilibrium, but they are overvalued when market players cannot differentiate across types.

2.3.3 Underpricing, timing and IPO waves

A general criticism posed by Tirole (2007) on signalling games is that they are plagued by a large multiplicity of equilibria. I thus consider two further refinements to obtain clear-cut predictions in terms of underpricing, the timing to go public and IPO waves.

Consider first the multiple separating equilibria derived in Proposition 3. Although all of these equilibria are optimal for issuers, they are not necessarily optimal for both issuers and outside investors when considered jointly. According to Proposition 3, issuers are indifferent between using underpricing or the remaining signals to reveal their type as long as they attain the same expected value in equilibrium. Nonetheless, outside investors benefit the most when issuers provide the highest levels of underpricing. The optimal separating equilibrium of the IPO game that maximizes total wealth is thus the one in which the highest amount of signalling costs is re-allocated from issuers to outside investors through underpricing. The current model thus provides a positive role for underpricing vis-à-vis the alternative signals of issuers to reveal firm type to outside investors.

\footnote{Note that they attain the same firm value once controlling for the time difference between one potential issuance strategy and another.}

\footnote{Several papers in the literature pose underpricing as an inefficient mechanism to reveal private information. For instance, Grimblatt and Hwang (1989) suggest that the percentage amount of shares issued is a more efficient signal than underpricing.}
Denote $\Delta W_j$ to be the loss of wealth due to asymmetric information for both issuers and outside investors of firm $j$ such that

$$\Delta W_j \big|_{X_t=x_1} = V^*_j - \left[ V^*_j + \alpha_{1j} (1 - \epsilon_{1j}) \frac{\theta_{1j}}{\delta} x_{1j} \right]$$

where $V^*_j$ is the value of the firm under perfect information and $V^*_j$ is the value of the firm under separating equilibria. It is straightforward to show that underpricing reduces the loss of wealth associated with asymmetric information such that

$$\frac{\partial \Delta W_j}{\partial \epsilon_{1j}} \bigg|_{X_t=x_1} > 0$$

This is illustrated in Figure 4. The aggregate wealth of all shareholders once the firm becomes public is the highest for the separating equilibrium with the lowest $\epsilon_{1j}$.

Market players need not behave in equilibrium according to what is best for all of them on aggregate. Nonetheless, the separating equilibrium with the highest underpricing is the only one that survives Bertrand competition in prices between firms of the same type. This has a direct implication for uniqueness. Consider the case where several issuers of the same type compete in the underpricing to ensure the placement of their shares. All else equal, outside investors demand those shares which ensure the greatest abnormal returns. It is then optimal for issuers to follow the strategy where they both signal their type to investors and provide the maximum amount of underpricing to outside investors. This result is supported by the empirical evidence which suggests that firms consistently underprice their shares.

**Proposition 5** The optimal separating equilibrium of the IPO game that maximizes total wealth and survives Bertrand competition among firms of the same type is the one where $\chi_{1H} \to 0^+$ and $\epsilon_{1H} < 1$ such that the highest amount of signalling costs is re-allocated from issuers to outside investors through underpricing.

Consider now the fact that there are both separating and pooling equilibria of the IPO game. Since each type of equilibrium has different implications for the strategy to go public,
a refinement concerning when each of these equilibria hold is relevant to derive empirical predictions. Consider the incentives of high types to separate from low types in equilibrium. Higher types have incentives to reveal their type in equilibrium as long as

\[
\left[ \frac{I_1 + F(\alpha_{1H})}{v - 1} \right] \geq \left[ \frac{I_1 + F(\alpha^*_1)}{v - 1} - \frac{\alpha^*_1 (1 - p) (\theta_{1H} - \theta_{1L})}{\delta} \right] \left( \frac{x_{1H}}{x^*_{1H}} \right)^v
\]

the option to go public in separating equilibria is more valuable than the option to go public when market players are unable to differentiate across types. Reordering terms, this condition provides an upper bound on the probability \( p \) of being a high type such that high types have incentives to separate from low types only if \( p \leq \bar{p} \). As the probability of the high type of firms increases, the ex post underpricing faced by high types in pooling equilibria decreases relative to the signaling costs under revelation. Figure 5 further illustrates the change in \( \bar{p} \) for a uniform distribution of types such that \( p = 0.5 \). When the difference in growth option types decreases, separating equilibria become more costly for high types and thus \( \bar{p} \) decreases.

**Proposition 6** Higher types only have incentives to separate in equilibrium when \( p \leq \bar{p} \) where

\[
\bar{p} = \left[ \frac{I_1 + F(\alpha_{1H})}{v - 1} \right] \left( \frac{x^*_{1H}}{x_{1H}} \right)^v - \left[ \frac{I_1 + F(\alpha^*_1)}{v - 1} - \frac{\alpha^*_1 (\theta_{1H} - \theta_{1L})}{\delta} \right] \left( \frac{x_{1H}}{x^*_{1H}} \right)^v
\]

reflects the ratio of the loss in firm value over the increase in market price when high types reveal their type to outside investors.

Given the refinements above, the model provides clear predictions concerning the optimal timing to go public under asymmetric information. In scenarios where there is a high share of low types in the market such that \( p \leq \bar{p} \), asymmetric information erodes the option value of waiting to issue. Conversely, when there is a high share of high types in the market, low types accelerate the decision to go public to increase the value of their equity. The implications for the fraction of shares issued and on underpricing are also straightforward. While low types always issue the optimal fraction of shares under perfect information, high types only issue as much when \( p > \bar{p} \). Concerning underpricing, low types are overvalued by market players when \( p > \bar{p} \); meanwhile, high types are usually subject to underpricing, and underpricing is optimally determined by managers when \( p \leq \bar{p} \).

The main results of the signaling game can be easily extended to explain IPO waves. Denote hot markets are those states of nature where adverse selection becomes less relevant

\[\text{An alternative refinement to rule out multiple equilibria in signalling games is that of Maskin and Tirole (1982). See Tirole (2007) for a review.}\]
such that $p > \pi$ (Tirole, 2007). Given a uniform distribution of types before the raise in IPO activity, the model predicts that the average age of firms going public decreases during hot markets. The model also predicts overvaluation of low quality firms and an average increase in fraction of shares issued during hot markets. Figures 6 – 7 provide supporting empirical evidence for these predictions.

**Proposition 7** Firms optimally pool during hot markets and reveal their type otherwise, such that during hot markets the average age of firms going public decreases, the average percentage amount of shares issued increases and there is overvaluation of low quality firms.

### 3 Further implications

#### 3.1 Optimal SEOs

When the firm has several growth options ($i > 1$), the current set-up also raises the question of which is the optimal number of public offerings to perform. This is because the option to do a SEO is triggered by the manager as long as it enhances firm value. The main result of this section is thus that the optimal number of public offerings to perform is endogenously determined and depends on firm characteristics.

Consider the case where the manager has $i = 2$ growth options to invest and denote by $n$ the total number of public offerings to be done by the firm. First, he can choose to do a single IPO to raise the necessary public capital to fund both growth options (case of $n = 1$). Alternatively, he can perform both an IPO and a SEO to raise public capital for its first and second growth options (case of $n = 2$). The optimal amount of public offerings to perform when firm $j$ has two growth options ($i = 2$) is endogenously determined such that

$$V_{1j} = \sup_T E \left[ e^{-r(T_{n=1} - \tau)} V_{1j}^{n=1} I_{T_{n=1} < T_{n=2}} + e^{-r(T_{n=2} - \tau)} V_{1j}^{n=2} I_{T_{n=2} < T_{n=1}} \right]$$

where $V_{1j}^{n}$ is the value of firm $j$ in Stage 1 when the firm performs $n$ public offerings.

In the case of perfect information, the optimal number of public offerings to perform depends on the investment plan of the firm and the structure underwriting fees. While a firm performing a single offering is subject to higher variable underwriting fees, a firm performing two public offerings pays fixed underwriting costs twice. In the presence of asymmetric information, the optimal strategy also depends on the signalling costs borne by higher types in equilibrium. Multi-stage offering strategies are thus partly intended to

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12 The assumption of a uniform distribution of types before the increase in IPO activity is a sufficient condition to prove that the average age of firms decreases during hot markets. See Appendix for derivation.
reduce uncertainty with respect to firm type, insofar they reveal more information about firm characteristics and thus reduce the issuance costs of asymmetric information. This is in line with Denis (1991) and Derrien and Kecksés (2007). In particular, separating equilibria are derived in the same way as in Section 2 yet considering $V_{ij}$ as in Equation (18). Table 3 illustrates the optimal equilibrium strategies for $i = 2$ when firms find it optimal to do both an IPO and a SEO.

3.2 Learning

The model can also be extended to cases where asymmetric information survives the announcement of the IPO. When the firm cannot convey enough information to the market about its growth options in a single public offering, the option to a SEO becomes relevant as it allows issuers to fully reveal their type in equilibrium. This section describes a separating equilibrium where firms fully reveal their type at the SEO; several implications follow.

Consider the case of $i = 4$ where the firm does both an IPO in Stage 1 and a SEO in Stage 2. In this set-up, asymmetric information survives the announcement of the IPO as long as the amount of signals that each firm can convey at the IPO are not enough to reveal all the information about the growth options of the firm. I first assume that both issuers and outside investors can commit intertemporally; I then assess whether this separating equilibrium of the IPO-SEO game is indeed commitment-proof.

In Stage 1, the problem faced by the manager of firm $H$ is the same as in the benchmark case of $i = 1$. The value of firm in Stage 1 depends on the value of the firm in Stage 2 because firm value is solved as a sequential investment problem. Most importantly, underpricing in Stage 1 depends on the value of the firm in Stage 2 since market beliefs are self-fulfilling in equilibrium.

In Stage 2, the problem of firm $H$ differs from the benchmark case as long as the underpricing in Stage 2 is a function the IPO strategy in Stage 1. This is because the signals observed by market players in Stage 1 determine the amount of asymmetric information before the SEO announcement. For instance, if the firm provides no informative signal to market players at the IPO, then no SEO strategy is fully revealing in equilibrium (since there are three signalling devices and four unknowns).

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13 See Appendix for derivation.
14 I assume that $n = 2$ and that the public funds raised at each stage are a fraction of the investment costs of that stage only. I do this without loss of generality, to illustrate the role of asymmetric information when $i > 3$.
15 See Fundemberg and Tirole (1991) and Baron and Besanko (1984).
16 Notice that all possible combinations of types have positive probability in Stage 1; once firms provide
There thus exists a fixed point in Stage 1 such that the optimal IPO strategy depends on the optimal SEO strategy and vice versa. On the one hand, firm value in Stage 1 depends on firm value in Stage 2 due to sequential investment. On the other hand, the optimal SEO strategy in Stage 2 depends on IPO strategy in Stage 1; the beliefs of market players in Stage 2 are a function of the signals observed in Stage 1. This fixed point can be defined as a learning constraint since it requires that the ex-ante beliefs by market players at the IPO (i.e., underpricing in Stage 1) be compatible with the ex-ante beliefs by market players at the SEO (once updating by the signals of the former offering).\textsuperscript{17}

Finally, reconsider the assumption that both issuers and outside investors can commit intertemporally. Due to the sequential nature of the investment problem, the relevant ICCs in Stage 1 are actually the intertemporal ICCs for the IPO game where firm value is the expected net present value of all future growth options and current assets in place. Then, the fixed point in Stage 1 actually ensures that the mechanism implemented in Stage 1 is incentive compatible for subsequent Stages.

Table 4 provides a numerical example of the optimal separating equilibrium strategies for $i = 4$. In line with Proposition 5, the calibrated figures assume that the Lagrange multipliers of the ICCs are such that $\chi_{ij} \rightarrow 0^+$ for $i = 1, 2$. In equilibrium, higher types issue earlier than lower types, and the percentage amount of shares issued is lower than the one for optimal individual strategies. Most importantly, underpricing is higher at the IPO than at the SEO. This supports Baron (1982), who posits that issues characterized by greater uncertainty tend to be more underpriced to compensate investors for learning about their values. In the current model, signalling costs are increasing in the amount of asymmetric information.

### 3.3 Abnormal Returns

All results on ex-ante and ex-post underpricing translate into announcement effects at the time of the public offering. When firms reveal their type in equilibrium, abnormal returns to market players are equal to the percentage underpricing provided by firms. When firms do not reveal their type in equilibrium, abnormal returns (either positive or negative) result from mispricing by market players.

\textsuperscript{17} As a remark, note that the result of having a fixed point in the first stage of the signalling game can be generalized to other set-ups other than the decision to public. This is because a fixed point arises each time that there is a sequential investment problem (to be solved by backward induction) with progressive learning by market players (given that the signals are informative in equilibrium).
Proposition 8 Abnormal returns in Stage $i$ at the announcement of the public offering of firm $j$ are given by

$$AR(x_{ij}) = \begin{cases} 1 - \epsilon_{ij}; & 1 - \frac{E[V_{i+1,j}]H_i}{V_{i+1,j}} \\ \end{cases}$$

where the first term applies when $p \leq \bar{p}$ and the second term applies otherwise.

Proposition 8 fully characterizes underpricing as a function of the incentives of firms to reveal their type. In separating equilibria, the timing to go public and abnormal returns optimally depend on each other. In pooling equilibria, abnormal returns are solely driven by beliefs by market players and are unrelated to the timing to raise public equity. Consequently, announcement effects for issuing firms can only be negative in pooling equilibria, when the lack of information by market players leads to overvaluation of low type firms. This result differs significantly from the usual approach in the real options literature where bad types are subject to negative announcement effects only when high types reveal their type in equilibrium (Carlson et al., 2006a).

Provided that abnormal returns are analogous to underpricing in separating equilibria, the welfare implications of underpricing derived in Section 2.3 can be restated in terms of abnormal returns. The optimal separating equilibrium for both market players and issuers in Proposition 5 is such that outside investors obtain the highest abnormal returns when high types still have incentives to separate.

For the case of multiple growth options, results in Section 3.2 also characterize underpricing in IPOs vis a vis underpricing in subsequent public offerings. When asymmetric information supersedes the IPO, then the abnormal returns of the SEO are lower than those of the IPO such that $\epsilon_{ij} < \epsilon_{2j}$. This implies that abnormal returns are larger in IPOs than in SEOs, in line with the empirical evidence (see Ritter, 2003, for a survey).

4 Empirical analysis

This paper provides several empirical predictions concerning the timing to raise public equity, the fraction of shares issued and on the underpricing of shares at public offerings, conditional on the incentives of firms to reveal their type in equilibrium. While the empirical evidence on the static signals has been traditionally considered in the IPO literature, this section provides complementary empirical evidence that has not been provided elsewhere by focusing on the timing of public issues. Proposition 7 is useful for the current section as it predicts that pooling equilibria occur in hot markets and separating equilibria occur otherwise.
The predictions of the model concerning the cash flow thresholds at which the firm raises public equity can be reinterpreted in terms of the expected firm age of the firm at public offerings. Denote by $x_{ij}$ the optimal threshold for firm $j$ to exercise its growth option in Stage $i$. Then the expected age of the firm $j$ at the beginning of Stage $i$ is given by

$$E \left[ T_{ij} \right] = \frac{\ln \left( \frac{x_{ij}}{X_0} \right)}{\mu}$$

(20)

where $\mu = (r - \delta) - \frac{\sigma^2}{2}$ is the drift of the lognormal process $\ln (X_t)$.

The empirical results in Chemmanur, He and Nandy (2007) for IPOs and Gomes and Phillips (2007) for SEOs are highly related to the results of this section. These papers estimate the probability of raising public equity as a function of firm characteristics. The current model provides a closed-form solution for the probability of performing public offerings that is functionally related to the expected age of raising public equity. In particular, the cumulative probability $\Pr_{(0,T_{ij})}$ that firm $j$ raises public equity in Stage $i$ is given by

$$\Pr_{(0,T_{ij})} = \Phi \left( \frac{-\ln \left( \frac{x_{ij}}{X_0} \right) + \mu T_{ij}}{\sigma \sqrt{T_{ij}}} \right) + e^{\frac{2\sigma^2}{X_0} x_{ij}} \Phi \left( \frac{-\ln \left( \frac{x_{ij}}{X_0} \right) - \mu T_{ij}}{\sigma \sqrt{T_{ij}}} \right)$$

where $\Phi$ is the standard normal cumulative probability distribution. The underlying parameters explaining the probability to raise public equity are thus the same as those explaining the expected time to go public. This section complements the available empirical evidence on probabilities by highlighting firm age as a relevant aspect of public issues.

4.1 Working database

The working sample considers US public equity issues from 1980 to 2005. The data on equity issues comes from the SDC new issues Platinum database. I then match the data obtained from this source to the merged CRSP-Compustat database, to obtain information on firms financials. I discriminate observations within the sample according to whether the issue occurred during the IPO bubble of the late 1990s or not. This relies on the working assumption that the IPO bubble in the late 1990s is the only period of hot markets in the sample.

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18 The derivation of the expected age of the firm is provided in Oksendahl (2000).
19 The closed-form solution for this probability can be computed from the hitting time distribution of the Brownian motion. See Harrison (1985).
20 This relies on the working assumption that the IPO bubble in the late 1990s is the only period of hot markets in the sample.
Table 1 describes the main summary statistics of the working sample for both IPOs and SEOs. For the period excluding the IPO bubble, the sample demonstrates the same general patterns that have been reported previously in the literature. The median underpricing in IPOs is 7.5% (i.e. Chen and Ritter (2002)) and that of SEOs is lower 1% (Ritter, 2003). Proceeds in IPOs are lower than in SEOs; meanwhile, the percentage amount of firm value sold in public offerings is higher in IPOs. This suggests that firms in their initial stages of their going public procedure are smaller in size.

Most importantly, Table 1 and Figure 6 provide supporting evidence that the average age of firms going public decreases during hot markets. In Table 1, the median age of firms going public during the IPO bubble is much lower (5 years) than the one out of the IPO bubble (8 years). In Figure 6, the distribution of the age of issuing firms in IPOs is more concentrated towards lower values. The relation between firm age and hot markets fades away for SEOs (Figure 7), suggesting that asymmetric information fades away in subsequent equity offerings (Section 3).

Table 2 further illustrates the pairwise correlations of the main variables related to public offerings for the working sample. For the period excluding the IPO bubble, firm age (AGE) is significantly correlated to the traditional moments of the going public decision. Younger firms going public are smaller in size as measured by log sales (LNSALES); this validates the inference of firm age out of cash flow thresholds in public offerings. Younger firms are subject to higher underpricing costs over proceeds (UNDP) in line with Proposition 3. The link between underpricing and firm age fades away for SEOs, suggesting the timing of SEOs is more related to alternative costs of raising public equity other than information asymmetry (Section 3). There is also a negative relation between ALPHA and UNDP for all issues (Proposition 3). Furthermore, PROCEEDS and UNDP are positively correlated, suggesting that underpricing facilitates the placements of larger amounts of shares in the market (Proposition 5). For the period of the IPO bubble, the correlations between AGE and all variables except UNDP are not significant, given that the timing of public issues becomes uninformative.\

4.2 Calibration

The choice of parameter values for calibration is determined in one of three ways using the basic IPO model for $i = 1$. The first group of parameters is determined by direct measurements conducted in other studies. These include the annual risk free rate $r$, the

\footnote{A priori, the model provides clear prediction concerning the ex post underpricing of firms in the market to explain the correlation between AGE and UNDP in Panels B and D.}
convenience yield on cash flows $\delta$, cash flow volatility $\sigma$, the annual probability of liquidity shocks $\lambda$ and the convexity of underwriting fees $\gamma$. The second set is selected so that the model matches relevant moments in the data. Finally, a third group of parameters is computed as functions of either the first or the second set. Table 3 summarizes the parameters used for the baseline calibration.

Starting with the first set of parameters, the annual risk free rate $r$, the convenience yield on cash flows $\delta$ and cash flow volatility $\sigma$ are in line with the baseline parametrization of a SEO model by Carlson et al (2006a). The annual probability of liquidity shocks $\lambda$ is assumed to be 5% as in Erickson and Renault (2002). Marginal underwriting fees have been assumed to be linear (i.e. $\gamma = 2$) as in Altinkılıç and Hansen (2000).

The second set of parameters consists of the level of investment $I$, the mark-up on firm cash-flows $\theta$, the volatility of these mark-ups $\sigma_\theta$, and the initial value of cash flows $X_0$. For simplicity, I assume a uniform distribution of firm types $\{\theta (1 - \sigma_\theta), \theta (1 + \sigma_\theta)\}$ such that $p = 0.5$. I have calibrated these four parameters so that the model matches the following moments of public offerings: (i) a median amount of proceeds of 30 million US dollars; (ii) a volatility of proceeds of 20 million US dollars; (iii) a median age 8 years at the moment of doing the IPO; and (iv) a median amount of (log) sales of 3 million US dollars before the IPO. All these quantities have been obtained from the sample statistics described in Table 1 for IPOs.

In sum, parameters $\theta$ and $\sigma_\theta$ determining the distribution of growth options are basically inferred from the ex-post distribution of proceeds in the working database. The timing to trigger the option to go public for medium type firms in the model is set to match the median age of firms going public. Finally, the initial value of cash flows $X_0$ is mainly inferred from the median amount of log sales of US firms at the moment of going public. The fixed costs of underwriting $f$, the disclosure costs $D$ and the scale factor on underwriting fees $c$ are computed as functions of the parameters described so far.

Table 4 shows the main results from the calibration of the basic IPO model. The

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22 The amounts chosen for the median amount of proceeds and its corresponding volatility are such that the assumed interquantile range goes from 10 MM to 50 MM. This is in line with Panel A in Table 1.

23 Altinkılıç and Hansen (2000) estimate fixed underwriting costs to be 10% of total underwriting fees paid by issuing firms; Corwin and Harris (2001) estimate total underwriting fees to be 6.5% of median proceeds for issuing firms. I thus consider $f$ to be 0.65% of the median amount of proceeds used for calibration. Corwin and Harris (2001) estimate disclosure or listing costs $D$ to be 1.5% of expected proceeds at the IPO. Finally, the cost of liquidity shocks $\eta$ is set as unity minus the average underpricing of 7% borne by high and medium type firms in the IPO. This is to acknowledge that costs of liquidity should be a function of asymmetric information between managers and market players. The scale factor on underwriting fees $c$ is defined as $\frac{c}{\gamma - 1}$ so that variable underwriting fees depend on the size of the growth option.
strategy under perfect information for high types in Panel A is significantly different from the separating strategies under asymmetric information in Panels B–C. Panel B illustrates the case where firms reveal their type in equilibrium but provide no underpricing to outside investors. Panel C illustrates the case of Proposition 4 where underpricing is the highest. In all separating equilibria, higher types choose to go public earlier than their first-best case under perfect information to make imitation more costly. The equilibrium strategies for the timing to go public in Panel C are the closest to those of Panel A; signalling costs in Panel C are mainly re-allocated to outside investors by means of underpricing.

Underpricing costs are 6% for high firms, in line with Corwin and Harris (2001) and Chen and Ritter (2002). Abnormal returns in the baseline calibration are thus 6%. Underwriting spreads between 7 – 9% match the sample spreads for IPOs of 5 – 10% reported by the empirical literature.\(^{24}\) In addition, the fraction of shares issued in equilibrium ranges from 7% to 16%. While Corwin and Harris (2001) suggest that the interquantile range of the value issued at IPOs is 25 – 63%, the current model assumes investment funding to be the only motive to go public. Alternative reasons to go public should also affect the optimal amount issued at the IPO (Roell, 1990).

The expected age to do both shelf registration and to go public for low types shown in Table 4 are far beyond real figures. The current model assumes all equity financed firms; the existence of alternative financing tools in the initial stages of the firm might delay the decision to go public to higher cash flow thresholds. Given that the availability of financing tools is correlated to firm type, an extension of this model combining alternative financing choices might provide longer waiting times to go public for higher types while keeping lower types out of public markets.\(^{25}\)

Table 5 shows the optimal equilibrium strategies for the extension with \(i = 2\). I consider the same baseline parameters as in the case of \(i = 1\) to understand the mechanics of the model. In addition, I focus on the optimal separating equilibrium where \(\chi_{1j}^e \to 0^+\) and \(\epsilon_{1j}^s < 1\). The optimal strategy for all firms is to do two public offerings instead of one. Proceeds in the SEO are larger than in the IPO, in line with the evidence in Table 1. The average expected time to go public for high firms is lower in Table 5 than in Table 4, reflecting that greater financing requirements to invest result in shorter waiting times to raise public equity.

Table 6 shows the optimal equilibrium strategies for all firms when \(i = 4\). The calibration shows that SEO underpricing is lower than IPO underpricing. This is in line with Baron

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\(^{24}\)See Ritter and Zhao (1996), Corwin and Harris (2001), and Chen and Ritter (2002)

\(^{25}\)This relates to models of funding choices under asymmetric information such as Stein (1992) and the empirical evidence by Gomes and Phillips (2007).
(1982) and Ritter and Zhao (1996) and confirms the underlying dynamic trade-off of the model, where the benefit of performing multi-stage offerings is that of lower underpricing costs. Also, abnormal returns are higher at the IPO than at the SEO (Ritter, 2003).

4.3 Empirical evidence

The main implications of the paper suggest that all the moments of the going public decision should be tested jointly, instrumenting for the endogeneity of the decision to raise public equity. I test these predictions using a simultaneous equation model for the main moments of going public. I consider firm age (AGE), underpricing over proceeds (UNDP), underwriting gross spreads (GSPREAD) and the percentage amount of shares issued (ALPHA) as the main moments of public offerings. With the exception of AGE, all variables are expressed in terms of percentages.

I use linear GMM generalized instrumental variable estimators as suggested by Hansen (1982) and Hansen and Singleton (1982). Since I apply the same instruments to all equations, the estimation technique yields the same results as 3SLS. An attractive feature of this method is that the results are robust to the fact that expected moments are tested using realized values. As a caveat, the estimation approach assumes linearity of the moments to go public with respect to the explanatory variables.

The construction of the system derives from the main underlying trade-offs of the model in separating equilibria. I then control for deviations during the IPO bubble in all equations. For each equation, I hereby describe the economic intuition behind its specification and then discuss the corresponding empirical results in Table 7 for IPOs. The coefficients \( \phi_{n,k} \) test the sensitivity of the variable \( n_k \) in equation \( k \). The signs written in parentheses under each coefficient are the signs predicted by either the theoretical model developed in this paper or related empirical evidence.

The first equation of the system is related to firm age (AGE) such that

\[
\text{AGE}_j = \phi_{01} + \phi_{11}\text{LNSALES}_j + \phi_{21}\text{TTLTD}_j + \phi_{31}\text{CAPX}_j + \phi_{41}\text{WK}_j + \phi_{51}\text{VENTURE}_j + \phi_{61}\text{BUBBLE}_j + \phi_{11}
\]

According to Propositions 1–3, the age of the firm at the public offering mainly depends on the cash flow threshold at the moment of going public \( x_{1j} \) and the initial cash flows of the firm \( X_0 \). All else equal, firms with more productive growth options go public earlier. In the equation above, I control both for \( x_{1j} \) using LNSALES and for \( X_0 \) using

\[26\text{See Hayashi (2000).}\]
alternative firm characteristics at the moment of the public offering.\textsuperscript{27} The remaining variables control for alternative sources of funds such as long term debt financing (TLTD) and uses of funds including investment (CAPX) and working capital (WK) over total assets (TA). The dummy VENTURE further controls for the fact that venture-backed IPOs may have higher incentives to go public; Tykova (2003) suggests that venture backed firms may go public earlier since the discount rate of managers is higher than that of the market. The dummy BUBBLE controls whether AGE is informative during hot markets. The dummy variables $I_j$ are applied to all equations in the system and control for industry fixed effects.\textsuperscript{28}

The first column of Table 7 provides the results for the AGE equation in IPOs. The timing to go public is positively related to LNSALES such that older firms go public at higher cash flow thresholds. Firms with higher book leverage ratios go public 11.2 years later than other firms; this suggests that the access to alternative financing tools does affect the going public strategy. Firms with higher investment over assets before the IPO go public 21.46 years earlier; this is in line with Propositions 1 – 3 and the investment motive assumed in the model. Venture-backed IPOs go public 4.02 years earlier than other firms; this is supporting empirical evidence for Tykova (2003). The dummy BUBBLE proves not to be significant during hot markets, in line with Proposition 4. Finally, the constant of estimation of the AGE equation (7.56) is close to the sample median in Table 1, suggesting that the heterogeneity in firm age with respect to firm characteristics is well captured in the system.

The second equation of the system (UNDP) models underpricing such that

$$\text{UNDP}_j = \phi_{02} + \phi_{12}\text{LNSALES}_j + \phi_{22}\text{NASDAQ}_j + \phi_{32}\text{NYSE}_j + \phi_{42}\text{BUBBLE}_j + \phi_{52}\text{AGE}_j + \phi_{62}I_j$$

All else equal, Proposition 5 suggests that firms with more productive growth options optimally choose higher underpricing rates to signal their type. I thus model UNDP a function of AGE, its relative size as measured by LNSALES and two control variables accounting for the exchange at which the offering takes place. The dummy BUBBLE further controls for potential deviations from fundamentals during the internet bubble.

\textsuperscript{27}By relating cash flow thresholds solely to LNSALES, I implicitly assume that the stochastic shocks in $X_t$ are mainly related by demand shocks in product markets. Chemmanur, He and Nandy (2006) find that firms with larger size, sales growth and total factor productivity are more likely to go public. Given that all these variables are functions of the sales of the firm, I hereby test a similar hypothesis relating firm AGE to LNSALES.

\textsuperscript{28}It seems straightforward to think of this model in terms of intra-industry dynamics, with firms that behave strategically within the same industry. I thus consider industry dummies to control for industry fixed effects.
The second column of Table 7 provides the results for UNDP in IPOs. The positive

The second column of Table 7 provides the results for UNDP in IPOs. The positive
correlation between age and LNSALES seen in Table 6 breaks down such that younger firms
discount their shares 0.7% more than older firms and, simultaneously, firms with larger
LNSALES provide higher discounts (2.8%). This is due to the fact that LNSALES is also
proxy for productivity.\footnote{Note that once the firm goes public cash flows are the product of the productivity coefficients \( \theta_{ij} \) and \( X_t \). Even if LNSALES is hereby measured before the IPO, both the empirical results and the model suggest that cash flows are related to both the age of the firm and to productivity. See Chemmanur, He and Nandy (2007).} Firms issuing during the internet bubble had a supplementary underpricing of 18.2%; this is in line with Loughran and Ritter (2004).

The third equation considers underwriting spreads (GSPREAD) as a function of ALPHA
and its squared value ALPHASQ, in line with the functional form assumed for \( F(\alpha_{1j}) \). The variable BRUNTOP assesses whether the choosing top underwriters implies higher underwriting fees for the issuer. The variable OVERALL tests whether the use of overallotment options by underwriters has an impact on underwriting spreads. The dummy BUBBLE further controls for potential changes in underwriting fees during hot markets.

\[
\text{GSPREAD}_j = \phi_{03} + \phi_{13} \text{ALPHA}_j + \phi_{23} \text{ALPHASQ}_j + \phi_{33} \text{BRUNTOP} + \phi_{43} \text{OVERALL} + \phi_{44} \text{BUBBLE} + \theta_{3j}
\]

The third column of Table 7 provides the results for GSPREAD in IPOs. An increase in
ALPHA raises underwriting spreads by 44.6%. Nonetheless, an increase in ALPHASQ decreases GSPREAD by 50.9%. This seems at odds with the convex functional form assumed for \( F(\alpha_{1j}) \); still the derivative of GSPREAD with respect to ALPHA is not monotone in the model. The choice of the underwriter measured in BRUNTOP is also significant such that top underwriters charge higher spreads to issuers. OVERALL and BUBBLE are not significant to explain gross spreads in IPOs.

Finally, the last equation of the system models the percentage amount of shares issued
in the public offering ALPHA, such that

\[
\text{ALPHA}_j = \phi_{04} + \phi_{14} \text{UNDP}_j + \phi_{24} \text{MOTCAPX} + \phi_{34} \text{MOTTD} + \phi_{44} \text{MOTW} + \phi_{54} \text{AGE} + \phi_{55} \text{INVGR} + \phi_{56} \text{BUBBLE} + \theta_{4j}
\]

According to Proposition 3, the optimal amount of shares issued depends on the op-
timal timing to IPO (AGE) and the costs of triggering the growth option at the offering.
These costs include the information asymmetry between firms and investors (UNDP), the
costs of underwriting (considered in the equation for GSPREAD) and future investment
costs. The current model assumes an investment motive to go public; the dummy variable
MOTCAPX thus tests whether those firms announcing that they raise public equity to fund investment effectively raise higher sums over proceeds relative to other firms. The explanatory variables (MOTTD) and (MOTWK) control for other motives to go public including debt repayment (MOTTD) and working capital requirements, respectively. The variable (INVGR) tests whether investment grade firms with lower costs of financing and further alternative financing tools issue as much as non investment grade firms with higher costs of financing. Finally, the variable BUBBLE tests whether firms issue more on average during hot markets, in line with Proposition 7.

The last column of Table 7 provides the results for ALPHA in IPOs. An increase of 1% in underpricing decreases the optimal alpha in 1.14%. Older firms issue higher percentage amounts of equity once controlling for UNDP; this is in line with results in Tables 3 – 5. The dummy variables controlling for the motives to go public MOTTD, MOTCAPX and MOTWK do not provide much explanatory power; this matches the evidence by Carlson et al (2006a). The dummy variable INVGR is significant and suggests that firms with lower financing costs issue lower percentage amounts of equity. Finally, the variable BUBBLE is positive and significant, suggesting that firms issue an 8% more during hot markets, all else equal.

Table 8 provides the corresponding estimation results for SEOs. The specification of the system is essentially the same as in IPOs; two additional variables are included to check whether the history former public offerings of the firm is related to the moments of the current public offering. YSISS checks whether the time elapsed since the last offering (either IPO or former SEO) is relevant for the contemporaneous SEO strategy. SEONO measures whether the number of public offerings performed since the IPO is relevant for the contemporaneous SEO strategy.

Overall, both the significance and the magnitude of the coefficients obtained for SEOs in Table 8 are the same as those described for IPOs. There are still two main observations to be done. First, the comparison between Tables 7 and 8 suggests that asymmetric information has a greater impact on IPOs than on SEOs. This is reflected in the magnitude of coefficient $\phi_{12}$ and in the decrease of the $\chi^2$ statistic for the joint test of significance of UNDP.30 Second, the coefficients on YSISS and SEONO suggest that the time elapsed since the last offering is significant in explaining AGE, UNDP and ALPHA. Both findings are supporting evidence for the economic trade-offs derived in Section 3 for multi-stage offering strategies.

30It is also reflected in the fact that the dummy BUBBLE is no longer significant for any of the equations; this is the reason why it does not show in Table 8.
5 Conclusions

This model addresses the option to raise public equity in a real options framework. The optimal strategy to raise public equity results from the firm’s value maximizing behavior and depends on its fundamentals, its long term investment plan and its issuance costs of public capital; it also depends on the strategy of other firms in the market.

From the theoretical standpoint, the paper provides several new insights to the current literature of real options and IPOs. First, the model provides a tractable approach to solve for signalling games in real options and relates the optimal timing to go public to IPO waves. In hot markets, firms have incentives to pool and the timing to go public is uninformative; in cold markets, asymmetric information erodes the option value of raising public equity. Second, the model provides a tractable approach to model underpricing in a real options set-up, suggesting that the option exercise of firms and underpricing must be solved jointly in equilibrium. Finally, the model provides a rational approach to relate underpricing to market timing of public offerings, and suggests that younger firms provide higher underpricing when they reveal their type in equilibrium.

From the empirical standpoint, the paper highlights that the age of the firm is significant to explain the decision to raise public equity and provides supporting evidence for the fact that firms go public earlier during hot markets. The paper also emphasizes the use of a simultaneous equation model to test the moments to go public jointly; the timing to raise public equity, the underpricing of shares, the percentage amount of shares issued and gross spreads of underwriting are all related and depend on the same underlying firm characteristics.

The model can be extended in several ways. First, a natural extension is incorporate discount effects in the model so as to provide a better rationale for hot markets and IPO waves (Pastor and Veronesi, 2003). Second, it would be interesting to understand the impact of alternative financing choices in the timing and structure of the going public strategy (Stein, 1991). Third, it would be relevant consider the impact of managerial entrenchment and corporate control in the decision to go public. As Myers (2000) has suggested, models should consider the rationale for outside equity financing when insiders are not inclined to act in outside investors’ interests. Finally, this paper assesses the option to go public from an investment perspective. Assessing the impact of alternative motives to go public should thus provide further insights on the underlying determinants of the going public strategy.
6 Appendix

6.1 Single crossing conditions

The relevant domains of \( \{x_{1j}, \alpha_{1j}, \epsilon_{1j}\} \) when used as signals by managers are given by

\[
0 < X_0 < x_{0j} < x_{1j} < \alpha_{1j} < 1 < \epsilon_{1j} < 1
\]

The timing to go public must be larger than both the threshold for shelf registration and the initial value of cash flows, which in turn is strictly positive by assumption. The percentage amount of shares issued is strictly lower than one due to the convexity of underwriting fees. Finally, managers effectively provide discounts on share prices as long as \( \epsilon_{1j} < 1 \); otherwise \( \epsilon_{1j} \) is not a signal.

For infinite-strategy signalling games with multiple signals, Cho and Sobel (1990) show that there exists a separating equilibrium as long as the single crossing conditions hold for each signal in isolation. Consider then the case where \( X_t = x_{1j} \). The single crossing conditions of the IPO game for any firm \( j \) are given by

\[
\frac{\partial}{\partial \theta} \left[ \frac{\partial V_{1j}}{\partial x_{1j}} \right] = \frac{(1 - \alpha_{1j} + \alpha_{1j} \epsilon_{1j})}{\alpha_{1j} \delta} > 0
\]

(21)

\[
\frac{\partial}{\partial \theta} \left[ \frac{\partial V_{1j}}{\partial \alpha_{1j}} \right] = -\frac{(1 - \epsilon_{1j})}{\alpha_{1j} \delta} < 0
\]

(22)

\[
\frac{\partial}{\partial \theta} \left[ \frac{\partial V_{1j}}{\partial \epsilon_{1j}} \right] = \frac{x_{1j}}{\delta} > 0
\]

(23)

The mechanism of the IPO game is implementable since the value function of the firm is linear in prices (Laffont, 1984). The incentive compatibility constraints are satisfied globally since both each component of the decision complies with the single crossing conditions above and the value function of the firm is linear in prices (Fudenberg and Tirole, 1991). There thus exists a separating equilibrium of the IPO game where firms truly reveal their type through signals \( \{x_{1j}, \alpha_{1j}, \epsilon_{1j}\} \).

6.2 The approach by Grenadier and Wang (2005)

The solution method presented in this paper to account for ICCs differs from Grenadier and Wang (2005). While both approaches yield the same result, the solution method
of this paper implies solving for firm value in the standard real options approach (Dixit and Pyndick, 1994) yet incorporating Lagrange multipliers in the boundary conditions. The proof that both approaches are analogous consists in showing that all the boundary conditions and ICCs are considered in a different order without altering results.

Consider first the analogy between the smooth pasting condition on firm value in the approach by Grenadier and Wang (2005) and the corresponding smooth pasting condition (14). Assume first that there is revelation in equilibrium such that

\[
U_{1H} = (1 - \epsilon_{1H})V_{2H}
\]

Once accounting for both value matching and smooth pasting conditions, firm value is given by

\[
V_{1H} = \frac{X_t}{\delta} + \left(\frac{1 - \alpha_{1H} + \alpha_{1H}\epsilon_{1H}}{\delta} \theta_{1H} - \frac{1}{x_{1H}} - (I_1 + F(\alpha_{1H}))\right) \left(\frac{X_t}{x_{1H}}\right)^\nu
\]

At the threshold where \(X_t = x_{1j}\), the expression above becomes

\[
V_{1H} = (1 - \alpha_{1H} + \alpha_{1H}\epsilon_{1H})V_{2H} - (I_1 + F(\alpha_{1H}))
\]

(24)

In the presence of asymmetric information, higher types are subject to ICCs. Then, the Lagrangian formulation à la Grenadier and Wang (2005) is such that

\[
\mathcal{L} = V_{1H} - \chi_{1H} \left[ \tilde{V}_{1L} - V_{1L} \right]
\]

The first order condition of the Lagrangian above with respect to \(x_{1j}\) is such that

\[
\frac{\partial \mathcal{L}}{\partial x_{1H}} = \frac{\partial V_{1H}}{\partial x_{1H}} - \chi_{1H} \left[ \frac{\partial \tilde{V}_{1L}}{\partial x_{1H}} - \frac{\partial V_{1L}}{\partial x_{1H}} \right]
\]

Now, note that the smooth pasting condition used to compute firm value is given by

\[
\frac{\partial V_{1H}}{\partial x_{1H}} = (1 - \alpha_{1H} + \alpha_{1H}\epsilon_{1H}) \frac{\partial V_{2H}}{\partial x_{1H}}
\]

So, an alternative expression for \(\frac{\partial \mathcal{L}}{\partial x_{1H}}\) is given by

\[
\frac{\partial \mathcal{L}}{\partial x_{1H}} = (1 - \alpha_{1H} + \alpha_{1H}\epsilon_{1H}) \frac{\partial V_{2H}}{\partial x_{1H}} - \chi_{1H} \left[ \frac{\partial \tilde{V}_{1L}}{\partial x_{1H}} - \frac{\partial V_{1L}}{\partial x_{1H}} \right]
\]

At the optimal separating equilibrium it holds that both

\[
\frac{\partial \mathcal{L}}{\partial x_{1H}} = 0
\]

\[
\chi_{1H} \left[ \tilde{V}_{1L} - V_{1L} \right] = 0
\]

(25)
Then it holds that

\[
\frac{\partial L}{\partial x_{1H}} = (1 - \alpha_{1H} + \alpha_{1H}\epsilon_{1H}) \frac{\partial V_{1H}}{\partial x_{1H}} - \chi_{1H} \left[ \frac{\partial \tilde{V}_{1L}}{\partial x_{1H}} - \frac{\partial V_{1L}}{\partial x_{1H}} \right]
\]

\[
\frac{\partial L}{\partial x_{1H}} = \frac{\partial V_{1H}}{\partial x_{1H}}
\]

which yields the smooth pasting condition under asymmetric information stated in Equation (??), namely

\[
\frac{\partial V_{1H}}{\partial x_{1H}} = (1 - \alpha_{1H} + \alpha_{1H}\epsilon_{1H}) \frac{\partial V_{2H}}{\partial x_{1H}} - \chi_{1H} \left[ \frac{\partial \tilde{V}_{1L}}{\partial x_{1H}} - \frac{\partial V_{1L}}{\partial x_{1H}} \right]
\]

The analogy between the value matching condition in my set-up and that of Grenadier and Wang (2005) follows using the same reasoning.

6.3 The age of firms in IPO waves

I hereby prove why the average age of a firm decreases in hot markets when \( p = 0.5 \) before there is increased IPO activity. Denote \( p_0 \) as the probability of being of the high type when firms separate and \( p_1 \) as the probability of being of the high type when firms pool in equilibrium such that \( p_0 \leq \bar{p} \leq p_1 \). Define \( T \) as the age of firms in pooling equilibrium and assume further that the age of high types is \( T - \Delta_H \) and the age of low types is \( T + \Delta_L \) in separating equilibria.

Consider first the case that \( \Delta_H = \Delta_L \). The average age of issuing firms decreases in pooling equilibria as long as \( p_0 < \frac{1}{2} \). This implies that the parameter requirement such that the average age decreases in hot markets is given by \( p_0 < \frac{1}{2} \leq \bar{p} \). This requirement seems reasonable as it implies that in a uniform distribution of types the average age of issuing firms does not vary between separating and pooling equilibria.

In the model, though, the maximum \( p_0 \) such that the average age decreases in pooling equilibria is higher than \( \frac{1}{2} \). This is because the separating equilibria of Proposition 3 are such that \( \Delta_H < \Delta_L \). This implies that even in a uniform distribution of types the average age of issuing firms decreases; this is already observed in Table 4. The requirement needed to state that firm age decreases during hot markets is then \( p_0 < \frac{1}{2} + \varepsilon \leq \bar{p} \) for some \( \varepsilon > 0 \). The formal proof that \( \Delta_H < \Delta_L \) is provided below and relies in comparing the optimal exercises strategies of firms under separating equilibria and pooling. For simplicity, I consider the separating equilibrium in Proposition 5.
Using the notation of the paper, $\Delta_H < \Delta_L$ can be restated in terms of the expected ages of firms going public, namely

$$E[T_{1H}^*] - E[T_{1L}] < E[T_{1L}^*] - E[T_{1H}]$$

Using the expression in (20), this implies

$$\frac{x_{1H}^*}{x_{1H}} < \frac{x_{1L}^*}{x_{1H}}$$

replacing by the results in Propositions 1-5,

$$\frac{I_1 + F_i(\alpha_1^*)}{\theta_{1H} - 1} \frac{(I_1 + F_i(\alpha_1^*))}{(1 - \alpha_{1H} + \alpha_{1H} \epsilon_{1H}) \theta_{1H} - 1} < \frac{\theta_{1H} - 1}{\theta_{1L} - 1}$$

since $\alpha_{1H} < \alpha_1^*$, a stronger requirement than the inequality $\Delta_H < \Delta_L$ is such that

$$\frac{(1 - \alpha_{1H} + \alpha_{1H} \epsilon_{1H}) \theta_{1H} - 1}{\theta_{1H} - 1} < \frac{(1 - \alpha_{1H}) \theta_{1H} - 1}{\theta_{1L} - 1}$$

which is always the true since $\theta_{1H} > \theta_{1L}$ and $\epsilon_{1H} < 1$. It then holds that $\Delta_H < \Delta_L$.

### 6.4 Extended models for SEOs

Consider first the case of $i = 2$ where the firm decides on the number $n$ of public offerings to perform. In the case of $n = 1$, the going public procedure consists of three stages. In Stage 1, the firm does the public offering, raises the necessary funds for its two investment projects and invests in the first one. Since all funds for current and future investment are raised at the time of the IPO, the marginal costs of the firm with respect to $\alpha_{1j}$ are set equal to the net present value of total investment costs in Stages 1 and 2.$^{31}$ Finally, in Stage 2 the firm has already raised the necessary capital to invest in earlier stages and exercises the option to invest in the second growth option. The value of firm $j$ in Stage 1 is then given by

$$V_{1j}^{n=1} = \frac{X_i}{\delta} + \left[ \frac{I_j + F_i(\alpha_{1j})}{v - 1} \right] \left( \frac{X_i}{\pi_{1j}} \right)^v + \left[ \frac{I_2}{v - 1} \right] \left( \frac{X_i}{\pi_{2j}} \right)^v$$

In the case of $n = 2$, the manager decides to raise public capital in two public offerings. In Stage 1, the manager raises proceeds for a percentage $\alpha_{1j}$ to finance a fraction $\alpha_{1j}I_1$ of its current investment costs and pays underwriting costs. With two signals and two unknown

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$^{31}$Note that the total investment costs of the firm should not be defined in expected terms, as long as the firm might not be able to meet its financing requirements for certain states in Stage 2.
types of real options, market players fully update their beliefs with a single IPO. In Stage 2, the manager raises a percentage $\alpha_{2j}$ to finance a fraction $\alpha_{2j}I_2$ of the investment costs of its second growth option and pays additional underwriting fees. In this case, the value of firm $j$ is given by

$$V_{1j}^{n=2} = \frac{X_t}{\delta} + \left[ \frac{I_1 + F(\alpha_{1j})}{v-1} \right] \left( \frac{X_t}{\pi_{1j}} \right)^{\nu} + \left[ \frac{I_2 + F(\alpha_{2j})}{v-1} \right] \left( \frac{X_t}{\pi_{2j}} \right)^{\nu}$$

The optimal amount of public offerings to perform when firm $j$ has two growth options ($i = 2$) is endogenously determined such that

$$V_{1j} = \sup_T E \left[ e^{-r(T_{n=1} - \tau)} V_{1j}^{n=1} I_{T_1=n=1} I_{T_1<n=2} + e^{-r(T_{n=2} - \tau)} V_{1j}^{n=2} I_{T_1=n=2} I_{T_1<n=1} \right]$$

where $V_{1j}$ is the value of firm $j$ in Stage 1 when the firm performs $n$ public offerings. In this case, the separating equilibrium is such that

$$\{x_{1H}, x_{2H}\} = \left\{ \frac{(I_1 + F(\alpha_{1H}))}{(1 - \alpha_{1H} + \alpha_{1H} \epsilon_{1H}) \theta_{1H} - 1} \frac{\delta v}{v - 1}, \frac{I_2 + [F(\alpha_{2H})] I_{n=2}}{\theta_{2H} - \theta_{1H}} \frac{\delta v}{v - 1} \right\}$$

the optimal percentage amount of shares to issue are given by

$$\{\alpha_{1H}, \alpha_{2H}\} = \left\{ \frac{(I_1 + \hat{I}_2 I_{n=2} - U_{1H})}{\gamma c}^{1/\gamma - 1}, \left( \frac{I_2}{\gamma c} \right)^{1/\gamma - 1} I_{n=2} \right\}$$

where $\hat{I}_2$ is the present value of $I_2$ in Stage 1 and $\epsilon_{1H}$ is given by

$$\tilde{V}_{1L} \bigg|_{X_t=x_{1H}} = V_{1L} \bigg|_{X_t=x_{1H}}$$

Consider now the case where $i = 4$ where the manager of the high type fully reveals firm type at the SEO. For simplicity, I assume further that $n = 2$ and that the public funds raised at each stage are a fraction of the investment costs of that stage only. I do this without loss of generality, to illustrate the role of asymmetric information when $i > 3$. Since it is a sequential investment problem, I solve it by backward induction and start considering Stage 2.

The problem faced by the manager of higher type firms in Stage 2 is solved in the same way as in Proposition 3. As a distinct feature, the underpricing in Stage 2 is a function the IPO strategy in Stage 1, namely

$$U_{2j} \bigg|_{X_t=x_{2j}} = (1 - \epsilon_{2j}) V_{2j}$$

This is because the signals observed by market players in Stage 1 determine the amount of asymmetric information before the SEO announcement.
Consider now the problem faced by the manager of firm \( j \) in Stage 1. Sequential investment solving is such that the optimal IPO strategy depends on the optimal SEO strategy. In addition, beliefs by market players at the IPO are self-fulfilling; the ex-ante underpricing determined by managers must be equal to the ex-post underpricing by market players in equilibrium. The problem faced by firms in Stage 1 is solved in the same way as Proposition 3, yet underpricing is such that it solves

\[
U_{1j} | X_t = x_{1j} = (1 - \epsilon_{1j}) V_{2j}
\]

The commitment-proof optimal separating equilibrium is such that the low type performs its optimal strategy under perfect information and the optimal strategy for the high type in Stages \( i = 1, 2 \) is such that

\[
\begin{align*}
X_{iH} &= \frac{(I_i + F(\alpha_{iH}))}{(1 - \alpha_{iH} + \alpha_{iH} \epsilon_{iH}) \theta_{iH} - 1} \delta v \\
\alpha_{iH} &= \left( \frac{I_i - U_{iH}}{C_{iH}} \right)^{\frac{1}{\gamma - 1}}
\end{align*}
\]

where \( \epsilon_{iH} \) is given by

\[
\delta V_{iL} | X_t = x_{iH} = V_{iL} | X_t = x_{iH}
\]

7 Database construction

The source for identifying IPOs and SEOs is the Securities Data Company’s (SDC) Deals Database. The sample considers common equity issues between January 1, 1980 and December 31, 2005 for all US firms excluding financial firms (SICs 6000-6999) and regulated industries (SICs 4900-4999). Then match the data obtained from this source to the merged CRSP-Compustat database, to obtain information on firms financials.

The main variables used throughout the empirical section that come from SDC are: (1) the date of the issue; (2) the date of the IPO; (3) the date when the company was founded; (4) the offer price of the deal; (5) the stock price at close of offer/first trade; (6) the principal amount traded, for all markets; (7) the primary use of proceeds; (8) the gross spread including total management fees, underwriting fees and selling concessions; (9) the venture capital backed IPO issue flag; (10) the stock exchange on which the issuer’s stock trades; (11) the underwriter performing the deal. PROCEEDS are thus equal to (1); UNDP is the difference between the (4) and (5) divided by (5); GSPREAD is (8); ALPHA is the ratio of (6) over the market value of equity after the offer ((5) times item25 from COMPUSTAT); MOTCAPX, MOTTD and MOTWK are dummies constructed based on
(7); NASDAQ and NYSE are dummies based on (10); finally, BRUNTOP is built based on (11).

The variable describing firm age (AGE) comes from two sources. First, for the firms no missing info on dates, AGE is the difference between the date of the issue (1) and the date when the firm was founded (3) as stated by SDC. For firms with (1) but no (3), I used the year when the company was founded are reported in the database [Age Data (.xsl)] built by Boyan Jovanovic and available online.32

Ex-ante firm characteristics used in the database are taken from the merged CRSP-Compustat database. LNSALES is the log of sales (item12); CAPX to TA is the ratio of capital expenditures (item128) and total assets (item6); TLTD is the sum total long term debt (item9) and long term debt in current liabilities (item34); WK is working capital (item179). The dummy INVGR has been built using the S&P long term credit rating reported in item280.

32See http://www.nyu.edu/econ/user/jovanovi/. I thank Bojan Jovanovic for posting the data online and Alexei Zdhanov for making me aware of this.
References


Figure 1: The going public procedure

In Stage 0, firms decide the time to do their shelf registration at the stock exchange. This figures illustrates the structure of the model derived in Sections 2-3. In Stage 1, firms decide the optimal time to do their IPO and invest in their first growth option. From Stage 2 on, firms may perform further public offerings and invest in further growth options depending on their investment plan, cost structure and asymmetric information in the market. The time at which the firm passes from one stage to the other $T_{ij}$ is endogenously determined and depends on firm characteristics.
Figure 2: The going public strategy for firm j

This figure depicts the optimal strategy to issue public equity as a function of its cash flows. The optimal strategy to raise public equity depends on the realizations of $X_t$. When $X_t$ hits the threshold $x_{ij}$, the firm passes from one stage to the other in the going public procedure. The age of the firm is thus informative about the stage in which the firm is for separating equilibria.
Figure 3: Comparative Statics for High Types in the basic IPO model

This figure illustrates the comparative statics between the different signals when $i=1$ and firms fully reveal their private information at the IPO. Denote underpricing as $(1 - \epsilon_{ij})$. The cash flow threshold $x_{1j}$ is increasing in underpricing; the timing to raise public equity is closer to that of perfect information when firms provide higher levels of underpricing to outside investors. The fraction of shares issued $\alpha_{ij}$ is decreasing in underpricing costs; firms optimally issue less when the issuance costs are higher. The Lagrange multiplier $\lambda_{1j}$ is increasing in $\epsilon_{ij}$; this reflects the duality between signalling costs and underpricing in the model. Finally, the cash flow threshold to raise public equity is decreasing in $\alpha_{1j}$; older (and larger) firms issue lower percentages of equity in equilibrium.
Figure 4: Signalling costs for High Types
in the basic IPO model

This figure illustrates the signalling costs for medium and high types in separating equilibria. The strategy of high types is that of Proposition 3 where the binding ICC considers the optimal strategy for low types under perfect information. In the presence of asymmetric information, the model predicts that aggregate wealth is higher for higher levels of underpricing.
Figure 5: Separating Equilibria
and the Strategy to Go Public

This figure illustrates it is more costly for higher types to separate when the two types are more similar. Consider a uniform distribution of types such that \( \sigma_\theta \) reflects the dispersion of the quality of growth options in the market. As \( \sigma_\theta \) decreases, the high types try to make the issue unappealing to low types by going public earlier, issuing a lower percentage amount of equity and underpricing more their shares. The upper bound on the probability of high types derived in Proposition 6 is increasing in \( \sigma_\psi \); this suggests that separating equilibria are more likely when signalling is less costly.
Figure 6: Distribution of Firm Age, Fraction of Shares Issued, Underpricing and Proceeds for IPOs in the Working Sample

This figure illustrates the kernel density distributions of AGE, ALPHA, UNDP and PROCEEDS for IPOs in the working sample. The dashed line corresponds to the observations during the IPO bubble and the solid line considers the remaining observations. The distribution of AGE, ALPHA, UNDP and PROCEEDS changes significantly during hot markets. During hot markets, AGE is more concentrated on lower values and PROCEEDS are higher (due to overvaluation). As a caveat, ALPHA decreases.
Figure 7: Distribution of Firm Age, Fraction of Shares Issued, Underpricing and Proceeds for SEOs in the Working Sample

This figure illustrates the distribution of AGE, ALPHA, UNDP and PROCEEDS in the working sample. The dashed line corresponds to the observations during the IPO bubble and the solid line considers the remaining observations. The distribution of AGE, ALPHA and UNDP does not change significantly during hot markets, suggesting that market players are better informed about firm type during SEOs (Section 3).
### Table 1: Sample Statistics

This table reports summary statistics on the working sample US public equity issues reported by SDC from 1980 to 2005. **PROCEEDS** are the total proceeds by the issuer in all markets. **UNDP** is the underpricing of issued shares after the first trading day. **GSPREAD** is the gross underwriting spread including management, underwriting and selling concession fees. **ALPHA** is the percentage amount of shares issued at the offering. **LNSALES** is the logarithm of the sales of the firm before the offering. **AGE** is the age of the firm at the moment of the public offering. **LNSALES** is the logarithm of the sales of the firm before the offering.

#### Panel A: IPOs

<table>
<thead>
<tr>
<th></th>
<th>80-97 and 01-05</th>
<th>98-00</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PROCEEDS</strong></td>
<td>4565</td>
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</tr>
<tr>
<td><strong>UNDP</strong></td>
<td>3775</td>
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</tr>
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<td><strong>GSPREAD</strong></td>
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<td><strong>AGE</strong></td>
<td>1732</td>
<td>14.859</td>
</tr>
<tr>
<td><strong>LNSALES</strong></td>
<td>4048</td>
<td>3.201</td>
</tr>
</tbody>
</table>

#### Panel B: SEOs

<table>
<thead>
<tr>
<th></th>
<th>80-97 and 01-05</th>
<th>98-00</th>
</tr>
</thead>
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<tr>
<td><strong>PROCEEDS</strong></td>
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<tr>
<td><strong>UNDP</strong></td>
<td>5235</td>
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<tr>
<td><strong>GSPREAD</strong></td>
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<tr>
<td><strong>ALPHA</strong></td>
<td>5274</td>
<td>0.181</td>
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<tr>
<td><strong>AGE</strong></td>
<td>2806</td>
<td>26.827</td>
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<tr>
<td><strong>LNSALES</strong></td>
<td>5430</td>
<td>4.823</td>
</tr>
</tbody>
</table>
Table 2: Pairwise Correlations of Main Variables

This table reports the pairwise correlations of the main variables of public offerings analyzed in Section 5. **PROCEEDS** are the total proceeds by the issuer in all markets **UNDP** is the underpricing of issued shares after the first trading day. **GRSPREAD** is the gross underwriting spread including management, underwriting and selling concession fees. **ALPHA** is the percentage amount of shares issued at the offering. **AGE** is the age of the firm at the moment of the public offering. **LNSALES** is the logarithm of the sales of the firm before the offering.

**Panel A: IPOs - 80-97 and 01-05**

<table>
<thead>
<tr>
<th></th>
<th>PROCEEDS</th>
<th>UNDP</th>
<th>GRSPREAD</th>
<th>ALPHA</th>
<th>AGE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>UNDP</strong></td>
<td>0.0217</td>
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<td></td>
</tr>
<tr>
<td><strong>GRSPREAD</strong></td>
<td>0.1721**</td>
<td>0.0166</td>
<td>-0.3292**</td>
<td>-0.0275</td>
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<tr>
<td><strong>ALPHA</strong></td>
<td>-0.0011</td>
<td>-0.3292**</td>
<td>-0.0275</td>
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<td></td>
</tr>
<tr>
<td><strong>AGE</strong></td>
<td>0.1726**</td>
<td>-0.0521*</td>
<td>0.0848**</td>
<td>-0.0428</td>
<td>1</td>
</tr>
<tr>
<td><strong>LNSALES</strong></td>
<td>0.4805**</td>
<td>0.0102</td>
<td>0.1796**</td>
<td>-0.1169**</td>
<td>0.3989**</td>
</tr>
</tbody>
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**Panel B: IPOs - 98-00**

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<tr>
<th></th>
<th>PROCEEDS</th>
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<th>GRSPREAD</th>
<th>ALPHA</th>
<th>AGE</th>
</tr>
</thead>
<tbody>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>GRSPREAD</strong></td>
<td>0.1186*</td>
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<td>-0.5486**</td>
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<td>1</td>
</tr>
<tr>
<td><strong>ALPHA</strong></td>
<td>0.0726</td>
<td>-0.5486**</td>
<td>-0.2108**</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td><strong>AGE</strong></td>
<td>0.0578</td>
<td>-0.1282*</td>
<td>-0.0098</td>
<td>0.0815</td>
<td>1</td>
</tr>
<tr>
<td><strong>LNSALES</strong></td>
<td>0.3782**</td>
<td>-0.1155*</td>
<td>0.0238</td>
<td>-0.0655</td>
<td>0.3652**</td>
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</table>

**Panel C: SEOs 80-97 and 01-05**

<table>
<thead>
<tr>
<th></th>
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<th>GRSPREAD</th>
<th>ALPHA</th>
<th>AGE</th>
</tr>
</thead>
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<tr>
<td><strong>UNDP</strong></td>
<td>0.0380*</td>
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<td></td>
</tr>
<tr>
<td><strong>GRSPREAD</strong></td>
<td>0.1495**</td>
<td>0.0256</td>
<td>-0.1409**</td>
<td>0.2623**</td>
<td>1</td>
</tr>
<tr>
<td><strong>ALPHA</strong></td>
<td>-0.0996**</td>
<td>-0.1409**</td>
<td>0.2623**</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td><strong>AGE</strong></td>
<td>0.1514**</td>
<td>-0.0130</td>
<td>0.0552*</td>
<td>-0.1915**</td>
<td>1</td>
</tr>
<tr>
<td><strong>LNSALES</strong></td>
<td>0.4725**</td>
<td>0.0185</td>
<td>0.1501**</td>
<td>-0.3517**</td>
<td>0.4654**</td>
</tr>
</tbody>
</table>

**Panel D: SEOs - 98-00**

<table>
<thead>
<tr>
<th></th>
<th>PROCEEDS</th>
<th>UNDP</th>
<th>GRSPREAD</th>
<th>ALPHA</th>
<th>AGE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>UNDP</strong></td>
<td>0.1280**</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>GRSPREAD</strong></td>
<td>0.2124**</td>
<td>0.0201</td>
<td>-0.1002*</td>
<td>0.0377</td>
<td>1</td>
</tr>
<tr>
<td><strong>ALPHA</strong></td>
<td>-0.0298</td>
<td>-0.1002*</td>
<td>0.0377</td>
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<tr>
<td><strong>AGE</strong></td>
<td>0.1615**</td>
<td>0.0249</td>
<td>0.1032*</td>
<td>-0.0826</td>
<td>1</td>
</tr>
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<td><strong>LNSALES</strong></td>
<td>0.4151**</td>
<td>0.1025*</td>
<td>0.2346**</td>
<td>-0.2420**</td>
<td>0.4321**</td>
</tr>
</tbody>
</table>

*significant at 5 percent; **significant at 1 percent
Table 3: Parameters for Calibration

This table reports the parameters used in Tables 4 to 6. Parameters X0, I, theta and sigmatheta have been calibrated to match the interquantile range of PROCEEDS, the median of AGE and the median of LNSALES of IPOs reported on Table 1.

<table>
<thead>
<tr>
<th>Input</th>
<th>Source</th>
<th>Parameter Choice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash flow premium</td>
<td>data</td>
<td>δ =0.025</td>
</tr>
<tr>
<td>Cash flow volatility</td>
<td>data</td>
<td>σx =0.25</td>
</tr>
<tr>
<td>Disclosure costs</td>
<td>data</td>
<td>D =0.75</td>
</tr>
<tr>
<td>Fixed costs of underwriting</td>
<td>data</td>
<td>f=0.20</td>
</tr>
<tr>
<td>Probability of liquidity shocks</td>
<td>data</td>
<td>λ=0.05</td>
</tr>
<tr>
<td>Risk free interest rate</td>
<td>data</td>
<td>r=0.06</td>
</tr>
<tr>
<td>Initial value of cash flows</td>
<td>calibrated</td>
<td>X0 =2.072</td>
</tr>
<tr>
<td>Investment costs</td>
<td>calibrated</td>
<td>I =25.393</td>
</tr>
<tr>
<td>Markup on cash flows</td>
<td>calibrated</td>
<td>θ =2.008</td>
</tr>
<tr>
<td>Variance of markup on cash flows</td>
<td>calibrated</td>
<td>σθ=0.148</td>
</tr>
<tr>
<td>Convexity of underwriting fees</td>
<td>assumption</td>
<td>γ =2</td>
</tr>
<tr>
<td>Cost of liquidity shock</td>
<td>assumption</td>
<td>η =0.930</td>
</tr>
</tbody>
</table>
Table 4: Calibrated IPO Model with i=1

This table illustrates the main predictions of the model for IPOs. Panel A reports the case of perfect information. Panel B reports the separating equilibrium with no underpricing. Panel C derived the optimal separating equilibrium where underpricing is the highest for all types. High types issue earlier and provide more underpricing to reveal their type in separating equilibria.

<table>
<thead>
<tr>
<th></th>
<th>Perfect Info</th>
<th>Asymmetric Info</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Panel A</td>
<td>Panel B</td>
</tr>
<tr>
<td></td>
<td>$\chi_1 = 0; \epsilon_1 = 1$</td>
<td>$\chi_1 &gt; 0; \epsilon_1 = 1$</td>
</tr>
<tr>
<td></td>
<td>L</td>
<td>H</td>
</tr>
<tr>
<td>Going Public Strategy</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_0$</td>
<td>0.177</td>
<td>0.139</td>
</tr>
<tr>
<td>$x_1$</td>
<td>3.943</td>
<td>2.150</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>16.35%</td>
<td>16.35%</td>
</tr>
<tr>
<td>$\epsilon_1$</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$\chi_1$</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>IPO</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proceeds</td>
<td>44.139</td>
<td>32.412</td>
</tr>
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<td>Underpricing</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Gross Spread</td>
<td>5.14%</td>
<td>7.01%</td>
</tr>
<tr>
<td>Age at $x_0$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Age at $x_1$</td>
<td>182</td>
<td>20</td>
</tr>
<tr>
<td>Abnormal Returns</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
</tbody>
</table>
Table 5: Calibrated SEO Model with \( i = 2 \)

This table illustrates the main predictions of the model for the SEO case with \( i = 2 \). Panel A reports the case of perfect information. Panel B reports the optimal separating equilibrium. The median age to go public in Panel B is lower with respect to Table 3; firms with multiple growth options go public earlier.

<table>
<thead>
<tr>
<th></th>
<th>Panel A</th>
<th>Panel B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \chi_1 = 0; , \epsilon_1 = 1 )</td>
<td>( \chi_1 \rightarrow 0; , \epsilon_1 &lt; 1 )</td>
</tr>
<tr>
<td></td>
<td>L</td>
<td>H</td>
</tr>
<tr>
<td><strong>Going Public Strategy</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x_0 )</td>
<td>0.126</td>
<td>0.107</td>
</tr>
<tr>
<td>( x_1 )</td>
<td>3.391</td>
<td>2.360</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>16.35%</td>
<td>16.35%</td>
</tr>
<tr>
<td>( \epsilon_1 )</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>4.586</td>
<td>3.828</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>16.35%</td>
<td>16.35%</td>
</tr>
<tr>
<td>( \epsilon_2 )</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td><strong>IPO</strong></td>
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<td></td>
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<tr>
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<td>58.058</td>
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</tr>
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<td>3.91%</td>
</tr>
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<td>Age at ( x_0 )</td>
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<td>0</td>
</tr>
<tr>
<td>Age at ( x_1 )</td>
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<td>15</td>
</tr>
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<td>Abnormal Returns</td>
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<td>0.00%</td>
</tr>
<tr>
<td><strong>SEO</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Underpricing</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Gross Spread</td>
<td>2.07%</td>
<td>2.07%</td>
</tr>
<tr>
<td>Age at ( x_2 )</td>
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<td>174</td>
</tr>
<tr>
<td>Abnormal Returns</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
</tbody>
</table>
This table reports results for the SEO case with $i=4$. Panel A reports the case of perfect information. Panel B derives the optimal separating equilibrium. Higher types provide higher discounts on share prices; underpricing decreases with reduction of asymmetric information across public offerings.

### Table 6: Calibrated SEO Model with $i = 4$

<table>
<thead>
<tr>
<th></th>
<th>Panel A</th>
<th>Panel B</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\chi_1 = 0; \epsilon_1 = 1$</td>
<td>$\chi_1 \rightarrow 0; \epsilon_1 &lt; 1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Going Public Strategy</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_0$</td>
<td>0.126</td>
<td>0.107</td>
<td>0.126</td>
<td>0.108</td>
</tr>
<tr>
<td>$x_1$</td>
<td>3.397</td>
<td>2.357</td>
<td>3.397</td>
<td>2.089</td>
</tr>
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<td>16.35%</td>
<td>16.35%</td>
<td>16.35%</td>
<td>7.39%</td>
</tr>
<tr>
<td>$\epsilon_1$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.873</td>
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<td>16.35%</td>
<td>16.35%</td>
<td>11.34%</td>
</tr>
<tr>
<td>$\epsilon_2$</td>
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<td>0.978</td>
</tr>
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<td><strong>IPO</strong></td>
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</tr>
<tr>
<td>Proceeds</td>
<td>118.242</td>
<td>94.653</td>
<td>118.242</td>
<td>32.297</td>
</tr>
<tr>
<td>Underpricing</td>
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<td>0.00%</td>
<td>12.66%</td>
</tr>
<tr>
<td>Gross Spread</td>
<td>1.92%</td>
<td>2.40%</td>
<td>1.92%</td>
<td>1.92%</td>
</tr>
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<td>0</td>
</tr>
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<td>Age at $x_1$</td>
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<td>14</td>
<td>142</td>
<td>2</td>
</tr>
<tr>
<td>Abnormal Returns</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>12.66%</td>
</tr>
<tr>
<td><strong>SEO</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>184.094</td>
<td>184.094</td>
<td>184.094</td>
<td>132.531</td>
</tr>
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<td>0.00%</td>
<td>0.00%</td>
<td>2.16%</td>
</tr>
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<td>Gross Spread</td>
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<td>3.49%</td>
<td>3.49%</td>
<td>2.41%</td>
</tr>
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<td>Age at $x_2$</td>
<td>222</td>
<td>174</td>
<td>222</td>
<td>157</td>
</tr>
<tr>
<td>Abnormal Returns</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>2.16%</td>
</tr>
</tbody>
</table>
Table 7: Estimation for IPOs

This table reports the estimation results for a sample of 1531 US IPOs of the simultaneous equation model described in Section 5. The description of the variables is provided in the Appendix. The endogenous variables of the system are \( \text{AGE}, \text{UNDP}, \text{GRSPREAD}, \text{ALPHA} \) and \( \text{ALPHASQ} \). Results confirm that \( \text{AGE} \) is a relevant moment of the going public decision and depends on firm characteristics.

<table>
<thead>
<tr>
<th>( \text{AGE} )</th>
<th>( \text{UNDP} )</th>
<th>( \text{GRSPREAD} )</th>
<th>( \text{ALPHA} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>LNSALES</td>
<td>2.572**</td>
<td>0.0280**</td>
<td>0.446**</td>
</tr>
<tr>
<td></td>
<td>(11.09)</td>
<td>(2.98)</td>
<td>(6.69)</td>
</tr>
<tr>
<td>VENTURE</td>
<td>-4.018**</td>
<td>ALPHASQ</td>
<td>-0.509**</td>
</tr>
<tr>
<td></td>
<td>(4.72)</td>
<td></td>
<td>(6.42)</td>
</tr>
<tr>
<td>TLTD/TA</td>
<td>11.186**</td>
<td>BRUNTOP</td>
<td>0.006*</td>
</tr>
<tr>
<td></td>
<td>(6.22)</td>
<td></td>
<td>(2.04)</td>
</tr>
<tr>
<td>CAPX/TA</td>
<td>-21.455**</td>
<td>OVERALL</td>
<td>0.011</td>
</tr>
<tr>
<td></td>
<td>(5.60)</td>
<td></td>
<td>1.440</td>
</tr>
<tr>
<td>BUBBLE</td>
<td>-1.161</td>
<td>BUBBLE</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>-0.930</td>
<td></td>
<td>0.640</td>
</tr>
<tr>
<td>WK/TA</td>
<td>-0.369</td>
<td>UNDP</td>
<td>-1.142**</td>
</tr>
<tr>
<td></td>
<td>-0.490</td>
<td></td>
<td>(11.23)</td>
</tr>
<tr>
<td></td>
<td>MOTCAPX</td>
<td></td>
<td>-0.054</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-1.490</td>
</tr>
<tr>
<td>NASDAQ</td>
<td>-0.036</td>
<td>MOTTD</td>
<td>0.033*</td>
</tr>
<tr>
<td></td>
<td>-1.880</td>
<td></td>
<td>(2.50)</td>
</tr>
<tr>
<td>NYSE</td>
<td>-0.036</td>
<td>MOTWK</td>
<td>-0.028</td>
</tr>
<tr>
<td></td>
<td>-1.340</td>
<td></td>
<td>-0.510</td>
</tr>
<tr>
<td>BUBBLE</td>
<td>0.182**</td>
<td>BUBBLE</td>
<td>0.085</td>
</tr>
<tr>
<td></td>
<td>(6.92)</td>
<td></td>
<td>(2.94)</td>
</tr>
<tr>
<td>AGE</td>
<td>-0.007**</td>
<td>AGE</td>
<td>-0.004**</td>
</tr>
<tr>
<td></td>
<td>(2.75)</td>
<td></td>
<td>(3.32)</td>
</tr>
<tr>
<td></td>
<td>IGRADE</td>
<td></td>
<td>-0.346**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(4.60)</td>
</tr>
<tr>
<td>Constant</td>
<td>7.565**</td>
<td>0.120*</td>
<td>Constant</td>
</tr>
<tr>
<td></td>
<td>(3.59)</td>
<td>(2.67)</td>
<td>(18.56)</td>
</tr>
<tr>
<td>Chi2</td>
<td>424.58</td>
<td>92.04</td>
<td>Chi2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>244.87</td>
</tr>
</tbody>
</table>
| Prob           | 0.0000        | 0.0000            | Prob           | 0.0000         | 0.0000

*significant at 5 percent; **significant at 1 percent
Table 8: Estimation for SEOs

This table reports the estimation results for a sample of 1912 US SEOs of the simultaneous equation model described in Section 5. The description of the variables is provided in the Appendix. Results confirm that AGE is a relevant moment of SEOs and depends on firm characteristics. Moreover, the number of public offerings done before the current issuance SEONO and the number of years passed since the last issuance YSISS are also significant to explain the moments of public offerings.

<table>
<thead>
<tr>
<th>AGE</th>
<th>UNDP</th>
<th>GSPREAD</th>
<th>ALPHA</th>
<th>GSPREAD</th>
</tr>
</thead>
<tbody>
<tr>
<td>LNSALES</td>
<td>3.061**</td>
<td>0.0196**</td>
<td>ALPHA</td>
<td>0.561**</td>
</tr>
<tr>
<td>TLTD/TA</td>
<td>8.230**</td>
<td>ALPHASQ</td>
<td>-0.920**</td>
<td></td>
</tr>
<tr>
<td>CAPX/TA</td>
<td>-20.827**</td>
<td>BRUNTOP</td>
<td>-0.002</td>
<td></td>
</tr>
<tr>
<td>WK/TA</td>
<td>-6.029*</td>
<td>OVERALL</td>
<td>0.001*</td>
<td></td>
</tr>
<tr>
<td>NASDAQ</td>
<td>-0.002</td>
<td>UNDP</td>
<td>-1.244**</td>
<td></td>
</tr>
<tr>
<td>NYSE</td>
<td>0.008</td>
<td>MOTCAPX</td>
<td>-0.016</td>
<td></td>
</tr>
<tr>
<td>MOTTD</td>
<td>0.028**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MOTWK</td>
<td>0.051*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IGRADE</td>
<td>-0.058**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AGE</td>
<td>-0.002*</td>
<td>AGE</td>
<td>0.001*</td>
<td></td>
</tr>
<tr>
<td>YSISS</td>
<td>1.111**</td>
<td>0.004*</td>
<td>YSISS</td>
<td>-0.000</td>
</tr>
<tr>
<td>SEONO</td>
<td>1.871**</td>
<td>0.003</td>
<td>SEONO</td>
<td>-0.000</td>
</tr>
<tr>
<td>Constant</td>
<td>1.109</td>
<td>-0.063*</td>
<td>Constant</td>
<td>0.007</td>
</tr>
<tr>
<td>Chi2</td>
<td>578.72</td>
<td>50.70</td>
<td>Chi2</td>
<td>103.35</td>
</tr>
<tr>
<td>Prob</td>
<td>0.0000</td>
<td>0.0001</td>
<td>Prob</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

*significant at 5 percent; **significant at 1 percent