Managerial Incentives, Endogeneity, and Firm Value

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Abstract

We examine the impact of firm and managerial characteristics on the value and incentive effects of executive stock options. Such a study hinges critically on using a model in which managerial behavior and firm value are endogenous, precluding the use of the Black-Scholes model. In our model, a self-interested manager makes an investment decision (choosing between a risky and a risk-free project) based on private information about investment opportunities. His decision is not only affected by the quality of his signal, but also by his risk aversion, his mix of wealth between stock, options, and shares, as well as the strike prices of his options (if he has any). We show that when a call option's value is $11.32 under an optimal investment policy, its value varies from $0 to $13.73, depending on the manager's risk aversion and wealth composition. We also show that the manager's delta is a poor proxy for incentives to invest, or agency costs. Managers of identical firms with nearly identical deltas may pursue vastly different investment strategies, due to differences in wealth composition and risk aversion.

These findings have important implications for empirical research on executive compensation and incentives. Specifically, our model shows that a manager's portfolio delta is not a sufficient statistic for his incentives to increase firm value. These incentives are multi-dimensional, suggesting that a univariate measure like delta will result in an omitted variables problem, and thus biased inferences. It is important that controls be included for executive risk aversion, wealth composition, investment opportunities, and firm risk.

JEL Classification: G31, G34, J33, L22.

1. Introduction

Performance-based pay can be used to overcome the agency problems between managers and shareholders that arise due to the separation of corporate ownership and control (Jensen and Meckling, 1976). At the heart of this idea is the notion that executives can directly and significantly affect firm value. Yet the Black-Scholes formula (Black and Scholes, 1973) - the "gold standard" for valuing many types of options, including executive stock options - assumes an exogenous stochastic process for the value of the asset underlying the option. Thus, in a Black-Scholes world, an executive's actions have no effect on his firm's shares.

But, the value of executive stock options is inextricably linked to executive actions and firm value. If options didn't affect managerial behavior and managerial behavior didn't affect firm value, why would firms choose to grant executive stock options? The question, then, is not whether the assumption of an exogenous stock price process is violated, but rather how serious are the violations of this assumption? Our paper's purpose is to assess the magnitude of this problem and its impact on compensation research.

A distinction is made in the compensation literature between the cost of issuing executive stock options (usually taken to be the Black-Scholes value) and the value of the options to an executive who receives them. For instance, Meulbroek (2001) shows that this value "wedge" can easily be 30% - 50% of the value of the options to a well-diversified investor. Similarly, Lambert, Larcker and Verrechia (1991) show that a risk-averse executive will assign a lower value to stock options than that implied by the Black-Scholes model. Ingersoll (2002) and Hall and Murphy (2002) draw similar conclusions. However, our point here is that the Black-Scholes option value is not even a good indicator of the value of the option to a well-diversified investor. The value of the option depends on who is holding it. If the holder of the option is unable to affect firm value, the option's value will not depend on the wealth and risk preferences of its owner, and the Black-Scholes model may give a reasonable estimate. However, if the option is held by an executive whose decisions affect the value of the underlying firm, then that executive's wealth and risk aversion can dramatically affect the value of that option. In this situation, the Black-Scholes formula provides a poor estimate of the option's value, regardless of whether we are interested in the value of the option to the executive or the cost of the option to the firm.

In a model of option valuation that accounts for the possibility of early option exercise or option termination, Carpenter (1998) describes how a manager's risk preferences and wealth can not only affect the value of the option to the executive, but also the cost of the option to the issuing firm. This is because the manager controls the exercise decision. Early exercise reduces the option's value to the executive, as well as the cost to the firm issuing the option. In an analogous way, when a manager who receives an option can affect firm value, say through his investment decision, his decision affects not only his valuation of the option, but also the option cost to his firm. A related point is that the *incentives* provided by an option (not just the value of that option) depend on characteristics of the individual holding the option and characteristics of the firm that he manages.

The logical extension of using the Black-Scholes model to value executive stock options is to use the option's Black-Scholes delta as a measure of the executive's incentive to increase stock price. It is common practice in the empirical compensation literature to measure executive incentives to increase firm value either by estimating the delta of the manager's options or the delta of his portfolio of equity-based compensation.[1] But delta captures the increase in the manager's options (or wealth) based on an increase in the share price. Because the Black-Scholes model is based on an exogenous share price process, it is assumed that the manager cannot affect the share price. However, it makes no sense to talk about the incentive for the manager to raise share price in a setting where it is assumed that the manager has no effect on share price! In fact what we are measuring with delta is how much the manager stands to benefit if the share price rises, assuming that the manager cannot affect share value, rather than the manager's incentive to increase the share price.

Finally, when managers are under-diversified relative to shareholders, and managers can alter their firms' investment strategies, the convexity of a manager's compensation/wealth is an important determinant of executive risk-taking incentives (Rajgopal and Shevlin, 2002 and Guay, 1999). But proxying for

1 See, for example, Jensen and Murphy (1990), Yermack (1995), Hall and Liebman (1998), Guay (1999), Core and Guay (1999), Johnson and Tian (2000), Brenner, Sundaram and Yermack (2000), Bliss and Rosen (2001), Datta, Iskandar-Datta, and Raman (2001), Perry and Zenner (2001), Core and Guay (2001), Barron and Waddell (2003), Aggarwal and Samwick (2003a, 2003b), Milbourn (2003) and Garvey and Milbourn (2003).

managerial incentives with delta ignores convexity. For example, consider two identical managers. Both managers have a wealth delta of 1. However, the first manager owns 1 share of stock while the second owns 2 call options with delta = 0.5. The first manager's wealth responds linearly to changes in share price, but the second manager has a highly convex payoff structure. These differences clearly matter if managers are given options to induce risk-taking, and the evidence that they are is quite strong.[2]

We construct a model in which stock and option prices are endogenously determined, based on the actions of a self-interested manager. Specifically, the manager maximizes his own utility and chooses between a risky and risk-free project, after observing a signal about the profitability of the risky project. The manager's investment strategy is characterized by setting a hurdle rate for the risky project that directly affects both share and option values. The manager's choice of a hurdle rate is directly affected by the parameters of his compensation contract (the mix of pay, the strike price of any options received, etc.), his wealth, the firm's investment opportunity set, and the level of the manager's risk aversion.

For simplicity and clarity we have not assumed the existence of any sort of agency problem. Thus, the optimal compensation scheme is to pay the manager cash, only, and forbid him from holding any equity-based securities.[3] But

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2 See, for example, Rajgopal and Shevlin (2002), DeFusco, Johnson and Zorn (1990), and Agrawal and Mandelker (1987). However, increasing the convexity of the manager's wealth will not necessarily increase his propensity to take risk; see Carpenter (2000) and Ross (2003).

3 In the presence of agency costs resulting from managerial career concerns, the optimal contract includes stock option compensation (See Nohel and Todd, 2003).

presumably there are agency problems (not modeled here) that might push the firm to compensate the manager with share-based pay, instead of cash.

The manager adds value to the firm because he makes his investment decision based on private information. This value is essentially the value of the real option to switch between the risky and risk-free projects, based on the signal that the manager receives. The value of the firm's shares and the value of options on the firm's shares are directly affected by the manager's actions.

We parameterize the model and solve it numerically. We document several interesting findings. First, though the equivalent of the Black-Scholes value of an at-the-money option in our setting is $13.73 when firm value is $100, the actual value of the option to a risk-neutral investor varies from $0 to $13.73, and equals $11.32 when the optimal investment policy is followed. The option value is a function of the manager's risk aversion and wealth composition because these impact the manager's investment rule (choice of hurdle rate).

Second, we show that the executive's portfolio delta, a value commonly used to measure incentives (or agency costs), is actually a very misleading measure. For example, in one parameterization of our model, managers of identical firms with nearly identical deltas set their hurdle rates more than 33 percentage points apart because of differences in wealth composition and risk aversion. Alternatively, managers of identical firms with nearly identical investment policies have quite different deltas. Our model yields qualitatively similar results with any reasonable parameterization.

These findings are of enormous importance for those doing empirical work in the compensation area. A manager's portfolio delta is not a sufficient statistic for his incentives to increase firm value. Performance-based incentives are multi-dimensional, suggesting that a univariate measure like delta will result in an omitted variables problem, and thus biased inferences. It is important that controls be included for executive risk aversion, wealth composition, investment opportunities, and firm risk.

The remainder of the paper is organized as follows. We formalize a model of managerial investment in Section 2. We present and analyze numerical solutions to our model in Section 3. Section 4 concludes.

2. A Model of Managerial Investment

In this section, we model the investment behavior of a risk-averse manager whose wealth consists of cash and equity-based wealth in the form of shares and call options on his firm's shares. We assume the manager maximizes the expected utility of his future wealth. The manager sets his firm's investment strategy based on private information he possesses about project payoffs.

2.1 Set-up and Information Structure of the Model

Consider a firm with I dollars in cash on its balance sheet at t = 0. The firm's investment opportunity set consists of two mutually exclusive projects. Each project requires an initial investment of I dollars. One project is riskless and thus earns the risk-free rate, which we set equal to 0 for ease of exposition. The other project is risky and generates a payoff that is uniformly distributed on the interval, $[V_L, V_H]$, with $V_H \geq I \geq V_L$. The low end of the distribution, V_L, is known and certain. The high end, V_H, is uncertain but known to be uniformly distributed on the interval, $[V_{H1}, V_{H2}]$.[4] Assume the relationship among I, V_L, V_{H1}, and V_{H2} is such that $I = (V_L + (V_{H1} + V_{H2})/2) / 2$.[5]

These investment opportunities are managed by an executive who maximizes his own preferences, rather than those of outside shareholders. Unbeknownst to shareholders, the manager is able to generate a signal about the profitability of the risky project, and he makes his investment decision following the realization of this signal.

The timing in our model is as follows: at t = 0, investors know that the firm has I in cash on its balance sheet. Furthermore, at t = 0, investors are aware of the firm's investment opportunities, and they are aware of the composition of the manager's wealth. His wealth consists of cash and incentive compensation in the form of shares and options on his firm's shares. At t = 0+, investors learn that the manager will receive a signal at time 1 and make his investment choice based on the realization of the signal. At t = 1, the manager receives a signal that identifies V_H. Based on this signal, the manager chooses between the firm's two investment opportunities. The signal is never revealed and thus

[4] The distribution of project outcomes and the information structure of our model are similar to Ross (1977). The binary nature of the investment choice is common in the literature. See, for example, John and John (1993), Lambert (1986), and Holmstrom and Ricart i Costa (1986).

[5] That is, *ex-ante*, both the riskless and the risky projects have zero NPV. This assumption is consistent with strong-form market efficiency. However, as long as $V_{H1} \geq I \geq V_L$, our results hold.

cannot be contracted on. At t = 2, the outcome of the investment decision is realized, the manager's shares and options vest, and the firm is liquidated.

Investors and the manager differ in their preferences towards risk. We assume investors are risk-neutral and the manager is risk-averse in wealth. This assumption is consistent with Carpenter (1998), Huddart and Lang (1996) and Hemmer, Matsunaga, and Shevlin (1996), who find that executives exercise their stock options early. We further assume the manager's utility function exhibits constant relative risk-aversion, as described below.[6]

$$U(w) = \frac{1}{1-b} w^{(1-b)},\ b \geq 0,\ b \neq 1 \tag{1}$$

Here, w is the level of the manager's wealth (both cash and share-based wealth).[7]

Let K denote the value of the manager's cash wealth. The manager's other wealth includes n shares, and m options with an exercise price of X.[8] Here n and m represent fractions of the shares outstanding in the manager's firm. We restrict K, n, and m to be non-negative and we assume shares and options vest at t = 2.

Investors are rational so the firm's value changes as new information is released. Starting at an initial value of I (at time 0), the firm's value adjusts

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6 The conclusions of our paper are valid with any utility function that exhibits decreasing absolute risk aversion. We chose the specification above since it is common in the literature; see Hall and Murphy (2000) and Lambert et al. (1991).

7 In the interests of simplicity, we model the manager's risky security holdings as cash equivalents. For a similar treatment, see Hall and Murphy (2000) and Lambert, et al. (1991).

8 In the interests of simplicity, we sidestep the more general problem of an executive who has a portfolio of options with varying strike prices.

sequentially to reflect information about the manager's investment opportunities (time 0+), his investment decision (time 1), and the final project outcome (time 2).

Due to the binary nature of the investment choice in our model, an investment strategy is defined as a cut-off point. Consider any $\hat{V}_H$, where $V_{H2} \geq \hat{V}_H \geq V_{H1}$. $[\hat{V}_H]$ denotes the strategy where the manager invests in the risky project whenever he observes a signal $V_H \geq \hat{V}_H$; otherwise he invests in the riskless project. Thus, if the manager follows investment strategy $[\hat{V}_H]$, he chooses the risky project with probability $(V_{H2} - \hat{V}_H)/(V_{H2} - V_{H1})$, and he chooses the riskless project with probability $1 - \frac{V_{H2} - \hat{V}_H}{V_{H2} - V_{H1}} = \frac{\hat{V}_H - V_{H1}}{V_{H2} - V_{H1}}$. By definition, the riskless project has zero NPV. By assumption, the risky project has NPV = 0. In contrast, the *opportunity* to take the risky project has NPV equal to $[(V_{H2} + \hat{V}_H)/2 + V_L]/2 - I$.[9]

Therefore, if the manager follows investment strategy $[\hat{V}_H]$, the value of the firm after it is revealed that the manager will receive a signal, V_{0+}, as a function of the manager's investment strategy, $[\hat{V}_H]$, is given by:

$$V_{0+}(\hat{V}_H) = I + \frac{(V_{H2} - \hat{V}_H)}{(V_{H2} - V_{H1})}\left(\frac{\frac{V_{H2} + \hat{V}_H}{2} + V_L}{2} - I\right) \tag{2}$$

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9 Note, this differs from the ex-ante NPV because here it is known that the manager gets a signal about V_H.

Risk-neutral shareholders strictly prefer the risky project as long as it offers an expected return that exceeds the risk-free rate of zero. This implies that their preferred investment strategy is to set $\hat{V}_H = 2I - V_L$.

2.2 Solution to the model

The manager develops his investment strategy [$\hat{V}_H$] by solving for the value of the signal that makes him indifferent between the two projects. Given his wealth (*K*, *n*, *m*, *X*) and a signal V_H, the manager can invest in the risky project and derive expected utility equal to:

$$E[U(K,n,m,X,V_H)] = \frac{1}{1-b}\left(\frac{1}{(V_H - V_L)}\right)\left(\int_{V_L}^{Min(X,V_H)} (nV + K)^{(1-b)} dV + \int_{Min(X,V_H)}^{V_H} (nV + m(V-X) + K)^{(1-b)} dV\right) \tag{3}$$

Note that we set the integral limits equal to Min(X, V_H) rather than X. When the option strike price is greater than the value of the manager's signal, the options are worthless and the manager makes his investment decision based only on the value of his cash and shares.

Alternatively, the manager can invest in the riskless project and derive (expected) utility equal to:

$$U(K,n,m,X,V_H) = \frac{1}{1-b}[nI + mMax(0, I - X) + K]^{1-b} \tag{4}$$

The manager sets his investment rule [$\hat{V}_H$] by solving for the value of V_H that equates (3) and (4); this is the value of V_H that solves (5):

$$\frac{1}{1-b}\left(\frac{1}{(V_H - V_L)}\right)\Big(\int_{V_L}^{Min(X,V_H)} (nV+K)^{(1-b)} dV$$

$$+ \int_{Min(X,V_H)}^{V_H} (nV + m(V-X) + K)^{(1-b)} dV) = \frac{1}{1-b} [nI + mMax(0, I-X) + K]^{1-b} \qquad (5)$$

Let V_H^* denote the value of V_H that solves (5). If the manager acts in his own interest, he follows investment strategy, $[V_H^*]$.

3. Results

In this section, we solve for the manager's preferred investment strategy, i.e., the $[V_H^*]$ that solves (5). We are interested in assessing how the manager's investment strategy varies with firm and manager characteristics. Additionally, we examine how stock option values deviate from Black-Scholes values.

We normalize the firm's initial value to 100. This is the cash the firm has on hand at t=0 and is sufficient to undertake either the risky or risk-free project at t=1. We consider three alternative risky projects: a low-risk project, a medium risk project and a high-risk project. As a percentage of invested capital, these projects have volatilities equal to 14.4%, 28.9%, and 43.3% respectively, and net present values equal to 3.125%, 6.25%, and 9.375% respectively. We believe these projects have risk/return characteristics that span the documented ranges on the first two moments of U.S. historical stock returns. As Dimson and Marsh (2001) show, for the period 1955 – 1999, the geometric equity risk premium for U.S. firms is 6.2%. Moreover, Ibbotson and Sinquefield (2001) report that over the period 1926-2000, stock returns had an average standard deviation of 28%, with firms in the largest decile averaging 19.05% and firms in the smallest decile averaging 45.82%.

Table 1a reports the manager's preferred investment strategy in the medium risk project for various wealth allocations and levels of managerial risk aversion. We restrict non-firm wealth to 25%, 50% and 75% of total wealth and assume this wealth is held in cash-equivalent assets. The balance of the manager's wealth is held in shares and at-the-money options. We convert the manager's cutoff value into a hurdle rate. Under shareholders' preferred investment strategy, the manager sets his hurdle rate at 0.00%. We shade regions in the table where over-investment occurs.

From Table 1a we see that a manager with cash and share wealth, only, is perfectly aligned with shareholders if he is risk-neutral. However, as the manager's coefficient of relative risk aversion increases, he becomes progressively more conservative. This conservatism becomes more pronounced as we shift the manager's wealth from cash to shares. In contrast, a manager with cash and option wealth, only, always takes the risky project, even if options represent ε% of his wealth, for arbitrarily small ε. In this situation, over-investment necessarily occurs because the managers' options are worthless under the riskless project.

When a manager holds cash, shares and option wealth, increasing the proportion of option wealth increases the manager's propensity to take risk. For example, with cash at 50% of wealth and a coefficient of risk aversion equal to 5, the manager lowers his hurdle rate from 17.02% to -25.00% as options move from 0% to 50% of his wealth (and shares move from 50% to 0% of his wealth). This flip from very conservative to very aggressive behavior is more pronounced at higher levels of risk aversion and lower amounts of cash. Clearly, similar mixes of wealth can produce considerable variation in risk-taking incentives, depending on the manager's risk aversion. Similarly, Tables 1b (the high-risk project) and 1c (the low-risk project) show that the riskiness of the manager's investment opportunities can also dramatically affect incentives.

Table 2 reports the per share value of the manager's call options. Keep in mind that these options are always structured with X=100, i.e., at-the-money at time 0. As in Table 1a, we assume the risky project payoffs follow the medium risk project. Hence, variation in option values here is entirely driven by the impact of the options on the manager's investment strategy. For example, if the manager's options are not sufficient to induce the manager to ever take the risky project, the option's value is $0.00. Alternatively, if the manager's wealth and risk preferences induce him to always take the risky project, the option's value is maximized at $13.73. In contrast, if the manager follows an optimal investment policy from the perspective of shareholders, the call options are worth $11.32 per share. Clearly, by assuming that the option's value is independent of managerial actions, one makes enormous errors. Given that the hurdle rate determines both firm value and firm risk, this error is a function of both the delta and the vega of the option.

Of course these valuation errors rely on our stylized model. More generally, how serious are the errors if we use a valuation framework that ignores the manager's incentives to affect firm value? We perform a "back of the envelope" calculation to estimate the economic significance of option valuation errors.

We have evidence from earlier studies (e.g., Yermack, 1997, DeFusco et al., 1990, and Brickley, Bhagat and Lease, 1985) that stock option awards result in stock price increases. Rajgopal and Shevlin (2002) and DeFusco et al. (1990) document increases in firm variances following the adoption of stock option plans. Based on the findings of DeFusco et al. (1990), the "average" firm shows a 4% share price increase and a 16% increase in variance following the adoption of an executive stock option plan. Consider a firm whose stock is trading at $30 per share and whose return volatility is 30% per year. Assume this firm awards its CEO at-the-money options that expire in 10 years. If the risk-free rate is 5%, these options have a Black-Scholes value of $15.77 per share. If the option grant pushes the share price up by 4% and the variance up by 16%, the option price will rise to $17.89 per share. Thus ignoring the endogeneity effect induces an error of approximately 13% of the option's value.[10]

In Table 3, we report firm value (share price) as a function of the manager's risk aversion and wealth composition. As in Tables 1a and 2, we

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[10] Consider this estimate a lower bound, because it is based on data from 1978 - 1982, when option grants were relatively small. Additionally, stock options are usually exercised long before maturity (see Murphy, 1999); with a shorter maturity, e.g., 4 years, this error increases to more than 19%.

assume the risky project payoffs follow the medium risk project. Comparing Tables 3 and 2, it is clear that maximizing firm and option values are two very different things. Option value is maximized when the manager pursues the most aggressive investment policy, whereas firm value is maximized when the manager sets his hurdle rate close to zero. However, the manager will choose an investment policy that maximizes his own utility, rather than firm or option values. Finally, notice that the most aggressive and most conservative policies both result in the same firm value, but for very different reasons. In these instances, the manager either never takes, or always takes the risky project. Thus, any real option value attributed to the manager is lost.

In Table 4, we calculate pay-performance sensitivity (PPS) measures, in a manner equivalent to Jensen and Murphy (1990). Our approach is detailed in Appendix B. The computed PPS measures, equivalent to the delta of the manager's wealth, tell us how much the manager's wealth increases when firm value increases by $1,000.

We first note that our PPS values, based on our stylized model and chosen parameter values, closely match the pay-performance sensitivities documented in studies by Aggarwal and Samwick (1999), Hall and Liebman (1998), and Jensen and Murphy (1990).

We next consider the relation between PPS and managerial investment strategy. Note that a manager with a coefficient of risk aversion equal to 3 and a wealth allocation of 25% cash and 75% shares has a portfolio delta of $7.21.

This is quite similar to the delta for a manager with a coefficient of risk aversion equal to 10 and a wealth allocation of 75% cash, 5% shares and 20% options. Table 1a informs us that the first manager sets his hurdle rate +14.76%, while the second the manager sets his hurdle rate at –18.46%. In other words, though these managers have nearly identical portfolio deltas, their preferred hurdle rates are more than 33 percentage points apart!

Alternatively, consider a manager with a coefficient of risk aversion equal to 1.5 and a wealth allocation of 50% cash, 10% shares and 40% options. This manager has a portfolio delta of \$14.72. In contrast, a manager with a coefficient of risk aversion equal to 5 and a wealth allocation of 75% cash, 5% shares and 20% options has a portfolio delta of \$7.34. Yet, Table 1a informs us these managers pursue nearly identical investment policies, with the first manager setting his hurdle rate at -19.87% and the second manager setting his hurdle rate at –19.64%.

In short, it is clear from Tables 1 and 4 that a manager's portfolio delta is a very poor proxy for his incentives to raise firm value, once we account for the endogenous relationship between compensation and investment. Therefore it is critical that researchers who use a manager's portfolio delta to proxy for managerial incentives include, as regressors, controls for managerial risk aversion, composition of managerial wealth, and proxies for firm investment opportunities and risk.

4. Conclusions

This paper studies the impact of firm and managerial characteristics on the value and incentive effects of executive stock options. Such a study hinges critically on using a model in which managerial behavior and firm value are endogenous, a condition that, alas, precludes the use of the option pricing model of Black and Scholes (1973), which assumes an exogenous stock price process.

If a manager who receives stock options can affect firm value, say through his investment decision, then the value he places on those options depends on his actions, just as the cost of those options to his issuing firm depends on his actions. The Black-Scholes option value is not even a good indicator of the option's value to a well-diversified investor. Option values depend on who holds them. Moreover, the *incentives* provided by stock options also depend on characteristics of the individual holding the options and characteristics of the firm that he manages.

Yet compensation research routinely relies on intuition gleaned from the Black-Scholes model and proxies for effects based on Black-Scholes-based comparative statics. It is common practice in the empirical compensation literature to measure executive incentives to increase firm value either by estimating the delta of a manager's stock options or the delta of a manager's equity-based compensation.

We construct a model wherein stock and option prices are endogenously determined based on the actions of a self-interested manager. Specifically, the manager chooses between a risky and risk-free project, based on a signal about the profitability of the risky project. The manager's investment strategy is characterized by setting a hurdle rate for the risky project. This hurdle rate is affected by manager- and firm-specific characteristics. In turn, this hurdle rate directly affects both share and option values.

We parameterize our model and solve it numerically. We find that the value of an at-the-money option in our setting varies from $0 to $13.73 per share (when firm value is $100 per share), and equals $11.32 when the optimal investment policy is followed. The option value is a function of the manager's risk aversion and wealth composition because these impact the manager's investment decision (choice of hurdle rate). We provide an estimate of the error induced by ignoring the endogenous relation between compensation and investment. This error might easily exceed 20% of an option's value.

We also show that an executive's portfolio delta is actually a very misleading measure of investment incentives or agency costs. In particular, managers of identical firms with nearly identical deltas set their hurdle rates more than 33 percentage points apart, because of differences in wealth composition and risk aversion. Alternatively, managers of identical firms pursuing nearly identical investment policies may have vastly different deltas. Our model yields qualitatively similar results with any reasonable parameterization.

These findings have enormous importance for those doing empirical work in the compensation area. A manager's portfolio delta is not a sufficient statistic to describe his incentives to increase firm value. These incentives are multi-dimensional, suggesting that a univariate measure like delta will result in an omitted variables problem, and thus biased inferences. It is important that controls be included for executive risk aversion, wealth composition, investment opportunities, and firm risk.

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Appendix A: Valuing a call option based on the investment strategy, [$\tilde{V}_H$]

Here we derive a formula for calculating the value of a call option, assuming the manager pursues the investment strategy, [$\hat{V}_H$]. If the manager takes the riskless project, the final stock price (at t = 2) is known with certainty. A call option on the entire firm with exercise price *X*, expiring at t=2, is worth *Max(0, I - X)*. When the manager takes the risky project, the value of a call option on the firm (ignoring compensation costs) is:

$$CallValue = \frac{1}{V_H - V_L}\int_{V_L}^{V_H} Max(0, V - X)dV \tag{A1}$$

Thus, if the manager follows investment policy, [$\hat{V}_H$], the call value at time 0 is:

$$C_0 = \frac{1}{V_{H2} - V_{H1}}\int_{V_{H1}}^{\hat{V}_H} Max(0, I - X)dV_H + \frac{1}{V_{H2} - V_{H1}}\int_{\hat{V}_H}^{V_{H2}}\left[\frac{1}{V_H - V_L}\int_{V_L}^{V_H} Max(0, V - X)dV\right]dV_H \tag{A2}$$

The first integral represents the call's payoff under the riskless project, while the second integral represents the call's expected payoff under the risky project. To simplify expression (A2), we separately consider the cases *X*<$\hat{V}_H$ and *X*>$\hat{V}_H$. If *X*<$\hat{V}_H$, then:

$$C_0 = \frac{\hat{V}_H - V_{H1}}{V_{H2} - V_{H1}} Max(0, I - X) + \frac{1}{V_{H2} - V_{H1}}\int_{\hat{V}_H}^{V_{H2}}\left[\frac{1}{V_H - V_L}\int_{X}^{V_H}(V - X)dV\right]dV_H \tag{A3}$$

Alternatively, if *X*>$\hat{V}_H$ then:

$$C_0 = \frac{\hat{V}_H - V_{H1}}{V_{H2} - V_{H1}} Max(0, I - X) + \frac{1}{V_{H2} - V_{H1}} \int_X^{V_{H2}} \left[\frac{1}{V_H - V_L} \int_X^{V_H} (V - X) dV \right] dV_H \tag{A4}$$

These can be integrated to give the following valuation formulae for the call option at t=0. If $X < \hat{V}_H$:

$$C_0 = \frac{\hat{V}_H - V_{H1}}{V_{H2} - V_{H1}} Max(0, I - X) + \frac{1}{2(V_{H2} - V_{H1})} * \left[\frac{(V_{H2} - V_L)^2}{2} - \frac{(\hat{V}_H - V_L)^2}{2} - 2(X - V_L)(V_{H2} - \hat{V}_H) + (X - V_L)^2 \ln\left(\frac{V_{H2} - V_L}{\hat{V}_H - V_L} \right) \right] \tag{A5}$$

If $X > \hat{V}_H$:

$$C_0 = \frac{\hat{V}_H - V_{H1}}{V_{H2} - V_{H1}} Max(0, I - X) + \frac{1}{2(V_{H2} - V_{H1})} * \left[\frac{(V_{H2} - V_L)^2}{2} - \frac{(X - V_L)^2}{2} - 2(X - V_L)(V_{H2} - X) + (X - V_L)^2 \ln\left(\frac{V_{H2} - V_L}{X - V_L} \right) \right] \tag{A6}$$

Appendix B: Pay-performance sensitivity

Following Jensen and Murphy (1990), we define pay-performance sensitivity (PPS) as the change in compensation value associated with a $1,000 change in firm value, for all possible liquidation values of the firm, given a fixed compensation package. The value of cash compensation is uncorrelated with changes in firm value, while the value of share and option compensation is positively correlated with changes in firm value. Thus, PPS is zero for cash compensation and positive for share- and option-based compensation. In our model, the firm's value is V_{0+} once the manager is hired. As firm value moves from V_{0+} to V (its liquidation value), the value of the manager's incentive compensation changes by:

$$(m*Max[V - X*,\ 0] + n*V + K*) - (m*C_{0+} + n*V_{0+} + K*) \qquad \text{(B1)}$$

Rearranging terms, the change in the value of the manager's compensation, as a function of the change in firm value from t = 0+ to t = 2, i.e., as firm value changes from V_{0+} to V, is:

$$\Delta Pay = \begin{cases} m*(V_{0+} - X* - C_{0+}) + (n*+m*)\Delta Value, & when \quad V > X* \\ -m*C_{0+} + n*\Delta Value, & when \quad V \leq X* \end{cases} \qquad \text{(B2)}$$

We define PPS as the coefficient of Δ*Value* in (B2), consistent with Jensen and Murphy (1990). It is straightforward to show that PPS is defined as:

PPS = n + [m * Prob(risky) * Prob(V > X* | risky)] (B3)

Here, Prob(risky) is the probability that the manager pursues the risky project, and Prob(V > X* | risky) is the probability that the risky project payoff exceeds the option strike price, given the risky project is selected.

Table 1a
Managerial investment strategy (hurdle rates) for the “medium-risk” project

% Share of managerial wealth as:			Coefficient of Risk Aversion, b				
Cash	*Shares*	*Options*	*b=0*	*b=1.5*	*b=3*	*b=5*	*b=10*
25	75	0	0.00	5.74	14.76	25.00	25.00
25	50	25	-13.69	-10.25	-4.40	11.43	25.00
25	25	50	-18.75	-16.59	-12.83	-3.46	25.00
25	0	75	-25.00	-25.00	-25.00	-25.00	-25.00
50	50	0	0.00	3.47	8.03	17.02	25.00
50	40	10	-10.56	-8.29	-5.26	0.66	25.00
50	30	20	-14.90	-13.19	-10.87	-6.29	25.00
50	20	30	-17.88	-16.70	-15.11	-12.06	2.70
50	10	40	-20.49	-19.87	-19.07	-17.65	-11.74
50	0	50	-25.00	-25.00	-25.00	-25.00	-25.00
75	25	0	0.00	1.59	3.39	6.22	16.68
75	20	5	-10.56	-9.56	-8.42	-6.63	-0.19
75	15	10	-14.90	-14.19	-13.37	-12.09	-7.51
75	10	15	-17.88	-17.41	-16.89	-16.08	-13.29
75	5	20	-20.49	-20.27	-20.02	-19.64	-18.46
75	0	25	-25.00	-25.00	-25.00	-25.00	-25.00

Table 1a reports hurdle rates implied by the manager’s preferred investment strategy. Under shareholders’ preferred investment strategy, the manager sets the hurdle rate at 0.00%. We assume the risky project cash flows are uniformly distributed on the interval (50, V_H) with V_H uniformly distributed on the interval (100, 200). We vary the manager’s coefficient of relative risk aversion from 0 (risk-neutral) to 10. We shade regions in the table where over-investment occurs.

Table 1b
Managerial investment strategy (hurdle rates) for the “high-risk” project

% Share of managerial wealth as:			Coefficient of Risk Aversion, b				
Cash	*Shares*	*Options*	*b=0*	*b=1.5*	*b=3*	*b=5*	*b=10*
25	75	0	0.00	14.84	37.50	37.50	37.50
25	50	25	-17.97	-9.16	9.80	37.50	37.50
25	25	50	-26.31	-21.01	-10.04	37.50	37.50
25	0	75	-37.50	-37.50	-37.50	-37.50	-37.50
50	50	0	0.00	8.30	21.42	37.50	37.50
50	40	10	-13.25	-7.76	0.53	20.51	37.50
50	30	20	-19.89	-15.80	-9.62	4.73	37.50
50	20	30	-24.81	-22.03	-17.94	-9.18	37.50
50	10	40	-29.35	-27.91	-25.95	-22.18	-5.39
50	0	50	-37.50	-37.50	-37.50	-37.50	-37.50
75	25	0	0.00	3.61	7.99	15.60	37.50
75	20	5	-13.25	-10.92	-8.14	-3.48	17.11
75	15	10	-19.89	-18.24	-16.27	-13.00	0.37
75	10	15	-24.81	-23.75	-22.49	-20.47	-12.90
75	5	20	-29.35	-28.83	-28.25	-27.35	-24.32
75	0	25	-37.50	-37.50	-37.50	-37.50	-37.50

Table 1b reports hurdle rates implied by the manager’s preferred investment strategy. Under shareholders’ preferred investment strategy, the manager sets the hurdle rate at 0.00%. We assume the risky project cash flows are uniformly distributed on the interval (25, V_H) with V_H uniformly distributed on the interval (100, 250). We vary the manager’s coefficient of relative risk aversion from 0 (risk-neutral) to 10. We shade regions in the table where over-investment occurs.

Table 1c
Managerial investment strategy (hurdle rates) for the “low-risk” project

% Share of managerial wealth as:			Coefficient of Risk Aversion, b				
Cash	*Shares*	*Options*	*b=0*	*b=1.5*	*b=3*	*b=5*	*b=10*
25	75	0	0.00	1.29	2.86	5.58	12.50
25	50	25	-8.21	-7.46	-6.45	-4.42	6.28
25	25	50	-10.24	-9.75	-9.05	-7.61	-1.04
25	0	75	-12.50	-12.50	-12.50	-12.50	-12.50
50	50	0	0.00	0.82	1.75	3.22	8.58
50	40	10	-6.80	-6.28	-5.68	-4.67	0.66
50	30	20	-8.72	-8.33	-7.86	-7.06	-3.78
50	20	30	-9.90	-9.63	-9.29	-8.73	-6.48
50	10	40	-10.88	-10.73	-10.56	-10.27	-9.21
50	0	50	-12.50	-12.50	-12.50	-12.50	-12.50
75	25	0	0.00	0.39	0.81	1.42	3.19
75	20	5	-6.80	-6.56	-6.31	-5.94	-4.79
75	15	10	-8.72	-8.55	-8.37	-8.10	-7.25
75	10	15	-9.90	-9.79	-9.67	-9.49	-8.95
75	5	20	-10.88	-10.83	-10.77	-10.68	-10.44
75	0	25	-12.50	-12.50	-12.50	-12.50	-12.50

Table 1c reports hurdle rates implied by the manager’s preferred investment strategy. Under shareholders’ preferred investment strategy, the manager sets the hurdle rate at 0.00%. We assume the risky project cash flows are uniformly distributed on the interval (75, V_H) with V_H uniformly distributed on the interval (100, 150). We vary the manager’s coefficient of relative risk aversion from 0 (risk-neutral) to 10. We shade regions in the table where over-investment occurs.

Table 2
Option values (per share)

% Share of managerial wealth as:			Coefficient of Risk Aversion, b				
Cash	*Shares*	*Options*	*b=0*	*b=1.5*	*b=3*	*b=5*	*b=10*
25	75	0	11.32	9.63	5.91	0.00	0.00
25	50	25	13.44	13.13	12.28	7.44	0.00
25	25	50	13.68	13.61	13.38	12.09	0.00
25	0	75	13.73	13.73	13.73	13.73	13.73
50	50	0	11.32	10.36	8.81	4.76	0.00
50	40	10	13.17	12.90	12.43	11.15	0.00
50	30	20	13.52	13.41	13.20	12.60	0.00
50	20	30	13.65	13.61	13.53	13.31	10.59
50	10	40	13.71	13.70	13.69	13.65	13.28
50	0	50	13.73	13.73	13.73	13.73	13.73
75	25	0	11.32	10.90	10.38	9.47	4.94
75	20	5	13.17	13.06	12.91	12.66	11.36
75	15	10	13.52	13.48	13.42	13.32	12.79
75	10	15	13.65	13.64	13.62	13.58	13.41
75	5	20	13.71	13.71	13.70	13.70	13.67
75	0	25	13.73	13.73	13.73	13.73	13.73

Table 2 reports option values (per share). We assume the risky project cash flows are uniformly distributed on the interval (50, V_H) with V_H uniformly distributed on the interval (100, 200). We vary the manager's coefficient of relative risk aversion from 0 (risk-neutral) to 10. We shade regions in the table where over-investment occurs.

Table 3
Firm value, given the manager's investment strategy

% Share of managerial wealth as:			Coefficient of Risk Aversion, b				
Cash	*Shares*	*Options*	*b=0*	*b=1.5*	*b=3.0*	*b=5.0*	*b=10.0*
25	75	0	106.25	105.92	104.07	100.00	100.00
25	50	25	104.38	105.20	106.06	104.94	100.00
25	25	50	102.73	103.50	104.60	106.13	100.00
25	0	75	100.00	100.00	100.00	100.00	100.00
50	50	0	106.25	106.13	105.60	103.35	100.00
50	40	10	105.14	105.56	105.97	106.25	100.00
50	30	20	104.03	104.51	105.07	105.85	100.00
50	20	30	103.05	103.46	103.97	104.80	106.18
50	10	40	102.05	102.30	102.61	103.14	104.87
50	0	50	100.00	100.00	100.00	100.00	100.00
75	25	0	106.25	106.22	106.13	105.86	103.47
75	20	5	105.14	105.34	105.54	105.81	106.25
75	15	10	104.03	104.24	104.46	104.79	105.69
75	10	15	103.05	103.22	103.40	103.67	104.48
75	5	20	102.05	102.14	102.24	102.39	102.84
75	0	25	100.00	100.00	100.00	100.00	100.00

Table 3 reports firm values under the manager's preferred investment strategy [See equation (2) in the text]. We assume the risky project cash flows are uniformly distributed on the interval (50, V_H) with V_H uniformly distributed on the interval (100, 200). We vary the manager's coefficient of relative risk aversion from 0 (risk-neutral) to 10. We shade regions in the table where over-investment occurs.

Table 4
Pay-performance sensitivity (PPS), under the manager's investment strategy

% Share of managerial wealth as:			Coefficient of Risk Aversion, b				
Cash	*Shares*	*Options*	*b=0.0*	*b=1.5*	*b=3.0*	*b=5.0*	*b=10.0*
25	75	0	7.06	7.08	7.21	7.50	7.50
25	50	25	12.72	12.32	11.72	10.54	5.00
25	25	50	19.37	18.89	18.06	16.21	2.50
25	0	75	27.31	27.31	27.31	27.31	27.31
50	50	0	4.71	4.71	4.73	4.84	5.00
50	40	10	6.85	6.74	6.61	6.39	4.00
50	30	20	9.33	9.17	8.96	8.58	3.00
50	20	30	11.99	11.82	11.62	11.22	9.57
50	10	40	14.82	14.72	14.58	14.33	13.31
50	0	50	18.20	18.20	18.20	18.20	18.20
75	25	0	2.35	2.35	2.36	2.36	2.42
75	20	5	3.42	3.40	3.37	3.33	3.21
75	15	10	4.66	4.63	4.59	4.54	4.34
75	10	15	5.99	5.96	5.93	5.87	5.69
75	5	20	7.41	7.39	7.37	7.34	7.23
75	0	25	9.10	9.10	9.10	9.10	9.10

Table 4 reports pay-performance sensitivity (PPS) measures, under the manager's preferred investment strategy. We assume the risky project cash flows are uniformly distributed on the interval (50, V_H) with V_H uniformly distributed on the interval (100, 200). We vary the manager's coefficient of relative risk aversion from 0 (risk-neutral) to 10. We shade regions in the table where over-investment occurs.