Detecting Determinants of Capital Structure*

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Abstract

The determinants of the leverage of firms are widely used as a means to study optimal capital structure decisions. However, empirical studies do not agree on the importance of several commonly proposed determinants. We provide a dynamic capital structure model which endogenously generates the indeterminacy found in empirical studies, and thus we explain why such studies are inherently deemed to be problematic. Specifically, we demonstrate how interdependence between leverage determinants can cause us to falsely reject important correlations. Our findings add to the insights of the challenges attached to understanding how firms choose their capital structure.

Keywords: Dynamic Structural Model, Firm Leverage Determinants, Capital Structure, Firm Fixed Effect

JEL subject codes: G30, G32, D81

1 Introduction

The capital structure of firms is one of the most studied topics in finance. The desire to understand capital structure is understandable as it provides insights into a firm's risk, future opportunities, operations, and ownership structures. Yet, despite the existing theories and a vast amount of empirical studies our understanding appears to be hindered by a layer of opaqueness regarding the underlying factors. The literature reports conflicting evidence concerning which leverage determinants are reliably important. Empirical studies typically examine capital structure decisions by regressing a company's leverage on specific determinants, thus resulting in a particular correlation structure. The sign and magnitude of the correlation coefficients link the various determinants to a particular theory, and thus the analysis provides either evidence in favour of a particular theoretical model or suggest that new models are needed to understand the firm's capital structure decisions better. In particular, Lemmon et al. (2008) question the importance of previously identified leverage determinants such as size, profitability, market-to-book, industry, etc. Lemmon et al. find, contrary to the findings in previous studies, that time-invariant leverage determinants are more important than time-varying determinants. Specifically, they show that including firm fixed effects in a leverage regression significantly improves the model fit, and their results therefore suggest that new theoretical models are needed to identify these time-invariant, persistent, determinants.

The goal of this paper is to address the disagreements in the empirical literature regarding the detection of capital structure determinants. To structure the analysis we provide a dynamic capital structure model which endogenously generates some of the indeterminacy found in empirical studies. In particular, we address the limited explanatory power of the previously identified determinants and we demonstrate how an empirical analysis misinterpret the significance of a correlation between leverage and specific factors. This misinterpretation occurs as determinants can be interdependent. Failing to account for interdependence may lead to a rejection of important leverage factors. Hence, the present paper addresses the results of existing empirical work and add to the insights of the challenges of understanding how firms choose their capital structure.

We show that the explanatory power of fundamental leverage determinants vanishes unless we know how to account properly for their interdependence. To facilitate the analysis our dynamic model includes two determinants of the firm's capital structure. The firm generates cash flows from assets in place, and the value of the amount produced depends on an underlying market-based profitability index. Thus, the firm's capital structure depends both on the assets and on profitability. The firm's assets are positively related to the firm's leverage as the investment in assets are funded by debt. The profitability index (which is a continuous state variable) is negatively correlated with the firm's leverage as an increase in the profitability increases the firm's total value, thereby decreasing the leverage. Finally, the model implies a positive correlation between the profitability index and the firm's assets as more earnings gives rise to further investment in assets. By construction, this model implies a negative relationship between the firm's leverage and profitability. However, failing to account for the firm's assets distorts this relation. The model imposes interdependence between the assets and profitability. Furthermore, the model implies that correlation between the firm's leverage and its assets differs in sign from the correlation between the firm's leverage and profitability. These model implications imply that we do not see the real correlation between the firm's leverage and profitability when we use a traditional regression. This causes an empirical analysis to reject profitability as an important leverage determinant, even when the correlation between profitability and leverage is high by construction.

We simulate a sample of firms that take optimal decisions according to our model. This enables us to study the relation between a firm's leverage and profitability. Performing a standard empirical analysis on the simulated data profitability has a limited explanatory power of leverage. However, if we condition the regression on the amount of assets for a particular firm profitability has a large significant impact of leverage. The regression fit also improves considerably. Indeed, the R-squared value increases from 0.045 to 0.99. Our analysis therefore shows that failing to control for assets decreases the explanatory power of profitability to a degree where it seems almost insignificant. Obviously, assets are a convenient determinant in our model and if firms were in practice as simple as our model empirical studies could easily control for assets. However, our model is set up to parsimoniously illustrate the key point. Firms have a much more complicated structure in reality, and if we cannot control for all dimensions in the interdependent relations, we are led to draw false conclusions.

The results presented in Lemmon et al. (2008) suggest that the majority of variation in leverage is driven by an unobserved time-invariant effect that generates a surprising stable capital structure. They find that a firm's initial leverage is an important determinant of future leverage levels. We test their predictions on our model by including initial leverage as a determinant. In line with Lemmon et al. we find that the explanatory strength of profitability becomes less pronounced when we include a proxy for the firm's initial leverage. Instead, the initial leverage appears to be the most significant leverage determinant. However, if we account for the interdependence between profitability and the firm's assets, the explanatory power of profitability increases while the significance of the initial leverage decreases. This finding points to that failing to account for interdependence enhances the importance of the initial leverage while reducing the influence of factors which are significant by construction. In general, there is no reason to believe that this finding should not carry over to other firm characteristics.

The present paper shows that even though regressions are a valuable tool for studying capital structure choice we need to account for the interdependence that exists between determinants. Indeed, we demonstrate how interdependence between leverage determinants causes the empirical analysis to falsely reject meaningful correlations. Hence, empirical studies are inherently deemed to be problematic if they cannot in great detail take interdependency and dynamic effects properly into account. Thus, our analysis suggest that it is perhaps time to consider whether a one dimensional measure—like leverage—is sufficient to describe the effects of a firm's capital structure determinants.

1.1 Literature overview

Understanding how firms' optimally choose their capital structure is a central question within finance. Among others, a company's capital structure provides insights into its risk, its future financing opportunities and operations, growth rates and ownership structures. Therefore, the factors that drive the capital structure decision has been described by both theoretical and empirical work since the modern theory of capital structure began with Modigliani and Miller (1958). Their basic theory states that in the absence of frictions, the value of a firm is unaffected by how it chooses to finance itself. Firms are, however, exposed to frictions such as taxes, bankruptcy costs, agency costs, and asymmetric information. In light of these frictions, Myers (1984) presents the capital structure puzzle and two ways to think about capital structure. First, a static trade-off framework, in which the firm is trading off the advantages and costs of debt to determine the optimal level. Second, a pecking order framework where the cost of financing increases with asymmetric information.

Following Myers (1984) a vast amount of empirical tests aim at providing support for a particular theory. The empirical evidence does, however, disagree over basic facts. Harris and Raviv (1991) find that the available studies agree that leverage increases with fixed assets, nondebt tax shields, growth opportunities, and firm size. In contrast, a study by Titman and Wessels (1988) finds no support for an effect on debt ratios arising from nondebt tax shields, volatility, collateral value, or future growth. These conflicting findings support the idea put forward by Frank and Goyal (2009) who suggest that it is often all too easy to provide some empirical support for almost any idea. Instead of supporting a specific capital structure theory Frank and Goyal contribute to our understanding of capital structure by examining which factors are reliably important for predicting leverage. They find the following six core factors that account for more than 27% of the variation in leverage: industry mean leverage, tangibility, profits, firm size, market-to-book asset ratio, and expected inflation.

Contrary to the findings presented by Frank and Goyal, Lemmon et al. (2008) show that the majority of variation in leverage ratios is driven by an unobserved time-invariant effect. This time-invariant effect does, in turn, generate surprising stable capital structures. This feature of leverage is largely unexplained by the core determinants identified by Frank and Goyal. Opposed to earlier empirical work, they suggest that the value of the known leverage factors may be more limited than previously thought. The disagreement between these studies is yet another example of conflicting empirical evidence.

Following Lemmon et al. (2008) there is a string of literature that aims at identifying the firm fixed effect. Menichini (2015) implements a structural model that can generate leverage ratios that reproduce the stability of leverage. Menichini argues that different fundamental characteristics such as capital elasticity and the volatility of profits cause the stability. With this finding Menichini (2015) adds to the list of time-varying capital structure determinants which may be difficult to support empirically. Hanousek and Shamshur (2011) also support leverage stability. Hanousek and Shamshur find that substantial changes in the economic environment do not affect the stability of a firm's leverage. The presence of credit constraints keeps the leverage stable. Hanousek and Shamshur suggest that annual information on ownership and ownership together with financial constraints have the potential to be the answer to the puzzle of stability in capital structure. The present paper differs from these as we do not search for determinants that can explain the persistence in firm leverage. Instead, we show how economically meaningful correlations are hard to detect when there exists interdependence between leverage determinants.

The idea that current empirical models misinterpret important correlations is also pointed out by Baranchuk and Xu (2011). They show how heterogeneity in the sample of firms drives the explanatory power of the firm fixed effect. To this end, the authors present an alternative way of empirically estimating the explanatory power of leverage determinants. Sorting firms based on their capital structure policies doubles the explanatory power of a standard capital structure regression. Baranchuk and Xu capture the heterogeneity by letting the data determine when and how the firms in the sample should be categorized, and hence they present an idea to possibly overcome the heterogeneity that may cause us to reject significant capital structure determinants falsely. However, the in-sample data driven work by Baranchuk and Xu does not provide any theoretical reasoning as to why a certain categorization is valuable. Contrary to Baranchuk and Xu (2011) we present a theoretical argument explaining why some empirical studies are inherently deemed to be problematic.

The remainder of the paper is organized as follows. Section 2 presents an illustrative example. We set up our dynamic capital structure model in Section 3. In Section 4 we simulate a sample of firms that behave according to the model. With this sample, we analyse the correlation between the firms' leverage and the profitability index. We show how the magnitude of this correlation depends on regression specifications. Furthermore, we show how the sign of the index changes when the firm optimally invests. Section 5 concludes.

2 An illustrative example

American Airlines Group is an example clearly illustrating a key point in the present paper. Using Compustat data from Q4-2005 until Q2-2016 we depict the firm's (market) leverage as the filled circles in Figure 1. We also depict earnings (operating income before depreciation, OIBDP) with open circles and a line as well as property plant, and equipment (PPENT) as squares in the figure. Considering leverage as a function of earnings over the full time horizon leads to a very negative relationship. Regressing leverage on earnings yields a coefficient at about -10 with an R^2 of 0.55. The regression line is shown as "Regression 1" in Figure 2. However, even though we obtain a significant negative correlation between leverage and the earnings for our data sample, it is clear that something happens at the end of 2013. This is particularly so, if we consider the firm's property plant and equipment. Indeed, the American Airlines Group was officially formed on December 9, 2013, as a combination of AMR Corporation and US Airways Group.

It is clear that the merger had an impact on the firm's assets (PPENT). Therefore, it is natural to assume that this also changed the correlation between the firm's leverage and earnings. We control for this effect by considering the period before and after the merger. The resulting regressions are depicted in Figure 2. The blue regression line corresponds to the period before the merger whereas the red regression line corresponds to the period after the merger. The coefficient on earnings (profitability) are change dramatically from

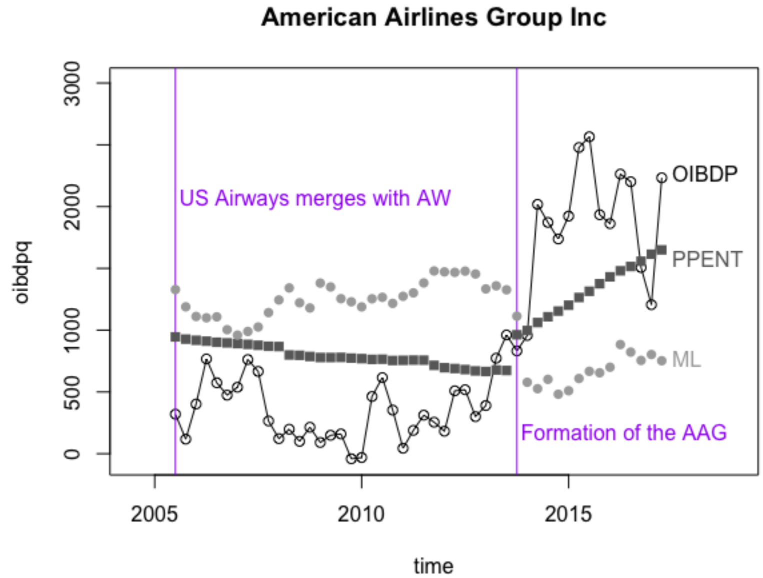


Figure 1: The evolution of the companies earnings, property plant and equipment, and market leverage after the merger with America West and until summer 2016. Note that ppent and ml is scaled to fit the frame

the initial regression. Before the merger the coefficient is about -1.3, after the merger it is about -4.6, and the explanatory power is much higher as R^2 increases to about 0.98. The increase in the goodness of fit suggests that the merger was, in fact, changing the relation between leverage and earnings to an extent where we could have misinterpreted their correlation.

While the relationship between leverage and earnings was easy to appropriately establish in this example, it was only so due to hindsight. We know exactly which event occurred, and we are therefore able to invert the event. This is not so in general. A merger is a very explicit form of exercising an expansion option, but expansion (and other) options are in reality often exercised without clear and easy to address signals useful in an empirical analysis. Below we set up a model incorporating this effect.

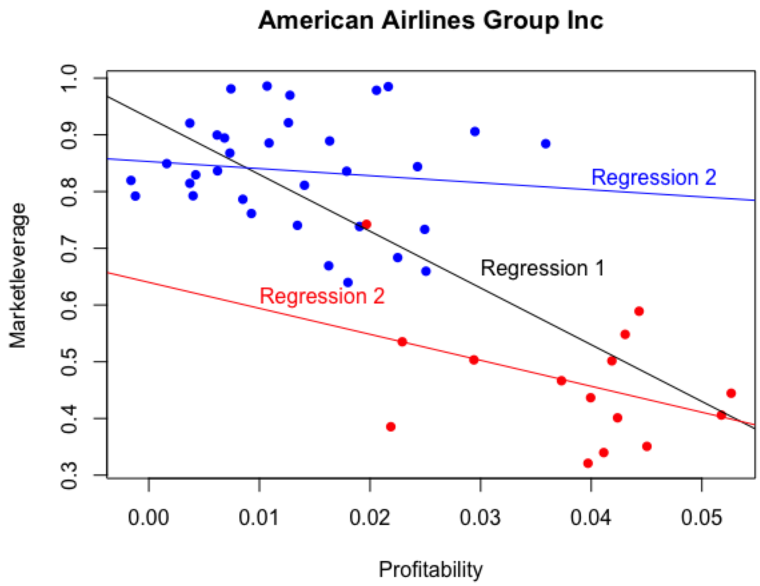


Figure 2: The relation between the market leverage and the profitability. The black line represents the fit from Table 9 and the blue lines represents the fit from Table 10 (tables are reported in the appendix).

3 The model

To demonstrate our point in a concise manner we set up a simple continuous-time dynamic capital structure model. The model is similar in spirit to standard models as, for example, Danis et al. (2014); Goldstein et al. (2001), but we additionally include assets as a determinant for the firm's cash flow Thus, the firm generates cash flows from assets in place, but the amount generated depends on an underlying market based state variable. The main driver for debt in our model is that debt is used to fund expansion of the firm's assets. Furthermore, the firm's cash flow is taxed, but debt provides to some extent a shield reducing the actual tax payment. The firm chooses its dynamic capital structure and investment strategy to maximize equity value. Below we set up the details of the model and describe the empirical predictions it generates.

We begin our analysis by assuming that the firm has assets of size K_0 in place which produce a good. The earnings received by selling this good is modelled by a market profitability index M_t at time t. We assume that $M = (M_t)_{t\geq 0}$ follows a geometric Brownian motion,

$$dM_t = \mu M_t dt + \sigma M_t dW_t, \tag{1}$$

where μ is the instantaneous (expected) growth rate, σ is the instantaneous volatility, and $W = (W_t)_{t\geq 0}$ is a Wiener process under the risk-neutral measure. Thus, the firm's earnings are driven by the two components K and M and the firm earns the cash flow

$$\pi_t = K_t M_t. \tag{2}$$

For a given level of the assets and the market profitability index the investment yields the present value

$$PV = \frac{KM}{r - \mu}. \tag{3}$$

It is important to realize that the market profitability index varies independently of the specific firm, whereas the size of the assets is controlled by the firm's manager. For simplicity we henceforth assume that the firm's manager acts in the best interest of equity holders and we will interchangeably denote the manager or the equity holders as the decision maker(s).

The firm is at the onset initiated by an investment of I. We assume that the investment cost is proportional to the present value. That is,

$$I(K, M) = i\frac{KM}{r - \mu}, \tag{4}$$

where $i \in (0, 1)$ is the fraction of the present value spent on the investment cost. Our assumption regarding i ensures that the investment has a positive net present value.

Since the firm's profit depends on the market profitability index and the size of the assets in a complementary sense, the firm has an incentive to buy more assets (Cooper, 2006; Hackbarth and Johnson, 2015). The point of our paper can be easiest illustrated if we allow the firm's growth opportunities to be embedded in sequential investment options. That is, we let the firm have one option at the time to expand its capacity. The investment increases the size of assets, so that the total assets increases to ΛK_0 after the investment. Expanding the model to a more elaborate investment opportunity set would make the model more complex, but it would not provide fundamental new insights.[1] If anything, a more rich investment opportunity set will make empirical research more challenging, and it would thus only further support the point of the present paper.

Furthermore, we include a standard tax structure and allow coupon payment to be tax deductible. Specifically, the firm can deduct C from its dividends to equity holders before taxes are paid. The effective corporate tax rate is τ_e. Debt holders have to pay the tax rate τ_i on the received coupon. Taxes give equity holders an incentive to issue more debt than what is required to fund the investment. However, since the exploitation of the tax system per se is not our focus, we let the value of the debt issuance be equal to

[1]For example, we could include other underlying determinants of the firm's cash flow; or we could allow the firm to also adjust its capital structure for tax shield reasons alone or for tuning its maturity or priority structure etc. (e.g.; Christensen et al., 2014; Flor and Lester, 2004; Gorbenko and Strebulaev, 2010; Hackbarth and Mauer, 2012; He and Xiong, 2012). To keep our model focused we omit adding these interesting features.

the investment cost. Therefore, we assume that the investment cost is covered by issuing debt. We follow the standard in the literature and consider a single class of perpetual debt (e.g.; Danis et al., 2014; Goldstein et al., 2001). The debt contract promises a coupon rate $C > 0$ and includes a covenant prohibiting further debt issuance. The debt is callable and hence the equity holders can change the amount of debt by calling the previously issued debt. We assume that debt holders are paid a premium if the debt is called. The premium is a fraction λ of the principal and we assume that debt is issued at par. The equity holders have an incentive to call the debt if the market profitability index becomes high enough because they want to exploit the investment opportunity. Let $u > 1$ be the scaling factor relative to the initial level of the market profitability index so that the firm decides to invest in more assets the first time $M = uM_0$. Our model leads to the following value matching conditions for debt and equity, respectively:

$$D(uM_0; K_0, M_0) = (1 + \lambda)D(M_0; K_0, M_0), \tag{5}$$

$$\begin{aligned} E(uM_0; K_0, M_0) = {} & E(uM_0; \Lambda K_0, uM_0) + D(uM_0; \Lambda K_0, uM_0) - I(\Lambda K_0, uM_0) \\ & - (1 + \lambda)D(M_0; K_0, M_0). \end{aligned} \tag{6}$$

On the other hand, if the market profitability index becomes low enough, the equity holders prefer to default on the debt. Default implies that equity holders give up on all their claims and surrender the firm to the debt holders. Default is costly and we assume that a fraction α of the assets is lost due to default costs. That is, the debt holders take over a firm with assets reduced to $(1 - \alpha)K$. Suppose default occurs the first time the market profitability index hits $M = dM_0$, where $1 > d > 0$ is the scaling factor relative to the initial level of the market profitability index. This gives us the value matching conditions

$$D(dM_0; K_0, M_0) = E(dM_0; (1-\alpha)K_0, dM_0), \tag{7}$$

$$E(dM_0; K_0, M_0) = 0. \tag{8}$$

Papers analyzing dynamic capital structure issues traditionally let the firm re-balance its capital structure to exploit the tax shield. This is not the focus of our paper, and therefore we require that the new investment is funded by debt.[2] Thus, for a given level of assets and the market profitability index we have

$$D(M; K, M) = I(K, M). \tag{9}$$

This condition implicitly defines the coupon.[3]

Our framework allows us to use a homogeneity property to simplify the conditions at investment and default, respectively. This allows us to obtain the following results.

Proposition 1. *For a given level of assets K_0, market profitability index M_0, and relative coupon rate $c = \frac{C}{K_0 M_0}$, the value of equity and debt, respectively, can be written on the*

[2]Of course, we could in principle allow for a mix of new equity and debt. It is well-known that a security issuance is costly (Hennessy and Whited, 2005), so we are basically assuming that debt for exogenous reasons is cheaper to issue than equity.

[3]With taxes (4) is given as

$$I(K, M) = (1-\tau_e) i \frac{KM}{r-\mu}$$

form

$$E(M;K_0,M_0) = (1-\tau_e)\left(\frac{K_0 M}{r-\mu} - \frac{cK_0M_0}{r}\right) + e_1 K_0 M_0^{1-\beta_1} M^{\beta_1} + e_2 K_0 M_0^{1-\beta_2} M^{\beta_2}, \quad (10)$$

$$D(M;K_0,M_0) = (1-\tau_i)\frac{c}{r} K_0 M_0 + d_1 K_0 M_0^{1-\beta_1} M^{\beta_1} + d_2 K_0 M_0^{1-\beta_2} M^{\beta_2}, \quad (11)$$

where $d_1, d_2, e_1,$ and e_2 are constants depending on the scaling factors d and u as well as the relative coupon c. $\beta_1 > 1$ ($\beta_2 < 0$) is the positive (negative) root satisfying $\sigma^2\beta(\beta - 1)/2 + \mu\beta - r = 0$. The coupon rate c solves the condition that debt is issued at par and is equal to the investment cost

$$(1-\tau_i)\frac{c}{r} = \frac{i}{r-\mu} - (d_1 + d_2). \quad (12)$$

The scaling factors are found by solving the smooth-pasting condition when the equity holders decide to invest

$$\left.\frac{\partial E(M;K_0,M_0)}{\partial M}\right|_{M=uM_0} = \left.\frac{\partial E(M;\Lambda K_0,M) + D(M;\Lambda K_0,M)}{\partial M}\right|_{M=uM_0} - i\frac{\Lambda K_0}{r-\mu}. \quad (13)$$

and when they decide to default.

$$\left.\frac{\partial E(M;K_0,M_0)}{\partial M}\right|_{M=dM_0} = 0. \quad (14)$$

Having set up our model with the market profitability index as the state variable and the size of assets as controlled by the firm, we proceed to consider the implications.

4 Model implications

To demonstrate how the explanatory power of leverage determinants depend on their interdependence we simulate a sample of companies based on the model described above. By construction, the model implies a significant negative relation between leverage and profitability when the amount of firm's book assets is constant. However, when the firm optimally invests, there is a significant positive relation between leverage and profitability. This change in the correlation between leverage and profitability distorts their relation in an ordinary OLS regression unless we control for the firm's book assets. In the following, we present the simulation procedure and demonstrate some insights of the challenges attached to understanding leverage determinants.

4.1 Simulation procedure and variable definitions

For the implementation of the model, we use parameter values that match the literature.[4] The parameter choices are presented in Table 1. The risk neutral drift is chosen to match the parametrization in Christensen et al. (2014). The instantaneous standard deviation of EBIT follows Flor and Lester (2004) and is a standard parameter choice in the literature. For the risk-free interest rate we use the average of the levels in Christensen et al. (2014) (4.5%), Hackbarth et al. (2006) (5.5%), and Strebulaev (2007) (5%). The tax rates on both interest and dividends, the debt call premium and the bankruptcy costs follow directly from Christensen et al. (2014). The parametrization for the investments cost, the investment scalar and the initial values for the level of assets and the index follows the literature see, e.g., (Cooper, 2006; Hackbarth and Johnson, 2015).

[4] We are aware that for most parameters of interest the empirical evidence points to time-varying ranges for the parameters. Our model assumes them to be constant. To account for this, we have performed unreported robustness checks which show that our results are not qualitatively affected by changing the parameters.

Table 1: **Parameter choices**

	Parameter choices	
Risk neutral drift of the EBIT process	μ	0.03
Volatility of the EBIT process	σ	0.2
Risk free interest rate	r	0.05
Tax rate on interest payments	τ_i	0.35
Effective tax rate on dividends	τ_e	0.50
Debt call premium	λ	0.075
Bankruptcy costs	α	0.25
Present value spent on investment cost	i	0.6
Investment scalar	Λ	1.75
Initial level of capital	K_0	1
Initial value for the profitability index	M_0	1

The table contains the values chosen for all parameters required to simulate the benchmark case of the model. The parameter choices are based on the literature for references, see Christensen et al. (2014), Hackbarth et al. (2006), Flor and Lester (2004), Strebulaev (2007), Cooper (2006), and Hackbarth and Johnson (2015).

At date zero all companies in the economy are "born" and choose their optimal capital structure. For the benchmark simulation, we decide to simulate 50 years of data for 1,000 firms. To minimize the impact of initial conditions, we drop the first ten years of data leaving the sample period at 40 years. This sample period length implies that the firms in the sample invest or default during the sample period.

For construction of our variables we follow Lemmon et al. (2008) and define

$$\text{Market Leverage} = \frac{\text{Total Debt}}{\text{Total Debt} + \text{Market Equity}} \tag{15}$$

$$\text{Profitability} = \frac{\text{Operating income}}{\text{Book assets}}. \tag{16}$$

In our setting,

$$\text{Market Leverage} = \frac{D(M; K_0, M_0)}{D(M; K_0, M_0) + E(M; K_0, M_0)} \tag{17}$$

$$\text{Profitability} = \frac{K_t M_t}{K_t} = M_t. \tag{18}$$

Table 2 presents summary statistics for our simulated sample of firms.[5] We note that the firms in our simulated sample have a high amount of leverage compared to empirical levels (0.28 in Lemmon et al. (2008)). In general, models based on the trade-off theory imply higher leverage levels than observed empirically. Table 2 show that this is also the case for

[5]Our model set-up implies that we could also define firm size as log of book asset ($\log(K_t)$), market-to-book as market equity and total debt to book assets ($\frac{D(M;K_0,M_0)+E(M;K_0,M_0)}{K_t}$), and book leverage as total debt to book asset ($\frac{D(M;K_0,M_0)}{K_t}$). The purpose of the model is not to match data but to show how interdependence between variables can distort significant correlations. To keep the analysis simple, we will not focus on how well the model matches data or include these additional variables in our analysis.

our model. The mean value of the profitability indicates that the firms' earnings increase, on average, over time. The increase in profitability is in line with the mean of the book assets above one as the rise in the profitability implies investments in assets.

Table 2: **Summary Statistics**

Variable	Mean [Median]	(SD)
Market leverage	0.532	(0.165)
	[0.491]	
Profitability	2.429	(4.671)
	[1.27]	
Book Assets	4.179	(8.969)
	[1.75]	
Observations	40,000	

The simulated sample is simulated based on our model with variable definitions given in (18) and (17). The table presents variable averages, medians (in brackets), and standard deviations (SD) for the entire sample.

4.2 Features of the model

Our model set-up and variable definitions imply that when the profitability, M_t approaches zero, the firm's leverage approaches one. Thus, for constant levels of book assets, a regression of market leverage on profitability has an intercept equal to one. Another feature of the model set-up is that the slope of a regression of market leverage on profitability is decreasing as the levels of book assets increases. Market leverage is bounded between zero and one and the difference between the default and investment boundary increases as the value of the book assets increases. Together, this implies the decreasing slopes. Thus, for higher levels of book assets, we expect a less steep slope.

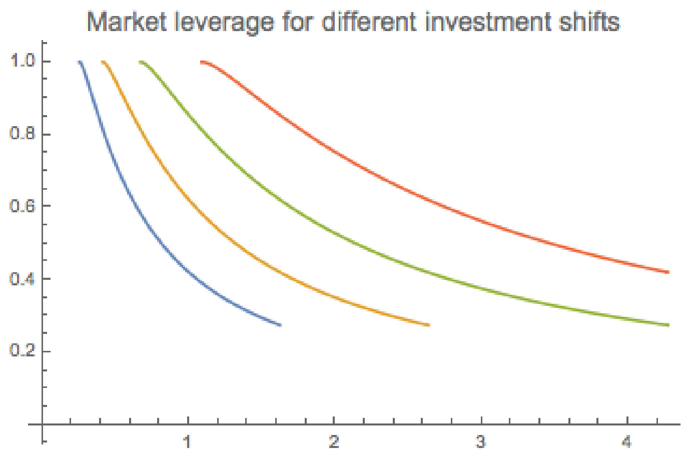


Figure 3: The Figure shows how the firm's leverage correlates with profitability for four different values of the firm's book assets. The blue line corresponds to the initial value of the firm's book assets. The red line corresponds to the value of the firm's book assets after investing once. The green line corresponds to the value after the firm invested twice and the red after it invested three times.

Figure 3 show how the firm's leverage correlates with profitability and depicts these two features. Each different colored line illustrates the relationship between profitability and leverage for a specific level of book assets. The blue line corresponds to the initial level of book assets, the yellow after one investment, the green after two, and the red after four. All four lines approach one as the profitability approaches the default boundary. Also, the distance between the default and investment boundary increases as more investment occurs. Thus, the slope of the blue line is the steepest and the slope of the red line the flattest.

4.3 Data analysis

Using our simulated sample of firms, we can create an example that resembles that of American Airlines Group. This highlights shows how an empirical analysis can be inherently problematic even for a dynamic model that endogenously generate a clear correlation between leverage and profitability. We generalize this for our entire sample.

First, we focus on a single firm in our simulated sample. To compare with the example of American Airlines Groups in Figure 1 we depict the firm's profitability, book assets, and market leverage as a function of time in Figure 4. This Figure shows how the correlation between profitability and market leverage is negative in most years. However, the sign of the correlation changes in year 12 where the value of the assets increases from the initial value of 1 to a value of 1.75. At this specific point in time there is a positive correlation between the firm's profitability and market leverage. Figure 4 shows how the interdependence between the profitability and the book assets distorts the correlation between the profitability and the market leverage the year the firm investment in book assets.

To be more specific, Figure 5 illustrates the correlation between the firm's leverage and its profitability. The black dots in Figure 5 represent the data points for leverage and the profitability. The figure shows the negative relationship between the index and leverage for constant levels of the book assets. However, as depicted in Figure 4 the value of assets increases at year 12. The increase in book assets implies that some of the distinct values of the profitability correspond to two different values of the market leverage. This feature extends to our entire sample. Figure 6a depicts the average level of market leverage on average profitability for our simulated sample. Clearly the same level of average earnings correspond to several levels of average leverage. Also, the same level of average leverage correspond to several levels of average earnings. Figure 6b show that the same feature occurs in data. Below we explain in more detail how this means that a standard OLS regression does not capture the real correlation between the firm's leverage and profitability in a satisfying manner.

First, we consider market leverage as a function of profitability over the full time

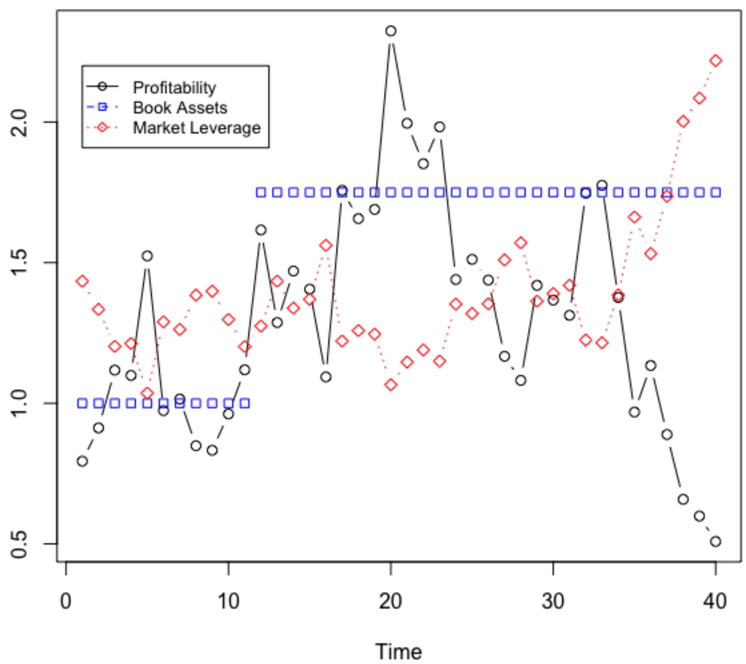


Figure 4: The firms profitability, book assets, and market leverage as a function of time for a single firm in our simulated sample of firms.

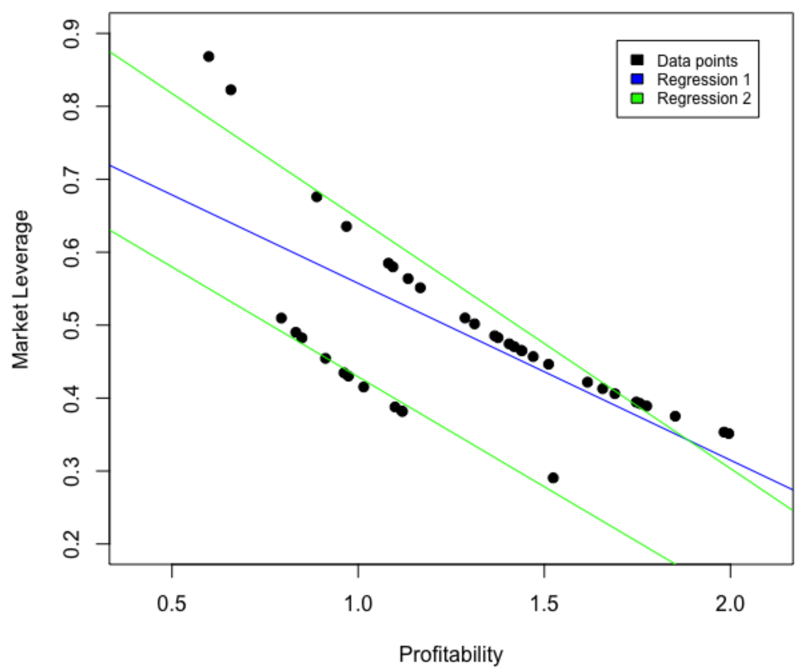
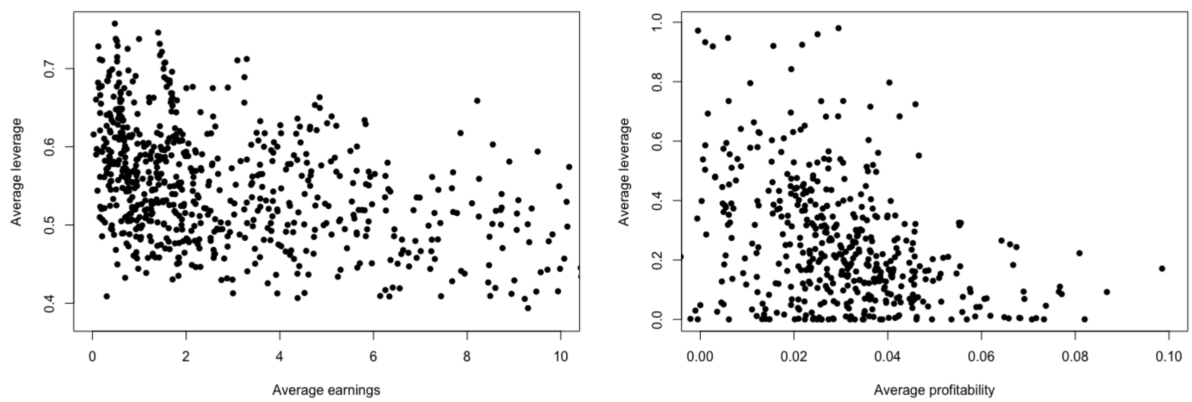


Figure 5: The firm's leverage as a function of the profitability. The black dots represent the data points for our simulated firm. The blue line represents an OLS regression with leverage as the dependent and profitability as the independent variable. The two green lines represent an OLS regression where we allow for distinct intercepts and slopes for each particular value of the firm's book assets.



(a) Simulated data sample

(b) Data from compustat

Figure 6: Average leverage as a function of average earnings. Figure 6a depicts the relationship for our simulated data sample while Figure 6b shows the relationship for compustat data.

horizon. The blue line in Figure 5 accounts for a standard OLS regression where we regress the firm's leverage on the profitability index. The regression implies an $R^2 = 0.52$ and is the best possible linear fit if we do not control for the company's book assets.

To extend this analysis for our entire sample of simulated firms we consider the following regression,

$$\text{Market leverage}_{it} = \alpha + \beta_1 \cdot \text{Profitability}_{it} + \varepsilon_{it} \tag{19}$$

in which we do not account for changes in the firm's environment that may occur over time. We are interested in the sign and magnitude of β_1 as well as the goodness of fit, R^2. Table 3 presents the results form regression (19). The negative sign of β_1 corresponds well to our model predictions. We do however notice that even though the index has a significant effect on the firm's leverage the magnitude of the coefficient is smaller than we expect from the model. Also, the low value of the R^2 suggests that the index only attributes a small proportion of the variation in leverage. Though this result is not what we expected from the model it does align with the findings by Lemmon et al. (2008); who show that profitability is a weak determinant of firm leverage.

Table 3: Regression (19)

	Dependent variable:
	Marketleverage
Profitability	-0.007^{***}
	(0.0002)
Constant	0.550^{***}
	(0.001)
Observations	40,000
R^2	0.045
Adjusted R^2	0.045
Residual Std. Error	0.161 (df = 39998)
F Statistic	$1,880.050^{***}$ (df = 1; 39998)
Note:	*p<0.1; **p<0.05; ***p<0.01

The table presents the results from applying regression (19) on our simulated sample of firms.

The intuition from Figure 4 tell us that the level of the firm's book assets is also a determinant of the market leverage. Thus, we consider the following regression,

$$\text{Market leverage}_{it} = \alpha + \beta_1 \cdot \text{Profitability}_{it} + \beta_2 \cdot \text{Book Assets}_{it} + \varepsilon_{it}. \tag{20}$$

According to our model specifications, we expect β_1 to be significant and negative and β_2 to be positive and significant. As is evident from Table 4 this is indeed the case. Profitability correlates significantly with leverage with a negative coefficient and the size of the firm's book assets correlates positively and significantly with the firm's leverage.

We also note that the size of β_1 is increased in Table 4 compared to Table 3. This increase in the magnitude of the coefficient on the index suggests that profitability has a larger negative impact on the firm's leverage when we also control for the value of the firm's book assets. However, the goodness of fit for the regression in (20) is still rather poor as suggested by the low value of the R^2. Since we know for sure that the model implies a strong positive correlation between the profitability index and the firm's leverage we would expect a much better goodness of fit.

Table 4: Regression (20)

	Dependent variable:
	Marketleverage
Profitability	-0.035^{***}
	(0.0004)
Assets	0.016^{***}
	(0.0002)
Constant	0.552^{***}
	(0.001)
Observations	40,000
R^2	0.155
Adjusted R^2	0.155
Residual Std. Error	0.152 (df = 39997)
F Statistic	$3{,}681.571^{***}$ (df = 2; 39997)
Note:	*p<0.1; **p<0.05; ***p<0.01

The table presents the results from applying regression (20) on our simulated sample of firms.

For our simple example, with a single firm, we achieve a much better fit when we regress leverage on profitability and allow each level of the firm's assets to have its distinct intercept and slope. To this end, we create a dummy variable corresponding for each different value of the firm's book assets. In this example, we have two dummies. One which is equal to 1 if the value of the firm's assets is 1 and zero otherwise. The other dummy is equal to 1 is the value of the firm's assets is 1.75 and zero otherwise. If we include an interaction of profitability with each of these dummies, this implies that we obtain different slope coefficients for the two different levels of the firm's assets. If we furthermore include the dummy for either of the asset levels, we have two intercepts, one for each value of the firm's assets. In sum, the green lines in Figure 5 represents

$$\text{Leverage}_t = \alpha + \text{dummy}_{\text{Assets}=1} + \text{d}_{\text{Assets}=1} * \text{Profitability} + \text{d}_{\text{Assets}=1.75} * \text{Profitability}. \tag{21}$$

with a goodness of fit, $R^2 = 0.96$. The increase in goodness of fit compared to the first regression suggests that we need to account for the firm's book assets to correctly identify the correlation between market leverage and profitability.

Again, we extend the simple example to our entire simulated data sample. To be able to detect the real size of the correlation between the firms' leverage and the profitability index we now allow each unique level of book assets to have its distinct slope and intercept. To be more precise, we create a dummy for each of the different values for the firms' book assets. This dummy is set to be equal for one for a unique size of the book assets and zero for all other. In our regression analysis, we include the dummies to account for different intercepts and add intersections of the dummies with the profitability to obtain different slopes. We, therefore, consider the following regression,

$$\begin{aligned}\text{Market leverage}_{it} = \alpha_1 d_{\text{Book Assets}=j_1} + ... + \alpha_N d_{\text{Book Assets}=j_N} + \\ \beta_1 \cdot (d_{\text{Book Assets}=j_1} \cdot \text{Profitability}_{it}) + ... \\ + \beta_N \cdot \left(d_{\text{Book Assets}=\text{j}_N} \cdot \text{Profitability}_{it}\right) + \varepsilon_{it}\end{aligned} \quad (22)$$

where $j = 1, ..., N$ correspond to the different levels of the firms' book assets. To ensure that there is sufficient data we leave out data with fever than 100 observations for a specific level of book asset. From our model-based illustrative example, we expect the regression analysis from (22) to provide a much better goodness of fit.

The results of the regressions analysis in (22) is presented in Table 5 and show that our intuition from the small example also holds for a large sample of simulated firms. The goodness of fit is much larger compared to Table 3 and 4. The high level of goodness of fit aligns with the model predictions as it suggests that profitability is an important determinant of the firms' leverage.

We note from the signs and magnitudes from the α's in Table 5 that these corresponds well to our model predictions, as suggested by Figure 3. The α's are close to one and significant. We also note that the slopes becomes less steep as the value of book assets increases. We have arranged the data such that for β_j, $j = 1, ...N$, an increase in j correspond to an increase in the value of the firm's book assets. The results in Table 5 does deviate from this pattern for some values of the β's. However, our analysis suggests that this might be due to simulation irregularities.

As for the example with American Airlines Group, the small example and the more formal analysis show how a standard OLS regression does not capture the real correlation between a firm's market leverage and profitability unless we account for a third variable. In

Table 5: Regression (22)

	Dependent variable: Marketleverage				*Dependent variable:* Marketleverage		
β_1	-8.318^{***}	α_1	0.983^{***}	β_{11}	-0.448^{***}	α_{11}	0.932^{***}
	(0.208)		(0.013)		(0.005)		(0.005)
β_2	-2.002^{***}	α_2	0.955^{***}	β_{12}	-0.121^{***}	α_{12}	0.961^{***}
	(0.013)		(0.003)		(0.001)		(0.002)
β_3	-4.643^{***}	α_3	0.933^{***}	β_{13}	-0.268^{***}	α_{13}	0.914^{***}
	(0.102)		(0.010)		(0.004)		(0.007)
β_4	-0.574^{***}	α_4	1.014^{***}	β_{14}	-0.074^{***}	α_{14}	0.953^{***}
	(0.002)		(0.001)		(0.0005)		(0.003)
β_5	-1.238^{***}	α_5	0.956^{***}	β_{15}	-0.165^{***}	α_{15}	0.914^{***}
	(0.008)		(0.003)		(0.005)		(0.014)
β_6	-2.294^{***}	α_6	0.832^{***}	β_{16}	-0.045^{***}	α_{16}	0.956^{***}
	(0.108)		(0.019)		(0.0004)		(0.004)
β_7	-0.326^{***}	α_7	0.975^{***}	β_{17}	-0.109^{***}	α_{17}	0.948^{***}
	(0.001)		(0.002)		(0.003)		(0.014)
β_8	-0.746^{***}	α_8	0.942^{***}	β_{18}	-0.028^{***}	α_{18}	0.954^{***}
	(0.006)		(0.004)		(0.0004)		(0.007)
β_9	-1.718^{***}	α_9	0.922^{***}	β_{19}	-0.015^{***}	α_{19}	0.872^{***}
	(0.058)		(0.016)		(0.0005)		(0.015)
β_{10}	-0.199^{***}	α_{10}	0.968^{***}				
	(0.001)		(0.002)				
Observations	39,773						
R^2	0.993						
Adjusted R^2	0.993						
Residual Std. Error	0.047 (df = 39735)						
F Statistic	$145{,}052.600^{***}$ (df = 38; 39735)						

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

The table presents the results from applying regression (22) on our simulated sample of firms.

the American Airlines Group example we controlled for the merger, and in the simulated data example, we accounted for the firm's book assets. With a simulated sample, the relationship between market leverage and profitability is easy to interpret as we know the model dynamics. The model implies that we know precisely in which years the firm's increase their book assets. Unfortunately, this simple model setting does not generalize to data. With empirical data, we cannot be sure to observe clear changes in a firm's book assets. Also, these increases may not only occur because of investments but also in case of, e.g., mergers or other strategic decisions. Thus, identifying changes in the firm's environment can be challenging. The goal of this paper is not to address the challenges linked to identifying firm's situations but rather to show how changes in firm's environment can distort the outcome of a standard OLS regression. To this end, we suggest that misinterpretation of actual correlation between a firm's leverage and leverage determinants could occur.

In general, the above analysis shows how the interdependence between profitability and the size of the firms' book assets affects the relationship between profitability and the firm's' leverage. The results in Table 3 and Table 4 suggest that correlation between the firms' leverage and profitability is of less importance. By construction, the model implies a strong correlation between leverage and profitability. Thus, the specifications in (19) and (20) cause us to reject an important correlation falsely. Our data behave according to a specified model, we therefore know how to control for the interdependence between profitability and the value of the firms' assets. With our specification in (22) we can see the real correlation between profitability and the firms' leverage in Table 5.

We expect that interdependencies exist among several of the empirically acknowledged leverage determinants. One example is tangibility that relates to investment in both operating assets and capital. Another example is sales that is related to inside as well as outside factors. Even though some of these factors might be picked up by including year fixed effects most of them are firm specific. We propose that it is this interdependence between capital structure determinants that are reflected by the significant firm fixed effect presented by Lemmon et al. (2008). We suggest that a significant firm fixed effect is evidence of a large amount of interdependence between different factors rather than persistence in leverage.

4.4 The sign of the profitability index

According to the model, we expect the correlation between leverage and profitability to be positive at all points in time where the firm finds it optimal to increase its book assets. The firm funds its investment in assets by debt which in turns implies this positive correlation. We test this particular model implication by regressing leverage on profitability interacted with a dummy equal to one when the firms are investing in assets. This procedure is similar to the one presented in Danis et al. (2014). Danis et al. (2014) empirically test the model predictions from Strebulaev (2007) who argues that there is a positive correlation between a firm's leverage and the firm's profit at the time of debt restructuring. In general, the trade-off theory suggests that firms increase their leverage when it is profitable to do so. That is when the benefits of the (the tax advantage) are larger than the costs of the debt (default costs).

We test the correlation between the firms' leverage and profitability at the time of investment on our entire sample of simulated firms. As we are mainly interested in this particular correlation, we do not include individual slopes and intersections for the unique asset values. Thus, we consider the following regression,

$$\text{Market leverage}_{it} = \alpha + \beta_1 \text{Index}_{it} + \beta_2 d_{\text{Investment}} \cdot \text{Index}_{it} + \varepsilon_{it} \tag{23}$$

with results presented in Table 6.

Table 6: Regression (23)

	Dependent variable:
	Marketleverage
Profitability	-0.006^{***}
	(0.0002)
I(restruc ∗Profitability)	0.001^{***}
	(0.0003)
Constant	0.545^{***}
	(0.001)
Observations	40,000
R^2	0.042
Adjusted R^2	0.042
Residual Std. Error	0.157 (df = 38997)
F Statistic	861.425^{***} (df = 2; 38997)
Note:	*p<0.1; **p<0.05; ***p<0.01

The table presents the results from applying regression (23) on our simulated sample of firms.

As expected, the results from (23) show that correlation between leverage and profitability is positive at points where the firms invest in book assets but negative elsewhere. This positive correlation aligns with our model predictions as companies use debt to fund their investments.

The magnitude of the coefficient on the profitability index aligns with the results in

Table 3. We note that the size of the coefficient on profitability is small even when we include a dummy indicating investment. This observation supports our findings in the previous section. We only observe the actual magnitude of the correlation between a firm's leverage and profitability when we allow for separate slopes and intercepts for each unique size of its book assets.

4.5 The role of initial leverage

The results presented in Lemmon et al. (2008) suggests that leverage is largely unexplained by previously identified determinants. Instead, the most important determinants are time invariant, and, e.g., initial leverage seems to be a good indication of future leverage levels. We follow the intuition from Lemmon et al. (2008) and test their prediction on our simulated sample of firms. Especially we wish to highlight how their analysis can be problematic if there exists interdependence among leverage determinants.

We follow the analysis in Lemmon et al. (2008) and regress leverage on the firm's initial leverage as well as profitability. We define a firm's initial leverage as the first leverage observation for each particular firm in our sample. This specification implies that each firm in our sample obtains a unique intercept which is related to the firm's initial leverage. We consider the following regression

$$\text{Market leverage}_{it} = \alpha + \beta_1 \text{Initial Leverage}_i + \beta_2 \text{Profitability}_{it} + \varepsilon_{it}, \tag{24}$$

and use this to determine whether firms' future leverage is closely related to their initial leverage. The results in Tabel 7 show that even though both factors are significant the coefficient on initial leverage is more than ten times as large in magnitude. Our findings follow the results from Lemmon et al. (2008) and suggest that initial leverage predicts future leverage levels better than profitability. Again, we see that profitability only has a small impact on leverage similar to the results from Table 19. This is the case even though our theoretical model predicts a strong correlation between the firm's leverage and profitability and no particular relation between the firm's initial and future leverage levels.

Table 7: Regression (24)

	Dependent variable:
	Marketleverage
InitialLeverage	0.098^{***}
	(0.005)
Profitability	-0.007^{***}
	(0.0002)
Constant	0.497^{***}
	(0.003)
Observations	40,000
R^2	0.053
Adjusted R^2	0.053
Residual Std. Error	0.161 (df = 39997)
F Statistic	$1{,}119.751^{***}$ (df = 2; 39997)
Note:	*p<0.1; **p<0.05; ***p<0.01

The table presents the results from applying regression (24) on our simulated sample of firms.

To see whether the magnitude of the firm's initial leverage persist if we control for the interdependence between firm's profitability and the value of book assets we consider the following regression,

$$
\begin{aligned}
\text{Market leverage}_{it} = \alpha_1 d_{\text{Book Assets}=j_1} + \dots + \alpha_N d_{\text{Book Assets}=j_N} \\
+ \beta_0 \cdot \text{Initial Leverage}_i + \beta_1 \cdot (d_{\text{Book Assets}=j_1} \cdot \text{Profitability}_{it}) + \dots \\
+ \beta_N \cdot \left(d_{\text{Book Assets}=\text{j}_N} \cdot \text{Profitability}_{it}\right) + \varepsilon_{it} \qquad (25)
\end{aligned}
$$

with results presented in Table 8. We note that even though the coefficient on the firm's initial leverage is significant, it has decreased in magnitude. We also note that the coefficients on the β's and α's do not differ much between Table 5 and Table 8. The small difference suggests that adding the firm's initial leverage to the regression in (25) does not add to explaining the firm's leverage.

Overall, our findings in Table 7 suggests a firm's initial leverage seems to be a good predictor for future leverage levels. This result occurs when we do not control for the interdependence between the firm's book assets and profitability. The results in Table 8 shows that allowing for each unique value of the firm's assets to have its different slope and intercept reduces the magnitude of the coefficient on initial leverage and depicts the expected relation between market leverage and profitability. This finding suggests that even in a model where there is no correlation between a firm's initial and future leverage the noise created by interdependence between leverage determinants can lead to spurious conclusions.

5 Conclusion

We show how regression analyses can be inherently problematic when we wish to investigate the relationship between leverage and leverage determinants. With a dynamic capital

Table 8: Regression (25)

	Dependent variable:				*Dependent variable:*		
	Marketleverage				Marketleverage		
InitialLeverage	0.008^{***}						
	(0.002)						
β_1	-8.333^{***}	α_1	0.978^{***}	β_{11}	-0.448^{***}	α_{11}	0.928^{***}
	(0.208)		(0.013)		(0.005)		(0.005)
β_2	-2.002^{***}	α_2	0.949^{***}	β_{12}	-0.121^{***}	α_{12}	0.957^{***}
	(0.013)		(0.003)		(0.001)		(0.002)
β_3	-4.660^{***}	α_3	0.931^{***}	β_{13}	-0.268^{***}	α_{13}	0.909^{***}
	(0.102)		(0.010)		(0.004)		(0.007)
β_4	-0.572^{***}	α_4	1.008^{***}	β_{14}	-0.074^{***}	α_{14}	0.949^{***}
	(0.002)		(0.002)		(0.0005)		(0.003)
β_5	-1.238^{***}	α_5	0.951^{***}	β_{15}	-0.166^{***}	α_{15}	0.911^{***}
	(0.008)		(0.003)		(0.005)		(0.014)
β_6	-2.287^{***}	α_6	0.825^{***}	β_{16}	-0.045^{***}	α_{16}	0.952^{***}
	(0.108)		(0.019)		(0.0004)		(0.004)
β_7	-0.325^{***}	α_7	0.970^{***}	β_{17}	-0.108^{***}	α_{17}	0.943^{***}
	(0.001)		(0.002)		(0.003)		(0.014)
β_8	-0.746^{***}	α_8	0.937^{***}	β_{18}	-0.028^{***}	α_{18}	0.951^{***}
	(0.006)		(0.004)		(0.0004)		(0.007)
β_9	-1.714^{***}	α_9	0.915^{***}	β_{19}	-0.015^{***}	α_{19}	0.868^{***}
	(0.058)		(0.016)		(0.0005)		(0.015)
β_{10}	-0.199^{***}	α_{10}	0.964^{***}				
	(0.001)		(0.002)				
Observations		39,773					
R^2		0.993					
Adjusted R^2		0.993					
Residual Std. Error		0.047 (df = 39734)					
F Statistic		$141{,}418.300^{***}$ (df = 39; 39734)					
Note:	$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$						

The table presents the results from applying regression (25) on our simulated sample of firms.

structure model, we show how the interdependence between determinants can cause us to reject meaningful correlations falsely. Our model specifications imply a strong correlation between a firm's leverage and a market-based state variable. A standard OLS regression does however not display this relation unless we control for the interdependence there exists between the state variable and the firm's control variable. We claim that this interdependence can explain some of the disagreements we have seen in the empirical capital structure literature.

We also consider the string of literature that looks at how well initial leverage predicts future leverage. We find that even for our model, a firm's initial leverage is, in fact, a good predictor for future leverage. This result occurs as interdependence exists among leverage determinants and is true even though our model, by construction, does not establish any correlation between initial and future leverage levels. This finding suggests that the same interdependence might exist in actual data. In general, we point out that empirical research must consider the effects of interdependence between leverage determinants when trying to quantify the firm's optimal capital structure choice.

Appendix

A An illustrative example

We report the regressions from the example with the American Airlines Group below.

Table 9: Market leverage = profitability

	Dependent variable:
	marlev
prof	-10.009^{***}
	(1.309)
Constant	0.930^{***}
	(0.034)
Observations	48
R^2	0.560
Adjusted R^2	0.550
Residual Std. Error	0.135 (df = 46)
F Statistic	58.510^{***} (df = 1; 46)
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 10: Marketleverage = $\text{size}_1 + \text{size}_2 + \text{size}_1 \cdot \text{profitability} + \text{size}_2 \cdot \text{profitability}$

	Dependent variable:
	marlev
size1	0.853^{***}
	(0.031)
size2	0.640^{***}
	(0.103)
I(size1 ∗prof)	-1.250
	(1.939)
I(size2 ∗prof)	-4.585^{*}
	(2.609)
Observations	48
R^2	0.984
Adjusted R^2	0.982
Residual Std. Error	0.100 (df = 44)
F Statistic	660.359^{***} (df = 4; 44)
Note:	*p<0.1; **p<0.05; ***p<0.01

B Proofs

B.1 Debt and equity values

The values of debt and equity is derived by considering their valuation in the continuation region, i.e., in the periods of time in which the firm does not actively change its assets. Suppose the initial level of assets is M_t. Recall from (2) that the firm earns the instantaneous cash flow

$$\pi_t = K_0 M_t. \tag{26}$$

Since the level of assets is constant in the continuation region and the profitability index follows a geometric Brownian motion, it follows that the cash flow follows a geometric Brownian motion:

$$d\pi_t = \mu \pi_t dt + \sigma \pi_t dW_t, \tag{27}$$

where $\pi_0 = K_0 M_0$. Following the usual steps in real options analysis (for example, Christensen et al., 2014; Dixit and Pindyck, 1994; Hackbarth and Mauer, 2012), a claim F depending on π and receiving a payment flow $h(\pi)$ must satisfy the differential equation

$$\frac{1}{2}\sigma^2 \pi^2 F''(\pi) + \mu F'(\pi) - rF(\pi) + h(\pi) = 0. \tag{28}$$

Assuming that the payment flow has the form $h(\pi) = \hat{h}_0 + h_1\pi$ for constants $\hat{h}_0$ and h_1, the solution can be written as

$$F(\pi) = \frac{h_1\pi}{r-\mu} + \frac{\hat{h}_0}{r} + \hat{f}_1\pi^{\beta_1} + \hat{f}_2\pi^{\beta_2}, \tag{29}$$

where β_i solves $\sigma^2\beta_i(\beta_i - 1)/2 + \mu\beta_i - r = 0$. Simple inspection of the parabola shows that $\beta_1 > 1$ and $\beta_2 < 0$. The constants $\hat{f}_1$ and $\hat{f}_2$ are to be determined by boundary conditions. Since we want to set up a model with a scaling property we require that $\hat{h}_0 = h_0\pi_0$. This makes it natural to rewrite (29) assuming that the model is linear in π when the firm actively changes its assets, that is,

$$F(\pi;\pi_0) = \frac{h_1\pi}{r-\mu} + \frac{h_0\pi_0}{r} + f_1\pi_0^{1-\beta_1}\pi^{\beta_1} + f_2\pi_0^{1-\beta_2}\pi^{\beta_2}, \tag{30}$$

implying that

$$F(\pi;\pi) = \left(\frac{h_1}{r-\mu} + \frac{h_0}{r} + f_1 + f_2\right)\pi. \tag{31}$$

A well specified model then requires that the constants f_1 and f_2 (as well as h_0 and h_1) are independent of π_0, i.e., K_0 and M_0.

We now turn to the debt and equity contracts in the model. The debt contract promises the debt holders a coupon C until default or the debt is called. The coupon is taxed with the rate τ_i. At the time of the debt issuance the level of the assets was K_0 and the level of the profitability index was M_0. Hence, the debt holders receive the after tax payment $(1-\tau_i)C = (1-\tau_i)cK_0M_0$, that is, $h_0 = (1-\tau_i)c$ and $h_1 = 0$ in (30). Using that $\pi = K_0M$

we can thus write the value of debt as

$$D(M; K_0, M_0) = \frac{(1-\tau_i)cK_0M_0}{r} + d_1K_0M_0^{1-\beta_1}M^{\beta_1} + d_2K_0M_0^{1-\beta_2}M^{\beta_2}. \tag{32}$$

Similar arguments allow us to write the value of equity as

$$E(M; K_0, M_0) = (1-\tau_e)\left(\frac{K_0M}{r-\mu} - \frac{cK_0M_0}{r}\right) + e_1K_0M_0^{1-\beta_1}M^{\beta_1} + e_2K_0M_0^{1-\beta_2}M^{\beta_2}, \tag{33}$$

where we need to show that the constants d_1, d_2, e_1, and e_2 are independent of K_0 and M_0. Inserting the debt value (32) in the value matching condition for debt (5), when the debt is called, gives us:

$$\begin{aligned} &\frac{(1-\tau_i)cK_0M_0}{r} + d_1K_0M_0^{1-\beta_1}(uM_0)^{\beta_1} + d_2K_0M_0^{1-\beta_2}(uM_0)^{\beta_2} \\ &\qquad = (1+\lambda)\left(\frac{(1-\tau_i)cK_0M_0}{r} + d_1K_0M_0^{1-\beta_1}M_0^{\beta_1} + d_2K_0M_0^{1-\beta_2}M_0^{\beta_2}\right), \end{aligned} \tag{34}$$

which we can reduce to

$$(1-\tau_i)\frac{c}{r} + d_1u^{\beta_1} + d_2u^{\beta_2} = (1+\lambda)\left((1-\tau_i)\frac{c}{r} + d_1 + d_2\right). \tag{35}$$

Using both the debt value (32) and the equity value (33) in the value matching condition for equity when the debt is called, (6), gives us:

$$\begin{aligned}
&(1-\tau_e)\left(\frac{K_0uM_0}{r-\mu}-\frac{cK_0M_0}{r}\right)+e_1K_0M_0^{1-\beta_1}(uM_0)^{\beta_1}+e_2K_0M_0^{1-\beta_2}(uM_0)^{\beta_2}\\
&\qquad=(1-\tau_e)\left(\frac{\Lambda K_0uM_0}{r-\mu}-\frac{c\Lambda K_0uM_0}{r}\right)\\
&\qquad\quad+e_1\Lambda K_0(uM_0)^{1-\beta_1}(uM_0)^{\beta_1}+e_2\Lambda K_0(uM_0)^{1-\beta_2}(uM_0)^{\beta_2}\\
&\qquad\quad+(1-\tau_i)\frac{c\Lambda K_0uM_0}{r}\\
&\qquad\quad+d_1\Lambda_0K_0(uM_0)^{1-\beta_1}(uM_0)^{\beta_1}+d_2\Lambda K_0(uM_0)^{1-\beta_2}(uM_0)^{\beta_2}\\
&\qquad\quad-(1-\tau_e)i\frac{\Lambda K_0uM_0}{r-\mu}\\
&\qquad\quad-(1+\lambda)\left((1-\tau_i)\frac{cK_0M_0}{r}+d_1K_0M_0^{1-\beta_1}M_0^{\beta_1}+d_2K_0M_0^{1-\beta_2}M_0^{\beta_2}\right).
\end{aligned} \tag{36}$$

Using (9) and rewriting gives us

$$\begin{aligned}
&(1-\tau_e)\left(\frac{u}{r-\mu}-\frac{c}{r}\right)+e_1u^{\beta_1}+e_2u^{\beta_2}\\
&\qquad=(1-\tau_e)\Lambda u\left(\frac{1}{r-\mu}-\frac{c}{r}\right)+\Lambda u\left(e_1+e_2\right)-(1+\lambda)\big((1-\tau_i)\frac{c}{r}+d_1+d_2\big).
\end{aligned} \tag{37}$$

Similarly, the value matching condition for debt at default (7) becomes

$$\begin{aligned}
&\frac{(1-\tau_i)cK_0M_0}{r}+d_1K_0M_0^{1-\beta_1}(dM_0)^{\beta_1}+d_2K_0M_0^{1-\beta_2}(dM_0)^{\beta_2}\\
&\qquad=\ (1-\tau_e)\left(\frac{(1-\alpha)K_0dM_0}{r-\mu}-\frac{c(1-\alpha)K_0dM_0}{r}\right)\\
&\qquad\quad+e_1(1-\alpha)K_0(dM_0)^{1-\beta_1}(dM_0)^{\beta_1}+e_2(1-\alpha)K_0(dM_0)^{1-\beta_2}(dM_0)^{\beta_2},
\end{aligned} \tag{38}$$

yielding

$$(1-\tau_i)\frac{c}{r}+d_1d^{\beta_1}+d_2d^{\beta_2}=(1-\tau_e)(1-\alpha)d\left(\frac{1}{r-\mu}-\frac{c}{r}+e_1+e_2\right). \tag{39}$$

Finally, the value matching condition for equity at default (8) becomes

$$(1-\tau_e)\left(\frac{K_0 d M_0}{r-\mu} - \frac{cK_0 M_0}{r}\right) + e_1 K_0 M_0^{1-\beta_1}(dM_0)^{\beta_1} + e_2 K_0 M_0^{1-\beta_2}(dM_0)^{\beta_2} = 0, \qquad (40)$$

and hence

$$(1-\tau_e)\left(\frac{d}{r-\mu} - \frac{c}{r}\right) + e_1 d^{\beta_1} + e_2 d^{\beta_2} = 0. \qquad (41)$$

Inspection of the reduced value matching conditions (35)–(41) shows that the equations do not depend on the level of the assets K_0 and the level of the profitability index M_0. Whence it follows that the constants d_1, d_2, e_1, and e_2 do not depend on K_0 and M_0. Furthermore, it follows that d_1, d_2, e_1, and e_2 satisfy a four dimensional linear equation system, and hence they can be easily calculated (given c, d, and u).

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