This paper tests the effectiveness of a newly proposed systemic risk tax to be levied on systematically important banks and have identified that such tax could force the banks to build up capital holdings in recessions, while it seems opposite in booms. The systemic tax might also affect the social welfare through the equilibrium loan rate, significantly influencing the non-systemically bank which is assumed to be the loan rate taker. As for the optimal capital requirements, we have discovered the existence of a countercyclical capital requirement (at around 2.3%) that requires a higher capital in booms, which is in line with Basel III. Moreover, we have also suggested that contagion effect, bank capital supply, deposit insurance might also influence the optimal capital requirements profoundly.

1. Introduction

Banking capital requirements play a role in avoiding banks’ insolvency that might cause an externality to the rest of the economy. The recent crisis implies that the systemic risk could also impair other financial institutions by macro-prudential effects in the event of failure of some institutions that are regarded as Too-Big-To-Fail or Too-Interconnected-To-Fail. However, Basel I and Basel II Accords, regarding on capital requirements, are designed to mitigate the micro-prudential effects of financial institutions but neglect the interconnections between these institutions. The new Basel III has considered the impact of globally systemically important financial institutions (SIFIs) and aims to mitigate greater risks they might pose to the financial system. These SIFIs are, accordingly, required with higher capacity at the amount of 1% to 2.5% additional capital requirements. Basel III Accord also aims to mitigate the negative effects of cyclical effects of the banking regulation that might allow banks to hold less capital buffers in booms. Basel III increases the capital requirements for both recessions and booms, and especially adding 0-2.5% countercyclical capital buffer in booms, during which period the systemic risk might be built up (BCBS 2011).

We introduce a two-bank model that comprises one systemically important bank and one non-systemically important bank. We have established a two-period investment environment and introduced two financial situations, booms and recessions, to analyse the impact of business cycle. The banks are unable to access the equity market and the business cycle determines loans’ probabilities of default. The banks can only collect their equities from their shareholders, to satisfy the capital requirements, at the beginning of the first period and
cannot reimburse the equities during the next periods. For simplicity, we assume that at the second period the banks would only hold the capital at the exact level set up by the capital requirements to reflect the fact that there are no further periods, and thus no capital buffer is necessary in case of potential economy shocks. Our study combines mathematical methods with empirical analyses to give the empirical guidance on banking regulations. We adopt the baseline parameters from U.S. and European data prior to the global financial crisis started in 2007 to estimate the economic situation within the business cycle.

We distinguish the systemically and non-systemically important bank throughout different treatments. Firstly, the systemically important bank is assigned with larger size, or at least the same, to the non-systemically important one. Secondly, the systemically important bank could cause a potential contagion effect to the rest of the banking system (the non-systemically important bank) in the case of bankruptcy, while the non-systemically important bank might not trigger this contagion effect due to its less systemically importance. Thirdly, the depositors of the non-systemically important bank might be less confident about the government’s rescue to their investing bank and thus would require higher deposit rates to compensate their potential loss.

Our main objective in this paper is to demonstrate the (optimal) capital requirements on the systemically important bank and non-systemically important bank. We have also tested the effectiveness of a newly proposed systemic tax, proposed by Freixas & Rochet (2013) and Acharya et al. (2017), to be levied on systematically important bank to mitigate the bankruptcy costs and negative economic effects of possible reductions in loans to comply with capital requirements. The systemic tax is tested to discover its impact on the systemically important bank’s capital holdings. Moreover, the systemic tax has been analysed to identify its effects on optimal capital requirements.

Our contributions are 1) evaluating the pros and cons of the aforementioned systemic tax; 2) estimating the optimal capital requirements for the systemically important and non-systemically important banks; 3) showing banks’ responses to the optimal capital requirements, and giving suggestions on regulating different banks. To our knowledge, there are no research working on the optimal capital requirements for banks based on their systemic importance, and no studies reveal their responses to the optimal capital requirements. Additionally, although some studies, such as Freixas & Rochet (2013), give the mathematical proof to support the effectiveness of systemic tax, as far as we are aware, no one has shown the exact merits and limitations of the systemic tax using empirical analysis. From our analysis, the systemic tax could force the systemically important bank to hold more capital buffers, but it would trigger potential pro-cyclical effects by introducing more capital buffers in recessions (2.8%) than in booms (1.2%). Moreover, we have also incorporated the depositors’ impacts, which is neglected by the majority of the studies, regrading banking regulations to make our analysis more realistic and convincing.

We firstly consider the systemically important bank and non-systemically important bank to identify capital requirements on different banks and their response to the capital requirements set up by the Basel Accords. We especially focus on banks’ capital holdings and shareholders’ net worth. Among all capital requirements, Laissez-faire regime (no minimum capital required), Basel I regime, Basel II regime and Basel III regime, the Basel III regime is the harshest that makes the systemically important banks retain capital holdings at 9.3%
(9.5%) and 10.2% (8.0%) for booms and recessions with systemic tax regime (without tax regime) respectively. The systemic tax could also force the systemically important banks to retain more capital holdings when their bank size increases. This finding indicates that systemic tax could help to mitigate the Too-Big-To-Fail concerns by introducing more capital holdings for larger banks. As for our equilibrium analysis, we consider the loan rate market that is determined by the systemically important, while is taken by the non-systemically important bank due to its less systemic importance. We have discovered a fact that systemic tax could influence the loan rate and thus demonstrate a transmission mechanism to the social welfare. Accordingly, the non-systemically importance bank’s capital holdings and optimal capital requirements is changed profoundly.

We have estimated the optimal capital requirements to be imposed on different banks. Our finding suggests that not only bankruptcy costs but also bank sizes and contagion effects should be considered, re-emphasizing the limitation of one-size-fit-all requirements proposed by Basel II Accord. However, this effect would be more significant after the introduction of the systemic tax. When the systemic tax regime is implemented, the capital requirements could be softened without incentivizing the banks to reduce capital holdings in recessions; however, this impact seems less significant in booms. Moreover, the systemic tax could increase social welfare, through reducing credit rationing, by allowing lower capital requirements in recessions, while this is opposite in booms. This finding thus suggests the systemic tax seems to be effective only when the financial economy goes worse.

For the analysis of optimal capital requirements, from the perspective of social welfare maximization, we have identified a countercyclical capital requirement that in booms the requirements should be 2.3% higher than that in recessions. This result proves the validation of the new Basel III regime that regulates the banks with a countercyclical buffer ranging from 0% to 2.5% when the overall economy grows. In addition, we have identified that systemically important bank seems to need a higher capital requirement than the non-systemically important ones. This finding re-confirms the limitation of one-size-fit-all principles because systemic importance could also be a factor for capital regulation. This effect has been also considered by the Basel III Accord which an additional 1% to 2.5% capital requirement to global systemically important banks (SIBs). In addition, we have also identified that contagion effect, bank capital supply and deposit insurance might also affect banks’ capital requirements.

Other papers that have discussed optimal capital requirements are Miles et al. (2012), Repullo & Suarez (2013), Nicolo et al. (2014) and Tian et al. (2013). Miles et al. (2012) have identified that 1% increase in firm’s cost of capital could result in 0.25% decrease in output, and the firm’s cost of capital (represented by interest rates of loans) are linked with bank’s capital structure. Miles et al. (2012) reveal that optimal bank capital structure could be introduced to maximize social welfare. Repullo & Suarez (2013) consider a dynamic equilibrium model and have discovered that optimal capital requirements seems to be cyclically varying, but less cyclical for high social costs of bank failure. Nicolo et al. (2014) setup a dynamic model to analyse micro-prudential regulation. They compare three capital regimes: unregulated, capital requirement at 4% and at 12%. The social welfare is highest when capital requirement set at 4%, and this insight suggests there exists an inverted U-shaped relationship between bank capital requirements and social welfare. Tian et al. (2013) develop a theoretical framework to link the contagion effect and bailout policy into bank’s
capital regulation, and have showed that optimal capital holdings decrease with the anticipated probability of bailout, suggesting the existence of moral hazard.

To mitigate moral hazard or risk-taking behaviour of the banks’ managers (or shareholders), some researchers has proposed several suggestions. Repullo (2004) presents a dynamic model where the banks can invest in a prudent or a gambling asset. He shows that the risk-based capital requirements could be effective in controlling risk-shifting incentives by penalizing investment in riskier assets. Freixas & Rochet (2013) propose levying a systemic tax and establishing a system risk authority to lessen managers’ risk-taking behaviours. They propose the systemically important financial institutions should not be permitted to fail or downsize due to their high systemic importance. They thus prove that capital regulation might have a very limited role in protecting banks from bankruptcy, and confirm that systemic tax might help to solve managers’ excess risk taking. Dewatripont and Tirole (2012) consider a scenario under which the banks face with macroeconomic shocks, and they maintain it is suboptimal to forbear banks by allowing lower capital ratios in recession, which might lead to banks’ gambling for resurrection. They have also identified that Basel III countercyclical capital buffer or dynamic provisioning are appropriate ways to deal with the macroeconomic shocks. However, banks’ risk-taking behaviour is not the focus of our analysis, and we just regard banks’ capital holdings as a proxy for measuring risk-taking behaviour because as Schepens (2016) has revealed, shareholders might be aware that they will lose more from bank failure if they have more equities investing in the bank. Thus, we just assume more capital holdings can be interpreted as lower shareholders’ (or managers’) risk-taking incentives.

As for cyclical capital regulation, Repullo and Suarez (2013) maintain that Basel II is more cyclical than Basel I by introducing more credit rationing in recessions. However, Basel II could make the bank safer and would be superior in social welfare. Ayuso et al. (2004) study Spanish business cycle from 1986 to 2000. They reveal the pro-cyclicality of capital buffers by showing that 1% point in GDP growth is likely to reduce capital buffers by 17% and this relationship might be asymmetric during upturns. Repullo (2013) presents a model of an economy with banks that could be funded with deposits and equity capital. He considers the effect of a negative shock to the supply of bank capital and suggests that optimal capital requirements should be lowered in recessions to avoid potential deduction in aggregate investment. Behn et al. (2016) study the effect of pro-cyclical capital regulations to banks’ lending and argue that 0.5% points increase in capital charge could result in 2.1%-3.9% points decrease in loan lending, suggesting cyclical capital regulation can have sizeable effects. Gordy and Howells (2006) suggest counter-cyclical indexing to change business mix for Basel II, and similarly, Repullo and Saurina (2009) suggest through-the-cycle PDs or GDP-growth-based multiplier to mitigate the pro-cyclicality of Basel II. Fonseca and Gonzalez (2010) have identified that capital buffers are positively influenced by the costs of deposits and bank market power.

Acharya et al. (2017) suggest marginal expected shortfall (MES) and systemic expected shortfall (SES) to measure banks’ systemic risk and recommend an optimal taxation policy based on systemic importance to mitigate the negative effects to the economy due to banks’ systemic importance. Gauthier et al. (2012) define macro-prudential capital requirements under which each bank’s capital requirement equals its contribution to the risk of the system. We consider a simplified model by distinguishing systemically important banks using bank
sizes and contagion effects and we estimate the optimal capital requirements regarding their systemic importance.

The rest of this paper is organized as follows. Section 2 introduces the participants of our model, and Section 3 describes the time periods which features participants’ investment actions. We setup our model in Section 4, and the first half part of which introduces the systemically important bank and its response of capital holdings to different capital requirement regimes, with and without the consideration of systemic tax. The second half part of Section 4 introduces the non-systemically bank by analysing deposit rate premium required by its depositors. Section 5 shows the social welfare analysis and compares the optimal capital requirements under different scenarios. Section 6 shows some extensions for our model by conducting robust checks. Section 7 concludes our paper. The appendix shows the calculation of non-systemically important bank’s deposit rate premium and the procedure of obtaining its social welfare analysis for calculating optimal capital requirements.

2. Participants

2.1 Banks

In our model, we assume there are two banks: one systemically important bank and one non-systemically important bank. However, given the fact that banks with large market share are generally treated as systemically important, and for simplicity, we call them large bank and small bank respectively in the remainder of our analysis. The banks are operated by their shareholders whose required return is \( \delta \), the shareholders invest the banks with equity and finance their banks by receiving deposits from depositors. The only option for bank’s investments is loans. Without loss of generality, we assume that banks lend all the deposits and equities in the form of loans (Acharya & Yorulmazer 2007). Thus, the balance sheet of the banks can be shown as \( \text{loans} = \text{deposits} + \text{equities} \). All the banks (the large and small bank) are regulated by the government and are required to adopt the capital requirement in order to be allowed to undertake banking activity. Failure to do so will force the bank to leave the market. To distinguish large bank’s systemically importance, we assume the large bank’s failure will cause a contagion effect to the rest of the banking system (to the small bank) by incurring additional social costs, while the small bank would not cause such contagion effect to the large bank.

2.2 Entrepreneurs

We assume that entrepreneurs borrow money from the banks in order to undertake their projects. However, the projects face the danger of failure. Following Repullo & Suarez (2013), we assume each project has two outcomes: success and failure. For each period, if the project is successful, each unit investment will yield a pledge-able return \( 1 + \lambda \) to the bank; if the project fails, the bank will get \( 1 - \lambda \) where \( 0 < \lambda < 1 \). The project’s return will be realized at the end of each period. The probability of default of the project is independent across the periods, and all of the projects have identical probability of failure denoted by \( p \). In line with Repullo & Suarez (2013), we assume this probability satisfies

\[
p = E(x) = \int_0^1 x \, dF(x)
\]  

(1)
where $x \sim \{0, 1\}$ is a random variable which denotes the fraction of failed projects for each period, and $F(x)$ is the cumulative distribution function of the variable $x$. As in Repullo & Suarez (2004), we assume the variable $x$ has the following distribution:

$$F(x) = \Phi(\sqrt{1 - \rho \Phi^{-1}(x)} - \Phi^{-1}(p))$$

(2)

where $p$ is conditional on the overall economic situation. Equation (2) is set up by value-at-risk foundation to the capital requirement. The notation $\Phi(\cdot)$ is the cdf of a normal random distribution and $\rho$ is a parameter that measures the dependence of individual defaults on the common risk factor (see Repullo & Suarez 2004).

2.3 Government

The government is expected to set up the optimal capital requirements in order to maximize social welfare. The government is also responsible for supervising the banks to ensure that they abide by the capital requirements, and taking over the banks if they fail. The government will also perform as a deposit insurance agency, and thus it is responsible for paying the guaranteed amount to the depositors, under the deposit insurance. This assumption has support from Diamond & Dybvig (1983) who maintain that private insurance companies might be constrained by their limited reserves to honour a deposit guarantee. The government will also pay for the bankruptcy costs no matter which bank fails. For the large bank that is regarded as systemic important, the government will additionally levy a systemic tax $T$ to cover the expected cost of interventions (Freixas & Rochet, 2013). Additionally, the government has access to obtain the information about the banks’ actual capital holdings at any time because of its supervision power.

2.4 Depositors

The public is restricted to equity investment and only has access to deposit investment. As a result, the only option for public investment is depositing. All the depositors are risk neutral. We assume all the banks’ depositors are under partial deposit insurance that is guaranteed by the government and the insured amount is at the portion of $q$. However, the large bank’s depositors are confident that they will be very likely to reclaim all their deposit because it might trigger a potential bank run to the rest of the banking system (the small bank) if the large bank’s depositors cannot reclaim their deposits in full. On the other hand, small bank’s deposit loss might not cause a bank run to the large bank. Without loss of generality, we assume the government will help to guarantee the large bank’s depositor confidence to avoid bank runs (Diamond & Dybvig 1983). On the other hand, the government might not assist the depositors of the small bank to achieve so. Accordingly, the depositors of the small bank will require higher deposit rates compared with the large bank to compensate for potential loss. All the depositors, due to asymmetric information, can only get access to banks’ capital holding from banks’ annual report that should be released at the end of each period.

3. Time Periods

We assume there are three time points: time 0, time 1 and time 2, which make up two investment time periods. The banks and entrepreneurs are born at time 0 and aim to proceed to time 2. Like Repullo & Suarez (2013) and Nicolo et al. (2014), we also assume that for
each time period there are two possible states: booms (low business failure) and recessions (high business failure), denoted by \( l \) and \( h \) respectively. Each state has different probabilities of failure, and the corresponding probabilities are estimated from empirical data. We denote the probability of failure in booms and recessions are \( p_l \) and \( p_h \), respectively. It is straightforward to accept that \( p_l < p_h \). In order to analyse bank’s short-run behaviour, we assume that these two periods are under the same market situation. Each participant knows the states of the business environment and assumes the financial situation will be unlikely to change within these two periods.

At time 0, each bank sets up its equity holding to satisfy the capital requirements defined by the government. Then, at time 1, each bank calculates its return based on the performance of its investment, and adjusts its capital holdings based on the capital requirement. After the return is realized, the bank itself will pay a dividend to the shareholders if the realized equity exceeds its adopted capital requirements. It will reduce the loan amount if the retained equity is less than the required level, and will be liquidated if the equity is below zero and thus this bank will not be allowed to continue its banking activity into the next investment period. For simplicity, we assume that once the bank has obtained its equity at the time 0, it cannot absorb additional equity during the next periods, while the banks could adjust its deposit holdings at time 1 to make their balance sheet break even, without any adjustment costs.

4. Model Setup

For our analysis, we assume the large bank and the small bank have total deposits of \( \frac{Q}{Q+1} \) and \( \frac{1}{Q+1} \), respectively. This means the ratio of the size of large bank to that of small bank is \( Q \). The capital requirements set up by the government for one unit of the deposits (invested as loans) is \( \gamma_L \) and \( \gamma_S \) for the large and small bank, respectively. The capital requirements are set up at time 0 and time 1, and no requirements are necessary for time 2 because there are no further periods. At time 0, these two banks lend to the entrepreneurs the amount of \( \frac{Q}{Q+1} \) and \( \frac{1}{Q+1} \) respectively, and will refinance the entrepreneurs at time 1 with their full available deposits and equities if they are allowed to stay in the banking market. Next, the banks will raise equity holdings, at the level of \( k_L \) and \( k_S \) respectively, to satisfy the capital requirements. It is clear that \( k_L \geq \gamma_L \) and \( k_S \geq \gamma_S \), and they will possibly keep a capital buffer \( k_L - \gamma_L > 0 \) or \( k_S - \gamma_S > 0 \) to cope with potential shocks. For simplicity, we normalize the risk-free rates to zero. For the second period, the bank would not hold any capital buffers and adopt their capital holdings at \( k_L \) and \( k_S \) respectively. The intuition for assuming so attributes to the fact that there are no further periods proceeded and the bank might find it unprofitable to hold any excess capital to secure the deposits.

4.1 Large Bank Analysis

At time 1, the large bank obtains return \( 1 + r_m \) from the fraction of the performing loans \( 1 - x \), and \( 1 - \lambda \) from the fraction of the defaulted loans \( x \). We assume that for the first period only, each bank will incur a setup cost to absorb deposits and pay for the related inner costs for some inner costs. This cost will not be caused at the second period because the large bank will not need to absorb deposits and depositors are less likely to change bank to deposit due
to switching costs\(^1\). The setup cost is \(\mu\). Recall that the large bank’s total loan outstanding is \(\frac{Q}{Q+1}\). After paying to the deposit holders at the amount of \(1 - k_L\), the net worth of the large bank at date 1, \(k_L'(x)\), is

\[
k_L'(x) = k_L + r_m - (r_m + \lambda)x - \mu
\]

(3)

where \(x\) is the random variable representing the fraction of failed loans in the first period.

To be able to proceed to the second investment period, the large bank must hold equity at least at the ratio of \(\gamma_L\), and for simplicity, we assume the banks will adopt their capital holdings exactly at the capital requirements. Due to the dependency on the bank in the second period, entrepreneurs’ demand in loan is inelastic and thus the second-period loan rate will be \(a\), assigning all the pledge-able return to bank.

There exists three possible outcomes of the large bank’s banking activities. First, if \(k_L'(x) < 0\), the bank will be termed as bankrupt. In this case, it will be liquidated and thus is not allowed to proceed into the next investment period. Second, if \(0 \leq k_L'(x) < \gamma_L\), the bank will be unable to undertake the full investment and it is required to liquidate some of its deposit to satisfy the capital requirements. As a result, credit rationing will be introduced. Third, if \(k_L'(x) > \gamma_L\), the bank is eligible to finance the project in full and will thus pay a dividend to the shareholders at the amount of \(k_L'(x) - \gamma_L\) so that its equity holdings are exactly \(\gamma_L\) at the beginning of the next investment period.

The above three outcomes depend on the realization of the default rate \(x\). It is straightforward to show that:

1. the bank fails when \(k_L'(x) < 0\), equivalent to \(x > \bar{x}_m\), where

\[
\bar{x}_m = \frac{k_L + r_m - \mu}{\lambda + r_m}
\]

(4)

2. the bank has insufficient lending capacity when \(0 \leq k_L'(x) < \gamma_L\), equivalent to \(\bar{x}_m' \leq x < \bar{x}_m\), where

\[
\bar{x}_m' = \frac{k_L + r_m - \mu - \gamma_L}{\lambda + r_m}
\]

(5)

3. the bank has excess lending capacity when \(x < \bar{x}_m'\).

4.1.1 Taxation to mitigate the systemic risk

Levying a systemic tax \(T\) to the large bank will help to mitigate the negative effects in case of large bank’s downsize (due to credit rationing) and bankruptcy. Without loss of generality, we assume this tax is only levied for the first period, and it is paid to the government at time 0. Recall that we only regard the large bank as a systemic important institution, and thus we

\(^1\) For simplicity, we neglect the switching costs in our model but assume the depositors will find it is unprofitable to change bank in the second period.
do not consider the corresponding taxation on the small bank. Freixas & Rochet (2013) argue that tax $T$ will be used to cover the expected cost of interventions. Unlike the small bank, large bank’s failure will not only trigger a proportional bankruptcy cost $c$ times its own size, but also a potential contagion to the rest of the economy. We assume that the proportional cost due to contagion will be at the ratio of $\varphi$, thus the contagion cost is

$$\frac{c\varphi}{Q + 1}$$  

(6)

The contagion effect might attribute to the fact that: 1) the large bank’s failure will possibly make the small bank’s depositors withdraw their money from the small bank, even if the small bank itself is still functioning (Diamond & Dybvig, 1983). 2) The large bank sells protection by using derivative products like credit default swaps (CDS), but big losses might be caused in the event of crisis (Dungey & Gajurel 2015 and Freixas & Rochet 2013). To determine this cost, we follow the assumption proposed by Freixas & Rochet (2013), but, for simplicity, we neglect the continuation value, restructuring cost and some other related costs$^2$. Thus, the systemic tax because of bankruptcy is:

$$\lambda_m = \frac{c(\varphi + Q)}{Q + 1} \left[1 - F(x_m)\right]$$  

(7)

where $x_m$ is defined in Equation (4), and the multiplier $(\varphi + Q)/(Q + 1)$ denotes the bankruptcy costs of the large bank and the contagion costs (denoted by $\varphi$) to the small bank.

In addition, Freixas & Rochet (2013) also argue that the downsize, due to the insufficient lending ability, of the large bank will also trigger potential bank run, and thus this downsize will also be taxed as a result. Additionally, Repullo & Suarez (2013) assign a non-pledge-able return $b = a$ to the developed and succeed projects, the practical implication of assuming this parameter is to introduce an additional cost with credit rationing. This non-pledge-able return could attribute to the large bank’s systemically importance to the social welfare, and the overall economy would suffer more from the large bank’s malfunctioning. We adopt this assumption in order to feature the large bank’s downsize cost and assume $b = a$. Our interpretation for this assumption is the large bank’s downsize would be an act of forgoing potential production, although no bankruptcy cost is caused. Thus, the social cost of the large bank’s downsize is

$$\vartheta_m = \frac{b(\varphi + Q)}{Q + 1} \int_{x_m}^{X_m} \left[1 - \frac{k_l(x)}{\gamma_L}\right]dF(x)$$  

(8)

The integrand of Equation (8) denotes the second period’s amount of downsize, as a function of $x$, due to first period’s credit rationing as a result of failing to satisfy capital requirements. The coefficient $b(\varphi + Q)/(Q + 1)$ denotes the proportional downsize cost.

$^2$Our treatments regarding these costs deserves comments, however, the estimation of these costs is exceedingly difficult because these costs might be subject to various factors, such as bank’s capital profile and government regulation accords. However, neglecting these costs would not lose the generality.
In all, Equation (8) calculates the expected downsize cost due to credit rationing at the end of the first time period. Thus, the total systemic tax to be levied on the large bank is

\[ T_m = \lambda_m + \theta_m \]  \hspace{1cm} (9)

### 4.1.2 Large Bank’s shareholder net present value

In line with the previous description, the net present value of the shareholders of the large bank will be

\[ v_{L,m}(k_L) = \frac{1}{1 + \delta} E[v_m(x)] - k_L - T_m \]  \hspace{1cm} (10)

where

\[ v_m(x) = \begin{cases} 
\pi_m + k'_L(x) - \gamma_L & \text{if } x < \tilde{x}_m \\
\pi_m k'_L(x) \gamma_L & \text{if } \tilde{x}_m < x < \tilde{x}_m \\
0 & \text{if } x > \tilde{x}_m 
\end{cases} \]  \hspace{1cm} (11)

and

\[ \pi_m = \frac{1}{1 + \delta} \int_0^1 \max\{\gamma_L + a - x' (\lambda + a), 0\} dF(x') \]  \hspace{1cm} (12)

In Equation (10), \( \delta \) denotes the required return by the shareholders, \( x' \) is the random variable representing the realization of the fraction of the non-performing loan during the second investment period, namely from date 1 to date 2. Equation (11) calculates the expected return for the bank, discounted by the required return, minus the initial capital holdings and systemic tax paid to the government and it summarizes three outcomes based on the realization of the projects. As denoted by Equation (5), the bank will have sufficient lending to proceed to the second time period when \( x < \tilde{x}_m \), and its return is the expected income of the second time period \( \pi_m \) plus the net worth at the end of first time period \( k'_L(x) \) minus \( \gamma_L \) which will be used to satisfy the capital requirement for the second period. When \( \tilde{x}_m < x < \tilde{x}_m \), the bank will only have insufficient lending and it can merely invest a fraction of \( k'_L(x) / \gamma_L \), making its gross return at \( \pi_m k'_L(x) / \gamma_L \). However, when \( x > \tilde{x}_m \), the bank fails, and its return is zero for the second period. Equation (12) denotes the bank’s expected income in the second period if no credit rationing was made at the end of the first period. Note that we neglect the setup costs for the second period and assume the bank’s capital holdings for the second period is \( \gamma_L \).

From Equation (11) we can show that the credit rationing due to bankruptcy and bank’s downsize will be

\[ CR_{L,m} = [1 - F(\tilde{x}_m)] + \int_{\tilde{x}_m}^{\tilde{x}_m} \left[ 1 - \frac{k'_L(x)}{\gamma_L} \right] F(x) \]  \hspace{1cm} (13)
The first term of Equation (13) is the large bank’s probability of failure while the second term, similar to the interpretation in Equation (8), is the expected credit rationing due to insufficient lending. We assume that the large bank’s aim is to maximize $v_{L,m}(k_L)$.

### 4.1.3 Equilibrium

For an equilibrium, we follow the assumption of Repullo and Suarez (2013) to define a zero net worth of the shareholders due to perfect equilibrium. Because the large bank would have the systemic importance which could influence the loan rate while the small bank takes it as given, we have the following equation.

$$v_{L,m}[k_{L,m}^*, r_m^*(Q)] = 0$$

for

$$k_{L,m}^* = \arg \max v_{L,m}[k_{L,m}^*, r_m^*(Q)]$$

under the condition that $k_{L,m} > \gamma_m$.

As the income is strictly increasing with $r_m^*(Q)$, there should exist only a unique $r_m^*(Q)$ which could satisfy the above equations, and the large bank will thus seek this loan rate as the equilibrium rate.

### 4.1.4 Large Bank’s Response to Capital Requirements

#### 4.1.4.1 Baseline parameters

Table 1 describes our baseline parameters of the model.

<table>
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<th>$a$</th>
<th>$\lambda$</th>
<th>$\mu$</th>
<th>$\delta$</th>
<th>$p_l$</th>
<th>$p_h$</th>
<th>$\rho$</th>
<th>$c$</th>
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</tbody>
</table>

Following Repullo & Suarez (2013), we adopt the rate of return $a$ as 0.04, which is approximately calculated by estimating the Total Interest Income of the banks minus the Total Interest Expense and the Total Deposits Income. Parameter $\lambda = 0.45$ denotes the loss given default (LGD) that a failed project yields. This value is based on the Basel II foundation Internal Ratings-Based (IRB) approach. The value $\mu$, the setup cost, is introduced to feature the banks’ inner cost at the first investment period. The required return $\delta$ set up by the equity holders is from Van den Heuvel (2008) estimates at the value of 3.16% as the lower bound for the cost of Tier 1 capital. Others like Iacoviello (2005) estimate this value at around 4%. To keep in line with Repullo & Suarez (2013), we double the required return to consider both the Tier 1 and Tier 2 capital setup by the shareholders, and thus we adopt the value at $\delta = 0.08$. Moreover, the values of $p_l$, $p_h$ and $\rho$ are adopted from Repullo & Suarez (2013), and then we take these for our baseline analysis. Nicolo et al. (2014) gives the estimated baseline bankruptcy cost at the level of 0.104, and they view this value as a lower bound for bankruptcy costs because this estimate is based on nonfinancial sector, while Repullo (2013) set up the social cost of bank failure at 0.2. Thus, we adopt its value at 0.20, nearly double of what Nicolo et al. (2014) have viewed for the lower bound for bankruptcy cost. The value of $\varphi$ is rather difficult to estimate as very limited literature has studied the
contagion effects so far. Dungey & Gajurel (2015) have studied the contagion effects in banking during 2007-2009, and they give the estimated likelihood of a systemic crisis through contagion at about 37 percent. Petmezas & Santamaria (2014) identify the fact of contagion effect within European sovereign debt crisis during 2007-2012. Based on this study, they have figured out the correlations between stock and bond markets range from -0.047 to 0.401. Greenwood et al. (2015) study the fire sale effect when banks are facing a negative shock to their equity and give the estimation that 40.1% of aggregate bank equity will be affected due to contagion within Europe. Thus, we take the value of $\varphi$ at 0.40.

4.1.4.2 Basel regulation regimes

As addressed previously, our analysis is based on what Basel regulations define as Tier 1 capital (principally, common equity), and, without loss of generality, we neglect the Tier 2 capital (including lower loss-absorbing capacity common equity, such as convertible and subordinated debt). In order to identify the bank’s response to different regulatory regimes, we consider the following four capital regulation regimes: lasiss-z-faire regime, Basel I regime, Basel II regime and Basel III regime. Under the lasiss-z-faire regime, we set up the capital requirements $\gamma_l = \gamma_h = 0$. In the Basel I regime we set $\gamma_l = \gamma_h = 0.04$, under the Basel Accord of 1988. In the Basel II regime, using the Basel II formula, the capital requirements should be

$$\gamma_m = \frac{\lambda}{2} \frac{\Phi^{-1}(p_m) + \Phi^{-1}(0.999)\sqrt{\rho(p_m)}}{\sqrt{1-\rho(p_m)}}$$  \hspace{1cm} (14)$$

where

$$\rho(p_m) = 0.12(2 - \frac{1 - e^{-50p_m}}{1 - e^{-50}})$$  \hspace{1cm} (15)$$

Equations (14) and (15) can be supported by BCBS (2004). Note in Equation (14) and (15) $m = l, h$, denoting booms and recessions, respectively. In Equation (14), the Tier 1 capital requirements are obtained by dividing by two for the overall capital requirements of Tier 1 + Tier 2 capital (Repullo and Suarez 2013), and similar to their calculation we also get $\gamma_l = 3.2\%$ and $\gamma_h = 5.5\%$. As a revision of Basel II Accords, Basel Committee on Banking Supervision (2011) has recently reformed the capital requirements regarding countercyclical buffer, with Basel III regime. The Basel III Accord has introduced an additional conservation buffer and a countercyclical buffer as a revision for Basel II regime. The conservation buffer (in the form of common equity within Tier 1 capital) is imposed at 2.5% and the suggested range of the countercyclical buffer is 0-2.5% (in the form of common equity) (See BCBS 2011). For simplicity, we use the mean of the suggested value, namely 1.3%, to be added for the capital requirements in booms, and the conservation buffer both for booms and recessions. Thus, under Basel III regime, the capital requirements are at 7% (=3.2%+2.5%+1.3%) for booms and 8% (=5.5%+2.5%) for recessions. Thus, we can see that

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3 This assumption can find support from BCBS, 2011 and Repullo & Suarez, 2013.
under this new Basel III regime, the capital requirements are harsher and less pro-cyclical than Basel II regime.

### 4.1.4.3 Quantitative Results

We set Q at different levels to identify the effect of bank size on the bank’s capital decisions. In addition, we have also considered the systemic tax (proposed by Acharya et al., 2017 and Freixas & Rochet, 2013) that aims to mitigate the large bank’s systemic risk.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Loan rate, capital buffers, systemic tax under different regulatory regimes and different bank sizes (all variables in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank Size: Q=1/Q=5/Q=10</td>
<td>Laissez-faire</td>
</tr>
<tr>
<td>Loan rate in state m</td>
<td></td>
</tr>
<tr>
<td>$r_t$</td>
<td>0.8/0.8/0.8</td>
</tr>
<tr>
<td>$r_h$</td>
<td>3.2/3.3/3.3</td>
</tr>
<tr>
<td>Capital holdings in state m</td>
<td></td>
</tr>
<tr>
<td>$k_t$</td>
<td>6.0/6.2/6.3</td>
</tr>
<tr>
<td>Capital buffer in state m</td>
<td></td>
</tr>
<tr>
<td>$\Delta_t = k_t - \gamma_t$</td>
<td>6.0/6.2/6.3</td>
</tr>
<tr>
<td>$\Delta_h = k_h - \gamma_h$</td>
<td>8.2/8.7/8.8</td>
</tr>
<tr>
<td>Systemic tax in state m</td>
<td></td>
</tr>
<tr>
<td>$T_t$</td>
<td>0.0/0.0/0.0</td>
</tr>
<tr>
<td>$T_h$</td>
<td>0.2/0.2/0.2</td>
</tr>
<tr>
<td>Loan rate under no tax in state m</td>
<td></td>
</tr>
<tr>
<td>$r_t'$</td>
<td>0.6/0.6/0.6</td>
</tr>
<tr>
<td>$r_h'$</td>
<td>2.6/2.6/2.6</td>
</tr>
<tr>
<td>Capital buffer under no tax in state m</td>
<td></td>
</tr>
<tr>
<td>$\Delta'_t = k'_t - \gamma_t$</td>
<td>4.6/4.6/4.6</td>
</tr>
<tr>
<td>$\Delta'_h = k'_h - \gamma_h$</td>
<td>2.5/2.5/2.5</td>
</tr>
<tr>
<td>Net Capital buffer with tax in state m</td>
<td></td>
</tr>
<tr>
<td>$\alpha_t = \Delta_t - \Delta'_t$</td>
<td>1.4/1.6/1.7</td>
</tr>
<tr>
<td>$\alpha_h = \Delta_h - \Delta'_h$</td>
<td>5.7/6.2/6.3</td>
</tr>
</tbody>
</table>

### 4.1.4.4 Loan rates

The equilibrium loan rates are higher in recessions than in booms because in recessions the probability of default increases and thus a higher rate is desirable to compensate the bank shareholders. Interestingly, the loan rates $r_m$ are higher than the $r_m'$ under all regulation regimes. Recall that loan rates is the return required by the bank from the entrepreneur, and thus the higher the loan rate the less the entrepreneur’s payoff will be. Thus, the systemic tax will not only worsen bank’s pro-cyclicality, but also affect the overall economy with a less loan demand or a lower investment payoff due to an increased loan rate.

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*Notice that a 1.3% is the countercyclical buffer required by the Basel III.*
4.1.4.5 Capital buffer and Net capital buffer increase

Similar to Repullo and Suarez (2013), under non-systemic tax regime, bank will hold more capital buffers in booms than in recessions, from $\Delta'_{m}$. However, we have not discovered a capital buffer pro-cyclicality of Basel II (Compared with Basel I) as we have assumed the short-term effects of banking regulation by ruling out the economy situation changes. When under systemic tax regime, the results are opposite, that is, the capital buffers are higher in recessions than in booms (except for Basel III). This result can be checked from $\Delta_{m}$ in the third line of Table 2. Interestingly, the bank under Basel III could effectively hold a higher capital buffer (3.6%=2.3%+1.3%) in booms because of a countercyclical buffer required at 1.3%. However, under other capital regimes, the buffers are higher in recessions, causing a potential pro-cyclicality to the economy.

Moreover, the net capital buffer increase $\alpha_{i}$ in the last line suggests although the systemic tax could effectively help to make the bank increase their capital holdings, it might not be useful in booms. For the Basel I and Basel III, the net increase of capital buffer is negative in booms, which means the systemic tax might discourage the bank from holding more capital due to the increased cost of paying systemic tax to the government. Thus, the systemic tax might not help to motivate the bank to become safer, but conversely jeopardize the bank’ capital holding decision, particularly in booms.

4.1.4.6 Capital holdings

When under non-systemic tax regime, the capital holdings are 4.6% (2.5%), 6.5% (5.2%), 5.7% (6.7%) and 9.5% (8.0%) respectively for Laissez-faire, Basel I, Basel II and Basel III regimes when in booms (recessions). We can notice that except Basel II, capital holdings are all higher in booms than in recessions due to a higher profitability in the second period. For Basel II, because of a higher capital requirement in recessions, the capital holdings are higher in recessions, but will make the bank safer in recessions because of a higher capital holding. However, in booms, the bank’s capital holdings is reduced and might cause a potential damage to the bank once when in long-term that economy situation change is possible. As for Basel III, due to countercyclical buffer that is set at 1.5%, the capital holding is still higher in booms even though the capital requirement is higher in recessions. This, in turn, proves that Basel III might be more effective in preventing bankruptcy and pro-cyclicality.

When considering systemic-tax, bank’s capital holdings changes significantly. From the second line of Table 2, bank’s capital holdings are at around 6.2% (8.7%), 6.1% (9.3%), 6.9% (9.5%) and 9.3% (10.2%) respectively for Laissez-faire, Basel I, Basel II and Basel III regimes when in booms (recessions). When compared with non-systemic tax regime, the systemic tax will definitely make bank safer in recessions (even with the consideration of loan rate income); however, the cyclical effect is more significant as more capital will be induced. However, in booms, the systemic tax seems ineffective in increasing bank’s capital holdings. This result implies that systemic tax will worsen the pro-cyclical effects within the systemically important bank and might not be useful when the overall economy is good.

4.2 Small Bank Analysis
Recall that the size of the lending amount of the small bank is \( \frac{1}{Q+1} \). In order to differentiate the size effect, we assume that \( Q \geq 1 \). Similar to the large bank, the small bank sets up the equity holdings at the ratio of \( k_S \) subject to the capital requirement that \( k_S \geq \gamma_S \).

### 4.2.1 Deposit Rate Premium

Because of small bank’s depositors’ low confidence of reclaiming full deposits in case of bankruptcy, they will request a deposit rate premium to deposit in the small bank. Under deposit insurance, only fraction of \( q \) will be reclaimed, and accordingly they request the premium to cover their expected loss. Without loss of generality, we assume the deposit premium is only quoted for the first period, but for the second period, due to depositors’ dependency and switching costs, they are not able to claim this premium (see Shy et al. (2016) and Repullo & Suarez (2013) for more details). This premium is paid to the depositors at time 1 only if the small bank does not fail. To determine the deposit premium, we assume the depositors do not know the actual capital holdings of the small bank at time 0 and thus they use the only available information: capital requirements \( \gamma_S \). In order to distinguish the large bank from the small bank, we assume that the small bank’s first period loan’s random default rate is \( x_S \) that follows the same distribution as the large bank’s. The latent value of the small bank, from the perspective of the deposit, \( K'_S \) is as follows

\[
K'_S(x_S) = \left[1 + r_m(Q)\right](1 - x_S) + (1 - \lambda)x_S - (1 + r_d)(1 - \gamma_S) - \mu
\]

\[
= \gamma_S + r_m(Q) - [r_m(Q) + \lambda]x_S - r_d + r_d\gamma_S - \mu
\]

(16)

To interpret Equation (16), notice the small bank retains \( \gamma_S \) at time 0 as the depositors have assumed. It will receive the gross return of the investments from the entrepreneurs at the value of \( [1 + r_m(Q)](1 - x_S) \) and \( (1 - \lambda)x_S \) for the performing loans and non-performing loans, respectively; pay back the depositors principals and interests (because of deposit rate premium) at the value of \( (1 + r_d)(1 - \gamma_S) \); pay off the setup cost \( \mu \). Recall that due to less systemically importance, the small bank will take the first-period loan rate as given, conditional on the large bank’s size, and similar to the large bank, the second-period loan rate is \( \alpha \).

Then, we can conclude the small bank fails if \( K'_S(x_S) < 0 \), equivalent to \( x_S > X_{Sm} \), where

\[
X_{Sm} = \frac{\gamma_S + r_m(Q) - \mu - r_d}{r_m(Q) + \lambda}
\]

(17)

Note that due to the insignificant value of \( r_d\gamma_S \), we drop it for simplicity. Recall that the depositors do not know the small bank’s actual capital holdings \( k_S \) at time 0, and thus \( X_{Sm} \) is the critical value of default from the view of the depositors, not the small bank’s actual critical value.

---

5 This might because at time 0, the depositors cannot know the small bank’s capital holdings from its annual report that should be released at time 1. It will also be impossible for depositors to know this from the government at time 0 due to asymmetric information.
To determine \( r_d \), we have assumed the depositors are risk-neutral and thus they would request \( r_d \) to cover their expected loss. Thus, we can get

\[
F(X_{sm}) r_d + [1 - F(X_{sm})](q - 1) = 0
\]  

(18)

Note that, as discussed before, once the small bank fails the residual value the depositors can only be able to reclaim is the portion of \( q \) of their deposits because the government might find it costly to pay for all their deposit loss due to the small bank’s lower systemically importance. Because we have assumed that the risk-free rate is zero, the depositors would thus require the deposit rate premium \( r_d \) to make their expected income zero to make their investment break even. Accordingly, the risk-neutral depositors will be indifferent in depositing in large or small bank with this deposit rate premium. The first part of Equation (18) is the depositors’ income from deposit rate premium if the small bank does not fail, and the second part is the depositors’ (negative) income when the small bank fails. However, it is impossible to give explicit solutions of Equation (18) because \( F(Y_m) \) also contains \( r_d \). However, we can present the following proposition for \( r_d \):

**Proposition 1:** There are at most two solutions for \( r_d \), however, under some circumstances there would be one or no solution. If there are two solutions, we take the smaller one because the bank’s effort to minimize its cost. If there is no solution, we will take \( r_d = \gamma_S + a - \mu \). This value is the maximum feasible rate the small bank could offer to the depositors once \( \gamma_S \) or \( q \) is too low that the depositors are aware they are under large exposure. We give the proof in the Appendix.

### 4.2.2 Small Bank’s shareholder net present value

For the small bank’s analysis, due to it lower systemically importance, it will not be levied for systemic tax, and thus the small bank’s shareholder net present value is as follows

\[
\nu_{Sm}(k_S) = \frac{1}{1 + \delta} E[\nu_{Sm}(x_S)] - k_S
\]  

(19)

The term \( \nu_{Sm}(x_S) \) in Equation (19) can be summarized as

\[
\nu_{Sm}(x_S) = \begin{cases} 
\pi_{Sm} + k_S'(x_S) - \gamma_S & \text{if } x_S < x_{Sm}' \\
\pi_{Sm} \frac{k_S'(x_S)}{\gamma_S} & \text{if } x_{Sm}' < x_S < x_{Sm} \\
0 & \text{if } x_S \geq x_{Sm}
\end{cases}
\]  

(20)

where

\[
\pi_{Sm} = \frac{1}{1 + \delta} \int_0^1 \max\{\gamma_S + a - x_S'(\lambda + a), 0\} dF(x_S')
\]  

(21)

Note that \( x_S' \) in Equation (21) denotes the random default variable of the second investment period. The shareholder’s net value at the end of first investment period is
\[ k_S'(x_S) = k_S + r_m(Q) - [r_m(Q) + \lambda x_S - \mu - r_d] \quad (22) \]

Additionally, we can get

\[ x_{S_m} = \frac{k_S + r_m(Q) - \mu - r_d}{\lambda + r_m(Q)} \quad (23) \]

and

\[ x_{S_m} = \frac{k_S + r_m(Q) - \mu - y_S - r_d}{\lambda + r_m(Q)} \quad (24) \]

The small bank’s aim is to adjust the capital holding \( k_S \) in order to maximize \( v_{S,m}(k_S) \). The credit rationing of the small bank due to bankruptcy and downsize will be as follows

\[ CR_{S,m} = [1 - F(x_{S_m})] + \int_{x_{S_m}}^{x_{S_m}'} \left[ 1 - \frac{k_S'(x_S)}{y_S} \right] F(x_S) \quad (25) \]

5. Social Welfare Analysis

In our model, social welfare can be measured by the sum of the expected net present value gained from the investment project. In order to identify the effect of the cost of credit rationing, and as assumed in Equation (8), we assume that the large bank will obtain an additional non-pledge-able return for succeed projects. However, the small bank could not obtain this return because of its lower contribution to the whole society. Thus, the overall social welfare, \( SW_{m} \), can be written as

\[ SW_{m} = E_m + GI_m + FC_m \quad (26) \]

where

\[ E_m = \frac{Q}{Q+1} \left\{ (1 - p_m)(a - r_m(Q) + b) + (1 - CR_{L,m})(1 - p_m)b \right\} + \frac{1}{Q+1} (1 - p_m)[a - r_m(Q)] \quad (27) \]

Equation (27) shows the pledge-able and non-pledge-able return of the large bank’s succeed investments over the two investment periods and the small bank’s loan net return to the entrepreneurs, where \( m = l, h \) denoting booms and recessions. The first term of Equation (27), \( E_m \), denotes the expected return of the successful projects for the first period, while the second term, \( GI_m \), is the expected non-pledge-able return for the second period if the bank is not credit rationed at the end of first period and the third term shows the small bank’s pledgeable return if it survive at time 1. The variable \( CR_{L,m} \) is defined by Equation (13). The second term of Equation (26), \( GI_m \), can be defined as follows
It denotes the net payoff to the government and the depositors during the bankruptcy, inclusive of the positive income of the taxation of the systemic risk $T_m$ (to the government) and the payoff of the deposit rate premium (to the depositors of the small bank). Thus, the second term of the Equation (28) shows the payoff of the small bank’s depositors: deposit rate premium $r_d$ if the bank succeeds after the first period and $q - 1$ if the bank fails at time 1 and time 2 respectively. The third term is the government’s negative payoff when the large bank fails. Recall that $\bar{x}_m = (\gamma_L + a)/\lambda$ and $\bar{x}_m' = (\gamma_S + a)/\lambda$ demonstrating the critical value of the default rate above which after the second-period large bank and small bank will fail. The fourth term shows the payoff $G_{S_m}/(Q + 1)$ to the government once the small bank fails. Proposition 2 gives the detailed calculation procedure.

**Proposition 2**

After simplifying Equation (28) we get obtain the following

\[
GL_m = T_m + \frac{1}{Q + 1} \left[ r_d F(\bar{x}_m') + (q - 1) \left[ 1 - F(\bar{x}_m) + (1 - CR_{S,m}) \left( 1 - F(\bar{x}_m') \right) \right] \right]
\]

\[
+ \frac{Q}{Q + 1} \left\{ \int_{\bar{x}_m}^{1} k'_L(x) dF(x) + (1 - CR_{L,m}) \int_{\bar{x}_m'}^{1} [\gamma_L + a - x(\lambda + a)] dF(x') \right\}
\]

\[
+ \frac{1}{Q + 1} G_{S_m}
\]

(28)

Additional parameters in Equation (26)

\[
FC_m = -c \left[ \frac{Q + \varphi}{Q + 1} \left[ 1 - F(\bar{x}_m) + (1 - CR_{L,m}) \left( 1 - F(\bar{x}_m') \right) \right] \right]
\]

\[
+ \frac{1}{Q + 1} \left[ 1 - F(\bar{x}_m') + (1 - CR_{S,m}) \left( 1 - F(\bar{x}_m') \right) \right]
\]

(31)

are the negative payoff to the whole society due to banks’ failure. The parameter $c$ indicates the proportional bankruptcy cost that is determined by Table 1. Then, the last two terms of
Equation (26) shows the banks’ shareholders’ net worth after modified by the bank size. Based on the equation above, we assume that the government will set up optimal capital requirements \((\gamma^*_L, \gamma^*_S)\), respectively to the large and the small bank, which maximizes \(SW\).

### 5.1 Optimal capital requirements and social welfare

Figure 1 depicts \(SW\) as a function of bank size \(Q\) for optimal capital requirements with and without systemic tax. The determination of value \(q\) seems difficult because different countries might be able to realize different ratios of deposit insurance coverage. Karas et al. (2013) use the data from Russia’s Deposit Insurance Agency (DIA) and report roughly 92.5% of the deposits has been insured since 2008. Chakrabarty (2011) analyses the Indian banking system and has reported that around 93 percent of the deposit accounts have been covered. Kroszner (2008) reports that since 2003 the small business deposits in USA have been largely insured and only 8.7% was under deposit exposure. Thus, we then take \(q = 0.9\) in our analysis, which means only 10% of the deposits is not guaranteed by the deposit insurance the depositors.

![Figure 1](image)

**Figure 1**

**Social welfare versus bank size, with and without systemic tax regime**

In Figure 1, \(m=h\) stands for the states in recessions; \(m=l\) stands for booms. This notation is the same for the following figures.

From Figure 1 we can notice that social welfare, computed by Equation (26), is higher when systemic tax is introduced for recessions. This might be attributed to the tax’s leverage effect which enables the government to stipulate the bank with lower capital requirements and thus helps to reduce the social costs by lowering credit rationing. This is confirmed by our findings regarding credit rationing that are around 0.0189 and 0.1045 with and without system tax. However, in booms it is the opposite: the credit rationing is higher with systemic tax regime (0.0722) than without (0.0376). This is attributed to the fact that although capital requirement is reduced, the capital holdings are not kept at a high level (around 8.3%)
compared with 14.0% with no systemic tax. Accordingly, the social welfare is slightly lower in systemic tax regime due to a higher credit rationing. Another insight from Figure 1 is that the social welfare increases when \( Q \) increases, due partially to the increase in the non-pledge-able return with the increase of the market share of the systemically important bank. However, this point is not our focus, which has been explained in the footnote, while our main objective of showing Figure 1 is the systemic tax could, under some circumstances, help to improve social welfare by allowing lower capital requirements.

5.2 Optimal capital requirements for the large and small bank

5.2.1 Large bank capital requirement

Figure 2 shows the optimal capital requirements for the large bank (with and without systemic tax) as a function of bank size (\( Q \)).

![Figure 2: Optimal capital requirements versus bank size for the large bank, with and without systemic tax regime](image)

When there is no systemic tax, the bank size do not influence the optimal capital requirements. The optimal capital requirements are fixed at 9.2% and 11.5%, obtained from Equation (26), respectively for recessions and booms for the large bank. From this result, we can notice that optimal capital requirement is higher in booms than in recessions, and this is different from the result of Repullo and Suarez (2013) as we only focus on banks’ short-term behaviour by ruling out the situation changes. We have thus verified the validation of Basel

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\(^6\) The increase in welfare due to the increase in \( Q \) is not the main target of our analysis, it is just an assumption issue: we have assumed only the large bank could obtain non-pledge-able return and the SW increases when the large bank’s share grows. Thus, we does not mean that society would be better off if we had very large banks.
BCBS (2010) which proposes a countercyclical buffer ranging from 0% to 2.5%, and our result suggests 2.3% (= 11.5% − 9.2%).

Nevertheless, when the systemic tax is introduced, the optimal capital requirements change when $Q$ increases, indicating that tax might help to incorporate bank size (and related contagion effect) into capital regulation. For both in booms and recessions, the optimal capital requirement is higher when $Q = 1$ (at 5.1% and 6.6% respectively) than $Q = 5$ (at 4.6% and 5.6%) because of the fact that contagion effect seems higher when large bank’s size is relatively small, say $Q = 1$. Thus, a higher capital requirement is necessary to minimize loss to the rest of the banking system due to a higher contagion effect. However, when the bank size increases, the size effect dominates and thus the optimal capital requirement rises again, especially for booms which increases again to 8.9% when $Q = 100$. However, for recessions this effect seems insignificant, the optimal capital requirements stay at 4.6% as the capital holdings (which will be shown later) have already been set up at a high level, and no harsher requirement is needed therefore. Overall, this finding reveals that capital regulation should also consider the Too-interconnected-To-Fail factor (proxy by contagion effect). In all, this result corroborates the limitation of one-size-fit-all principle, recognized by Repullo and Suarez (2013). Our results suggest that capital requirements should be set up not only according to the financial situations, *like Basel II regime*, cyclical effects (*Basel III regime*) but also based on bankruptcy costs, proposed by Repullo & Suarez (2013), and its bank sizes and contagion effects to the other banks, from our findings.

### 5.2.2 Small bank capital requirement

Table 3 shows the optimal capital requirements of the small bank. Recall that the small bank will take the loan rate determined by the large bank and adjust their capital holdings accordingly, and we give the equilibrium loan rate to identify its effect to the small bank regulations.

When under systemic tax, the equilibrium loan rate is at 0.7% and 3.8% respectively for booms and recessions. More interestingly, the loan rate is 0.5% and 4.0% when under non-systemic tax regime. This finding thus confirms our conclusion made before that systemic tax seems more effective in recessions as the loan rate is lower (3.8% compared with 4.0%), and thus the economy will obtain a higher return as the loan rate required from the bank (the cost to the entrepreneurs) is reduced, while this is opposite in booms. Accordingly, the effectiveness of systemic tax seems insignificant in booms. When under systemic tax regime, the optimal capital requirements demonstrate a similar countercyclical effect as in the large bank, that is the capital requirement is higher (6.0%) in booms than in recessions (4.2%). The countercyclical buffer is at 1.8% (= 6.0% − 4.2%), slightly smaller than that of the large bank (at 2.3%). However, when under non-systemic tax, this countercyclical effect vanishes and the requirement in recessions is at 6.6%, higher than in booms (at 6.0%). This can be explained by the increase in the loan rates (4.0%) compared with 3.8% under systemic tax regime, which means the pledge-able return is reduced and thus a higher capital requirement is required to prevent future loss, making a countercyclical buffer effect insignificant. The optimal capital requirements regarding the small bank suggests that, due to less systemically
importance and less credit rationing cost to the economy\(^7\), the countercyclical buffers seems not optimal for the small bank.

Table 3 Small bank optimal capital requirement and loan rates taken from the large bank (all variables in %)

<table>
<thead>
<tr>
<th></th>
<th>With systemic tax</th>
<th>Without systemic tax</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(m=l)</td>
<td>(m=h)</td>
</tr>
<tr>
<td>Capital requirement (\gamma_m)</td>
<td>6.0</td>
<td>6.0</td>
</tr>
<tr>
<td>Loan rate (r_m)</td>
<td>0.7 (0.6 when (Q=1))</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>3.8</td>
<td>4.0</td>
</tr>
</tbody>
</table>

Since the bank size \(Q\) does not change the result significantly in the above table, we only report it once to avoid repetition, but we have reported the different values when appropriate.

5.3 Capital requirements versus capital holdings

5.3.1 Large bank capital holdings

We have already discussed the capital requirements set up by the government whose aim is to maximize overall social welfare. Now, we turn to the analysis of the banks’ capital holdings as a response of optimal capital requirements. Figure 3 answers this question for the large bank.

---

\(^{7}\) Recall that when calculating social welfare we do not assign a non-pledge-able return to small bank’s invested and succeeded project due to its less systemically importance.
The results of Figure 3 are based on Equation (10) and Equation (19). From Figure 3, we can notice that the capital holdings in recessions seems to be the same with and without systemic tax regime, which is around 9.3%. This proves that systemic tax is effective in recessions to make the bank hold a high capital while allowing a lower capital requirement. Moreover, the reason why the capital holdings (with systemic tax) when \( Q = 1 \) is slightly lower is that the systemic tax is little bit higher due to a higher contagion effect for this bank size, and thus discourages the bank from raising a higher capital holdings. However, this effectiveness seems insignificant in booms that once the systemic tax is introduced the capital holdings of the small bank drop dramatically from 14% to around 9.0%, failing to making the bank safer. This finding explains what we have discovered in Part 5.1 that the credit rationing is higher (0.0722) with systemic tax than without (0.0376). This is because the capital holdings is reduced with the introduce of systemic tax and thus fails to protect the bank from suffering from a credit rationing.

### 5.3.2 Small bank capital holdings

<table>
<thead>
<tr>
<th></th>
<th>( m=l )</th>
<th>( m=h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>With systemic tax</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital requirement</td>
<td>6.0</td>
<td>4.2</td>
</tr>
<tr>
<td>Capital holdings</td>
<td>6.0</td>
<td>6.7</td>
</tr>
<tr>
<td>Loan rate</td>
<td>0.7 (0.6 when ( Q = 1 ))</td>
<td>3.8</td>
</tr>
<tr>
<td>Credit rationing</td>
<td>0.15 (0.14 when ( Q = 1 ))</td>
<td>8.94</td>
</tr>
<tr>
<td>Shareholder Value</td>
<td>0.29 (0.19 when ( Q = 1 ))</td>
<td>-0.68</td>
</tr>
<tr>
<td>( r_{d,m} )</td>
<td>0.007</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Without systemic tax</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital requirement</td>
<td>6.0</td>
<td>6.6</td>
</tr>
<tr>
<td>Capital holdings</td>
<td>8.5</td>
<td>6.6</td>
</tr>
<tr>
<td>Loan rate</td>
<td>0.5</td>
<td>4.0</td>
</tr>
<tr>
<td>Credit rationing</td>
<td>0.13</td>
<td>15.06</td>
</tr>
<tr>
<td>Shareholder Value</td>
<td>0.088</td>
<td>0.15</td>
</tr>
<tr>
<td>( r_{d,m} )</td>
<td>0.007</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Since the bank size does not change the results significantly, we only report the results once but specify different value where appropriate.

For the small bank which is demonstrated in Table 4 we can show that credit rationing in recessions is reduced (from 15.06% to 8.94%) once systemic tax is introduced. However, the credit rationing is insignificant when in booms. As for the capital holdings, we can identify that, similar to the large bank, the systemic tax might be effective in maintaining a capital buffers as the capital holdings under non-systemic tax is higher in booms at 8.5% than recessions (at 6.6%). However, once the systemic tax is introduced, the capital holdings fall dramatically to 6.0% for booms, lower than that of recessions in booms (at 6.7%). As for shareholder value, systemic tax seems create a cyclical effect of the shareholder’s net worth, changes from 0.29% (in booms) to -0.68% (in recessions), while it is 0.088% and 0.15% respectively in booms and recessions without systemic tax. This can be largely due to bank’s loan rates, capital requirements and deposit rates under different tax regimes. Overall, unlike the large bank, the impacts of systemic tax to the small bank seems
ambiguous, as various factors are changed and thus hard to give a direct interpretation from the above table.

5.4 Too-Interconnected-To-Fail and optimal capital requirements

We now turn to analyse the optimal capital requirements regarding Too-Interconnected-To-Fail for the large bank. We change the baseline parameter $\varphi$ to discover the optimal capital requirements for the large bank. Figure 4 depicts the result.

![Graph showing optimal capital requirements for large bank](image)

**Figure 4**
**Too-Interconnected-To-Fail and optimal capital requirements with and without systemic tax regime**
This figure depicts the optimal capital requirements as the function of large bank’s contagion effect ($\varphi$). The bank size $Q$ equals 10.

To facilitate comparison, the bank size $Q$ is set equal to 10. The results of Figure 4 are obtained from Equation (26). The Too-Interconnected-To-Fail consideration might have a marginal impact on the optimal capital requirements for both with (without) tax regime: they range from 4.6% (9.2%) to 4.7% (9.3%) in recessions and from 5.8% (11.5%) to 6.2% (11.6%) in booms. This insight is in line with the fact that the capital requirement should be set up to incorporate the contagion effects (the Too-Interconnected-To-Fail concerns) into banking regulation.

6. Extensions

In this section, we analyse the impacts of negative shocks to the supply of bank capital and the effects of the deposit insurance to the small bank regarding their optimal capital requirements. Repullo (2013) argues in recessions the supply of bank capital would be reduced and thus the optimal capital requirements should be lowered to avoid a large reduction in aggregate investment. We now turn to evaluate the validation of this argument.

---

8 In our unreported tables, we have identified that loan rates are fixed at our baseline results when $\varphi$ changes.
Repullo (2013) considers the overall amount of the supply of the bank capital and assumes it is fixed but will be reduced in recessions. However, it seems impossible to follow this assumption totally in our model but we simplify this assumption and argue that in recessions the shareholders (the providers of the bank capital) will require higher required return to compensate for higher risks they might take. The degree of increased required return depends on the overall amount of supply of bank capital and shareholders’ risk appetites (Repullo, 2013), however, this is not the objective of our paper. We just show the results for different required return and evaluate the argument regarding the optimal capital requirements. Figure 5 gives this answer.

![Figure 5](image.png)

**Figure 5**
**Optimal capital requirements regarding shareholders’ required return**
This figure depicts the optimal capital requirements as the function of shareholders’ required return ($\delta$) which is used as a proxy for the supply of bank capital. The bank size $Q$ equals 10.

The result from Figure 5 is based on Equation (26). For the large bank, the optimal capital requirements should be lowered to cope with the increase in shareholders’ required return due to a negative shock to the supply of bank capital. This result verifies the findings from Repullo (2013) that optimal capital requirements should be lowered in order to stimulate the economy. As for the impacts of systemic tax, the requirements decrease slightly from 4.6% to 3.7%, while the requirements drop more significantly from 9.2% to 7.8% under no systemic tax regime. This insight reveals that systemic tax might help to stabilize the optimal capital requirements in recessions, and would be effective in making banks safer when the negative shock of supply of bank capital is high: when $\delta = 0.12$, the optimal capital requirement (without systemic tax) is merely at 3.7%. The result for the small bank seems ambiguous and it provides an opposite conclusion (for systemic tax): it reaches its peak at 6.5% (when $\delta = 0.09$) and the optimal capital requirements are higher than our baseline value ($\delta = 0.08$). This ambiguity might attribute to the effects of systemic tax that makes the bank raise the loan rates from 0.038 ($Q = 1$) to 0.040 ($Q = 100$), which makes a higher capital requirements necessary to limit potential loss (an increase in loan rates will reduce economy
pledge-able return). However, when without systemic tax, the optimal capital requirements for the small bank decreases with the increase of the required return as the loan rates is fixed at 0.04. Accordingly, the results from Figure 5 suggests it seems optimal to lower the capital requirements to the large bank in recessions to stimulate the economy. However, due to effects of systemic tax on the loan rates, the optimal capital requirements seems ambiguous to the small bank regarding whether lowering the requirements or not.

Additionally, in order to discover the impacts of deposit premium (this is exclusive to the small bank in our paper), we alter the guaranteed amount $q$ to analyse its effect regarding the optimal capital requirements. Recall that in our previous analysis we assume the government (the insurance agency) will be more likely to assist the large bank’s depositors to reclaim all their deposits once bankrupts to avoid potential contagion effects, even their deposits are also under partial insurance. Figure 6 gives this result.

![Figure 6](image)

**Figure 6**

*Optimal capital requirements regarding deposit insurance accords*

This figure depicts the optimal capital requirements as the function of the guaranteed portion of deposit insurance ($q$). The bank size $Q$ equals 10.

The results from Figure 6 are adopted from Equation (26). For booms, the optimal capital requirement for with and without tax regimes is both at 6.0% and we combine them into one curve for simplicity. It thus seems that, in booms, the optimal capital requirement might not be significantly influenced by the deposit insurance due to less loss once the bank fails. However, for recessions, the impact of deposit insurance changes the optimal capital requirement dramatically due largely to the potential high loss once bankrupts. The figure demonstrates an asymmetric result that optimal capital requirements increases dramatically to 7.2% (7.0%) from 4.2% (3.5%) when $q = 0.8$ when under systemic tax (non-systemic tax) regime, but increases a little to 5.0% (4.3%), compared with its corresponding lowest value 4.2% (3.5%), when the full deposit insurance is in effective (namely $q = 1$). We can also notice that, no matter with systemic tax or not, the optimal capital requirements demonstrate a U-shaped curve respects to the deposit insurance. This is because when $q =$
the depositors will face a larger loss once the small bank fails, and thus a sufficiently high capital requirement is necessary to limit the potential bankruptcy probability in order to lower the depositors’ loss. However, when the full deposit insurance is guaranteed, the government might also raise the capital requirement to limit its expected (negative) payoff to the depositors as the role of deposit insurer. This result suggests that deposit insurance regime should also be considered when making capital requirements, especially for the banks which is more sensitive to the deposit insurance for example the non-systemically important banks.

7. Concluding remarks

In this paper, we analyse the impact of a systemic tax to systemically important banks and estimate the optimal capital requirements to systemically important and non-systemically important banks. Our model enables the cyclical analysis that gives suggestions for the business cycle. We evaluate the newly proposed systemic tax to be imposed on the systemically important banks and reveal its merits in regulating the banks despite the fact that it might introduce pro-cyclical effects and less effectiveness in booms. Besides affecting the banks’ capital holdings directly, we have also shown that systemic tax might influence the social welfare through the equilibrium loan rates. In addition, we analyse the Basel Accords, including Basel III, and compare the differences between Basel Accords and optimal capital requirements. The optimal capital requirements are shown based on bank size, systemic importance and contagion effect. Moreover, our results confirm the countercyclical capital requirement, from the perspective of social welfare maximization, and thus proves the validation of a countercyclical buffer targeted by Basel III.

Overall, our results confirm that the systemic tax (proposed by Freixas & Rochet, 2013 and Acharya et al., 2017) would force systemically important banks to hold more capital in recessions, although it might not be useful in booms. Our results regarding optimal capital requirements corroborate the limitation of one-size-fit-all principle, argued by Repullo and Suarez (2013), and suggest the adoption of optimal capital requirements should also consider systemic importance (Gauthier et al., 2012). Our results thus reveal some facts that worth the consideration of the policy-makers for the banking system. The systemic tax seems to incorporate Too-Big-To-Fail and Too-Interconnected-To-Fail concerns into the capital requirements, and it will increase the social welfare in recessions by allowing a lower capital requirement without reducing the banks’ incentive to retain equities. We are delighted to see that Basel III Accord has assigned a higher capital requirement to the systemically important bank as our result suggests that systemically important banks need a higher capital requirement than the other banks. In addition, systemically and non-systemically important banks might need different treatment during recessions where the overall supply of bank capital is constrained.

We conclude with some paths to future research. Firstly, our paper merely focuses on the capital requirements, while the liquidity requirements also worth future study to investigate its impacts to the banking regulation. Secondly, some resolution policies, like prompt corrective action (PCA), and the evaluation of these policies should be conducted. In addition, optimal bailout policy carried out by the government might affect the adoption of capital requirements because the bailout policy will stabilize the economy in case of the
bankruptcy, but will cause potential risk-taking behaviours that makes banks more reliant on the bailout action. Thus, a feasible optimal bailout policy is highly recommended to improve social welfare through banking regulation.

Appendix

Proof of Proposition 1

Equation (18) shows that \( F(X_{S,m}) r_d + [1 - F(X_{S,m})](q - 1) = 0 \). After rearranging the Equation (18) we can obtain

\[
F(X_{S,m}) = \frac{1 - q}{r_d - q + 1}
\]

From Equation (2) and Equation (17), we can show that

\[
\Phi \left[ \sqrt{1 - \rho \Phi^{-1} \left( \frac{Y_s + a - \mu - r_d}{a + \lambda} \right) - \Phi^{-1}(p)} \right] = \frac{1 - q}{r_d - q + 1}
\]

Adding \( \Phi^{-1}(\cdot) \) to both sides of the above equation, we can obtain

\[
\sqrt{1 - \rho} \Phi^{-1} \left( \frac{Y_s + a - \mu - r_d}{a + \lambda} \right) = \Phi^{-1} \left( \frac{1 - q}{r_d - q + 1} \right) + \Phi^{-1}(p)
\]

Next, we assume the function \( X(r_d) \) as

\[
X(r_d) = \Phi^{-1} \left( \frac{1 - q}{r_d - q + 1} \right) + \frac{\Phi^{-1}(p)}{\sqrt{\rho}} - \frac{1 - \rho}{\sqrt{\rho}} \Phi^{-1} \left( \frac{Y_s + a - \mu - r_d}{a + \lambda} \right)
\]

Thus, our aims turn to find the solutions to \( X(r_d) = 0 \). Making differentiation to \( X(r_d) \) in terms of \( r_d \), we can show that

\[
\frac{dX(r_d)}{dr_d} = \frac{1}{a + \lambda} \frac{\sqrt{1 - \rho}}{\sqrt{\rho}} \frac{d\Phi^{-1} \left( \frac{Y_s + a - \mu - r_d}{a + \lambda} \right)}{d\left( \frac{Y_s + a - \mu - r_d}{a + \lambda} \right)} - \frac{1 - q}{(r_d - q + 1)^2} \frac{d\Phi^{-1} \left( \frac{1 - q}{r_d - q + 1} \right)}{d\left( \frac{1 - q}{r_d - q + 1} \right)}
\]

It is straightforward to show that \( \frac{d\Phi^{-1}(z)}{dz} \) is always positive because \( \Phi^{-1}(z) \) is an increasing function. Additionally, we can notice \( r_d \) can only range from 0 to \( Y_s + a - \mu \) because the definition domain of \( \Phi^{-1}(x) \) is from 0 to 1. When \( r_d \) increases from zero, \( \frac{dX(r_d)}{dr_d} \) is negative infinity as \( \frac{d\Phi^{-1}(z)}{dz} \) is positive infinity when \( z \) approaches to 1, and when \( r_d \) approaches to \( Y_s + a - \mu \), \( \frac{dX(r_d)}{dr_d} \) is positive infinity because \( \frac{d\Phi^{-1}(z)}{dz} \) is also infinity when \( z \) approaches to 0. We can conclude that when \( r_d \) changes from 0 to \( Y_s + a - \mu \), \( \frac{dX(r_d)}{dr_d} \) changes from negative infinity to positive infinity. Thus, the function \( X(r_d) \) is a U-shaped curve and it reaches its minimum level where \( \frac{dX(r_d)}{dr_d} = 0 \). It is also easy to notice that when \( r_d = 0 \) and \( r_d = Y_s + a - \mu \), \( X(r_d) \) is positive infinity. Namely, when \( r_d \) changes from 0 to \( Y_s + a - \mu \), \( X(r_d) \) starts from positive infinity; decreases to its minimum; increases back to positive infinity. Thus, for appropriate value sets, the minimum of \( X(r_d) \) can be
negative, making there are two solutions, and we choose the smaller value of \( r_d \) for deposit rate premium. However, if \( q \) or \( y_S \) is too small, making \( X(r_d) \) high above zero, there will exist no solutions to make \( X(r_d) \) zero. Under this circumstance, we will let \( r_d = y_S + a - \mu \). Because if \( q \) or \( y_S \) are too small, the depositors will find their deposits are under larger exposure and thus we assign the highest feasible deposit rate premium to the depositors

**Proof of Proposition 2**

We have assumed that when the small bank fails, the government will take over it, and repay the depositors the promised value \( e \). Thus, for the first period, the government’s payoff for taking the failed bank is

\[
GS_{m}^{\text{First}} = \int_{x_{S_{m}}}^{1} \left[ k_S + r_m(Q) - [r_m(Q) + \lambda]y - \mu + 1 - q \right] dF(x_S)
\]

Notice that in case of bankruptcy, the small bank is not responsible for paying the deposit rate premium \( r_d \), and it is dropped out. The above equation shows the negative payoff to the government for the first period. It is clear that when the bank fails but the loss is not significant, when \( y \) ranges from \( x_{S_{m}} \) to \( x_{S_{m}'} \) where \( x_{S_{m}} = (k_S + r_m(Q) - \mu + 1 - q)/[r_m(Q) + \lambda] \), the bank still has some positive revenue due to the partial deposit insurance regime. Thus, the government is exempted from the payment for a portion of \( 1 - q \). We can simplify the above equation and get the following results

\[
GS_{m}^{\text{First}} = \int_{x_{S_{m}}}^{1} \left[ k_S + r_m(Q) - [r_m(Q) + \lambda]x_S - \mu \right] dF(x_S) + (1 - q)[1 - F(x_{S_{m}})]
\]

Then, for the second period, if the small bank fails, the depositors will still only be able to receive partial payback of their deposits. Then we can get

\[
GS_{m}^{\text{Second}} = (1 - CR_{S,m}) \left[ \int_{x_{S_{m}'}^{1}} \left[ y_S + a - (a + \lambda)x_S' + 1 - q \right] dF(x_S') \right]
\]

Similar to the first period, we can simplify \( GS_{m}^{\text{Second}} \) and get the following

\[
GS_{m}^{\text{Second}} = (1 - CR_{S,m}) \left[ \int_{x_{S_{m}'}^{1}} \left[ y_S + a - (a + \lambda)x_S' \right] dF(x_S') + (1 - q)[1 - F(x_{S_{m}'}^{1})] \right]
\]

Thus, the value of \( GS_m \) should be the sum of \( GS_{m}^{\text{First}} \) and \( GS_{m}^{\text{Second}} \), then we can get

\[
GS_m = GS_{m}^{\text{First}} + GS_{m}^{\text{Second}}
\]

Thus, we can simplify Equation (28)

\[
GI_m = T_m + \frac{1}{Q + 1} \left[ r_d F(x_{S_{m}}) + (q - 1) \left[ (1 - F(x_{S_{m}}) + (1 - CR_{S,m}) (1 - F(x_{S_{m}'})) \right] \right]
\]

and get

\[
GI_m = T_m + \frac{1}{Q + 1} r_d F(x_{S_{m}}) + \frac{1}{Q + 1} \left[ \int_{x_{S_{m}}}^{1} [k_S + r_m(Q) - [r_m(Q) + \lambda]x_S - \mu] dF(x_S) \right]
\]
\[ + (1 - CR_{S,m}) \left\{ \int_{x_{S,m}'}^{1} [y_S + a - (a + \lambda)x_S]dF(x_S') \right\} \]
\[ + \frac{Q}{Q + 1} \left\{ \int_{x_m}^{1} k'_S(x)dF(x) + (1 - CR_{L,m}) \int_{x_{m}'}^{1} [y_L + a - x(\lambda + a)]dF(x') \right\} \]

If we replace \( k_S + r_m(Q) - [r_m(Q) + \lambda]x_S - \mu \) with \( \tilde{k}'_S(x_S) \), we can obtain Equation (29).

**References**


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