Systemic Importance and Optimal Capital Regulation

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This paper tests the effectiveness of a newly proposed systemic risk tax to be levied on systematically important banks and highlights that such tax could force the banks to build up capital holdings and help to regulate the banks with Too-Big-to-Fail and Too-Interconnected-to-Fail consideration. However, this tax might cause pro-cyclical effects by introducing more capital increase in recessions (3.1%) than in booms (1.4%). As for the optimal capital requirements, systemically important banks seem to need higher but more cyclical requirements than the non-systemically ones. As responses to optimal capital requirements, non-systemically important banks are prone to hold much less capital than the systemically important ones in recessions.

1. Introduction

Banking capital requirements play a role in avoiding banks’ insolvency that might cause an externality to the rest of the economy. The recent crisis implies that the systemic risk could also impair other financial institutions by macro-prudential effects in the event of failure of some institutions that are regarded as Too-Big-To-Fail or Too-Interconnected-To-Fail. However, Basel I and Basel II Accords, regarding on capital requirements, are designed to mitigate the micro-prudential effects of financial institutions but neglect the interconnections between these institutions. The new Basel III has considered the impact of global systemically important financial institutions (SIFIs) and aims to mitigate greater risks they might pose to the financial system. These SIFIs are, accordingly, required with higher capacity at the amount of 1% to 2.5% additional capital requirements. Basel III Accord also aims to mitigate the negative effects of cyclical effects of the banking regulation that might allow banks to hold less capital buffers in booms. Basel III increases the capital requirements for both recessions and booms, and especially adding 0-2.5% countercyclical capital buffer in booms, during which period the systemic risk might be built up (BCBS 2011).

We introduce a two-bank model that comprises one systemically important bank and one non-systemically important bank. We have established a two-period investment environment and introduced two financial situations, booms and recessions, to analyse the impact of

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business cycle. The banks are unable to access the equity market and the business cycle determines loans’ probabilities of default. The banks can only collect their equities from their shareholders, to satisfy the capital requirements, at the beginning of the first period and cannot reimburse the equities during the next periods. For simplicity, we assume that at the second period the banks would only hold the capital at the exact level set up by the capital requirements to reflect the fact that there are no further periods, and thus no capital buffer is necessary in case of potential economy shocks. Our study combines mathematical methods with empirical analyses to give the empirical guidance on banking regulations. We adopt the baseline parameters from U.S. and European data prior to the global financial crisis started in 2007 to estimate the economic situation within the business cycle.

We distinguish the systemically and non-systemically important bank throughout different treatments. Firstly, the systemically important bank is assigned with larger size, or at least the same, to the non-systemically important one. Secondly, the systemically important bank could cause a potential contagion effect to the rest of the banking system (the non-systemically important bank) in the case of bankruptcy, while the non-systemically important bank might not trigger this contagion effect due to its less systemically importance. Thirdly, the depositors of the non-systemically important bank might be less confident about the government’s rescue to their investing bank and thus would require higher deposit rates to compensate their potential loss.

Our main objective in this paper is to demonstrate the (optimal) capital requirements on the systemically important bank and non-systemically important bank. We have also tested the effectiveness of a newly proposed systemic tax, proposed by Freixas & Rochet (2013) and Acharya et al. (2017), to be levied on systemically important bank to mitigate the bankruptcy costs and negative economic effects of possible reductions in loans to comply with capital requirements. The systemic tax is tested to discover its impact on the systemically important bank’s capital holdings. Moreover, the systemic tax has been analysed to identify its effects on optimal capital requirements.

Our contributions are 1) evaluating the pros and cons of the aforementioned systemic tax; 2) estimating the optimal capital requirements for the systemically important and non-systemically important banks; 3) showing banks’ responses to the optimal capital requirements, and giving suggestions on regulating different banks. To our knowledge, there are no research working on the optimal capital requirements for banks based on their systemic importance, and no studies reveal their responses to the optimal capital requirements. Additionally, although some studies, such as Freixas & Rochet (2013), give the mathematical proof to support the effectiveness of systemic tax, as far as we are aware, no one has shown the exact merits and limitations of the systemic tax using empirical analysis. From our analysis, the systemic tax could force the systemically important bank to hold more capital buffers, but it would trigger potential pro-cyclical effects by introducing more capital buffers in recessions (3.1%) than in booms (1.4%). Moreover, we have also incorporated the depositors’ impacts, which is neglected by the majority of the studies, regrading banking regulations to make our analysis more realistic and convincing.

We firstly consider the systemically important bank and non-systemically important bank to identify capital requirements on different banks and their response to the capital requirements set up by the Basel Accords. We especially focus on banks’ capital holdings and
shareholders’ net worth. Among all capital requirements, Laissez-faire regime (no minimum capital required), Basel I regime, Basel II regime and Basel III regime, the Basel III regime is the harshest that makes the systemically important banks retain capital holdings at 7.0% (7.0%) and 11.9% (9.5%) for booms and recessions with systemic tax regime (without tax regime) respectively. In addition, the Basel III regime helps to mitigate banks’ cyclical effects to 4.9% from 5.5% (Basel II regime), but this mitigation is at the expense of banks’ shareholder welfare. However, the systemic tax could also force the systemically important banks to retain more capital holdings when their bank size increases. This finding indicates that systemic tax could help to mitigate the Too-Big-To-Fail concerns by introducing more capital holdings for larger banks.

We have estimated the optimal capital requirements to be imposed on different banks. Our finding suggests that not only bankruptcy costs but also bank sizes and contagion effects should be considered, re-emphasizing the limitation of one-size-fit-all requirements proposed by Basel II Accord. However, this effect would be more significant after the introduction of the systemic tax. When the systemic tax regime is implemented, the capital requirements could be softened without incentivizing the banks to reduce capital holdings. Moreover, the systemic tax could reduce banks’ cost of holding equities by allowing lower capital requirements, and thus improve social welfare.

Our model is for short-run analysis that assumes the economy situation would not change. Our results confirm the pro-cyclicality that makes the banks prone to retain less capital holdings in booms, posing potential threats to the whole economy once the financial situations become worse, and the capital buffers retained might not be sufficient to rescue the banks. We have identified that under optimal capital requirements the systemically important banks might need more cyclically varying capital requirements (8.7% and 2.1% for recessions and booms) than the non-systemically important ones with 6.6% (1.7%) for recessions (booms). This finding re-confirms the limitation of one-size-fit-all principles because systemic importance could also be a factor for capital regulation. This effect has been considered by the Basel III Accord at which an additional 1% to 2.5% capital requirements are imposed to global systemically important banks (SIBs), thus verifying the validation of Basel III.

Our analysis, unlike other mathematical models (Dewatripont and Tirole 2012, Freixas and Rochet 2013, Repullo 2013), focuses on the banks’ actual capital holdings, not just on the capital requirements. Our results confirm that capital requirements cannot represent the banks’ capital holdings. The banks might hold higher capital even if the capital requirements are relatively low. For example, when under systemic tax regime although the optimal capital requirements are at 5.4% for the systemically important banks in recessions, they might hold the capital at around 10.3%, higher than that of 10.2% when the capital requirements was set at 8.7% under non-systemic tax regime. Thus, this insight proves that capital requirements might not be effective proxies for banks’ actual capital holdings, cyclical behaviours of capital adjustments, and thus the probabilities of default.

Other papers that have discussed optimal capital requirements are Miles et al. (2012), Repullo & Suarez (2013), Nicolo et al. (2014) and Tian et al. (2013). Miles et al. (2012) have identified that 1% increase in firm’s cost of capital could result in 0.25% decrease in output, and the firm’s cost of capital (represented by interest rates of loans) are linked with
bank’s capital structure. Miles et al. (2012) reveal that optimal bank capital structure could be introduced to maximize social welfare. Repullo & Suarez (2013) consider a dynamic equilibrium model and have discovered that optimal capital requirements seems to be cyclically varying, but less cyclical for high social costs of bank failure. Nicolo et al. (2014) setup a dynamic model to analyse micro-prudential regulation. They compare three capital regimes: unregulated, capital requirement at 4% and at 12%. The social welfare is highest when capital requirement set at 4%, and this insight suggests there exists an inverted U-shaped relationship between bank capital requirements and social welfare. Tian et al. (2013) develop a theoretical framework to link the contagion effect and bailout policy into bank’s capital regulation, and have showed that optimal capital holdings decrease with the anticipated probability of bailout, suggesting the existence of moral hazard.

To mitigate moral hazard or risk-taking behaviour of the banks’ managers (or shareholders), some researchers has proposed several suggestions. Repullo (2004) presents a dynamic model where the banks can invest in a prudent or a gambling asset. He shows that the risk-based capital requirements could be effective in controlling risk-shifting incentives by penalizing investment in riskier assets. Freixas & Rochet (2013) propose levying a systemic tax and establishing a system risk authority to lessen managers’ risk-taking behaviours. They propose the systemically important financial institutions should not be permitted to fail or downsize due to their high systemic importance. They thus prove that capital regulation might have a very limited role in protecting banks from bankruptcy, and confirm that systemic tax might help to solve managers’ excess risk taking. Dewatripont and Tirole (2012) consider a scenario under which the banks face with macroeconomic shocks, and they maintain it is suboptimal to forbear banks by allowing lower capital ratios in recession, which might lead to banks’ gambling for resurrection. They have also identified that Basel III countercyclical capital buffer or dynamic provisioning are appropriate ways to deal with the macroeconomic shocks. However, banks’ risk-taking behaviour is not the focus of our analysis, and we just regard banks’ capital holdings as a proxy for measuring risk-taking behaviour because as Schepens (2016) has revealed, shareholders might be aware that they will lose more from bank failure if they have more equities investing in the bank. Thus, we just assume more capital holdings can be interpreted as lower shareholders’ (or managers’) risk-taking incentives.

As for cyclical capital regulation, Repullo and Suarez (2013) maintain that Basel II is more cyclical than Basel I by introducing more credit rationing in recessions. However, Basel II could make the bank safer and would be superior in social welfare. Ayuso et al. (2004) study Spanish business cycle from 1986 to 2000. They reveal the pro-cyclicality of capital buffers by showing that 1% point in GDP growth is likely to reduce capital buffers by 17% and this relationship might be asymmetric during upturns. Repullo (2013) presents a model of an economy with banks that could be funded with deposits and equity capital. He considers the effect of a negative shock to the supply of bank capital and suggests that optimal capital requirements should be lowered in recessions to avoid potential deduction in aggregate investment. Behn et al. (2016) study the effect of pro-cyclical capital regulations to banks’ lending and argue that 0.5% points increase in capital charge could result in 2.1%-3.9% points decrease in loan lending, suggesting cyclical capital regulation can have sizeable effects. Gordy and Howells (2006) suggest counter-cyclical indexing to change business mix for Basel II, and similarly, Repullo and Saurina (2009) suggest through-the-cycle PDs or GDP-growth-based multiplier to mitigate the pro-cyclicality of Basel II.
Acharya et al. (2017) suggest *marginal expected shortfall* (MES) and *systemic expected shortfall* (SES) to measure banks’ systemic risk and recommend an optimal taxation policy based on systemic importance to mitigate the negative effects to the economy due to banks’ systemic importance. Gauthier et al. (2012) define macro-prudential capital requirements under which each bank’s capital requirement equals its contribution to the risk of the system. We consider a simplified model by distinguishing systemically important banks using bank sizes and contagion effects and we estimate the optimal capital requirements regarding their systemic importance.

The rest of this paper is organized as follows. Section 2 introduces the participants of our model, and Section 3 describes the time periods which features participants’ investment actions. We setup our model in Section 4, and the first half part of which introduces the systemically important bank and its response of capital holdings to different capital requirement regimes, with and without the consideration of systemic tax. The second half part of Section 4 introduces the non-systemically bank by analysing deposit rate premium required by its depositors. Section 5 shows the social welfare analysis and compares the optimal capital requirements under different scenarios. Section 6 shows some extensions for our model by conducting robust checks. Section 7 concludes our paper. The appendix shows the calculation of non-systemically important bank’s deposit rate premium and the procedure of obtaining its social welfare analysis for calculating optimal capital requirements and gives some additional results for systemically important banks.

2. Participants

2.1 Banks

In our model, we assume there are two banks: one systemically important bank and one non-systemically important bank. However, given the fact that banks with large market share are generally treated as systemically important, and for simplicity, we call them large bank and small bank respectively in the remainder of our analysis. The banks are operated by their shareholders whose required return is $\delta$, the shareholders invest the banks with equity and finance their banks by receiving deposits from depositors. The only option for bank’s investments is loans. Without loss of generality, we assume that banks lend all the deposits and equities in the form of loans (Acharya & Yorulmazer 2007). Thus, the balance sheet of the banks can be shown as $loans = deposits + Equities$. All the banks (the large and small bank) are regulated by the government and are required to adopt the capital requirement in order to be allowed to undertake banking activity. Failure to do so will force the bank to leave the market. To distinguish large bank’s systemically importance, we assume the large bank’s failure will cause a contagion effect to the rest of the banking system (to the small bank) by incurring additional social costs, while the small bank would not cause such contagion effect to the large bank.

2.2 Entrepreneurs

We assume that entrepreneurs borrow money from the banks in order to undertake their projects. However, the projects face the danger of failure. Following Repullo & Suarez (2013), we assume each project has two outcomes: success and failure. For each period, if the project is successful, each unit investment will yield a pledge-able return $1 + \alpha$ to the bank; if the project fails, the bank will get $1 - \lambda$ where $0 < \lambda < 1$. The project’s return will be...
realized at the end of each period. The probability of default of the project is independent across the periods, and all of the projects have identical probability of failure denoted by $p$. In line with Repullo & Suarez (2013), we assume this probability satisfies

$$p = E(x) = \int_0^1 x \, dF(x)$$

(1)

where $x \sim [0,1]$ is a random variable which denotes the fraction of failed projects for each period, and $F(x)$ is the cumulative distribution function of the variable $x$. As in Repullo & Suarez (2004), we assume the variable $x$ has the following distribution:

$$F(x) = \Phi\left(\sqrt{1 - \rho \Phi^{-1}(x) - \Phi^{-1}(p)}\right)$$

(2)

where $p$ is conditional on the overall economic situation. Equation (2) is set up by value-at-risk foundation to the capital requirement. The notation $\Phi(\cdot)$ is the cdf of a normal random distribution and $\rho$ is a parameter that measures the dependence of individual defaults on the common risk factor (see Repullo & Suarez 2004).

2.3 Government

The government is expected to set up the optimal capital requirements in order to maximize social welfare. The government is also responsible for supervising the banks to ensure that they abide by the capital requirements, and taking over the banks if they fail. The government will also perform as a deposit insurance agency, and thus it is responsible for paying the guaranteed amount to the depositors, under the deposit insurance. This assumption has support from Diamond & Dybvig (1983) who maintain that private insurance companies might be constrained by their limited reserves to honour a deposit guarantee. The government will also pay for the bankruptcy costs no matter which bank fails. For the large bank that is regarded as systemic important, the government will additionally levy a systemic tax $T$ to cover the expected cost of interventions (Freixas & Rochet, 2013). Additionally, the government has access to obtain the information about the banks’ actual capital holdings at any time because of its supervision power.

2.4 Depositors

The public is restricted to equity investment and only has access to deposit investment. As a result, the only option for public investment is depositing. All the depositors are risk neutral. We assume all the banks’ depositors are under partial deposit insurance that is guaranteed by the government and the insured amount is at the portion of $q$. However, the large bank’s depositors are confident that they will be very likely to reclaim all their deposit because it might trigger a potential bank run to the rest of the banking system (the small bank) if the large bank’s depositors cannot reclaim their deposits in full. On the other hand, small bank’s deposit loss might not cause a bank run to the large bank. Without loss of generality, we assume the government will help to guarantee the large bank’s depositor confidence to avoid bank runs (Diamond & Dybvig 1983). On the other hand, the government might not assist the depositors of the small bank to achieve so. Accordingly, the depositors of the small bank will
require higher deposit rates compared with the large bank to compensate for potential loss. All the depositors, due to asymmetric information, can only get access to banks’ capital holding from banks’ annual report that should be released at the end of each period.

3. Time Periods

We assume there are three time points: time 0, time 1 and time 2, which make up two investment time periods. The banks and entrepreneurs are born at time 0 and aim to proceed to time 2. Like Repullo & Suarez (2013) and Nicolo et al. (2014), we also assume that for each time period there are two possible states: booms (low business failure) and recessions (high business failure), denoted by \( l \) and \( h \) respectively. Each state has different probabilities of failure, and the corresponding probabilities are estimated from empirical data. We denote the probability of failure in booms and recessions are \( p_l \) and \( p_h \), respectively. It is straightforward to accept that \( p_l < p_h \). In order to analyse bank’s short-run behaviour, we assume that these two periods are under the same market situation. Each participant knows the states of the business environment and assumes the financial situation will be unlikely to change within these two periods.

At time 0, each bank sets up its equity holding to satisfy the capital requirements defined by the government. Then, at time 1, each bank calculates its return based on the performance of its investment, and adjusts its capital holdings based on the capital requirement. After the return is realized, the bank itself will pay a dividend to the shareholders if the realized equity exceeds its adopted capital requirements. It will reduce the loan amount if the retained equity is less than the required level, and will be liquidated if the equity is below zero and thus this bank will not be allowed to continue its banking activity into the next investment period. For simplicity, we assume that once the bank has obtained its equity at the time 0, it cannot absorb additional equity during the next periods, while the banks could adjust its deposit holdings at time 1 to make their balance sheet break even, without any adjustment costs.

4. Model Setup

For our analysis, we assume the large bank and the small bank have total deposits of \( \frac{Q}{Q+1} \) and \( \frac{1}{Q+1} \) respectively. This means the ratio of the size of large bank to that of small bank is \( Q \). The capital requirements set up by the government for one unit of the deposits (invested as loans) is \( \gamma_L \) and \( \gamma_S \) for the large and small bank, respectively. The capital requirements are set up at time 0 and time 1, and no requirements are necessary for time 2 because there are no further periods. At time 0, these two banks lend to the entrepreneurs the amount of \( \frac{Q}{Q+1} \) and \( \frac{1}{Q+1} \) respectively, and will refinance the entrepreneurs at time 1 with their full available deposits and equities if they are allowed to stay in the banking market. Next, the banks will raise equity holdings, at the level of \( k_L \) and \( k_S \) respectively, to satisfy the capital requirements. It is clear that \( k_L \geq \gamma_L \) and \( k_S \geq \gamma_S \), and they will possibly keep a capital buffer \( k_L - \gamma_L > 0 \) or \( k_S - \gamma_S > 0 \) to cope with potential shocks. For simplicity, we normalize the risk-free rates to zero. For the second period, the bank would not hold any capital buffers and adopt their capital holdings at \( k_L \) and \( k_S \) respectively. The intuition for assuming so attributes to the fact that there are no further periods proceeded and the bank might find it unprofitable to hold any excess capital to secure the deposits.
4.1 Large Bank Analysis

At time 1, the large bank obtains return $1 + \alpha$ from the fraction of the performing loans $1 - x$, and $1 - \lambda$ from the fraction of the defaulted loans $x$. We assume that for the first period only, each bank will incur a setup cost to absorb deposits and pay for the related inner costs for some inner costs. This cost will not be caused at the second period because the large bank will not need to absorb deposits and depositors are less likely to change bank to deposit due to switching costs. The setup cost is $\mu$. Recall that the large bank’s total loan outstanding is $\frac{Q}{Q+1}$. After paying to the deposit holders at the amount of $1 - k_L$, the net worth of the large bank at date 1, $k'_L(x)$, is

$$
\begin{align*}
  k'_L(x) &= k_L + \alpha - (\alpha + \lambda)x - \mu \\
\end{align*}
$$

(3)

where $x$ is the random variable representing the fraction of failed loans in the first period.

To be able to proceed to the second investment period, the large bank must hold equity at least at the ratio of $\gamma_L$, and for simplicity, we assume the banks will adopt their capital holdings exactly at the capital requirements. At time 1, there might exist three possible outcomes of the large bank’s banking activities. First, if $k'_L(x) < 0$, the bank will be termed as bankrupt. In this case, it will be liquidated and thus is not allowed to proceed into the next investment period. Second, if $0 \leq k'_L(x) < \gamma_L$, the bank will be unable to undertake the full investment and it is required to liquidate some of its deposit to satisfy the capital requirements. As a result, credit rationing will be introduced. Third, if $k'_L(x) > \gamma_L$, the bank is eligible to finance the project in full and will thus pay a dividend to the shareholders at the amount of $k'_L(x) - \gamma_L$ so that its equity holdings are exactly $\gamma_L$ at the beginning of the next investment period. Next, the banks will adjust their deposit amounts to make their balance sheet satisfy $\text{Loans} = \text{Deposits} + \text{Equities}$.

The above three outcomes depend on the realization of the default rate $x$. It is straightforward to show that:

1. the bank fails when $k'_L(x) < 0$, equivalent to $x > \bar{x}_m$, where

$$
\begin{align*}
  \bar{x}_m &= \frac{k_L + \alpha - \mu}{\lambda + \alpha} \\
\end{align*}
$$

(4)

2. the bank has insufficient lending capacity when $0 \leq k'_L(x) < \gamma_L$, equivalent to $\bar{x}_m^r \leq x < \bar{x}_m$, where

$$
\begin{align*}
  \bar{x}_m^r &= \frac{k_L + \alpha - \mu - \gamma_L}{\lambda + \alpha} \\
\end{align*}
$$

(5)

3. the bank has excess lending capacity when $x < \bar{x}_m^r$.

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2 For simplicity, we neglect the switching costs in our model but assume the depositors will find it is unprofitable to change bank in the second period.
4.1.1 Taxation to mitigate the systemic risk

Levying a systemic tax $T$ to the large bank will help to mitigate the negative effects in case of large bank’s downsize (due to credit rationing) and bankruptcy. Without loss of generality, we assume this tax is only levied for the first period, and it is paid to the government at time 0. Recall that we only regard the large bank as a systemic important institution, and thus we do not consider the corresponding taxation on the small bank. Freixas & Rochet (2013) argue that tax $T$ will be used to cover the expected cost of interventions. Unlike the small bank, large bank’s failure will not only trigger a proportional bankruptcy cost $c$ times its own size, but also a potential contagion to the rest of the economy. We assume that the proportional cost due to contagion will be at the ratio of $\phi$, thus the contagion cost is

$$\frac{c\phi}{Q + 1}$$

The contagion effect might attribute to the fact that: 1) the large bank’s failure will possibly make the small bank’s depositors withdraw their money from the small bank, even if the small bank itself is still functioning (Diamond & Dybvig, 1983). 2) The large bank sells protection by using derivative products like credit default swaps (CDS), but big losses might be caused in the event of crisis (Dungey & Gajurel 2015 and Freixas & Rochet 2013). To determine this cost, we follow the assumption proposed by Freixas & Rochet (2013), but, for simplicity, we neglect the continuation value, restructuring cost and some other related costs. Thus, the systemic tax because of bankruptcy is:

$$\lambda_m = \frac{c(\phi + Q)}{Q + 1} [1 - F(x_m)]$$

(7)

where $x_m$ is defined in Equation (4), and the multiplier $(\phi + Q)/(Q + 1)$ denotes the bankruptcy costs of the large bank and the contagion costs (denoted by $\phi$) to the small bank.

In addition, Freixas & Rochet (2013) also argue that the downsize, due to the insufficient lending ability, of the large bank will also trigger potential bank run, and thus this downsize will also be taxed as a result. Additionally, Repullo & Suarez (2013) assign a non-pledgeable return $b = a$ to the developed and succeed projects, the practical implication of assuming this parameter is to introduce an additional cost with credit rationing. This non-pledgeable return could attribute to the large bank’s systemically importance to the social welfare, and the overall economy would suffer more from the large bank’s malfunctioning. We adopt this assumption in order to feature the large bank’s downsize cost and assume $b = a$. Our interpretation for this assumption is the large bank’s downsize would be an act of forgoing potential production, although no bankruptcy cost is caused. Thus, the social cost of the large bank’s downsize is

$$\theta_m = \frac{b(\phi + Q)}{Q + 1} \int_{x_m}^{x_m} \left[1 - \frac{k_L(x)}{\gamma_L} \right] dF(x)$$

3 Our treatments regarding these costs deserves comments, however, the estimation of these costs is exceedingly difficult because these costs might be subject to various factors, such as bank’s capital profile and government regulation accords. However, neglecting these costs would not lose the generality.
The integrand of Equation (8) denotes the second period’s amount of downsize, as a function of $x$, due to first period’s credit rationing as a result of failing to satisfy capital requirements. The coefficient $b(\varphi + Q)/(Q + 1)$ denotes the proportional downsize cost. In all, Equation (8) calculates the expected downsize cost due to credit rationing at the end of the first time period. Thus, the total systemic tax to be levied on the large bank is

$$T_m = \lambda_m + \vartheta_m$$

(9)

4.1.2 Large Bank’s shareholder net present value

In line with the previous description, the net present value of the shareholders of the large bank will be

$$v_{L,m}(k_L) = \frac{1}{1 + \delta} E[v_m(x)] - k_L - T_m$$

(10)

where

$$v_m(x) = \begin{cases} 
\pi_m + k_L'(x) - \gamma_L & \text{if } x < \tilde{x}_m \\
\pi_m - \frac{k_L'(x)}{\gamma_L} & \text{if } \tilde{x}_m < x < \tilde{x}_m \\
0 & \text{if } x > \tilde{x}_m 
\end{cases}$$

(11)

and

$$\pi_m = \frac{1}{1 + \delta} \int_0^1 \max\{\gamma_L + a - x'(\lambda + a), 0\} dF(x')$$

(12)

In Equation (10), $\delta$ denotes the required return by the shareholders, $x'$ is the random variable representing the realization of the fraction of the non-performing loan during the second investment period, namely from date 1 to date 2. Equation (11) calculates the expected return for the bank, discounted by the required return, minus the initial capital holdings and systemic tax paid to the government and it summarizes three outcomes based on the realization of the projects. As denoted by Equation (5), the bank will have sufficient lending to proceed to the second time period when $x < \tilde{x}_m$, and its return is the expected income of the second time period $\pi_m$ plus the net worth at the end of first time period $k_L'(x)$ minus $\gamma_L$ which will be used to satisfy the capital requirement for the second period. When $\tilde{x}_m < x < \tilde{x}_m$, the bank will only have insufficient lending and it can merely invest a fraction of $k_L'(x)/\gamma_L$, making its gross return at $\pi_m k_L'(x)/\gamma_L$. However, when $x > \tilde{x}_m$, the bank fails, and its return is zero for the second period. Equation (12) denotes the bank’s expected income in the second period if no credit rationing was made at the end of the first period. Note that we neglect the setup costs for the second period and assume the bank’s capital holdings for the second period is $\gamma_L$. 

10
From Equation (11) we can show that the credit rationing due to bankruptcy and bank’s downsize will be

\[ CR_{L,m} = \left[ 1 - F(x_m) \right] + \int_{x_m}^{\infty} \left[ 1 - \frac{k_L(x)}{\gamma_L} \right] F(x) \] (13)

The first term of Equation (13) is the large bank’s probability of failure while the second term, similar to the interpretation in Equation (8), is the expected credit rationing due to insufficient lending. We assume that the large bank’s aim is to maximize \( v_{L,m}(k_L) \).

### 4.1.3 Large Bank’s Response to Capital Requirements

#### 4.1.3.1 Baseline parameters

Table 1 describes our baseline parameters of the model.

<table>
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<th>Parameter</th>
<th>( a )</th>
<th>( \lambda )</th>
<th>( \mu )</th>
<th>( \delta )</th>
<th>( p_t )</th>
<th>( p_h )</th>
<th>( \rho )</th>
<th>( c )</th>
<th>( \varphi )</th>
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<td>0.04</td>
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</tbody>
</table>

Following Repullo & Suarez (2013), we adopt the rate of return \( a \) as 0.04, which is approximately calculated by estimating the Total Interest Income of the banks minus the Total Interest Expense and the Total Deposits Income. Parameter \( \lambda = 0.45 \) denotes the loss given default (LGD) that a failed project yields. This value is based on the Basel II foundation Internal Ratings-Based (IRB) approach. The value \( \mu \), the setup cost, is introduced to feature the banks’ inner cost at the first investment period. The required return \( \delta \) set up by the equity holders is from Van den Heuvel (2008) estimates at the value of 3.16% as the lower bound for the cost of Tier 1 capital. Others like Iacoviello (2005) estimate this value at around 4%. Based on these estimations, we setup \( \delta = 0.04 \). Differently from Repullo & Suarez (2013), we only consider Tier 1 capital for our analysis and thus will not double the required return. Moreover, the values of \( p_t \), \( p_h \) and \( \rho \) are adopted from Repullo & Suarez (2013), and then we take these for our baseline analysis. The bankruptcy cost is adapted from Nicolo et al. (2014) who gives the baseline bankruptcy cost at the level of 0.104, and for approximation, we set it at 0.10. The value of \( \varphi \) is rather difficult to estimate as very limited literature has studied the contagion effects so far. Dungey & Gajurel (2015) have studied the contagion effects in banking during 2007-2009, and they give the estimated likelihood of a systemic crisis through contagion at about 37 percent. Petmezas & Santamaria (2014) identify the fact of contagion effect within European sovereign debt crisis during 2007-2012. Based on this study, they have figured out the correlations between stock and bond markets range from -0.047 to 0.401. Greenwood et al. (2015) study the fire sale effect when banks are

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4 Repullo & Suarez (2013) adopt the required return at the value of 0.08 because they have considered the Tier 2 capital and have assumed the Tier 2 capital has the same size as Tier 1 capital. However, this assumption might not be acceptable for Basel III as it requires more Tier 1 capital than Tier 2 capital (BCBS 2011) and thus it is not valid to assume equal size of Tier 1 and Tier 2 to analyse the Basel III. Moreover, when using \( \delta = 0.08 \), the bank might hold slightly lower capital holdings due to increased cost of holding capital, but the result is fundamentally the same as the situation when \( \delta = 0.04 \). The result is shown in the Appendix.
facing a negative shock to their equity and give the estimation that 40.1% of aggregate bank equity will be affected due to contagion within Europe. Thus, we take the value of \( \varphi \) at 0.40.

### 4.1.3.2 Basel regulation regimes

As addressed previously, our analysis is based on what Basel regulations define as Tier 1 capital (principally, common equity), and, without loss of generality, we neglect the Tier 2 capital (including lower loss-absorbing capacity common equity, such as convertible and subordinated debt)s. In order to identify the bank’s response to different regulatory regimes, we consider the following four capital regulation regimes: lasissz-faire regime, Basel I regime, Basel II regime and Basel III regime. Under the lasissz-faire regime, we set the capital requirements \( \gamma_l = \gamma_h = 0 \). In the Basel I regime we set \( \gamma_l = \gamma_h = 0.04 \), under the Basel Accord of 1988. In the Basel II regime, using the Basel II formula, the capital requirements should be

\[
\gamma_m = \frac{\lambda}{2} \Phi\left(\Phi^{-1}(p_m) + \Phi^{-1}(0.999)\sqrt{\rho(p_m)}\right) / \sqrt{1 - \rho(p_m)}
\]

(14)

where

\[
\rho(p_m) = 0.12\left(2 - \frac{1 - e^{-50p_m}}{1 - e^{-50}}\right)
\]

(15)

Equations (14) and (15) can be supported by BCBS (2004). Note in Equation (14) and (15) \( m = l, h \), denoting booms and recessions, respectively. In Equation (14), the Tier 1 capital requirements are obtained by dividing by two for the overall capital requirements of Tier 1 + Tier 2 capital (Repullo and Suarez 2013), and similar to their calculation we also get \( \gamma_l = 3.2\% \) and \( \gamma_h = 5.5\% \). As a revision of Basel II Accords, Basel Committee on Banking Supervision (2011) has recently reformed the capital requirements regarding countercyclical buffer, with Basel III regime. The Basel III Accord has introduced an additional conservation buffer and a countercyclical buffer as a revision for Basel II regime. The conservation buffer (in the form of common equity within Tier 1 capital) is imposed at 2.5% and the suggested range of the countercyclical buffer is 0-2.5% (in the form of common equity) (See BCBS 2011). For simplicity, we use the mean of the suggested value, namely 1.3%, to be added for the capital requirements in booms, and the conservation buffer both for booms and recessions. Thus, under Basel III regime, the capital requirements are at 7% (=3.2\%+2.5\%+1.3\%) for booms and 8% (=5.5\%+2.5\%) for recessions. Thus, we can see that under this new Basel III regime, the capital requirements are harsher and less pro-cyclical than Basel II regime.

### 4.1.3.3 Quantitative Results

We set Q at different levels to identify the effect of bank size on the bank’s capital decisions. In addition, we have also considered the systemic tax (proposed by Acharya et al., 2017 and Freixas & Rochet, 2013) that aims to mitigate the large bank’s systemic risk.

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5 This assumption can find support from BCBS, 2011 and Repullo & Suarez, 2013.
Table 2
Capital buffers, systemic tax and bank’s net income under different regulatory regimes and different bank sizes (all variables in %)

<table>
<thead>
<tr>
<th>Bank Size: Q=1/Q=5/Q=10</th>
<th>Laissez-faire</th>
<th>Basel I</th>
<th>Basel II</th>
<th>Basel III</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Capital Holdings in state m</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_l$</td>
<td>3.1/3.3/3.3</td>
<td>5.5/5.5/5.5</td>
<td>4.7/4.7/4.7</td>
<td>7.0/7.0/7.0</td>
</tr>
<tr>
<td>$k_h$</td>
<td>7.5/8.0/8.1</td>
<td>9.0/9.5/9.6</td>
<td>9.7/10.2/10.3</td>
<td>11.9/11.9/11.9</td>
</tr>
<tr>
<td><strong>Capital buffer in state m</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta_l = k_l - \gamma_l$</td>
<td>3.1/3.3/3.3</td>
<td>1.5/1.5/1.5</td>
<td>1.5/1.5/1.5</td>
<td>0.0/0.0/0.0</td>
</tr>
<tr>
<td>$\Delta_h = k_h - \gamma_h$</td>
<td>7.5/8.0/8.1</td>
<td>5.0/5.5/5.6</td>
<td>4.2/4.7/4.8</td>
<td>3.9/3.9/3.9</td>
</tr>
<tr>
<td><strong>Systemic tax in state m</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_l$</td>
<td>0.0/0.0/0.0</td>
<td>0.0/0.0/0.0</td>
<td>0.0/0.0/0.0</td>
<td>0.0/0.0/0.0</td>
</tr>
<tr>
<td>$T_h$</td>
<td>0.1/0.1/0.1</td>
<td>0.1/0.1/0.1</td>
<td>0.1/0.1/0.1</td>
<td>0.1/0.1/0.1</td>
</tr>
<tr>
<td><strong>Capital buffer minus tax in state m</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_l = \Delta_l - T_l$</td>
<td>3.1/3.3/3.3</td>
<td>1.5/1.5/1.5</td>
<td>1.5/1.5/1.5</td>
<td>0.0/0.0/-0.1</td>
</tr>
<tr>
<td>$\beta_h = \Delta_h - T_h$</td>
<td>7.4/7.9/8.0</td>
<td>4.9/5.4/5.5</td>
<td>4.1/4.6/4.7</td>
<td>3.8/3.8/3.8</td>
</tr>
<tr>
<td><strong>Capital buffers under no tax in state m</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta'_l = k'_l - \gamma_l$</td>
<td>1.7/1.7/1.7</td>
<td>0.0/0.0/0.0</td>
<td>0.0/0.0/0.0</td>
<td>0.0/0.0/0.0</td>
</tr>
<tr>
<td>$\Delta'_h = k'_h - \gamma_h$</td>
<td>2.0/2.0/2.0</td>
<td>1.5/1.5/1.5</td>
<td>1.5/1.5/1.5</td>
<td>1.5/1.5/1.5</td>
</tr>
<tr>
<td><strong>Net Capital buffers with tax in state m</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_l = \beta_l - \Delta'_l$</td>
<td>1.3/1.6/1.6</td>
<td>1.4/1.4/1.4</td>
<td>1.4/1.4/1.4</td>
<td>0.0/0.0/-0.1</td>
</tr>
<tr>
<td>$\alpha_h = \beta_h - \Delta'_h$</td>
<td>5.4/5.9/6.0</td>
<td>3.4/3.9/4.0</td>
<td>2.6/3.1/3.2</td>
<td>2.3/2.3/2.3</td>
</tr>
<tr>
<td><strong>Bank’s net income in state m with tax</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v_{L,l}$</td>
<td>3.6/3.6/3.6</td>
<td>3.4/3.4/3.4</td>
<td>3.4/3.4/3.4</td>
<td>3.2/3.2/3.2</td>
</tr>
<tr>
<td>$v_{L,h}$</td>
<td>1.2/1.2/1.2</td>
<td>0.8/0.8/0.8</td>
<td>0.7/0.7/0.7</td>
<td>0.6/0.6/0.6</td>
</tr>
<tr>
<td><strong>Bank’s net income in state m without tax</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v'_{L,l}$</td>
<td>3.7/3.7/3.7</td>
<td>3.4/3.4/3.4</td>
<td>3.5/3.5/3.5</td>
<td>3.3/3.3/3.3</td>
</tr>
<tr>
<td>$v'_{L,h}$</td>
<td>1.4/1.4/1.4</td>
<td>1.0/1.0/1.0</td>
<td>0.9/0.9/0.9</td>
<td>0.7/0.7/0.7</td>
</tr>
</tbody>
</table>

4.1.3.4 Bank Size Effect

Under Systemic tax regime, and from Table 2, we can identify that bank size might play a role in influencing the bank’s capital holdings, especially in recessions. When in recessions, except for Basel III, bank will be more likely to hold more capital holdings when bank size increases from 1 to 10. For Basel III, its requirement is too harsh and thus the large bank would find it unprofitable to increase its capital holdings. After dropping $T_m$ in Equation (10), we can obtain that when under no tax regime bank’s capital holdings are fixed at around 2.0% (Laissez-faire regime), 5.5% (Basel I regime), 7.0% (Basel II regime) and 9.5% (Basel III regime). This finding confirms that systemic tax could help to force larger bank to hold more capital due to too-large-to-fail.

4.1.3.5 Capital Requirement Regimes

Unlike Repullo & Suarez (2013) who find that banks might hold more capital buffers in booms our results demonstrate the opposite. This can be explained by the different
treatments of the investment periods which Repullo & Suarez (2013) assumes the second period’s economic situation will change from the first’s, while our analysis assumes they are the same\(^6\). The intuition for assuming so is to identify the short-run behaviour of the bank’s capital holdings. In addition, Repullo & Suarez (2013) demonstrate the results that the loan rates in boom is only at 1.3%, which is nearly one third of the rate in the recession (3.3%). This fact is also likely to force the large bank to increase its capital holdings in booms due to reduced revenues. However, our results are line with Ayuso et al. (2004) who identify a reduction in capital buffers when the economy is experiencing booms.

The *Basel III regime* was the harshest regulation between other three regimes. The large bank would be likely to hold the highest capital holdings, regardless in the boom or recession. We can see that Basel III significantly increase capital holdings in booms to 7% (from 4.7% under Basel II), while 1.7% increase in recessions, confirming Basel III’s aim to add countercyclical buffers in booms. As for cyclical capital regulation, Basel II might be more cyclical than Basel I for its softened requirements in booms. However, *Basel III regime* could help to mitigate the cyclical effects, compared with *Basel II regime*, to around 4.9% (\(=11.9%-7\%\)), from 5.5% (\(=10.2%-4.7\%\)).

### 4.1.3.6 Systemic Tax

Systemic taxes are slightly higher in recessions due to higher probabilities of default. To identify the effectiveness of systemic tax, we denote \(\alpha_m = \beta_m - \Delta'_m\) as the net capital buffers increase because of systemic tax. \(\beta_m\) is the bank’s capital buffer after deduction of systemic tax, and \(\Delta'_m\) is the bank’s capital buffer without systemic tax regime. We can identify, from the sixth column of Table 2, the systemic tax will help to increase capital holdings, although the increase effect might be less significant for *Basel III regime* due to its harsh requirements. For the value of \(\alpha_m\)\(^7\), we can see that under *Laissez-faire regime*, around 1.6% and 5.9% increase in capital buffers will be introduced by the tax during booms and recessions, respectively. Under *Basel I regime*, the net capital buffer increase is 1.4% and 3.9%, and this figure is 1.4% and 3.1% under *Basel II regime*. For Basel III, the net increase in recessions is around 2.3%, while no increase in booms. However, we can notice \(\alpha_m\) is higher than \(T_m\), which means the systemic tax could perform as a leverage, and a small amount of tax could introduce higher increase in capital buffers. We then call it the tax’s leverage effect, and this effect tends to be asymmetric and more significant in booms. However, this leverage effect might be insignificant under Basel III, especially when in booms, due to Basel III’s harsh treatments on the capital regulation, making the bank unprofitable to hold more capital. This confirms us with that systemic tax might be effective when the regulations is relatively soften, but it might have limited implication when *Basel III regime* is fully implemented. Moreover, this tax might yield pro-cyclical effects by introducing more capital buffers, which can be verified by the fact that \(\alpha_m\) is higher in recessions under all circumstances.

### 4.1.3.7 Bank’s shareholder net worth

\(^6\) When considering situation changes, as Repullo & Suarez (2013), the bank might retain more capital holdings in booms in case of encountering recessions in the second period. However, we ignore this assumption not only for simplicity but also for emphasising the bank’s short-run reactions to banking regulation.

\(^7\) For the ease of comparison, we adopt the average value of \(\alpha_m\) as \(Q = 5\).
As for the net income, we can show that the bank’s net income will be reduced after the tax is levied, although this decrease is not significant, only at around 0.2%. Then we add the income of the bank and the tax income of the government to compare the net income of the whole economy with and without the systemic tax. We thus add $T_m$, tax income of the government, and $v_{L,m}$, bank’s net income, to compare with the income $v'_{L,m}$ under the scenario without tax. We can confirm that the tax could not be welfare-increasing as under all scenarios $T_m + v_{L,m}$ are nearly the same as $v'_{L,m}$, which means the tax could perform as the function of transfer payment from the bank to the government. More importantly, this tax could make the bank safer: net increase in capital buffer shown by $\alpha_t$.

4.2 Small Bank Analysis

Recall that the size of the lending amount of the small bank is $\frac{1}{Q+1}$. In order to differentiate the size effect, we assume that $Q \geq 1$. Similar to the large bank, the small bank sets up the equity holdings at the ratio of $k_S$ subject to the capital requirement that $k_S \geq \gamma_S$.

4.2.1 Deposit Rate Premium

Because of small bank’s depositors’ low confidence of reclaiming full deposits in case of bankruptcy, they will request a deposit rate premium to deposit in the small bank. Under deposit insurance, only fraction of $q$ will be reclaimed, and accordingly they request the premium to cover their expected loss. Without loss of generality, we assume the deposit premium is only quoted for the first period, but for the second period, due to depositors’ dependency and switching costs, they are not able to claim this premium (see Shy et al. (2016) and Repullo & Suarez (2013) for more details). This premium is paid to the depositors at time 1 only if the small bank does not fail. To determine the deposit premium, we assume the depositors do not know the actual capital holdings of the small bank at time 0 and thus they use the only available information: capital requirements $\gamma_S^8$. In order to distinguish the large bank from the small bank, we assume that the small bank’s first period loan’s random default rate is $x_S$ that follows the same distribution as the large bank’s. The latent value of the small bank, from the perspective of the deposit, $K_S'$ is as follows

$$K_S'(x_S) = (1 + a)(1 - x_S) + (1 - \lambda)x_S - (1 + r_d)(1 - \gamma_S) - \mu$$

$$= \gamma_S + a - (a + \lambda)x_S - r_d + r_d\gamma_S - \mu$$

(16)

To interpret Equation (16), notice the small bank retains $\gamma_S$ at time 0 as the depositors have assumed. It will receive the gross return of the investments from the entrepreneurs at the value of $(1 + a)(1 - x_S)$ and $(1 - \lambda)x_S$ for the performing loans and non-performing loans, respectively; pay back the depositors principals and interests (because of deposit rate premium) at the value of $(1 + r_d)(1 - \gamma_S)$; pay off the setup cost $\mu$.

Then, we can conclude the small bank fails if $K_S'(x_S) < 0$, equivalent to $x_S > \bar{X}_{S,m}$, where

---

8 This might because at time 0, the depositors cannot know the small bank’s capital holdings from its annual report that should be released at time 1. It will also be impossible for depositors to know this from the government at time 0 due to asymmetric information.
\[
\tilde{X}_{Sm} = \frac{\gamma_S + a - \mu - r_d}{a + \lambda}
\]  

(17)

Note that due to the insignificant value of \(r_d\gamma_S\), we drop it for simplicity. Recall that the depositors do not know the small bank’s actual capital holdings \(k_S\) at time 0, and thus \(\tilde{X}_{Sm}\) is the critical value of default from the view of the depositors, not the small bank’s actual critical value.

To determine \(r_d\), we have assumed the depositors are risk-neutral and thus they would request \(r_d\) to cover their expected loss. Thus, we can get

\[
F(\tilde{X}_{Sm}) r_d + [1 - F(\tilde{X}_{Sm})](q - 1) = 0
\]  

(18)

Note that, as discussed before, once the small bank fails the residual value the depositors can only be able to reclaim is the portion of \(q\) of their deposits because the government might find it costly to pay for all their deposit loss due to the small bank’s lower systemically importance. Because we have assumed that the risk-free rate is zero, the depositors would thus require the deposit rate premium \(r_d\) to make their expected income zero to make their investment break even. Thus, the first part of Equation (18) is the depositors’ income from deposit rate premium if the small bank does not fail, and the second part is the depositors’ (negative) income when the small bank fails. However, it is impossible to give explicit solutions of Equation (18) because \(\tilde{Y}_m\) also contains \(r_d\).

However, we can present the following proposition for \(r_d\):

**Proposition 1:** There are at most two solutions for \(r_d\), however, under some circumstances there would be one or no solution. If there are two solutions, we take the smaller one because the bank’s effort to minimize its cost. If there is no solution, we will take \(r_d = \gamma_S + a - \mu\). This value is the maximum feasible rate the small bank could offer to the depositors once \(\gamma_S\) or \(q\) is too low that the depositors are aware they are under large exposure. We give the proof in the Appendix.

### 4.2.2 Small Bank’s shareholder net present value

For the small bank’s analysis, due to it lower systemically importance, it will not be levied for systemic tax, and thus the small bank’s shareholder net present value is as follows

\[
v_{S,m}(k_S) = \frac{1}{1 + \delta} E[v_{Sm}(x_S)] - k_S
\]  

(19)

The term \(v_{Sm}(x_S)\) in Equation (19) can be summarized as

\[
v_{Sm}(x_S) = \begin{cases} 
\pi_{Sm} + k_{Sm}'(x_S) - \gamma_S & \text{if } x_S < \tilde{x}_{Sm}^r \\
\pi_{Sm} \frac{k_{Sm}'(x_S)}{\gamma_S} & \text{if } \tilde{x}_{Sm}^r < x_S < \tilde{x}_{Sm}^- \\
0 & \text{if } x_S > \tilde{x}_{Sm}^-
\end{cases}
\]  

(20)
where

\[ \pi_{Sm} = \frac{1}{1 + \delta} \int_0^1 \max\{y_S + a - x_s'(\lambda + a), 0\} \, dF(x_s') \]

(21)

Note that \(x_s'\) in Equation (21) denotes the random default variable of the second investment period. The shareholder’s net value at the end of first investment period is

\[ k'_S(x_S) = k_S + a - (a + \lambda)x_S - \mu - r_d \]

(22)

Additionally, we can get

\[ x_{S_m} = \frac{k_S + a - \mu - r_d}{\lambda + a} \]

(23)

and

\[ x_{S_m} = \frac{k_S + a - \mu - \gamma_S - r_d}{\lambda + a} \]

(24)

The small bank’s aim is to adjust the capital holding \(k_S\) in order to maximize \(v_{S,m}(k_S)\). The credit rationing of the small bank due to bankruptcy and downsize will be as follows

\[ CR_{S,m} = [1 - F(x_{S_m})] + \int_{x_{S_m}}^{x_{S_m}'} \left[ 1 - \frac{k'_S(x_S)}{y_S} \right] F(x_S) \]

(25)

5. Social Welfare Analysis

In our model, social welfare can be measured by the sum of the expected net present value gained from the investment project. In order to identify the effect of the cost of credit rationing, and as assumed in Equation (8), we assume that the large bank will obtain an additional non-pledge-able return for succeed projects. However, the small bank could not obtain this return because of its lower contribution to the whole society. Thus, the overall social welfare, \(SW_m\), can be written as

\[ SW_m = E_m + Gl_m + FC_m + \frac{Q}{Q + 1}v_{L,m}(k_L) + \frac{1}{Q + 1}v_{S,m}(k_S) \]

(26)

where

\[ E_m = \frac{Q}{Q + 1}[(1 - p_m)b + (1 - CR_{L,m})(1 - p_m)b] = \frac{Qb}{Q + 1}(2 - CR_{L,m})(1 - p_m) \]

(27)

Equation (27) shows the non-pledge-able return of the large bank’s succeed investments over the two investment periods, where \(m = l, h\) denoting booms and recessions. The first term of Equation (27), \(E_m\), denotes the expected return of the successful projects for the first period,
while the second term, $G_{I_{m}}$, is the expected non-pledge-able return for the second period if the bank is not credit rationed at the end of first period. The variable $CR_{L_{m}}$ is defined by Equation (13). The second term of Equation (26), $G_{I_{m}}$, can be defined as follows

$$G_{I_{m}} = T_{m} + \frac{1}{Q + 1} \left\{ r_{d} F(x_{S_{m}}) + (q - 1) \left[ (1 - F(x_{S_{m}})) + \left( 1 - CR_{S,m} \right) \left( 1 - F(x_{S_{m}'}) \right) \right] \right\}$$

$$+ \frac{Q}{Q + 1} \int_{x_{S_{m}}}^{1} \left\{ k'_{L}(x)dF(x) + \left( 1 - CR_{L,m} \right) \int_{x_{S_{m}'}}^{1} [y_{L} + a - x(\lambda + a)]dF(x') \right\}$$

$$+ GS_{m} \right\} \right\}$$

(28)

It denotes the net payoff to the government and the depositors during the bankruptcy, inclusive of the positive income of the taxation of the systemic risk $T_{m}$ (to the government) and the payoff of the deposit rate premium (to the depositors of the small bank). Thus, the second term of the Equation (28) shows the payoff of the small bank’s depositors: deposit rate premium $r_{d}$ if the bank succeeds after the first period and $q - 1$ if the bank fails at time 1 and time 2 respectively. The third term is the government’s negative payoff when the large bank fails. Recall that $x_{S_{m}} = (\gamma_{L} + a)/(\lambda + a)$ and $x_{S_{m}'} = (\gamma_{S} + a)/(\lambda + a)$ demonstrating the critical value of the default rate above which after the second-period large bank and small bank will fail. The fourth term shows the payoff $GS_{m}$ to the government once the small bank fails. Proposition 2 gives the detailed calculation procedure.

**Proposition 2**

*After simplifying Equation (28) we get obtain the following*

$$G_{I_{m}} = T_{m} + \frac{1}{Q + 1} \left\{ r_{d} F(x_{S_{m}}) \right\}$$

$$+ \frac{1}{Q + 1} \left\{ \int_{x_{S_{m}}}^{1} k'_{S}(xS) dF(xS) \right\}$$

$$+ \left( 1 - CR_{S,m} \right) \int_{x_{S_{m}'}}^{1} [y_{S} + a - xS'(\lambda + a)]dF(xS') \right\}$$

$$+ \frac{Q}{Q + 1} \left\{ \int_{x_{S_{m}}}^{1} k'_{L}(x)dF(x) + \left( 1 - CR_{L,m} \right) \int_{x_{S_{m}'}}^{1} [y_{L} + a - x'(\lambda + a)]dF(x') \right\}$$

(29)

*where*

$$k'_{S}(xS) = k_{S} + a - (a + \lambda)xS - \mu$$

(30)

Additionally, in Equation (26)

$$FC_{m} = -(c + d) \left\{ \frac{Q + \varphi}{Q + 1} \left[ 1 - F(x_{S_{m}}) + \left( 1 - CR_{L,m} \right) \left[ 1 - F(x_{S_{m}'}) \right] \right] \right\}$$

$$+ \left( 1 - CR_{S,m} \right) \int_{x_{S_{m}'}}^{1} [y_{S} + a - xS'(\lambda + a)]dF(xS') \right\}$$

(31)
are the negative payoff to the whole society due to banks’ failure. The parameter $c + d$ indicates bank’s failure not only causes a direct social welfare loss, denoted by $c$, but also triggers a loss for the future due to reduced production activities, denoted by $d$. For simplicity, we assume that $d = c = 0.10$, determined by Table 1. Then, the last two terms of Equation (26) shows the banks’ shareholders’ net worth after modified by the bank size. Based on the equation above, we assume that the government will set up optimal capital requirements $(\gamma^*_L, \gamma^*_S)$, respectively to the large and the small bank, which maximizes $SW$.

5.1 Optimal capital requirements and social welfare

Figure 1 depicts $SW$ as a function of bank size $Q$ for optimal capital requirements with and without systemic tax. The determination of value $q$ seems difficult because different countries might be able to realize different ratios of deposit insurance coverage. Karas et al. (2013) use the data from Russia’s Deposit Insurance Agency (DIA) and report roughly 92.5% of the deposits has been insured since 2008. Chakrabarty (2011) analyses the Indian banking system and has reported that around 93 percent of the deposit accounts have been covered. Kroszner (2008) reports that since 2003 the small business deposits in USA have been largely insured and only 8.7% was under deposit exposure. Thus, we then take $q = 0.9$ in our analysis, which means only 10% of the deposits is not guaranteed by the deposit insurance the depositors.

Figure 1
Social welfare versus bank size, with and without systemic tax regime

In Figure 1, $m=h$ stands for the states in recessions; $m=l$ stands for booms. This notation is the same for the following figures.

From Figure 1 we can notice that social welfare, computed by Equation (26), might be higher after the implantation of systemic tax regime, both in booms and recessions. This effect might be more significant in recessions. This might be attributed to the tax’s leverage effect which enables the government to stipulate the bank with lower capital requirements reducing bank’s
cost of holding equities. Another insight from Figure 1 is that the social welfare increases when $Q$ increases, due partially to the increase in the non-pledge-able return with the increase of the market share of the systemically important bank.\(^9\) However, this point is not our focus, which has been explained in footnote, while our main objective of showing Figure 1 is the systemic tax could help to improve social welfare by allowing lower capital requirements.

### 5.2 Optimal capital requirements for the large and small bank

Figure 2 shows the optimal capital requirements for the large bank (with and without systemic tax) and the small bank, as a function of bank size ($Q$).

![Figure 2](image)

**Figure 2**

Optimal capital requirements versus bank size for the large bank, with and without systemic tax regime, and for the small bank

When there is no systemic tax, the bank size do not influence the optimal capital requirements. The optimal capital requirements are fixed at 8.7% and 2.1%, obtained from Equation (26), respectively for recessions and booms for the large bank. Optimal capital requirements might be more pro-cyclical than *Basel III regime*. This result suggests that optimal capital requirements might not consider the cyclical effect of the banking behaviour. Although the capital requirement is slightly higher than *Basel III regime* (8.0%) for recessions, the softened requirement in booms ignores potential financial crisis.

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\(^9\) The increase in welfare due to the increase in $Q$ is not the main target of our analysis, it is just an assumption issue: we have assumed only the large bank could obtain non-pledge-able return and the SW increases when the large bank’s share grows. Thus, we does not mean that society would be better off if we had very large banks.
Nevertheless, when the systemic tax is introduced, the optimal capital requirements change when $Q$ increases, indicating that tax might help to incorporate bank size (and related contagion effect) into capital regulation. After the introduction of the systemic tax, the capital requirements are reduced, especially when $Q$ is large (larger than 5). We can notice that when $Q$ equals to 1, the capital requirements might be higher, at the ratio of 7.3% and 1.3% respectively for recessions and booms. When the bank size $Q$ increases, the capital requirements decrease as response, and the rationale behind this might be that when the systemically important bank’s market share is relatively low (when $Q=1$), the contagion effect domains, and thus higher requirements are needed for mitigating contagion effect. Recall that we have assumed that large bank will trigger a proportional contagion cost at $\phi$ to the rest of the banking system (the small bank), and the contagion effect might be more significant if the small bank’s share is comparatively high (the same as the large bank when $Q = 1$). However, when the large bank’s share becomes higher, the contagion effect might be less significant, although the large bank’s share is higher. Thus, this finding reveals that capital regulation should also consider the Too-interconnected-To-Fail factor (proxy by contagion effect). In all, this result corroborates the limitation of one-size-fit-all principle, recognized by Repullo and Suarez (2013). Our results confirms and maintains that capital requirements should be set up not only according to the financial situations, like Basel II regime, cyclical effects (Basel III regime) but also based on bankruptcy costs, proposed by Repullo & Suarez (2013), and its bank sizes and contagion effects to the other banks, from our findings.

We can also notice that under tax regime the capital requirements are lower than that without tax, confirming that the tax could force the bank to hold more capital and thus the capital requirements could be lowered. Additionally, when without tax, the optimal capital requirements for recessions are at 8.7%, while under the tax regime, the capital requirements for recessions are still slightly higher than 5.5% (Basel II regime). This finding reemphasises it is not optimal to reduce the capital requirements in recessions merely to stimulate the economy where the overall probability of default of investment is higher, and would thus increase bank’s incentive of gambling in investing projects, making the effects of recession even worse (Dewatripont and Tirole, 2012). Thus, this also suggests the stricter requirements proposed by Basel III regime might be an appropriate choice for regulation in recessions. However, when in booms, the optimal capital requirements are all below 3.0%, which is even lower than Basel I and Basel II regimes. This might ignore potential situation changes into recessions, suggesting it might be suboptimal to discuss optimal capital requirements when in booms.

We only compare the small bank’s optimal capital requirements with large bank without systemic tax to avoid the impact of systemic tax. We can notice that the small bank’s capital requirements are all lower than that of the large bank, both in recessions and booms, showing the large bank’s systemically importance. However, the small bank’s capital requirements (from 1.7% to 6.6%) are less cyclical than the large bank (from 2.1% to 8.7%). However, the small bank’s capital requirements in recessions are still higher than that quoted in Basel II, reinforcing the appropriateness of Basel III. In addition, their requirement differences are 0.4% (=2.1%-1.7%) and 2.1% (=8.7%-6.6%) in booms and recessions, which
means the large bank’s requirements are much higher than the small one when in recessions. This means, in recessions, the large bank ought to be more supervised and harsher regulated due to its systemically importance.

5.3 Capital requirements versus capital holdings

We have already discussed the capital requirements set up by the government whose aim is to maximize overall social welfare. Now, we turn to the analysis of the banks’ capital holdings as a response of optimal capital requirements. Figure 4 answers this question.

![Figure 3: Capital holdings versus bank size for the large bank and small bank](image)

The results of Figure 3 are based on Equation (10) and Equation (19). From Figure 3, we can notice that these two banks are likely to retain the same capital holdings in booms, at around 3.6%. When in recessions, however, the systemically important bank, is more likely to keep more capital holdings (around 10.3%) than the non-systemically important bank (around 8.6%). Even though the systemically important bank is regulated with higher capital requirements, as depicted by Figure 2, it might still hold even higher capital holdings. The non-systemically important bank, on the other hand, might hold less capital holdings, in recessions, due partially to the fact that it has to pay its depositors the deposit rate premium, making it prone to hold less capital to reduce the cost of holding more equities. This result reveals the small bank is more likely to fail in recessions, based on its low capital holdings and higher cost due to deposit rate premium. This result might indirectly suggests the Too-Many-To-Fail problem, namely when the majority of the non-systemically important banks tend to hold comparatively lower capital, as we revealed, in recessions, and perform as herding, the economy might also suffer.
We can also identify the impact of systemic tax. Even though the capital requirements on the systemically important bank is lower when systemic tax regime is implemented (from Figure 2), the bank’s capital holdings are nearly the same, and its capital holdings are even slightly higher under the systemic tax regime. This result suggests the systemic tax might help to force the systemically important bank to retain more capital holdings, and thus the capital requirements could be lowered if the systemic tax is introduced, and the softened capital requirements might not add banks’ incentives in reducing capital holdings.

5.4 Too-Interconnected-To-Fail and optimal capital requirements

We now turn to analyse the optimal capital requirements regarding Too-Interconnected-To-Fail for the large bank. We change the baseline parameter \( \varphi \) to discover the optimal capital requirements for the large bank. Figure 4 depicts the result.

![Figure 4](image)

**Figure 4**
Too-Interconnected-To-Fail and optimal capital requirements with and without systemic tax regime

This figure depicts the optimal capital requirements as the function of large bank’s contagion effect (\( \varphi \)). The bank size \( Q \) equals 10.

To facilitate comparison, the bank size \( Q \) is set equal to 10. The results of Figure 4 are obtained from Equation (26). Under the systemic tax regime, the Too-Interconnected-To-Fail consideration might have a marginal impact on the optimal capital requirements: they range from 5.3% to 5.6% in recessions and from 0.7% to 1.2% in booms. However, when under no systemic regime, the optimal capital requirements are fixed at 8.7% and 2.1% for recessions and booms, respectively. This insight is in line with the fact that the systemic tax might help to incorporate the contagion effects (the Too-Interconnected-To-Fail concerns) into banking capital regulation. Thus, the tax could lessen the negative impacts caused by the contagion effects.

6. Extensions
In this section, we analyse the impacts of negative shocks to the supply of bank capital and the effects of the deposit insurance to the small bank regarding their optimal capital requirements. Repullo (2013) argues in recessions the supply of bank capital would be reduced and thus the optimal capital requirements should be lowered to avoid a large reduction in aggregate investment. We now turn to evaluate the validation of this argument. Repullo (2013) considers the overall amount of the supply of the bank capital and assumes it is fixed but will be reduced in recessions. However, it seems impossible to follow this assumption in our model but we simplify this assumption and argue that in recessions the shareholders (the providers of the bank capital) will require higher required return to compensate for higher risks they might take. The degree of increased required return depends on the overall amount of supply of bank capital and shareholders’ risk appetites (Repullo, 2013), however, this is not the objective of our paper. We just show the results for different required return and evaluate the argument regarding the optimal capital requirements. Figure 5 gives this answer.

Figure 5
Optimal capital requirements regarding shareholders’ required return
This figure depicts the optimal capital requirements as the function of shareholders’ required return (δ) which is used as a proxy for the supply of bank capital. The bank size \( Q \) equals 10.

The result from Figure 5 is based on Equation (26). We can notice that for the large bank, the optimal capital requirements should be lowered to cope with the increase in shareholders’ required return due to a negative shock to the supply of bank capital. This result verifies the findings from Repullo (2013) that optimal capital requirements should be lowered in order to stimulate the economy. As for the impacts of systemic tax, the requirements decrease slightly from 5.4% to 4.8%, while the requirements drop significantly from 8.7% to 4.3% under no systemic tax regime. This insight reveals that systemic tax might help to stabilize the optimal capital requirements in recessions, and would be effective in making banks safer when the negative shock of supply of bank capital is extremely high: when \( \delta = 0.08 \), the optimal capital requirement (without systemic tax) is merely at 4.3%, even lower than that when
under systemic tax regime (at 4.8%). This finding also confirms the systemic tax could make the large bank to hold more equities when the shareholders’ required return is relatively high (caused by the shortage of supply of bank capital). The result for the small bank seems ambiguous and it provides an opposite conclusion: it reaches its peak at 8.7% (when \( \delta = 0.06 \)) and the optimal capital requirements are higher than our baseline value (\( \delta = 0.04 \)). This ambiguity might attribute to the effects of depositors who require a deposit premium that makes the small bank’s shareholders more unwilling to raise capital holding, or keep a capital buffer, when they are aware that they need higher required return for their equities. Thus, the capital requirements should be increased to mitigate the small bank’s shareholders incentives to hold less capital. Accordingly, this result corroborates the limitation of one-size-fit-all principles that reducing capital requirements in recessions might be inappropriate to all banks, and the capital requirements should not be softened for the small banks whose costs are higher (due to deposit premium in our analysis).

Additionally, in order to discover the impacts of deposit premium (this is exclusive to the small bank in our paper), we alter the guaranteed amount \( q \) to analyse its effect regarding the optimal capital requirements. Recall that in our previous analysis we assume the government (the insurance agency) will be more likely to assist the large bank’s depositors to reclaim all their deposits once bankrupts to avoid potential contagion effects, even their deposits are also under partial insurance. Figure 6 gives this result.

![Figure 6](image-url)

**Figure 6**

**Optimal capital requirements regarding deposit insurance accords**

This figure depicts the optimal capital requirements as the function of the guaranteed portion of deposit insurance (\( q \)). The bank size \( Q \) equals 10.

The results from Figure 6 are adopted from Equation (26). For booms, the optimal capital requirement is around 1.7% and raises to 2.1% when \( q \) equals to 1.00, where the full deposit insurances is realized. This increase is due to the government (the deposit agency) might take more responsibility to honour the insurance (it has to pay more to the depositors when the small bank fails) and thus a higher capital requirement would be adopted. For recessions,
the optimal capital requirement shows a U-shaped relationship with the guaranteed insurance amount. When the guaranteed value is only at 0.80, the depositors might under larger exposure when bank fails, and thus a higher requirement (at 9.5%) will be set up to limit potential loss caused to the depositors. Thus, a higher capital requirement will help to make the depositors confident about the solvency of the small bank. Recall the conclusion of Proposition 1 that a lower \( \gamma_m \) or \( q \) will make the depositors require higher premium, and a higher capital requirement will compensate the depositors for a lower guaranteed insurance value. However, when the guaranteed amount increases to 0.95 or 1.00, the government (the insurance agency) will need a higher payoff to the depositors and; accordingly, the optimal capital requirement should be increased to limit its potential payment as a deposit insurer. More importantly, when \( q = 1.00 \) the requirement is at 7.9%, nearly equal but slightly lower than that of the large bank at 8.7% (shown by Figure 2). This means due to higher systemically importance the large bank still need a higher optimal capital requirement despite the small bank’s depositors are fully insured. Overall, the result from Figure 6 reveals that the deposit insurance, to some degree, influence the optimal capital requirements to the small bank, and the impact seems more complex for recessions.

7. Concluding remarks

In this paper, we analyse the impact of a systemic tax to systemically important banks and estimate the optimal capital requirements to systemically important and non-systemically important banks. Our model enables the cyclical analysis that gives suggestions for the business cycle. We evaluate the newly proposed systemic tax to be imposed on the systemically important banks and reveal its merits in regulating the banks despite the fact that it might introduce pro-cyclical effects. In addition, we analyse the Basel Accords, including Basel III, and compare the differences between Basel Accords and optimal capital requirements. The optimal capital requirements are shown based on bank size, systemic importance and contagion effect. We have concluded that systemically important banks might need more cyclical optimal capital requirements than the non-systemically ones. In addition, the extensions of our model reveal it is suboptimal to lower capital requirements to all banks in recessions to stimulate the economy.

Overall, our results confirm that the systemic tax (proposed by Freixas & Rochet, 2013 and Acharya et al., 2017) would force systemically important banks to hold more capital and limit their potential risk-taking behaviour. The pro-cyclical effects, revealed by Ayuso et al. (2004) and Repullo and Suarez (2013), has also been confirmed by our analysis that makes banks to hold lower equities in booms by neglecting future crises. Our results regarding optimal capital requirements corroborate the limitation of one-size-fit-all principle, argued by Repullo and Suarez (2013), and suggest the adoption of optimal capital requirements should also consider systemic importance (Gauthier et al., 2012). As for the capital requirements in recessions, our findings agree with Dewatripont and Tirole (2012) according to which is suboptimal to ‘ignore’ recessions. It is inappropriate to lower optimal capital requirements to all banks in recessions; on the contrary, the capital requirements on non-systemically banks should be raised to mitigate its shareholders’ unwillingness to hold more capital in the event of a negative shock of the bank capital.
Our results reveal some facts that worth the consideration of the policy-makers for the banking system. The systemic tax might be an effective tool to force the bank to hold more capital and will help to incorporate Too-Big-To-Fail and Too-Interconnected-To-Fail concerns into the capital requirements. Moreover, the tax will stabilize the capital requirements in the event of the shortage of bank capital supply. We are delighted to see that Basel III Accord has assigned higher capital requirements to the systemically important bank, which is the core argument of our paper. In addition, systemically and non-systemically important banks might need different treatment during recessions where the overall supply of bank capital is constrained.

We conclude with some paths to future research. Firstly, our paper merely focuses on the capital requirements, while the liquidity requirements also worth future study to investigate its impacts to the banking regulation. Secondly, some resolution policies, like prompt corrective action (PCA), and the evaluation of these policies should be conducted. In addition, optimal bailout policy carried out by the government might affect the adoption of capital requirements because the bailout policy will stabilize the economy in case of the bankruptcy, but will cause potential risk-taking behaviours that makes banks more reliant on the bailout action. Thus, a feasible optimal bailout policy is highly recommended to improve social welfare through banking regulation.

Appendix

Proof of Proposition 1

Equation (18) shows that \( F\left(X_{5_m}\right)r_a + \left[1 - F\left(X_{5_m}\right)\right](q - 1) = 0. \) After rearranging the Equation (18) we can obtain

\[
F\left(X_{5_m}\right) = \frac{1 - q}{r_a - q + 1}
\]

From Equation (2) and Equation (17), we can show that

\[
\Phi\left[\sqrt{1 - \rho} \Phi^{-1}\left(\frac{\gamma_S + a - \mu - r_d}{\sqrt{\rho}}\right) - \Phi^{-1}(p)\right] = \frac{1 - q}{r_a - q + 1}
\]

Adding \( \Phi^{-1}(\cdot) \) to both sides of the above equation, we can obtain

\[
\sqrt{1 - \rho} \Phi^{-1}\left(\frac{\gamma_S + a - \mu - r_d}{\sqrt{\rho}}\right) = \Phi^{-1}\left(\frac{1 - q}{r_a - q + 1}\right) + \Phi^{-1}(p)
\]

Next, we assume the function \( X(r_d) \) as

\[
X(r_d) = \Phi^{-1}\left(\frac{1 - q}{r_a - q + 1}\right) + \Phi^{-1}(p) - \sqrt{1 - \rho} \Phi^{-1}\left(\frac{\gamma_S + a - \mu - r_d}{\sqrt{\rho}}\right)
\]

Thus, our aims turn to find the solutions to \( X(r_d) = 0 \). Making differentiation to \( X(r_d) \) in terms of \( r_d \), we can show that
\[
\frac{dX(r_d)}{dr_d} = \frac{1}{a + \lambda} \frac{d\Phi^{-1}\left(\frac{\gamma_S + a - \mu - r_d}{a + \lambda}\right)}{dr_d} = \frac{1 - q}{(r_d - q + 1)^2} \frac{d\Phi^{-1}\left(\frac{1 - q}{r_d - q + 1}\right)}{dr_d}
\]

It is straightforward to show that \(\frac{d\Phi^{-1}(z)}{dz}\) is always positive because \(\Phi^{-1}(z)\) is an increasing function. Additionally, we can notice \(r_d\) can only range from 0 to \(\gamma_S + a - \mu\) because the definition domain of \(\Phi^{-1}(x)\) is from 0 to 1. When \(r_d\) is zero, \(\frac{dX(r_d)}{dr_d}\) is negative infinity as the slope of \(\frac{d\Phi^{-1}(z)}{dz}\) is infinity when \(z\) approaches to 1, and when \(r_d = \gamma_S + a - \mu\), \(\frac{dX(r_d)}{dr_d}\) is positive infinity because the slope of \(\frac{d\Phi^{-1}(z)}{dz}\) is infinity when \(z\) approaches to 0. We can conclude that when \(r_d\) changes from 0 to \(\gamma_S + a - \mu\), \(\frac{dX(r_d)}{dr_d}\) changes from negative infinity to positive infinity. Thus, the function \(X(r_d)\) is a U-shaped curve and it reaches its minimum level where \(\frac{dX(r_d)}{dr_d} = 0\). It is also easy to notice that when \(r_d = 0\) and \(r_d = \gamma_S + a - \mu\), \(X(r_d)\) is positive infinity. Namely, when \(r_d\) changes from 0 to \(\gamma_S + a - \mu\), \(X(r_d)\) starts from positive infinity; decrease to its minimum; increase back to positive infinity. Thus, for appropriate value sets, the minimum of \(X(r_d)\) can be negative, making there are two solutions, and we choose the smaller value of \(r_d\) for deposit rate premium. However, if \(q\) or \(\gamma_S\) is too small, making \(X(r_d)\) high above zero, there will exist no solutions to make \(X(r_d)\) zero. Under this circumstance, we will let \(r_d = \gamma_S + a - \mu\). Because if \(q\) or \(\gamma_S\) are too small, the depositors will find their deposits are under larger exposure and thus we assign the highest feasible deposit rate premium to the depositors.

**Proof of Proposition 2**

We have assumed that when the small bank fails, the government will take over it, and repay the depositors the promised value \(q\). Thus, for the first period, the government’s payoff for taking the failed bank is

\[
GS_m^{first} = \int_{x_{Sm}}^{1} [k_S + a - (a + \lambda) y - \mu + 1 - q] dF(x_S)
\]

Notice that in case of bankruptcy, the small bank is not responsible for paying the deposit rate premium \(r_d\), and it is dropped out. The above equation shows the negative payoff to the government for the first period. It is clear that when the bank fails but the loss is not significant, when \(y\) ranges from \(x_{Sm}\) to \(x_{Sm'}\), where \(x_{Sm} = (k_S + a - \mu + 1 - q)/(a + \lambda)\), the bank still has some positive revenue due to the partial deposit insurance regime. Thus, the government is exempted from the payment for a portion of \(1 - q\). We can simplify the above equation and get the following results

\[
GS_m^{first} = \int_{x_{Sm}}^{1} [k_S + a - (a + \lambda)x_S - \mu] dF(x_S) + (1 - q)[1 - F(x_{Sm})]
\]

Then, for the second period, if the small bank fails, the depositors will still only be able to receive partial payback of their deposits. Then we can get

\[
GS_m^{second} = (1 - CR_{Sm})(\int_{x_{Sm'}}^{1} [\gamma_S + a - (a + \lambda)x_S' + 1 - q] dF(x_S'))
\]

Similar to the first period, we can simplify \(GS_m^{second}\) and get the following

\[
GS_m^{second} = (1 - CR_{Sm})(\int_{x_{Sm'}}^{1} [\gamma_S + a - (a + \lambda)x_S'] dF(x_S') + (1 - q)[1 - F(x_{Sm'})])
\]
Thus, the value of $G_S^m$ should be the sum of $G_S^m$\textsuperscript{First} and $G_S^m$\textsuperscript{Second}, then we can get

$$G_S^m = G_S^m\textsuperscript{First} + G_S^m\textsuperscript{Second}$$

Thus, we can simplify Equation (28)

$$GI_m = T_m + \frac{1}{Q+1} \left[ r_d F(x_{m\textsuperscript{L}}) + (q-1) \left[ \left( 1 - F(x_{m\textsuperscript{L}}) \right) \left( 1 - F(x_{m\textsuperscript{L}}) \right) \right] \right]$$

$$+ \frac{Q}{Q+1} \left\{ \int_{x_{m\textsuperscript{L}}}^{1} k'_L (x) dF(x) + \left( 1 - CR_{L,m} \right) \left[ \int_{x_{m\textsuperscript{L}}}^{1} [\gamma_L + a - \lambda \cdot x] dF(x') \right] \right\} + G_S^m$$

and get

$$GI_m = T_m + \frac{1}{Q+1} \left[ r_d F(x_{m\textsuperscript{L}}) + \frac{1}{Q+1} \left\{ \int_{x_{m\textsuperscript{L}}}^{1} [k_S + a - (a + \lambda) x_S - \mu] dF(x_S) \right\} \right]$$

$$+ \left( 1 - CR_S^m \right) \left[ \int_{x_{m\textsuperscript{L}}}^{1} [\gamma_S + a - (a + \lambda) x_S - \mu] dF(x_S') \right]$$

$$+ \frac{Q}{Q+1} \left\{ \int_{x_{m\textsuperscript{L}}}^{1} k'_L (x) dF(x) + \left( 1 - CR_{L,m} \right) \left[ \int_{x_{m\textsuperscript{L}}}^{1} [\gamma_L + a - \lambda \cdot x] dF(x') \right] \right\}$$

If we replace $k_S + a - (a + \lambda) x_S - \mu$ with $k_S'(x_S)$, we can obtain Equation (29).

Large bank’s responses to different capital requirement regimes (when $\delta = 0.08$)

As noted by Footnote 3, we give the following Table A1 to show the results by using the shareholders’ required return at 0.08 that is used by Repullo and Suarez (2013) which has considered the impact of Tier 2 capital. We can confirm that the results are fundamentally the same as what we have demonstrated in Table 2 despite that the shareholders might retain lower capital holdings due to the increased cost of holding capital when under higher required return.

**Table A1**

Capital buffers, systemic tax and bank’s net income under different regulatory regimes and different bank sizes when $\delta = 0.08$ (all variables in %)

<table>
<thead>
<tr>
<th>Bank Size: Q=1/Q=5/Q=10</th>
<th>Laissez-faire</th>
<th>Basel I</th>
<th>Basel II</th>
<th>Basel III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital Holdings in state $m$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_L$</td>
<td>2.4/2.6/2.6</td>
<td>4.4/4.6/4.6</td>
<td>3.9/4.1/4.1</td>
<td>7.0/7.0/7.0</td>
</tr>
<tr>
<td>$k_H$</td>
<td>6.1/6.6/6.9</td>
<td>7.9/8.0/8.1</td>
<td>8.4/9.4/9.4</td>
<td>9.5/10.0/10.2</td>
</tr>
<tr>
<td>Capital buffer in state $m$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta_L = k_L - \gamma_L$</td>
<td>2.4/2.6/2.6</td>
<td>0.4/0.6/0.6</td>
<td>0.7/0.9/0.9</td>
<td>0.0/0.0/0.0</td>
</tr>
<tr>
<td>$\Delta_H = k_H - \gamma_H$</td>
<td>6.1/6.6/6.9</td>
<td>3.9/4.0/4.1</td>
<td>2.9/3.9/3.9</td>
<td>1.5/2.0/2.2</td>
</tr>
<tr>
<td>Systemic tax in state $m$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_L$</td>
<td>0.1/0.1/0.1</td>
<td>0.1/0.1/0.1</td>
<td>0.1/0.1/0.1</td>
<td>0.0/0.1/0.1</td>
</tr>
<tr>
<td>$T_H$</td>
<td>0.2/0.2/0.2</td>
<td>0.1/0.2/0.2</td>
<td>0.1/0.1/0.1</td>
<td>0.2/0.2/0.2</td>
</tr>
<tr>
<td>Capital buffer minus tax in state $m$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_L = \Delta_L - T_L$</td>
<td>2.3/2.5/2.5</td>
<td>0.3/0.5/0.5</td>
<td>0.6/0.8/0.8</td>
<td>0.0/-0.1/-0.1</td>
</tr>
<tr>
<td>$\beta_H = \Delta_H - T_H$</td>
<td>5.9/6.4/6.7</td>
<td>3.8/3.8/3.9</td>
<td>2.8/3.8/3.8</td>
<td>1.3/1.8/2.0</td>
</tr>
</tbody>
</table>
Capital buffers under no tax in state $m$

\[ \Delta'_{l} = k'_{l} - \gamma_{l} \]

\[ \Delta'_{h} = k'_{h} - \gamma_{h} \]

Net Capital buffers with tax in state $m$

\[ \alpha_{l} = \beta_{l} - \Delta'_{l} \]

\[ \alpha_{h} = \beta_{h} - \Delta'_{h} \]

Bank’s net income in state $m$ with tax

\[ v'_{L,l} = 3.4/3.4/3.4 \]

\[ v'_{L,h} = 1.3/1.3/1.3 \]

Bank’s net income in state $m$ without tax

\[ v_{L,l} = 3.3/3.3/3.3 \]

\[ v_{L,h} = 0.8/0.8/0.8 \]

References


Chakrabarty, K.C. 2011. Empowering deposit insurance entities to face challenges posed by an emerging financial landscape-global and Indian experience. BIS central bankers’ speeches.


