Pricing Deposit Insurance Premiums with Moral Hazard and Hedging with Credit Default Swaps

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Abstract

Moral hazard may emerge as a result of deposit insurance schemes that guarantee all deposits because managers of banks and depositors bear no consequences for banks' risk-taking in the pursuit of higher yields. In this paper, we propose a method for modeling moral hazard with deposit rate spreads and quantify the impact of moral hazard on deposit insurance premiums. Thus, we provide a closed-form solution for deposit premiums that incorporates moral hazard, early closure, capital forbearance, and a stochastic risk-free interest rate under a risk-based option pricing framework. Furthermore, we use credit default swaps to investigate a market-based method to estimate bank risk, and we present a hedging concept can be used by deposit insurance corporations to diversify the risk of deposit insurance via credit derivatives.

Keywords: deposit insurance, moral hazard, capital forbearance, credit default swap

1. Introduction

The management objectives of financial institutions are to protect the rights and benefits of depositors, to maintain financial order, and to promote financial stability and development. Given today's globalized financial environment, the ex-ante supervision of financial institutions and ex-post remedies are equally important. Governments must establish sound and complete financial management systems and offer deposit insurance such as that provided in the United States by the Federal Deposit Insurance Corporation (FDIC). Deposit insurance provides basic safeguards against bank runs and contagion proliferation that can lead to global financial crises. In addition, the commitments and the resulting safety net provided by deposit insurance corporations can strengthen market confidence for investors and prevent avoid panic, particularly in a global recession.

Since Merton (1977), deposit insurance has typically been modeled as a European put option, i.e., a put contract that is issued by the deposit insurer and written on bank assets that features a strike price equal to the deposit amount and maturity at the audit date. Therefore, the insurance claim is regarded as the option payoff, which is the shortfall between the bank's asset value and the deposit account, and the premium is calculated under the Black-Scholes framework (Ronn and Verma, 1986; Thomson, 1987; Episcopos, 2004). However, this Merton-type setting ignores the possibility of early closure or capital forbearance. Thus, the omission of these possibilities is at odds with reality.

Banks conforming to the risk-based capital standards of the Basel II regulations can increase their insurance subsidy by concentrating their lending and off-balance sheet activities (Pennacchi, 2006). When banks fail to meet the applicable capital standard, deposit insurers may provide these undercapitalized financial institutions capital forbearance and require them to take prompt corrective actions to recapitalize during a limited period or close early (Hellmann et al., 2000; Kane, 2001; Nagarajan and Sealey, 1995). Duan and Yu (1994) propose a multi-period deposit insurance pricing model that simultaneously incorporates these capital standards and the possibility of forbearance. Moreover, Duan and Yu employ GARCH option pricing techniques in determining the value of deposit insurance. In a simple model, Lee et al. (2005) derive a closed-form solution for deposit insurance under capital forbearance as an option to delay the resolution of undercapitalized financial institutions.

In addition to capital forbearance, recent studies incorporate early closure policies and stochastic interest rates into the deposit insurance pricing formula using a Merton-type setting. Hwang et al. (2009) apply the “down-and-out” put option formula to determine the regulatory threshold – defined as the lower barrier of the option for the deposit insurance – while explicitly considering bankruptcy costs and closure policies. Based on the calibration of pricing parameters, Chuang et al. (2009) measure the deposit insurance premium under stochastic interest rates for Taiwan’s banks by applying the two-step maximum likelihood estimation method. These methods attempt to consider the various risks that the deposit scheme faces when providing fair risk-based deposit insurance premium.

Although deposit insurance reduces the risk of bank runs, it simultaneously decreases the incentive for depositors to monitor a bank’s asset choices (Laeven 2002; Demirgüc-Kunt and Detriagache 2002). Wheelock and Kumbhakar (1995) attempt to discern whether insured banks are riskier because of moral hazard, adverse selection or both and found that the Kansas deposit insurance system appears to suffer from problems related to both adverse selection and moral hazard. The moral hazard problem is likely to plague deposit insurance schemes because it creates incentives for banks to take on greater risk and engage in risky activities with impunity (Laeven, 2002). So and Wei (2004) observe that the impact of the moral hazard on the fair insurance premium is more significant than a bank’s equity value and charter value. The insurer should have the ability to deter banks’ risky behavior and close problematic banks when necessary. A fair pricing framework of deposit insurance is therefore crucial to mitigating moral hazard problems (VanHoose, 2007).

In this paper, we consider two sources of moral hazards and attempt to measure the corresponding risk-based deposit insurance premium incurred by the particular moral hazard. The first source of moral hazard arises from the deposit rate spread (the difference between the deposit rate and the risk-free interest rate). Deposit insurance schemes guarantee compensation for depositor loss and thereby encourage depositors to choose banks that offer higher deposit rates, which means that banks can absorb more deposit liability and increase the bank's own size by increasing the deposit rate spread. However, this scheme also gives banks incentives to increase risk-taking due to their limited liability, which results in banks taking on more risky loans and higher interest-rate risk exposure to achieve higher net interest margins (Angbazo, 1997). In other words, the higher the deposit rate spread, the more incentive banks have to take greater risks than they otherwise would have taken by paying the deposit rate spread while maintaining its profit level.

We propose two ideas to reflect the risk of moral hazard regarding deposit premiums and to temper the moral hazard arising from the deposit rate spread. One idea characterizes the deposit rate spread as a risk function that involves a bank’s asset risk. When a bank provides a higher deposit rate spread, the deposit insurance corporation should charge higher insurance premiums due to the higher probability of risk-taking operations of the bank that will increase the bank’s risk. The other idea is to allow depositors to assume part of the risk when they pursue higher deposit rate spreads to achieve excess profits. Depositors save their money in the bank that provides a higher deposit rate when the deposit is completely insured, which, of course, increases the moral hazard incentives of the bank. To mitigate the moral hazard problem, we design a moral hazard multiplier with a deductible proportion to model the penalty for depositors when the insured bank closes or is taken over.

The second source of moral hazard occurs in the grace period of capital forbearance. If the insured bank's asset value cannot meet the capital standard but does not fall below the forbearance threshold at the time of the audit, the insured bank can extend its operations for a certain grace period (Nagarajan and Sealey, 1995; Kane, 2001; Lee et al., 2005). During the grace period, the insured bank is asked to adopt more aggressive financial operations to increase its earnings and thus to satisfy the adequacy requirement. In this case, the moral hazard operation may occur. The insured bank can adjust the underlying holdings of its securities positions by increasing the number of higher-yielding securities it holds or by changing its asset allocation and increasing the weight of investment positions, which typically have higher yields than loan positions and can offer quick profits. Both operations will lead to higher bank asset volatility, and the deposit premiums should take the additional risk during the grace period into account.

In this study, we examine deposit insurance premiums with moral hazard risks arising from two sources, and we incorporate early closure, capital forbearance, and stochastic interest rates into our investigation. Moreover, using the credit default swap (CDS) market, we develop a market-based method to calibrate the volatility parameter of future bank assets by using the bank's CDS position, and we suggest a hedging strategy for deposit insurance corporations to diversify the risk of deposit insurance via the CDS market.

The remainder of this paper is organized as follows: Section 2 proposes a deposit insurance scheme that considers closure policies and moral hazard risks. Section 3 derives a closed-form solution for risk-based deposit insurance premiums and constructs a market-based calibration concept of bank risk to value deposit insurance premiums through the CDS mechanism. Section 4 presents several numerical experiments that analyze the source of premiums relative to Merton's deposit insurance put and shows how banks maintain ratios, debt-to-asset ratios, asset allocation, and moral hazard relative to premiums. Moreover, we analyze the deposit premiums of specific large banks with extreme closure policies and the hedging effect of using credit derivatives. Section 5 concludes.

2. Deposit insurance schemes and bank balance sheets

2.1 Bank asset model

Before modeling the deposit insurance schemes, we must verify the assumptions of the risk-free interest rate (hereafter referred to as the risk-free rate), the bank's deposit liabilities and the consistency of the bank's assets. The stochastic risk-free rate $r(t)$ is adopted from the Vasicek (1977) model and leads to the following explicit formula:

$$dr(t) = \kappa(\theta - r(t))dt + \sigma_r dW_r^P(t) \tag{1}$$

where κ represents the mean-reverting force measurement, θ stands for the long-term mean of the risk-free rate, σ_r is the volatility of the risk-free rate and $W_r^P(t)$ is a Wiener process. Therefore, we define the riskless money market account $M(t) = \exp\{\int_0^t r(s)ds\}$ as the numeraire for the pricing deposit premium.

Banks provide the deposit rate for drawing funds based on their own financial condition. Therefore, the dynamics of deposit liabilities $D(t)$ should grow with the deposit rate $r(t)+\varepsilon$, where ε is the difference between the deposit rate and the risk-free rate, and we call this measure the deposit rate spread. Specifically,

$$dD(t) = \left(r(t)+\varepsilon\right)D(t)dt \tag{2}$$

where the dynamic of deposit liabilities has no uncertainty risk except for the risk-free rate risks. However, this does not mean that investing money in the deposit is without risk. The return on a riskless asset is a risk-free rate that prevents any arbitrage opportunity.

As documented in the literature, deposit insurance schemes generate a moral hazard problem (Grossman, 1992, Wheelock and Kumbhakar, 1995, Gropp and Versala, 2004, Cull et al. 2005, and Beck et al., 2006). When there is a moral hazard, a bank may engage in excessive risk taking to realize additional profit to cover the extra deposit interest. Thus, the bank's risk should include the risk of moral hazard caused by the deposit rate spread. In addition, the deposit may suffer a default risk; as we discuss later, that risk cannot be completely covered through the deposit insurance scheme designed in our model. Given the specification of riskless money market accounts $M(t)$ and a bank's outstanding deposit liabilities, $D(t)$, we can express deposit liabilities as

$$D(t) = D(0)\exp\left\{\int_0^t r(s)ds + \varepsilon t\right\} = D(0)M(t)\exp\{\varepsilon t\} \tag{3}$$

Table I presents the statistical reports of the financial statements of FDIC-insured institutions from the FDIC website[1] and presents the asset allocation of all commercial banks insured by the FDIC from 1994 to 2013. Table 1 demonstrates that the total percentage of bank assets in reserve, securities, and loans[2] is greater than 90%, i.e., the risk of the bank's assets is found mostly in these three components.

[Insert Table I here]

[1]http://www2.fdic.gov/SDI/SOB/

[2]In the table, reserves are defined as "cash and due from depository institutions" of a banking report, and the term securities represents "securities", "federal funds sold & reverse repurchase agreements," and the "trading asset account" of the balance sheets from the banking report at the FDIC.

Because bank assets fluctuate closely and stably around a given financial strategy, we assume in this paper that the allocation of bank assets consists of the following: (1) the amount held in reserves and cash, $R(t)$; (2) the loan position, with price process $L(t)$; and (3) the investment position, with price process $S(t)$. The investment position includes securities and trading account assets, hereafter referred to as securities. The reserve position is allocated among a fixed proportion, γ, of bank assets, where ω is the fraction of bank assets invested in securities, and the remaining fraction $1-\gamma-\omega$ consists of outstanding loans. The dynamics of bank assets, $A(t)$, evolve according to the following:

$$\frac{dR(t)}{R(t)} = r(t)dt \tag{4}$$

$$\frac{dL(t)}{L(t)} = \left(r(t)+\varepsilon+\lambda-\phi\kappa(\theta-r(t))\right)dt+\phi dr(t)+\sigma_c dW_L^P(t) \tag{5}$$

$$\frac{dS(t)}{S(t)} = \mu dt+\sigma_S dW_S^P(t) \tag{6}$$

$$\frac{dA(t)}{A(t)} = \gamma\frac{dR(t)}{R(t)}+\omega\frac{dS(t)}{S(t)}+(1-\gamma-\omega)\frac{dL(t)}{L(t)} \tag{7}$$

where ϕ denotes the instantaneous interest rate elasticity of the loans; λ is the interest rate spread, which is the interest rate paid by banks on loans to sector customers minus the interest rate paid by savings deposits; $W_L^P(t)$ represents the Wiener process under the physical probability measure and independent of $W_r^P(t)$; and σ_c denotes the constant credit risk, orthogonal to the interest rate risk. According to Duan et al. (1995) and Chuang et al. (2009), the total loan risk can be expressed as $\sigma_L=\sqrt{\phi^2\sigma_r^2+\sigma_c^2}$. The security dynamics follow Black-Scholes dynamics, with an instantaneous rate of return, $\mu>0$; the volatility parameter for the securities market is σ_S. The term $W_S^P(t)$ is the Wiener process under the physical probability measure representing the securities market risk, which is independent of $W_L^P(t)$ and $W_r^P(t)$. The dynamics of reserves and cash positions are increased with the risk-free interest rate and are held at a constant reserves-to-assets ratio, γ.

The bank's asset dynamics result from equations (4)-(6) because the bank's assets are weighted by reserves and cash, investments and loan positions. The assets can be derived as follows:

$$\frac{dA(t)}{A(t)} = \left(\omega\mu + (1-\omega)r(t) + (1-\gamma-\omega)(\varepsilon+\lambda)\right)dt + \sigma dW_A^P(t) \tag{8}$$

where $\sigma = \sqrt{\omega^2\sigma_S^2 + (1-\gamma-\omega)^2\sigma_L^2 + g(\varepsilon)}$ refers to the total risk of bank assets, and $W_A^P(t)$ is a Wiener process. The function $g(\varepsilon)$ denotes that under this assumption, the bank will experience increasing risk because of the additional deposit rate spread based on the moral hazard argument. We define $g(\varepsilon)$ as the excess risk function of the deposit rate spread.[3]

2.2 A deposit insurance contract with closure policies

Valuation models such as those of Merton and Ronn and Verma assume that the regulatory authorities can only monitor a bank's assets at the maturity of the insurance contract, i.e., at the time of audit. According to Brockman and Turtle (2003) and Episcopos (2008), bank creditors and depositors will not wait for the maturity date of debt or deposits. A bank can go bankrupt before the audit date if the bank's asset value

[3] The explicit form of the excess risk function of the deposit rate spread can be assumed in any form of the deposit rate spread used by insurers. In our model, we assume the excess risk function to be the simple linear function of the deposit rate spreads, and the relative analysis is shown in section 4.2.

is lower than a specific threshold, which triggers debt holders and depositors to withdraw their money. For deposit insurers, the bankruptcy premium occurs during the audit window period and should be considered when evaluating the risk of early closure.

To explicitly model regulations for early closure, we assume that the maintenance ratio, $\eta \in (0,1)$, represents the minimum asset-to-debt ratio required to keep the bank functioning. In other words, if the bank's asset value is lower than its maintenance working capital level $\eta D(t)$ during the contract period, the bank is regarded as bankrupt. Let τ be the first-time bank assets drop to breach the maintenance working capital level. We define the threshold as follows:

$$\tau = \inf\left\{ t > 0 \middle| A(t) \le \eta D(t) \right\} \tag{9}$$

If τ is not later than the auditing time, T_1, the bank goes bankrupt or is taken over. The deposit insurance payoff occurs at τ. If the bank goes into liquidation at time $\tau < T_1$, the deposit insurer compensates the depositor for the difference between the bank's asset value and its deposit debt. At this point, the bank's asset value equals maintenance working capital; therefore, the deposit insurance payoff at time τ can be expressed as follows:

$$P(\tau) = (1-\eta)D(\tau) \tag{10}$$

which is a threshold option on the underlying asset whose price breaches the maintenance working capital level, resulting in default. The insurer pays deposit insurer compensation $(1-\eta)D(\tau)$ to depositors when the bank defaults. Merton's (1977) original deposit insurance pricing model and other deposit insurance pricing models (Ronn and Verma, 1986; Lee et al., 2005) do not allow for premature default because default can only occur when a claim matures. In our setting, we first investigate passage time structural models corresponding to various specifications of the basic components of a credit risk model.

So and Wei (2004) specify that capital forbearance and the capital ratio play an important role in determining the deposit insurance premium. In our model, if the bank can operate until audit time T_1, the regulatory authority examines the bank's asset value at that time. The assumptions of capital forbearance are similar to those of Duan and Yu (1994): The regulator offers the insured bank capital forbearance for a grace period, Δ, if its asset value cannot meet the capital standard, $\alpha D(T_1)$, but does not fall below the capital forbearance threshold $\beta D(T_1)$, where β is greater than maintenance ratio η; otherwise, capital forbearance loses its meaning. The financially distressed bank can extend its operations until the time of the next audit, T_2, which is $T_1 + \Delta$ if the insuring agent promises to restore the asset value to a level higher than the bank's outstanding deposit liabilities, $D(T_2)$. Once the bank's value drops below the capital forbearance threshold, $\beta D(T_1)$, at time T_1, or $D(T_2)$ at time T_2, the regulator will take over the depository institution.

In the context of deposit insurance, regardless of whether the insured bank closes or is taken over, the liquidation of insured deposits must restore the asset value. The payoffs of the deposit insurance contract at time T_1 without default before the audit can be expressed as follows:

$$P(T_1) = \begin{cases} 0 & if \quad A(T_1) > \alpha D(T_1) \\ F(T_1, T_2) & if \quad \alpha D(T_1) \geq A(T_1) > \beta D(T_1) \\ D(T_1) - A(T_1) & if \quad otherwise \end{cases} \tag{11}$$

where $F(T_1, T_2)$ denotes the value of the capital forbearance and the grace period with maturity T_2 at audit time T_1. Equation (11) can be regarded as the payoff for a type of compound option that is a generalized writer-extendible put option. A pricing model of retractable and extendible bonds was presented by Brennan and Schwartz

(1977) and Ananthanaray and Schwartz (1980). Longstaff (1990) extended those authors' work to develop holder- and writer-extendible options and applied those options to evaluate real estate options, warrants, extendible bonds and American options. In this paper, we model the deposit insurance scheme as a general writer-extendible option when the bank works until audit time T_1. If the bank's assets are higher than the capital standard, the payoff of deposit insurance equals zero. However, if the asset value cannot satisfy the capital standard, the payoff of deposit insurance is a type of writer-extendible put option in which the underlying asset is the bank asset value, and the strike price is the capital forbearance threshold $\beta D(T_1)$. If the asset value is lower than $\beta D(T_1)$ at time T_1, the deposit insurer pays the difference between the deposit liabilities and the bank's assets to cover the deposit losses; otherwise, the results lead to another put option with maturity T_2. Longstaff (1990) showed that a deposit insurance payoff can degenerate into a writer-extendible put option with a time-variant strike price if parameter α tends to infinity and $\beta = 1$.

Based on capital forbearance for deposit insurance, the general extendible put option is in force because the bank's assets are lower than $\alpha D(T_1)$ but do not fall below the capital forbearance threshold, $\beta D(T_1)$. The financially distressed bank can extend its operations until the time of the next audit, T_2, if the insuring agent promises to restore the asset value of the bank's outstanding deposit liabilities, $D(T_2)$. The option is extended with time to maturity, Δ, and the strike price, which is the bank's outstanding deposit liabilities at T_2. Therefore, the payoff at time T_2 can be written as follows:

$$F(T_2, T_2) = \begin{cases} 0 & if \quad A(T_2) \geq D(T_2) \\ D(T_2) - A(T_2) & if \quad otherwise \end{cases} \tag{12}$$

When asset value cannot increase above deposit liabilities at audit time T_2, the claim amount is the difference between the bank's deposit liabilities and its asset value. According to the Basel Accord, bank operations are required to maintain a minimum of 8% of their capital, based on a percentage of risk-weighted assets. For simplicity, we assume the risk-weighted asset value at the current time is $A(t)$ and total deposit debt is $D(t)$. Because $A(t)-D(t)$ over $A(t)$ must be greater than 8% to meet the capital adequacy ratio, the capital standard $A(t)/D(t)$ is required to be greater than 1.087. Therefore, in this paper, the capital standard parameter α is set at 1.087 (see also Lee et al., 2005).

2.3 A design for reducing the moral hazard of the deposit insurance scheme

The deposit insurance scheme incentivizes banks' moral hazard and excessive risk-taking, which encourages both depositors and banks to assume excessive risk. Without deposit insurance, banks would compete for deposits because depositors prefer safe banks over risky banks to protect their money. With deposit insurance, banks can take excessive risks because depositors are not worried about their deposits' safety. Based on deposit insurance, we assume that a bank increases its deposit rate spread to attract depositors and then increases its own size by absorbing more deposit liability. However, the bank must engage in excessive risk-taking operations to pay the deposit rate spread while maintaining its profit level. To reduce the moral hazard incentives, the deposit insurance corporation should charge higher insurance premiums when a bank provides a higher deposit rate spread. Moreover, we define the ratio $\xi_{bank} > 1$ as the proportion of risk-based premiums that include a penalty for a bank's moral hazard. The ratio ξ_{bank} is essentially a function of the deposit rate spread and is determined by deposit insurer flexibility.

In addition to the banks' moral hazard problem, depositors generate their own moral hazard because they will save their money in the banks that provide a higher deposit interest rate if the deposit is completely insured, which indirectly incentivizes the moral hazard of banks raising the deposit rate spread and undertaking riskier operations. We assume that the depositor should bear a portion of the losses when an insured bank closes or is taken over, i.e., the depositor should only receive a ratio $\xi_{depositor} < 1$ of his/her deposit when an insured bank fails. In this design, the depositor shares the bank's default risk with the deposit insurance corporation, thereby reducing the premiums charged by the insurer. To reflect a fair risk-based premium, the higher the deductible proportion of the depositor is, the lower the multiplier ratio, $\xi_{depositor}$. Therefore, we take the ratio $\xi_{depositor}$ as the penalty multiplier of the depositors' moral hazard.

Based on the above argument, the moral hazard-adjusted deposit insurance premium $P(0)$ can be expressed as follows:

$$P(0) = E^Q\left[\frac{\xi_\varepsilon P(\tau)}{M(\tau)} I_{\{\tau<T_1\}}\right] + E^Q\left[\frac{\xi_\varepsilon P(T_1)}{M(T_1)} I_{\{\tau\geq T_1\}}\right] \tag{13}$$

where $E^Q[\cdot]$ denotes the expected value under the measure risk-neutral measure Q, and $I_{\{\cdot\}}$ is the indicator function. $\xi_\varepsilon = \xi_{bank} \cdot \xi_{depositor}$ is the combined multiplier designed to reduce the moral hazard for both the bank and its depositors. We define the ξ_ε as the penalty multiplier of the deposit insurance premium.

3. Risk-based deposit insurance premiums

3.1 Closed-form formula for deposit insurance premiums

To price the initial deposit premium, we adopt the standard practice of adjusting the probability measure that incorporates the risk premium into the risk-neutral probability measure. The dynamics of bank assets under the risk-neutral measure are written as follows:

$$dA(t) = r(t)A(t)dt + \sigma A(t)dW_A^Q(t) \tag{14}$$

where $\sigma = \sqrt{\omega^2\sigma_S^2 + (1-\gamma-\omega)^2\sigma_L^2 + g(\varepsilon)}$ is referred to as the total risk of bank assets and $W_A^Q(t)$ represents the Wiener processes under the risk-neutral probability measure.

In this paper, the asset-to-debt ratio is used to determine the insurance payment. The relative values of assets and debts eliminate the effect on returns caused by risk-free rates, and the risk-free rates' influence only appears in the volatility of bank assets. Applying Ito's lemma, we can write the relative dynamics of a bank's assets as follows:

$$d\frac{A(t)}{D(t)} = \frac{A(t)}{D(t)}\left(-\varepsilon dt + \sigma dW_A^Q(t)\right) \tag{15}$$

Following equation (13), the risk-based premium that the insured bank pays to the deposit insurer can be further divided into the following three components:

$$P(0) = E^Q\left[\frac{\xi_\varepsilon P(\tau)}{M(\tau)} I_{\{\tau<T_1\}}\right] + E^Q\left[\frac{\xi_\varepsilon P(T_1)}{M(T_1)} I_{\left\{\tau\ge T_1, \frac{A(T_1)}{D(T_1)}\le\beta\right\}}\right] + E^Q\left[\frac{\xi_\varepsilon P(T_1)}{M(T_1)} I_{\left\{\tau\ge T_1, \beta<\frac{A(T_1)}{D(T_1)}<\alpha\right\}}\right] \tag{16}$$

The closed-form solution for the risk-based deposit insurance premium consists of an audit window component (the first term in equation (16), denoted as P^a), a capital forbearance component (the second term in equation (16), P^c) and a grace period component (P^Δ , the third term in equation (16)). These are expressed as follows:

$$P^a = \xi_\varepsilon(1-\eta)D(0)e^{\frac{v-u}{\sigma^2}B(\eta)}\left[\Phi\left(c_1(\eta,u)\right) + e^{\frac{2u}{\sigma^2}B(\eta)}\Phi\left(c_1(\eta,-u)\right)\right] \tag{17}$$

$$P^{c}=\xi_{\varepsilon}D(0)e^{\varepsilon T_1}\begin{bmatrix}\left(\Phi\left(c_1(\beta,v)\right)\right)-\Phi(c_1(\eta,v))\right) \\ -e^{\frac{2v}{\sigma^2}B(\eta)}\left(\Phi\left(c_2(\beta,\eta,v)\right)-\Phi\left(-c_1(\eta,-v)\right)\right)\end{bmatrix}$$
$$-\xi_{\varepsilon}A(0)e^{\varepsilon T_1}\begin{bmatrix}\left(\Phi\left(c_1(\beta,\tilde{v})\right)-\Phi\left(c_1(\eta,\tilde{v})\right)\right) \\ -e^{\left(\frac{2v}{\sigma^2}+2\right)B(\eta)}\left(\Phi\left(c_2(\beta,\eta,\tilde{v})\right)-\Phi\left(-c_1(\eta,-\tilde{v})\right)\right)\end{bmatrix} \tag{18}$$

$$P^{\Delta}=\xi_{\varepsilon}e^{\varepsilon T_1}\left(D(0)\begin{Bmatrix}N\left(c_1(\alpha,v),e_1(v),\delta\right)-N\left(c_1(\beta,v),e_1(v),\delta\right) \\ -e^{\frac{2vB(\eta)}{\sigma^2}}\left[N\left(c_2(\alpha,\eta,v),e_2(v),\delta\right)-N\left(c_2(\beta,\eta,v),e_2(v),\delta\right)\right]\end{Bmatrix}\right.$$
$$\left.+A(0)\begin{Bmatrix}N(c_1(\alpha,\tilde{v}),e_1(\tilde{v}),\delta)-N(c_1(\beta,\tilde{v}),e_1(\tilde{v}),\delta) \\ -e^{\left(\frac{2v}{\sigma^2}+2\right)B(\eta)}\left[N(c_2(\alpha,\eta,\tilde{v}),e_2(\tilde{v}),\delta)-N(c_2(\beta,\eta,\tilde{v}),e_2(\tilde{v}),\delta)\right]\end{Bmatrix}\right) \tag{19}$$

where $v=-\varepsilon-\dfrac{\sigma^2}{2}$, $\tilde{v}=v+\sigma^2$, $u=\sqrt{v^2-2\sigma^2\varepsilon}$, $\sigma=\sqrt{\omega^2\sigma_S^2+(1-\gamma-\omega)^2\sigma_L^2}$,

$\sigma_L=\sqrt{\phi^2\sigma_r^2+\sigma_c^2}$, $c_1(x,z)=\dfrac{B(x)-zT_1}{\sigma\sqrt{T_1}}$, $c_2(x,y,z)=\dfrac{B(x)-2B(y)-zT_1}{\sigma\sqrt{T_1}}$, $e_1(v)=\dfrac{B(1)-vT_2}{\sigma\sqrt{T_2}}$,

$B(x)=\ln\dfrac{xD(0)}{A(0)}$, $e_2(v)=\dfrac{B(1)-2B(\eta)-vT_2}{\sigma\sqrt{T_2}}$, $\delta=\sqrt{\dfrac{T_1}{T_2}}$, $N(c,e,\delta)=\int_{-\infty}^{c}\Phi(\dfrac{e-\delta Z}{\sqrt{1-\delta^2}})\varphi(Z)dZ$.

$\varphi(\cdot)$ and $\Phi(\cdot)$ are the probability density and cumulative distribution functions of the standard normal distribution, respectively. More detail is provided in the appendix.

P^a is denoted as the early closure component, which has the purpose of evaluating the present value of the payment for banks that reach the default threshold before audit time T_1 such that the regulator must implement the bankruptcy process. Similarly, P^c is presented as the capital forbearance component because its value is similar to the value of a down-and-out put option whose strike price is the capital forbearance threshold $\beta D(T_1)$ at maturity time T_1. Although the bank may not go bankrupt, if it reaches the regulatory closure point, $\beta D(T_1)$, the regulator will take over the bank. The depositor can still obtain a rebate based on the difference between the regulatory closure point and the deposit insurance amount. The rebate should be reflected in the premium as P^c. P^Δ is regarded as a grace period component because its value depends on regulatory delay. An undercapitalized institution can improve its financial position by continuing to work during the grace period.

3.2 The moral hazard in the grace period

Under the forbearance policy on capital regulation, the bank is asked to adopt a more aggressive financial operation to increase its earnings to satisfy the adequacy requirement, particularly during the grace period. In this case, two possible moral hazard operations are present. One operation adjusts the underlying holdings of the bank's security positions to increase the number of higher-yielding securities and to incorporate higher volatility. The other operation adjusts the bank's asset allocation, increasing the weight of security positions because those positions typically have higher yields than loan positions and can offer quick profits.

This study analyzes risk taking in a simple, stylized manner, i.e., by adjusting the asset portfolio, because we argue that asset allocation is a crucial determinant of moral hazard. The bank manager tends to process a risk-taking operation by increasing the weight of an investment position; in other words, the bank may adjust its asset allocation with a larger value ω, particularly in the grace period. Thus, our model adopts another portfolio share $\tilde{\omega}$ of securities during the grace period, where $\tilde{\omega} > \omega$. The difference between ω and $\tilde{\omega}$ reflects the risk of moral hazard. According to a derivation similar to that shown for premium in the appendix, $P(0) = P^a(\omega) + P^c(\omega) + P^\Delta(\tilde{\omega})$, where $P^a(\omega)$ and $P^c(\omega)$ are the same as in equations (17) and (18). The term $P^\Delta(\tilde{\omega})$ is similar to equation (19), except that the volatility of bank assets gives greater weight to securities: $\sigma = \sqrt{\tilde{\omega}^2 \sigma_S^2 + (1-\gamma-\tilde{\omega})^2 \sigma_L^2 + g(\varepsilon)}$.

3.3 Calibration and hedging

Because bank risk derives from the volatility of bank assets, which greatly influences the insurance premium deposit, a proper estimation of the volatility of bank risk is critical in pricing the insurance premium deposit. We believe that calibrating the volatility of bank assets through market trading information such as the CDS market is an appropriate method for pricing the insurance premium deposit. CDSs are described by referring to the cash flows of the premium leg and the default leg. The premium leg is obtained from the present value of all payments made by the protection buyer:

$$CDS_{premium} = K \sum_{i=1}^{n} B(0,t_i) Q(t_i) \tag{20}$$

where K is the fixed insurance payment dependent on the period, $B(0,t_i)$ represents the present value of a zero coupon bond with maturity t_i and $Q(t_i)$ denotes the probability of survival.

In the bank CDS market, the volatility of bank assets can be calibrated from the CDS payment with the compounding loss rate by assuming the following model structure:

$$CDS_{default} = \int_0^T (1-R) B(0,t)(1-Q(t)) dt \tag{21}$$

where T denotes the maturity period of the CDS and R reflects the recovery rate in case of a credit event. Based on equation (21), the present value of the default leg can be derived via the first passage time theory under the debt-to-asset ratio dynamics described in equation (15). To simplify, we assume the discount rate for CDS payment when a default event occurs to be a constant number, which is the initial risk-free rate, r_0.

$$CDS_{default} = (1-R)\left(\frac{D(0)}{A(0)}\right)^{\frac{v-\tilde{u}}{\sigma^2}}\left[\Phi\left(\frac{\ln(\frac{D(0)}{A(0)})-\tilde{u}T}{\sigma\sqrt{T}}\right)+\left(\frac{D(0)}{A(0)}\right)^{\frac{2\tilde{u}}{\sigma^2}}\Phi\left(\frac{\ln(\frac{D(0)}{A(0)})+\tilde{u}T}{\sigma\sqrt{T}}\right)\right] \tag{22}$$

where $\tilde{u} = \sqrt{v^2 + 2\sigma^2 r_0}$.

Moreover, we can obtain the recovery rate in Moody's Default & Recovery Database. The default probability curve can be calculated by the bootstrap procedure either by using the zero interest rate curve and the spread of CDS market quotes or simply by approximating the average default intensity, $\bar{\lambda}$, which is measured by the spread of the corporate bond yield divided by $1-R$. Therefore, given the fixed portfolio of bank assets and the bank's initial assets and debts, we can use a series of CDSs to calibrate the volatility of security, credit risks and loan interest rate elasticity by setting the premium leg equal to the default probability curve and the default leg to equation (22).

Deposit insurance corporations provide deposit insurance that guarantees the safety of depositor accounts and charges depository institutions with insurance premiums that maintain the deposit insurance fund. Because the deposit insurance corporation ultimately bears the risk of a bank default, it carries a substantial amount of risk if many banks default, as occurred during the financial crisis. Therefore, deposit insurance corporations can transfer banks' default risk to the credit markets via credit derivatives. A numerical experiment is designed to verify the hedge effect of deposit insurance by buying CDSs and to determine the optimal hedging ratio (Section 4.3).

4. Numerical experiments

4.1 Closure policies, interest rate risk and bank asset allocation

In this section, we numerically investigate the proposed model to understand how the value of deposit insurance under closure policies that consider early closure and forbearance varies with respect to critical parameters such as the capital forbearance threshold, the length of the grace period and the cross effects of the two. Moreover, we analyze how asset allocation, moral hazard and stochastic interest rate factors affect deposit insurance premiums. Based on the same parameters used by Lee et al. (2005) and Chuang et al. (2009) unless otherwise specified, the following parameters are used throughout: $A(0)=100$, $D(0)=90$, $\gamma=0.1$, $\omega=0.25$, $\sigma_S=0.3$, $\sigma_c=0.1$, $\sigma_r=0.01$, $\kappa=0.1$, $\theta=0.05$, $\phi=-0.5$, $T_1=1$, $\Delta=0.5$, $\alpha=1.087$, $\beta=0.97$ and $\eta=0.8$.

To clearly identify the effects of closure policies, risk-free rate elasticity and a bank's asset allocation on insurance premiums, we provisionally ignore the deposit rate spread by setting $\varepsilon=0$, such that the excess moral hazard risk of a bank's assets is zero, that is, $g(\varepsilon)=0$.

[Insert Table II here]

Table II depicts the relation between the deposit insurance premium and closure policies, crossed with the debt-to-asset ratio. The closure policies comprise the early closure and capital forbearance provisions. A regulator cannot examine a bank's business operations until the audit period ends, unless the bank cannot function. Therefore, the probability and cost of bankruptcy depend on maintaining the ratio η. As the sensitivity analysis demonstrates in Table II, the deposit insurance premium decreases with the maintenance ratio but increases with the debt-to-asset ratio. The lower maintenance ratio decreases the opportunity of early closure but increases future risk. Therefore, lower maintenance lowers the premium of the early closure component but increases the premium of the capital forbearance and grace components. Integrating these three components, we observe that premiums and maintenance ratios are adversely related. However, the premium is dominated by the debt-to-asset ratio because that ratio most directly reflects the risks of operations and bankruptcy. A higher debt-to-asset ratio represents a higher risk of bankruptcy and thereby requires a higher premium per deposit.

The forbearance provision is interpreted through β and Δ, which are the capital forbearance threshold and the grace period length, respectively. According to the analysis in Table II, the premium formula shows that when the value of β is lower, the risk of bankruptcy at audit time is lower, but the risk at the end of the grace period is higher. Combining these two components, we find that the premium decreases with higher capital forbearance thresholds. Conversely, due to the grace period component, the higher the Δ, the higher the premium is. This case can be considered a European put with a longer time to maturity and higher costs. Furthermore, if moral hazard in the grace period is considered, the premium rises because it increases the weight of bank assets allocated in investment positions and raises banks' operational risk during the grace period.

[Insert Table III here]

Table III provides an analysis of the risk-free rate controlled by risk-free rate elasticity, ϕ, and risk-free rate volatility, σ_r. In the case of fixed interest rate elasticity and fixed weights on securities, the deposit premium increases with risk-free rate volatility because it raises the risk of loan positions. However, the risk of variation in the risk-free rate appears less influential as elasticity approaches zero, whether it is positive or negative while approaching zero. We can infer that in the case of higher interest rate elasticity, the premium is more sensitive to interest rate volatility. As the debt-to-asset ratio increases, the influences of both risk-free rate elasticity and volatility on the premium are relatively high because the variation of bank assets becomes more sensitive when the probability of bankruptcy increases.

[Insert Table IV here]

Deposit premiums for the credit market (loan position) and the securities market (investment position) that cross alternative portfolio shares for bank asset allocation ω are reported in Table IV. Overall, an increase in credit risk or security market risk volatility is reflected in higher deposit premiums. It is apparent that if the weight of securities decreases and most assets are allocated in loan positions, credit risk has a significant impact on premiums. As expected, security market risk appears to have a growing influence as the weight of securities increases. This finding reveals that asset allocation is an important determinant of the deposit premium in the case of fixed market risk.

4.2 Scenario analysis of the deposit insurance premium for moral hazard risks

In this section, we focus on the deposit rate spread and its consequent excess risk, which increases the bank asset's volatility because of moral hazard. We assume the excess risk function of the deposit rate spread to be simply a constant proportion of the deposit rate spread; thus, $g(\varepsilon) = x\varepsilon$. The constant *x* represents the excess risk per unit of the deposit rate spread, which we call the moral hazard multiplier. Further, assuming that $x = 2$,[4] we determine the relation of deposit insurance to the deposit rate spread and the deposit insurance premium multiplier, ξ_ε.

[Insert Figure I here]

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[4] We will analyze the effect of the moral hazard multiplier on the deposit insurance premium in Table V.

The upper left of Figure I is the overall deposit insurance premium; the upper right, lower left and lower right of Figure I are the early closure component, the capital forbearance component and the grace period component, respectively. Under a fixed deposit rate spread ε, all the premium components are proportional to ξ_{ε} because ξ_{ε} is the penalty multiplier of the risk-based premium. Conversely, given a fixed premium multiplier, the overall deposit insurance premium and early closure component are increased with ε because the deposit rate spread includes the excess risks in a bank's assets into our model, thereby raising the premium.

However, with an increase in the deposit rate spread, the premiums of the capital forbearance and grace period components are first increased and then decreased. As the deposit rate spread begins to increase from 0, the premiums of the capital forbearance and grace period components increase because of the excess risk resulting from the deposit rate spread, thereby increasing the risks related to bank assets. When the volatility of the bank's assets rises to a specific level, the probability of early closure becomes large and dominates the probability of bankruptcy at the time of the audit and entrance capital forbearance. Thus, the premiums of these two components decrease due to the reduction of risk exposure, whereas the premiums of the early closure component increase. Because the premium increase increment of early closure is larger than the decreased increment of the capital forbearance and grace period components, the overall deposit insurance premium is positively related to the deposit rate spread.

[Insert Table V here]

A scenario analysis of deposit insurance that crosses the deposit rate spread, the moral hazard multiplier and the penalty multiplier is shown in Table V. As previously noted, the penalty multiplier ξ_{ε} is the result of multiplying the penalty ratio for the bank's moral hazard ξ_{bank} by the depositor's moral hazard, $\xi_{depositor}$. We assume the depositor's moral hazard multiplier is $1-10\varepsilon$; in other words, if the depositor wishes to save her money in the bank with the deposit rate spread ε, then she must bear 0.1% of the deposit loss in the event that the bank enters bankruptcy, per the deposit rate spread. This result implies that the higher the deposit rate spread, the higher the deductible proportion is that a depositor must bear.

The third column in Table V is the scenario for $\xi_{depositor} = 1-10\varepsilon$ and $\xi_{bank} = 1$, which ignores the penalty of the bank's moral hazard, such that $\xi_{\varepsilon} = 1-10\varepsilon$. We see that when the moral hazard multiplier is 0 (which implies that the deposit rate spread will not cause any excess risk for the bank's assets), the premium decreases with an increase in the deposit rate spread. This decrease occurs because the depositor takes on a part of the deposit exposure, which reduces the exposure of the deposit insurance corporation. However, in the case of moral hazard, the multiplier equals 1 or 3, and the deposit rate spread brings excess risk to the bank's assets; in other words, the volatility of the bank's assets increases as the deposit rate spread increases. At this point, because of the increase in the bank's assets, the increased premium increment is greater than the decreased increment due to the depositor bearing a part of the risk. Therefore, the premium increases with the deposit rate spread. Moreover, in the case of a depositor bearing the fixed deductible risk, premiums increase with the moral hazard multiplier because the higher moral hazard multiplier indicates a higher risk to the bank's assets.

The premiums are shown in the second column of Table V: $\xi_{depositor} = 1-10\varepsilon$ and ξ_{bank} is set as the reciprocal of $\xi_{depositor}$, such that $\xi_{\varepsilon} = 1$. This scenario is used to explain that the premium should decrease when the depositor bears a part of the deductible risk. However, if the insurer ignores the moral hazard multiplier (that is, if it treats ξ_{ε} as 1), the premium charged by the insurer actually includes the penalty of the bank's moral hazard because of the deposit rate spread. For instance, if the spread is 10 basis points (bps) and the moral hazard multiplier is 1, the risk-based premium should be 139.55 bps per dollar of deposit if $\xi_{\varepsilon} = \xi_{depositor} = 1 - 10\varepsilon$. When the insurer charges the premiums in the case of $\xi_{\varepsilon} = 1$, those premiums increase to 140.96 bps, and the difference, 1.41 bps, is the penalty for the bank's moral hazard.

[Insert Table VI here]

From previous analysis, we know that premiums are positively correlated with the spread and penalty multiplier of moral hazard, ξ_{ε}. We can further realize the composition of premiums with respect to the implied risk between penalty multipliers and the deposit rate spread by studying Table VI. Given the penalty multiplier of a bank's moral hazard and a deposit rate spread, the premium is decreased as the deductible proportion is increased because the insurer's exposure decreases. Conversely, given a deductible proportion, premiums do not increase or decrease consistently along with the deposit rate spreads. In our model, the penalty multiplier of a depositor's moral hazard is assumed to be $\xi_{depositor} = 1 - d\varepsilon$, where $d > 0$ is the deductible ratio. When the deposit rate spread increases, the penalty multiplier of the depositor's moral hazard decreases because of the lower $\xi_{depositor}$. However, increases in the deposit rate spread also raise the bank's asset risk because of the bank's risk-taking operations.

When $\xi_{depositor} = 1 - 10\varepsilon$, the premium is positively correlated with the deposit rate spread because the premium increment from the bank's risk is greater than the premium reductions due to the depositor's penalty multiplier. Because $\xi_{depositor} = 1 - 100\varepsilon$, the depositor bears much greater deductive risks. In this case, when the deposit rate spread increases, the premium increments from the bank's risk increase become smaller compared to the premium decrease due to the reduction of $\xi_{depositor}$.

We must emphasize that premiums are determined not only by the penalty multiplier of moral hazard ξ_ε but also by its composition. For example, when $\xi_{bank}=1$, $\xi_{depositor}=1-10\varepsilon$, $\varepsilon=50$ bps and $\xi_\varepsilon=0.95$, the premium is 189.47 bps. When $\xi_{bank}=1$, $\xi_{depositor}=1-50\varepsilon$ and $\varepsilon=10$ bps, ξ_ε also equals 0.95, and the premium is then 133.91 bps. This result is also based on the risk that results from the deposit rate spread and is dispersed by depositor deductibles.

4.3 Hedging analysis

This section designs a numerical experiment to test the hedging effect for deposit insurance corporations using CDS in the case of no capital forbearance. The parameters are set in accordance with Section 4.1, except that the minimum capital requirement is $\alpha=1$, the capital forbearance threshold is $\beta=1$, the grace period is $\Delta=0$, and the deposit rate spread is $\varepsilon=0$. Moreover, the recovery rate and average default intensity throughout the numerical analysis are $R=0.8$ and $\bar{\lambda}=0.5$, respectively. The default probability within the contract period $T=1$ is, therefore, $Q(t)=e^{-\bar{\lambda}t}$. For a Monte Carlo simulation, we generate 100,000 sample paths to simulate the payments as the secured bank becomes insolvent. The hedge ratio, h, stands for the ratio of the CDSs it holds for the initial value of the bank's asset value. The optimal hedge ratio of the deposit insurance cooperation can be written as follows:

$$\begin{aligned}&\min_h Var(P(0)-h\cdot CDS)\\&s.t.\ \ P(0)=P^a+P^c\end{aligned} \tag{21}$$

[Insert Figure II here]

In Figure II, the solid line represents the standard deviation of the cash flow that the deposit insurance corporation must pay when the secured bank fails; it falls within the range of 1 to 6. The left and right *y*-axis values represent the standard deviation and mean, respectively, of the discounted profit for 100,000 simulations. The *x*-axis value is the hedge ratio, which represents the ratio of the total notional amount of CDSs accounted for during the bank's initial asset valuation. The payment that the deposit insurance corporation must make if the secured bank fails is equivalent to the fair value of the deposit insurance premium because the premium charged should equal the expected loss under risk-neutral measures. The dashed line is the mean of the payment's expected loss for the deposit insurance corporation, which falls within the range of -4 to 3. If the payment is negative, the deposit insurance corporation has a cash inflow if the secured bank fails because it holds many of the failed bank's CDSs. However, the more CDSs the deposit insurance corporation holds, the higher its costs are. The fair value of CDS cost equals the difference between the payment in the corresponding hedge ratio and the payment without the hedge. Because we consider the cost of holding the CDSs, the expected payment of the deposit insurance corporation will adjust to the horizontal dotted line. Figure II shows that, in the case of equal deposit insurance payments, as the mean of deposit insurance payment approaches zero, the variance in payments decreases. This intuitive result reveals that the deposit insurance corporation can transfer the risk of bank failure by buying the corresponding secured bank's CDSs with the entire deposit insurance premium received.

5. Conclusion

Analyzing the premium considering the necessary conditions required by deposit insurance – including minimum capital requirements, capital forbearance thresholds, grace periods, early closure regularity, and the prevention of moral hazard – is a common issue in the literature. This paper constructs an explicit deposit insurance scheme and derives a closed-form pricing formula for fair premiums, which is a general form that breaks down into four special models in the literature.

Using an alternative debt-to-asset ratio, we conduct an analysis of the impact of policy instruments and interest rate elasticity and volatility on premiums that consists of three components: early closure, capital forbearance and grace period. We also verify the source of bank risk that consists of the weights of asset allocation and the risks of various markets, and we demonstrate the influence of these factors on premiums.

The numerical results show that the insurance premium increases quickly with the debt-to-asset ratio when the additional moral hazard in the grace period and the ratio of asset allocations in the investing position are considered because these factors will significantly raise bank risk. Moreover, the premium increases as the capital forbearance threshold decreases when the increment of the grace period component is larger than the decrement of the capital forbearance component.

Moral hazard is an important issue when considering deposit insurance. In this study, we quantify the moral hazard arising from the deposit rate spread and design a penalty mechanism both for banks (deriving from bank risk related to higher deposit premiums) and depositors (bearing a part of the deposit risk). Given a fixed deposit rate spread, the analysis shows that the premiums are positively correlated with a penalty multiplier that consists of the penalty ratio for a bank's moral hazard and the depositors' deductible proportion thereof. Moreover, the premiums increase with moral hazard multipliers and the penalty multiplier of a bank's moral hazard but decrease with the deductible ratio thereof. Because the increment of premiums due to moral hazard has a different impact for each premium component, we argue that the moral hazard should be evaluated under comprehensive deposit regulations to be able to demonstrate the exact risk to the premiums.

Scott Richardson documents these concepts in his *Financial Times* article, "A Market-Based Plan to Regulate Banks," and argues that the FDIC should use CDS prices to charge more premiums from riskier banks and determine the proper premium for its insurance via the active credit derivatives market. In light of this concept, this study indirectly evaluates the deposit insurance premium through CDSs. We capture a bank's risk through the active credit market in advance and then calculate the premium under Merton's pricing framework that considers the deposit insurance scheme. This market-based method estimates a bank's risk as the parameters that reflect the appropriate premium pricing formula.

The deposit insurance corporation issues deposit insurance without hedging and acts as the ultimate risk taker. We suggest that the deposit insurance corporation can transfer the default risk of banks to credit markets via credit derivatives, and we design a numerical experiment to verify the hedging effect of purchasing CDSs on deposit insurance. If the insurance corporation uses greater deposit insurance premiums to buy a secured bank's CDSs, the numerical results show that the risk it undertakes is smaller and that the use of credit derivatives can indeed transfer the risk of deposit insurance.

Appendix: The Closed-Form Solution for Deposit Insurance Premium

The risk-based premium of deposit insurance paid by the deposit insurer to the insured bank is the expected discounted insurance payoff under a risk-neutral probability measure and can be decomposed into three parts, as shown in equation (14). The first part is the default premium of the insured bank before the time of the audit. Second, the default premium of the insured bank will be made when the bank's value drops below the capital forbearance threshold at the time of the audit. Finally, the deposit insurance must pay the default premium when the regulator takes over the depository institution during or at the end of the grace period:

$$P(0) = E^Q\left[\frac{P(\tau)}{M(\tau)} I_{\{\tau<T_1\}}\right] + E^Q\left[\frac{P(T_1)}{M(T_1)} I_{\left\{\tau\ge T_1, \frac{A(T_1)}{D(T_1)}<\beta\right\}}\right] + E^Q\left[\frac{P(T_1)}{M(T_1)} I_{\left\{\tau\ge T_1, \beta<\frac{A(T_1)}{D(T_1)}<\alpha\right\}}\right].$$

To price the risk-based premium of deposit insurance, we show the asset value and the asset-debt ratio of the discounted bank, according to the model assumptions:

$$\frac{A(t)}{M(t)} \overset{d}{=} A(0)\exp\left\{-\frac{1}{2}\sigma^2 t + \sigma W^Q(t))\right\} \tag{A.1}$$

and

$$\frac{A(t)}{D(t)} \overset{d}{=} \frac{A(0)}{D(0)}\exp\left\{\left(-\frac{\sigma^2}{2} - \varepsilon\right)t + \sigma W_A^Q(t)\right\} = \frac{A(0)}{D(0)}\exp\left\{vt + \sigma W_A^Q(t)\right\},$$

where $D(t) = D(0)e^{\varepsilon t}M(t)$ denotes the deposit liability that increases with the money market account, $v = -\varepsilon - \sigma^2/2$, $\sigma = \sqrt{\omega^2\sigma_S^2 + (1-\gamma-\omega)^2\sigma_L^2 + g(\varepsilon)}$ represents the total risk of the bank's assets as weighted by loan position and investing position, $\sigma_L = \sqrt{\phi^2\sigma_r^2 + \sigma_c^2}$ represents the risk of the loan position that incorporates interest rate risk, σ_S is the secondary market risk, and $\overset{d}{=}$ indicates equal in distribution. The three components of the deposit insurance premium are derived in the following lemmas.

Lemma 1: The premium of the audit window component is given by the following:

$$E^Q\left[\frac{P(\tau)}{M(\tau)}I_{\{\tau<T_1\}}\right]$$

$$=\xi_\varepsilon(1-\eta)D(0)e^{\frac{v-u}{\sigma^2}\ln\left(\frac{\eta D(0)}{A(0)}\right)}\left[\Phi\left(\frac{\ln\frac{\eta D(0)}{A(0)}-uT_1}{\sigma\sqrt{T_1}}\right)+e^{\frac{2u}{\sigma^2}\ln\left(\frac{\eta D(0)}{A(0)}\right)}\Phi\left(\frac{\ln\frac{\eta D(0)}{A(0)}+uT_1}{\sigma\sqrt{T_1}}\right)\right]$$

where $v=-\varepsilon-\sigma^2/2$ and $u=\sqrt{v^2-2\sigma^2\varepsilon}.$

Proof:

According to the definition of early closure that $\tau=\inf\{t|A(t)\le\eta D(t)\}$ comes before T_1 and the theorem of the first passage time, we obtain the probability of default before the time of the audit as follows:

$$\Pr^Q\left(\tau<T_1\,|\,\frac{A(0)}{D(0)}>\eta\right)=\Pr^Q\left(\min_{0\le s\le T_1}\frac{A(s)}{D(s)}<\eta\left|\frac{A(0)}{D(0)}>\eta\right.\right)$$

$$=\Pr^Q\left(\min_{0\le s\le T_1}\left(vs+\sigma W^Q(s)\right)<\ln(\frac{\eta D(0)}{A(0)})\left|\frac{A(0)}{D(0)}>\eta\right.\right)$$

$$=\Phi\left(\frac{\ln(\frac{\eta D(0)}{A(0)})-vT_1}{\sigma\sqrt{T_1}}\right)+e^{\frac{2v}{\sigma^2}\ln\left(\frac{\eta D(0)}{A(0)}\right)}\Phi\left(\frac{\ln(\frac{\eta D(0)}{A(0)})+vT_1}{\sigma\sqrt{T_1}}\right)$$

Let $f_\tau(t)=\Pr^Q[\tau\in dt]=\frac{1}{\partial T_1}\partial\Pr^Q\left(\tau<T_1\,|\,\frac{A(0)}{D(0)}>\eta\right)\Bigg|_{\tau=T_1}$ be the density function of τ that occurs instantaneously. A further straightforward calculation yields $f_\tau(t)=\frac{|y|}{\sqrt{2\pi\sigma^2t^3}}e^{\frac{(y-vt)^2}{-2\sigma^2t}}$, where $y=\ln(\frac{\eta D(0)}{A(0)})$.

$$E^Q\left[\frac{P(\tau)}{M(\tau)}I_{\{\tau<T_1\}}\right]$$

$$=\xi_\varepsilon(1-\eta)D(0)E^Q\left[e^{\varepsilon\tau}I_{\{\tau<T_1\}}\right]$$

$$=\xi_\varepsilon(1-\eta)D(0)\int_0^{T_1}e^{\varepsilon t}\frac{|y|}{\sqrt{2\pi\sigma^2t^3}}e^{\frac{(y-vt)^2}{-2\sigma^2t}}dt$$

$$=\xi_\varepsilon(1-\eta)D(0)e^{\frac{v-u}{\sigma^2}\ln\left(\frac{\eta D(0)}{A(0)}\right)}\left[\Phi\left(\frac{\ln\frac{\eta D(0)}{A(0)}-uT_1}{\sigma\sqrt{T_1}}\right)+e^{\frac{2u}{\sigma^2}\ln\left(\frac{\eta D(0)}{A(0)}\right)}\Phi\left(\frac{\ln\frac{\eta D(0)}{A(0)}+uT_1}{\sigma\sqrt{T_1}}\right)\right]$$

Lemma 2: The deposit insurance premium of a capital forbearance component is calculated as follows:

$$E^{Q}\left[\frac{P(T_1)}{M(T_1)} I_{\left\{\tau\ge T_1,\frac{A(T_1)}{D(T_1)}<\beta\right\}}\right]=\xi_{\varepsilon} D(0)e^{\varepsilon T_1}\begin{bmatrix}\left(\Phi\left(c_1(\beta,v)\right)\right)-\Phi(c_1(\eta,v))\big) \\ -e^{\frac{2vB(\eta)}{\sigma^2}}\left(\Phi\left(c_2(\beta,\eta,v)\right)-\Phi\left(-c_1(\eta,-v)\right)\right)\end{bmatrix}$$

$$-A(0)\begin{bmatrix}\left(\Phi\left(c_1(\beta,\tilde{v})\right)-\Phi\left(c_1(\eta,\tilde{v})\right)\right) \\ -e^{\left(\frac{2v}{\sigma^2}+2\right)B(\eta)}\left(\Phi\left(c_2(\beta,\eta,\tilde{v})\right)-\Phi\left(-c_1(\eta,\tilde{v})\right)\right)\end{bmatrix}$$

where $B(x)=\ln\left(\frac{xD(0)}{A(0)}\right)$, $c_1(x,z)=\frac{B(x)-zT_1}{\sigma\sqrt{T_1}}$, $c_2(x,y,z)=\frac{B(x)-2B(y)-zT_1}{\sigma\sqrt{T_1}}$, $v=-\varepsilon-\sigma^2/2$, and $\tilde{v}=v+\sigma^2$.

Proof:

To compute the joint probability of the no-default event before the time of the audit and the default event at the time of the audit, we can also evaluate the joint probability of the asset-debt ratio $A(T_1)/D(T_1)$ lower than b_1 at maturity and the minimum of the ratio $\min_{0\le s\le T_1}\left(A(s)/D(s)\right)$ higher than b_2 before maturity under the risk-neutral measure as follows:

$$\Pr^{Q}\left(\frac{A(T_1)}{D(T_1)}<b_1,\ \min_{0\le s\le T_1}\frac{A(s)}{D(s)}>b_2\right)$$
$$=\Pr^{Q}\left(vT_1+\sigma W^{Q}(T_1)<\ln\left(\frac{b_1D(0)}{A(0)}\right),\ \min_{0\le s\le T_1}\left(vs+\sigma W^{Q}(s)\right)>\ln\left(\frac{b_2D(0)}{A(0)}\right)\right)$$
$$=\left[\Phi\left(\frac{\ln\left(\frac{b_1D(0)}{A(0)}\right)-vT_1}{\sigma\sqrt{T_1}}\right)-\Phi\left(\frac{\ln(\frac{b_2D(0)}{A(0)})-vT_1}{\sigma\sqrt{T_1}}\right)\right] \tag{A.2}$$
$$-e^{\frac{2v}{\sigma^2}\ln\left(\frac{b_2D(0)}{A(0)}\right)}\left[\Phi\left(\frac{\ln\left(\frac{b_1D(0)}{A(0)}\right)-2\ln\left(\frac{b_2D(0)}{A(0)}\right)-vT_1}{\sigma\sqrt{T_1}}\right)-\Phi\left(\frac{-\ln\left(\frac{b_2D(0)}{A(0)}\right)-vT_1}{\sigma\sqrt{T_1}}\right)\right]$$

where $\Phi(\cdot)$ denotes the cumulative function of standard normal distribution.

The default premium of the insured bank when the bank's value drops below the capital forbearance threshold at the time of the audit can be derived as follows:

$$E^{Q}\left[\frac{P(T_1)}{M(T_1)}I_{\left\{\tau\geq T_1,\frac{A(T_1)}{D(T_1)}<\beta\right\}}\right]$$

$$=E^{Q}\left[\xi_{\varepsilon}D(0)e^{\varepsilon T_1}I_{\left\{\tau>T_1,\frac{A(T_1)}{D(T_1)}<\beta\right\}}\right]-E^{Q}\left[\xi_{\varepsilon}A(0)e^{vT_1+\sigma W^{Q}(T_1)}I_{\left\{\tau>T_1,\frac{A(T_1)}{D(T_1)}<\beta\right\}}\right]$$

$$=\xi_{\varepsilon}e^{\varepsilon T_1}\left(D(0)\Pr^{Q}\left(\frac{A(T_1)}{D(T_1)}<\beta,\min_{0\leq s\leq T_1}\frac{A(s)}{D(s)}>\eta\right)-A(0)\Pr^{\tilde{Q}}\left(\frac{A(T_1)}{D(T_1)}<\beta,\min_{0\leq s\leq T_1}\frac{A(s)}{D(s)}>\eta\right)\right)$$

According to the Girsanov theorem, $\tilde{Q}$ is another measure related to the Q measure, and the Brownian motion under $\tilde{Q}$ will be $dW_{A,t}^{Q}=dW_{A,t}^{\tilde{Q}}+\sigma dt$. Appling the results of equation (A.2), we have the following:

$$E^{Q}\left[\frac{P(T_1)}{M(T_1)}I_{\left\{\tau\geq T_1,\frac{A(T_1)}{D(T_1)}<\beta\right\}}\right]$$

$$=\xi_{\varepsilon}D(0)e^{\varepsilon T_1}\left[\left(\Phi\left(c_1(\beta,v)\right)-\Phi(c_1(\eta,v))\right)-e^{\frac{2vB(\eta)}{\sigma^2}}\left(\Phi\left(c_2(\beta,\eta,v)\right)-\Phi\left(-c_1(\eta,-v)\right)\right)\right]$$

$$-\xi_{\varepsilon}A(0)e^{\varepsilon T_1}\left[\left(\Phi\left(c_1(\beta,\tilde{v})\right)-\Phi\left(c_1(\eta,\tilde{v})\right)\right)-e^{\left(\frac{2v}{\sigma^2}+2\right)B(\eta)}\left(\Phi\left(c_2(\beta,\eta,\tilde{v})\right)-\Phi\left(-c_1(\eta,-\tilde{v})\right)\right)\right]$$

Hence, we complete the proof of lemma 2.

Lemma 3: The deposit insurance premium of the grace period component is represented as the following:

$$E^{Q}\left[\frac{P(T_1)}{M(T_1)}I_{\left\{\tau>T_1,\beta<\frac{A(T_1)}{D(T_1)}<\alpha\right\}}\right]$$

$$=\xi_{\varepsilon}e^{\varepsilon T_1}\left(D(0)\begin{Bmatrix}N\left(c_1(\alpha,v),e_1(v),\delta\right)-N\left(c_1(\beta,v),e_1(v),\delta\right)\\ -e^{\frac{2vB(\eta)}{\sigma^2}}\left[N\left(c_2(\alpha,\eta,v),e_2(v),\delta\right)-N\left(c_2(\beta,\eta,v),e_2(v),\delta\right)\right]\end{Bmatrix}\right.$$

$$\left.+A(0)\begin{Bmatrix}N(c_1(\alpha,\tilde{v}),e_1(\tilde{v}),\delta)-N(c_1(\beta,\tilde{v}),e_1(\tilde{v}),\delta)\\ -e^{\left(\frac{2v}{\sigma^2}+2\right)B(\eta)}\left[N(c_2(\alpha,\eta,\tilde{v}),e_2(\tilde{v}),\delta)-N(c_2(\beta,\eta,\tilde{v}),e_2(\tilde{v}),\delta)\right]\end{Bmatrix}\right)$$

where $N(c,e,\delta)=\int_{-\infty}^{c}\Phi(\frac{e-\delta Z}{\sqrt{1-\delta^2}})\varphi(Z)dZ$, $\varphi(Z)$ represents the probability density function of standard normal distribution, $\delta=\sqrt{T_1/T_2}$, $v=-\varepsilon-\sigma^2/2$, $\tilde{v}=v+\sigma^2$,

$B(x)=\ln\frac{xD(0)}{A(0)}$, $c_1(x,z)=\frac{B(x)-zT_1}{\sigma\sqrt{T_1}}$, $c_2(x,y,z)=\frac{B(x)-2B(y)-zT_1}{\sigma\sqrt{T_1}}$, $e_1(v)=\frac{B(1)-vT_2}{\sigma\sqrt{T_2}}$, and $e_2(v)=\frac{B(1)-2B(\eta)-vT_2}{\sigma\sqrt{T_2}}$.

Proof:

$$E^Q\left[\frac{P(T_1)}{M(T_1)}I_{\left\{\tau>T_1,\beta<\frac{A(T_1)}{D(T_1)}<\alpha\right\}}\right]$$

$$=E^Q\left[\frac{\xi_\varepsilon}{M(T_1)}E^Q\left[\frac{M(T_1)}{M(T_2)}\max\{D(T_2)-A(T_2),0\}\middle|\beta<\frac{A(T_1)}{D(T_1)}<\alpha,\min_{0\le s\le T_1}\frac{A(s)}{D(s)}>\eta\right]\right]$$

$$=E^Q\left[\xi_\varepsilon D(0)e^{\varepsilon T_1}\Phi(b_2)I_{\left\{\tau>T_1,\beta<\frac{A(T_1)}{D(T_1)}<\alpha\right\}}\right]-E^Q\left[\xi_\varepsilon A(0)e^{vt+\sigma W^Q(t)}\Phi(b_1)I_{\left\{\tau>T_1,\beta<\frac{A(T_1)}{D(T_1)}<\alpha\right\}}\right]$$

$$=\xi_\varepsilon D(0)e^{\varepsilon T_1}E^Q\left[\Phi(b_2)I_{\left\{\tau>T_1,\beta<\frac{A(T_1)}{D(T_1)}<\alpha\right\}}\right]-\xi_\varepsilon A(0)e^{\varepsilon T_1}E^{\tilde{Q}}\left[\Phi(b_1)I_{\left\{\tau>T_1,\beta<\frac{A(T_1)}{D(T_1)}<\alpha\right\}}\right]$$

$$=\xi_\varepsilon e^{\varepsilon T_1}\left(D(0)\int_{B(\beta)}^{B(\alpha)}\int_{B(\eta)}^{x}\Phi(b_2)f(x,y)dydx-A(0)\int_{B(\beta)}^{B(\alpha)}\int_{B(\eta)}^{x}\Phi(b_1)f'(x,y)dydx\right)\qquad\text{(A.3)}$$

where $b_1=-\frac{\ln\frac{A(T_1)}{D(T_1)}-v\Delta}{\sigma\sqrt{\Delta}}$, $b_2=b_1-\sigma\sqrt{\Delta}$, and $B(z)=\ln\frac{zD(0)}{A(0)}$.

The $f(x,y)$ takes the first derivative of equation (A.2) with respect to $a^*=\ln\left(\frac{b_1D(0)}{A(0)}\right)$ and $b^*=\ln(\frac{b_2D(0)}{A(0)})$, and then we obtain the joint probability density as follows:

$$f(x,y)=\frac{\partial^2\Pr\left(vT_1+\sigma W^Q(T_1)<a^*,\min_{0\le s\le T_1}\left(vs+\sigma W^Q(s)\right)>b^*\right)}{\partial a^*\partial b^*}\Bigg|_{a^*=x,\ b^*=y}$$

$$=\frac{2(x-2y)}{\sigma^2T_1\sqrt{2\pi\sigma^2T_1}}e^{-\frac{(x-2y)^2}{2\sigma^2T_1}+\left(\frac{vx}{\sigma^2}-\frac{v^2T_1}{2\sigma^2}\right)}$$

$f'(x,y)$ is the joint probability density function of equation (A.2) under another measure $\tilde{Q}$, where the relative asset dynamic under the $\tilde{Q}$ measure is

$\frac{A(t)}{D(t)} = \frac{A(0)}{D(0)} \exp\left\{(\nu+\sigma^2)t + \sigma W^{\tilde{Q}}(t))\right\}$. Thus, as with $f(x,y)$, we can verify the following:

$$f'(x,y) = \frac{2(x-2y)}{\sigma^2 T_1 \sqrt{2\pi\sigma^2 T_1}} \exp\left\{-\frac{(x-2y)^2}{2\sigma^2 T_1} + \left(\frac{(\nu+\sigma^2)x}{\sigma^2} - \frac{(\nu+\sigma^2)^2 T_1}{2\sigma^2}\right)\right\}.$$

The first term of equation (A.3) can be computed as follows:

$$\xi_\varepsilon D(0) e^{\varepsilon T_1} \int_{B(\beta)}^{B(\alpha)} \int_{B(\eta)}^{x} \Phi(b_2) f(x,y) dy dx$$

$$= \xi_\varepsilon D(0) e^{\varepsilon T_1} \int_{B(\beta)}^{B(\alpha)} \int_{B(\eta)}^{x} \Phi\left(-\frac{\ln \frac{A(T_1)}{D(T_1)} + \nu\Delta}{\sigma\sqrt{\Delta}}\right) \left(\frac{2(x-2y)}{\sigma^2 T_1 \sqrt{2\pi\sigma^2 T_1}} e^{-\frac{(x-2y)^2}{2\sigma^2 T_1} + \left(\frac{\nu x}{\sigma^2} - \frac{\nu^2 T_1}{2\sigma^2}\right)}\right) dy dx$$

$$= \xi_\varepsilon D(0) e^{\varepsilon T_1} \int_{B(\beta)}^{B(\alpha)} \Phi\left(\frac{B(1) - x - \nu\Delta}{\sigma\sqrt{\Delta}}\right) \frac{1}{\sqrt{2\pi\sigma^2 T_1}} e^{\frac{\nu x}{\sigma^2} - \frac{\nu^2 T_1}{2\sigma^2}} \left(\int_{B(\eta)}^{x} \frac{2(x-2y)}{\sigma^2 T_1} e^{-\frac{(x-2y)^2}{2\sigma^2 T_1}} dy\right) dx$$

$$= \xi_\varepsilon D(0) e^{\varepsilon T_1} \left\{\begin{array}{l} \int_{B(\beta)}^{B(\alpha)} \Phi\left(\frac{B(1) - x - \nu\Delta}{\sigma\sqrt{\Delta}}\right) \frac{e^{-\frac{(x-\nu T_1)^2}{2\sigma^2 T_1}}}{\sqrt{2\pi\sigma^2 T_1}} dx \\ -e^{\frac{2\nu B(\eta)}{\sigma^2}} \int_{B(\beta)}^{B(\alpha)} \Phi\left(\frac{B(1) - x - \nu\Delta}{\sigma\sqrt{\Delta}}\right) \frac{e^{-\frac{(x-2B(\eta)-\nu T_1)^2}{2\sigma^2 T_1}}}{\sqrt{2\pi\sigma^2 T_1}} dx \end{array}\right\}$$

$$= \xi_\varepsilon D(0) e^{\varepsilon T_1} \left\{\begin{array}{l} \int_{c_1(\beta,\nu)}^{c_1(\alpha,\nu)} \Phi\left(\frac{B(1) - \sigma\sqrt{T_1} Z - \nu T_2}{\sigma\sqrt{\Delta}}\right) \frac{e^{-\frac{Z^2}{2}}}{\sqrt{2\pi}} dZ \\ -e^{\frac{2\nu B(\eta)}{\sigma^2}} \int_{c_2(\beta,\eta,\nu)}^{c_2(\alpha,\eta,\nu)} \Phi\left(\frac{B(1) - 2B(\eta) - \sigma\sqrt{T_1}\tilde{Z} - \nu T_2}{\sigma\sqrt{\Delta}}\right) \frac{e^{-\frac{\tilde{Z}^2}{2}}}{\sqrt{2\pi}} d\tilde{Z} \end{array}\right\}$$

$$= \xi_\varepsilon D(0) e^{\varepsilon T_1} \left\{\begin{array}{l} \left[N\left(c_1(\alpha,\nu), e_1(\nu), \delta\right) - N\left(c_1(\beta,\nu), e_1(\nu), \delta\right)\right] \\ -e^{\frac{2\nu B(\eta)}{\sigma^2}} \left[N\left(c_2(\alpha,\eta,\nu), e_2(\nu), \delta\right) - N\left(c_2(\beta,\eta,\nu), e_2(\nu), \delta\right)\right] \end{array}\right\}$$

where $Z = \left(x - \nu T_1\right) / \sigma\sqrt{T_1}$ and $\tilde{Z} = \left(x - 2B(\eta) - \nu T_1\right) / \sigma\sqrt{T_1}$ are changing variables to simplify the integrations. Similarly, the second term in equation (A.3) can be derived as the following:

$$\xi_\varepsilon A(0)e^{\varepsilon T_1}\int_{B(\beta)}^{B(\alpha)}\int_{B(\eta)}^{x}\Phi(b_1)f'(x,y)dydx$$

$$= \xi_\varepsilon A(0)e^{\varepsilon T_1}\left\{\begin{aligned}&N(c_1(\alpha,\tilde{v}),e_1(\tilde{v}),\delta) - N(c_1(\beta,\tilde{v}),e_1(\tilde{v}),\delta)\\ &-e^{\left(\frac{2v}{\sigma^2}+2\right)B(\eta)}\left(N(c_2(\alpha,\eta,\tilde{v}),e_2(\tilde{v}),\delta) - N(c_2(\beta,\eta,\tilde{v}),e_2(\tilde{v}),\delta)\right)\end{aligned}\right\}$$

Hence, we complete the calculation of lemma 3.

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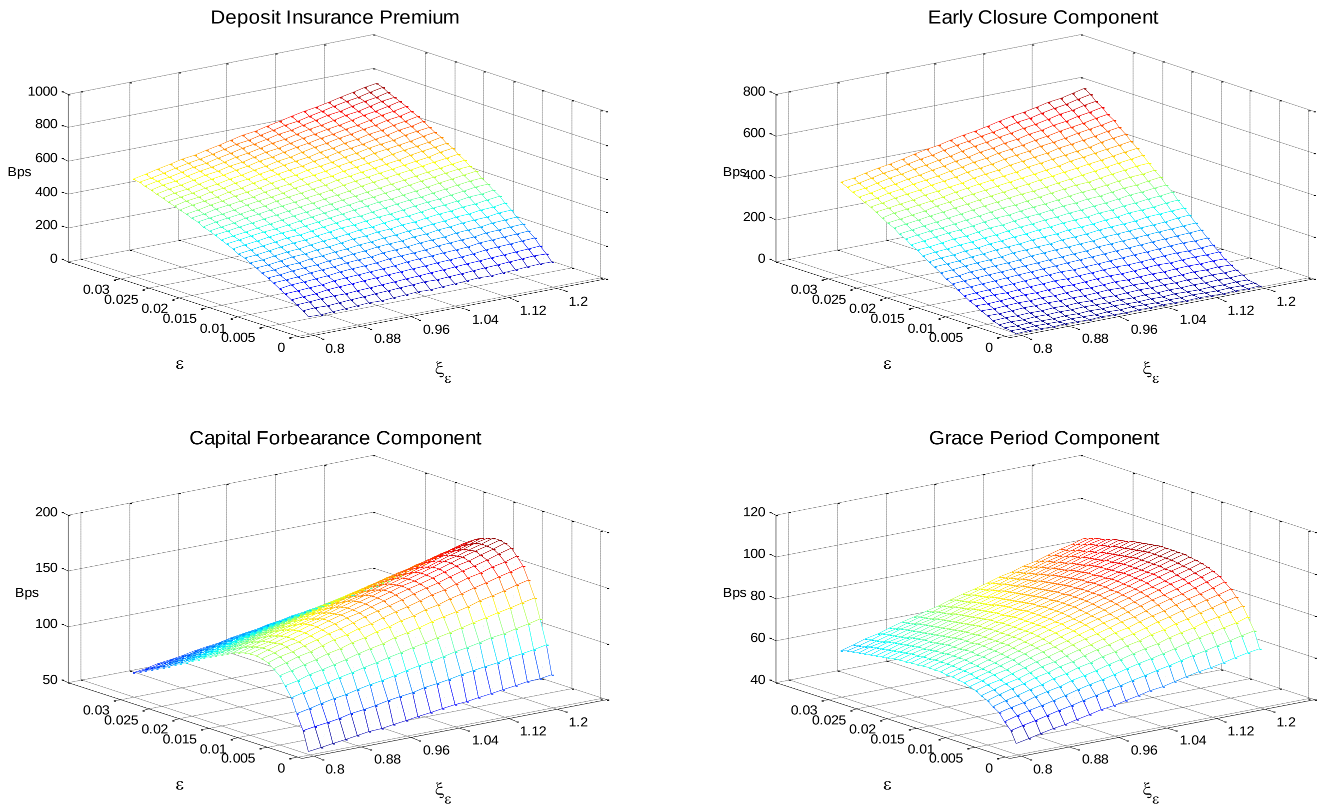

Figure I. Deposit Insurance Premium Crossing Deposit Rate Spreads and Penalty Multiplier

The upper left is the overall deposit insurance premium and the upper right, lower left, and lower right of Figure I are the early closure component, the capital forbearance component and the grace period component, respectively.

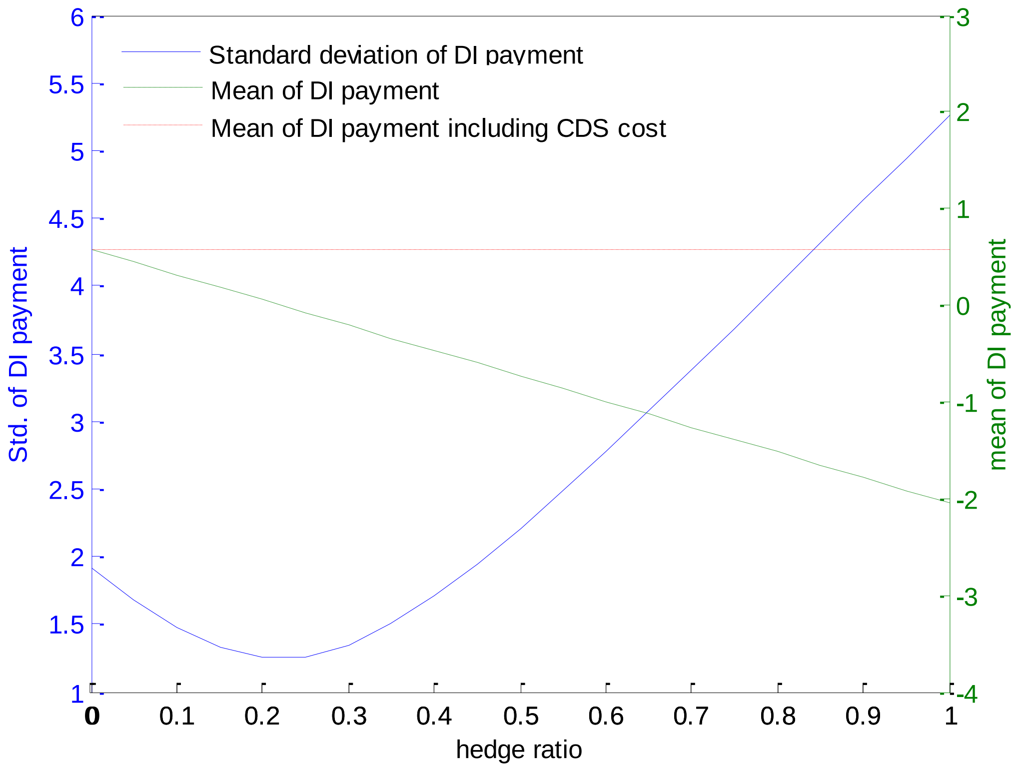

Figure II: The Standard Deviation and Average of the Payment Made by the Deposit Insurance Corporation under Various Hedge Ratios.

In Figure II, the solid line represents the standard deviation of the cash flow that the deposit insurance corporation must pay if the secured bank fails, which falls within the range of 1 to 6. The dashed line is the mean of the payment's expected loss to the deposit insurance corporation, which falls within the range of -4 to 3. The left and right *y*-axis values represent the standard deviation and mean of the discounted profit for 100,000 simulations, respectively, and the *x*-axis value is the hedge ratio, which represents the ratio of the total notional amount of CDSs accounted for during the bank's initial asset valuation. Because we consider the cost of holding the CDSs, the expected payment of the deposit insurance corporation will adjust to the horizontal dotted line.

Table I: Asset Allocation of FDIC-insured Commercial Banks from 1994 to 2013

Table I presents the asset allocation of all commercial banks the FDIC insured from 1994 to 2013 and indicates that the total percentage of bank assets in reserves, securities, and loans is greater than 90%. The data source is the statistical reports of the financial statements of FDIC-insured institutions from the FDIC website, and the dollar figures are in billions. The numbers in parentheses represent the proportion of the correspondent items to total assets. “Reserves” represents the “cash and due from depository institutions” from the banking report, and “Securities” includes the “securities”, “Federal funds sold & reverse repurchase agreements”, and “trading asset account” of the balance sheet from the FDIC banking report.

	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
Total assets	4,012	4,315	4,582	5,019	5,443	5,735	6,246	6,552	7,077	7,602
Reserves	304	307	336	355	357	366	370	390	384	387
	(7.57)	(7.10)	(7.33)	(7.08)	(6.55)	(6.39)	(5.92)	(5.96)	(5.42)	(5.10)
Securities	1,166	1,209	1,205	1,430	1,544	1,530	1,663	1,793	2,044	2,237
	(29.06)	(28.02)	(26.31)	(28.50)	(28.36)	(26.68)	(26.63)	(27.37)	(28.88)	(29.42)
Loans	2,308	2,553	2,761	2,920	3,179	3,430	3,751	3,812	4,079	4,352
	(57.52)	(59.15)	(60.27)	(58.18)	(58.42)	(59.81)	(60.06)	(58.18)	(57.64)	(57.25)

	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
Total assets	8,416	9,041	10,092	11,176	12,309	11,823	12,065	12,649	13,391	13,670
Reserves	388	400	433	482	1,042	977	923	1,196	1,334	1,632
	(4.61)	(4.43)	(4.29)	(4.31)	(8.46)	(8.26)	(7.65)	(9.45)	(9.96)	(11.94)
Securities	2,441	2,515	2,815	3,104	3,374	3,308	3,530	3,713	3,978	3,753
	(29.00)	(27.81)	(27.90)	(27.78)	(27.41)	(27.98)	(29.25)	(29.36)	(29.71)	(27.46)
Loans	4,833	5,313	5,913	6,537	6,682	6281	6,377	6,540	6,896	7,120
	(57.43)	(58.77)	(58.59)	(58.49)	(54.28)	(53.13)	(52.85)	(51.71)	(51.50)	(52.08)

Table II: A Scenario Analysis of the Deposit Insurance Premium with Closure Policies

The early closure provision is determined by the maintain ratio, η, and the forbearance provision is interpreted by β and Δ, which are the capital forbearance thresholds. DI refers to the insurance premium per deposit in basis points. The basic setting of the deposit insurance contract's time to maturity is assumed to be one year, the minimum capital requirement α is set at 1.087, the capital forbearance threshold β = 0.97, the maintain ratio η = 0.8, and the grace period Δ=0.5. The bank's asset allocation in its investment position is the proportion of ω = 0.25 and its reserve asset ratio γ = 0.1. The volatility of the security market, credit market and interest rate are σ_s = 0.3, σ_c = 0.1, and σ_r = 0.01, respectively. The interest rate elasticity is φ = -0.5. The weight on securities is $\tilde{\omega}$ = 0.35 during the grace period when there is a moral hazard. Ignore the impact of the deposit rate spread; in other words, ε = 0.

	Maintain ratio (η)			Forbearance threshold (β)			Grace period (Δ)		
	0.85	0.9	0.95	0.9	0.95	1	0.25	0.5	1
Debt-to-asset ratio=0.88									
DI premium	88.52	88.23	80.52	92.75	91.15	78.45	70.72	88.52	117.26
Early closure component	5.99	21.17	38.93	0.97	0.97	0.97	0.97	0.97	0.97
Capital forbearance component	36.94	21.79	2.48	12.97	33.45	48.49	41.95	41.95	41.95
Grace period component	45.60	45.28	39.11	78.80	56.73	28.99	27.80	45.60	74.34
DI with moral hazard	127.73	127.40	117.66	136.09	132.64	111.31	111.53	127.73	157.21
Debt-to-asset ratio=0.90									
DI premium	125.49	124.88	111.54	131.61	129.21	112.13	103.96	125.50	159.98
Early closure component	11.96	37.56	61.95	2.21	2.21	2.21	2.21	2.21	2.21
Capital forbearance component	56.69	31.12	3.14	22.94	54.39	75.26	66.44	66.44	66.44
Grace period component	56.84	56.20	46.45	106.46	72.60	34.65	35.31	56.85	91.33
DI with moral hazard	172.79	172.07	155.13	185.41	179.95	150.58	153.23	172.80	208.01
Debt-to-asset ratio=0.92									
DI premium	172.03	170.80	148.92	180.42	177.00	155.26	147.24	172.05	211.47
Early closure component	22.49	62.96	93.54	4.71	4.71	4.71	4.71	4.71	4.71
Capital forbearance component	82.15	41.67	3.68	38.20	83.81	111.18	99.93	99.93	99.93
Grace period component	67.39	66.18	51.70	137.50	88.48	39.37	42.60	67.41	106.83
DI with moral hazard	226.16	224.67	196.87	244.12	235.98	197.84	203.68	226.18	266.29

Table III: Deposit Insurance Premium with Stochastic Risk-free Rate

This table provides an analysis of the risk-free interest rate controlled by interest rate elasticity, ϕ, and interest rate volatility, σ_r. DI refers to the insurance premium per deposit in basis points. The contract time to maturity is assumed to be one year, the initial debt-to-asset ratio is 0.9, the weights on investment position are ω = 0.25, and the reserve asset ratio is $\gamma = 0.1$. The maintain ratio is $\eta = 0.8$, the minimum capital requirement, α, is set at 1.087, β= 0.97, and the length of the grace period is $\Delta = 0.5$. The volatility of the security market, the credit market and the interest rate are σ_s= 0.3 and $\sigma_c = 0.1$, respectively. The deposit rate spread is $\varepsilon = 0$.

Interest rate elasticity (φ)	-0.5	-0.5	-0.5	-0.6	-0.3	0	0.3
Interest rate volatility (σ_r)	0.01	0.05	0.1	0.01	0.01	0.01	0.01
Debt-to-asset ratio = 0.88							
DI premium	88.52	91.84	102.18	88.58	88.43	88.38	88.43
Early closure component	0.97	1.15	1.85	0.98	0.97	0.97	0.97
Capital forbearance component	41.95	44.05	50.62	41.99	41.89	41.86	41.89
Grace period component	45.60	46.64	49.71	45.62	45.57	45.56	45.57
Debt-to-asset ratio = 0.90							
DI premium	125.50	129.44	141.63	125.57	125.39	125.33	125.39
Early closure component	2.21	2.56	3.90	2.22	2.20	2.20	2.20
Capital forbearance component	66.44	69.08	77.14	66.49	66.37	66.33	66.37
Grace period component	56.85	57.80	60.59	56.86	56.82	56.81	56.82
Debt-to-asset ratio = 0.92							
DI premium	172.05	176.57	190.43	172.14	171.93	171.86	171.93
Early closure component	4.71	5.37	7.76	4.72	4.69	4.68	4.69
Capital forbearance component	99.93	102.98	112.11	99.98	99.84	99.80	99.84
Grace period component	67.41	68.22	70.56	67.43	67.39	67.38	67.39

Table IV: Deposit Insurance Premium with Asset Allocation

The deposit premiums for the credit market and security market risk cross alternative portfolio shares, ω, for the bank assets in investment position. DI refers to insurance premium per deposit in basis points. The contract time to maturity is assumed to be one year, and the other parameters are set as follows: $A(0) = 100$, $D(0) = 90$, and the initial debt-to-asset ratio is 0.9. The minimum capital requirement, α, is set at 1.087, β= 0.97, the length of the grace period is $\Delta = 0.5$, and the maintain ratio is $\eta = 0.8$. The reserve asset ratio is $\gamma = 0.1$. The volatility of the interest rate is σ_r= 0.01, and the interest rate elasticity is φ= -0.5. The deposit rate spread is $\varepsilon = 0$.

Credit risk (σ_c)	0.1	0.1	0.1	0.05	0.1	0.2
Security market risk (σ_s)	0.05	0.1	0.2	0.3	0.3	0.3
Weights on securities $\omega = 0.1$						
DI premium	70.65	71.87	76.77	11.24	84.91	205.92
Early closure component	0.10	0.11	0.16	0.00	0.29	18.70
Capital forbearance component	30.19	30.95	34.03	1.63	39.28	114.86
Grace period component	40.36	40.81	42.57	9.61	45.35	72.37
Weights on securities $\omega = 0.3$						
DI premium	29.04	39.42	83.17	112.11	153.78	218.24
Early closure component	0.00	0.00	0.26	1.27	5.65	23.23
Capital forbearance component	7.79	12.57	38.15	57.42	84.97	120.82
Grace period component	21.25	26.85	44.77	53.42	63.16	74.19
Weights on securities $\omega = 0.5$						
DI premium	8.13	33.13	152.21	304.17	318.51	341.77
Early closure component	0.00	0.00	5.40	70.46	80.78	98.82
Capital forbearance component	0.94	9.58	83.97	149.45	152.15	155.41
Grace period component	7.19	23.54	62.84	84.26	85.58	87.53

Table V: Deposit Insurance Premium with Penalty Multiplier and Moral Hazard Multiplier

DI refers to the insurance premium per deposit in basis points. The contract time to maturity is assumed to be one year, the minimum capital requirement, α, is set at 1.087, the capital forbearance threshold is β = 0.97, the maintain ratio is η = 0.8, and the length of the grace period is Δ = 0.5. The weights on investment position are ω = 0.25, and the reserve asset ratio is γ = 0.1. The volatility of the security market, credit market and interest rate are σ_s = 0.3, σ_c = 0.1, and σ_r = 0.01, respectively. The interest rate elasticity is φ = -0.5.

	$\xi_\varepsilon = 1 - 10\varepsilon$				$\xi_\varepsilon = 1$			
Deposit rate spread ε	0	1 bps	10 bps	50 bps	0	1 bps	10 bps	50 bps
Moral hazard multiplier = 0								
DI premium	125.5	125.39	124.42	119.63	125.5	125.52	125.68	125.92
Early closure component	2.21	2.2	2.12	1.77	2.21	2.2	2.14	1.87
Capital forbearance component	66.44	66.34	65.4	61.2	66.44	66.4	66.06	64.43
Grace period component	56.85	56.85	56.9	56.65	56.85	56.91	57.48	59.63
Moral hazard multiplier = 1								
DI premium	125.5	126.95	139.72	190.3	125.5	127.07	141.13	200.31
Early closure component	2.21	2.34	3.66	14.06	2.21	2.34	3.7	14.8
Capital forbearance component	66.44	67.38	75.57	105.11	66.44	67.45	76.33	110.64
Grace period component	56.85	57.23	60.49	71.13	56.85	57.29	61.1	74.87
Moral hazard multiplier = 3								
DI premium	125.5	130.05	169.37	311.18	125.5	130.18	171.08	327.56
Early closure component	2.21	2.62	8.56	77.63	2.21	2.62	8.65	81.72
Capital forbearance component	66.44	69.45	94.28	147.82	66.44	69.52	95.23	155.6
Grace period component	56.85	57.98	66.53	85.73	56.85	58.03	67.2	90.24

Table VI: Deposit Insurance Premium Across the Deposit Rate Spread and Penalty Multiplier

DI refers to the insurance premium per deposit in basis points. The contract time to maturity is assumed to be one year, the minimum capital requirement, α, is set at 1.087, the capital forbearance threshold is β = 0.97, the maintain ratio is η = 0.8, and the length of the grace period is Δ = 0.5. The weights on the investment position are ω = 0.25, and the reserve asset ratio is γ = 0.1. The volatility of the security market, credit market and interest rate are σ_s = 0.3, σ_c = 0.1, and σ_r = 0.01, respectively. The interest rate elasticity is φ = -0.5. The moral hazard multiplier is x=1

	$\xi_{bank}=1$			$\xi_{bank}=1.2$		
Deposit rate spread ε	1 bps	10 bps	50 bps	1 bps	10 bps	50 bps
$\xi_{depositor}=1-10\varepsilon$						
DI penalty multiplier ξ_ε	0.999	0.99	0.95	1.1988	1.188	1.14
DI premium	126.95	139.72	190.30	152.34	167.66	228.35
Early closure component	2.34	3.66	14.06	2.80	4.40	16.87
Capital forbearance component	67.38	75.57	105.11	80.85	90.68	126.13
Grace period component	57.23	60.49	71.13	68.68	72.59	85.35
$\xi_{depositor}=1-50\varepsilon$						
DI penalty multiplier ξ_ε	0.995	0.95	0.90	1.194	1.14	0.90
DI premium	126.44	134.07	150.23	151.73	160.89	180.28
Early closure component	2.33	3.51	11.10	2.79	4.22	13.32
Capital forbearance component	67.11	72.51	82.98	80.53	87.02	99.58
Grace period component	57.00	58.05	56.15	68.40	69.66	67.38
$\xi_{depositor}=1-100\varepsilon$						
DI penalty multiplier ξ_ε	0.95	0.75	0.50	1.188	1.08	0.60
DI premium	125.80	127.02	100.16	150.96	152.42	120.19
Early closure component	2.31	3.33	7.40	2.78	4.00	8.88
Capital forbearance component	66.77	68.70	55.32	80.13	82.44	66.38
Grace period component	56.72	54.99	37.43	68.06	65.99	44.92