Optimal Stress Tests and Diversification*

Keshav Dogra[†]
Federal Reserve Bank of New York

Keeyoung Rhee[‡]
Korea Development Institute

January 15 2017

Preliminary and Incomplete

Abstract

We present a model to study whether a regulator should reveal imperfect information about banks' financial health or not. Stress tests can restore market confidence in banks. However, the regulator's information is inevitably imperfect, thereby misclassifying some financially sound banks as risky. To avoid being misclassified, banks will choose portfolios that the regulator deems safe, making the financial system less diversified than without the stress tests. As a consequence, while it is always ex post optimal to reveal stress test results, this policy is not ex ante optimal. We show that the ex-ante optimal policy is non-monotonic in the underlying economic state: information should be revealed either in very good times or in very bad times.

JEL classification: D82, G11, G18

Keywords: Stress tests, optimal disclosure, portfolio diversification

*The views expressed in this paper are those of the authors and do not necessarily represent those of the Federal Reserve Bank of New York, the Federal Reserve System, or Korea Development Institute. Heehyun Lim provided able research assistance.

[†]Keshav.Dogra@ny.frb.org

[‡]ky.rhee829@gmail.com

1 Introduction

Since the 2008 financial crisis, stress tests have become a key tool of supervisory policy. The 2009 Supervisory Capital Assessment Program (SCAP) tested whether the largest U.S. bank holding companies would have adequate capital in the event of a severe recession. The bank-specific results of the test, including projected losses by asset type and capital shortfalls, were disclosed to the public. One goal of stress tests is to reduce investors' uncertainty about banks' future losses and capital requirements, making them more willing to invest; and indeed, several authors credit the SCAP with restoring confidence in the financial system (Bernanke (2013); Morgan, Peristiani and Savino (2014)). Following the success of the SCAP, the Dodd-Frank Act requires the Federal Reserve to conduct annual stress tests of large financial institutions.

But stress tests are inevitably imperfect, because regulators have limited information about both the risks facing the economy, and the exposure of particular asset classes to these risks.[1] Frame, Gerardi and Willen (2015) present a cautionary example; the Office of Federal Housing Enterprise Oversight (OFHEO) conducted a risk-based capital stress test for government-sponsored enterprises (GSE) such as Fannie Mae and Freddie Mac, but it failed to detect the GSEs' risk and insolvency. Frame, Gerardi and Willen (2015) show that the OFHEO was too optimistic, both about aggregate risk (their adverse house price scenario was less severe than the actual housing bust) and about Fannie and Freddie's exposure to this risk (their model substantially underpredicted defaults).

Because stress test results are imperfect, releasing them may induce banks to make inefficient investment decisions. Since banks may be incorrectly classified as 'risky' by the regulator, they may choose their portfolios to avoid this penalty. As a result, banks may not make investment decisions based on their own information. Rather, they may withdraw their (possibly superior) model of measuring financial risk and adapt to the regulator's model. Such a "model monoculture" may not only lead to inefficient investments, but also induce banks to herd in financial markets, increasing systemic risks.[2]

We present a model to ask whether stress tests can have these unintended consequences;

[1] In his speech at the "Maintaining Financial Stability: Holding a Tiger by the Tail" financial markets conference, Ben Bernanke pointed out another limit of stress tests, namely that they struggle to measure bank-specific risk:

> Another challenge is that our stress scenarios cannot encompass all of the risks that banks might face. For example, although some operational risk losses, such as expenses for mortgage put-backs, are incorporated in our stress test estimates, banks may face operational, legal, and other risks that are specific to their company or are otherwise difficult to estimate.

[2] Gillian Tett in the Financial Times claimed that routinization of stress tests may induce banks to share a similar view on how to measure and manage their financial risks, but regulators may not take this possibility into account:

> The point about a model monoculture is that it makes risk models pretty useless. As Donald MacKenzie of Edinburgh University notes, what models cannot measure is the chance that banks all act as a herd – creating financial panics.

and, if so, how the regulator should optimally disclose stress test results. In the model, outside investors are fully rational, risk averse, and their incentives are aligned with the banks. Banks choose ex ante whether to invest in a 'good' project, or a 'diversifying' project, which pays off in a different state of the world. There are also a random measure of 'bad' banks, who act mechanically, mimicking good and diversifying banks in order to borrow from outside investors and invest in unproductive projects. To initiate projects, banks need to borrow from outside investors in a capital market, but they face an adverse selection problem. Specifically, the outside investors cannot observe which type each bank is – whether it is good, diversifying, or bad – but only know the fraction of each type present in the market.

A key assumption is that a regulator can provide additional – but imperfect – information to the investors. Specifically, we assume that the regulator can correctly identify good banks, but cannot distinguish between diversifying and bad banks. The regulator can disclose its superior information when the banks go to the market to fund their projects, but such disclosure has different effects on adverse selection for 'good' and 'diversifying' banks. On the one hand, the regulator's information fully separates the good banks from the others – in particular, from bad banks – and removes the adverse selection problem faced by good banks. Thus good banks can successfully fund their projects in the capital market. On the other hand, the diversifying banks are lumped together with only the bad banks. Consequently, if the population of the bad banks is large, the diversifying banks may not be able to borrow in the capital market due to the severe adverse selection problem.

We analyze what the regulator's optimal disclosure policy will be from both an ex ante and an ex post perspective. We first show that if the regulator ex post decides whether to release its superior information, it is always (weakly) socially desirable to publicize the information. If there are few bad banks in the market, adverse selection problems will be mild, whether the information is released or not, so both 'good' banks and 'diversifying' banks can borrow in capital markets. By contrast, suppose there are so many bad banks that good and diversifying banks cannot finance their projects in the market. In this case, the regulator can improve social welfare by releasing its superior information, because this removes the lemons problem faced by the good banks, restoring their market access.

However, if the regulator can commit, before the banks choose projects, whether to share its information in particular states of the world, it is not always optimal to share information. Releasing the information only alleviates the adverse selection problem faced by 'good' banks, and therefore encourages most banks to invest in the 'good' project ex ante. This under-diversified portfolio exposes the economy to a higher risk – in the event that the 'good' project fails – than would obtain without the regulator's information, thereby reducing ex-ante social welfare.

Our main finding is that the ex-ante optimal disclosure policy is non-monotonic with respect to economic conditions, captured by the population of bad banks in the market. Specifically, we show that the regulator should commit to release its superior information if and only if the adverse selection problem at the capital market is either relatively mild (the fraction of the 'bad' banks is below a threshold) or relatively severe (the fraction of the bad banks is above another threshold). The regulator commits to release information in states of the world where adverse selection problems are severe, in order to partially activate the capital market for the 'good' banks. However, such a disclosure policy benefits only the 'good' banks, which inefficiently increases banks' choice of the 'good' projects. To mitigate this problem, the regulator refrains from releasing its information in states with moderate adverse selection problems.

Interestingly, it is also optimal for the regulator to disclose its information in states of the world where there are very few bad banks in the market. In these states, telling investors that a bank is not 'good' does not necessarily imply that the bank is 'bad'; instead, it is very likely to be 'diversifying'. After receiving the regulator's information, the investors prefer to fund these 'not good' banks because 'diversifying' banks are relatively scarce (relative to an equilibrium without any information revelation). Knowing this, the regulator commits to release its information in the states with mild adverse selection problems, as a way to increase diversification.

Besides reducing opacity, another important justification for stress tests is that low levels of bank capital impose costs on the wider economy, which are not internalized by individual banks. In order to ask whether this affects our main result – that regulators should not always disclose stress test results, because this would reduce diversification – we modify our model, assuming that when bank' projects fail, they must sell productive assets in order to repay depositors. This creates a pecuniary externality, because banks do not internalize that the more assets they sell, the lower the price other banks receive for their assets, and the greater the efficiency losses from misallocation of the productive assets (Lorenzoni (2008); Korinek (2011)). We show that this externality makes the social planner more risk-averse than private agents, strengthening our main result: in the economy with externalities, it is optimal to release stress tests even less often, and induce even more diversification, than in the model without externalities.

Our main result, that regulators should commit to sometimes refrain from releasing stress test results, depends on the assumption that stress tests are imperfect, misclassifying some diversifying banks as risky. It is therefore interesting to ask: given that stress tests inevitably contain *some* errors, what *kind* of errors are most harmful? To answer this question, we augment the model by allowing the banks to choose between different signal structures, which can be interpreted as different stress test scenarios. Ex post, the regulator always prefers to use more discriminating signals, which clearly separate good banks from bad. Ex ante, however, it may be desirable to use less discriminating signals, which misclassify some bad banks as good. By allowing some bad banks to pass the stress test, this increases the average quality of those who fail, reducing the penalty associated with investing in a diversifying project (which always fails the stress test).

1.1 Related literature

A growing theoretical literature discusses the benefits and costs of disclosing stress test results. Goldstein and Sapra (2013) and Leitner (2014) survey four arguments against disclosure of stress test results: full disclosure of stress test results may reduce risk-sharing among financial firms a la Hirshleifer (1971) (Goldstein and Leitner, 2015); forcing firms to disclose their financial status too often may encourage short-termism (Gigler et al., 2014); public information provided by stress tests may crowd out private information held by individual creditors (Morris and Shin, 2002); and a regulator's disclosure of stress tests may adversely affect its ability to learn from the market because the public information reduces incentives for investors in the market to acquire private information, making market prices less informative (Bond and Goldstein, 2015). We complement this literature by focusing on a different potential cost of stress tests, namely that by misclassifying some banks, they may undermine diversification.

Our finding that disclosure is optimal in a crisis, but not in normal times, is shared with a number of recent papers which arrive at this result for different reasons. (Goldstein and Leitner, 2015) also find that no disclosure is optimal in normal times, because of the Hirshleifer effect: while the financial system has enough capital to share risk, revealing banks' liquidity shocks would preclude insurance against such risks. Partial disclosure is optimal in a crisis, because banks do not have enough capital to share risk. Bouvard, Chaigneau and Motta (2015) study how disclosure affects financial stability when banks face rollover risk. Disclosure is desirable in a crisis, because it can prevent bank runs, but undesirable in normal times, because it can cause bank runs. These papers argue that disclosure is not always optimal ex post, taking the distribution of banks' 'type' as given. Our contribution is to ask how the anticipation of stress tests might change banks' investment in projects ex ante. To this end, we present a model in which disclosure is always optimal ex post, and argue that it may not be optimal ex ante, since it reduces diversification.

While our main result - that releasing public information reduces diversification - may seem reminiscent of Morris and Shin (2002), the mechanism in our paper is different. In Morris and Shin (2002), public information reduces diversification because agents' actions are strategic complements, and they coordinate on the public signal. In our paper, banks have no intrinsic coordination motive. If anything, they have an intrinsic motive for diversification, because risk-averse households reward banks for investing in assets that pay off when consumption is low. However, disclosure of public information causes outside investors to rationally punish banks for investing differently from the herd, because those banks may be risky.

Our results also relate to the recent empirical literature on stress tests. Morgan, Peristiani and Savino (2014) and Flannery, Hirtle and Kovner (2015) present evidence that the Supervisory Capital Assessment Program (SCAP) stress tests in 2009 were informative: banks with larger capital gaps experienced more negative unexpected returns. This empirical finding is consistent with one of the ex-post optimality of full revelation of the regulator's information in our model. On the contrary, Glasserman and Tangirala (2015) document that routinized stress tests – such as the Comprehensive Capital Analysis and Review (CCAR) and the Dodd-Frank Act Stress Testing (DFAST) program – become less informative over time. Their empirical evidence suggests that unconditional releasing of the regulator's information may not be effective as intended, consistent with our main finding that the ex-ante optimal disclosure is non-monotonic to the severity of the adverse selection problem in the market.

Some researchers analyze influences of other policy instruments on the effectiveness of bank stress tests. Spargoli (2013) argues that disclosure of negative stress test results either reduces lending or requires costly bailouts. Faria-e Castro, Martinez and Philippon (2015) make a similar point, arguing that disclosure can create inefficient bank runs, unless there is a fiscal backstop.

2 Model

2.1 Setup

Consider a three-period economy ($t = 0, 1, 2$) in which *(i)* a continuum of banks can undertake long-term financial projects which create stochastic returns at $t = 2$; *(ii)* a continuum of investors fund those projects by purchasing bonds issued by the banks; *(iii)* a regulator, who seeks to maximize social welfare, decides whether or not to reveal information about banks at date 1. Nature randomly draws an underlying state of the economy $\omega \in \{G, D\}$ where $\omega = G$ with probability $p \in (0, 1)$. No one in this economy knows what the true state is until date 2.

At $t = 0$, a continuum of banks with measure one individually choose which project to undertake. Each individual bank can choose one of two projects $\theta \in \{\gamma, \delta\}$: project γ returns one unit of cash flow at $t = 2$ if $\omega = G$ while it returns nothing if $\omega = D$; project δ returns one unit of cash flow at $t = 2$ only in the state $\omega = D$. After the bank makes its selection, a continuum of 'bad' banks enter into the economy. Their projects – henceforth called type β – return nothing in any state to the economy. However, bad banks earn private benefits by undertaking their projects which do not contribute to social welfare. The fraction of these bad banks $b > 0$ is a random variable with distribution $F(\cdot)$ over support $[0, \infty)$. The true value of b is drawn after the banks other than the bad ones choose their projects. For convenience of exposition, each bank's financial project is hereafter referred as its type.

At $t = 1$, the banks – including bad ones – must raise $x > 0$ dollars in order to continue their projects. They raise these funds by issuing bonds to a continuum of outside investors with measure one at the bond market. The outside investors simultaneously decide whether or not to fund each bank's financial project. If an investor decides to fund, then she competes with other investors willing to fund the same bank a lá Bertrand by offering repayment terms R at $t = 2$ per unit of capital.

There are several assumptions on the agents' preferences and the information structure. First, every investor exhibits the same expected utility function $U(\cdot)$ as follows:

$$U(c_1, c_2) = c_1 + \mathbb{E}[u(c_2)] = c_1 + pu(c_G) + (1-p)u(c_D),$$

where c_1 is consumption at $t = 1$, c_G is consumption in the state $\omega = G$ at $t = 2$, and c_D is consumption in the state $\omega = D$ at $t = 2$. Throughout the paper, we assume $u(x) = \log x$; we show in section 4 that our results are not altered if we assume CRRA utility. Since investors are risk averse, they have an incentive to purchase bonds issued by both types of bank in order to smooth consumption across states G and D. Second, we assume that every investor owns an equal amount of shares issued by all banks. This implies that social welfare is simply the aggregated expected utility of the investors. Third, except the bad banks, there is no moral hazard between the banks and the shareholders in this economy: either a type γ bank or a type δ bank behaves in the best interest of its shareholders. Fourth, we assume that each bank's type is private information, which creates an adverse selection problem at the bond market.

Finally, the key assumption is that there exists a regulator who maximizes social welfare – i.e., the investors' expected utility – by choosing whether to reveal his superior but imperfect information about the banks' private types at date 1, before the bond market opens. More specifically, we assume that the regulator can distinguish type γ banks from the other types, but he cannot distinguish type δ banks from type β banks. The regulator receives this information before the banks enter the bond market. Moreover, he can commit whether or not to disclose its information, contingent on the realization of b, before banks choose their type.

2.2 Interpretation

Type γ banks represent financial institutions that are investing in socially desirable, low-risk projects that can be identified as such by the regulator's stress test models. Type δ banks represent institutions that are investing in *different* desirable, low-risk projects. These projects are especially socially desirable - if most banks are type γ - because they diversify the economy's aggregate portfolio. Crucially, we assume that type δ cannot be identified as low-risk by the regulator's stress test models. The justification for this is that regulators' risk models are necessarily imperfect.

Bad banks are introduced in order to create an adverse selection problem, so that there is a potential benefit to releasing stress test results in order to prevent adverse selection and market breakdown. However, releasing stress test results also has an ex ante cost, in that it increases the relative cost to banks of investing in socially valuable projects that are not identified as such by regulators.

One interpretation of these assumptions is that the regulator understands that two kinds of crisis could occur: state G, and state D. She understands relatively well how economic and financial market variables would evolve in state G, and can project how the value of bank assets, loan charge-offs, and so on, would behave in this state. In particular, looking at a particular bank, the regulator can judge whether or not it would have adequate capital in state G. In contrast, while the regulator knows that *other* kinds of crises could occur (state D), she does not know how aggregate variables, and particular banks' portfolios, would behave in these crises.

The regulator can run a stress test based on scenario G. Doing so would distinguish the banks who would perform well in this scenario from the other. However, the banks who 'fail' this stress test do so for different reasons. Some of them ('diversifying' banks) would perform well in the other, less well-understood kinds of crisis. But some of them ('bad' banks) simply have risky portfolios that would perform badly in any adverse scenario.

An alternative interpretation is that the regulator understands how macroeconomic and financial market variables will behave in a crisis, but is uncertain about how particular asset classes would evolve in this crisis. The best the regulator can do is to identify a class of assets which, in her judgment, will perform well with probability p. She expects that with probability $1 - p$, her judgment will turn out to be incorrect, and some other assets will pay off. However, she cannot distinguish the 'diversifying' assets – which would perform well if her model is wrong – from the 'bad' assets, which will perform badly in any crisis.

3 Partial equilibrium

Throughout the paper, we restrict our attention to a symmetric equilibrium in which *(i)* every bank chooses type γ with probability $g \in [0,1]$; *(ii)* every investor enters into the bond market with probability $\phi \in [0,1]$. In this section, we describe the partial equilibrium at $t = 1$ to analyze how investors and banks strategically interact in the bond market, given the measure of banks with type γ projects g, δ projects $d = 1 - g$, and β projects b.

Bank decisions. Type γ and δ banks enter date 1 with pre-existing projects of total size 1. A type γ bank needs to invest $x < 1$ new funds in order to produce 1 unit of output in state G at date 2, and similarly δ-banks must invest x to produce one unit of output in state D. Thus the return on investment is $\bar{R} := 1/x > 1$.

Investor decisions. Due to asymmetric information, the investors do not know whether a bank is type γ, δ, or β. Once entering the bond market, the investors meet type-γ banks with probabilities $\pi_g = \dfrac{g}{g+d+b}$ and type-δ banks with probability $\pi_d = \dfrac{d}{g+d+b}$, respectively. Competition among other investors in terms of repayments induces each investor to buy bonds from $(1 + b)$ banks on average for the equilibrium repayment terms $R^* x$. At $t = 2$, each investor also receives state-dependent dividends y_ω in each state $\omega \in \{G, D\}$.

Given the equilibrium repayment terms Rx, each investor solves the following participation decision problem:

$$\max_{\phi \in [0,1]} -\phi x(1+b) + pu(y_G + (1+b)\pi_g Rx\phi) + (1-p)u(y_D + (1+b)\pi_d Rx\phi).$$

The investor (weakly) prefers to go into the bond market – i.e., $\phi \geq 0$ – if and only if

$$1 \leq [pu'(c_G)\pi_g + (1-p)u'(c_D)\pi_d]R.$$

If the inequality is strict, then $\phi = 1$. If the inequality does not hold, then $\phi = 0$. Dividends distributed to the investors in states G, D are $y_G = [1 - R^*x]\pi_g(1+b) = [1 - R^*x]g$, $y_D = [1 - R^*x]\pi_d(1+b) = [1 - R^*x]d$ respectively. Consumption in the two states must be equal to the total returns of the banks' projects, thus we have $c_G = g$ and $c_D = d$.

3.1 Equilibrium Structure

We consider an equilibrium with the following structure:

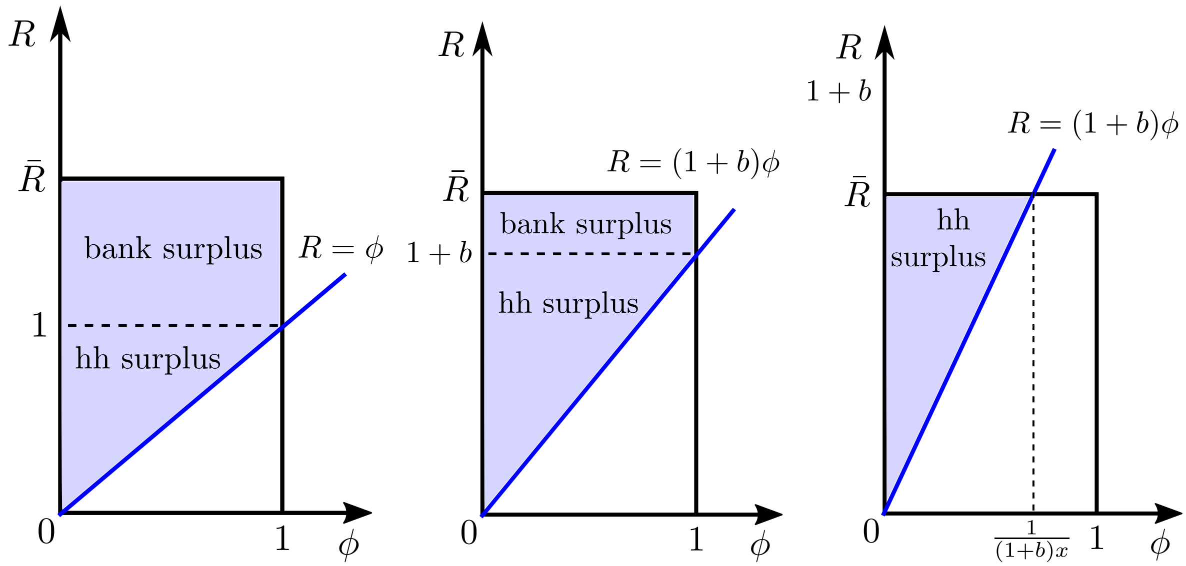
Definition 3.1. *An equilibrium is R, ϕ such that given g, d, and b,*

$$\frac{1}{1+b} = [pu'(\phi g)\pi_g + (1-p)\pi_d u'(\phi d)]R,$$
$$\begin{cases} \phi \in [0,1) & \text{if } Rx = 1, \\ \phi = 1 & \text{if } Rx < 1. \end{cases}$$

This equilibrium concept captures credit rationing in the capital market caused by adverse selection. If the fraction of bad banks in the market is sufficiently small, investors will demand only a small spread in order to hold bonds, and the interest rate will be low enough that banks have some equity in their project at date 2, after repaying bondholders: $R \leq \overline{R} \equiv 1/x$. As a result, all investors will enter the capital market and compete to obtain funding of their project. However, if there are too many bad banks, investors will demand such a high spread ($R > \overline{R}$) to compensate them for the risk of funding a bad bank that banks would have no equity left in the market after borrowing at these rates. If investors charged such a high interest rate, good and diversifying banks would drop out of the market, leaving only bad banks. As good and diversifying banks drop out of the market, investors become poorer at date 2, and become willing to lend to banks at a lower interest rate. In equilibrium, $R = \overline{R}$ and lending to banks is rationed, so that each bank can obtain funding with probability $\phi \in (0,1)$.

Lemma 3.2. *If $pu'(g)g + (1-p)gu'(d) \geq x(g+d+b)$, $\phi = 1$ and $R = [pu'(g)\pi_g + (1-p)\pi_d u'(d)]^{-1}$.*
If $pu'(g)g + (1-p)gu'(d) < x(g+d+b)$, ϕ is the solution to

$$pu'(\phi g)g + (1-p)u'(\phi d)d = x(g+d+b),$$



(a) $b = 0$ **(b)** Low b **(c)** High b

Figure 1 Equilibrium in the bond market

and $R = [pu'(g)\pi_g + (1-p)\pi_d u'(d)]^{-1}$.

For the remainder of this section, unless otherwise specified, we specialize to the case of log utility:

Assumption 3.3. *(i)* $u(c) = \ln c$*; (ii)* $p > x$*; and (iii)* $g + d = 1$*.*

To simplify notation, in what follows we denote $b_x := x(1+b)$.

Lemma 3.4. *Suppose Assumption 3.3 holds. Then*

(i) If $b_x \leq 1$, $\phi = 1$, $R = b_x/x$.

(ii) If $b_x > 1$, $R = 1/x$, $\phi = 1/b_x$.

Ex-ante welfare is

$$U_0(g, b_x) = pu(\phi g) + (1-p)u(\phi d) - \phi x(1+b) = w(g) + \ln \phi - \phi x(1+b) = w(g) - \min\{b_x, 1 + \ln b_x\},$$

where we define $w(g) := p \ln g + (1-p)\ln(1-g)$*.*

Three subfigures in Figure 1 illustrate the equilibrium in the bond market for different values of b. In all cases, the supply of bonds - measured by ϕ, the share of banks that can be financed - is an increasing function of the interest rate. The demand for credit is a decreasing function of the interest rate. When rates are low, banks each inelastically demand to refinance all their projects. When $R = \bar{R} = 1/x$, demand is perfectly elastic: banks are indifferent between funding and not funding. If $R > \bar{R}$, demand for credit is zero, as the interest rate is higher than the internal rate of return.

In the leftmost figure, there are no bad banks, $b = 0$. Moving to the middle panel, as we increase b, the slope of the bond supply function becomes steeper, as investors demand a higher interest rate to compensate them for the risk of lending to a bad bank. This increases equilibrium interest rates. Eventually, interest rates increase all the way to $\bar{R}$. Above this point, further increases in b are lead to a reduction in credit rather than an increase in interest rates, as shown on the rightmost figure. In this region, a higher b leads to more rationing, and a smaller ϕ. For any finite b, though, banks are able to obtain some financing.

3.2 The Ex-Post Optimal Information Disclosure

We now suppose that the regulator reveals her information to the public. Recall that the regulator can tell apart γ-type banks from others, while she cannot distinguish δ-types from β-types. The following assumption ensures that it would be efficient to finance type γ projects.

Assumption 3.5. *$p\ u'(g) > x$*

Investors can now perfectly identify γ banks, while there are a remaining set of banks that could be either type δ or type β. Thus investors can now hold two distinct securities. They can invest in γ banks, receiving R_g in state G, and nothing in state D. Or they can invest in the other banks, receiving $\pi_d R_d$ in state D (by the law of large numbers argument described above). Now the fraction of δ banks in the second market is $\frac{d}{d+g+b}$.

Date 1 equilibrium is defined as follows.

Definition 3.6. *An equilibrium with stress tests is R_g, R_d, ϕ_1 such that, given g, d, b, $\pi_d = \frac{d}{d+b}$,*

$$1 = pu'(g)R_g,$$
$$1 = (1-p)u'(\phi d)\pi_d R_d,$$
$$\begin{cases} \phi_1 \in [0,1) & \text{if} \quad R_d x = 1, \\ \phi_1 = 1 & \text{if} \quad R_d x < 1. \end{cases}$$

The following Lemma states that releasing the signal increases rationing for those banks who are rationed.

Lemma 3.7. *$\phi_1 \leq \phi_0$ for every $\beta \geq x$, where inequality strictly holds for $\phi_0 < 1$.*

Proof. In the rationing regime with no signal, ϕ_0 is defined by

$$pu'(\phi_0 g)g + (1-p)u'(\phi_0 d)d = b_x$$

In the rationing regime with a signal, ϕ_1 is defined by

$$(1-p)u'(\phi_1 d)d = b_x - xg$$

Define $\phi(\alpha)$ by

$$(1-\alpha)pu'(\phi(\alpha)g)g + (1-p)u'(\phi(\alpha)d)d = b_x - \alpha xg$$

By definition, $\phi(0) = \phi_0, \phi(1) = \phi_1$. By the Implicit function theorem,

$$\phi'(\alpha) = \frac{[pu'(\phi(\alpha)g) - x]g}{(1-\alpha)pu''(\phi(\alpha)g)g^2 + (1-p)u''(\phi(\alpha)d)d^2} < 0$$

So $\phi_1 < \phi_0$: when there is rationing in both regimes, there is more rationing (lower ϕ) with the signal. □

Lemma 3.8. *Suppose Assumption 3.3 holds. Then*

(i) If $b_x \leq \breve{b}_x(g) := 1 - p + xg$, $\phi_1 = 1$.

(ii) If $b_x > \breve{b}_x(g)$, $\phi_1 = \dfrac{1-p}{b_x - xg}$.

The ex-ante welfare is

$$U_1(g, b_x) = w(g) + (1-p)\ln\phi_1 - xg - \phi_1 x(d+b) = w(g) - \min\left\{b_x, 1 - p + xg + (1-p)\ln\left(\frac{b_x - xg}{1-p}\right)\right\}.$$

The following Lemma states formally that it is always ex post optimal to reveal the signal.

Lemma 3.9. *We have the following properties of U:*

(i) U_0, U_1 are strictly decreasing in g when $g > g^*$.

(ii) For the difference $U_1(g, b_x) - U_0(g, b_x)$, we have

$$\text{For all } \ b_x \leq \breve{b}_x(g), \ U_1(g, b_x) - U_0(g, b_x) = 0$$

$$\text{For all } \ b_x > \breve{b}_x(g), \ U_1(g, b_x) - U_0(g, b_x) > 0 \ \text{ and } \ \frac{\partial}{\partial b_x}[U_1(g, b_x) - U_0(g, b_x)] > 0.$$

Proof. (i) When $g > g^*$,

$$\frac{\partial U_0}{\partial g} = w'(g) < 0,$$

$$\frac{\partial U_1}{\partial g} = \begin{cases} w'(g) < 0 & \text{if } \ b_x < \breve{b}_x(g), \\ w'(g) - x\left[1 - \dfrac{1-p}{b_x - xg}\right] < 0 & \text{otherwise.} \end{cases}$$

(ii) When $b_x \leq \check{b}_x(g)$, $U_1 - U_0 = 0$. When $b_x \in [\check{b}_x(g), 1]$,

$$U_1(g, b_x) - U_0(g, b_x) = b_x - xg - 1 - p - (1-p)\ln\left(\frac{b_x - xg}{1-p}\right),$$
$$\implies \frac{\partial[U_1(g, b_x) - U_0(g, b_x)]}{\partial b_x} = 1 - \frac{1-p}{b_x - xg} > 0.$$

When $b_x > 1$,

$$U_1(g, b_x) - U_0(g, b_x) = p + \ln b_x - xg - (1-p)\ln\left(\frac{b_x - xg}{1-p}\right),$$
$$\implies \frac{\partial[U_1(g, b_x) - U_0(g, b_x)]}{\partial b_x} = \frac{1}{b_x} - \frac{1-p}{b_x - xg} > 0,$$

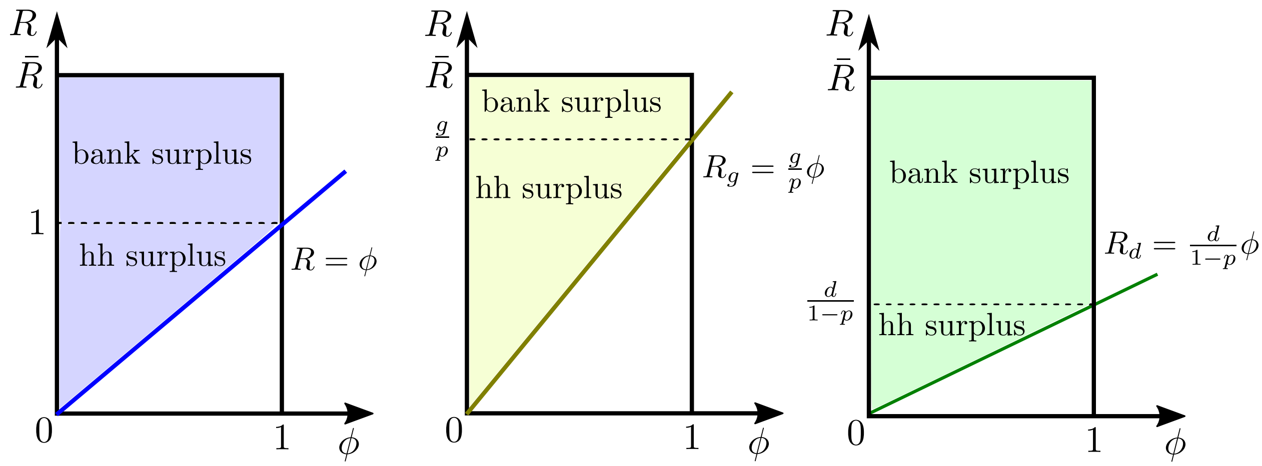
where the last inequality holds because $b_x > 1 > xg/p$. So $U_1(g, b_x) - U_0(g, b_x) > 0$ for $b_x > \check{b}_x(g)$. □

The effect of releasing the signal depends on the number of bad banks in the market, and also on banks' prior investments in g and d projects.

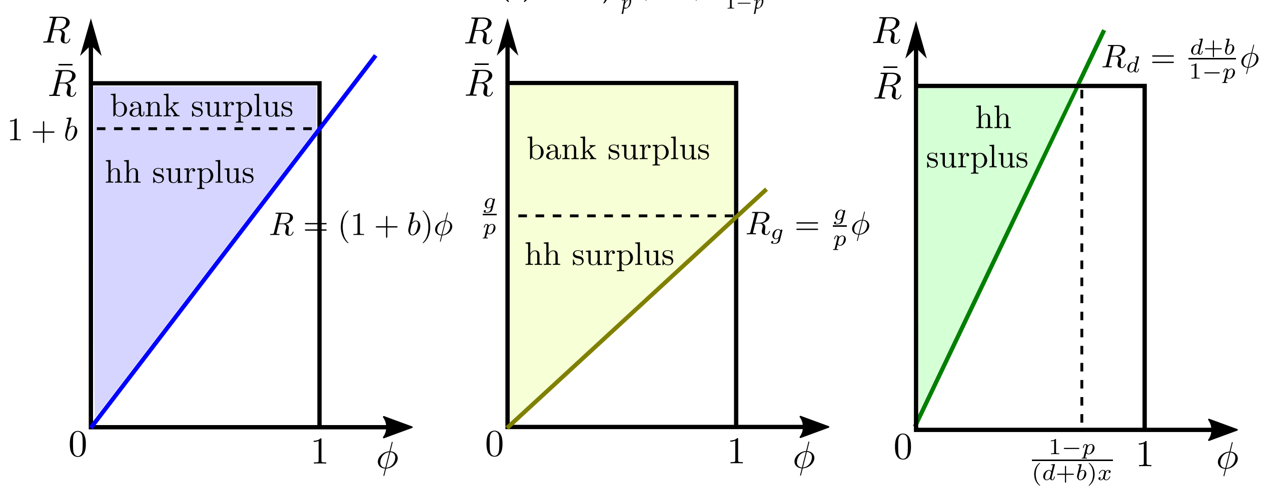
The simplest case to consider is where there are no bad banks ($b = 0$) and investment is efficient ($g = p, d = 1 - p$). In this case, the fair price of γ and δ bank debt is the same, and is equal to 1. Thus releasing the signal does not change market equilibrium in any way. Each bank can still borrow at $R = 1$.

Next, suppose that there are still no bad banks ($b = 0$), but there is inefficiently high investment in g banks: $\frac{g}{p} > 1 > \frac{d}{1-p}$. In this case, δ banks would have a higher fair value than γ banks, and would be able to borrow at a lower interest rate, under full information. This is because investors are poorer in state D, and value assets that pay off in this state more highly. Figure 2a illustrates the effect of releasing the signal in this case. The leftmost panel shows the equilibrium in the integrated market in the absence of information. Once information is released, the market segments. The middle panel shows equilibrium in the market for γ banks. The supply curve shifts up, and equilibrium interest rates are higher for g banks. The rightmost panel shows the equilibrium for δ banks. Their supply curve shifts down, and their equilibrium interest rates are lower. In this environment, then, releasing the signal benefits δ banks at the expense of γ banks. Note, however, that social surplus is unchanged.

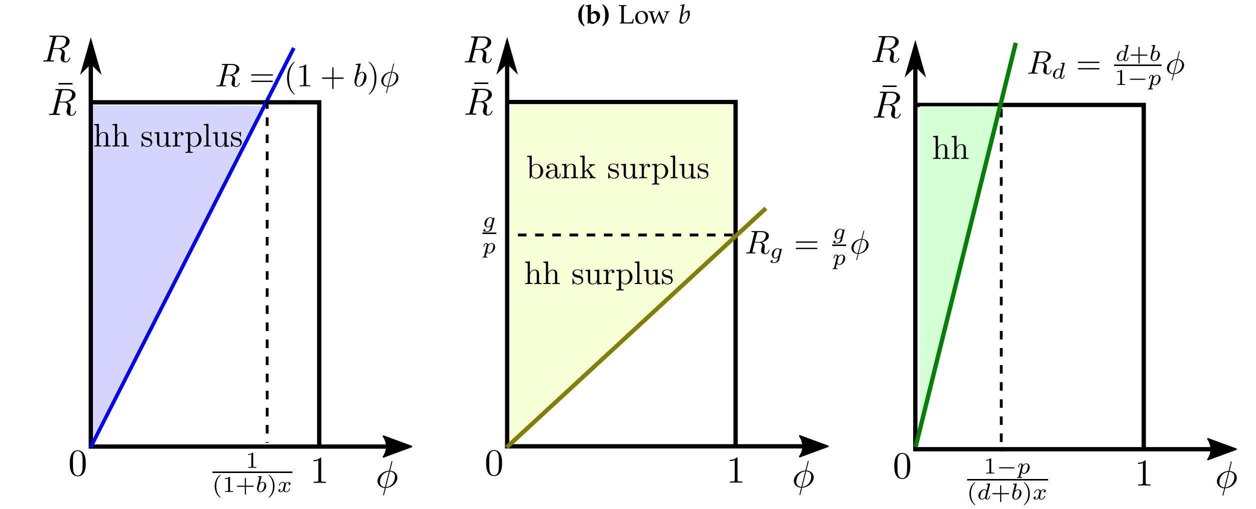
Next, suppose there are some bad banks ($b > 0$), but not so many that there is rationing in the integrated bond market ($(1 + b)x < 1$). Figure 2b illustrates the effect of releasing the signal in this case. Again, the leftmost panel shows the equilibrium without the signal, and the middle and rightmost panels show the equilibrium for G and D banks, respectively, if the signal is released. In this case, the supply curve for G banks shifts down, as investors learn that these banks are definitely not type B. At the same time, the supply curve for banks who receive a negative signal shifts up, as investors believe it is more likely that these banks are type B. In



(a) $b = 0$, $\frac{g}{p} > 1 > \frac{d}{1-p}$



(b) Low b



(c) High b

Figure 2 Equilibrium with and without disclosure the case illustrated, this upward shift is large enough that it leads to rationing in the market for type D banks. Social surplus increases on average, since releasing the signal reduces inefficient investment in bad banks. In addition, the surplus of G banks increases, while the surplus of D banks falls to zero.

Finally, Figure 2c illustrates a case in which there are so many bad banks that there is rationing in the integrated bond market ($(1+b)x > 1$). In the integrated equilibrium, shown in the leftmost panel, all banks are rationed and make zero surplus. After revelation of the signal, the supply curve of G banks shifts down, and these banks make positive surplus; the supply curve of D banks shifts up, and these banks are rationed further, and still face zero surplus. Again, social surplus increases on average. In this case, revealing the signal prevents rationing, at least for some banks. Note that while D banks are made no worse off by revelation, G banks are made strictly better.

Summing up: if the regulator shares her information with the public, this affects both bank profits and social welfare. Releasing the signal has no effect on welfare when b is low, and always increases welfare when b is high, as it prevents investment in bad banks, and increases investment in at least some good banks. When b is low, releasing the signal benefits D banks, if there is inefficiently low investment in these banks (or G banks, if there is inefficiently low investment in these banks). However, when b is high, releasing the signal always harms D banks, relative to G banks. This will affect ex ante investment, to which we now turn.

4 The Full Equilibrium Analysis

4.1 The Ex-Ante Bank Decision

As described above, at date 0, a continuum of banks with measure 1 choose to be either γ or δ. The measure of b_x banks b - or equivalently, $b_x = x(1+b)$ - follows a distribution $F(b_x)$, and is realized at $t = 1$. The policymaker commits at date 0 to an *information policy*, denoted by a set B: the policymaker reveals her signal if and only if $b_x \in B$. Throughout this section, we assume Assumption 3.3 holds (households have log preferences).

If the signal is not released, the payoff to a type γ bank is

$$\max\left\{\frac{p}{g}[1-Rx], 0\right\} = \max\left\{\frac{p}{g}[1-b_x], 0\right\}$$

and to a type δ bank:

$$\max\left\{\frac{1-p}{d}[1-b_x], 0\right\}$$

Note that banks only get payoffs when there is full entry in the bond market, which simplifies these expressions.

The *relative gain* to being a type γ is

$$\Delta_0(g, b_x) := \lambda(g) \max\{1 - b_x, 0\}$$

where we define $\lambda(g) = w'(g) = \dfrac{p}{g} - \dfrac{1-p}{1-g}$. Both $\lambda(g)$ and $\Delta_0(g, b_x)$ are decreasing in g and equal zero at $g^* := p$. (Δ_0 is obviously zero everywhere if $b_x > 1$).

If the signal is released, type γ banks get interest rate $R_g = g/p$ and payoff $\dfrac{p}{g} - x$. Type δ banks may be rationed. They face interest rate

$$R_d = \phi(d + b)/(1 - p)$$

and get payoff

$$\max\left\{\frac{1-p}{d} - x\frac{d+b}{d}, 0\right\}$$

The relative gain to being a type γ is

$$\Delta_1(g, b_x) := \frac{p}{g} - x - \max\left\{\frac{1-p}{1-g} - x\frac{1-g+b}{1-g}, 0\right\} = \min\left\{\lambda(g) + \frac{b_x - x}{1-g}, \frac{p}{g} - x\right\}$$

Ex ante, taking into account that the signal will be released in states $b_x \in B$, the expected gain from being a type γ is

$$H(g, B) := \int_\Omega \Delta_0(g, b_x) f(b_x)\, \mathrm{d}b_x + \int_B (\Delta_1(g, b_x) - \Delta_0(g, b_x)) f(b_x)\, \mathrm{d}b_x$$

In an equilibrium with $g \in (0, 1)$, banks must be indifferent between choosing to be type γ or type δ at date 0. Consequently, we must have $H(g, B) = 0$. Assuming this equation has a unique solution for g, then by choosing her information policy B, the regulator implicitly determines the fraction of type γ banks g. The following Lemma states that with log preferences, for any B, there is a unique, interior equilibrium.

Lemma 4.1. *(Uniqueness.) The constraint*

$$H(g, B) := \int_\Omega \Delta_0(g, b_x) f(b_x)\, \mathrm{d}b_x + \int_B (\Delta_1(g, b_x) - \Delta_0(g, b_x)) f(b_x)\, \mathrm{d}b_x = 0$$

uniquely defines g as a function of B, $g(B)$.

Proof. H is continuous and decreasing in g with $H(0, B) = \infty$, $H(1, B) = -\infty$. □

4.2 The Ex-Ante Optimal Disclosure Policy

In this section, we describe the optimal disclosure of information under commitment. Again, the regulator's problem is to choose a set $B \subset \Omega$ in which to release the signal, which in turn uniquely determines the fraction of γ banks $g \in (0,1)$. We can equivalently let the regulator choose both g and B, subject to the banks' indifference condition. Formally, the regulator's problem is

$$\max_{g,B} \int_\Omega U_0(g,b_x)f(b_x)\,\mathrm{d}b_x + \int_B (U_1(g,b_x) - U_0(g,b_x))f(b_x)\,\mathrm{d}b_x$$
$$\text{s.t.} \int_\Omega \Delta_0(g,b_x)f(b_x)\,\mathrm{d}b_x + \int_B (\Delta_1(g,b_x) - \Delta_0(g,b_x))f(b_x)\,\mathrm{d}b_x = 0$$

where

$$\begin{aligned}
U_0(g,b_x) &= w(g) - \min\{b_x, 1 + \ln b_x\} \\
U_1(g,b_x) &= w(g) - \min\{b_x, 1 - p + xg + (1-p)\ln\left(\frac{b_x - xg}{1-p}\right)\} \\
\Delta_0(g,b_x) &= \lambda(g)\max\{1 - b_x, 0\} \\
\Delta_1(g,b_x) &= \min\left\{\lambda(g) + \frac{b_x - x}{1-g}, \frac{p}{g} - x\right\} \\
w(g) &= p\ln g + (1-p)\ln(1-g),\ \lambda(g) = w'(g)
\end{aligned}$$

and f has support $\Omega \subseteq [x, \infty)$.

The following Proposition characterizes the optimal disclosure policy.

Proposition 4.2. *Define $\hat{b}_x(g) = xg/p, \check{b}_x(g) = xg + 1 - p$. The optimal strategy $\{g^{**}, B^{**}\}$ satisfies $g^{**} > g^*$, and for some $\bar{b}_x \in \Omega$,*

$$B^{**} = [x, \hat{b}_x(g^{**})] \cup [\bar{b}_x, \infty)$$

4.3 Interpretation

Before turning to the proof of Proposition 4.2, we describe the qualitative properties of the optimal policy. As a benchmark, it is useful to consider the unconstrained program in which the regulator ignores the constraint $H(g,B) = 0$, and can directly choose the level of diversification g. In this relaxed program, it would be optimal to set $g = g^*$, $B = \Omega$. Banks' investment is efficiently diversified between the two projects, because households are risk averse and value diversification. The regulator's private signal is always revealed, because this always increases social surplus.

The constraint $H(g,B) = 0$ means that this allocation is not implementable. Banks' entry decisions induce a tradeoff between efficient information revelation and diversification. Releasing information generally harms type δ banks, because they are misclassified as bad, and lose access to bond markets. Banks anticipate this ex ante, and will be less willing to choose δ projects. In equilibrium, the fraction of δ projects must fall until these projects are so scarce, and households are so poor in state D, that the expected benefit of investing in these projects compensates for the stigma of being misclassified as a bad bank.

Given this tradeoff, might it be better to reveal no information at all, and set $B = \emptyset$? No, because the benefit from information revelation is first-order, while the cost of deviating from efficient diversification is second order.

Given that it is optimal to disclose information sometimes, when should it be disclosed? The proposition states that disclosure is optimal both when b_x (the normalized fraction of bad banks) is below some threshold, and when b_x is above a threshold. In this sense, the optimal disclosure strategy is nonmonotonic. Take the upper threshold first. The intuition for this result is that the benefits of disclosure are increasing in b_x: when there are more bad banks, adverse selection is more severe, and the benefits from reducing adverse selection are larger. But the costs of disclosure are (weakly) decreasing in b_x. In the absence of disclosure, type δ banks would earn higher profits than type γ banks, because since $g > g^*$ in equilibrium, δ banks are relatively scarce. Releasing information removes this scarcity rent, discouraging δ-investment. But when b_x is high, these scarcity rents are small in any case, because *both* γ and δ banks pay higher interest rates. Thus the cost of releasing information is smaller. Putting these results together, the net benefit from disclosure is clearly increasing in b_x.

Next, consider the lower cutoff. Why can it be optimal to reveal the signal for low values of b_x? Take the extreme case where there are no bad banks, $b_x = x$. In this case, by revealing her signal, the regulator perfectly distinguishes γ and δ banks. As in Figure 2a above, this increases the interest rate charged to γ banks, and decreases the rate charged to δ banks. Intuitively, δ banks are valued more highly than γ banks, because they are relatively scarce. Distinguishing these banks, if possible, increases their profits, and encourages investment in δ projects ex ante. This is desirable, because $g > g^*$ in equilibrium, so the regulator would like to use all means to keep g as low as possible.

If b_x is only slightly larger than x, the same argument goes through - by Bayes' rule, the signal that a bank is *not* type γ still implies that the bank is extremely likely to be type δ, so the average 'not γ' bank is still more valuable than a γ bank. As b_x gets larger, eventually this is no longer true, and a 'not γ' bank is less valuable than a γ bank in expectation. At this point it is optimal to stop releasing the signal.

4.4 Proof

Next, we prove Proposition 4.2.

Lemma 4.3. *(Properties of Δ.). When $g > g*$:*

1. $\Delta_1(g, b_x) - \Delta_0(g, b_x) < 0$ *for* $b_x < \hat{b}_x(g)$. $\Delta_1(g, b_x) - \Delta_0(g, b_x) > 0$ *for* $b_x > \hat{b}_x(g)$.

2. $\Delta_1(g, b_x) - \Delta_0(g, b_x)$ *is increasing in* b_x *for* $b_x < \check{b}_x(g)$*, decreasing for* $b_x \in [\check{b}_x(g), 1]$*, and constant for* $b_x > 1$.

Proof. By calculation. □

Lemma 4.4. *(Feasible g.) In any feasible allocation,* $g \geq g^*$*. If* $g = g^*$*, then B has measure zero.*

Proof. Suppose by contradiction that $g < g^*$. Then $\Delta_0(g, b_x)$ is clearly nonnegative, and positive for some b_x. $\Delta_1(g, b_x) - \Delta_0(g, b_x)$ is increasing in b_x, and is positive for all $b_x > x$. So we must have

$$\int \Delta_0(g, b_x) f(b_x)\, \mathrm{d}b_x + \int_B [\Delta_1(g, b_x) - \Delta_0(g, b_x)] f(b_x)\, \mathrm{d}b_x > 0$$

for any B.

$\Delta_0(g^*, b_x) = 0$, $\Delta_1(g^*, b_x) = \min\{(b_x - x)/(1 - g^*), 1 - x\} > 0$ with probability 1, since $b_x > x$. So if (g^*, B) is feasible, B has measure zero. □

Lemma 4.5. *(Optimal g.)* $g^{**} > g^*$ *for any optimal policy* $\{g^{**}, B^{**}\}$.

Proof. Suppose by contradiction that $g = g^*$ is optimal: then by Lemma 4.4, B has measure zero. Consider the following class of deviations, indexed by $\varepsilon > 0$: Set $B = [1, 1 + \varepsilon)$. $g(\varepsilon)$ is defined by

$$\lambda(g) \int_x^1 (1 - b_x)\, \mathrm{d}F(b_x) + \left[\frac{p}{g} - x\right] \int_1^{1+\varepsilon} \mathrm{d}F(b_x) = 0$$

as a continuous, differentiable function of ε. The utility from such a deviation is then

$$V(\varepsilon) = w(g(\varepsilon)) - \int_x^\infty \min\{b_x, 1 + \ln b_x\}\, \mathrm{d}F(b_x) + \int_1^{1+\varepsilon} [U_1(g(\varepsilon), b_x) - U_0(g(\varepsilon), b_x)]\, \mathrm{d}F(b_x)$$

Taking derivatives and setting $\varepsilon = 0$, we have

$$V'(0) = w'(g(0))g'(0) + f(1)[U_1(g(0), 1) - U_0(g(0), 1)] = f(1)[U_1(g^*, 1) - U_0(g^*, 1)] > 0$$

where we use the facts that $g(0) = g^*$ and $w'(g^*) = 0$. Since small deviations increase utility, $g = g^*$ cannot be optimal. □

Lemma 4.6. *(Low* b_x*.) The optimal strategy* $\{g^{**}, B^{**}\}$ *satisfies*

$$[x, \hat{b}_x(g^{**})] \subset B^{**}, (\hat{b}_x(g^{**}), \check{b}_x(g^{**})) \cap B^{**} = \emptyset.$$

Proof. Suppose to the contrary that there exists an interval $[b_x^1, b_x^2] \subset [x, \hat{b}_x(g)]$ but $[b_x^1, b_x^2] \not\subset B^{**}$. Consider the following class of deviations, indexed by $\varepsilon > 0$: Augment B^{**} with $[b_x^1, b_x^1 + \varepsilon)$, and define $g(\varepsilon)$ as a solution to

$$I(g, B) := H(g, B) + \int_{b_x^1}^{b_x^1 + \varepsilon} (\Delta_1(g, b_x) - \Delta_0(g, b_x)) f(b_x)\, \mathrm{d}b_x = 0,$$

where $I(g, B)$ is differentiable with respect to ε. Since $I_g < 0$, $I_\varepsilon < 0$ (since $\Delta_1 < \Delta_0$ when $b_x < \hat{b}_x$), thus $g'(\varepsilon) < 0$.

Such a deviation increases utility to

$$V(\varepsilon) = \int_\Omega U_0(g(\varepsilon), b_x) f(b_x)\, \mathrm{d}b_x + \int_{B^{**}} (U_1(g(\varepsilon), b_x) - U_0(g(\varepsilon), b_x)) f(b_x)\, \mathrm{d}b_x$$

where we use the feature from Lemma 3.9 that $U_0 = U_1$ when $b_x < \check{b}_x(g)$. Since $g'(\varepsilon) < 0$ and U_0, U_1 are decreasing in g (by Lemma 3.9), this small deviation increases utility, so the original allocation cannot be optimal.

Similarly, one can show that B^{**} cannot intersect $(\hat{b}_x(g), \check{b}_x(g))$ because $\Delta_1 > \Delta_0$ for every $\beta \in (\hat{b}_x(g), \check{b}_x(g))$. □

Lemma 4.7. *(High b_x.) $B^{**} \backslash [x, \check{b}_x(g))$ is an interval $[\bar{b}_x, \infty)$ for some $\bar{b}_x \geq \check{b}_x(g)$.*

Proof. First, we show that B^{**} must contain some $b_x \geq \check{b}_x(g)$. Suppose not; then by Lemma 4.6, $B^{**} = [x, \hat{b}_x(g)]$ for some $g > g^*$, and the associated expected welfare is $\int_\Omega U_0(g, b_x)\, \mathrm{d}F(b_x)$ since $U_0 = U_1$ for $b_x < \hat{b}_x(g)$. Consider the following deviation: set $B = \emptyset$ and $g = g^*$. Since U_0 is decreasing in g by Lemma 3.9, clearly this deviation strictly increases utility. Therefore, the original disclosure rule $[x, \hat{b}_x(g)]$ cannot be optimal.

Next, we show that B^{**} must be 'monotonic' above $\check{b}_x(g)$. Suppose to the contrary that there exist two sets $B_1, B_2 \subset [\check{b}_x(g), \infty)$ such that $\mathbb{P}(b_x \in B_1), \mathbb{P}(b_x \in B_2) \neq 0$, $B_1, B_2 \subset B$, and $b_x^1 < b_x^2$ for all $b_x^1 \in B_1, b_x^2 \in B_2$. Let $\overline{b_{x1}} := \max\{b_x | b_x \in B_1\}$ and $\underline{b_{x_2}} := \min\{b_x | b_x \in B_2\}$. For a given g, consider

$$H(B^{**}) = \int_{B^{**}} [\Delta_1(g, b_x) - \Delta_0(g, b_x)] f(b_x)\, \mathrm{d}b_x =: \int_{B^{**}} h(b_x)\, \mathrm{d}b_x$$

Find nonempty subsets $B_1' \subset B_1$ and $B_2' \subset (\overline{b_{x1}}, \underline{b_{x_2}})$ such that $\mathbb{P}(b_x \in B_1'), \mathbb{P}(b_x \in B_2') \neq 0$ and $H(B_1') = H(B_2') \neq 0$. Consider the following deviation: augment B with B_2', but remove B_1'. By construction, this keeps the value of the constraint unchanged, so the same value of g remains feasible.

Define

$$k(b_x) = \frac{U_1(g, b_x) - U_0(g, b_x)}{\Delta_1(g, b_x) - \Delta_0(g, b_x)}$$

$k(b_x)$ is positive and increasing for $b_x > \check{b}_x(g)$ by Lemmas 3.9 and 4.3. The expected welfare of this new disclosure rule is

$$
\begin{aligned}
&\int_{B_2'} [U_1(g,b_x) - U_0(g,b_x)] f(b_x)\,\mathrm{d}b_x - \int_{B_1'} [U_1(g,b_x) - U_0(g,b_x)] f(b_x)\,\mathrm{d}b_x \\
&= \int_{B_2'} k(b_x) h(b_x)\,\mathrm{d}b_x - \int_{B_1'} k(b_x) h(b_x)\,\mathrm{d}b_x \\
&> \min_{b_x \in B_2'} \{k(b_x)\} \cdot \int_{B_2'} h(b_x)\,\mathrm{d}b_x - \max_{b_x \in B_1'} \{k(b_x)\} \cdot \int_{B_1'} h(b_x)\,\mathrm{d}b_x \\
&\geq \min_{b_x \in B_2'} \{k(b_x)\} \cdot \left(\int_{B_2'} h(b_x)\,\mathrm{d}b_x - \int_{B_1'} h(b_x)\,\mathrm{d}b_x \right) = 0,
\end{aligned}
$$

which shows that this deviation increases the expected social welfare while it does not affect the banks' incentive for portfolio choice. We can thus conclude the original disclosure rule is not optimal. □

4.5 More general preferences

Disclosing the regulator's information is costly because households are risk averse. It is therefore of interest to describe how the optimal policy changes as households' risk aversion changes. To do this, we need to solve for optimal policy with more general preferences. This introduces additional complications.

When households are less risk-averse than log, multiple equilibria are possible, as is standard in adverse selection models with risk-neutral agents. If many δ-banks enter the market, the average 'not γ' bank is probably type δ, and is valuable, thus investors will buy its bonds at date 1; this confirms that it was a good idea for these banks to invest. If few such banks enter, the average 'not γ' bank is probably a bad bank, and is not valuable, thus investors will not buy its bonds, confirming that this it was a good idea for banks *not* to choose project δ. Mathematically, adverse selection tends to make the gain to being a type γ bank *increasing* in g. On the other hand, risk aversion makes the return decreasing in g: when δ banks are scarce, they are valued more highly by the market. It so happens that with log utility, risk aversion always outweighs adverse selection, and there is a unique equilibrium. When households are less risk averse than log, however, multiple equilibria are possible.

In this section, we prove nonetheless that our main result goes through, with the following modification. If we allow the regulator to choose B and to select g among the equilibria consistent with B, optimal policies have the form described above. In particular, the regulator always wants to choose the equilibrium with the lowest g (closest to g^*). However, such policies now involve the risk that the economy may coordinate on the bad equilibrium with less diversification.

Let $u(c) = \dfrac{c^{1-\sigma}}{1-\sigma}$, $\sigma \in (0,\infty)$. Define $w_G(g) = pg^{1-\sigma}$, $w_D(g) = (1-p)(1-g)^{1-\sigma}$, $w(g) = w_G(g) + w_D(g)$.

Without the signal, there is partial entry if $b_x > w(g)$. The interest rate satisfies $xR =$

$\min\left\{\frac{b_x}{w(g)}, 1\right\}$. With the signal, there is partial entry if $b_x > \check{b}_x(g) := w_D(g) + xg$. The interest rate satisfies $xR_D = \frac{b_x - xg}{w_D(g)}$. The profit of G banks in this case is $pg^{-\sigma} - x$. The profit of D banks is

$$(1-p)(1-g)^{-\sigma} - \frac{b_x - xg}{1-g}$$

Also define $\lambda(g) = pg^{-\sigma} - (1-p)(1-g)^{-\sigma}$.

$$\begin{aligned}
U_0(g, b_x) &= \max\left\{\frac{w(g)}{1-\sigma} - b_x, \frac{\sigma b_x^{1-\frac{1}{\sigma}} w(g)^{\frac{1}{\sigma}}}{1-\sigma}\right\} \\
U_1(g, b_x) &= \max\left\{\frac{w(g)}{1-\sigma} - b_x, \frac{w_G(g)}{1-\sigma} - xg + \frac{\sigma[b_x - xg]^{1-\frac{1}{\sigma}} w_D(g)^{\frac{1}{\sigma}}}{1-\sigma}\right\} \\
\Delta_0(g, b_x) &= \lambda(g) \max\left\{1 - \frac{b_x}{w(g)}, 0\right\} \\
\Delta_1(g, b_x) &= \min\left\{\lambda(g) + \frac{b_x - x}{1-g}, pg^{-\sigma} - x\right\}
\end{aligned}$$

Again, the regulator's problem is

$$\begin{aligned}
&\max_{g,B} \int_\Omega U_0(g, b_x) f(b_x)\, \mathrm{d}b_x + \int_B (U_1(g, b_x) - U_0(g, b_x)) f(b_x)\, \mathrm{d}b_x \\
&\text{s.t. } \int_\Omega \Delta_0(g, b_x) f(b_x)\, \mathrm{d}b_x + \int_B (\Delta_1(g, b_x) - \Delta_0(g, b_x)) f(b_x)\, \mathrm{d}b_x = 0
\end{aligned}$$

Note that by allowing the regulator to choose any g, B consistent with the entry condition, we essentially assume away the multiple equilibria issue, allowing the regulator to implement whichever g she prefers consistent with the same B.

With this caveat, the following Proposition characterizes the optimal disclosure policy, showing that our result for the log utility case can be generalized to CRRA utilities.

Proposition 4.8. *For general CRRA perferences, the optimal allocation rule $\{g^{**}, B^{**}\}$ satisfies (i) $g^{**} > g^*$; and (ii) $B^{**} = [x, \hat{b}_x(g^{**})] \cup [\bar{b}_x, \infty)$ for some $\bar{b}_x \in \Omega$.*

Proof. In the Appendix. □

Figure 3 illustrates numerically how the optimal policy depends on households' risk aversion. The measure of bad banks, b, is uniformly distributed over $[0, 10]$. We set the cost of investing, x, equal to 0.2, and the probability that state G realizes, p, equal to 0.5. Figure 3 plots the optimal value of g, its first best value g^*, and the optimal lower and upper bounds, $F(\hat{b}_x(g))$ and $F(\bar{b}_x)$, as functions of σ, households' risk aversion. The lower and upper bounds are normalized to represent probabilities, so the area in between the blue and black lines represents the probability that the regulator does not reveal her signal.

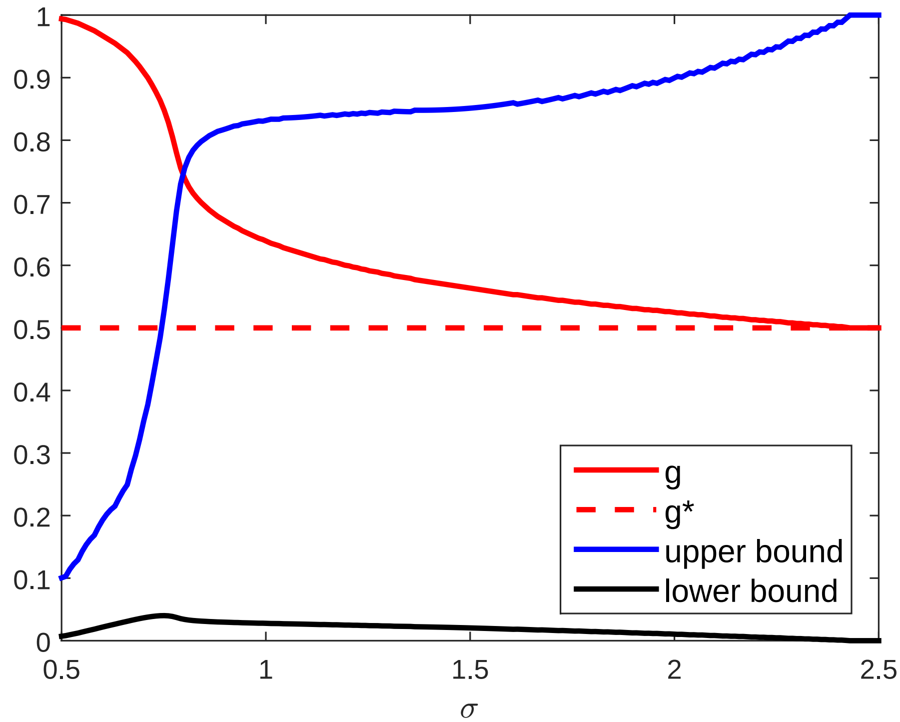


Figure 3 The optimal policy as a correspondence of σ

When households are not too risk averse, it is optimal to reveal the signal in most states of the world. This involves a large gap between the efficient level of investment in γ banks, $g^* = 0.5$, and the actual level, which is close to 1. But because households are not too risk-averse, they are willing to under-diversify in return for the efficiency gains associated with revealing the signal in bad states of the world and reducing inefficient investment. Moving left to right, as households become more risk averse, it is optimal to reveal the signal less often in order to induce a lower level of g, closer to the first-best level of $g^* = 0.5$.

5 Externalities

A key justification of stress tests (and financial regulation and supervision more broadly) in practice is that in a crisis, low levels of bank capital have negative effects on the wider economy, which are not internalized by individual banks. We have abstracted from such externalities in the model thus far, in order to highlight our main point - even absent other frictions, imperfect stress tests can reduce diversification, and regulators should sometimes refrain from releasing stress test results in order to increase diversification. However, it is clearly important to ask whether this result is robust to the presence of the kind of pecuniary externalities which motivate stress tests in practice.

To answer this question, in this section we assume that when banks' projects fail, they are forced to sell productive assets in order to repay depositors. This introduces a pecuniary externality, because an individual bank does not internalize that his asset sales reduce the prices other banks receive for their assets (Lorenzoni (2008); Korinek (2011)). Asset sales reduce welfare, because banks are more productive managers of assets. A social planner would like banks to diversify their projects across different states of the world, rather than concentrating losses in states where fire sales are more severe.

More concretely, we modify the model as follows. There are 4 dates, $t = 0, 1, 2, 3$. Before date 0, banks each owe 1 dollar to households at date 2 (we will sometimes refer to these as deposits), and each hold L legacy projects which pay off at date 3. A legacy project pays off 1 in the hands of a household and $A > 1$ in the hands of a bank.

At date 0, banks choose whether to invest in a type γ project or a type δ project.

At date 1, the fraction of bad banks is realized, and the regulator either releases or does not release its private information. Banks must pay a cost x to keep each project alive. They raise funds by issuing equity claims on the output of their projects at date 2 (we previously called these bonds, but here it is useful to think of them as equity).

At date 2, the aggregate state $s \in \{G, D\}$, and thus the return on bank projects, is realized. Banks must each pay depositors 1 dollar. To do this, they can use the output of their project (after repaying the holders of equity claims), if the project was successful. Banks can also sell their legacy assets to households at price q_G or q_D (depending on the state). We assume the banks' endowment of legacy projects L is large enough that there is always an interior solution.

At date 3, banks receive a payoff A on each of the legacy projects they retained; households also receive a payoff of 1 on each of the legacy projects they acquired. This is the only date at which banks consume. The social welfare function we use to define optimal allocations is a simple sum of household and banker utility; one interpretation is that banks' payoffs are returned to households at date 3.

Households have preferences

$$c_1 + \mathbb{E}[\ln c_2 + c_3]$$

Each bank seeks to maximize its expected date 3 payoff

$$\mathbb{E}A(L - \tilde{k})$$

where $\tilde{k}$ denotes the number of legacy projects the bank is forced to sell to pay its depositors.

5.1 Equilibrium without revelation of the signal

Consider first the case in which the regulator does not reveal her information to the public. At date 1, the household chooses equity purchases e and legacy projects to buy in each state of the world k_G, k_D to solve

$$\max_{e,k_G,k_D} -q_e e + p[\ln(1 + \pi_g e - q_G k_G) + k_G] + (1-p)[\ln(1 + \pi_d e - q_D k_D) + k_D]$$

This yields first order conditions

$$q_e = \frac{p\pi_g}{1 + \pi_g e - q_G k_G} + \frac{(1-p)\pi_d}{1 + \pi_d e - q_D k_D}$$
$$\frac{q_G}{1 + \pi_g e - q_G k_G} = 1$$
$$\frac{q_D}{1 + \pi_d e - q_D k_D} = 1$$

At date 1, banks must issue equity $\tilde{e}$ to cover maintenance costs x:

$$q_e \tilde{e} = x$$

which means that the fraction of a project that must be sold to equityholders if the project is to remain alive, $\tilde{e} = \dfrac{x}{q_e}$, is exogenous from the perspective of an individual bank. At date 2, a bank whose project was successful can pay its depositors 1 with the proceeds from its project, net of payments to shareholders, $1 - \tilde{e}$, and by selling $\tilde{k}$ assets at price q_s:

$$1 = 1 - \tilde{e} + q_s \tilde{k}$$

A bank whose project was unsuccessful must rely on asset sales:

$$1 = q_s \tilde{k}$$

To see why this can be interpreted as a model of fire sales, suppose for concreteness that g is close to 1 - most banks invest in project γ - and state D realizes. All γ banks suffer a capital shortfall, are liquidity constrained, and must sell legacy assets to repay their debt. Since almost all banks are selling these assets, their price will be severely depressed, which in turn forces each bank to sell more assets.

In fact, even if state G realizes, there will be some fire sales in equilibrium, given household preferences, and given that banks cannot buy legacy assets from other banks. This keeps the analysis in this section similar to our baseline model. In a more realistic model, there would only be fire sales when aggregate consumption and bank capital are sufficiently low - in the example just given, this would happen in state D, but not in state G. Such a model would strengthen our main result - banks tend to under-diversify, and optimal policy should increase diversification relative to a world without externalities - but would take us too far away from the baseline model presented above.

Banks' only objective is to minimize sales of the legacy asset, $\tilde{k}$. Consequently, they will be indifferent between continuing their project and letting it die when $\tilde{e} = \frac{x}{q_e} = 1$, and they will prefer to abandon the project if $\tilde{e} > 1$. As before, we assume that when $\frac{x}{q_e} = 1$, banks are rationed in the equity market, and randomize, financing their project with a probability $\phi \in (0, 1)$ that is the same across γ, δ and b_x banks.

Market clearing implies that

$$\begin{aligned} 1 + \pi_g e - q_G k_G &= \phi g, \\ 1 + \pi_d e - q_D k_D &= \phi d, \\ \pi_g &= \frac{g}{1+b}, \\ \pi_d &= \frac{d}{1+b} \end{aligned}$$

So we have

$$q_e = \frac{1}{\phi(1+b)}, q_G = \phi g, q_D = \phi d$$

In particular, note that the price of legacy assets equals household consumption. If, for example, households are poor in state D, because few banks invested in δ projects paying off in that state, then households will only buy legacy assets at a low price.

As before, if $x(1+b) \leq 1$, there is an equilibrium with $\phi = 1$ and $q_e = \frac{1}{1+b}$; if $x(1+b) > 1$, we have $\phi = \frac{1}{x(1+b)}$, $q_e = x$. If there is no rationing, $e = (1+b)x = b_x$.

It remains to solve for sales of the legacy project at date 2. If there is rationing in equilibrium, all banks must meet the whole of their debt by selling legacy assets (since even if their projects pay off, the proceeds are all sold to equityholders), and we have $1 = q_s k_s$, i.e.

$$k_G = \frac{1}{\phi g} = \frac{b_x}{g}, k_D = \frac{1}{\phi d} = \frac{b_x}{d}$$

If there is no rationing, then - say, in state G - γ banks receive return $1 - b_x$ from the project (after repaying shareholders), and only need to raise b_x by selling assets. So

$$k_G = \frac{1 - g + b_x g}{g} = \frac{1}{g} + b_x - 1$$

By a symmetric argument,

$$k_D = \frac{1}{1-g} + b_x - 1$$

If there is no rationing, ex ante expected social welfare equals

$$U_0(g, b_x) = w(g) - b_x + AL - (A-1)\left\{\frac{p}{g} + \frac{1-p}{1-g} - (1-b_x)\right\}$$

where as before we define $w(g) = p\ln g + (1-p)\ln(1-g)$. If there is rationing, welfare is

$$U_0(g, b_x) = w(g) - (1 + \ln b_x) + AL - (A-1)\left\{b_x\frac{p}{g} + b_x\frac{1-p}{1-g}\right\}$$

Welfare in the economy with externalities is identical to welfare in the original model, with two changes. First, there is now a constant term AL representing the value produced by banks' retained legacy projects; this plays no role in the analysis that follows. Second, there is now the term in curly brackets, which equals expected sales of the legacy asset at date 2. If legacy projects are equally profitable in the hands of households, $A = 1$, this term disappears: higher sales of the legacy asset reduce bank payoffs, but benefit households by the same amount. However, if banks have an advantage in managing these projects, $A > 1$, then sales of the legacy project are inefficient, and reduce social welfare.

It is clear that expected asset sales are a convex function of g, which is increasing in g for $g > g^* = p$. When g is high, the economy invests very little in δ projects which pay off in state D, and households are very poor in this state. This means they are only willing to buy legacy assets at a very low price; which, in turn, means more assets must be liquidated to repay depositors, reducing welfare. The convexity of marginal utility means that the social cost of lower prices in state D outweighs the benefit of higher prices in state G.

Finally, the net benefit of being a γ bank is

$$\Delta_0(g, b_x) = A\lambda(g)\max\{1 - b_x, 0\}$$

where as before, $\lambda(g) = w'(g) = \frac{p}{g} - \frac{1-p}{1-g}$. This is the same as in the economy without externalities, up to a multiplicative constant which does not affect the equilibrium, since $\Delta_0(g, b_x) = 0$.

The marginal benefit to the social planner of having one extra γ bank, rather than a δ bank, is

$$\frac{\partial U_0(g, B)}{\partial g} = \lambda(g) + (A-1)\max\{1, b_x\}\left[\frac{p}{g^2} - \frac{1-p}{(1-g)^2}\right] < \lambda(g)$$

when $g > g^* = p$. Intuitively, the social planner is more risk-averse than private agents. As in the pecuniary externalities literature (Korinek (2011)), the reason is that individual banks do not internalize that when they invest in portfolios exposed to systemic risk, they add to this risk, by increasing fire sales in the event of a systemic crisis, which causes more resources to be diverted away from the productive banking sector.

5.2 Equilibrium with revelation of the signal

If the regulator releases her signal at date 1, the household can buy separately equity issued by γ banks, e_γ, and equity issued by δ (and b_x) banks, e_δ. These securities trade at different prices, q_γ and q_δ. The household's problem is

$$\max_{e_\gamma, e_\delta, k_G, k_D} -q_\gamma e_\gamma - q_\delta e_\delta + p[\ln(1 + e_\gamma - q_G k_G) + k_G] + (1-p)[\ln(1 + \pi_d e_\delta - q_D k_D) + k_D]$$

Since the derivation of equilibrium follows the main text and the previous subsection, we leave a full description to the Appendix, and focus on social welfare and the net benefit of being a γ bank.

If there is no rationing in equilibrium, welfare is the same as if the regulator does not release the signal:

$$U_1(g, b_x) = w(g) - b_x + AL - (A-1)\left\{\frac{p}{g} + \frac{1-p}{1-g} - (1-b_x)\right\}$$

If δ banks are rationed, welfare is

$$U_1(g, b_x) = w(g) - (1-p)\ln\left(\frac{b_x - xg}{1-p}\right) - xg - (1-p) + AL - (A-1)\left\{\frac{p}{g} - p + xg + \frac{b_x - xg}{1-g}\right\}$$

Again, this is (modulo the irrelevant constant) identical to the model without externalities, except for a new negative term represents expected sales of the legacy asset. The net benefit of being a γ bank is

$$\Delta_1(g, b_x) = A \min\left\{\lambda(g) + \frac{b_x - x}{1-g}, \frac{p}{g} - x\right\}$$

5.3 Optimal disclosure policy

The regulator's problem is defined as before:

$$\begin{aligned}
\max_{g,B} & \int_\Omega U_0(g, b_x) f(b_x)\, \mathrm{d}b_x + \int_B (U_1(g, b_x) - U_0(g, b_x)) f(b_x)\, \mathrm{d}b_x \\
\text{s.t.} & \int_\Omega \Delta_0(g, b_x) f(b_x)\, \mathrm{d}b_x + \int_B (\Delta_1(g, b_x) - \Delta_0(g, b_x)) f(b_x)\, \mathrm{d}b_x = 0
\end{aligned}$$

Let $(g^{**}_{ext}, B^{**}_{ext})$ denote the solution. The main result of this section is that in the model with externalities, it is optimal to induce a higher level of g, and release stress test results less often. In the presence of fire sale externalities, private agents under-diversify relative to the social planner. Since releasing stress test results can reduce diversification, it is even more important to refrain from releasing stress test results when there is already under-diversification in equilibrium.

Proposition 5.1. *The optimal level of g is lower in the economy with externalities than in the baseline model: $g^{**}_{ext} < g^{**}$*

The proof rests on two observations. First, when $g > p$, expected capital sales are a convex, increasing function of g. If g is very high, households are very poor in state D, and pay a low price for the legacy assets, leading to inefficiently high fire sales. Second, releasing the regulator's information always directly increases expected sales of the legacy asset. A sketch of the proof then proceeds as follows. Start with the optimal allocation (g^{**}, B^{**}) in the economy without externalities. To first order, any small, feasible change in g and B has no effect on welfare in the model without externalities. Suppose we shrink the set B slightly, and reduce g, giving a new feasible allocation (g, B). Both the reduction in g and the shrinking of B reduce expected sales of legacy assets, to first order. This strictly increases welfare.

6 Information design

In our model so far, we assume the regulator only has access to one source of information, which mistakenly classifies a certain type of socially efficient financial project as inefficiently risky. We show that always releasing this imperfect information reduces diversification. In this section, we raise a more general question: given that regulators make *some* errors when measuring the risk of each bank, what *kind* of errors are most harmful for diversification and social welfare?

To answer this question, we allow the regulator to have an alternative source of information about each bank's private type, and study under which circumstances the regulator prefers to use this new information. Specifically, the regulator has another signal which can misclassify both γ and δ types as type β banks with a positive probability. More formally, the regulator can choose whether to release one of two signals $\{\sigma_1, \sigma_2\}$. The realization of either signal, for a particular bank, is equal to either γ or δ. The signals perfectly identify good and diversifying banks, but they differ in the way they classify bad banks.

The first signal is the one considered in the model thus far, which always misclassifies bad banks as 'diversifying': for $\sigma_1 \in \{\gamma, \delta\}$, $\mathbb{P}(\sigma_1 = \gamma|\theta = \gamma) = \mathbb{P}(\sigma_1 = \delta|\theta = \delta) = 1$ but $\mathbb{P}(\sigma_1 = \delta|\theta = \beta) = 1$. The second signal, instead, misclassifies bad banks as 'good' with probability κ, and 'diversifying' with probability $1 - \kappa$, That is, for $\sigma_2 \in \{\gamma, \delta\}$, $\mathbb{P}(\sigma_2 = \gamma|\theta = \gamma) = \mathbb{P}(\sigma_2 = \delta|\theta = \delta) = 1$ but $\mathbb{P}(\sigma_2 = \gamma|\theta = \beta) = \kappa \in (0, 1)$. Consequently, outside investors' beliefs about a bank's type θ, after observing either of these signals, are: $\mathbb{P}(\theta = \gamma|\sigma_1 = \gamma) = 1$ and $\mathbb{P}(\theta = \delta|\sigma_1 = \delta) = \frac{1-g}{1-g+b}$; $\mathbb{P}(\theta = \gamma|\sigma_2 = \gamma) = \frac{g}{g+\kappa b}$ and $\mathbb{P}(\theta = \delta|\sigma_2 = \delta) = \frac{1-g}{1-g+(1-\kappa)b}$.

In what follows, we study which choice of information structure - σ_1, σ_2, or no information disclosure at all - maximizes social welfare, both from an ex post and an ex ante perspective.

6.1 Ex post optimal information disclosure

To find an ex post optimal information structure, fix the fraction of type γ banks as $g \in (0,1)$. For any $b \in [0,\infty)$,[3] the date-1 equilibrium under the second signal σ_2 can be characterized as follows:

Lemma 6.1. *If the signal σ_2 is revealed, there exists a date-1 equilibrium in which type γ banks are funded with probability $\phi_{2g} =\in (0,1]$ with repayment terms R_{2g} and type δ banks are funded with probability $\phi_{2d} \in (0,1]$ with repayment terms R_{2d}, where*

$$\begin{aligned}
\phi_{2g} &= \min\left\{1, \frac{p}{x\,(g+\kappa b)}\right\}, \\
R_{2g} &= \min\left\{\frac{1}{x}, \frac{g+\kappa b}{p}\right\}, \\
\phi_{2d} &= \min\left\{1, \frac{1-p}{x\,((1-g)+(1-\kappa)b)}\right\}, \\
R_{2g} &= \min\left\{\frac{1}{x}, \frac{(1-g)+(1-\kappa)b}{1-p}\right\}.
\end{aligned}$$

The corresponding ex-post social welfare U_2 is

$$U_2(g,b) = w(g) + p\log\phi_{2g} + (1-p)\log\phi_{2d} - \phi_{2g}x(g+\kappa b) - \phi_{2d}x((1-g)+(1-\kappa)b),$$

where $w(g) = p\log g + (1-p)\log(1-g)$.

Similar to the date-1 equilibrium without information disclosure, both γ and δ banks can suffer from adverse selection at the bond market when there are a large fraction of β banks, which worsens the repayment terms as well as the probability of selling the bonds. We next find an optimal choice of the signal with respect to the degree of the adverse selection $b \in [0,\infty)$.

Proposition 6.2. *Given $g \in (0,1)$, it is ex post optimal for the regulator to choose and release the first signal σ_1 to the outside investors. Particularly, σ_1 induces a higher social welfare than σ_2 if and only if b exceeds a threshold.*

Proof. If the signal σ_1 is released, we have the ex-post social welfare $U_1 := w(g) + (1-p)\log\phi_1 - xg - \phi_1 x((1-g)+b)$, where $\phi_1 = \min\left\{1, \frac{1-p}{1-g+b}\right\}$. For each $b \in [0,\infty)$, we have four possible cases: *(i)* $\phi_1 = \phi_{2g} = \phi_{2d} = 1$; *(ii)* $\phi_1 = \phi_{2d} = 1$ but $\phi_{2g} < 1$; *(iii)* $\phi_1 < \phi_{2d} = \phi_{2g} = 1$; *(iv)* $\phi_1 < \phi_{2d} < 1$ and $\phi_{2g} < 1$. If $\phi_1 = \phi_{2g} = \phi_{2d} = 1$, then $U_2 - U_1 = 0$. In all other cases, we have $U_2 - U_1 < 0$ and $\frac{d}{db}(U_2 - U_1) < 0$. Since $\phi_1 = \phi_{2g} = \phi_{2d} = 1$ if and only if $b \le \min\left\{\frac{1}{\kappa}\left(\frac{p}{x} - g\right), \left(\frac{1-p}{x} - (1-g)\right)\right\}$, we have the desired result. □

[3]For convenience of our analysis, we hereafter withdraw our abbreviated notation $b_x \equiv (1+b)x$ and reuse b as the degree of the adverse selection problem at the financial market.

Proposition 6.2 states that the new signal does not improve the ex-post social welfare. To provide intuition, we may need to return to roles of the information released by the regulator. When b is high, the regulator's information improves the ex-post welfare through two channels: the information reduces the inefficient investment by crowding β-typed banks out; the information also boosts efficiency in investment by partially mitigating the adverse selection faced by either γ-typed or δ-typed banks. Those improvements in the ex-post welfare can be maximized when the regulator partially mitigates the adverse selection for one type but leaving the other type with β-typed banks. However, the new signal σ_2 pools β-typed banks not only with δ-typed banks but also with γ-typed banks. Such a pooling of two banks dampens the ex-post effects of the regulator's information.

6.2 Ex ante optimal information disclosure

Even if it is never optimal to use signal σ_2 ex post, can it ever be optimal ex ante? If banks expect the regulator to release signal σ_1 when b is high, they know they will likely be misclassified as bad banks if they choose project δ. As shown in Section 3, the regulator can mitigate this, to some extent, by committing to reveal the information σ_1 when the fraction of bad banks, b, is very low: the outside investors perceive that banks who are *not* 'good' are more likely to be δ banks rather than β banks, and this perception improves the funding terms offered to δ banks, given their scarcity in the market.

We now argue that this effect can be enhanced if the regulator uses the second signal σ_2 because

$$\mathbb{P}(\theta = \delta | \sigma_2 = \delta) = \frac{1-g}{1-g+(1-\kappa)b} > \mathbb{P}(\theta = \delta | \sigma_1 = \delta) = \frac{1-g}{1-g+b}$$

If investors observe that $\sigma_2 = \delta$ for a particular bank, they will form a more favorable impression of that bank than if they observe that $\sigma_1 = \delta$. Thus releasing σ_2 rather than σ_1 ameliorates adverse selection problems for δ banks, but worsens adverse selection problems for γ banks, and overall makes investors more inclined to fund δ banks. Since there is inefficiently low investment in δ projects from a social planner's perspective, this is desirable - especially states of the world where adverse selection problems are not too severe.

To see this more formally, we calculate the bank's payoff of choosing γ project under the signal σ_2:

$$\min\left\{\frac{p}{g}\left(1-\frac{g+\kappa b}{p}x\right),0\right\},$$

and the bank's payoff of choosing δ project

$$\min\left\{\frac{1-p}{1-g}\left(1-\frac{(1-g)+(1-\kappa)b}{1-p}x\right),0\right\}.$$

For every $b > 0$, the net benefit from choosing γ under the signal σ_2 is

$$\Delta_2(g,b) := \min\left\{\frac{p}{g}\left(1-\frac{g+\kappa b}{p}x\right),0\right\} - \min\left\{\frac{1-p}{1-g}\left(1-\frac{(1-g)+(1-\kappa)b}{1-p}x\right),0\right\}.$$

If $\Delta_2(g,b) > 0$, then the banks strictly prefer to choose γ project, and vice versa. The following lemma states that the regulator can always encourage the banks to choose δ project by replacing σ_1 with the new signal σ_2:

Proposition 6.3. *Suppose the regulator commits to reveal either σ_1 or σ_2 if b is in a subset B with a positive measure. Then σ_2 induces more selection of δ project than σ_1 does.*

Proof. Fix $g \in (0,1)$. Then we algebraically have $\Delta_2(g,b) - \Delta_1(g,b) < 0$ for every $b > 0$. Combining the above inequality with Proposition 6.2, we have the desired result. □

An important implication from Proposition 6.3 is that the new signal σ_2 improves the ex-ante efficiency in portfolio diversification more than the old signal σ_1. To provide intuition of this result, we may need to recall the main result from the original model without σ_2. In the original model, the regulator can improve the ex-post adverse selection problem by releasing its information for high b. However, this revelation policy makes δ project an unattractive choice from the perspective of banks, which inefficiently lowers the fraction of δ project at the market. To boost ex ante selection of δ project, the regulator also needs to release the information for low values of b. For low b, a bank owning "not γ" project is perceived as being δ type rather than β type. Given that δ project is scarce at the market, such a distinguishing effect increases the expected profit of choosing δ-typed bank for low values of b. This in turn encourages the banks to make ex ante selection of δ project, compared to the case of the information revelation for high b only.

The same effect can be strengthened if the regulator uses the new signal σ_2. Indeed, σ_2 pools β-typed banks with γ-typed banks as well as δ-typed banks. Thus when σ_2 is released, the investors believe that "not γ" banks are more likely to be δ-typed banks than they would when σ_1 is released for the same b. Therefore, releasing σ_2 in good states of the economy (i.e. low b) encourages the banks to select δ project more than σ_1 would do.

From Proposition 6.2 and Proposition 6.3, we can design an optimal information revelation policy. On the one hand, the regulator uses the same signal as in the original model to alleviate the ex-post adverse selection problem which becomes severe in the bad states of the economy (or high b). On the other hand, the regulator commits to reveal the new signal in good states of the economy (or low b) in order to improve the ex-ante portfolio diversification.

Corollary 6.4. *An ex-ante optimal information structure is: (i) revealing σ_2 for $b \in [0, \underline{b}_2^{**})$; (ii) no revelation for $b \in [\underline{b}_2^{**}, \overline{b}_2^{**})$; (iii) revealing σ_1 for $b \in (\overline{b}_2^{**}, \infty)$.*

7 Conclusion

Stress tests have moved from an exceptional measure of crisis management to a routine part of financial regulation. In a crisis, stress tests can reduce bank opacity and prevent market shutdowns. But as a routine policy, stress tests may encourage banks to mimic regulators' models, pass the tests, and ignore their own measures of risk. This reduces diversification, leaving the financial system vulnerable to the risk that regulators' models turn out to be wrong. We presented a simple model to understand this tradeoff, and showed that the optimal policy is to release stress test results only in severe crises.

References

Bernanke, Ben. 2013. "Stress Testing Banks: What Have We Learned?" Remarks by Chairman Ben S. Bernanke at the 'Maintaining Financial Stability: Holding a Tiger by the Tail' financial markets conference sponsored by the Federal Reserve Bank of Atlanta, Stone Mountain, Georgia.

Bond, Philip, and Itay Goldstein. 2015. "Government intervention and information aggregation by prices." *The Journal of Finance*, 70(6): 2777–2812.

Bouvard, Matthieu, Pierre Chaigneau, and Adolfo De Motta. 2015. "Transparency in the Financial System: Rollover Risk and Crises." *The Journal of Finance*, 70(4): 1805–1837.

Faria-e Castro, Miguel, Joseba Martinez, and Thomas Philippon. 2015. "Runs versus Lemons: Information Disclosure and Fiscal Capacity." National Bureau of Economic Research Working Paper 21201.

Flannery, Mark, Beverly Hirtle, and Anna Kovner. 2015. "Evaluating the Information in the Federal Reserve Stress Tests." Federal Reserve Bank of New York Staff Reports 744.

Frame, W. Scott, Kristopher S. Gerardi, and Paul S. Willen. 2015. "The Failure of Supervisory Stress Testing: Fannie Mae, Freddie Mac, and OFHEO." Federal Reserve Bank of Boston Working Papers 15-4.

Gigler, Frank, Chandra Kanodia, Haresh Sapra, and Raghu Venugopalan. 2014. "How Frequent Financial Reporting Can Cause Managerial Short-Termism: An Analysis of the Costs and Benefits of Increasing Reporting Frequency." *Journal of Accounting Research*, 52(2): 357–387.

Glasserman, Paul, and Gowtham Tangirala. 2015. "Are the Federal Reserve's Stress Test Results Predictable?" Office of Financial Research, US Department of the Treasury Working Papers 15-02.

Goldstein, Itay, and Haresh Sapra. 2013. "Should Banks' Stress Test Results be Disclosed? An Analysis of the Costs and Benefits." *Foundations and Trends in Finance*, 8(1): 1–54.

Goldstein, Itay, and Yaron Leitner. 2015. "Stress Tests and Information Disclosure." Federal Reserve Bank of Philadelphia Working Paper.

Hirshleifer, Jack. 1971. "The Private and Social Value of Information and the Reward to Inventive Activity." *American Economic Review*, 61(4): 561–74.

Korinek, Anton. 2011. "Systemic Risk-taking: Amplification Effects, Externalities, and Regulatory Responses." European Central Bank Working Paper Series.

Leitner, Yaron. 2014. "Should regulators reveal information about banks?" *Business Review*, , (Q3): 1–8.

Lorenzoni, Guido. 2008. "Inefficient Credit Booms." *The Review of Economic Studies*, 75(3): 809–833.

Morgan, Donald P., Stavros Peristiani, and Vanessa Savino. 2014. "The Information Value of the Stress Test." *Journal of Money, Credit and Banking*, 46(7): 1479–1500.

Morris, Stephen, and Hyun Song Shin. 2002. "Social Value of Public Information." *American Economic Review*, 92(5): 1521–1534.

Spargoli, Fabrizio. 2013. "Bank Recapitalization and the Information Value of a Stress Test in a Crisis." Universitat Pompeu Fabra Working Paper.

Tett, Gillian. 2015. "Stress Tests for Banks Are a Predictable Act of Public Theatre." The Financial Times.