Expected inflation and other determinants of Treasury yields

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Objective and main results

- Goal: produce facts to help calibrate, estimate macro-finance term structure models

- 10-20% of var of quarterly shocks to nominal Treasury yields is news about average expected inflation over life of bond

- Robust statistically, across time, across parsimonious/highly flexible models

- Cannot reliably break down remainder into news about future short term real rates and term premia shocks
Needed: facts about joint dynamics of inflation, nominal yields

Ang, Bekaert, and Wei, JF 2008

“A large empirical literature has yielded surprisingly few generally accepted facts . . . empirical estimates for the real interest rate process vary between constancy . . ., mean-reverting behavior, or a unit root process . . . There seems to be more consensus on the fact that real rate variation, if it exists at all, should only affect the short end of the term structure whereas the variation in long-term interest rates is primarily affected by shocks to expected inflation . . .”

“. . . we seek to establish a comprehensive set of stylized facts regarding real rates, expected inflation, and inflation risk premiums . . .” [Followed by list of seven results]
Measuring the quantity of inflation risk in yields

- Need narrow definition, or all yield risk is inflation risk

- My approach: quantity of inflation risk is fraction of variance of yield shock attributed to news about inflation over life of the bond


\[
\text{Time-}t\ \text{yield shock} = \text{news about expected } \pi \text{ during bond life} \\
+ \text{news about expected short real rate during bond life} \\
+ \text{shock to term premia}
\]
Variance decompositions

- **Direct measure of inflation risk**

\[
\text{direct measure} \equiv \frac{\text{Var} (\pi \text{ news from } t + 1 \text{ to } t + m)}{\text{Var} (\text{shock to } m\text{-mat yield at } t)}.
\]

- **Indirect measure**

  Adds twice covariances of inflation news with real-rate news, term premia shock

- **Computation requires observations of investors’ predictions of yields, inflation at many horizons, or a dynamic model of inflation expectations and yields**
Measures from the existing literature

Population properties of five-year bond yield, inflation expectations implied by models

<table>
<thead>
<tr>
<th>Source</th>
<th>Sample Period</th>
<th>Variance ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>Campbell and Viceira (2001)</td>
<td>1952–1996</td>
<td>0.99 0.60</td>
</tr>
<tr>
<td>Campbell and Viceira (2001)</td>
<td>1983–1996</td>
<td>0.01 0.10</td>
</tr>
<tr>
<td>Ang et al. (2008)</td>
<td>1952–2004</td>
<td>0.50 0.50</td>
</tr>
<tr>
<td>Chernov and Mueller (2012)</td>
<td>1971–2008</td>
<td>0.17 0.03</td>
</tr>
<tr>
<td>Haubrich et al. (2012)</td>
<td>1982–2010</td>
<td>0.15 0.17</td>
</tr>
</tbody>
</table>
Gaussian state space approach

- Length $N$ state vector, Gaussian dynamics

$$x_{t+1} = \mu + K x_t + \Sigma \epsilon_{t+1}, \quad \epsilon_{t+1} \sim MVN(0, I)$$

- Affine mapping to yields

$$y_t^{(n)} = A_n + B'_n x_t + \eta_{n,t}$$

- Absence of arbitrage not imposed

No cross-sectional restrictions on $A_n, B_n$
Survey forecasts of inflation (1)

- Mean forecasts of GDP inflation one quarter ahead and three quarters ahead, from Survey of Professional Forecasters
- Assume affine function of the state

\[ E_t^S(\pi_{t+j}) = A_{\pi,j} + B'_{\pi,j} x_t + \eta_{\pi,j,t} \]

Following Pennacchi (1991), use of surveys in DTSM estimation is common

- Realized inflation not used – differs from usual approach
Survey forecasts of inflation (2)

- Assume equal to true expectations

\[ E_t(\pi_{t+j}) = A_{\pi,j} + B'_{\pi,j}x_t \]

Ang et al. (2007), Faust and Wright (2009), Croushore (2010): survey forecasts are more accurate than econometric forecasts

Faust and Wright (2012): “...purely judgmental forecasts of inflation are right at the frontier of our forecasting ability”

- Sticky information
  - Coibion and Gorodnichenko (2012): mean survey forecasts respond to info with a lag
  - Here: closer look at data does not support this conclusion
Estimation

- Sample periods

- $N = 3, 4, \text{ and } 5 \text{ factor versions}$

- Impose consistency of inflation expectations ($N + 1 \text{ constraints}$)

- Kalman filter

- Confidence bounds from Monte Carlo simulation using outer product estimate of covariance matrix of params
### Decomposition of variances of yield shocks

Four factors, full sample. Sum of ratios equals one, 95% bounds in brackets

<table>
<thead>
<tr>
<th>Maturity (years)</th>
<th>Direct</th>
<th>Direct + indirect</th>
</tr>
</thead>
<tbody>
<tr>
<td>One</td>
<td>0.15</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>[0.10 0.21]</td>
<td>[−0.15 0.23]</td>
</tr>
<tr>
<td>Five</td>
<td>0.13</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>[0.07 0.20]</td>
<td>[−0.28 0.25]</td>
</tr>
<tr>
<td>Ten</td>
<td>0.16</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>[0.07 0.31]</td>
<td>[−0.56 0.36]</td>
</tr>
</tbody>
</table>
## Real rate and term premium components

Four factors, full sample. Sum of ratios equals one, 95% bounds in brackets

<table>
<thead>
<tr>
<th>Maturity (years)</th>
<th>Average expected real rate</th>
<th>Term premium</th>
<th>2 Covar</th>
</tr>
</thead>
<tbody>
<tr>
<td>One</td>
<td>0.70</td>
<td>0.03</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>[0.44 1.18]</td>
<td>[0.01 0.16]</td>
<td>[−0.31 0.45]</td>
</tr>
<tr>
<td>Five</td>
<td>0.30</td>
<td>0.29</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td>[0.06 1.40]</td>
<td>[0.08 1.04]</td>
<td>[−1.12 0.51]</td>
</tr>
<tr>
<td>Ten</td>
<td>0.15</td>
<td>0.53</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>[0.03 1.53]</td>
<td>[0.17 1.51]</td>
<td>[−1.46 0.49]</td>
</tr>
</tbody>
</table>
Robustness: parsimonious/flexible models

1968Q4–2013Q4, fraction of variance shock directly attributable to news about average expected inflation

<table>
<thead>
<tr>
<th>Maturity (years)</th>
<th>3 factors</th>
<th>5 factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>One</td>
<td>0.18</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>[0.12 0.24]</td>
<td>[0.09 0.22]</td>
</tr>
<tr>
<td>Five</td>
<td>0.20</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>[0.12 0.30]</td>
<td>[0.06 0.22]</td>
</tr>
<tr>
<td>Ten</td>
<td>0.23</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>[0.11 0.40]</td>
<td>[0.06 0.41]</td>
</tr>
</tbody>
</table>

Number of free params: 47 (3-factor), 59 (4-factor), 72 (5-factor)
Robustness: sample period

Fraction of variance shock directly attributable to news about average expected inflation

<table>
<thead>
<tr>
<th>Sample</th>
<th>Maturity (years)</th>
<th>3 factors</th>
<th>4 factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1968-1987</td>
<td>One</td>
<td>0.19</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>Five</td>
<td>0.25</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>Ten</td>
<td>0.24</td>
<td>0.15</td>
</tr>
<tr>
<td>1987-2013</td>
<td>One</td>
<td>0.11</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>Five</td>
<td>0.09</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>Ten</td>
<td>0.10</td>
<td>0.10</td>
</tr>
</tbody>
</table>
Shocks to components of the 5-yr Treasury yield

- **A.** Avg expected short real rates
- **B.** Avg expected inflation
- **C.** Term premium

Fitted shocks from 4-factor model

Top row: full-sample estimation

Bottom row: spliced split-sample estimation
Impulse responses for a shock to expected inflation

A. Three quarter ahead expected inflation

B. Ex ante real short rate

C. Five-year nominal yield

D. Term premium for five-year nominal yield

Population responses to one standard deviation shock to 3-q-ahead inflation

Sample 1968Q4–2013Q4

95% confidence bounds are dashed blue lines
Impulse responses for a shock to the real short rate

A. Ex ante real short rate
B. Three quarter ahead expected inflation
C. Five-year nominal yield
D. Term premium for five-year nominal yield

- Population responses to one standard deviation shock to real short rate
- Sample 1968Q4–2013Q4
- 95% confidence bounds are dashed blue lines
Long-term inflation expectations

- Four-factor model (black line) implied ten-year average GDP inflation
- Trend-cycle model (dotted-dashed line) implied ten-year average GDP inflation
- Colored dots are different measures of ten-year average inflation, primarily CPI
Wrapping up

1. Robust conclusion is shocks to expected inflation over a bond’s life are small relative to other shocks to nominal yields.

2. Not enough info in data to distinguish reliably role of expected real short rates from role of term premium.

3. Recommendation is that macro-finance models be calibrated/estimated using size of expected inflation shocks.