The Bank Lending Channel of Financial Integration

Roman Inderst*  Sebastian Pfeil†  Falko Fecht‡

May 2014 [STILL PRELIMINARY]

Abstract

We show how even absent regulatory impediments, banks may have insufficient incentives to contribute to greater financial integration by interbank lending or cross-border mergers. Notably, where banks are funded largely from insured deposits, our model predicts a high degree of fragmentation, even though integration would lead to greater regional diversification of risk and greater allocative efficiency. Interbank lending is particularly insufficient across regions where it would generate greater diversification of risk. The key mechanism at work in our model is that a reallocation of funds across regionally segmented markets involves a positive coinsurance externality on depositors. Our model helps to explain why financial integration is still far from complete even in de jure homogeneous areas such as the Euro zone and it provides novel implications, notably on bank integration and the size of interbank connectedness. Our results also suggest that current re-regulation initiatives that would impede interbank lending and bank mergers may negatively impact on risk diversification and allocative efficiency across regional markets.

*Johann Wolfgang Goethe University Frankfurt and Imperial College London. E-mail: inders@finance.uni-frankfurt.de.
†Johann Wolfgang Goethe University Frankfurt.
‡Frankfurt School of Finance and Management.
1 Introduction

There is currently ample dissatisfaction, among policymakers and regulators but also shared in the wider public, about the performance of the global financial system. This has lead to widespread calls for a reduction in the size of the financial system, notably through taxing financial transactions, discouraging the use of certain financial instruments, and reducing the web of interconnections between its participants. While this seems largely justified to the extent that it can considerably reduce the risk of future crises and moral hazard, in particular fueled by expectations of bail outs, the pursuit of this agenda should also not interfere with the beneficial roles that the financial system and notably banks perform. In this paper we focus on the role of the financial system of reallocating resources across otherwise geographically segmented markets.\footnote{Cf. Merton and Bodie (1995) or Allen and Gale (2001).} In a stylized model of segmented funding and lending markets we argue that even when interbank lending and integration through forming larger banking groups is not hampered by regulatory interference, there is a strong tendency towards too little rather than too much integration. This implies that there is indeed a danger of large welfare losses when regulation overshots in reducing the scope for financial integration. Our theory also leads to various novel implications on the determinants of interbank lending and integration through cross-border mergers of financial institutions.

Our paper also contributes to the wider literature that tries to understand the forces and patterns of global financial integration. Such greater financial integration yields potentially large welfare benefits given cross-regional differences in net savings, in productivity, and in exposures to output shocks both on a global scale but also within relatively homogenous areas such as the Euro zone and the U.S.\footnote{For evidence and measurements see, for instance, ECB (2013a, p. 96-107), Kalemli-Ozcan et al. (2003), or Bonfiglioli (2008).} Various researchers have, however, noted that the extent to which such financial integration has been achieved is still limited. Surprisingly, this observation seems to apply not only to global financial integration, which is still restrained by regulation, but also to the financial integration in the Euro area, where de jure obstacles to financial integration have been removed.\footnote{For a discussion of the evidence for the limited effect of financial globalization see Stulz (2005). Lane (2009) and more recently van Beers et al (2014) discuss this for the Euro area.} To understand this puzzle, it is paramount to understand the incentives of banks as they play a key role both in collecting funds from households, notably through deposits, and in investing, notably in smaller and
medium-sized companies where local proximity is (still) of major importance. This is where our paper contributes as well.

The role of banks for financial integration, both through cross-border asset holdings and interbank lending as well as through cross-border mergers, has indeed been largely documented in the literature. Globally, Milesi-Ferretti and Tille (2011) argue that the cross-border activity of banks plays a dominant role for financial integration. Even within the Euro area the pre-crisis growth in cross-border asset holdings and financial integration was predominantly driven by the internationalization of European banks (cf. van Beers et al. 2014) and interbank lending (Sapir and Wolff 2013). We show how even absent impediments by regulation, reallocation of resources through both channels, namely interbank lending and cross-border mergers, will remain inefficiently low, preventing the efficient channeling of resources to their most productive use and limiting geographic diversification. Interestingly, we also show that integration becomes deeper and resource allocation more efficient when local economies are more closely aligned (as represented by a higher correlation). And we show how, notably for deposit-taking institutions, interbank lending becomes the more preferred channel, rather than fully integrating the allocation of resources through a merger. As our model also applies to regionally segmented funding and deposit markets, we predict a high level of fragmentation for such institutions even on a national level. This is different for financial institutions that are predominantly financed from non-insured sources.

As noted above, the predominant view in the present discussion on the re-regulation of the banking industry is that bank size has to be limited and interbank lending restrained. Several policy initiatives therefore strive to discourage interbank lending and aim at either directly limiting bank size or imposing additional levies on larger banks. Clearly, "too-big-to-fail" as well as "too-connected-to-fail" can generate moral hazard problems, which need to be restrained. An important insight of our analysis is, however, that there may also be strong disincentives working the opposite way and those effects need to be considered when determining the optimal degree of regulation.

---

4See, for instance, Petersen and Rajan (1994) and more recently Degryse and Ongena (2005).
5Cf. notably Figure 1 in Fecht et al. (2012) for the role of interbank lending.
6There is also a large literature showing that the deregulation of cross-regional banking improved diversification and capital allocation even though other financial markets were already de facto integrated before. See, for instance, Black and Strahan (2002), Acharya et al. (2006), and Acharya et al. (2010).
7According to the BIS (2011) banks considered as global systemically important financial institutions (G-SIFIs) will be required to hold up to 3.5% additional equity against their risk based assets. Whether a bank is considered a G-SIFI depends among other things on its wholesale funding ratio. On limiting the size of banks see also the respective provisions in the Dodd Frank Act, Section 622.
Our results are derived from a stylized model of segmented funding and lending markets, which may be regional or national. Local banks play a key role in identifying profitable local investment opportunities. We assume that a bank has loan making opportunities in its home region that have decreasing marginal returns and are subject to regional systematic shocks. A reallocation of funds across regions is beneficial as regions differ in the amount of funds made available from households. Because of the decreasing marginal returns from local lending an efficient allocation would entail the same expected marginal returns on bank loan portfolios. How can a reallocation of funds across regions be ensured? Overall, we consider three different channels and their interaction: 1) Banks can provide each other loans - and, in our model, these are not hampered by frictions due to informational asymmetries; 2) banks can merge, so that the integrated bank inherits both the access to local funding in the respective regions and loan-making opportunities; 3) absent integration, banks can also compete for funds in their non-domestic region, albeit they thereby have to overcome the impediment of switching costs due to the small granularity of retail deposits. The focus of our analysis (and, therefore, also of the subsequent summary of results) is on channels 1) and 2). It turns out that typically both are used insufficiently.

Take the case of interbank lending. Interbank loans expose banks to the regional systematic risk of other regions. Given that regional shocks are not perfectly correlated interbank loans improve the diversification of banks’ portfolios. However, improved diversification leads to a positive coinsurance externality to banks’ depositors. This is one of our key insights, as from this observation there follows a strong tendency towards inefficient reallocation of resources and inefficient diversification. If retail deposits are insured, then bank funding costs do not respond to the coinsurance, so that diversification generates only a windfall profit to the deposit insurance at the expense of bank shareholders. However, even if deposits are uninsured bank shareholders will underinvest in interbank lending. This is so as arguably banks cannot commit to a certain degree of diversification through interbank lending, so that the link between interbank lending and funding costs must remain tenuous. Greater diversification through interbank lending then again generates a windfall profit to depositors at the expense of shareholders.

With a cross-border merger depositors of the different regions are repaid out of the proceeds of the diversified loan portfolio. This further increases the windfall to depositors from cross-border activity and makes cross-border mergers unattractive for bank shareholders, both in general and, in particular, relative to interbank lending if deposits are insured. Notably, when a bank’s funding base consists largely out of insured deposits,
incentives to merge are largely absent, even though - as we also show - once a bank was integrated, then this could lead to a more efficient allocation of resources across regional markets. In this case, notably when also competition for retail deposits across regions is largely ineffective due to switching costs, interbank lending remains the sole - though also potentially largely ineffective - channel to bridge regional differences in funding and investment opportunities. However, if much of banks’ funding is uninsured, the anticipation of greater diversification and investment efficiency in an integrated bank can lead to lower funding costs. The merger serves in other words as a commitment device to diversify. Thus only if the share of uninsured deposits is sufficiently large mergers can be attractive for shareholders. This finding also has an important policy implication: It shows that the implicit and explicit insurance of bank debt holders are a subtle disincentive to bank mergers. It also suggests that the new EU Bank Recovery and Resolution Directive that increases the bail-in of bank debt holders, should make bank funding costs become more risk sensitive and could thereby foster Euro area banks’ incentives to merge.

A further prediction of our model is that the benefits of cross-border interbank lending and mergers depend on the asymmetries in the regional deposit base and on the cross-regional correlation in loan portfolio risks. In particular, the higher the cross-regional correlation in credit risks are, the lower the coinsurance externality and as a consequence the more do banks lend to each other. This aspect of our paper adds a new insight to the analysis of correlated risk taking, here notably of interconnected banks. Acharya and Yorulmazer (2006, 2007) argue that banks have an incentive to invest in correlated risks in order to benefit from the time inconsistency of a no bail-out policy in case of a widespread banking crisis.\footnote{In a similar vein Farhi and Tirole (2012) argue that banks jointly expose themselves to liquidity risks in order to benefit from central banks’ liquidity provisions.} Along this line of argument, also Wagner (2010) argues that banks may excessively diversify, which increases their exposure to a systemic shock. Our paper gives a very different perspective: Here banks leave diversification potentials unexploited and ultimately bear inefficiently large idiosyncratic risks the higher the cross-regional correlation. This also implies that in our model a bail-out in case of a joint bank failure might actually correct for the incentives to underinvest in interbank lending.

Our model also shows that the risk premium required by uninsured depositors can decrease in the size of banks. If banks grow larger due to mergers, depositors can expect to benefit from the coinsurance and will charge a lower deposit rate. If one bank has a larger deposit base relative to other banks its marginal return from lending in its own region is low and the return from cross-border lending relatively high. It increases interbank
lending even though this partially benefits depositors. Depositors bear lower risk and charge a lower premium. Thus the deposit rate is sensitive to bank size even if too-big-to-fail guarantees do not play a role. Studies such as O’Hara and Shaw (1995) and Penas and Unal (2004) that study the market distortions due to too-big-to-fail guarantees by regressing bank funding costs on bank size may thus provide a misleading picture.

Our paper also contributes to the general literature analyzing the interbank market. The majority of these papers, such as Bhattacharya and Gale (1987), Allen and Gale (2000), and Freixas et al (2000), stress the role of the interbank market in liquidity risk sharing. Our model focuses on the allocation of funds through interbank lending.\textsuperscript{9} Our comparison of the efficiency of resource allocation when banks are integrated and when they are separate relates more broadly to the large literature on internal capital markets in firms. Yet, apart from our focus on overall economic efficiency, there are also major differences to the extant literature, relating in particular to the fact that we explicitly consider banks and, in case of non-integration, lending between independent institutions as an alternative way to channel funds to different investment projects. A large number of empirical papers on multinational banks draws, however, largely on the extant theoretical literature on internal capital markets, thereby potentially ignoring the specificities of the banking sector.\textsuperscript{10} One special feature of the banking industry is that funding costs are not necessarily risk sensitive and that having unimpeded access to particular (retail) funding may require integration. Also the fact that multinational banks with internal capital markets have to be traded-off against cross-border interbank lending is largely specific to the financial sector. As our analysis also shows, these specificities have important implications for how regulation affects efficiency.

The rest of this paper is organized as follows. In Section 2 we introduce our model of segmented funding and lending markets. The analysis with non-integrated banks is contained in Section 3. Section 4 considers the allocation with an integrated bank and makes the decision whether to integrate or not endogenous. Section 5 concludes with a summary of the key positive and normative implications of our model.

\textsuperscript{9}From that perspective our approach is more closely related to the framework used in Diamond and Rajan (2005, 2006). Building on the banking model proposed by Calomiris and Kahn (1991) and Diamond and Rajan (2001), they show that a spot money market might deter liquidation incentives, while our focus is on the risk sharing and reallocation decision.

\textsuperscript{10}See, for instance, Campello (2006) and Cetorelli and Goldberg (2012).
2 The Model

Based on the motivation provided in the Introduction, we consider a stylized model of segmented lending and funding markets. We also build into our model a role for banks in both collecting savings from households and making informed investment decisions through loans. The various assumptions that we thereby make follow closely the large extant literature on banking,\(^{11}\) which is why the following presentation of our model focuses on those ingredients that are more novel and decisive for our subsequent results.

There are two locally segmented markets, \(n = A, B\). Each market is populated by a mass one of households. In market \(A\), each household has funds of size \(M_A\). As there is a mass one of households, this also represents the measure of the total funding potential when funds are raised solely in market \(A\). In market \(B\), each household has funds of size \(M_B\). We assume without loss of generality that \(M_A \geq M_B \geq 0\). The interesting case will be that where the local funding potential differs across markets. To derive for this a convenient measure, we denote total available funding by \(M_A + M_B = 2M\) and write \(M_A = M + z\) and \(M_B = M - z\) with \(z \geq 0\). When analyzing the role of banks to allocate funding across markets, we will conduct a comparative statics analysis in \(z\).

To streamline the model, we abstract from modelling consumption and saving decisions of households and thus take as given that households set aside the respective funds \(2M\) for later consumption. Next to a storage technology, which simply preserves the value of funds, we introduce a risky investment technology in each of the two markets. For this we suppose that in each market there is one penniless entrepreneur who has access to a "real" investment opportunity, as specified next, and that there is at the same time a large number of fraudulent entrepreneurs who will abscond with any funds that they receive. By specifying that only one locally active bank has the necessary "soft" information to screen out fraudulent entrepreneurs, we grant each local bank monopoly power in the lending market and also preclude any forms of non-intermediated financing. When we later consider also the case of an integrated bank, we suppose that the integrated bank inherits this knowledge across both markets.\(^{12}\)

The project of the "real" entrepreneur, on whom we can focus, is risky as it only succeeds with probability \(p\). In case of success, when having received funds of size \(F\), the project pays back \(L(F)\), while it pays back zero when it fails. The (production) function

\(^{11}\)See, for instance, Freixas and Rochet (2009).

\(^{12}\)Hence, we abstract from any agency related inefficiencies that larger banks could have in generating and processing the necessary local information (cf. Stein 1997).
\( L(\cdot) \) satisfies \( L' > 0 \) and \( L'' < 0 \). As we stipulate that the monopolistic local bank can make a take-it-or-leave-it offer to the identified local entrepreneur, the function \( L(F) \) also represents the local lending (or loan-making) potential. By assuming that it is symmetric across markets, we can focus our analysis on banks’ role to bridge funding differences across markets. A crucial parameter in our analysis, however, will be the extent to which the performance of loans in the two markets is correlated. We denote the respective correlation coefficient by \( \rho \) and allow for values \( 0 \leq \rho \leq 1 \). As is immediate, the likelihood with which loans in both markets perform is then given by \( p^2 + \rho p(1-p) \), which becomes \( p^2 \) when projects are fully independent (\( \rho = 0 \)) and \( p \) when projects are perfectly (positively) correlated (\( \rho = 1 \)).\(^{13}\) Next to \( z \), which captures the difference in the local funding base, \( \rho \) will be our main comparative variable in what follows. We further want to focus our analysis on the case where local funding is never in excess, so that we assume throughout that

\[
pl'(M + z) > 1. \tag{1}
\]

Further, by ensuring that loans are not "too" profitable, we create scope for default and contagion when interbank loans are not repaid. A sufficient condition for this is that

\[
L(M) < 2M. \tag{2}
\]

In words, when only half of all available funding, \( M \), is invested, then in case of success the resulting payoff is insufficient to pay back all available funding, \( 2M \).

**Strategies and Timing.** With this background, we now consider the following funding and investment game. In \( t = 1 \) funding can be collected from households. Given our preceding discussion, households will either invest in the storage technology or invest in risky projects through one of the two banks. In our baseline analysis, we further stipulate that households in market \( n \) can only invest through bank \( n \), albeit in Appendix B we allow both banks to compete for funding across markets.\(^{14}\) Our key assumption is that households’ claims on banks’ assets will be senior to those of shareholders. We comment shortly on this assumption. We will refer to these claims as deposits, so that in our baseline

\(^{13}\)Note at this point that our specification of a single loan opportunity in each market can also be interpreted as a perfect positive correlation for loans in a local market. What is essential for our following arguments is that, in this case, loans in the bank’s own portfolio are more correlated than loans across banks’ loan portfolios.

\(^{14}\)There, we still endow the local bank with an advantage: Households who invest in a non-local bank will incur switching costs.
model at \( t = 1 \) bank \( n \) offers a deposit rate \( r_n \) in its local market, thereby attracting deposits of total size \( R_n \leq M_n \).

Banks’ only additional source of funding is interbank lending, which can be arranged in \( t = 2 \). Interbank lending prescribes a transfer of funds \( W_n \) from bank \( n' \) to bank \( n \) in exchange for a promised repayment \( w_n \). To make our baseline analysis as transparent as possible, we first stipulate that bank \( A \), which from \( M_A \geq M_B \) has a larger deposits base, can in \( t = 2 \) make a take-it-or-leave-it offer to bank \( B \). However, we show in Appendix C that results are unchanged when we stipulate instead a game of Nash bargaining with a more symmetric distribution of bargaining power.

In \( t = 3 \) banks extend loans in their local market. Payoffs are realized in \( t = 4 \). All parties are risk neutral and we abstract from discounting. Note finally that banks are run in the interest of shareholders as residual claimants. Below we adapt our analysis to the case with only a single integrated bank operating across the two markets. There, we also allow for integration to arise endogenously in our model.

Before proceeding to the analysis, it remains to comment on our specification of banks’ contracts with households. The key assumption in what follows is that these claims are senior, both to those of shareholders and to any interbank loan. The assumption of such deposit financing is again shared with a large literature in banking. Though it is there often assumed exogenously as well, it is well known that seniority can be given a microfoundation, e.g., through appealing to an agency problem between the bank and its lenders (cf. Diamond and Rajan 2005).\(^{15}\) However, we abstain from enriching our model in such ways, thereby focusing on what is novel in our analysis compared to the extant literature. Recall also from our discussion in the Introduction that the seniority of retail deposits as well as the immediacy with which they can be withdrawn are not necessarily undone by households’ inertia compared to institutional investors, notably in our model the creditor bank in case of an interbank loan. One reason for this is the discussed longer maturity of these claims.

**Plan of Analysis and Overview of Results.** The plan of our further analysis is as follows. Section 3 contains our main analysis of interbank lending to bridge funding differences between segmented banking markets. Our main result will be that of a persistent gap despite the operation of a frictionless interbank lending market (Section 3.1). Funding differences will only be smoothed out completely when a further (marginal) increase of in-

\(^{15}\)Other aspects of deposit financing, such as a "first-come-first-serve" feature that gives rise to bank runs or liquidity in a Diamond-Dybvig model, are however not at the core of our analysis.
terbank lending no longer offers coinsurance benefits to depositors, which will be the case either when an interbank loan is already large enough so as to have a contagious effect or when loan portfolios are perfectly correlated. These insights further give rise to various comparative statics results, notably for the size of interbank lending, depending on the characteristics of banks’ segmented funding and lending markets (Section 3.2). Funding differences are also persistent when deposit markets are no longer fully segmented, but - given the low granularity of retail deposits - subject to frictions in the form of switching costs. To streamline the exposition of results, however, we relegate an extension to competitive retail deposit markets to Appendix B, where we confirm our key comparative results.

Section 4 considers an integration of banks as an alternative way to allow for the reallocation of funds across segmented markets, other than through competition on the deposit market or through interbank lending. Importantly, a bank merger does not per se reduce frictions in the allocation of funds, as the interbank lending market is notably not plagued by adverse selection or moral hazard in our model. Still, we show how it can substantially alter the allocation of funds across local markets (Section 4.1). The fundamental difference is that with integration depositors in both markets, A and B, have a claim that needs to be repaid out of the proceeds of the bank’s lending in both markets, rather than only in one market as in the case of separate banks and no interbank lending. While integration and the resulting reallocation of funds within the integrated bank can improve efficiency compared to interbank lending, the additional coinsurance benefits that such a merger would provide to depositors can make such mergers unprofitable from shareholders’ perspective (Section 4.2). In fact, when depositors are insured, we show that such a merger will never arise, such that the banking industry would remain highly (regionally) fragmented. Neither bank mergers nor the operation of an interbank market would then ensure the efficient allocation of funds to bridge funding differences.

When depositors (or a sufficiently large part of all funding) are, however, uninsured, then the promise of coinsurance across not fully correlated loan portfolios may allow an integrated bank to obtain better funding conditions, rendering integration profitable. There is then, however, a trade-off given that, as we show, integration can also open up additional risk-shifting opportunities that independent banks would not realize through interbank lending alone. Banks may then still choose to remain non-integrated so as to thereby commit - vis-à-vis non-insured depositors - not to engage in such opportunistic risk taking. The analysis of integration when depositors are non-insured also allows us to derive
additional implications on the relation of bank size and funding costs.

3 Baseline Analysis

3.1 Shortfall of Interbank Lending

In this Section we consider the determination of interbank lending in $t = 2$. Hence, we presently take as given the retail deposit funds $R_n$ that each bank $n$ has already attracted through promising an interest rate $r_n$. As noted above, we will later consider the stage where $R_n$ and $r_n$ are determined. Note also that for the present analysis it does not matter whether deposits are insured or not, though this will prove decisive further below. The main result in this Section will be a characterization of optimal interbank lending and its key determinants.

As is intuitive (and formally derived in the proof of Proposition 1), in equilibrium there will be at most one interbank loan, i.e., in our model there is no scope for both a loan of bank $A$ to bank $B$ and vice versa. As the purpose of interbank lending is to better align banks’ funding with their loan-banking opportunities, it is equally intuitive that an interbank loan will be made, if at all, by the bank with higher initial funding $R_n$ to that with lower funding. We presently suppose that this is bank $A$, so that $R_A \geq R_B$. Intuitively, this will also hold when funding is endogenized, given that bank $A$ has access to a (weakly) larger local depositor base. Denote thus by $W \geq 0$ the interbank loan that bank $A$ makes to bank $B$ and by $w \geq 0$ the respective agreed repayment. Recall that in this Section we stipulate that the terms of interbank lending are determined through a take-it-or-leave-it offer made by bank $A$ to bank $B$ (cf. however Appendix C).

Banks are managed in shareholders’ interest. Take first bank $A$. For given (remaining) funding, $F_A = R_A - W$, provided that this is then used to make a loan of the same size, the expected profits of shareholders are

$$
\pi_A = \left[ p^2 + \rho p(1-p) \right] \left[ L(F_A) + w - R_A(1 + r_A) \right] + \rho (1-p)(1-\rho) \left[ \max \{0, L(F_A) - R_A(1 + r_A)\} + \max \{0, w - R_A(1 + r_A)\} \right].
$$

(3)

The first line in (3) accounts for the state where all loans are successful. That is, with the respective probability, $p^2 + p(1-p)\rho$, the loan portfolios of both bank $A$ and bank $B$ perform. This also enables bank $B$ to repay $w$ to bank $A$. Clearly, the respective probability is higher when loan portfolios are more highly correlated (higher $\rho$). When loan portfolios are independent, so that $\rho = 0$, the respective probability becomes simply
Note that we implicitly assume that the total repayment to bank A, arising from both its own (corporate) loan and the loan made to bank B, is sufficient to cover the repayment that bank A promised to its depositors, \( R_A(1 + r_A) \). Naturally, this will always be the case in equilibrium. The second line in (3) accounts jointly for two states that are equally likely: that where only the loans of bank A perform (captured by the first part) and that where only the loans of bank B perform (captured by the second part). When both loan portfolios do not perform, then clearly shareholders of bank A realize zero profits.

Expression (3) for the payoff of bank A’s shareholders thus contains various case distinctions, depending on whether the repayment of the bank’s own loans, the repayment of its loan to bank B, or only both together are sufficient to cover claims to its own depositors, \( R_A(1 + r_A) \). When \( L(F_A) > R_A(1 + r_A) \), then there is a positive payout to the shareholders of bank A even when bank B cannot repay its interbank loan. This case is more likely if bank A’s funds are mostly invested locally, i.e., in its corporate loan, so that the size of the interbank loan \( W \) and consequently also the respective promised repayment \( w \) are small relative to \( L(F_A) \). The other subcases are those where a failure of repayment from bank B causes default of bank A, i.e., interbank lending can then have a contagious effect. While then the proceeds from its own loans, \( L(F_A) \), allow bank A to make some repayment to depositors, when its loan to bank B is not paid back this is no longer sufficient to allow for a payout to shareholders as well. Proposition 1 below characterizes the outcome for all possible cases. Which case arises in equilibrium is determined further below, as it depends on the initial allocation of funds \( R_n \) as well as the interest rates promised to depositors \( r_n \).

We next state the profits of shareholders of bank B,

\[
\pi_B = p \left[ L(F_B) - w - R_B(1 + r_B) \right],
\]

where \( F_B = R_B + W \). Shareholders of bank B only receive a positive payout when the bank’s own loans perform.\(^{16}\)

Given the presently assumed take-it-or-leave-it offer by bank A, we have next that

\[
w = L(F_B) - L(R_B).
\]

Hence, in case there is a loan of size \( W \) from bank A to bank B, the respective repayment \( w \), as specified in (5), ensures that bank B’s profits are just equal to the "standalone payoff" \( L(R_B) \). This reflects our present specification of a take-it-or-leave-it offer by the creditor bank.

\(^{16}\)That profits are positive in this case will naturally arise in equilibrium, so that we can safely restrict consideration to this case.
Proposition 1  Consider stage $t = 2$, where banks can arrange for an interbank loan $W$ from bank $A$, which has more retail funding as $R_A \geq R_B$, to bank $B$. When $W > 0$, then there are two cases to consider.

Case 1: The loan size $W$ and the repayment $w$ are chosen sufficiently small so that a failure of repayment does not itself cause the insolvency of the creditor bank $A$, as $L(R_A - W) \geq R_A(1 + r_A)$. Then, $W = W_1^*$ uniquely solves

$$pL'(R_A - W_1^*) = [p^2 + pp(1 - p)]L'(R_B + W_1^*).$$

(6)

Case 2: $W$ and $w$ are, instead, sufficiently large so that from $L(R_A - W) < R_A(1 + r_A)$ a failure of repayment causes insolvency also of the creditor bank $A$. Then, $W = W_2^*$ uniquely solves

$$L'(R_A - W_2^*) = L'(R_B + W_2^*).$$

(7)

Proof. See Appendix.

For a discussion, note first that an efficient reallocation of funds through an interbank loan would require that $W = W^{**}$ with $W^{**}$ solving $L'(R_A - W^{**}) = L'(R_B + W^{**})$ - or, expressed differently, $W^{**} = (R_A - R_B)/2$, so that the same amount of funding is allocated to either market. This is the case in condition (7), where thus $W = W_2^* = W^{**}$, but not in condition (6), where $W = W_1^* < W^{**}$. In fact, there are altogether two cases in which the efficient outcome will not obtain. The first case is that when $W = 0$. We discuss below when this case arises. The other case is that referred to as Case 1 in the Proposition 1, where the respective value $W = W_1^*$ solves instead (6). There, unless banks’ loan portfolios are perfectly correlated, so that $\rho = 1$, the respective size of the interbank loan $W_1^*$ always remains inefficiently low. As a consequence, more of the total available funding, $R_A + R_B$, is allocated to loans in market $A$ than to loans in market $B$.

As a particular case, suppose for an illustration that loan performance across the two banks is independent ($\rho = 0$). Then, the negotiated interbank loan, if positive at all, is such that at this level the non-risk-adjusted return from loans of the creditor bank $A$ is equal to the risk-adjusted return from loans of the debtor bank $B$: $L'(R_A - W_1^*) = pL'(R_B + W_1^*)$. In Case 2 of Proposition 1, instead, the resulting allocation is efficient. We explain next how the difference between the cases arises.

The results of Proposition 1 arise from the incentives of leveraged shareholders to engage in risk shifting. Precisely, as long as the correlation between the corporate loan portfolios of bank $A$ and $B$ is not perfect, as $\rho < 1$, interbank lending diversifies the overall
loan exposure of bank A. That is, when bank A’s own (corporate) loans fail, depositors can still be (partly) paid back when the loans in market B perform, as this will result in a repayment of the interbank loan. Thus, the resulting diversification that reduces bank A’s own loan portfolio but generates an exposure to loans in market B tends to make the claims of its depositors safer and thus more valuable. In Case 1 of Proposition 1, this positive externality of diversification for bank A’s depositors generates a wedge between the allocation of funds that would be efficient (through choosing $W = W^{**}$) and the allocation of funds that results as an outcome of optimal interbank lending in shareholders’ interest ($W = W^*_1 < W^{**}$). This wedge is intuitively smaller when banks’ loan portfolios become more (positively) correlated, in which case depositors of bank A have less to gain from such coinsurance of their deposits through interbank lending.\footnote{Of course, under full deposit insurance these benefits would be reaped rather by the deposit insurance institution than by insured depositors themselves, other than in the case without deposit insurance. These considerations will prove important later when the equilibrium interest rate is determined.} Consequently, the optimally arranged interbank loan $W^*_1$ increases in Case 1 as banks’ loan portfolios become more correlated. The characterization of Case 1 would thus predict a positive correlation between the size of interbank lending and the correlation of the local loan portfolios of the involved banks. The resulting increase in $W$ enhances efficiency.

\textbf{Corollary 1} Suppose that Case 1 from Proposition 1 applies. Then, as the correlation between banks’ loan portfolios increases (higher $\rho$), the size of the interbank loan $W = W^*_1$ increases as well.

Corollary 1 conducts a comparative analysis only for Case 1. Once we have derived the equilibrium for the full game (and also for the case with competition for retail deposits), we will show that our model predicts a robust positive relationship between interbank lending and the correlation of banks’ loan portfolios. For now, however, we postpone a further discussion of this implication.

The allocation of funding becomes efficient in Case 2 of Proposition 1. The reason is as follows. In this case the exposure of bank A to the risk of bank B is sufficiently large such that failure of repayment of the interbank loan would make bank A insolvent as well, regardless of the performance of its own loan portfolio. Then, $W = W^*_2$ solves (7). Intuitively, once the interbank loan is sufficiently large, so that a failure of repayment has such a "contagious effect", a marginal adjustment of the loan has no longer the discussed positive externality on depositors of bank A. We turn next to the question when the different cases apply in equilibrium.
3.2 Equilibrium Analysis

To conduct a further comparative analysis in terms of the model’s primitives, we turn to stage \( t = 1 \) of our model. Recall that we presently still assume that banks can only access their own local deposit market. Also, we take as given that banks are non-integrated. Taken together, presently the only way to reallocate funds between the two markets is thus through interbank lending, as analyzed in the preceding Section.

Given locally segmented retail deposit markets, bank \( A \) has access to funds of size \( M_A = M + z \) and bank \( B \) to funds of size \( M_B = M - z \). Hence, by continuously varying \( z \in [0, M] \) we obtain a more asymmetric allocation of retail deposits across the two markets.

As discussed in the Introduction, such a more or less asymmetric situation may arise both between different countries and between different regions, depending for instance on local demographics that will then be reflected in different savings rates. The further primitive that we consider in what follows is the correlation between banks’ loan portfolios, which derives from the respective correlation between their local real investment opportunities.

To close the model, recall that at \( t = 1 \) banks attract funds by promising an interest rate \( r_n \). As we presently abstract from competition and as, for simplicity only, we ignore discounting, \( r_n = 0 \) results with deposit insurance. (Recall that households have always access to a safe storage technology.) The respective interest rate \( r_n \) that prevails without deposit insurance will be determined so that depositors just break even in equilibrium. Without interbank lending, we have \( r_n = 1/p - 1 \), while from the preceding discussion we will have \( r_A < 1/p - 1 \) when the depositors of bank \( A \) can expect that the bank diversifies through making a loan to bank \( B \). For bank \( B \)’s depositors we will always have \( r_B = 1/p - 1 \).\(^{18}\)

Proposition 2 Suppose that retail deposit markets are fully segmented, so that \( R_n = M_n \). Then the following comparative result obtains for the resulting equilibrium. The (generically uniquely determined) interbank loan \( W \) is (weakly) increasing in both the difference in the size of the retail deposit markets, \( M_A - M_B = 2z \), and in the correlation between local loan portfolios \( (\rho) \). Precisely, we have the following results:

i) Comparative analysis in \( \rho \): Take \( z > 0 \) as given. Then, there are two thresholds \( 0 \leq \rho_1 \leq 1 \) and \( \rho_1 \leq \rho_2 \leq 1 \) such that \( W = 0 \) when \( \rho < \rho_1 \), \( W = W_1^* \) and strictly increasing in \( \rho \) when \( \rho_1 < \rho < \rho_2 \), and \( W = W_2^* = z \) when \( \rho \geq \rho_2 \).

\(^{18}\)Note that from (1) also under autarky there is no excess supply of deposits in any given market. Otherwise, we would have to consider the additional case where, notably without deposit insurance, bank \( A \) may "ration" the inflow of deposits.
ii) Comparative analysis in \(z\): Take \(\rho < 1\) as given. Then, there are two thresholds \(0 < z_1 \leq M\) and \(z_1 < z_2 \leq M\) such that \(W = 0\) when \(z < z_1\), \(W = W_1^*\) and strictly increasing in \(z\) when \(z_1 < z < z_2\), and \(W = W_2^* = z\) when \(z \geq z_2\).

**Proof.** See Appendix.

Consider first the comparative analysis in \(\rho\), where the difference in the size of retail deposits is taken as given (assertion i) in Proposition 2). As we increase the correlation between loan portfolios in the two markets, interbank lending always increases weakly - and strictly so at two instances: First, when we are in Case 1 of Proposition 1 and, second, as we proceed to Case 2 for still higher \(\rho\), in which case \(W\) strictly jumps upwards (namely at \(\rho = \rho_2\)). We should thus be likely to see either relatively high interbank exposures, namely when banks’ loan portfolios are highly correlated, and relatively low exposures, namely when banks’ loan portfolios are less correlated. This is further illustrated below in an example. Note at this point that we will further below collect all implications from the model.

Take now assertion ii) of Proposition 2. As \(z\) increases, there are two reasons for why the interbank loan should increase in size, holding now the correlation \(\rho\) fixed. A larger interbank loan is then needed to reduce the gap between the size of retail deposits in the two markets and, thereby, increase efficiency. The second reason is that as \(z\) increases, we are more likely to be in Case 2 with an efficient interbank loan of size \(W = W_2^* = z\). To be specific, when \(r_n = 0\) as there is deposit insurance (and no competition for retail deposits), this is the case when \(z > L(M) - M\).\(^{19}\)

**Illustration.** Take a linear-quadratic loan-value function, \(L(F) = bF - aF^2\), for which we can now explicitly derive both the resulting interbank loan and the allocation of funds as well as when the different cases arise. For this, we normalize the size of funds so that, when there is symmetry, each bank has a potential deposit base of mass one: \(M = 1\).\(^{20}\)

\(^{19}\)This is, however, only a necessary condition for that Case 2 applies, but not sufficient (cf. the proof of Proposition 2 for details).

\(^{20}\)Precisely, we obtain for the thresholds \(z_1\) and \(z_2\), as used in Proposition 2, that

\[
\begin{align*}
z_1 &= \left(\frac{1}{2a}\right) \frac{(b - 2a)(1 - p)(1 - \rho)}{1 + p + (1 - p)\rho}, \\
z_2 &= \left(\frac{1}{2a}\right) \frac{(b - 2a)^2}{1 + p + (1 - p)\rho} - \frac{4a + 8a^2 - 8ab + b^2}{4a}.
\end{align*}
\]

Both thresholds are also a function of \(\rho\) and we can also solve from this for the respective thresholds \(\rho_1\) and \(\rho_2\). Note that these thresholds need not always be interior.
Figure 1: This graph plots the critical levels $z_1$ and $z_2$ as functions of $\rho$. There will be interbank lending for $z \geq z_1$ (light grey) and lending will be contagious for $z \geq z_2$ (dark grey). Parameter values are $p = 7/8$, $a = 1/16$, and $b = 9/5$.

For Figure 1 we choose the success probability $p = 7/8$ and depict the three regions for when the different cases apply.

For the following Figure 2, we choose again $p = 7/8$, but now two different values for the initial funding gap: $z = 0.5$, and $z = 0.9$. The case with contagious interbank lending only arises when the asymmetry of retail deposits is sufficiently large. In this case, note the aforementioned discrete jump of the interbank loan as the correlation $\rho$ crosses the respective boundary, $\rho_2$, so that $W = W^*_2$.

We return to the implications of our analysis in Section 5, where we collect the various predictions from our model.

## 4 Integration of Banks

### 4.1 Allocation of Funds within an Integrated Bank

We now suppose that a single bank operates across both markets, $A$ and $B$. We will ask how the resulting allocation of funds differs from that achieved when markets are served by separate banks. While the present analysis will be of interest on its own, as we notably derive conditions for when an integrated bank may either achieve more or less efficiency in its lending, it will also form the background for our subsequent analysis of endogenous
integration.

When a single bank, $AB$, operates, the question of whether retail deposit markets are fully segmented or not becomes superfluous. Also, as the repayment of all deposits is served by all of the bank’s assets, as long as all depositors obtain the same level of deposit insurance (or not), in $t = 1$ the integrated bank will now offer the same interest rate $r_{AB}$ to depositors in both markets. As there is no interbank lending, the game then proceeds to $t = 3$, where the bank allocates its aggregate funds over the two segmented lending markets, choosing $F_A$ and $F_B$. Payoffs are again realized in $t = 4$.

The integrated bank’s shareholders’ profits are given by

$$
\pi_{AB} = \left[ p^2 + p (1 - p) \rho \right] \left[ L(F_A) + L(F_B) - R_{AB} (1 + r_{AB}) \right] + p (1 - p) (1 - \rho) \max \{ 0, L(F_A) - R_{AB} (1 + r_{AB}) \}.
$$

(8)

Note first that without loss of generality we restrict attention to cases where weakly more funds are allocated to market $A$: $F_A \geq F_B$. The first line in (8) accounts for the outcome where loans in both markets are successful. With respect to the second line in (8), note first that the case where the repayment from loan market $B$ alone would already be sufficient for the integrated bank to remain solvent is excluded by assumption (2)

\[\text{Again, as in the case of separate banks, we abbreviate the analysis by stipulating that in this case the bank can indeed fully repay its depositors. This will clearly be the case in equilibrium.}\]
and by $F_B \leq F_A$ and thus $F_B \leq M$. Hence, when only one loan portfolio performs, then from $F_A \geq F_B$ shareholders of the integrated bank can only expect to receive a payout when loans in market $A$ perform. The case distinction in the second line of (8) is then whether this is indeed sufficient or not to make depositors whole, i.e., whether $L(F_A) > R_{AB}(1 + r_{AB})$ indeed holds or not.

**Proposition 3** Take the case where an integrated bank, $AB$, operates and consider the optimal allocation of total funds $R_{AB}$ across the two loan markets $A$ and $B$ in $t = 2$: $F_A^*$ and $F_B^*$, where $F_A^* + F_B^* = R_{AB}$. Assume wlog that the amount invested in loan portfolio $A$ is weakly larger than the amount invested in loan portfolio $B$, $F_A^* \geq F_B^*$. If there is positive investment in both markets, so that also $F_B^* > 0$, then there are two cases to consider: In Case 1, where $L(F_A^*) \geq R_{AB}(1 + r_{AB})$, it holds that

$$pL'(F_A^*) = [p^2 + p(1 - p)\rho] L'(F_B^*),$$

(9)

while in Case 2, where $L(F_A^*) < R_{AB}(1 + r_{AB})$, it holds that

$$L'(F_A^*) = L'(F_B^*).$$

(10)

Further, there are two thresholds for the correlation, $\rho_1^* \leq \rho_2^* \leq 1$, so that $F_B^* = 0$ when $\rho < \rho_1^*$, Case 1 applies when $\rho_1^* < \rho < \rho_2^*$, and Case 2 when $\rho > \rho_2^*$. Overall, as the correlation increases, the allocation becomes (weakly) more efficient ($F_A^* - F_B^*$ decreases).

**Proof.** See Appendix.

Our main analysis in this section concerns a comparison of the allocation obtained under integration with that obtained when banks are separate. Before turning to this, a few comments on Proposition 3 are in order. Case 2 represents that where the allocation is efficient, while too much is invested into one market in Case 1 or when even $F_A^* = R_{AB}$ and $F_B^* = 0$. In cases 1 or 2, provided that they apply, the same allocation of funds is realized both with an integrated bank and with separate banks, as can be seen from inspecting the first-order conditions for $F_n^*$ and $W^*$, respectively. Also the rationale for the inefficiency in Case 1 is the same for an integrated bank as it was in case of two separate banks and a possible interbank loan. In both instances, a more diversified allocation of funds across the two markets, that is by the integrated bank or by the creditor bank in case of an interbank loan, has a positive coinsurance externality on the respective depositors. Leveraged shareholders have, instead, an incentive to (inefficiently) increase risk, which results in an inefficiently high allocation of funds in one market. As we explore next, the incentives for doing so may overall be either higher or lower in the integrated bank.
Comparing the Allocation of Funds. So as to isolate more clearly the differences between interbank lending and the allocation inside an integrated bank, we again abstract from the operation of a retail deposit market in case of non-integrated banks. To understand the key differences between the allocation of funds through the interbank market and that in an integrated bank, the difference in the treatment of depositors is key. When banks are non-integrated, it is only through the interbank loan from A to B that a deposit may be repaid both from loans in market A and loans in market B. When no interbank loan is made, deposits in bank A and deposits in bank B will only be repaid when the loan in the respective local market performs. Instead, all deposits in the integrated banks represent senior claims, compared to those of shareholders, to the proceeds from loans in both market A and market B. We show that this difference in the "status quo" of depositors’ claims has two contrasting implications, depending notably on the correlation of loans in the two markets: The allocation of funds in the integrated bank can become both more efficient, representing ultimately less risk-shifting, as well as less efficient, as then an even larger share of funds is allocated to only one market.

Recall now that, at least from the perspective of marginally adjusting the interbank loan, in the case of separate banks the size of the interbank loan is efficient only if it can be "contagious" for bank A: Both the outstanding claims of its depositors and the size of the loan are then large enough so that failure of repayment of the interbank loan would result in insolvency of bank A even when the loans in market A performed. When there is integration, the case where the performance of only one loan portfolio is sufficient to repay depositors becomes less likely. Hence, with integration the contagious case is obtained even when the allocation of funds between the two markets is still more asymmetric, implying that the full incremental benefits from further increasing efficiency by diversifying loan-making across markets accrue all to shareholders. Consequently, with integration it is now more likely that the efficient allocation (Case 2) is obtained.

Recall next that when correlation is low, without integration we obtain $W = 0$, so that the allocation remains inefficiently asymmetric. With integration, however, in this case the outcome will be still less efficient, as even less of all funds will be allocated to market B, so that $F_B$ is further reduced and $F_A$ further increased. Though shareholders are also hurt by the resulting inefficiency, they gain as this reduces the value of depositors’ claims through reducing coinsurance. Given that the "status-quo" in the case of integration is that all deposits are secured by all assets, this can thus also induce greater risk shifting compared to the case of non-integrated banks.
Note now that for the following proposition we relabel the thresholds for the case distinction with separate banks from Proposition 2 by $\rho_1^S$ and $\rho_2^S$. Recall in particular that $\rho_2^S$ denotes the threshold for the correlation between loan portfolios so that for $\rho \geq \rho_2^S$ interbank lending is large enough to make the allocation of funds efficient. With integration the corresponding threshold for when efficiency is obtained was denoted by $\rho_2^I$ in Proposition 3.

**Proposition 4** The allocation of funds when banks are integrated is (weakly) more efficient than that achieved between separate banks through interbank lending when the correlation of loan portfolios in the two markets is high (high $\rho$). It is however (weakly) less efficient when the correlation is low (low $\rho$). Precisely:

i) When there is interbank lending in case of separate banks ($W > 0$), which is the case for all $\rho > \rho_1^S$, then the allocation is always (weakly) more efficient when banks are integrated, i.e., (weakly) more funds are allocated to market $B$ when banks are integrated. In particular, from $\rho_2^I \leq \rho_2^S$ it is more likely that the allocation is fully efficient with $F_A = F_B = M$.

ii) When there is no interbank lending in case of separate banks ($W = 0$ as $\rho \leq \rho_1^S$), then the allocation is more efficient when banks are separate, as greater risk-taking in the integrated bank increases the difference $F_A - F_B > 0$.

**Proof.** See Appendix.

The comparison in Proposition 4 derives clear-cut conditions for when an allocation of funds inside an integrated bank is more efficient than that achieved through interbank lending between two separate banks. To our knowledge, such a comparison has not yet been undertaken. Though our analysis is admittedly highly stylized, the respective simplifications allow to clearly isolate incentives for risk shifting by leveraged shareholders as the driving force between the difference in allocations. Incentives for risk shifting, as manifested by a more asymmetric allocation of funds between the two markets, can both be lower and higher in an integrated bank. As discussed previously, the difference lies in the "status quo" regarding the treatment of deposits, which for separate banks means that each bank’s deposits are secured only by the assets of this bank, while for an integrated bank depositors in either market have senior access to repayments of loans made in both markets.
4.2 Endogenous Integration

As discussed previously, integration can - at least when correlation of loan portfolios is sufficiently high - lead to a greater reallocation of funds from depositors in market $A$, which has a larger deposit base, to loans made in market $B$. On the other hand, we showed as well how integration can lead to greater risk shifting. We now ask whether integration arises endogenously and, if so, when this coincides with the realization of efficiency gains. It turns out that the answer to this relies crucially on the extent to which deposits are insured. For this we analyze first the cases where either all deposits are insured or where there is no deposit insurance at all.

Besides affecting the interest rates that prevail in the market, the absence of deposit insurance makes the following key qualitative difference. Without deposit insurance, interest rates positively react to the extent to which depositors’ claims are coinsured by investments into both markets $A$ and $B$. When banks are separated, this is only the case for the depositors of bank $A$ and only when subsequently an interbank loan is made. Likewise, in cases where integration leads to greater risk taking, this will be equally anticipated by depositors and lead to higher funding costs. Overall, such a feedback channel between funding costs and the decision to integrate is fully absent when depositors are insured. Note next that shareholders of bank $A$ and bank $B$ can clearly realize the same efficiency gains from reallocating resources as the shareholders of an integrated bank $AB$ - the relevant question is rather whether this is in their own best interest! And observe as well that, unless all resources are allocated into one market, integration always leads to a positive coinsurance gain for depositors. These observations together lead to the following stark result.

**Proposition 5** Consider the case where all deposits are insured and suppose that retail markets are always fully segmented. Then as long as $z < M$, so that there is a positive deposit base in both markets, banks will remain separate as integration would strictly reduce shareholders’ joint profits.

A key prediction of Proposition 5 is that banks that are (largely) financed by insured deposits are likely to remain small and to resist mergers.\footnote{Note that one reason why we have now fully abstracted from possible competition for deposits without integration is that then integration of banks would trivially lead to benefits, namely by lowering funding costs. We conjecture that the non-profitability result survives as long as competition is not too intense without integration - or, likewise, in case integration would not sufficiently reduce deposit rates as the two considered banks face competition also from other financial institutions.} As discussed in the Introduction, there seems to be indeed a tendency towards fragmentation among smaller traditional
savings and loans banks. Our argument is that this is the case as these banks can reap the benefits from reallocating resources also through interbank loans, to the extent that they wish to do so, but without at the same time providing additional coinsurance benefits to their (insured) depositors. When integration thus does not take place and when loan portfolios are not sufficiently correlated \( (\rho < \rho_2^S) \), local banking markets remain insufficiently integrated: The differences in the local funding base are not sufficiently compensated for, so that from the perspective of overall efficiency, there is both excess loan-making in one market and insufficient diversification across markets.

We now turn to the case where deposits are uninsured. Here, the fact that anticipated coinsurance benefits as well as additional risk shifting under integration are both internalized through a lower or higher deposit rate leads to the following result.

**Proposition 6** Consider the case where all deposits are uninsured and suppose that retail markets are always fully segmented. When loan portfolios are sufficiently uncorrelated \( (\rho < \rho_1^S) \), banks will strictly prefer to remain non-integrated, while for \( \rho > \rho_1^S \) they weakly prefer to become integrated (and strictly so for \( \rho \in (\rho_2^l, \rho_2^S) \)).

**Proof.** See Appendix.

5 Conclusion: Collecting the Implications

In this Section we provide in a more descriptive way a final collection of the main implications that we have derived from our stylized model of locally segmented funding and lending markets. In our model, as (local) banks have an advantage in making loans, to achieve a more efficient allocation when there are differences in local funding it is necessary that funds are either reallocated through interbank lending or within an integrated bank that operates across markets.\(^{23}\) We derive implications both for loans made between banks and for whether and when we should observe integration that could facilitate the reallocation of funds.

**Implication 1.** The size of an interbank exposure should increase both with the difference in banks’ local funding base and with the correlation in their own loan portfolios.

Note again that the interdependence between interbank lending and the correlation in their loan portfolios is, in particular, not due to banks’ anticipation of collective bail-out.\(^{23}\)

---

\(^{23}\)Notably, also retail competition alone is insufficient as long as a local bank still enjoys an advantage also on the funding side, e.g., due to switching costs of depositors; cf. Appendix B.
In fact, note that in our model we have taken the risk of loans as well as the correlation between loans as exogenous, thereby focusing on the reallocation of funds across markets as the only channel through which a bank can increase or decrease aggregate risk.

**Implication 2.** *We should expect to see a tendency towards either low (or zero) interbank exposure, notably when loan portfolios have relatively low correlation, and relatively high interbank exposure that could have a "contagious" effect, notably when loan portfolios have relatively high correlation.*

Note first again that the results that underpin Implication 2 are not obtained from the possibility of a bail-out and banks’ corresponding strategic consideration. Recall also that the predicted clustering of observations follows from the fact that a contagious effect reduces (or, in our model, renders zero) the positive coinsurance externality that a further increase in interbank lending has on depositors through diversification. Hence, when an interbank exposure is already sufficiently large so as to be contagious in this sense, then a further increase becomes even more profitable.

**Implication 3.** *A bank that operates in different markets, both on the funding and on the lending side, can have a more or less efficient allocation of funds across the different markets compared to the combined operations of non-integrated banks that can reallocate funds through interbank lending. The allocation of the integrated bank is more diversified and more efficient when the correlation between the loans across markets is relatively high, it is less diversified and less efficient when the correlation is relatively low.*

Recall that the key insight for Implication 3, where one compares allocative efficiency and diversification across markets, is the following: In an integrated bank that secures funding from various markets all deposits represent claims to all assets, i.e., to all loans made in different markets, whereas for non-integrated banks the respective deposits are only secured by local loans, unless there is interbank lending as well. This difference can lead to more or less risk taking, namely by concentrating lending in one market, when banks are integrated. Implication 3 allows to predict when we should observe a more efficient allocation.

**Implication 4.** *Banks that rely sufficiently on insured deposits have no incentives to integrate, even when this leads to inefficiently low reallocation of funds through interbank lending.*
When deposits are insured, shareholders can not benefit through lower funding costs from higher coinsurance of deposits when integration would lead to greater diversification of loan-making. Unless integration leads to other gains, such as reduced competition in the deposit market, it will not materialize when banks rely sufficiently on insured deposits. From Implication 4 the market segment of banks with this feature should thus remain heavily fragmented. This is different for banks that rely more on uninsured funding.

**Implication 5.** *Banks that rely sufficiently on uninsured funding have an incentive to integrate when this allows to realize greater efficiency in reallocating funds across markets, compared to interbank lending, as they also benefit from lower funding costs that flow from greater diversification.*

We also derived the following as a straightforward corollary to Implication 5.

**Implication 6.** *An integrated bank that generates more reallocation of funds across markets, compared to non-integrated banks that rely on interbank lending, enjoys on average lower costs of uninsured funding.*

The relation between size, albeit crucially arising from operations in different markets, and funding costs in Implication 6 is once again not driven by the expectations of a bailout (e.g., through becoming "too-big-to-fail"). We would further want to highlight the following aggregate implication of our analysis, which relates to the empirical motivation in the Introduction.

**Implication 7.** *Overall, when the reallocation of funding across local markets relies crucially on banks and their specific ability to collect funds from households or invest in local business, then there is a strong tendency for too little financial integration, i.e., both through (inefficiently low) interbank lending and through (inefficiently rare) integration and the subsequent reallocation with the integrated bank.*

Implications 1 to 7 generate new predictions for empirical work and both help to explain existing puzzles and throw new light on existing evidence. There are however also various reasons for why these predictions and the underlying results are important also from a normative perspective. In a nutshell, one upshot of our analysis is that, as a baseline result, there can be a tendency for banks to be too little interconnected and also to remain too small. As noted in the Introduction, our analysis however abstracts from the very reasons why presently researchers as well as policymakers consider banks to
be too large and too interconnected, namely the possibility of a bail-out and the thereby arising problems of moral hazard. Still, when policymakers target, e.g., through levies on interbank exposure or bank size, a particular level of interconnectedness, they should bear in mind the following two observations that are at the heart of our present analysis: First, interbank lending plays a key role in financial integration and, thereby, in reallocating funds to where they are most efficiently used; second, absent any other considerations that could arise, for instance, from an anticipated bail-out, there can be a strong tendency towards too little interconnectedness, as well as too few mergers that could provide a substitute through reallocating funds internally.
Appendix A: Omitted Proofs

**Proof of Proposition 1.** We argue first that an interbank loan can arise only when this leads to an overall more efficient allocation. When $R_A \geq R_B$ this implies $W \geq 0$, but also that $W$ does not exceed a threshold where $R_A - W \leq R_B + W$. This follows from combining the following observations. Note that these arguments hold irrespective of whether, as is presently assumed, bank $A$ can make a take-it-or-leave-it offer to bank $B$ or whether we assume a different form of how net surplus is shared, as we only make use of individual rationality of shareholders of both banks. When bank $n$ provides a loan to bank $n'$, then this implies that when the loans of bank $n'$ perform so that the interbank loan can be repaid, then in this state shareholders of bank $n'$ and thus by their seniority also its depositors can not be worse off than without an interbank loan. This implies now that when the terms of an interbank loan satisfy this condition, as implied by optimality of shareholders, then from total expected repayment to depositors can not be lower when an interbank loan is made and it must be strictly higher when also $< 1$.

We are now left with the following possible cases, which are implicit in expression (3) for the profits of shareholders of bank $A$: Case 1 with $L(F_A) \geq R_A (1 + r_A)$ and $w < R_A (1 + r_A)$, Case 2 with $L(F_A) < R_A (1 + r_A)$ and $w < R_A (1 + r_A)$, and Case 3 with $L(F_A) \geq R_A (1 + r_A)$ and $w \geq R_A (1 + r_A)$. We treat these cases in turn.

Consider first Case 1 where, after substituting $w = L(R_B + W) - L(R_B)$, it follows from (3) that the profits of bank $A$’s shareholders are given by

$$\pi_{A1} = p \left[ L(R_A - W_1^*) - R_A(1 + r_A) \right] + \left[ p^2 + \rho p (1 - p) \right] L(R_B + W_1^*) - L(R_B)$$

and the first-order condition yields (6). Note that the program is strictly concave in this case.

In Case 2 shareholder profits are from (3) equal to

$$\pi_{A2} = \left[ p^2 + \rho p (1 - p) \right] \left[ L(R_A - W_2^*) - R_A(1 + r_A) + L(R_B + W_2^*) - L(R_B) \right]$$

and the first-order condition yields (7). Note that also in this case the program is strictly concave.

Finally, consider Case 3, where (3) becomes

$$\pi_{A3} = p \left[ L(R_A - W_3^*) - R_A(1 + r_A) + L(R_B + W_3^*) - L(R_B) \right]$$

and the first order condition would imply that $R_A - W_3^* = R_B + W_3^* = M$. We now argue that if the interbank loan is sufficiently low so that repayment from its own loans is
sufficient to repay $A$’s depositors, the repayment from the interbank loan can not at the same time also be sufficiently high to repay depositors of bank $A$. Hence, we argue that

$$L(R_B + W) - L(R_B) - R_A(1 + r_A) < 0. \quad (11)$$

Condition (11) is for $R_B = 0$ implied by assumption (2). It also holds for $R_B \in (0, M)$ since the partial derivative of (11) with respect to $R_B$ is, using $R_A = 2M - R_B$, given by

$$1 + r_A - L'(R_B) \leq \frac{1}{p} - L'(R_B),$$

which is strictly negative for $R_B \in [0, M]$ due to concavity of $L$ and condition (1). (Recall also that $r_A \leq \frac{1}{p} - 1.$) Q.E.D.

**Proof of Proposition 2.** It is convenient to prove the assertions by considering separately the comparative analysis in $\rho$ and $z$.

**Comparative analysis in $\rho$:** Starting from $W = 0$ we have from (3) the following derivative of $\pi_A$:

$$d\pi_A = \begin{cases} 
[p^2 + \rho p (1 - p)] L'(R_B + W) - p L'(R_A - W) & \text{if } L(R_A - W) \geq R_A(1 + r_A) \\
[p^2 + \rho p (1 - p)] [L'(R_B + W) - L'(R_A - W)] & \text{if } L(R_A - W) < R_A(1 + r_A) 
\end{cases}$$

where $R_A = M + z$ and $R_B = M - z$. Suppose now first that even when $W = W_2^* = z$, so that $F_n = M$, Case 2 does not apply as

$$L(M) \geq (M + z)(1 + r_A), \quad (13)$$

Clearly, (13) holds when $z = 0$. When, for given $p$, it also holds at $z = M$ so that

$$L(M) > 2M(1 + r_A), \quad (14)$$

then only Case 1 or $W = 0$ can apply. Otherwise, there is a cutoff $\tilde{z}$ defined by

$$L(M) = (M + \tilde{z})(1 + r_A), \quad (15)$$

so that we can altogether rule out Case 2 if and only if $z \leq \tilde{z}$. From inspection of expression (6) in Proposition 1, note next that $W_1^*$ strictly increases in $\rho$. Using strict concavity, we can define for given $z > 0$ and $p$ a value $\rho_1$ so that $W_1^* > 0$ only if $\rho > \rho_1$:

$$\rho_1 = \frac{1}{1 - p} \left( \frac{L'(M + z)}{L'(M - z)} - p \right), \quad (16)$$
where further
\[
\frac{d\rho_1}{dz} = \frac{1}{1-p} \frac{L''(M+z)L'(M-z) + L'(M+z)L''(M-z)}{L'(M-z)^2} < 0.
\]

Suppose now that Case 2 is feasible as \( z > \bar{z} \). Clearly, in Case 2 \( W_2^* \) no longer depends on \( \rho \). Also, it holds that \( W_2^* > W_1^* \) (unless \( \rho = 1 \), so that there is perfect positive correlation). The crux is now that the objective function for the maximization problem with respect to \( W \) is now altogether no longer quasiconcave as we shift between different cases. Denote now \( \pi_{A0} \) with \( W = 0 \), \( \pi_{A1} \) with \( W = W_1^* \), and \( \pi_{A2} \) with \( W = W_2^* \). We thus consider in the following how the differences \( \pi_{A2} - \pi_{A0} \) and \( \pi_{A2} - \pi_{A1} \) change in \( \rho \). For this we consider first
\[
\frac{d}{d\rho} (\pi_{A2} - \pi_{A0}) = p (1-p) \left[ 2L(M) - L(M-z) - (M+z)(1+r_A) \right]
\]
and show that this is strictly positive. Note first that this is surely the case for \( z = 0 \) (cf. the much stronger condition (1)). Next, differentiating the expression with respect to \( z \), it is strictly increasing when \( L'(M-z) > 1 \), which is also implied by (1). Next, we also show that \( \pi_{A2} - \pi_{A1} \) is increasing in \( \rho \). Making use of the first-order condition for \( W_1^* \), we have
\[
\frac{d}{d\rho} (\pi_{A2} - \pi_{A1}) = p (1-p) \left[ 2L(M) - L(M-z+\bar{W}_1) - (M+z)(1+r_A) \right].
\]

To confirm that this is strictly positive, it is sufficient to do so at the highest value \( L(M-z+\bar{W}_1) \) that is still compatible with Case 1, which in turn is the lowest value at which still \( L(M+z-\bar{W}_1) = (M+z)(1+r_A) \). But then the sign of the derivative is determined by
\[
2L(M) - L(M-z+\bar{W}_1) - L(M+z-\bar{W}_1) > 0,
\]
where we used strict concavity of \( L \).

We will now confirm that this still holds if deposits are uninsured. Then, for a given interbank loan \( W \) and repayment \( w \), the interest rate that has to be promised to depositors of bank \( A \) is given by the following break even condition:
\[
R_A = \left[ p^2 + p(1-p)\rho \right] (M+z)(1+r_A) + p(1-p)(1-\rho) \left[ w + \min \left\{ (M+z)(1+r_A), L(M+z-W) \right\} \right].
\]

The first line in (17) captures the state where corporate loans in both markets are successful and depositors of \( A \) receive their principal plus interest. The second line captures
two equally likely states in which only loans in one market are successful. First, if only
loans in market $B$ are successful, depositors receive - by seniority - the repayment from
the interbank loan, $w$. Second, when only loans in market $A$ are successful, this may
be sufficient to make depositors whole (Case 1) or not, in which case depositors receive
repayment $L (M + z - W)$ (Case 2). We can now substitute $r_A$ from (17) with $W = W_2^*$
into (22) and differentiate to get
\[
\frac{d}{d\rho} (\pi_{A2} - \pi_{A1}) \frac{1}{p(1-p)} = 2L (M) - L (M - z + W_1^*) - R_A (1 + r_A)
+ \frac{p(1-p)(1-\rho^2)}{p^2 + p(1-p)\rho} [2L (M) - L (M - z) - R_A (1 + r_A)].
\]
Again, it suffices to show that this is strictly greater than zero for the lowest value
$L (M + z - W_1^*)$ that is still compatible with Case 1 as $L (M + z - W_1^*) = (M + z) (1 + r_A)$.
Then, by strict concavity we have that $2L (M) - L (M - z + W_1^*) - L (M + z - W_1^*) > 0$
and, a fortiori, $2L (M) - L (M - z) - L (M + z - W_1^*) > 0$.

Comparative analysis in $z$: We are now rather brief as the analysis is largely analogous to
that of the comparative analysis in $\rho$. Taking first Case 1, note that from (16) we can now
define, for given $\rho$, a cutoff $z_1$ so that indeed $W_1^* > 0$ when $z > z_1$, where $z_1 < M$ when
\[
\rho < \frac{1}{1-p} \left( \frac{L'(2M)}{L'(0)} - p \right).
\] 
(18)
When $W_1^* > 0$, it is also strictly increasing in $z$. Next, note that now also $W_2^*$ is strictly
increasing in $z$, as
\[
\frac{d}{dz} (\pi_{A2} - \pi_{A0}) = [p^2 + p(1-p)\rho] L'(M - z) - pL'(M + z) + p(1-p)(1-\rho)
> p(1-p)(1-\rho),
\]
since for $\rho > \rho_1$, it holds that $[p^2 + p(1-p)\rho] L'(M - z) > pL'(M + z)$ and
\[
\frac{d}{dz} (\pi_{A2} - \pi_{A1}) = p(1-p)(1-\rho).
\]
Again, we show that this still holds for the situation without deposit insurance, where
\[
\frac{\partial}{\partial z} (\pi_{A2} - \pi_{A1}) = p(1-p)(1-\rho) \left[ (1 + r_A) + (M + z) \frac{dr_A}{dz} \right]
\]
This is strictly positive as from (17) with $W = W_2^*$, we get that
\[
(1 + r_A) + (M + z) \frac{dr_A}{dz} = \frac{1 - p(1-p)(1-\rho) L'(M - z)}{p^2 + p(1-p)\rho}
\]
\[
> \frac{1 - p(1-p)(1-\rho)}{p^2 + p(1-p)\rho} \frac{2}{p + \rho(1-p)}.
\]
where we used (18) together with $L'(M - z) < L'(0)$ by concavity and that by assumption (2) $L'(2M) < 2$. Hence the sign of the derivative is determined by the following expression:

$$p + \rho (1 - p) - p (1 - p) (1 - \rho) 2 > p (2p - 1).$$

To see that this is nonnegative, observe that by assumption (1) $L(M) > M \frac{1}{p}$. Therefore, it has to hold that $p > \frac{1}{2}$ since otherwise we would have $L(M) > 2M$, thus violating assumption (2). Q.E.D.

**Proof of Proposition 3.** Note first that by symmetry of the loan-making opportunities in the two markets we can indeed wlog restrict attention to the case where $F_A \geq F_B$. Take first the case where $L(F_A) \geq R_{AB} (1 + r_{AB}) > L(F_B)$, such that the integrated bank’s profit function becomes

$$\pi_{AB1} = p [L(F^*_A) - R_{AB} (1 + r_{AB})] + [p^2 + p (1 - p) \rho] L(F^*_B),$$

and first order condition (9). If instead $L(F_A) < R_{AB} (1 + r_{AB})$ the integrated bank’s profit function becomes

$$\pi_{AB2} = [p^2 + p (1 - p) \rho] [L(F^*_A) + L(F^*_B) - R_{AB} (1 + r_{AB})],$$

with first order condition (10). Note first that from inspection of (9), $F^*_{A1}$ strictly decreases and $F^*_{B1}$ in Case 1. Using strict concavity, we can further define a value $\rho^I_1$ (in analogy to (16)), so that $F_B > 0$ if $\rho > \rho^I_1$:

$$\rho^I_1 = \frac{1}{1 - p} \left( \frac{L'(2M)}{L'(0)} - p \right).$$

Next, observe that Case 2 is always possible to achieve for the integrated bank, as from assumption (2) and $r_A \geq 0$, it follows that

$$L(M) < 2M(1 + r_A).$$

Clearly, $F^*_{A2}$ and $F^*_{B2}$ do no longer depend on $\rho$ when Case 2 applies and, as long as $\rho < 1$, $F^*_{A2} = M < F^*_{A1}$ and $F^*_{B2} = M > F^*_{B1}$. Now, as was noted in the proof of Proposition 2, the program is not quasiconcave in $W$ over all cases, so that we need to pin down the transition from Case 1 to Case 2. Since the problem of the integrated bank is technically equivalent to that of separate banks with $z = M$ (cf. Proposition 2), we will be brief. Consider first

$$\frac{d}{d\rho} (\pi_{AB2} - \pi_{AB0}) = p (1 - p) [2L(M) - 2M(1 + r_{AB})].$$
which is strictly positive by the much stronger condition (1). Next, making use of the first-order condition for \( F_{A1}^* \) and \( F_{B1}^* \), we have

\[
\frac{d}{d\rho}(\pi_{AB2} - \pi_{AB1}) = p(1-p)[2L(M) - L(F_{B1}^*) - 2M(1 + r_{AB})].
\]

To confirm that this is strictly positive, it is sufficient to do so at the highest value \( L(F_{B1}^*) \) that is still compatible with Case 1, which in turn is the lowest value at which still \( L(F_{A1}^*) = 2M(1 + r_{AB}) \). But then the sign of the derivative is determined by

\[
2L(M) - L(F_{B1}^*) - L(F_{A1}^*) > 0,
\]

where we used strict concavity of \( L \).

We will now confirm this result for the situation without deposit insurance, where \( r_{AB} \) is determined by the break even constraint of depositors which, for corporate loans of size \( F_A \) and \( F_B \), is given by

\[
R_{AB} = \left[p^2 + p(1-p)\rho\right]R_{AB}(1 + r_{AB}) + p(1-p)(1-\rho)[L(F_B) + \min\{R_{AB}(1 + r_{AB}), L(F_A)\}].
\]

To pin down \( \rho^*_2 \), consider the difference \( \pi_{AB2} - \pi_{AB1} \) for a given interest rate \( r_{AB} \) where we use the equilibrium values when one of the cases applies: \( \pi_{AB1} \) with \( F_{A1}^* \) and \( F_{B1}^* \), and \( \pi_{AB2} \) with \( F_{A2}^* = F_{B2}^* = M \)

\[
\pi_{AB2} - \pi_{AB1} = \left[p^2 + p(1-p)\rho\right][2L(M) - L(F_{B1}^*) - R_{AB}(1 + r_{AB})] - p[L(F_{A1}^*) - R_{AB}(1 + r_{AB})].
\]

We can now substitute \( r_{AB} \) from (20) for \( F_A = F_B = M \) into (21) and differentiate with respect to \( \rho \) to get

\[
\frac{d}{d\rho}(\pi_{AB2} - \pi_{AB1})\frac{1}{p(1-p)} = 2L(M) - L(F_{B1}^*) - R_{AB}(1 + r_{AB})
\]

\[
+ p(1-p)(1-\rho)\left[2L(M) - R_{AB}(1 + r_{AB})\right].
\]

To confirm that this is strictly positive, it is again sufficient to show it is positive at the lowest value \( L(F_{A1}^*) \) that is still compatible with Case 1 as \( L(F_{A1}^*) = R_{AB}(1 + r_{AB}) \). But then, the expression is strictly positive because, by concavity, \( 2L(M) - L(F_{B1}^*) - L(F_{A1}^*) > 0 \). Q.E.D.

**Proof of Proposition 4.** Recall first that technically the problem of the integrated bank’s shareholders to determine the optimal \( F_A \) and \( F_B \) is the limiting case (with \( z = M \)
of the separate banks’ shareholders’ problem to determine the optimal interbank loan. Therefore, we will establish the result by showing that both values $\rho_1^S$ and $\rho_2^S$ are strictly decreasing in $z$ and thus $\rho_1^I$ and $\rho_2^I$ constitute their respective lower bound.

Recall that the value above which there will be reallocation over the interbank market ($W > 0$) is given by

$$\rho_1^S = \frac{1}{1-p} \left( \frac{L'(M+z)}{L'(M-z)} - p \right),$$

while the integrated bank will invest a positive amount in loan portfolio $B$ only if $\rho$ is greater than

$$\rho_1^I = \frac{1}{1-p} \left( \frac{L'(2M)}{L'(0)} - p \right).$$

Clearly, $\rho_1^S \geq \rho_1^I$ with equality for $z = M$. Therefore, whenever $\rho_1^I \leq \rho < \rho_1^S$, we have that

$$\frac{L'(F_A^*)}{L'(F_B^*)} < \frac{L'(M+z)}{L'(M-z)},$$

implying that the allocation achieved by the integrated bank is less efficient than the status quo of the separate banks (a fortiori so for $\rho < \rho_1^I$ where $F_A = 2M$).

Recall now from Proposition 2 that the critical $\rho_2^S$ is determined by

$$\pi_{A2} - \pi_{A1} = \left[ p^2 + p(1-p)\rho_2^S \right] [2L(M) - L(M - z + W_1^*)] \tag{22}$$

$$+ p(1-p) \left( 1 - \rho_2^S \right) R_A (1 + r_A) - pL(M + z - W_1^*),$$

and that further, $\frac{\partial}{\partial z} (\pi_{A2} - \pi_{A1}) > 0$ and $\frac{\partial}{\partial \rho} (\pi_{A2} - \pi_{A1}) > 0$. Hence, we conclude that

$$\frac{\partial \rho_2^S}{\partial z} = -\frac{\frac{\partial}{\partial \rho} (\pi_{A2} - \pi_{A1})}{\frac{\partial}{\partial z} (\pi_{A2} - \pi_{A1})} < 0.$$ 

For $\rho \in (\rho_2^I, \rho_2^S)$, the integrated bank therefore achieves a strictly more efficient allocation (Case 2) than the two separate banks (Case 1). Finally, for $\rho \in (\rho_1^S, \rho_2^S]$ as well as $\rho \geq \rho_2^S$, both market structures achieve the same allocations. Q.E.D.

**Proof of Proposition 6.** We will now compare joint profits $\pi_A + \pi_B$ to the integrated bank’s profits $\pi_{AB}$ for different regions of $\rho$. Since we consider the situation without deposit insurance, interest rates are given by $r_B = \frac{1}{p} - 1$ as well as $r_A$ from (17) and $r_{AB}$ from (20) for the respective cases.

We then get for $\rho < \rho_1^I$:

$$\pi_{A0} + \pi_B - \pi_{AB0} = p \left[ L(R_A) + L(R_B) - L(2M) \right]$$

$$> 0$$
by strict concavity.

For \( \rho_1^l \leq \rho < \rho_1^S \), we get

\[
\pi_{A0} + \pi_B - \pi_{AB1} = p [L (R_A) + L (R_B)] - p [L (F^*_{A1}) + L (F^*_{B1})] - 2M (1 - p (1 + r_{AB})).
\]

(23)

Note that \( \rho < \rho_1^S \) can also be expressed as \( z < z_1 \) and that for \( z = z_1 \), we have

\[
L (M + z - W^*_1) = L (F^*_{A1}) \quad \text{and} \quad L (M - z + W^*_1) = L (F^*_{B1}) \quad \text{with} \quad W^*_1 = 0.
\]

Therefore, at \( z = z_1 \), we get \( \pi_{A0} + \pi_B - \pi_{AB1} = 0 \). Differentiating (23) with respect to \( z \) yields

\[
\frac{d}{dz} (\pi_{A0} + \pi_B - \pi_{AB1}) = p [L' (R_A) - L' (R_B)] < 0,
\]

implying that \( \pi_{A0} + \pi_B - \pi_{AB1} > 0 \) for \( z < z_1 \).

Now consider \( \rho_1^S \leq \rho < \rho_2^l \), where the difference in profits becomes

\[
\pi_{A1} + \pi_B - \pi_{AB1} = p [L (R_A - W^*_1 - R_A (1 + r_A)) + p [L (R_B) - R_B (1 + r_B)]
\]

\[
+ \left[p^2 + p (1 - p) \rho \right] [L (R_B + W^*_1) - L (R_B)]
\]

\[
- p [L (F^*_{A1}) - R_{AB} (1 + r_{AB})] - \left[p^2 + p (1 - p) \rho \right] L (F^*_{B1})
\]

\[
= 0,
\]

where we used the fact that \( R_A - W^*_1 = F^*_{A1} \) and \( R_B + W^*_1 = F^*_{B1} \).

For \( \rho_2^l \leq \rho < \rho_2^S \), we get

\[
\pi_{A1} + \pi_B - \pi_{AB2} = p [L (R_A - W^*_1) - R_A (1 + r_A)] + p [L (R_B) - R_B (1 + r_B)]
\]

\[
+ \left[p^2 + p (1 - p) \rho \right] [L (R_B + W^*_1) - L (R_B)]
\]

\[
- \left[p^2 + p (1 - p) \rho \right] [2L (M) - R_{AB} (1 + r_{AB})]
\]

\[
= pL (R_A - W^*_1) + pL (R_B + W^*_1) - p2L (M)
\]

\[
< 0,
\]

by strict concavity.

Finally, for \( \rho \geq \rho_2^S \), we get

\[
\pi_{A2} + \pi_B - \pi_{AB2} = p (1 - p) (1 - \rho) L (R_B)
\]

\[
- pR_A (1 + r_A) - pR_B (1 + r_B)
\]

\[
+ \left[p^2 + p (1 - p) \rho \right] R_{AB} (1 + r_{AB})
\]

\[
= 0.
\]

Q.E.D.
Appendix B: Market for Retail Deposits

Model of the Retail Deposit Market. In this Appendix we allow for active retail competition in stage $t = 1$ and show that our results are robust to a potential reallocation of funds via the retail deposit market. It is convenient, however, to first consider the case where only the retail market is active as, for instance due to regulatory constraints, there is no subsequent interbank lending market: $W = 0$ and thus $F_n = R_n$. This analysis allows us to isolate some key features of how the retail deposit market works in our model. We then solve the model where both retail competition and interbank lending interact and show, amongst other things, how then the key implications from Proposition 2 still hold.\footnote{Huang and Ratnovski (2011) also analyze the interaction between retail and wholesale funding of banks, albeit their focus is very different. Wholesale funding provides peer monitoring and disciplines bank managers through the threat of withdrawals, though short maturity can lead to overreaction to publicly available signals and thereby triggering inefficient liquidations.}

If a household residing in the local market of bank $n$ deposits not with the local bank but instead with bank $n'$, it incurs a switching cost $s \geq 0$. For each depositor the respective value of $s$ represents an independent random draw from the cumulative distribution function $G(s)$. The assumption of switching costs that are, for all depositors with a draw $s > 0$, non-negligible relative to their savings reflects the small granularity of retail deposit financing. Given $M_A \geq M_B$, it is intuitive that, in equilibrium switching will occur at most out of market $A$. For depositors in market $A$ we can then determine a critical switching cost level, $s^*$, so that only depositors with draws $s \leq s^*$ take up the offer of the rival bank $B$. If interior, with deposit insurance this yields $s^* = r_B - r_A$ and without deposit insurance $s^* = p(r_B - r_A)$. The respective attracted deposit volumes are then given by $R_A = M_A[1 - G(s^*)]$ and $R_B = M_B + M_AG(s^*)$.

If both banks choose an interest rate above the participation constraint of depositors, $r_n = 0$ or $r_n = 1/p - 1$ for the cases with and without deposit insurance, then these are pinned down by the respective first-order conditions

$$
\frac{dR_n}{dr_n} [L'(R_n) - (1 + r_n)] - R_n = 0.
$$

Here, we have $\frac{dR_n}{dr_n} = pg(s^*)M_A$ without deposit insurance and $\frac{dR_n}{dr_n} = g(s^*)M_A$ with deposit insurance. We restrict attention to cases where $g(0)$ is sufficiently large to ensure that the equilibrium deposit rates are characterized by the respective conditions (24).\footnote{A corner solution may arise when, starting from the monopoly interest rates, an increase will induce instead too little switching.} In order to facilitate the exposition, we stipulate a uniform distribution $s \in [0, \bar{s}]$ with $g(s) = 1/\bar{s}$ and thus suppose that $\bar{s}$ is not too large.
As is immediate, competition will not fully bridge the differences in the deposit base. Formally, this is most immediately seen from the first-order conditions (24): Given that \( \frac{dR_A}{dr_A} = \frac{dR_B}{dr_B} \) and given that at \( r_A = r_B \) it holds that \( R_A = M_A > R_B = M_B \), these can indeed only jointly hold for bank A and bank B when \( r_B > r_A \) though still \( R_A > R_B \). The reason for this is the low granularity of retail deposits combined with inertia, as modeled by switching costs. There is, however, an interesting twist to this observation when we now briefly compare the outcomes with and without deposit insurance. We find that the resulting allocation remains more asymmetric when there is no deposit insurance than when there is deposit insurance. Essentially, deposit insurance leads to a decrease in the cost of attracting deposits. This intensifies competition and ensures that the outcome more closely reflects the different marginal profitability in loan making at the two banks, depending on the attracted and invested funds.

**Proposition 7** Suppose only for now that interbank loans are not possible, e.g., due to regulatory constraints, but that there is competition for retail deposits across local markets, albeit hampered by frictions due to switching costs. Then, a difference in the size of the deposit base, \( M_A > M_B \), still leads to different volumes of attracted deposits, \( R_A > R_B \), and there is less reallocation of funds across markets (larger \( R_A - R_B \)) under deposit insurance than without deposit insurance.

**Proof.** It is now convenient to set up the banks’ problem slightly differently for the proof. We suppose that banks compete for depositors by promising some value \( v_n \), so that \( v_n = p_n(1 + r_n) \) without deposit insurance and \( v_n = (1 + r_n) \) with insurance. Note that we can then always express the cutoff as \( s^* = v_B - v_A \), provided that we still restrict wlog the analysis to the case where \( M_A \geq M_B \). Without deposit insurance the first-order condition wrt \( v_n \) is obtained from maximizing

\[
\pi_n = pL(R_n) - v_n R_n
\]

and thus equal to

\[
pL'(R_n) \frac{dR_n}{dv_n} - R_n - v_n \frac{dR_n}{dv_n} = 0.
\]

With deposit insurance we have, instead,

\[
\pi_n = pL(R_n) - pv_n R_n
\]

and thus

\[
pL'(R_n) \frac{dR_n}{dv_n} - pR_n - pv_n \frac{dR_n}{dv_n} = 0.
\]
Note next that
\[
\frac{dR_n}{dv_n} = M_A g(s^*) = M_A \frac{1}{s},
\]
making use also of the uniform distribution of switching costs. If we now subtract the
first-order condition for \( A \) from that for \( B \), we obtain without deposit insurance
\[
pM_A \frac{1}{s} [L'(R_B) - L'(R_A)] = (R_B - R_A) + M_A \frac{1}{s} (v_B - v_A),
\]
(25)
where, as a function of \( v_B - v_A \), the left-hand side is strictly decreasing and the right-hand
side strictly increasing, noting that \( R_B - R_A \) is strictly increasing in \( v_B - v_A \) and that \( L'' < 0 \).

The only change when there is deposit insurance is that the right-hand side of (25) is
multiplied by \( p \), which obtains
\[
pM_A \frac{1}{s} [L'(R_B) - L'(R_A)] = \left[ (R_B - R_A) + M_A \frac{1}{s} (v_B - v_A) \right] p.
\]
(26)
Starting from the equilibrium without deposit insurance, where \( R_B < R_A \) so that both
sides are strictly positive, and multiplying the right-hand side by \( p < 0 \), to restore equality
so as to obtain (26), we must increase \( v_B - v_A \) and thus increase \( R_B - R_A \), as asserted in
the Proposition.\(^{26}\) Q.E.D.

**Reallocation of Funds through Both the Deposit Market and Interbank Lending.** Suppose now again that interbank lending is feasible, though in contrast to the
baseline analysis, funds can now also be reallocated through the retail deposit markets.
We presently stipulate that there is full deposit insurance (covering, for concreteness, also
the promised interest rate \( r_n \)). Also, we focus again on the case where \( \bar{s} \) is not too large,
so that there is indeed competition in equilibrium.

How does the operation of a retail deposit market and interbank lending interact?
Suppose first that in equilibrium ultimately Case 1 applies, so that \( W = W_1^* > 0 \). Hence,
if this case prevails, a reallocation of funds is indeed obtained both through the retail and
through the interbank channel. The first thing to note is that when \( W > 0 \), the final
allocation of funds, as given by \( F_A \) and \( F_B \), does not depend on the outcome of retail
competition. Still, even though this does not affect the final allocation in this case, banks
have an incentive to acquire a larger fraction of the total retail deposit market. We obtain
from the respective first-order conditions for \( t = 1 \) the requirements

\(^{26}\)This result can also be established by using (26) and implicitly differentiating \((v_B - v_A)\), obtaining
thus that \( d(v_B - v_A)/dp < 0 \).
We stipulate that the respective problems are strictly quasiconcave so that the best responses are uniquely determined. In both expressions in (27) the respective (first) term in rectangular brackets describes the first-order condition when there is no subsequent interbank lending. The second term in bank $A$'s first-order condition captures the profits that bank $A$ will extract from an interbank loan. Notably, using the first-order condition for $W = W_A^*$ this term is indeed strictly positive. This makes bank $A$ relatively more aggressive in the retail deposit market, compared to the benchmark situation where an interbank loan was exogenously ruled out.\footnote{We can show that when the terms of the interbank loan are determined by symmetric Nash bargaining, then this observation still holds for bank $A$, while then, in addition, bank $B$’s incentives to attract funds through the retail deposit market are muted. Taken together, the subsequently reported results then still hold.} As a consequence, the anticipation of interbank lending reduces the reallocation of funds through retail deposit competition. Interestingly, this will be even more pronounced when Case 2 applies. Then, by rearranging the respective first-order condition for bank $A$ we obtain

\[
\frac{d\pi_A}{dr_A} \frac{1}{p^2 + \rho p(1-p)} = \left[ (L'(R_A) - (1 + r_A)) \frac{dR_A}{dr_A} - R_A \right] + \left[ (p + \rho (1-p)) L'(R_B) - L'(R_A) \right] \frac{dR_A}{dr_A} = 0, 
\]

\[
\frac{d\pi_B}{dr_B} \frac{1}{p^2 + \rho p(1-p)} = \left[ L'(R_B) - (1 + r_B) \right] \frac{dR_B}{dr_B} - R_B = 0. 
\]

The first-order condition for bank $B$ remains the same. The difference between (27) and (28) is that the repayment of an interbank loan from bank $B$ is now no longer weighted by $[p + \rho (1-p)]$ in the first-order condition. This reflects the fact that bank $A$’s shareholders benefit more from a marginal increase in the interbank loan when this leads to contagion as then there is no coinsurance externality on its depositors. Consequently, compared to Case 1, the incentives for bank $A$ to attract deposits - relative to the incentives of bank $B$ - further increase in Case 2. As in Case 2 the overall reallocation of funds increases
compared to Case 1, as then \( F_A = F_B \) prevails, and as we have noted that there will be less reallocation of funds through retail deposit competition, the size of interbank lending is then much higher.

**Proposition 8** The characterization of the equilibrium in Proposition 2 still applies when, next to interbank lending, funds can be reallocated through competition in the retail deposit market. Precisely, the size of interbank lending \( W \) is still increasing in both the difference in the deposit base \((2z)\) and the correlation of loan portfolios \((\rho)\), and \( W \) again jumps upwards when (at the respective levels \( z_2 \) or \( \rho_2 \)) the interbank loan becomes sufficiently large to be contagious.

**Proof.** We consider first the outcome when Case 1 applies. When we add up the two first-order conditions in (27), we obtain, using also \( s^* = r_B - r_A \) and \( \frac{dR_n}{dr_n} = R' = M_{1/\hat{\pi}} \), the condition

\[
y = R's^* - (R_A - R_B) + R'[L'(R_A) - L'(R_B)] + R' [(p + \rho(1-p)) L'(R_B) - L'(R_A)] = 0.
\]

As we stipulated strict concavity for both objective functions \( \pi_n \), we have also that \( \frac{\partial y}{\partial s^*} > 0 \).\(^{28}\) Recall finally that the term in rectangular brackets is strictly positive and that for the case without interbank lending this term disappears.\(^{29}\) With respect to a comparative analysis of \( \rho \), it is now more convenient to conduct this in \( p_{AB} = p^2 + \rho p(1-p) \).

For this we first implicitly differentiate condition (29). We have

\[
\frac{\partial y}{\partial p_{AB}} = \frac{1}{p} L'(R_B) R' > 0,
\]

so that altogether

\[
\frac{ds^*}{dp_{AB}} = -\frac{\partial y}{\partial p_{AB}} \frac{\partial p_{AB}}{\partial s^*} < 0,
\]

from which \( dR_A/dp_{AB} > 0 \) and \( dR_B/dp_{AB} < 0 \). On the other hand, we have from expression (6) (cf. also Corollary 1) that \( dF_A/dp_{AB} < 0 \) and \( dF_B/dp_{AB} > 0 \), so that for given \( R_n \) it holds that \( dW_1^*/dp_{AB} > 0 \).

\(^{28}\)Precisely, note again that this is the difference of \( \frac{\partial \pi_A}{\partial s_A} \frac{1}{p} \) and \( \frac{\partial \pi_B}{\partial s_B} \frac{1}{p} \). We then use that from strict concavity these are both strictly decreasing in \( r_n \) while \( ds^*/dr_A = 1 \) and \( ds^*/dr_B = -1 \).

\(^{29}\)We can more formally rewrite \( y(\theta, s^*) \) as the sum of the first line, which we denote by \( A(s^*) \) and \( \theta \) times the second line, which we write as \( \theta B(s^*) \). As \( \partial y(\theta = 1)/\partial s^* > 0 \) and \( \partial A/\partial s^* > 0 \), we have that \( \partial y(s^*)/\partial s^* > 0 \) for all \( 0 \leq \theta \leq 1 \). We then have, using uniqueness, \( ds^*/d\theta < 0 \), which formalizes that \( s^* \) is lower and thus \( R_A \) higher (\( R_B \) lower) under subsequent wholesale funding.
Next, consider a comparative analysis in \( z \), where \( M_A = M + z \) and \( M_B = M - z \). For this we first rewrite condition (29) as follows. Instead of \( s^* \), it is now convenient to think of \( R_A \) as the variable for which to solve this condition, noting also that \( R_B = 2M - R_A \). Note that clearly \( \partial y / \partial R_A < 0 \). Making use of the uniform distribution, we can also obtain explicitly
\[
s^* = \bar{s} - \frac{\bar{s} \cdot R_A}{M + z}.
\]
Now note also that \( R' = \frac{M + z}{\bar{s}} \). Consequently, we now have that
\[
\frac{\partial y}{\partial z} = \frac{\bar{s}(3R_A - 2M)}{(M + z)^2} > 0,
\]
so that now
\[
\frac{ds^*}{dz} = -\frac{\partial y / \partial z}{\partial y / \partial s^*} < 0.
\]
Again, we thus have that \( dR_A / dz > 0 \) and \( dR_B / dz < 0 \), while now both \( F_n \) remain unchanged.

Suppose next that Case 2 applies in equilibrium. The argument why \( R_A \) is now even larger than in Case 1 and thus, in particular, larger than without subsequent interbank lending is completely analogous to the preceding argument. Precisely, we can now express the difference of first-order conditions as
\[
Y = y + \frac{p - p_{AB}}{p} L'(R_B) \frac{dR_A}{dr_A}
\]
From strict concavity of the objectives we have again \( \partial Y / \partial s^* > 0 \) and can apply the same arguments as before. Q.E.D.

Proposition 8 thus confirms that our previous comparative analysis for the size of interbank lending is robust also to the operation of a market for retail deposits. This is intuitive given that the market for retail deposit does not provide an adequate substitute due to a combination of switching costs and the low granularity of individual deposits.\(^{30}\) Finally, also with regard to its regulatory implications, it is worthwhile to stress the following result:

\(^{30}\)The preceding discussion as well as the proof of Proposition 8 entail in addition implications that relate directly to the operation of the retail deposit market, which we presently do not stress. For a given difference in the local deposit base, as captured by \( M_A - M_B = 2z > 0 \), the respective difference in attracted retail deposits, \( R_A - R_B < M_A - M_B \), is strictly higher when banks’ loan portfolios are less correlated (lower \( \rho \)). Interestingly, however, as the difference in the deposit base \( z \) increases, while interbank lending is always strictly increasing, this may not hold for the volume of retail deposits that bank \( B \) attracts in market \( A \).
Corollary 2 Compared to the outcome when interbank lending is exogenously restricted, e.g., through regulation, once it is made feasible it will always (weakly) increase the extent to which funds are reallocated between the two local markets. That is, when \( z > 0 \) and thus \( M_A > M_B \), interbank lending will reduce or even fully close the gap \( F_A - F_B > 0 \) that persists also when retail deposit competition is active.

Appendix C: Nash Bargaining over Interbank Lending

We will now relax the assumption that bank \( A \) can make a take-it-or-leave-it offer and show that our results are robust when the surplus generated by interbank lending is allocated according to axiomatic Nash bargaining. For this recall first that there is no asymmetric information, so that bargaining proceeds under common knowledge. When banks do not come to an agreement, we denote their respective outside options by \( \pi_{n0} \), which are obtained by substituting \( W = 0 \) and \( w = 0 \) into (3). We next derive the bargaining (or Pareto) frontier. For some given (feasible) value of profits \( \tilde{\pi}_{n'} \) for bank \( n' \) this entails finding the interbank loan that maximizes the other bank’s profits, \( \pi_n \), subject to the constraint that \( \pi_{n'} \geq \tilde{\pi}_{n'} \). It is inessential which banks we choose as \( n \) or \( n' \). For specificity, suppose we maximize \( \pi_A \).

As is immediate, the constraint \( \pi_B \geq \tilde{\pi}_B \) will be binding so that the maximization problem gives rise to a function \( \pi_A = \psi(\pi_B = \tilde{\pi}_B) \). Note, in particular, that this notation suppresses the optimal choice of an interbank loan that, for given \( \pi_B \), maximize \( \pi_A \). There are two cases to distinguish. In the first case, interbank lending is not optimal from the banks’ shareholders’ perspective, so that there does not exist a pair \((\pi_A, \pi_B)\) with \( \pi_A \geq \pi_{A0} \) and \( \pi_B \geq \pi_{B0} \), where at least one holds strictly. In the second case, such a pair exists. Then, if \( \psi \) is concave (it is, in fact, linear, as we show below), the symmetric Nash solution is characterized as follows: The uniquely obtained solution \((\pi_A, \pi_B)\) maximizes the (symmetric) Nash product \([\psi(\pi_B) - \pi_{A0}][\pi_B - \pi_{B0}]\), which from the first-order condition is the case if

\[
\frac{\psi'(\pi_B) - \pi_{A0}}{\pi_B - \pi_{B0}} = -\psi'(\pi_B).
\]

The derivation of the Nash bargaining solution - or, more precisely, the derivation of the frontier \( \pi_A = \psi(\pi_B) \) - is complicated by the following feature. The bargaining frontier does not have slope of minus one, as in the most standard case where risk-neutral players can simply make a fixed transfer. This results from the fact that a debtor bank can make its contractual repayment only if its own corporate loans perform. And even when a
creditor bank receives such payment, it may go straight to depositors rather than banks’ equity holders (owners) when the bank becomes insolvent. Thus the discussed coinsurance externality on the creditor bank’s depositors is still present under symmetric Nash bargaining.

It is now convenient to solve explicitly for $w$ from the binding constraint $\pi_B \geq \tilde{\pi}_B$, so that in this case

$$w = L(R_B + W) - R_B(1 + r_B) - \tilde{\pi}_B \frac{1}{p}.$$  \hspace{1cm} (31)

Hence, in case there is a loan of size $W$ from bank $A$ to bank $B$, then the repayment $w$ as specified in (31) ensures that bank $B$’s profits are just equal to $\tilde{\pi}_B$. Substituting for $w$, $W \geq 0$ is then chosen so as to maximize

$$\pi_A = \left[p^2 + \rho p (1 - p)\right] \left[L(R_A - W) - R_A(1 + r_A) + L(R_B + W) - \tilde{\pi}_B \frac{1}{p_B} - R_B(1 + r_B)\right] + p(1 - p - \rho (1 - p)) \max\{0, L(R_A - W) - R_A(1 + r_A)\}.$$  \hspace{1cm} (32)

Importantly, this implies that $\tilde{\pi}_B$ does not affect the optimal choice of $W$. As a consequence, under Nash bargaining the same optimal $W$ obtains as when one bank can make a take-it-or-leave-it offer. That the corresponding repayment level $w$, as given by (31), is different does not affect our results qualitatively (albeit it affects the thresholds for $z$ and $\rho$ from which Cases 1 and 2 apply).

42
References


