Understanding Volatility Risk

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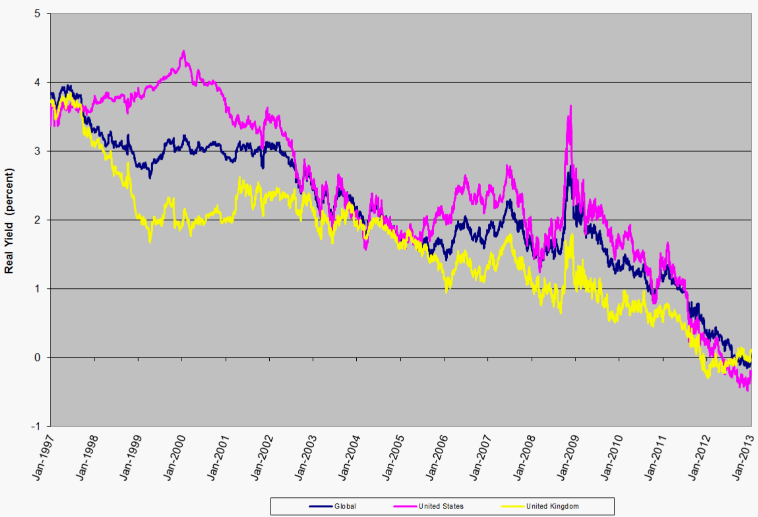
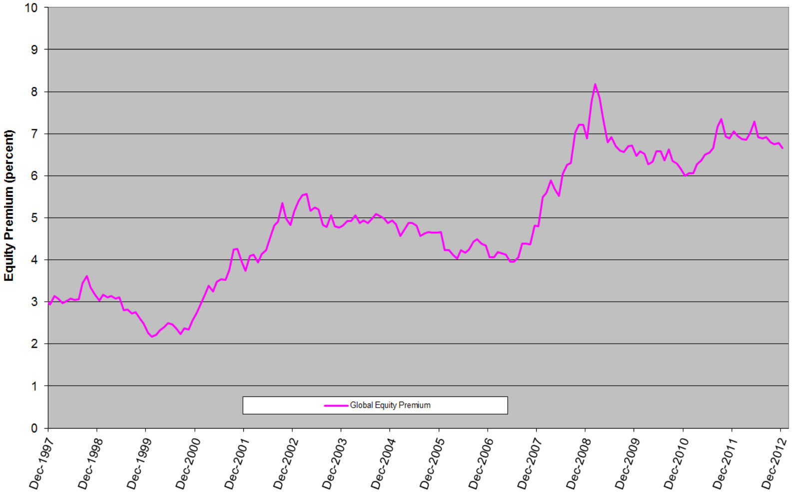
Financial Markets Are Interesting!

- Investment opportunities are not static, but change importantly over time
- The 10-year riskless real interest rate has fallen from an average of 3.5% in the 1990s to around 0% today
- The equity premium has risen from a historic low at the turn of the millennium to roughly the historic norm today
- Volatility was low in the mid-1990s and mid-2000s, high and unstable today

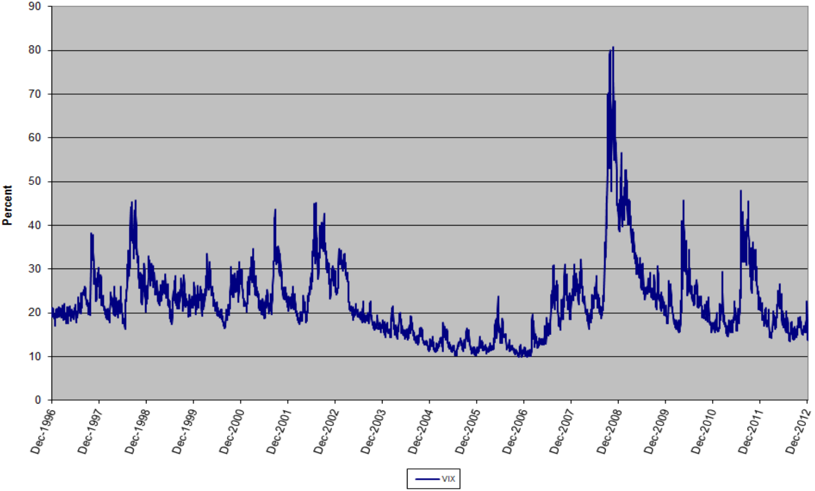
The Real Interest Rate



The Equity Premium



Unstable Volatility



What Does This Mean for Investors?

- Changing investment opportunities have many implications
- In a world of low safe real rates,
  - Claims to safe real income (DB pensions) are far more valuable than before
  - Institutions and individuals living on investment income must reduce return expectations, increase risk, or both
  - This requires unprecedented flexibility
- Long-term investors must plan for the inevitable fluctuations in investment opportunities that will occur in the future
  - Declining real rates are bad news
  - Declining expected stock returns are bad news
  - Increasing volatility is bad news

Intertemporal Hedging

- How can long-term investors hedge against these shocks to investment opportunities?
  - Merton (1973) intertemporal CAPM
  - Over the past 20 years I have developed the empirical implications in a series of papers with Chan, Giglio, Polk, Turley, Viceira, and Vuolteenaho, and a book with Viceira
- Long-term asset classes are natural hedges
  - Bonds hedge against interest rate declines
  - Stocks hedge against declines in the expected stock return
- Within the stock market, growth stocks also seem to have hedge value
  - Campbell-Vuolteenaho (2004) break the CAPM beta into two components
  - Beta with permanent cash-flow shocks to the market ("bad beta") should have a premium $\gamma = RRA$ times higher than beta with temporary discount-rate shocks to the market ("good beta")
  - Value stocks have relatively high bad betas; growth stocks have relatively high good betas

Hedging Volatility

- What about hedging against shocks to volatility?
- The desire to hedge volatility may explain many patterns in asset returns
  - Low returns on options ("variance risk premium")
  - High returns on corporate bonds
  - Low returns on growth stocks
- However there are challenges to understanding this
  - We need to find a **tractable** intertemporal model with stochastic volatility
  - There must be **persistent** variation in volatility for intertemporal hedging to be important
- Campbell, Giglio, Polk, and Turley, "An Intertemporal CAPM with Stochastic Volatility" (2013), takes on the challenge

Our Model

- We use Epstein-Zin preferences and substitute consumption out of the stochastic discount factor to derive an ICAPM
  - The alternative is to substitute out market returns to derive an extended CCAPM as in the "long-run risk" literature
  - Our approach is closer to the way investors themselves perceive risk
- A stock's risk is determined not only by its betas with market cash flows and discount rates, but also by its beta with news about future market volatility
- Although our model has three dimensions of risk, all three risk prices are determined by a single free parameter, RRA γ
  - The EIS ψ matters in the extended CCAPM (which requires $\psi > 1/\gamma$ to get aversion to long-run risk), but not in the ICAPM

Our Empirical Findings

- Novel low-frequency movements in market volatility can be tied to the default spread
- The negative post-1963 CAPM alphas of growth stocks are justified because these stocks hedge long-term investors against both declining expected stock returns, and increasing volatility
- The addition of volatility risk to the model helps it to deliver a moderate, economically-reasonable value of risk aversion
- The same preference parameters fit average returns on risk-sorted equity portfolios
- Volatility hedging is also relevant for equity index options, corporate bonds, and currency portfolios

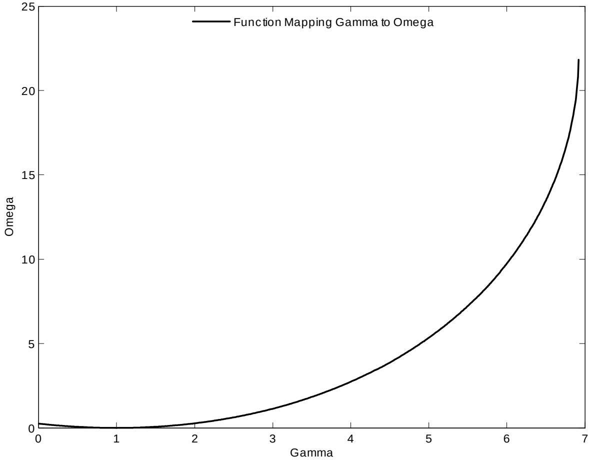
Summary of the Model

- Epstein-Zin (1989, 1991) preferences with discount factor δ, risk aversion γ, and EIS ψ
- Use the budget constraint to substitute consumption out of the log SDF
- VAR for market return, variance, and state variables: defines news about **long-run discounted values** of cash flows (N_{CF}), discount rates (N_{DR}), and variance (N_V)
- Variances move in proportion for all elements of the VAR (affine stochastic volatility)

$$\mu_{i,t} = \gamma \mathrm{Cov}_t \left[r_{i,t+1}, N_{CF,t+1} \right] + \mathrm{Cov}_t \left[r_{i,t+1}, -N_{DR,t+1} \right] - \frac{1}{2} \omega \mathrm{Cov}_t \left[r_{i,t+1}, N_{V,t+1} \right].$$

Function Mapping Gamma to Omega

$$\omega\sigma_t^2 = (1-\gamma)^2 \text{Var}_t\left[N_{CF_{t+1}}\right] + \omega(1-\gamma)\text{Cov}_t\left[N_{CF_{t+1}}, N_{V_{t+1}}\right] + \omega^2 \frac{1}{4}\text{Var}_t\left[N_{V_{t+1}}\right]$$



Our Empirical Implementation

- Explain simple expected returns
- Condition down
- Express in terms of betas

$$E[R_i - R_f] = \gamma\sigma_M^2\beta_{i,CF_M} + \sigma_M^2\beta_{i,DR_M} - \frac{1}{2}\omega\sigma_M^2\beta_{i,V_M}$$

where

$$\begin{aligned}
\beta_{i,CF_M} &\equiv \frac{Cov(r_{i,t}, N_{CF,t})}{Var(r_{M,t} - E_{t-1}r_{M,t})}, \\
\beta_{i,DR_M} &\equiv \frac{Cov(r_{i,t}, -N_{DR,t})}{Var(r_{M,t} - E_{t-1}r_{M,t})}, \\
\beta_{i,V_M} &\equiv \frac{Cov(r_{i,t}, N_{V,t})}{Var(r_{M,t} - E_{t-1}r_{M,t})}.
\end{aligned}$$

The Paper's Three Empirical Steps

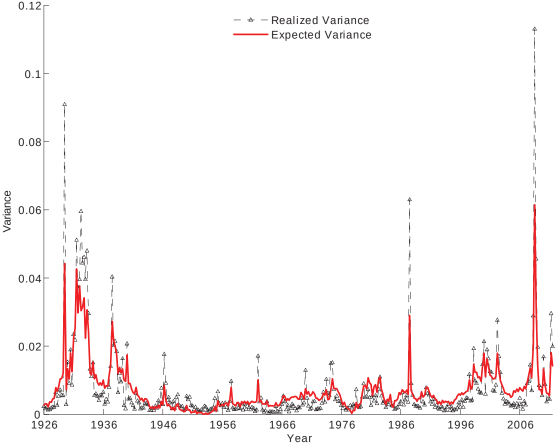
1. Estimate the market's cash-flow, discount-rate, and variance news
2. Using the estimated series, measure the cash-flow, discount-rate, and variance betas for various test assets
3. See how these betas explain average returns, and compare the premia to those predicted by the theory

VAR Data: 1926:2-2011:4

Six variables:

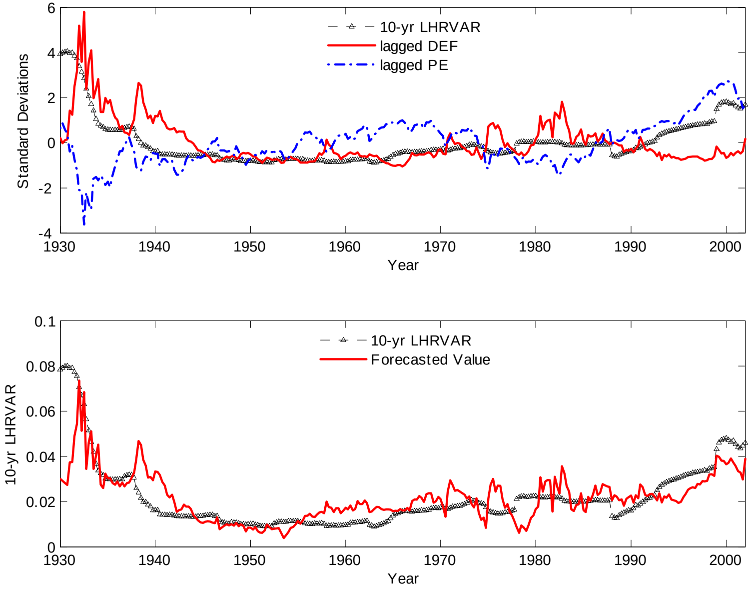
- Log real return on the CRSP value-weighted index (r_M)
- Expected market return variance ($EVAR$) generated from a regressing forecasting within-quarter realized variance ($RVAR$)
- Log ratio of S&P index to 10-year smoothed earnings (avoiding earnings interpolation) (PE)
- Term spread in Treasury yields (10 years to 3 months) (TY)
- Small-stock value spread (difference in log B/M for small growth and small value portfolios) (VS)
- Default spread (BAA to AAA bonds) (DEF)

Forecasting Realized Variance



Constant	$r_{M,t}$	$RVAR_t$	PE_t	TY_t	DEF_t	VS_t	$R^2\%$
-0.020	-0.004	0.394	0.006	0.000	0.006	0.001	36.88%
(0.008)	(0.005)	(0.064)	(0.002)	(0.001)	(0.001)	(0.002)	

Forecasting 10-Year Realized Variance

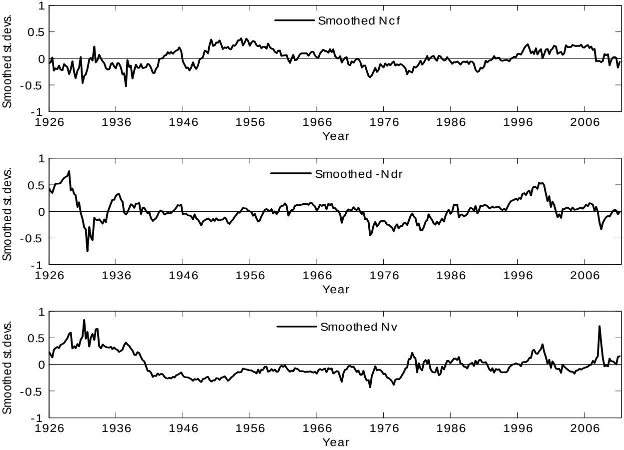


Confirming Heteroskedasticity Assumptions

Heteroskedastic Shocks

Squared, second-stage, unscaled residual	Constant	$EVAR_t$	$R^2\%$
$r_{M,t+1}$	-0.003	1.912	19.78%
	[0.004]	[0.309]	
$EVAR_{t+1}$	0.000	0.004	5.86%
	[0.000]	[0.001]	
PE_{t+1}	-0.004	1.937	19.61%
	[0.004]	[0.310]	
TY_{t+1}	0.205	15.082	1.67%
	[0.085]	[7.323]	
DEF_{t+1}	-0.117	27.841	26.12%
	[0.045]	[3.718]	
VS_{t+1}	0.004	0.472	5.47%
	[0.002]	[0.138]	

Implied News Histories



News volatilities: $\sigma(N_{CF}) = .05$, $\sigma(N_{DR}) = .09$, $\sigma(N_V) = .10$
News correlations: $\rho(N_{CF}, N_{DR}) = -0.10$, $\rho(N_{CF}, N_V) = -0.22$, $\rho(N_{DR}, N_V) = -0.09$

Test Asset Data: 1931:3-2011:4

- 25 size- and BE/ME-sorted portfolios from Ken French
  - Series begin in July 1931 as some portfolios are empty before that
  - Daniel and Titman (1997, 2012) and Lewellen, Nagel, and Shanken (2010) argue that characteristic-sorted portfolios are likely to show some spread in betas identified as risk by almost any model
- In response, we form 6 risk-sorted portfolios using backward-looking estimates of market and volatility betas
- We also examine the returns on an S&P100 index option straddle, Fama-French risky bond factors and RMRF, SMB, and HML, and interest-rate sorted currency portfolios

Subsamples

- Previous work has shown that
  - The CAPM betas of value stocks are high in the first part of our sample, and low in the second
  - The CAPM fits the characteristic-sorted portfolios well in the first part of the sample, and very poorly in the second
- Accordingly we break our sample into two subsamples, early (1931:3-1963:2), and modern (1963:3-2011:4)
  - We would like our models to explain both subsamples with stable preference parameters
  - Given limited time I will only show modern-period results

Characteristic-Sorted Betas: Modern Period

$$\beta_{i,CF} \equiv \frac{\text{Cov}(r_{i,t}, N_{CF,t})}{\text{Var}(r_{M,t} - E_{t-1} r_{M,t})}, \ \beta_{i,DR} \equiv \frac{\text{Cov}(r_{i,t}, -N_{DR,t})}{\text{Var}(r_{M,t} - E_{t-1} r_{M,t})}, \ \beta_{i,V} \equiv \frac{\text{Cov}(r_{i,t}, N_{V,t})}{\text{Var}(r_{M,t} - E_{t-1} r_{M,t})}$$

$\widehat{\beta}_{CF}$	Growth		2		3		4		Value		Diff	
Small	0.23	[0.06]	0.24	[0.05]	0.24	[0.04]	0.23	[0.04]	0.26	[0.05]	0.03	[0.03]
2	0.22	[0.05]	0.22	[0.04]	0.24	[0.04]	0.24	[0.04]	0.26	[0.05]	0.04	[0.03]
3	0.20	[0.05]	0.22	[0.04]	0.22	[0.04]	0.23	[0.04]	0.24	[0.04]	0.05	[0.03]
4	0.19	[0.04]	0.21	[0.04]	0.22	[0.04]	0.22	[0.04]	0.24	[0.04]	0.05	[0.03]
Large	0.13	[0.03]	0.17	[0.03]	0.16	[0.03]	0.17	[0.03]	0.19	[0.04]	0.05	[0.03]
Diff	-0.10	[0.04]	-0.07	[0.03]	-0.08	[0.02]	-0.06	[0.02]	-0.07	[0.03]		

$\widehat{\beta}_{DR}$	Growth		2		3		4		Value		Diff	
Small	1.31	[0.10]	1.06	[0.08]	0.89	[0.07]	0.83	[0.07]	0.87	[0.09]	-0.44	[0.08]
2	1.21	[0.09]	0.97	[0.07]	0.85	[0.06]	0.76	[0.07]	0.80	[0.08]	-0.42	[0.08]
3	1.14	[0.07]	0.89	[0.05]	0.77	[0.06]	0.72	[0.06]	0.72	[0.07]	-0.42	[0.08]
4	1.03	[0.06]	0.85	[0.05]	0.74	[0.06]	0.72	[0.06]	0.75	[0.07]	-0.28	[0.08]
Large	0.84	[0.05]	0.71	[0.04]	0.60	[0.05]	0.59	[0.06]	0.64	[0.06]	-0.20	[0.06]
Diff	-0.46	[0.10]	-0.35	[0.08]	-0.29	[0.06]	-0.24	[0.07]	-0.23	[0.08]		

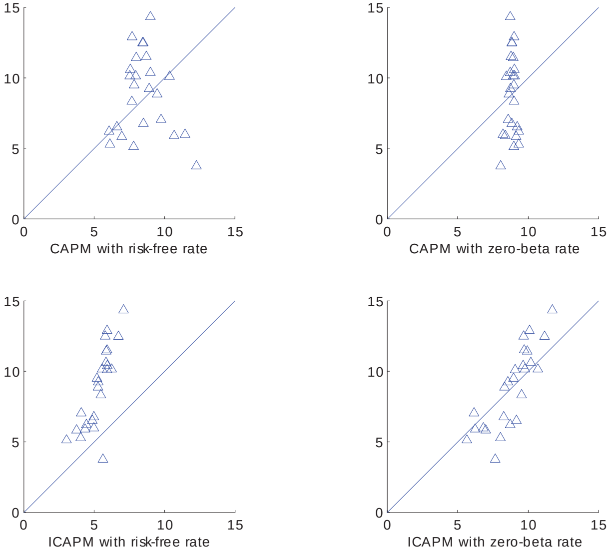
$\widehat{\beta}_{V}$	Growth		2		3		4		Value		Diff	
Small	0.18	[0.07]	0.12	[0.06]	0.08	[0.06]	0.07	[0.05]	0.03	[0.07]	-0.15	[0.03]
2	0.19	[0.07]	0.12	[0.06]	0.08	[0.05]	0.06	[0.06]	0.04	[0.06]	-0.15	[0.03]
3	0.19	[0.06]	0.11	[0.05]	0.08	[0.05]	0.04	[0.06]	0.06	[0.04]	-0.13	[0.03]
4	0.17	[0.06]	0.11	[0.05]	0.06	[0.06]	0.05	[0.06]	0.04	[0.06]	-0.13	[0.03]
Large	0.13	[0.05]	0.10	[0.04]	0.06	[0.04]	0.04	[0.05]	0.04	[0.05]	-0.09	[0.02]
Diff	-0.05	[0.03]	-0.02	[0.03]	-0.03	[0.02]	-0.03	[0.02]	0.01	[0.03]		

Characteristic-Sorted Model Estimates: Modern Period

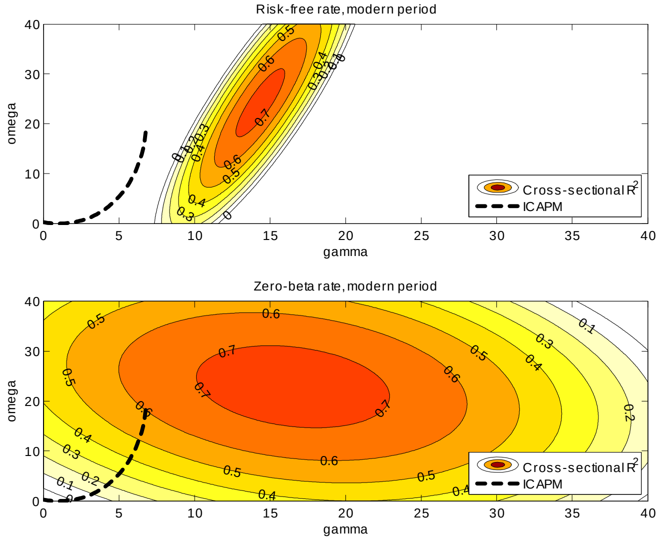
$$E[R_i - R_f] = \gamma\sigma_M^2\beta_{i,CF} + \sigma_M^2\beta_{i,DR} - \frac{1}{2}\omega\sigma_M^2\beta_{i,V}$$

Parameter	CAPM		2-beta ICAPM		3-beta ICAPM		Constrained		Unrestricted	
R_{zb} less R_f (g_0)	0	0.027	0	-0.019	0	0.011	0	-0.004	0	-0.005
% per annum	0%	10.62%	0%	-7.71%	0%	4.50%	0%	-1.66%	0%	-2.00%
Std. err. A	0	[0.014]	0	[0.013]	0	[0.012]	0	[0.013]	0	[0.013]
Std. err. B	0	(0.014)	0	(0.019)	0	(0.015)	0	(0.015)	0	(0.015)
$\widehat{\beta}_{CF}$ premium (g_1)	0.020	-0.004	0.074	0.161	0.047	0.054	0.112	0.128	0.175	0.199
% per annum	7.98%	-1.67%	29.41%	64.39%	18.78%	21.49%	44.65%	51.35%	70.18%	79.55%
Std. err. A	[0.010]	[0.019]	[0.047]	[0.070]	[0.024]	[0.013]	[0.050]	[0.071]	[0.070]	[0.084]
Std. err. B	(0.010)	(0.019)	(0.087)	(0.113)	(0.040)	(0.053)	(0.114)	(0.116)	(0.124)	(0.126)
$\widehat{\beta}_{DR}$ premium (g_2)	0.020	-0.004	0.008	0.008	0.008	0.008	0.008	0.008	-0.018	-0.020
% per annum	7.98%	-1.67%	3.11%	3.11%	3.11%	3.11%	3.11%	3.11%	-7.30%	-7.83%
Std. err. A	[0.010]	[0.019]	[0.002]	[0.002]	[0.002]	[0.002]	[0.002]	[0.002]	[0.023]	[0.025]
Std. err. B	(0.010)	(0.019)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.054)	(0.055)
$\widehat{\beta}_{VAR}$ premium (g_3)					-0.039	-0.081	-0.094	-0.089	-0.002	0.009
% per annum					-15.51%	-32.47%	-37.65%	-35.60%	-0.72%	3.62%
Std. err. A					[0.039]	[0.024]	[0.063]	[0.069]	[0.092]	[0.094]
Std. err. B					(0.091)	(0.151)	(0.356)	(0.349)	(0.399)	(0.387)
$\widehat{R}^2$	-36.51%	5.22%	25.10%	39.97%	-108.63%	62.74%	73.90%	74.45%	76.46%	77.25%
Pricing error	0.110	0.107	0.058	0.042	0.210	0.042	0.027	0.025	0.026	0.023
5% critic. val. A	[0.050]	[0.034]	[0.061]	[0.056]	[0.503]	[0.101]	[0.051]	[0.037]	[0.046]	[0.031]
5% critic. val. B	(0.049)	(0.033)	(0.096)	(0.083)	(0.492)	(0.119)	(0.104)	(0.078)	(0.065)	(0.049)
Implied γ	N/A	N/A	9.5	20.7	6.0	6.9	N/A	N/A	N/A	N/A
Implied ω	N/A	N/A	N/A	N/A	10.0	20.9	N/A	N/A	N/A	N/A

Characteristic-Sorted Model Comparison: Modern Period



Why a Free Zero-Beta Rate Helps the ICAPM



Summary of Remaining Results

- The same preference parameters fit risk-sorted portfolios and interest-rate sorted currency portfolios
- The model explains about a third of the extremely low average returns on a straddle portfolio
- The distinction between long-run variance and short-run variance is key
    - In the modern sample, we estimate that the aggregate stock market has a positive beta with N_V even though it has a negative beta with realized short-run variance and the VIX
- We explore variations of the basic VAR specification:
    - Results are robust to different estimation methods, to different measures of the market's valuation ratio, and to including CAY or a GARCH volatility forecast in the VAR
    - R^2s remain reasonable for excess zero-beta rates that are as low as 40 bps/quarter

Conclusions

- We extend the approximate closed-form intertemporal capital asset pricing model of Campbell (1993) to allow for stochastic volatility
  - A conservative long-horizon investor will wish to hedge against both a decline in the equity premium and an increase in market volatility
  - Though our model has three dimensions of risk, a single free parameter, the relative risk aversion coefficient, determines all risk prices
- We uncover new persistent variation in market volatility via DEF/PE
- We justify the negative post-1963 CAPM alphas of growth stocks
  - These stocks hedge long-term investors against both declining expected stock returns, and increasing volatility
  - The addition of volatility risk helps deliver an ICAPM with a moderate, economically reasonable value of risk aversion
- We confirm that the same preference parameter also explains the average returns on risk-sorted equity portfolios
- We show that our measure of volatility risk is also relevant for equity index option, corporate bond, and currency returns

Open Questions

We assume a rational long-term investor always holds 100% of his or her assets in equities. Consider two ways to justify that assumption:

- Test the model conditionally: Real interest rates and market volatility should move in exactly the right way to keep the equity premium proportional to market volatility
    - Work by Campbell (1987) and Harvey (1989, 1991) rejects this proportionality restriction
- Invoke binding leverage constraints
    - Consistent with this interpretation, modern-sample estimates of the excess zero-beta rate in our three-beta ICAPM are positive, statistically significant, and economically large
    - However, we need to check when leverage constraints should bind given the risk aversion coefficient we estimate