On Setting Adequate Capital Ratios: A Study of Changing Patterns between Leverage and Risk-Based Capital Ratios

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Abstract
Basel regulators have received widespread criticism for failing to prevent two credit crises that hit the U.S. over the last two decades. Yet, banks prior to the onset of the subprime crisis of 2007-2009, were considerably overcapitalized compared to those who had undergone the 1990-1991 recession. Hence, if the capital requirements were largely met for the second crisis, how could the Basel framework be blamed again for having accelerated if not caused another credit crunch? In this paper, we find that the answer lies in the changing dynamics between two different regulatory requirements, the capital ratio and the leverage ratio. Indeed, we argue that the change in risk-weight on residential mortgages which was introduced in the Basel II framework was sufficient in reversing the correlation pattern between both ratios. This inherently changed the binding constraints on banks between the two crises. Our reasoning is based on a formula linking the two ratios together which is derived from the sensitivity of the risk-based capital ratio to a change in its risk-weight(s). The implication of our work on the phasing in of Basel III consists in validating the newly established capital increments in a mathematical rather than heuristical approach.

JEL classification: G2 E5

Keywords: Capital Ratio, Leverage, Basel

1. Introduction

After the U.S. witnessed two credit crunches in a span of less than twenty years, the Basel Committee on Banking Supervision (BCBS) has been widely criticized by governments and academics alike for not meeting its safety objective. Because these governments, including the U.S., were at the forefront of the G-10 signatories who ratified the Basel agreements, the bulk of the criticism had to come from practitioners and academic researchers. Despite leading towards the structuring of a new Basel framework, Basel III, the blame was nonetheless not as strong after the subprime crisis as it was following the 1990-1991 recession. Hereafter, we refer to these two episodes as the first and second crunch based on chronological order.

In fact, the storyline that the mainstream literature agreed on after the first crunch was much simpler than the one(s) that followed the second. At that time, banks were struggling to meet the...
newly established Basel I capital requirements through shifting their portfolio asset composition to boost their capital levels. In other words, their main concern was to achieve the target risk-based capital ratios (CRs) set forth by the Basel regulators which resulted in a contraction in lending.

In contrast, since the Basel II framework did not change the pre-established CR requirements which were maintained at 4% and 8% for Tier 1 and Total CR, respectively, it seems that banks had willingly increased their CRs beyond the target thresholds prior to the second crunch. Still, one change to the risk-weight (RW) on a particular category of assets, residential mortgages, is believed by some to have driven banks towards increased portfolio positions in that asset category. Unable to unwind their positions when defaults began their domino effect, banks saw their capital cushions rapidly shrink, triggering the second credit crunch. It was therefore not the insufficiency of capital requirements, but the quality of their make-up, which became the main culprit.

Incentivized by their overcapitalized positions, banks, however, fell short of the leverage ratio (LR) requirements established by the main U.S. regulators (OCC, FDIC and FED) for well-capitalized institutions. Note that the latter had enforced the LR years before the first crunch; yet, the impact it had on the second was much more severe. For instance, Gilbert (2006) states that up until mid-2005 only the two largest U.S. banks did not fall below the 5% leverage requirement.

On the other hand, liquidity as well as various other “regulatory loopholes”\(^2\) in the general framework were pinpointed by those who questioned the regulators’ goal of maintaining a bank “level-playing field”. However, the fact that these shortcomings were not directly related to capital requirements was one of the main reasons for the above mentioned “leniency” towards Basel regulators. Today, the related amendments form a major part of the overhaul which took place as part of the Basel III process. Among other reasons is the fact that the main players during the second crunch, such as Credit Rating Agencies and Special Purpose Vehicles, were not formally bound by any jurisdiction under the Basel II agreement. This too is currently being reassessed from a macro-prudential perspective.

Overall, the literature has studied the impact of capital on two main aspects of bank behavior: the first is related to growth (Bernanke and Lown (1991); Baer and McElravey (1993); Peek and Rosengren (1992, 1994, 1995a,b); Barajas et al. (2004); Cathcart et al. (2012)) whereas the second focuses on risk (Koehn and Santomero (1980); Furlong and Keely (1987); Kim and Santomero (1988); Furlong and Keely (1989); Keely and Furlong (1990); Gennette and Pyle (1991); Shrieves and Dahl (1992); Calem and Rob (1999); Blum (1999); Montgomery (2005); Berger and Bouwman (2010)). While opinions remain mixed as to the effect capital can have on each of these behavioral elements, other inter-linkages have been found between a bank’s capital position and other related features such as size. Herein, the conventional wisdom seems to be that larger banks have smaller CRs (Hall (1993), Estrella et al. (2000), Gilbert (2006) and Demirguc-Kunt et al. (2010))\(^3\).

In this research, we look at matters from a different angle through revealing how a change in RW(s) can determine which of the CR or the LR is the banks’ binding constraint. This can shed light on the controversy highlighted by various authors (Hall (1993); Thakor (1996); Blum (2008); Buehler et al. (2010); Blundell-Wignall and Atkinson (2010); Kiema and Jokivuolle (2010) to name a few) on the benefit(s) of combining the two capital measures. In doing so, we reflect upon changes we find in the correlation patterns between the two ratios over the different crunch periods.

\(^1\) Some components of the CR, however, did change between both regulations (BCBS (2004)).

\(^2\) These terms were attributed to the items that were considered missing from the Basel framework.

\(^3\) Still, this finding depends on the choice of capital: Tier 1 VS Total (Demirguc-Kunt et al. (2010)).
Indeed, correlation between the CR and the LR can reveal distinctive features of a certain crunch. The intuition for that lies in the definition of the CR which is influenced by the proportion of assets held in risky versus safe categories. Note that it is a common empirical mistake for some authors to confuse correlation for causality, an opinion shared with Furfine (2000). We therefore assess the implied causality before arguing that the co-movements of the two ratios influence which one is more binding for a given bank.

To validate this hypothesis, we establish the following scenario. In section 2, we showcase the impact each requirement had on bank failures for each crunch. This allows us to establish the different binding constraint on banks during these periods. Our analysis also looks at the degree to which banks fell below these requirements through an overview of capital shortfall. This can be seen as a contribution to the Basel III efforts in setting the new CRs and the backstop LR. In section 3, our next step is to explore the correlation patterns between the two ratios, as well as with different market signals. This allows us to ascertain the loan category mostly correlated with the second crunch. Finally, in section 4, we explain the correlation reversals between the two crises with a closed-form equation which binds the two capital requirements, together with a commonly used credit risk measure and a newly defined factor. The solution of this equation is essential in the setting by policy makers of adequate CRs.

2. The Change in the Binding Capital Constraint

2.1. Equity as the Common Numerator

As stated earlier, the literature has so far considered two streams related to the Basel regulatory impacts: growth and risk. While one might consider bank failure as being the adverse consequence of excessive risk-taking, not all failures relate back to a bank’s risk-taking behavior in relation to capital positions. After 2009, the literature turned its attention towards investigating causal linkages to the subprime crisis outside the realm of risk-based capital requirements such as leverage, liquidity, securitization and others. Our study re-emphasizes the effects of capital requirements on failures in an aim to fill the gap.

The CR and LR are the most popular measures of capital adequacy in the U.S. It is argued, albeit, that each can have very different effects on a bank’s behavior depending on which of the two is the binding constraint (Berger and Udell (1994), Hancock and Wilcox (1994), Peek and Rosen-gren (1994), Chiuri et al. (2002), Barajas et al. (2004), Blundell-Wignall and Atkinson (2010)). While the two measures differ mainly with respect to their denominator (Risk-Weighted versus Un-Weighted Assets), the common numerator, Tier 1 capital, has been under wide investigation following the second crunch. Hence, the latter can be characterized, from the perspective of either ratio. We opt for the LR in this case.

According to Greenspan et al. (2010), over a span of less than two centuries, leverage has fallen to almost a fifth of its initial value. This is probably due to the perception by regulators, and the banks themselves, that so much excess capital was unnecessary. Hence, the necessary

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4Operational risk has been at the helm of many investigations: fraud (Daiwa, Sumitomo), rogue trading (Barings Bank), as well as more common reasons such as lack of competitiveness.

5See section 4

6With reasonable approximation based on the risk-based capital definitions for Prompt Corrective Action (PCA) as posted by the FDIC.

7This corresponds to an increase in the common sense of Debt/Equity.
amount had to be optimized and reduced in an effort to increase competitiveness. Focusing on the previous decade, an unprecedented event highlighted by Greenspan et al. (2010), is that the market valued LR was almost double its book value up until the start of the subprime crisis. Yet, at the peak of this crunch, the latter actually exceeded the former. Since one can easily cast out the hypothesis of an increase in the market value of assets during crisis times, the only possible explanation relates to depreciation in the value of capital. That is mainly because of the failure of some capital components to assume their pre-defined role as primary layer for loss absorption. As a matter of fact, this has given Basel III regulators the incentive to scrap the Tier 3 capital layer and remove all elements in Tier 2 that cannot fulfill this purpose.

2.2. CR VS LR: The Binding Constraint

In order to point out which of the LR or CR was primarily responsible for destabilizing banks’ safety positions during both crunches we start by reviewing the history of their implementation. As noted earlier, in view of the common (equity) capital feature embedded in both ratios, any changes between the two can be attributed to changes in their differentiating (denominator) components, risk-weighted VS unweighted assets. We start with the first crunch which attracted most of the attention in terms of failure analysis with respect to capital regulation.

First, it is important to recognize that the substitution of the 1980s flat rate for the risk-based capital (RBC) standard under Basel I meant that banks accounting for a quarter of total assets failed the newly imposed regulation according to Avery and Berger (1991). Based on Berger and Udell (1994), this amounted to a 20% increase in non-abiding banks. Moreover, Peek and Rosengren (1994) emphasized that, towards mid-1991, from the 20 largest First District commercial and savings banks, the numbers violating the targets on Tier 1 capital, Total capital and a 5% Leverage target were zero, seven and nine, respectively.

On the other hand, the choice of which leverage threshold one must chose to compare between banks is each author’s discretion. This is because CAMEL ratings are not disclosed in order to infer the actual discretionary requirement imposed by the national regulators. This point is emphasized by Hall (1993) who demonstrated that if the average LR were assumed at 3%, the CR becomes the more likely culprit since most banks are able to fulfill the first but not the second requirement. However, if the LR were established at a level of 5% then at least 18% of these would fail the leverage requirement.

Two studies which investigated the impact of CR and LR on failures during the first crunch are Avery and Berger (1991) and Estrella et al. (2000). Although the latter study came at a much later time than the former, it only pointed out the critical regions at which banks were affected by one ratio or the other. Hence, no consideration was given to the combined effect of the two. Still, we reflect on an important observation we make from the authors’ analysis: at least one year prior to its failure, a bank can have the same LR in the critical region as one which eventually survived. This would mean that the LR has no embedded predictive power in contrast to the CR which is in line with the authors’ conclusion. Surely, this would make sense should the CR emerge as the binding constraint as in Avery and Berger (1991). However, this is not always the case as we argue for the second crunch.

8Mainly some types of preferred stock categorized under hybrid instruments.
9Under this rating scheme, the safest banks, attributed the best rating of 1, are given a leverage target of 3%. Depending on their condition, all other banks are set a target of either 1 to 2 percentage points higher.
10Even if it were known, the function underlying the “CAMEL-to-Leverage” specification is arguably not bijective.
What is noteworthy regarding the similar assessment made by Avery and Berger (1991) is their breakdown and combination of the CR and LR requirements that banks could fail at the introduction point of Basel I requirements. This was matched against the number of banks that went bankrupt just before the start of the first crunch given that these banks had earlier failed to meet one or more of the new regulations. For example, almost a third of the 6% of banks which could not meet the targets for Tier 1 capital, Total capital or leverage failed over the next 2 years\(^{11}\). On the other hand, 50% of banks failing Tier 1 eventually went bankrupt putting this requirement at pole position in terms of forecasting power.

We continue on the same path as the previous authors to reflect on the relationship between the new Basel capital standards and bank failures for the second crunch. That is not to say that other authors did not attempt to establish some sort of relationship during this period. For instance, Berger and Bouwman (2010) observed that a one standard deviation decrease in capital more than doubles the probability of bankruptcy. Yet their result shows this trend as being linear although it is understood from the previously surveyed literature on risk and capital that capital shortfalls weigh more on bank survival than surpluses.

Our results, which are compiled from the FDIC Call report\(^{12}\) database over the period 2004Q3-2009Q2 are presented in Table 1. This period extends from the date Basel II was initially published until the established end of the subprime crisis (King (2012)\(^{13}\)). Although some components might have changed, the CR and LR thresholds were maintained across the first two Basel frameworks\(^{14}\). Hence, instead of looking at changes before and after the Basel II capital standards were brought in, this study uses three intervals (pre, mid and end of the crisis), in order to gauge the evolution in meeting these standards along with the leverage requirements as the crisis unfolded. As per Avery and Berger (1991), we look at a range of leverage targets (3%, 4% and 5%) instead of assuming an arbitrary one\(^{15}\). More importantly, we look at how combinations of both standards impact on bankruptcies. Similarly, we also account for bank percentage in terms of number (%B) and asset (%A) for each of our measurement criteria.

Table 1 shows a number of compelling features. First, with respect to the 4% (median) leverage requirement, Avery and Berger (1991) obtained a 94% estimate of the proportion of banks that passed all three requirements prior to the first crunch. This is still well below the corresponding 99% at the onset of the second crunch. This confirms the result in Greenspan et al. (2010). What’s more is that the percentage of banks failing any of the standards was at least an order of magnitude less than those in Avery and Berger (1991)’s first crunch sample. This confirms that, in quantitative terms, banks were holding capital well in excess of the targets (overcapitalization) prior to the second crunch.

Nevertheless, failing any of the standards this time had more serious repercussions since a much greater proportion of the pre-crunch pool (> 50%) ended up bankrupt. Ultimately, all banks failing Tier 1 CR or a 3% LR went bankrupt. This obviously brings back into question the purpose for dual requirements. Moreover, we point to the increase in the failure to meet any of the requirements

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\(^{11}\)Note that in the Basel framework, if a bank fails Tier 1 it automatically fails the Total requirement as the regulators impose that Tier 2 cannot exceed 50% of Tier 1.

\(^{12}\)Also known as Reports of Condition and Income taken from the Federal Financial Institutions Examination Council (FFIEC).

\(^{13}\)see Cathcart et al. (2012) for more details on the technical aspects of how this study period was defined.

\(^{14}\)The period under consideration does allow, however, for some delay before the change in RW(s) took place. This will become important later on.

\(^{15}\)This is also the case in Hall (1993).
over time. This contrasts with a simultaneous decrease in bankruptcy rate, specifically between the start and end of the period under observation. The first finding stresses the weakened capital

<table>
<thead>
<tr>
<th>Standard</th>
<th>Pre-crisis</th>
<th>Mid-crisis</th>
<th>End-crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>%B</td>
<td>%A</td>
<td>%Bkrpt</td>
</tr>
<tr>
<td>Tier1(1)</td>
<td>0.02</td>
<td>0.00</td>
<td>100.00</td>
</tr>
<tr>
<td>Total(2)</td>
<td>0.10</td>
<td>0.12</td>
<td>55.56</td>
</tr>
<tr>
<td>Lev3(3)</td>
<td>0.03</td>
<td>0.02</td>
<td>100.00</td>
</tr>
<tr>
<td>Lev4(4)</td>
<td>0.07</td>
<td>0.05</td>
<td>66.67</td>
</tr>
<tr>
<td>Lev5(5)</td>
<td>0.14</td>
<td>0.15</td>
<td>69.23</td>
</tr>
<tr>
<td>(1)</td>
<td></td>
<td>(2)</td>
<td></td>
</tr>
<tr>
<td>(1)&amp;(2)&amp;(4)</td>
<td>0.02</td>
<td>0.00</td>
<td>100.00</td>
</tr>
<tr>
<td>Pass</td>
<td>99.86</td>
<td>99.85</td>
<td>7.02</td>
</tr>
</tbody>
</table>

The data in this table is based on Call Reports for the period 2004Q3-2009Q2 broken down into three consecutive periods. Each row consists of a different target ratio. Numbers in brackets are for use in the last rows as combinations of the previous single standards. The last row is for banks who passed all standards (with a 4% LR). In that case, the first two columns in each period can be retrieved with the identity: Prob(Pass) = 1 - Prob(Fail any (combination of) standard(s)).

position of banks eroded by losses throughout the crunch period. The second finding relates to corporate finance theory in that survival rates increase for banks which can endure more phases of a crunch as this relates to macroeconomic fundamentals (Klapper and Richmond (2011)).

A striking feature is that during all three phases, all banks that failed Tier 1, and obviously Total, capital also failed the average leverage requirement of 4%. This has crucial implications on Basel III as it suggests that the choice of imposing a backstop 3% requirement could be overly conservative. More importantly, increasing leverage by 1% always resulted in an average doubling of failure rate across all phases. These reasons are why leverage emerges as the binding constraint for this crunch; thus statistically corroborating the statements in Gilbert (2006) and Blundell-Wignall and Atkinson (2010). This is in contrast with the first crunch where banks who failed a requirement were mostly struggling to meet their CR requirements.

2.3. **Basel III Upgrades from the perspective of the subprime crisis capital shortfall**

When quantifying the magnitude of failing a specific standard one must relate back to the concept of surpluses/shortfalls. As a matter of fact, Brinkmann and Horvitz (1995) stress that capital requirements are not just in terms of meeting a certain standard. They emphasize that the importance of surpluses lies in that regulators should not only look at how many banks are likely to fail a newly introduced standard but also, and foremost, by how much their excess capital cushion would vary. Hence, one motivation for performing the following study is to assess the Basel regulators’ setting of the new Basel III standards.

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16See next section.

17Though the author uses a different definition for the binding capital requirement based on surpluses rather than the actual number of banks that achieved the given target.

18Regulators classify institutions into four main capital surplus/shortfall categories: Adequately/Under capitalized and Significantly/Critically undercapitalized.
Focusing on shortfall is arguably a better choice than surplus. Firstly because, as was evoked earlier, the fact that banks were overly capitalized prior to the crunch did not fare well for some of them during the crunch. In other words, while size does matter (for the regulators) it does not reflect quality of capital which means that surplus could be a biased signal for the health of the banking sector. Secondly, shortfall is more amenable to the idea of setting minimum capital requirements. Hence, from the previous dataset, and in the same spirit as Hancock and Wilcox (1994), we calculate the average shortfall in risk-based capital for banks failing to meet any of the CRs. This turned out to be equal to 1.5% for Tier 1 and 1.4% for Total Capital\(^{19}\) (conditional on having met Tier 1).

The first result is actually in line with the current steps taken by the Basel III Committee to increase the Tier 1 requirement by just above that amount (2%). On the other hand, while the Total CR is to remain at the existing 8% target level according to the BCBS, this ratio is being supplemented by other capital buffers\(^{20}\) bringing the overall requirement well over the shortcomings during the recent crunch (between 10.5% and 15%).

Finally, if the Basel III regulators had decided to abide by the median leverage requirement implemented in the U.S. (i.e. 4%), this could end up short of expectations as the crunch revealed a close to 1% shortage in leverage. This could explain why they chose a conservative 3% target for compliance of all banks. In this context, the introduction of the new Basel framework could mean the rating standard would become obsolete. Otherwise, our finding suggests that the CAMEL ratings system should be revised downward by the same shortage amount to reflect the change in the median bank’s capital safety.

3. The Change in CR and LR Correlation Patterns

Having illustrated one crucial change between the two crises with respect to the established binding capital constraint, we now move on to another differentiating aspect. To find out if there is any pattern in the co-movements of banks’ LR and CR we compute the correlation between both over each crunch period in Figures 1 and 2.

Various authors have measured this correlation over specific periods without mention if the obtained pattern is likely to change or be persistent over time. For instance, Estrella et al. (2000) perform their calculations for the first crunch only. Their yearly values coincide to a large extent with the ones we obtain for the first quarter of each year in Figure 1. To our knowledge, they are the first to have observed an imperfect correlation between the two capital measures which hints to the fact that each ratio can provide independent information on capital adequacy for a given bank.

Our estimates are based on Call Report data and calculated on the basis of the whole bank sample as well as for each bank size category\(^{21}\). Note how in both illustrations, the small bank correlation pattern moves closely with that of ALL banks put together. This is merely a feature of the data as small banks accounted for almost 72% and 44% of banks in the sample on average.

\(^{19}\)Note that the average shortage also decreased towards the end of the crunch due to the increase in number of failed banks.

\(^{20}\)Mainly the conservation, countercyclical and TBTF systemic buffers.

\(^{21}\)As per Berger and Udell (1994), in terms of Un-Weighted Assets, large banks were those with more than $1 billion in assets, medium were those with assets between $100 million and $1 billion, while small banks were the ones remaining.
Figure 1: Correlation Pattern between LR and CR for different size banks during the 1990-1991 crunch

Figure 2: Correlation Pattern between LR and CR for different size banks during the 2007-2009 crunch
Figure 3: Relation between LR/CR Correlation Pattern and Loan Growth for different loan categories during the 1990-1991 crunch.

Figure 4: Relation between LR/CR Correlation Pattern and Loan Growth for different loan categories during the 2007-2009 crunch.
during the first and second crunch, respectively. On the other hand, the fall in the number of small banks between crises was compensated by a rise in medium banks from 25% to 48%. However, owing to the static nature of our choice of SIZE thresholds dating back to the first crunch, we do not account for survivorship bias in our sample as many banks were either removed or merged with other medium/large banks. This explains why medium banks were somewhat reflective of the overall correlation pattern in the first crunch while not as much in the second. In particular, they exhibited features of large banks during the second crunch.

Nonetheless, the salient feature is with regard to the different overall correlation pattern trends for each of the crises: during the first crunch (1990Q1-1991Q2), the trend is monotonously decreasing while the opposite is true for the second (2007Q3-2009Q2)\(^\text{22}\). The intuition for this result lies in the CR definition which, by construction is affected by the balance of assets between risky and safe categories (see equation 1). Note that, with regard to the size of the fluctuations, during the shorter crunch, the difference between inflexion points is much more pronounced: 0.65 VS 0.27.

In addition, it seems as though each crunch period’s correlation pattern, (referred to as LR/CR), is itself correlated with other distinctive fundamentals, starting with loan growth. In abidance with Berger and Udell (1994) and Shrieves and Dahl (1995), we categorize lending growth type into three major groups: real estate (LNRE), commercial and industrial (LNCIUSD) and consumer (LNCONOTH) lending. We also add the aggregate (LNSGR). As is apparent during the first crunch, the correlation between LR/CR and the growth pattern is positive (Figure 3). The opposite is true during the second crunch where the two are negatively correlated (Figure 4)\(^\text{23}\).

Although one cannot infer ex-ante from observing these figures which of the loan categories is mostly correlated with the business cycle, the overall picture depicting whether the lending situation is worsening or not is clear. We now try to capture the loan category mostly linked to each of the crises by computing the correlation of each category with the LR/CR correlation pattern. The results for each category are shown in Table 2. While total loans and consumer lending dispute the first place in each crunch, they are followed by real-estate and commercial lending. Note as well that, in relative terms, the LR/CR correlation with the real estate category is closer to that of the overall loan portfolio during the second crunch compared with that of the first. This illustrates the differential role this category played during each crunch.

### Table 2: LR/CR Crunch Correlation

<table>
<thead>
<tr>
<th>Loan Category</th>
<th>Crunch 1</th>
<th>Crunch 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>LNRE</td>
<td>0.44</td>
<td>-0.49</td>
</tr>
<tr>
<td>LNCIUSD</td>
<td>0.28</td>
<td>-0.32</td>
</tr>
<tr>
<td>LNCONOTH</td>
<td>0.80</td>
<td>-0.54</td>
</tr>
<tr>
<td>LNSGR</td>
<td>0.75</td>
<td>-0.61</td>
</tr>
</tbody>
</table>

The data in this table is based on Call Reports for each crunch period. The values refer to the correlation between various loan categories and the observed LR/CR pattern.

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\(^{22}\)This excludes the systematic reversals in the first and last quarters of each crunch. The reasons for these reversals will become apparent shortly.

\(^{23}\)Market reversals at each period’s end-points are re-visited at the end of this section.
Moreover, to determine whether the LR/CR process influences the loan growth pattern or vice versa, we postulate the following causal relationships before proceeding to their empirical validation. We start by considering two case scenarios for the value of the RW on real-estate loans: the first being that of a relatively high RW, $W_{LNRE}$ while the second is that of a low RW, $w_{LNRE}$. What we are assuming is that in the case of a high RW, as growth in real-estate increases, the overall CR tends towards the LR. This results, ceteris paribus, in a positive correlation (referred to as “Correl”), between the LR/CR correlation pattern (denoted $\rho$) and lending growth. This is the case, for instance, of the 1990-1991 crunch. In contrast, during the 2007-2009 crunch which saw the RW fall from its earlier level, the CR dissociates from the LR resulting in a negative correlation pattern.

$$W_{LNRE} : \frac{\Delta L}{L} \Rightarrow CR \rightarrow LR \Rightarrow Correl > 0$$

$$w_{LNRE} : \frac{\Delta L}{L} \Rightarrow CR \rightarrow LR \Rightarrow Correl < 0$$

Of course, one needs to substantiate that a potential association or dissociation between LR and CR effectively results in a positive or negative correlation pattern (“Correl”). This will be verified empirically via Granger causality tests. We start by running a basic VAR regression between loan growth and $\rho$. The results in Table 3 show the causal effect of each depending on which one is depicted as the dependent/independent variable. The sign of “Correl” will effectively be given by the sign of the slope between the two variables. Also, the loan growth category mostly correlated with the LR/CR ($\rho$) pattern is determined via the Akaiki Criterion. We perform this analysis for each crunch with the following interpretation.

During the first crunch, the causal effect stems consistently from any of loan growth categories towards the LR/CR pattern, $\rho$. This can be seen through the p-values (and $R^2$) which are, with the exception of the LNCIUSD VAR, significantly lower (higher) than those of the reverse causal relation. Hence, we interpret these values as a rejection of the Granger hypothesis of non-causality. Note that, the AIC and $\beta$ coefficient are the highest in magnitude in the case of the LNRE category as expected from our initial postulates. What is more important is in fact the positive sign in the relationship given by $\beta$ which is a clear indicator of the pattern we observe in Figure 3.

The picture is not as clear during the second crunch because for most cases, the causal relation seems to have been reversed. In other words, it is the LR/CR correlation pattern, $\rho$, that is determining loan growth. Yet, what matters to us is that in the preferred AIC case of LNRE, the first crunch relationship is preserved. More importantly, the $\beta$ sign becomes negative which points to the opposite correlation pattern we observe in Figure 4. We deduce from these two cases that the change in RW could well have been at the heart of this transformation in correlation patterns.

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24 We employ the word “relatively” as we consider the actual values for both crunch periods (i.e. 50% and 35%). However, the inferences we make can be better understood intuitively using the limit RW values (i.e. 100% and 0%).

25 Using only 1 lag to limit the time effect of any variable on the next since they appear to vary simultaneously. Moreover, in spite of the small number of observations, the results we obtain are consistent with our reasoning.

26 This comes from the basic econometric relation $\beta = Correl(x, y) \times \frac{\sigma_y}{\sigma_x}$ where the variances $\sigma$ are positive.
Table 3: Causal link between Loan Growth LR/CR Correlation ($\rho$)

### Panel A: Crunch 1

<table>
<thead>
<tr>
<th>Dep</th>
<th>Indep</th>
<th>$\beta$</th>
<th>t-stat</th>
<th>p-val</th>
<th>$R^2$</th>
<th>AIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>LNRE</td>
<td>33.86</td>
<td>12.93</td>
<td>0.000</td>
<td>0.97</td>
<td>-9.56</td>
</tr>
<tr>
<td>LNRE</td>
<td>$\rho$</td>
<td>-0.02</td>
<td>-1.81</td>
<td>0.070</td>
<td>0.47</td>
<td>-9.56</td>
</tr>
<tr>
<td>$\rho$</td>
<td>LNCIUSD</td>
<td>29.49</td>
<td>1.87</td>
<td>0.062</td>
<td>0.42</td>
<td>-6.16</td>
</tr>
<tr>
<td>LNCIUSD</td>
<td>$\rho$</td>
<td>-0.00</td>
<td>-0.13</td>
<td>0.894</td>
<td>0.82</td>
<td>-6.16</td>
</tr>
<tr>
<td>$\rho$</td>
<td>LNCONOT</td>
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<td>19.08</td>
<td>0.000</td>
<td>0.99</td>
<td>-9.20</td>
</tr>
<tr>
<td>LNCONOT</td>
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<td>-2.55</td>
<td>0.011</td>
<td>0.66</td>
<td>-9.20</td>
</tr>
<tr>
<td>$\rho$</td>
<td>LNLSGR</td>
<td>-0.03</td>
<td>-1.65</td>
<td>0.098</td>
<td>0.50</td>
<td>-9.53</td>
</tr>
</tbody>
</table>

### Panel B: Crunch 2

<table>
<thead>
<tr>
<th>Dep</th>
<th>Indep</th>
<th>$\beta$</th>
<th>t-stat</th>
<th>p-val</th>
<th>$R^2$</th>
<th>AIC</th>
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<tbody>
<tr>
<td>$\rho$</td>
<td>LNRE</td>
<td>-12.84</td>
<td>-4.13</td>
<td>0.000</td>
<td>0.74</td>
<td>-10.40</td>
</tr>
<tr>
<td>LNRE</td>
<td>$\rho$</td>
<td>-0.00</td>
<td>-0.13</td>
<td>0.893</td>
<td>0.70</td>
<td>-10.40</td>
</tr>
<tr>
<td>$\rho$</td>
<td>LNCIUSD</td>
<td>3.16</td>
<td>2.12</td>
<td>0.034</td>
<td>0.47</td>
<td>-6.62</td>
</tr>
<tr>
<td>LNCIUSD</td>
<td>$\rho$</td>
<td>-0.30</td>
<td>-3.64</td>
<td>0.000</td>
<td>0.84</td>
<td>-6.62</td>
</tr>
<tr>
<td>$\rho$</td>
<td>LNCONOT</td>
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<td>-1.35</td>
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<td>0.33</td>
<td>-7.05</td>
</tr>
<tr>
<td>LNCONOT</td>
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<td>0.003</td>
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<td>-0.48</td>
<td>0.634</td>
<td>0.20</td>
<td>-7.74</td>
</tr>
<tr>
<td>LNLSGR</td>
<td>$\rho$</td>
<td>-0.05</td>
<td>-0.80</td>
<td>0.422</td>
<td>0.81</td>
<td>-7.74</td>
</tr>
</tbody>
</table>

The data in this table is based on Call Reports for each crunch period. The dependent and independent columns refer to the variables in the VAR model. $\beta$ is the slope coefficient while the Akaiki (AIC) column gives a sense of the overall fit.

Finally, to argue that the effects of the reversals do not invalidate our results, we perform a similar analysis to the previous one under the assumption that these reversals are only an artifact of the choice we make related to the exact start and end dates of each crunch. This time we use a macroeconomic variable, GDP growth. One difference with the previous case is that one can argue that in a financial crisis, macro-effects take longer to appear in the economy than at the micro-bank level. For this reason, the lagged LR/CR pattern is chosen instead of the concurrent one. Again, notwithstanding the observation in 2008Q2, the patterns match to a large extent in the first ($\rho = 0.83$) and second ($\rho = -0.47$) crises. This completes the picture given by Tables 2 and 3.

4. Explaining the Changes using the CR sensitivity to RW

In order to explain the reversals we witnessed in the abovementioned figures, we need to assess if any of the two ratios influences the other. In fact, Furfine (2000) claims that the same magnitude change in either ratios can lead to drastically opposite effects in terms of portfolio behavior. Similarly, Gilbert (2006) states that changing the RWs in the CR would impact the number of banks bound by the LR. More specifically, using the exact scenario that ensued prior to the subprime

---

27Bearing in mind that the LR is insensitive to a change in RW, the subtle implication here is that this is done through reallocating assets in a non-linear fashion to different asset categories.
Figure 5: Relation between LR/CR Correlation Pattern and GDP for different loan categories during the 1990-1991 crunch.

Figure 6: Relation between LR/CR Correlation Pattern and GDP for different loan categories during the 2007-2009 crunch.
crunch (i.e. a reduction in the RW attributed to first-lien residential mortgages\textsuperscript{28}), the author shows that the lowering of RW lead to an increase in the number of banks bound by the LR. This is a clear illustration of how the interaction between the two ratios can lead to a change in the binding constraint. In what follows, we undertake a mathematical approach in order to explain the LR/CR correlation pattern reversals.

4.1. Deriving the relationship between CR and LR

We start with the variable in equation (1) which is commonly used as a proxy for credit risk: RWA/TA. The measure is bound between 0 and 1 in increasing order of risk because Risk-Weighted Assets tend towards Total (Unweighted) Assets as the proportion of risky assets (high RW) increases. Note that this tendency drives the CR toward the LR, which explains how the two can move together. However, what is not noted in most of the recent literature who use this ratio (Van-Roy (2005), Hassan and Hussain (2006), Berger and Bouwman (2010)) is that it boils down to an interaction between CR and LR, irrespective of Tier 1 capital K\textsuperscript{29}. This will be useful in deriving the formula presented next.

\[
\frac{\text{RWA}}{\text{TA}} = \frac{K}{K} \frac{\text{CR}}{\text{LR}} = \frac{\text{LR}}{\text{CR}}
\]

(1)

In the next step, the change in CR is derived with respect to a change in RW, denoted as \(w_i\), affecting a certain asset category \(i\) out of a pool of \(N\) categories\textsuperscript{30}. As can be seen from Equation (3), this change is negatively related to the product of the CR and a second term which is dubbed “asset proportion” (\(AP_i\)). This term refers to the “proportion” (in currency amount) held by the asset whose risk-weight is being changed vis-a-vis the total amount of risk-weighted assets.

\[
\frac{\delta \text{CR}}{\delta w_i} = \frac{\delta}{\delta w_i} \left( \frac{\sum_{i=1}^{N} w_i A_i}{\sum_{i=1}^{N} w_i A_i} \right) = K \times \frac{\delta}{\delta w_i} \left( \frac{1}{\sum_{i=1}^{N} w_i A_i} \right) \\
= -K \times \frac{1}{\sum_{i=1}^{N} w_i A_i} \times \frac{A_i}{(\sum_{i=1}^{N} w_i A_i)^2} = -\frac{K}{\sum_{i=1}^{N} A_i} \times \frac{\sum_{i=1}^{N} A_i}{\sum_{i=1}^{N} w_i A_i} \times \frac{A_i}{\sum_{i=1}^{N} w_i A_i} \\
= -LR \times \frac{1}{\frac{\text{RWA}}{\text{TA}}} \times \frac{A_i}{\text{RWA}} = -LR \times \frac{1}{\frac{\text{CR}}{\text{LR}}} \times AP_i = -CR \times AP_i
\]

(2)

(3)

The formula makes sense intuitively as the product of terms is always positive and hence the change in CR resulting from a change in \(w_i\) is always negative since an increase in RW means more risky assets which implies a negative (positive) shock to the CR numerator (denominator) resulting

\textsuperscript{28}Although his specification changes the original value of 50% to half its value, rather than the one chosen by Basel of 35%.

\textsuperscript{29}Kamada and Nasu (2000) are the closest to reach this result due to the fact that they use Total capital in the definition of the CR versus Tier 1 capital for the LR in deriving a different, albeit related, concept: the asset quality index.

\textsuperscript{30}Prior to Basel II, \(N=4\) for \(i\in\{0, 20, 50, 100\}$. 
in an overall decrease. Hence, in anticipation of this, Basel regulators should have increased the CR after lowering the RW on residential real-estate loans under Basel II to maintain the adequate capital buffers. Instead they kept the same CR at 4% and 8% for Tier 1 and Total capital. This is currently being accounted for indirectly under Basel III with an increase in CRs.

Note that this division into multiple product variables to yield the overall change in CR is in the same spirit as Van-Roy (2005) and Hassan and Hussain (2006). What is more useful for us at this stage is the interaction with the LR in equation (2) which brings out the inverse of the RWA/TA variable. In other words, the change in RW affects the CR in a manner proportional to the overall credit “safety” of the portfolio. This provides the mathematical intuition for why this variable is so commonly used for the purposes outlined above. More importantly, it reveals the co-dependence of the CR on the LR, affected by a negative sign for the case of a change with respect to a single RW category, \( w_i \).

Note that the Basel II framework looked at shifting various RWs by introducing new possible categories. It is easy to show that the relationship between the CR and LR can be extended to all \( N \) categories which yields the following form in equations (4) and (5). The last two terms were adapted from the previous single RW case. Our focus now becomes on what used to be a negative sign, which now changes to a sinusoidal pattern of positive/negative signs depending on the number of RW categories being affected. This captures, along with the factorial term, the interactions between different changes in RWs.

\[
\frac{\delta CR}{\delta w_1 \ldots \delta w_N} = (-1)^N \times N! \times LR \times \frac{1}{\text{RWA/TA}} \times \prod_{i=1}^{N} AP_i
\]

\[
= (-1)^N \times N! \times CR \times \prod_{i=1}^{N} AP_i
\]

One reason why these formulae are meaningful is that if we single out one major change between Basel I and II as being the decrease in RW on residential real-estate mortgages, then the “new” capital ratio effectively becomes sensitive to an additional RW category. This takes \( N \) in the previous equations from four to five which is sufficient to reverse the sign in these equations. In turn, this forces the reversal in correlation patterns as seen in the previous section. Note that the validity of this statement depends on the total number and direction (positive/negative) of all possible changes affecting the RW categories.

### 4.2. Model Verification and Policy Implications

#### 4.2.1. Impact of a change in RW

In this section, we set out to test whether the 3-factor equation in (4) can explain the sensitivity of the CR to a change in RW. The first issue we face is that the change in RW on residential mortgages did not occur on a specific date owing to the fact that the regulation was not formally

\[\text{However, the authors’ derivations are with respect to CR itself, i.e. CR growth rather than with respect to a change in RW.}\]

\[\text{Those were 35\%, 75\%, 150\% and 300\%.}\]

\[\text{This term arises from the successive derivations with respect to the RWs.}\]

\[\text{Note that this implies the need to take into consideration the behavior of the function as N tends to +\infty but this is not an issue for a few number of RW categories as is normally the case.}\]
applied in the U.S. However, as per Cathcart et al. (2012) we prognosticate that the change was accounted for sometime between the setting out of Basel II in 2004Q3 and the crunch that followed in 2007Q3.

In order to substantiate this change, we run a cross-sectional analysis at each quarter between 2004Q3 and the end of the crunch in 2009Q2 and observe how the regression parameters differ for the banks denoted by subscript $j$ with respect to an absolute lowering of 0.15% (0.50-0.35) on the residential loan asset class $i$. To disentangle the effect of each factor in the equation we take logarithms and run the following regression using robust estimators:\(^35\):

$$
\ln \left( \frac{\Delta CR}{\Delta w_i} \right)_j = \alpha + \beta_1 \ln(LR)_j + \beta_2 \ln(InvCrR)_j + \beta_3 \ln(AP)_j + \epsilon_j
$$

The results are shown in Figure 7 below. As can be seen, the model’s $R^2$ improves markedly over the pre-crunch period, confirming that banks were taking into account the change in RW as part of their CR calculations. The designated beginning of the crunch period saw the end of this adaptation as banks’ capital cushions slowly began to be eroded by the (exogenous) losses that ensued. On the other hand, the factor mostly responsible for that increased sensitivity appears to be the (trend in) LR\(^36\). Moreover, the Shapiro-Wilk test shows that the null hypothesis for normality can be rejected for the LR variable but not for the other two factors. We obtain similar results even after changing from loans on a consolidated basis to domestic office loans.

\(^35\)The negative sign is absorbed by the intercept.

\(^36\)In fact, our Wald tests show significant values for the $\alpha$ intercept. This is most probably due to the fact that the LR and CR do not move in total freedom due to the constraint imposed by the regulators’ minimum thresholds.
4.2.2. Linking the CR to the LR

One criticism of Basel I was that the RW on residential mortgages was not suitable for the risk inherent in that category (Hancock et al. (2005)). Another important implication which stems from equation (5) regarding Basel III is that instead of selecting a heuristic threshold for the CR such as 4%, regulators can start by deriving the new CR that should hold after applying any changes in RW(s). In fact, Equation (5) is simply a homogeneous PDE which can be resolved in closed form. The derivations are stated in the Appendix.

In the case of a single RW change, the relationship boils down to:

\[ CR = LR \times e^{\sum N_i [AP_i(1-w_i)]} \]  

(7)

Since the exponential term is always positive, the CR should always be greater than the LR. This means that for LR = 3% (backstop measure), the old Tier 1 CR (= 4%) is reasonable but for the less conservative 5% it is not. Hence, this could induce wrongful behavior on the part of banks. However, with the CR climbing to 6% in the new regulation, the formula maintains the relationship between CR and LR.

The last step is to verify how much this formula holds in the periods after the Basel regulations were introduced. We use data for the post-Basel I (1990Q1-1992Q3) and the post-Basel II (2004Q3-2009Q2) eras, and test the following regression using robust estimators:

\[ \ln \left( \frac{CR}{LR} \right)_{jt} = \alpha + \beta \sum N_i [AP_i(1-w_i)]_{jt} + \epsilon_{jt} \]  

(8)

To account for the presence of outliers, we run the regression for the panel of data using different sample composition levels as shown by the different percentiles in Table 4. Our results remain valid across the two sample periods with all estimates significant at the 1% level. Indeed, the \( R^2 \) rapidly converges to almost 90% in both periods which stipulates that the relationship does hold for the majority of the bank sample. What’s more is that the intercept converges to 0 (1 in anti-logarithmic terms\(^38\)) despite the fact that the Wald tests reject the null hypothesis for \( \alpha = 0 \) at all significance levels. This means that banks operate on a lower threshold basis of approximately CR=LR; after that, they increment their respective RBC positions by some portion of their weighted asset proportions captured by the exponential term in equation 7.

Table 4: Testing CR formula applicability

<table>
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<tr>
<th>Period</th>
<th>Crunch 1</th>
<th>Crunch 2</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>0%</td>
<td>0.1%</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.459</td>
<td>0.916</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.109</td>
<td>0.467</td>
</tr>
<tr>
<td>CONST</td>
<td>0.446</td>
<td>0.157</td>
</tr>
</tbody>
</table>

The data in this table is based on Call Reports for each crunch period after running the regression model for equation 8. The Sample row denotes the percentage remaining from the original sample after removal of outliers.

\(^{37}\)Note that data is not available in Call reports prior to 1990.

\(^{38}\)This can be seen by taking the Taylor expansion on CONST from 90th percentile (10% column) of the sample, i.e.: \( \exp(0.021) \approx 1+0.021 = 1.021 \)
Using the 99.9 percentile (0.1% column) of banks, we run a Chow test to verify that the coefficients are stable across sample periods. The null hypothesis for stability is rejected at the 1% significance for the two mentioned periods despite the coefficients being equal up to two decimal places. Nonetheless, at the 0.1% significance, this rejection cannot be made for the post Basel II period. Assuming as before that the change in RW took effect some time before the crunch, this means that despite the fact that the second period accounts for both a pre-crunch and crunch period, the sensitivity captured by $\beta$ in equation 8 remains the same at a high significance level\(^{39}\). In other words, banks change their asset proportions in such a way that the formula above remains valid.

While these results show that the Basel requirements fall along the lines predicted by our formulae, in the future, there is room to improve on the choice of threshold levels by making them more stringent using a dataset of representative banks to calibrate a generalized model for banks. Alternatively, this could create the possibility for having endogenous bank-specific requirements rather than a one-size fits-all guideline; a point called for by some critics since the birth of the Basel “one-size-fits-all” concept.

**Conclusion**

In this paper, we showcase the drastic impact a small change in RW can have on the behavior of banks towards adjusting their CRs and LRs. Comparing the 1990-1991 to the 2007-2009 credit crunch, it seems the standings were reversed in terms of which requirement was more to blame for triggering the recession: the CR or the LR. From a mathematical standpoint, we establish that the latter cannot be changed without affecting the sensitivity of the former to a change in one or more of its RW categories. This finding is essential in substituting for the use of heuristics with regard to threshold selection in the case of both ratios.

In so doing, we first define which of the two ratios was the binding constraint on banks prior to each crunch period. Our analysis of the subprime crunch complements work done by Avery and Berger (1991) for the first crunch and shows the incidence of the crises on bank capital cushions (and vice versa). It also enables us to gauge the efforts by the Basel III workforce in quantifying the increments to the CR and selecting an adequate LR.

On the other hand, we illustrated the reversal in correlation patterns between the two ratios which we deem is at the heart of the change in binding constraint. The patterns are seemingly related to loan growth (microeconomic) and GDP (macroeconomic with appropriate lag) market signals. In fact, this reversal has its roots set in a mathematical relation emerging from the sensitivity of the CR to a change in RW(s). Singling out the change with regard to the residential mortgage asset class which happened before the onset of the second crunch can help explain the change in behavioral patterns.

What’s more is that our formula relates the movements in CR to three crucial elements: the LR, the inverse of the credit risk ratio (which prior to this paper was used solely based on intuitive reasoning), and a new factor conveniently dubbed “asset proportion”. An extension of the formula gives way to a first-order homogeneous PDE governing the behavior of the CR. We solve for single and multiple changes in RWs which fit into a closed form solution. This can allow for setting adequate CRs which reflect changes in RWs while bearing in mind the impact of its counterpart

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\(^{39}\)Calculated $\chi^2 (7.97)$ is less than the critical value (10.83)
capital measure, the LR. More importantly, this can be done in almost as simple a way as using the Basel pre-established ratios.

In sum, during the course of this research, we have identified elements which are promising in terms of the current efforts towards the phase-in of Basel III. We also address an area which can be improved on regarding the target choice of CR which needs to constantly factor in newly introduced RW buckets to improve the granularity of the RW scheme. We hope these findings have helped clarify one Basel paradox with regard to the diverse impact of capital positions on credit crunches in an effort to limit any further regulatory “em-Basel-ment”.

**Appendix A. Solution to the CR equation**

Assuming the CR is a function defined on $]0, 1[^N$ with $N$ possible RWs ($w_i$), the solution to the PDE in equation (5) is solved in the exponential form $Ae^{\sum_{i=1}^{N} c_i w_i}$ where $c_i$ are arbitrary constants to be found. Let $g(w_1, ..., w_N)$ be another function defined on the same support as CR and representing the product term in the equation ($\prod_{i=1}^{N} AP_i$). Substituting into (5) we get:

$$\prod_{i=1}^{N} c_i = (-1)^N \times N! \times g(w_1...w_N) \quad (A.1)$$

As state earlier, the only boundary condition we have is regarding the sensible approximation that Tier 1 CR$(1,...,1) = Ae^{\sum_{i=1}^{N} c_i} = LR$. Denoting by n the subset of N asset categories with respect to which we are calculating the sensitivity of the CR, this yields a system of two equations with $n+1$ unknowns. we solve for the cases of $n=1$, $n=2$ and $n=N$.

**Appendix A.1. Solution with $n=1$**

The system of equations for the case of a single RW change becomes:

$$\begin{cases} c_i = -g(w_i) \\ LR = Ae^{\sum_{i=1}^{N} c_i} \end{cases} \quad (A.2)$$

By substitution:

$$LR = Ae^{\sum_{i=1}^{N} c_i} \rightarrow A = LR \times e^{-\sum_{i=1}^{N} c_i}$$

$$CR = LR \times e^{-\sum_{i=1}^{N} c_i} \times e^{\sum_{i=1}^{N} c_i w_i} = LR \times e^{-\sum_{k \neq i}^{N} c_k + AP_i} \times e^{\sum_{k \neq i}^{N} c_k w_k - AP_i w_i}$$

$$= LR \times e^{-\sum_{k \neq i}^{N} [c_k (1-w_k)] + AP_i (1-w_i)} \quad (A.5)$$

By symmetry, the same form applies for a change in asset j which gives:

$$CR = LR \times e^{-\sum_{k \neq j}^{N} [c_k (1-w_k)] + AP_j (1-w_j)}$$

$$= LR \times e^{-\sum_{k \neq i}^{N} [c_k (1-w_k)] + AP_i (1-w_i) + \sum_{k \neq j}^{N} [c_k (1-w_k)] - AP_j (1-w_j)} \quad (A.7)$$

By the ratio of the two changes in assets we get the following identity:

$$1 = e^{-\sum_{k \neq i}^{N} [c_k (1-w_k)] + AP_i (1-w_i) + \sum_{k \neq j}^{N} [c_k (1-w_k)] - AP_j (1-w_j)}$$

$$= LR \times e^{-\sum_{k \neq i}^{N} [c_k (1-w_k)] + AP_i (1-w_i) + \sum_{k \neq j}^{N} [c_k (1-w_k)] - AP_j (1-w_j)}$$

(A.8)
Taking logarithms at both ends and applying the principle of linearity we get: \( c_k = -AP_k \) for all asset classes. This gives the final version of the CR equation given below. Note how the riskiest RW class has no bearing on the differential between CR and LR in the same way that the safest RW category has no impact on total RWA.

\[
CR = LR \times e^{\sum_{i=1}^{N}[AP_i(1-w_i)]}
\]  
(A.9)

**Appendix A.2. Solution with \( n=2 \)**

The boundary condition remains the same. Hence, using symmetry to overcome the under-specification in the case of 3 RW categories, the system of equations for the case of any two RW changes becomes:

\[
\begin{align*}
& c_i c_j = 2 \times g(w_i, w_j) = 2 \times AP_i AP_j \\
& c_j c_k = 2 \times g(w_j, w_k) = 2 \times AP_j AP_k \\
& c_k c_i = 2 \times g(w_k, w_i) = 2 \times AP_k AP_i
\end{align*}
\]  
(A.10) (A.11) (A.12)

Combining these equations together we get: \( c_i^2 = 2AP_i^2, c_j^2 = 2AP_j^2, c_k^2 = 2AP_k^2 \). This gives out two possible solutions which epitomizes how a change in RW can have opposite effects on the CR. However the second solution is discarded as the CR is decreasing in \( w_i \) which is counter-intuitive.

\[
CR = LR \times e^{-\sum_{i=1}^{N}[\sqrt{2}AP_i(1-w_i)]}
\]  
(A.13)

\[
CR = LR \times e^{\sum_{i=1}^{N}[\sqrt{2}AP_i(1-w_i)]}
\]  
(A.14)

**Appendix A.3. Solution with \( n=N \)**

Again using symmetry, we obtain the general solution as below.

\[
CR = LR \times e^{-\sum_{i=1}^{N}[\sqrt{N}!AP_i(1-w_i)]}
\]  
(A.15)

In the case where \( n \) is even, for the same reason as in \( n=2 \), we discard the second solution of the form:

\[
CR = LR \times e^{\sum_{i=1}^{N}[\sqrt{N}!AP_i(1-w_i)]}
\]  
(A.16)

**References**


