Single-bank proprietary platforms*

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Abstract

This paper models strategic trading in a market with a single informed dealer willing to trade a risky asset at public bid and ask prices. As is typical in currency markets, uninformed clients have liquidity need incentives either to increase or decrease their holdings of the risky asset providing the dealer's price is acceptable. Given the market power of the dealer and the fact that all the demand and supply are satisfied at the quoted prices, it is unclear whether the prices she sets reveal any information about the asset. Equilibria depend on the proportions and prior beliefs of uninformed clients. Even the possibility of speculative attack by informed clients does not necessarily ensure that the equilibrium price spread contains the true asset value. In fact, in some instances even a strong equilibrium refinement does not ensure that the danger of such a speculative intervention would make the price spread bracket the true value. Conversely, the equilibrium refinement does imply that the speculative attack suffered by the dealer cannot be too large and thus that the dealers' prices are at least minimally informative, providing there exist at least some informed clients. Finally, the no-trade outcome is eliminated: Although wealth is transferred from uninformed to informed clients, only equilibria in which at least half of the uninformed traders' wealth is retained by the dealer are credible.

Keywords: FX (foreign exchange) customer-dealer and retail trading; single-dealer market; strategic trading; asymmetric information; speculative attack.

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1 Introduction

This article proposes a theory of strategic trading in a two-sided single-dealer market. As in Akerlof (1970), a risky asset is individually traded for a riskless one between an informed dealer and (informed and/or uninformed) clients. The informed agents know the future liquidation value of the risky asset. Under market two-sidedness, the dealer stands ready to trade with any client, buying and selling at her public bid and ask quote respectively. The model applies directly to a setting in which (1) this dealer is always able to satisfy clients' demand/supply; (2) clients trade only with the dealer; and (3) each uninformed client has a '*liquidity*[1]' incentive to increase or decrease his holdings of the risky asset by a fixed amount providing the dealer's price is acceptable – this determines whether the uninformed client will buy, sell, or not trade.[2]

The most obvious example of such a market is a foreign exchange (FX) single-bank proprietary platform, also called FX *single-dealer platform (SDP)*.[3] Even relatively large banks not directly acting in any of the FX inter-dealer trading systems maintain control of local markets, in effect reselling access to the top-tier inter-dealer platforms provided by big multinational/global banks. In practice, global banks provide liquidity (acting as dealers) to their own customers – that is, 'other financial institutions' (e.g. 'local' banks, insurance companies, central banks, mutual, money market, pension, hedge and currency funds, etc.) – via SDP in the so-called 'customer-dealer' markets. Local banks – which are not able to guarantee market liquidity on top-tier SDPs – in turn act as monopoly dealers for their own local clients in retail FX markets.

FX trading volumes have increased rapidly; between 2004 and 2007 they increased by 72% and between 2007 and 2010 by a further 20%, reaching a daily average turnover of $4 trillion in April 2010 – more than ten times that of the NYSE. Much of this growth is due to the increased participation of 'other financial institutions' and retail investors, who mainly use SDPs and provided 85% of the entire FX daily average volume increase between 2007 and 2010. Indeed, during this period the largest SDPs have triplicated their volume of trades while inter-dealers trades have waned. This rise is linked to substantial investments in proprietary SDPs by the largest FX dealers (e.g., Barclays, Deutsche Bank, and UBS). By contrast, many smaller banks have become clients of the largest platform operators.[4]

Compared to the competing FX multi-dealer platform, the success of SDPs can be at least partially attributed to the following two characteristics. Different dealers provide this service over standard web browsers to enterprises, financial firms, and other end-users.[5] More important, it allows for *internalization of flows*.[6] However, practitioners are concerned about the accuracy of SDP price spreads – whether or not prices reflect the expectations

[1] This need to increase or decrease the client's holding of the asset arises outside the market itself, and is thus distinct from speculative demand or supply.

[2] In contrast to a *one-sided* single-dealer market, in a *two-sided* market the dealer facilitates an exchange among clients (setting both bid and ask prices). The current model is not quite a two-sided market in the conventional sense (see Rochet and Tirole, 2006) of a network in which trade between two distinct groups of agents is facilitated by a 'platform' intermediary. In the market considered here each group of clients interacts directly with the dealer; the sign of her trade, if any, is decided at equilibrium.

[3] Due to the predominance of banks acting as single dealers, these two terms are used synonymously. To be precise, few spread-betting companies (e.g., CMC Markets and GFT) also act as single dealer when engaging in contracts for difference (CFD) retail trading. However, their clients have no liquidity incentives, as their positions are automatically closed soon after the order is sent.

[4] For details, see King and Rime (2010).

[5] E.g., since 2008, Unicredit has used the Caplin Trader browser-based single-dealer portal for FX trading by Caplin, a leading SDP technology provider also serving global banks such as USB and Royal Bank of Scotland.

[6] E.g., Barclays Capital reports that a substantial proportion of trades on its own BARX platform – on which the 80% of its FX spot business is made – occur with minimal price impact (Source: Euromoney, October 2009).

of informed traders. In this respect, there have been few attempts to develop conceptual models of the strategic interactions arising in single-dealer platforms.[7]

Interestingly, this kind of currency transaction dates back to the 12th-13th century. The *bill of exchange* was the most important financial innovation of the High Middle Ages, and therefore one of the main determinants of the banking system's development (see Hunt and Murray, 1999, pp.64-66). As in present single-dealer markets, in which clients pay dealers for liquidity, this historical financial instrument was set up in a way that gave the issuer a reasonable profit on the deal.[8]

'Uninformed' clients do have private information, but not about the real asset value. Each knows the direction and magnitude of her own immediate liquidity need that, if satisfied, gives the client an intrinsic benefit – in this way the no-trade theorem problem (Milgrom and Stokey, 1982) is avoided. However, in equilibrium this information attracts no rents and does not affect the outcome. In particular, uninformed clients are assumed to be strategic and to gather information from quoted prices. As a result, even traders with a liquidity need for one currency might buy a different currency – or even sell their existing stocks if the currency in question appears sufficiently over-valued. Since quoted prices are assumed to be anonymous (available to all clients on a given SDP) and constant (independent of the scale of the transaction), what matters to the dealer is the aggregate volume of bids and asks, rather than whether individual clients need to buy or sell. Of course, if aggregate demand equals aggregate supply, the dealer satisfies Condition 1 without recourse to outside markets or her own portfolio (except temporarily as a matter of timing). Even when the net trade differs from zero (either because liquidity needs are not matched within the SDP or due to the presence of informed demand) there is no way for the dealer or the client to profit by knowing the sign and extent of each client's liquidity need.

The dealer can set any prices. Even in equilibrium, the expected (true, by our assumptions) asset value may lie outside the price spread provided all clients are uninformed. To reduce this indeterminacy – and to reflect natural information heterogeneity among clients – a group of informed speculators is introduced.[9] Even the *possibility* of their existence might be expected to affect the equilibrium outcome on the ground that whenever these informed clients' expect the asset value to lie outside the quoted price spread, they will speculate at the expense of the dealer.[10] Surprisingly, when informed speculative clients exist, it can well be that the asset value lies outside the equilibrium bid-ask spread. This result also survives a strong equilibrium refinement.

The model may provide a rationale for the emergence and persistence of asset price bubbles in terms of the uninformed clients' initial beliefs and the market structure. In particular, when all clients are uninformed liquidity traders, the monopolistic nature of the market and the (assumed) balanced liquidity needs for the asset allow the dealer to extract the maximum surplus even without taking into account her private information. Prices that do not convey information may thus be completely uncorrelated with underlying fundamentals. However, uninformed clients' beliefs leading to equilibrium price spreads that

[7] The Glosten and Milgrom (1985) and Kyle (1985) models do not take the market power of single-dealers into account, but instead assume competitive prices that give the market maker (in expectation) zero profits.

[8] Medieval banking owed its origins to these money-changers rather than money-lenders. This financial innovation was the first not to be regarded as usury, and hence to be tolerated by the Church.

[9] The model is thus not a standard signaling model, in that the payoff-relevant actions of informed agents are not all common knowledge.

[10] Since the dealer does not know whether a given client is informed or not, and is in any case unable to price discriminate, even a modest probability that some of the clients are informed may invalidate some price spreads. This is obvious when the dealer makes no net trade in equilibrium; profits depend only on the gross volume of trade and the bid-ask spread, so a price spread that includes the true value – and is thus immune to speculation – weakly dominates one that does not.

do not contain the true asset value can trigger speculative attacks by informed clients, as long as they exist. This reduces dealer's revenues. Indeed, the dealer's monopoly rents can be reduced or even eliminated by the combined effect of uninformed clients' initial beliefs and the presence of informed speculators.

To reduce the set of equilibria, and especially to eliminate those that militate against using this kind of market (and are thus counterfactual), we invoke the Intuitive Criterion (Cho and Kreps, 1987). The equilibria eliminated in this way include those that involve excessive speculation and price inefficiency of concern to dealers and regulators, respectively. On one side, this refinement discards equilibria vulnerable to intense speculative attack, strengthening the argument that the existence of informed clients increases informational efficiency without massive losses. On the other, the survival of uninformative equilibria tells us that in many instances there is no reason why the price spread will bracket the real value.

The paper is organized as follows. The next section initially considers the case of a dealer (or market maker, henceforth) providing liquidity to uninformed clients (or traders, henceforth). In a second sub-section potential attacks by informed speculative traders entering the single-dealer market are considered. Then a refinement of the equilibrium is presented. Finally, the case of naïve uninformed clients is proposed. In the last section we conclude.

2 The model

In the beginning, the informed dealer knows the true value of the currency, but the uninformed traders have only a common distribution. For this model, we take that distribution to be exogenous and common knowledge.

Each uninformed trader's liquidity need is private information to this trader, but the dealer knows the volume of trade by clients with a need to buy or sell. For simplicity, we assume that the total volume of trade due to liquidity preference nets out to zero, so the dealer can clear the liquidity market without using his own resources. Treating liquidity need incentives to buy or sell symmetrically is fine in FX markets at the steady state, when considering the exchange of currencies with the same connectivity in terms of trade.

The dealer sets a buy and a sell price, and accepts all orders at those prices. There are fixed transaction limits, so (as a result of the assumed preferences) each uninformed trader will buy or sell the maximum (limit) amount or will neither buy nor sell. Therefore, when traders are subject to the same position limit, the "balanced trade" assumption on liquidity trades translates to equal numbers of liquidity buyers and seller.

When the clients expect the true value to lie between two buy and sell prices, as long as the price spread is not too large, they all satisfy their liquidity needs. Conversely, when the price spread is too large, or when they expect the value not to lie between the observable buy and sell prices, they might have a speculative motive which could cause them to depart from their liquidity-determined buy/sell plans. The dealer will take this into account, which might affect the equilibrium behavior.

For instance, suppose that they expect the true value to lie just above the bid price, but definitely below the ask price. Even clients with a moderate need to sell satisfy their liquidity needs; however, clients with a need to buy do not trade, unless the incentive to buy is so strong to overwhelm the expected monetary loss. When the expected true value lies below the bid price, the speculative opportunity strengthens the incentive to sell by a client with sell plans, at the expenses of the dealer. Clients with a need to buy might change their plan: they start selling as well, unless their need to buy is stronger than their desire to speculate.

At the equilibrium, when uninformed traders satisfy their liquidity needs, the dealer obtains a profit equal to the difference in prices multiplied by one-half the volume of trade (equivalently, the volume of purchases or the volume of sales by uninformed traders). For example, suppose the true value is at the lower extreme of the distribution, but the expected value lies in the center. The dealer has an incentive to set prices around the expected value, in such a way that clients satisfy their liquidity needs. In fact, her loss from buying the currency at a high bid price is more than compensated by the gain from selling the same currency at an even higher ask price.

Timing and market participants The market maker, M, fixes prices and commits on providing liquidity at these prices. Then uninformed (liquidity) traders and informed (speculative) traders, unaware of other clients's decisions, decide whether to trade at these prices (buying or selling the risky asset) or not. Each trader does not learn whether other traders are undertaking exchanges with M. The *ex-post* liquidation value (or true value) of the asset, denoted $\widetilde{v} \in \mathcal{V}$, is a random variable (specifically, the results in the paper always refer to non-degenerate random variables; the case of a degenerate one is straightforward). All the agents know $f(\widetilde{v})$. In particular, the market maker and the group of informed speculative traders also know $\widetilde{v}{=}v$.

Space of actions and payoffs All the agents in the market are rational strategic utility-maximizers.
The n-th trader knows to be of type $\widetilde{s}_n{=}\{-\varepsilon, \varepsilon, \eta\}$ when she decides to trade $x_n \in [-c_n, c_n] \subset \mathbb{Z}$, positive if she buys, negative if she sells, zero otherwise.[11] Type $\widetilde{s}_n{=}\varepsilon$ and $\widetilde{s}_n{=}{-\varepsilon}$ are uninformed types that have a liquidity benefit from buying and selling respectively; type $\widetilde{s}{=}\eta$ is an informed speculator. Everybody knows that the proportion of uninformed agents' aggregate trading capacity in the market is $\alpha \in (0,1]$. This proportion is equally split among traders of type $\widetilde{s}_n{=}\varepsilon$ and $\widetilde{s}_n{=}{-\varepsilon}$. To simplify the notation, there is no loss in generality in setting $c_n = 1$, and refer to α as the probability that a trader is uninformed.
The market maker fixes $\underline{p}$ and $\overline{p}$, namely the price at which she buys or sells respectively. In particular, $\underline{p} \leq \overline{p}$.[12] When clearing the market, M is not subject to any capital constraint (alternative, we can say that M's trading capacity is sufficiently large).
Define the vector of functions P by $P{=}\langle\, \underline{P}{,}\overline{P}\,\rangle$, where P is M's pricing strategy. The function X_n is the n-th trader trading strategy. In particular: $\underline{P} : \mathcal{V} \to (-\infty, \overline{p}]$; $\overline{P} : \mathcal{V} \to [\underline{p}, \infty)$; $X_n : \{-\varepsilon, \varepsilon, \eta\} \times (-\infty, \overline{p}] \times [\underline{p}, \infty) \to [-c_n, c_n]$; $\underline{p}{=}\underline{P}(\widetilde{v}{=}v)$; $\overline{p}{=}\overline{P}(\widetilde{v}{=}v)$; $x_n{=}X_n(\widetilde{s}_n{=}\,\cdot\,, \underline{p}, \overline{p})$.
The n-th trader's expected utility, U_n, is equal to:

$$U_n{=}\begin{cases} x_n \,\{\, E[\widetilde{v}\mid \underline{p}, \overline{p}]{-}\overline{p}{+}\zeta\kappa\varepsilon \,\} & \text{if } x_n{>}0, \\ 0 & \text{if } x_n{=}0, \\ -x_n \,\{\, \underline{p}{-}E[\widetilde{v}\mid \underline{p}, \overline{p}]{-}\zeta(1-\kappa)\varepsilon \,\} & \text{if } x_n{<}0, \end{cases}$$

where $U_n(\widetilde{s}_n{=}\varepsilon){=}U_n(\kappa{=}1, \zeta{=}1)$, $U_n(\widetilde{s}_n{=}{-\varepsilon}){=}U_n(\kappa{=}0, \zeta{=}1)$, $U_n(\widetilde{s}_n{=}\eta){=}U_n(\zeta{=}0, \widetilde{v}{=}v)$, and $\varepsilon \in \mathbb{R}_{++}$ is finite.
The profits that a market maker of types $\widetilde{v}{=}v$ expects to extract from the n-th client present in the market are equal to:

$$\pi{=}\sum_{\widetilde{s}_n} \Pr(\widetilde{s}_n{=}\,\cdot\,)[\rho(\widetilde{s}_n{=}\,\cdot\,)],$$

[11] $c_n{>}0$ is finite and common knowledge, but does not necessary have to be equal among traders.
[12] If $\overline{p}{<}\underline{p}$, the n-th trader maximizes her profits for instance by buying at $\overline{p}$ and selling at $\underline{p}$, repeating this procedure an infinite number of times. M facing an infinite loss deviates.

where

$$\rho(\widetilde{s}_n{=}\ \cdot\ ){=}\begin{cases} (\overline{p}{-}v) & \text{if } X_n(\widetilde{s}_n{=}\ \cdot\ ,\ \cdot\ ){>}0, \\ 0 & \text{if } X_n(\widetilde{s}_n{=}\ \cdot\ ,\ \cdot\ ){=}0, \\ -(v{-}\underline{p}) & \text{if } X_n(\widetilde{s}_n{=}\ \cdot\ ,\ \cdot\ ){<}0. \end{cases}$$

To emphasize the dependence of π on P, X_n, and $\widetilde{v}{=}v$, we generally write $\pi(P, X_n, v)$.

Equilibrium definition The equilibrium concept we use is the Perfect Bayesian Equilibrium (PBE). Here, it is defined as follows:

(i) a pricing strategy by the market maker that maximizes her final payoff, given the traders' trading strategy and the information she has;

(ii) a trading strategy for each client that maximizes her expected final payoff, given the strategy of M and the price pair;

(iii) uninformed traders use Bayes' rule in order to update their beliefs from the prices they observe in the financial market;

(iv) each player's belief about the other players' strategies is correct in equilibrium.

Due to the use of PBE – rather than Sequential Equilibrium (Kreps and Wilson, 1982) –, the beliefs are the same for all the uninformed traders.

From now on, for brevity sake, when we say that the n-th trader satisfies her (liquidity) needs, we mean the following. If she is of type $\widetilde{s}_n{=}\varepsilon$, then she trades $x_n{=}c_n$; if $\widetilde{s}_n{=}{-}\varepsilon$, she trades $x_n{=}{-}c_n$.

Lemma 1 *(Case of $(1-\alpha) = 0$: Identification of traders' best response) Consider a market without informed traders. At the equilibrium, uninformed traders always satisfy their liquidity needs.*

Proof of Lemma 1. See Appendix. ■

Proposition 1 *(Case of $(1-\alpha) = 0$) Consider a market with no informed traders. A pooling and a separating equilibrium in which any M of type $\widetilde{v} = v$ sets $\underline{p} = E[\widetilde{v}] - \varepsilon$, $\overline{p} = E[\widetilde{v}] + \varepsilon$ and $\underline{p} = v - \varepsilon, \overline{p} = v + \varepsilon$ respectively always exist. Partial- and semi-pooling equilibria are also a possibility. M always extracts a surplus equal to ε per quantity traded.*

Proof of Proposition 1. See Appendix. ■

Uninformed traders primarily have liquidity needs. If they managed to gather information from prices, they would exploit it, trading strategically. However, at the equilibrium the dealer's prices are set in such a way that they are not informative. In fact a dealer revealing information would reduce her profits. The uninformative price spread has to be non-excessively width, and appropriately positioned in the light of the uninformed traders' initial beliefs. In this way uninformed clients are willing to satisfy their liquidity needs at the highest cost. The reasoning applied is the sequential equilibrium one. The dealer's strategy, function of the private information, is common knowledge to clients. In fact they have an initial belief about which type of dealer sets a particular pair of prices or another. Clients' strategies are function of the signal observed, namely the two prices. Therefore, agents' strategies depend on uninformed clients' initial beliefs, which are subject to a Bayesian revision.

Interestingly, even if the n-th trader cannot gather any meaningful information about $\widetilde{v}{=}v$, the separating equilibrium coincides with the equilibrium outcome in a market where all the agents possess information. The pooling equilibrium conversely coincides with the equilibrium in a market in which all participants are uninformed.

2.1 Providing liquidity under informed speculative attacks

Now we consider a situation in which informed speculators exist.[13] These clients are as informed as the dealer. Depending on the uninformed clients' expectations, the dealer might be in a situation in which she cannot extract the full surplus from traders with liquidity needs, and simultaneously avoid a speculative attack.

The uninformed traders are aware about the existence of these speculative clients, and interpret prices in the light of this extra element. Under some circumstances, the dealer is vulnerable to speculative attacks. The extreme cases are the ones in which the speculative clients' trading volume is so high (or alternatively, the asset value is so above the ask or below the bid price) that the attack suffered by the dealer would overwhelm the benefit deriving from providing liquidity to uninformed traders. This depends on the uninformed traders' beliefs. In these instances, the dealer prefers to give up profits deriving from providing liquidity, setting a price spread around the real currency value that makes nobody willing to trade (therefore avoiding speculative attacks).

Each equilibrium derived is self-consistent. However, the uninformed traders' beliefs that support some equilibria fail an equilibrium refinement argument. Interestingly we show that, although the dealer can set the prices she likes, even a strong refinement does not eliminate the danger of a speculative attack.

Finally, we consider the case of naïve uninformed traders, that is traders that do not try to extract information from prices.

Lemma 2 *(Case of $(1-\alpha) \neq 0$: Identification of type $\widetilde{s}_n \neq \eta$ best response) Consider a market with informed speculative traders and uninformed liquidity ones. The liquidity traders' best response to a specific pair $\underline{p}, \overline{p}$ can be restricted to two different choices: satisfying their own liquidity needs; alternatively (only when informed speculators do not trade), not trading.*

Proof of Lemma 2. See Appendix. ■

Proposition 2 *(Case of $(1-\alpha) \neq 0$: Equilibrium) Consider a market with informed speculative traders and uninformed liquidity ones.*

There always exists a separating equilibrium with informed speculators not trading, uninformed traders satisfying their liquidity needs, and M extracting $\alpha\varepsilon$ per unit traded.

Depending on $f(\widetilde{v})$, two classes of pooling equilibria might also exist. In one uninformed traders always satisfy their liquidity needs, in the other they never do so. A variety of partial- and semi-pooling equilibria are also possible. Due to the presence of informed speculators, at some of these equilibria M might earn a non-negative amount less than $\alpha\varepsilon$ per unit traded.

A characterization is presented in the proof below.[14]

[13] In principle, if M were able to distinguish informed from uninformed clients, she could discriminate the former, offering them better price conditions aside the market. However, they would only accept to trade at better prices. This would imply a larger loss for M, and no advantage in terms of higher surplus extraction from liquidity traders. Thus, for M this option is not interesting.

[14] Adopting the notion of Sequential Equilibrium (rather than PBE) is equivalent to studying all the different uninformed clients' empirical distributions of liquidity needs. Providing this is symmetric around zero – namely, whenever liquidity needs are balanced – the equilibrium outcomes are in line with those proposed herein, with three differences. First notice that large spreads do not allow M to extract any surplus

Proof of Proposition 2. See Appendix. ■

While with α=1 a pooling equilibrium with M setting the same pair of prices and uninformed traders satisfying their liquidity needs always exists, when $\alpha \neq 1$ this kind of equilibrium needs $\mathcal{V}$ to be bounded on both sides. Consider any specific $f(\widetilde{v})$ satisfying this requirement, and the pooling equilibrium arising for a sufficiently high $\alpha \in (0,1)$ and a sufficiently large ε. By progressively reducing either α or ε, we always end up in a situation in which this pooling equilibrium is not sustainable any more. This happens whenever the loss suffered by M because of speculative traders' intervention is not overwhelmed by the liquidity traders' surplus extraction. More in general, holding $f(\widetilde{v})$ fixed, the highest number of types of M earning positive profits by setting the same pair of prices is non-decreasing in α and ε.

2.2 Equilibrium refinement

To refine this plethora of equilibria, the Intuitive Criterion (Cho and Kreps, 1987) can be invoked. The current model belongs to a class of dynamic games of incomplete information in which actions are publicly observable and players all share the same initial beliefs about any other player's type, which is what allows the PBE concept to be involved. Since (common) beliefs about an informed player's type only change in response to (commonly observed) actions taken by that informed player, in PBE an uninformed player's beliefs are identical to those of any other uninformed player not only on but also off the equilibrium path. Specifically, in our model the uninformed traders will all revise their beliefs about v in the same way following any observed pair of prices; this revision will follow Bayes' rule if the price pair occurs with positive probability in equilibrium, and can be arbitrary otherwise – but is in any case the same for all uninformed traders. The strategy of each type of dealer takes these off-path beliefs into account when evaluating deviations from a particular equilibrium. Thus a (pooling, separating or other) strategy can be sustained in equilibrium if and only if we can find beliefs formed in response to a deviation – and best replies by uninformed traders holding those beliefs – that make the deviation unprofitable. Since these beliefs are arbitrary, it is natural to ask whether the 'threats' that sustain the equilibrium can be justified by 'reasonable' beliefs formed by uninformed traders after observing an unexpected price pair.

The Intuitive Criterion constrains beliefs in the following way. By definition of equilibrium, an unexpected price pair does not generate higher profits for any type of dealer, taking into account the uninformed traders' threatened response. But this unexpected price pair *might* be profitable *for some types of M* if the uninformed traders behaved differently: the support of reasonable posterior beliefs following a deviation should therefore only include types of dealer who could benefit offering those prices if the uninformed traders adopted – instead of the demand behavior threatened in equilibrium – a best reply given beliefs concentrated on those types.

To make the Intuitive Criterion more explicit, say that a pair of responses (one for liquidity traders needing to buy, one for those needing to sell) to a given price pair is *undominated* if they could be best replies of *informed* liquidity traders for at least one value of v. An

from clients with small liquidity needs; a reduction in the spread width implies a decrease in the surplus extraction from each uninformed client who trades with M, but can also imply an increase in the number of these clients. Thus, in equilibrium the following holds. (i) The spread can be larger or smaller than 2ε depending on the proportion of clients with a high incentive to buy (or sell) and how strong this need is. (ii) Not all uninformed clients suffer from a complete surplus extraction. Consequently, (iii) when there are informed clients, the size of the speculative attack that M can face is smaller.

initial restriction on beliefs formed in response to a specific out-of-equilibrium price pair is that their support should exclude (that is, they should assign probability zero to) any type of M who would not get more than her equilibrium payoff by setting these prices given that liquidity clients play *any* undominated pair of strategies. The equilibrium fails the IC if there exists a type who would inevitably do better than in equilibrium by setting this specific pair given this restriction on uninformed traders' beliefs and taking into account that the beliefs of both types of uninformed client (though not, of course, their actions) must be the same.

Proposition 3 *Every Perfect Bayesian Equilibrium with at least one type of M earning less than $\frac{\alpha\varepsilon}{2}$ fails the Intuitive Criterion (IC).*

Proof of Proposition 3. See Appendix. ■

The above proposition follows because an out-of-equilibrium price spread that is just slightly less than ε wide gives a dealer exactly in the middle of this spread almost $\frac{\alpha\varepsilon}{2}$; consequently, any PBE with at least one type of dealer earning less than $\frac{\alpha\varepsilon}{2}$ fails the IC – this type would inevitably do better by setting the out-of-equilibrium price spread specified above.

What follows is defined for clarity/brevity sake. Given an out-of-equilibrium price pair whose spread is $\delta \in [\varepsilon, 2\varepsilon)$ wide, notice that all the types of dealer against whom the satisfaction of own liquidity needs represents *an* undominated pair of strategies lie over a segment located between the out-of-equilibrium prices – call this segment $\mathcal{S}_{\delta}^{[]}$. With the exclusion of its lower- and upper-bound, every type that turns out to lie over the remaining part of this segment is a dealer against whom the satisfaction of own liquidity needs represents *the sole* undominated pair of strategies – call this sub-set $\mathcal{S}_{\delta}^{()}$. Specifically, when the price spread is ε wide, $\mathcal{S}_{\delta=\varepsilon}^{()}$ is almost ε wide, and shrinks to a single point centred inside the out-of-equilibrium price spread when the spread is almost 2ε wide.

A second result of Proposition 3 is that a PBE with dealers earning at least $\frac{\alpha\varepsilon}{2}$ fails the IC *only if* (but not *if*) there exists at least one out-of-equilibrium price spread that is δ wide such that the only types that can do better than in equilibrium are those over $\mathcal{S}_{\delta}^{()}$ (in fact, when this is the case, those dealers inevitably earn at least $\frac{\alpha\varepsilon}{2}$). This implicit requisite is made explicit in the next proposition, which lists six alternative conditions for this to happen. Specifically, the existence of an out-of-equilibrium price pair such that at least one of these conditions hold is sufficient to make the PBE fail the IC, provided that this price spread is large enough to make at least one of those dealers earn inevitably more than in equilibrium. It is clear that when α=1 no PBE can ever fail the IC, as no dealer does inevitably better than in equilibrium by setting any out-of-equilibrium pair that is δ wide.

Proposition 4 *Consider a specific PBE with all types of M earning at least $\frac{\alpha\varepsilon}{2}$. Given an out-of-equilibrium price pair $\underline{p}, \overline{p}$ such that $\overline{p}$=$\underline{p}$+δ, where $\delta \in [\varepsilon, 2\varepsilon)$, if there exists at least a M of type v such that $\overline{p}-\varepsilon<v<\underline{p}+\varepsilon$ and $\alpha\frac{\delta}{2}>\pi^*(v)$, then this BPE fails the IC, provided that either Condition I or II or III or IV or V or VI (defined in the Appendix) are also satisfied.*

Derivation of Proposition 4. See Appendix. ■

In different instances the speculative attack by informed clients survives the IC refinement. The IC failure generally depends on three factors: the specific support $\mathcal{V}$, and the uninformed clients' initial beliefs and proportion. In particular, while their beliefs only affect the dealers' profits at the equilibrium, their proportion impacts both on the equilibrium and out-of-equilibrium profits of those types of dealers lying outside the equilibrium and the out-of-equilibrium price spread respectively.

A first remark related to Proposition 4 is the following. Consider a combination of these three factors and a specific out-of-equilibrium price spread δ wide such that uninformed client's beliefs assign positive probability only to types belonging to the set $\mathcal{S}_{\delta}^{()}$; suppose however that none of these types earns inevitably more than in equilibrium. Ceteris paribus, their minimum out-of-equilibrium profits do not increase by widening the price spread to $\delta' \in (\delta, 2\varepsilon)$ via a shift of the out-of-equilibrium ask or bid price, unless this wider out-of-equilibrium spread still cause beliefs to assign positive probability only to dealers belonging to $\mathcal{S}_{\delta'}^{()}$. Since this set linearly shrinks as the out-of-equilibrium price spread widens, the problem is twofold. By shrinking, this set might cause some types that originally lied inside it to fall outside; at the same time, other types that originally lied outside the out-of-equilibrium pair might fall inside the wider spread. Specifically, when an out-of-equilibrium price spread causes a generic type v to fall between the out-of-equilibrium bid price included and the lower-bound of $\mathcal{S}_{\delta'}^{[]}$ excluded (or between the upper-bound of $\mathcal{S}_{\delta'}^{[]}$ excluded and the out-of-equilibrium ask price included) without bracketing the upper-bound (resp., lower-bound) of $\mathcal{V}$, no IC failure via the selection of this specific out-of-equilibrium pair δ' wide can be identified. In fact, out of equilibrium this generic type v earns more than $\alpha\varepsilon$, that is more than what she can even make in equilibrium (see *Scenario A* and *B* respectively). This intermediate finding leads to the following general result. As long as the support $\mathcal{V}$ is unbounded on both sides and continuous (so that $\mathcal{V} \equiv \Re$), any PBE with all types of M earning at least $\frac{\alpha\varepsilon}{2}$ never fails the IC. In fact, given any out-of-equilibrium price spread at least ε but less than 2ε, continuity guarantees that the generic type v falling where specified above always exists; an unbounded support guarantees that the out-of-equilibrium bid (or ask) price never lies below (resp., above) the lower-bound (resp., upper-bound) of $\mathcal{V}$.

Second, consider a situation in which there is at least one type of dealer above and one below an out-of-equilibrium price spread at least ε but less than 2ε wide. When this is the case, the liquidity clients' undominated pairs given beliefs concentrated on these types are respectively the following: both liquidity clients simultaneously buy, and both sell. Specifically, even in the extreme case in which the out-of-equilibrium price spread is only ε wide, the dealer lying above (or below) this spread always earns more than in equilibrium when both liquidity clients sell (resp., buy), provided that $\alpha > \frac{1}{2}$ (see *Condition I*, requirements *R.a* and *R.b*). This second intermediate finding leads to another general result. As long as the support $\mathcal{V}$ is unbounded on both sides and $\alpha > \frac{1}{2}$, any PBE with all types of M earning at least $\frac{\alpha\varepsilon}{2}$ never fails the IC. In fact, an unbounded support always guarantees the existence of types of dealer both above and below any out-of-equilibrium price spread.

An important consequence of the two intermediate findings presented above is the following. Whenever $\widetilde{v}$ is a continuous random variable, or alternatively, whenever $\alpha > \frac{1}{2}$, the investigation on whether a PBE with dealers earning at least $\frac{\alpha\varepsilon}{2}$ fails the IC can be restricted to cases in which an out-of-equilibrium price spreads that is δ wide brackets either the lower- or the upper-bound of $\mathcal{V}$ (on one side checking for *Condition III* and *IV* or *V* and *VI* respectively, on the other for *Condition II* depending on the case).

A third remark concerns the process that uninformed clients adopt to put reasonable restriction on beliefs given an out-of-equilibirum price spread. Where every single type of dealer is located – with respect to a specific out-of-equilibrium price pair – can be as important as assessing whether this type earns more than in equilibrium by setting this pair. For instance, consider a PBE with types of dealer lying over a support that is sufficiently large and bounded at least from below. Starting from an out-of-equilibrium price pair that is δ wide and causes $\underline{v} = \min \mathcal{V}$ to lie exactly on the lower-bound of $\mathcal{S}_{\delta}^{[]}$, by widening this spread to δ' via a negligible decrease of the out-of-equilibrium bid price causes $\underline{v}$ to lie just above the lower-bound of $\mathcal{S}^{()}_{\delta'}$. This negligible shift implies a decrease in the number of undominated pairs of strategies, with clear consequences on the requirements for an IC failure to occur (see *Condition III* and *IV*).[15]

2.3 The case of naïve liquidity traders

For completeness, the case of naïve liquidity traders is finally considered. Naïve liquidity traders ignore the informational content of prices (they decide whether to trade or not considering the unconditional expected value of the asset). When no informed speculative trader is present, the unique outcome is the following. Any type of M sets $\underline{p}=E[\widetilde{v}]-\varepsilon$,$\overline{p}=E[\widetilde{v}]+\varepsilon$, satisfying the naïve traders' liquidity needs. When informed traders exist, depending on M's type an (asymmetric) increase in the price spread occurs. In particular, this outcome does not depend on the proportion of informed traders in the market. The general case of an asymmetric proportion of uninformed traders with a liquidity need to buy or sell is considered here. This extra element does not impact on the final outcome.

Proposition 5 *(Naïve uninformed traders) Consider a market with informed speculative traders and naïve uninformed ones. In particular, type* $\widetilde{s}_n=\varepsilon$ *aggregate trading capacity accounts for* $\alpha\beta$ *of the total trading capacity in the market, while type* $\widetilde{s}_n=-\varepsilon$ *aggregate trading capacity accounts for* $\alpha(1-\beta)$*, where* $\beta\in[0,1]$*. At the equilibrium:*

i) if $v<-\varepsilon+E[\widetilde{v}]$*, M sets* $\underline{p}\leq v<E[\widetilde{v}]-\varepsilon\wedge\overline{p}=E[\widetilde{v}]+\varepsilon$ *(asymmetric price spread), type* $\widetilde{s}_n=\varepsilon$ *buys; type* $\widetilde{s}_n\neq\varepsilon$ *does not trade.*

ii) if $v\in[-\varepsilon+E[\widetilde{v}],E[\widetilde{v}]+\varepsilon]$*, M sets* $\underline{p}=E[\widetilde{v}]-\varepsilon\wedge\overline{p}=E[\widetilde{v}]+\varepsilon$*, uninformed traders satisfy their liquidity needs, informed speculative traders do not trade.*

iii) if $v>E[\widetilde{v}]+\varepsilon$*, M sets* $\underline{p}=E[\widetilde{v}]-\varepsilon\wedge\overline{p}\geq v>E[\widetilde{v}]+\varepsilon$ *(asymmetric price spread), type* $\widetilde{s}_n=-\varepsilon$ *sells; type* $\widetilde{s}_n\neq-\varepsilon$ *does not trade.*

Proof of Proposition 4. See Appendix. ■

3 Concluding remarks and future research

The paper considers the case of a 'two-sided' single-dealer market, with an informed dealer providing liquidity to a group of informed and/or uninformed clients.

Regardless of whether the informed speculative clients exist or not, we show that the prices set by the dealer might turn out to be uncorrelated with 'real' currency values known to the dealer. In particular, they can be extremely stable even when the fundamental value of the asset is unstable. In contrast to conventional models, it is not necessarily the case that the 'real' currency value in the single-dealer market lies between the public bid and ask quotes. A wide variety of equilibria are possible. Not only do several classes of equilibria with bubbles arise even when informed clients exist in the market; they also survive a strong equilibrium refinement. This means that the dealer's dual advantage from possessing private information and providing liquidity via this particular monopolistic platform can be mitigated by the consequences of the informed clients' speculative behavior when specific uninformed clients' beliefs cause a substantial price distortion.

[15] While *Condition III* and *IV* are interesting only when $\mathcal{V}$ is bouded at least from below, symmetrically *Condition V* and *VI* can be applied only when $\mathcal{V}$ is bouded at least from above.

The aggregate trading capacity of clients with a liquidity need incentive to buy or sell is assumed to be identical. An extension to this paper consists of relaxing such an assumption. Understanding what happens in the 'one-sided' single-dealer market is a good starting point to investigate this problem.

Beyond this, considering uninformed traders' liquidity needs correlated with the future value of the asset may lead to interesting outcomes. In fact uninformed traders could update their beliefs on whether they have a need to buy or sell.

Informed clients are mere speculators; uninformed clients in equilibrium have no speculative motive for trading, and simply satisfy their liquidity needs. Introducing informed clients that suffer for specific liquidity needs constitutes another natural extension.

Further modifications to the present model could be considered. A big one consists of endowing the dealer and the informed clients with different degrees of noisy private information.

4 Appendix

Proof of Lemma 1. Six cases corresponding to the relevant sub-classes of the trading strategy X_n can be identified. We study them, to identify which ones can be part of an equilibrium.

Case 1 Consider $X_n : \widetilde{s}_n{=}\varepsilon \rightarrow x_n{>}0, \widetilde{s}_n{=}{-}\varepsilon \rightarrow x_n{<}0$. The expected profits for M of type $\widetilde{v}{=}v$ setting $\underline{p}, \overline{p}$ are the following: $\pi(P, X_n, v){=}\frac{1}{2}(\overline{p} - v) + \frac{1}{2}(v - \underline{p}){=}\frac{\overline{p}-\underline{p}}{2}{=}w(\underline{p}, \overline{p})$.

Case 2 Consider $X_n : \widetilde{s}_n{=}\varepsilon \rightarrow x_n{>}0, \widetilde{s}_n{=}{-}\varepsilon \rightarrow x_n{=}0$. The profits that any type of M achieves when traders behave according to Case 2 are always smaller than those achieved in Case 1. We prove it by contradiction. Suppose that $\pi(P, X_n, v) \geq w(\underline{p}, \overline{p}) \therefore \frac{\overline{p}-v}{2} \geq \frac{\overline{p}-\underline{p}}{2} \therefore \underline{p} \geq v$. When this the case, from type $\widetilde{s}{=}\cdot$ perspective $E[\widetilde{v} \mid \underline{p}, \overline{p}]{=}\ E[\widetilde{v} \mid \underline{p} \geq v]{=}\underline{p} - \varphi$, where $\varphi \not< 0$. Type $\widetilde{s}{=}{-}\varepsilon$ does not deviate (she continue not to trade) if $0 \geq \underline{p} - E[\widetilde{v} \mid \underline{p} \geq v]{+}\varepsilon$, which is clearly false. A deviation occurs. Therefore this case is not of interest.

Case 3 Consider $X_n : \widetilde{s}_n{=}\varepsilon \rightarrow x_n{=}0, \widetilde{s}_n{=}{-}\varepsilon \rightarrow x_n{<}0$. This case is symmetric to the previous one (Case 2).

Case 4 Consider $X_n : \widetilde{s}_n{=}\varepsilon \rightarrow x_n{=}0, \widetilde{s}_n{=}{-}\varepsilon \rightarrow x_n{=}0$. Notice that $\pi(P, X_n, v){=}0$. M can always set an alternative pair of prices $\underline{p}, \overline{p} : \overline{p} - \underline{p}{>}0$ that makes uninformed traders willing to satisfy their liquidity needs (Case 1). For instance, given the uninformed traders beliefs, suppose that M sets $\underline{p}, \overline{p} : \overline{p} - \underline{p} \simeq 0 \wedge E[\widetilde{v} \mid \underline{p}, \overline{p}] \in (\underline{p}, \overline{p})$. Since ε is strictly positive, traders earn positive profits from satisfying their liquidity needs; every type of M earns an equal and positive amount. Therefore Case 4 is not of interest.

Case 5 Consider $X_n : \widetilde{s}_n{=}\varepsilon \rightarrow x_n{>}0, \widetilde{s}_n{=}{-}\varepsilon \rightarrow x_n{>}0$. The profits that M achieves when traders undertake X_n and she sets $\underline{p}, \overline{p}$ are always smaller than $w(\underline{p}, \overline{p})$. Suppose that this is not the case, and that $\pi(P, X_n, v) \geq w(\underline{p}, \overline{p}) \therefore \overline{p} - v \geq \frac{\overline{p}-\underline{p}}{2} \therefore \frac{\overline{p}+\underline{p}}{2} \geq v$. From type $\widetilde{s}{=}\cdot$ perspective $E[\widetilde{v} \mid \underline{p}, \overline{p}]{=}E[\widetilde{v} \mid \frac{\overline{p}+\underline{p}}{2} \geq v]{=}\frac{\overline{p}+\underline{p}}{2} - \varphi$. Type $\widetilde{s}{=}{-}\varepsilon$ does not deviate (she continue to buy) if $E[\widetilde{v} \mid \underline{p}, \overline{p}] - \overline{p} \geq \underline{p} - E[\widetilde{v} \mid \underline{p}, \overline{p}] + \varepsilon \therefore E[\widetilde{v} \mid \underline{p}, \overline{p}] \geq \frac{\overline{p}+\underline{p}+\varepsilon}{2} \therefore -\varphi \geq \frac{\varepsilon}{2}$, which is false. A deviation occurs. This case is not of interest.

Case 6 Consider $X_n : \widetilde{s}_n{=}\varepsilon \rightarrow x_n{<}0, \widetilde{s}_n{=}{-}\varepsilon \rightarrow x_n{<}0$. This case is symmetric to the previous one (Case 5).

There is no need to consider the cases of $X_n : \widetilde{s}_n{=}\varepsilon \rightarrow x_n{<}0, \widetilde{s}_n{=}{-}\varepsilon \rightarrow x_n{>}0$; $X_n : \widetilde{s}_n{=}\varepsilon \rightarrow x_n{<}0$, $\widetilde{s}_n{=}{-}\varepsilon \rightarrow x_n{=}0$; $X_n : \widetilde{s}_n{=}\varepsilon \rightarrow x_n{=}0, \widetilde{s}_n{=}{-}\varepsilon \rightarrow x_n{>}0$. Since at least one type $\widetilde{s}{=}\cdot$ deviates, these strategy profiles cannot be part of an equilibrium. ■

Proof of Proposition 1. Notice that: (i) $w(\underline{p} \neq \overline{p})>0, \forall v \in \mathcal{V}$; (ii) $\frac{\partial w(\underline{p},\overline{p})}{\partial(\overline{p}-\underline{p})}>0$, $\forall \underline{p},\overline{p}$: $\overline{p}-\underline{p}=\varphi$; (iii) $w(\underline{p},\overline{p} : \overline{p}-\underline{p}=\varphi)$ is the same for any M of type $\widetilde{v}=v$. Condition (i) and (ii) imply that a profit maximizer M sets the highest possible spread, provided that traders are still willing to trade. In particular, they are happy to satisfy their liquidity needs at any $\underline{p},\overline{p}$ such that $\underline{p}+\varepsilon \leq E[\widetilde{v} \mid \underline{p},\overline{p}] \leq \overline{p}-\varepsilon$ and $\frac{\underline{p}+\overline{p}-\varepsilon}{2} \leq E[\widetilde{v} \mid \underline{p},\overline{p}] \leq \frac{\underline{p}+\overline{p}+\varepsilon}{2}$; in fact, when the former condition is satisfied, type $\widetilde{s}_n=\varepsilon$ (or type $\widetilde{s}_n=-\varepsilon$) prefers to buy (sell resp.) rather than not trading; when the latter is satisfied, she does not want to sell (buy resp.). When setting a pair $\underline{p},\overline{p}$ such that $\underline{p}=E[\widetilde{v} \mid \underline{p},\overline{p}]-\varepsilon$ and $\overline{p}=E[\widetilde{v} \mid \underline{p},\overline{p}]+\varepsilon$, M of type $\widetilde{v}=v$ extracts the maximum surplus, which is equal to $w(\underline{p}=E[\widetilde{v} \mid \underline{p},\overline{p}]-\varepsilon, \overline{p}=E[\widetilde{v} \mid \underline{p},\overline{p}]+\varepsilon)=\varepsilon$ per unit traded. Because of condition (iii), any type v setting $\overline{p}-\underline{p}=2\varepsilon$ when traders are satisfying their liquidity needs cannot do any better by setting any other pair of prices. The width of the spread does not convey any information to the traders.

A pooling equilibrium always exists. At this equilibrium uninformed traders' (posterior) beliefs are such that $E[\widetilde{v} \mid \underline{p},\overline{p}]=E[\widetilde{v}], \forall \underline{p},\overline{p}$; any type of M sets $\underline{p},\overline{p} : \underline{p}+\varepsilon=E[\widetilde{v}]=\overline{p}-\varepsilon$; traders satisfy their liquidity needs.

A separating equilibrium always exists. At this equilibrium the uninformed traders' beliefs are such that $E[\widetilde{v} \mid \underline{p},\overline{p}]=\underline{p}+\varepsilon=\overline{p}-\varepsilon$; M of type $\widetilde{v}=v$ sets $\underline{p}=v-\varepsilon$, $\overline{p}=v+\varepsilon$, and uninformed traders satisfy their liquidity needs.

A variety of partial- and semi-pooling equilibria with M earning ε per unit traded and uninformed traders satisfying their liquidity needs might exist.

Each partial-pooling equilibrium presents the following characteristics. Every type of M selects a pure strategy. More than one of them (but not all) set the same pair $\underline{p}=\underline{p}_{\cdot},\overline{p}=\overline{p}_{\cdot}$: $\overline{p}_{\cdot}-\varepsilon=E[\widetilde{v} \mid \underline{p}=\underline{p}_{\cdot},\overline{p}=\overline{p}_{\cdot}]=\underline{p}_{\cdot}+\varepsilon$, where $\cdot = \{ .., i, .., j, .. \}$. Uninformed traders use Bayes' rule to update the prior belief into posterior.[16] In particular, $\mathcal{V}_{\cdot} \cap \mathcal{V}_{\neg\cdot}=\varnothing$, where $\mathcal{V}_{\cdot} \subset \mathcal{V}$ is the group of M setting the same pair $\underline{p}=\underline{p}_{\cdot},\overline{p}=\overline{p}_{\cdot}$; it follows that $E[\widetilde{v} \mid \underline{p}=\underline{p}_{\cdot},\overline{p}=\overline{p}_{\cdot}]=E[\widetilde{v} \mid v \in \mathcal{V}_{\cdot}]$. With semi-pooling equilibria, at least one type of M sets more than one pair of prices with positive probability less than one (she selects mixed strategies). At the equilibrium, if it happens that M of type $\widetilde{v}=v$ selecting mixed strategies sets a pair that nobody else sets, this pair is equal to $\underline{p}=v-\varepsilon, \overline{p}=v+\varepsilon$. More than one type of M sets the same pair $\underline{p}=\underline{p}'_{\cdot},\overline{p}=\overline{p}'_{\cdot}$: $\overline{p}'_{\cdot}-\varepsilon=E[\widetilde{v} \mid \underline{p}=\underline{p}'_{\cdot},\overline{p}=\overline{p}'_{\cdot}]=\underline{p}'_{\cdot}+\varepsilon$. The way uninformed traders update beliefs is more sophisticated than in the case of partial-pooling equilibria. In particular, since $\exists \mathcal{V}_{\cdot} : \mathcal{V}_{\cdot} \cap \mathcal{V}_{\neg\cdot} \neq \varnothing$, they consider a probability distribution over actions for each type v. As long as $\mathcal{V}$ contains at least two possible events, semi-pooling equilibria in which all types of M randomize exist.[17] Even when only two events are possible, infinite different semi-pooling equilibria arise.

For each of the equilibria described above, all pairs of prices different from the prescribed ones represent an off-path strategy for M. For instance, if M set $\underline{p}<E[\widetilde{v} \mid \underline{p},\overline{p}]-\varepsilon$, then type $\widetilde{s}_n=-\varepsilon$ would not trade any more, and M would earn less (since the equilibrium price spread is positive, the case of both uninformed agents not trading is not of interest). Therefore M would deviate. Consider now the case of M setting $\underline{p}$ such that $\underline{p}>E[\widetilde{v} \mid \underline{p},\overline{p}]-\varepsilon$. Even if type $\widetilde{s}_n=-\varepsilon$ was still willing to sell, M's profits would be smaller (we already showed that the case of type $\widetilde{s}_n=-\varepsilon$ aiming to buy is not part of an equilibrium). In conclusion, any pair of prices $\underline{p},\overline{p} : \underline{p}+\varepsilon \neq E[\widetilde{v} \mid \underline{p},\overline{p}] \vee E[\widetilde{v} \mid \underline{p},\overline{p}] \neq \overline{p}-\varepsilon$ represents an off-path strategy for M.

In particular, consider an equilibrium in which there exists one type of M setting a pair

[16] The posterior density of any type v charging $\underline{p}=\underline{p}_{\cdot},\overline{p}=\overline{p}_{\cdot}$ must be equal to the prior density divided by the total prior probability of types charging the same pair $\underline{p}=\underline{p}_{\cdot},\overline{p}=\overline{p}_{\cdot}$.

[17] When informed speculative traders exist, this is not necessarily true any more (see Proposition 2).

(or more than one pair) of prices that differs from the one(s) set by another type of M. If the former deviated and selected the pair that the latter is setting, she would earn the same profits as before (in fact each equilibrium price spread equals 2ε). However, this cannot be the case. Remember that uninformed traders' beliefs about each type of M' strategies must be correct in equilibrium. For instance, considering the separating equilibrium, any pair $\underline{p}, \overline{p} : \underline{p} \neq v - \varepsilon \vee \overline{p} \neq v+\varepsilon$ represents an off-path strategy for M of type $\widetilde{v}=v$, even when $\overline{p} - \underline{p}=2\varepsilon$. ■

Proof of Lemma 2. First notice that, regardless of what type $\widetilde{s}_n=\eta$ does, none of the following strategy profiles can be part of an equilibrium: $X_n : \widetilde{s}_n=\varepsilon \rightarrow x_n<0, \widetilde{s}_n=-\varepsilon \rightarrow x_n>0$; $X_n : \widetilde{s}_n=\varepsilon \rightarrow x_n<0, \widetilde{s}_n=-\varepsilon \rightarrow x_n=0$; and $X_n : \widetilde{s}_n=\varepsilon \rightarrow x_n=0, \widetilde{s}_n=-\varepsilon \rightarrow x_n>0$.

Now consider the three main cases of type $\widetilde{s}_n=\eta$ not trading, buying, or selling.

When $\widetilde{s}_n=\eta$ does not trade, six sub-cases corresponding to the relevant uninformed agents' trading strategies can be identified.

Cases from 1.a to 6.a The analysis of these six sub-cases is similar to the one in Lemma 1, and left to the reader. In each sub-case, the extra condition of M setting $\underline{p}, \overline{p} : \underline{p} \leq v \leq \overline{p}$ must be imposed. This is necessary to avoid a deviation by type $\widetilde{s}_n=\eta$. In particular, when both uninformed traders satisfy their liquidity needs, M earns $\alpha w(\underline{p}, \overline{p} : \underline{p} \leq v \leq \overline{p})$. The only two sub-cases not leading to a deviation by one type of uninformed trader are the one in which they both satisfy their liquidity needs, and the one in which none of them trade. Depending on the uninformed traders' beliefs and $f(\widetilde{v})$, it can be that the pair of prices that makes uninformed agents willing to trade is such that $v<\underline{p} \vee \overline{p}<v$ (clearly, adjusting the price spread to include v and still provide liquidity to one type of uninformed trader is not a possibility: a deviation would occur). When $v<\underline{p} \vee \overline{p}<v$, type $\widetilde{s}_n=\eta$ speculates. In this case, we show that all uninformed agents's strategy profiles but the one consisting of them satisfying their liquidity needs are not part of any equilibrium (see below). If type $\widetilde{s}_n=\eta$'s speculation overwhelmed the benefit that M of type $\widetilde{v}=v$ has from providing liquidity to uninformed traders, M sets prices that makes nobody trade (in this way her profits equal zero). In particular, there always exist beliefs and distributions of $\widetilde{v}$ such that this is the case, for any positive price spread such that types $\widetilde{s}_n \neq \eta$ satisfy their liquidity needs. Thus, different from the case without speculators, selecting prices that make nobody trade is a strategy that can arise at the equilibrium.

When type $\widetilde{s}_n=\eta$ buys, six sub-cases corresponding to the relevant uninformed agents' strategy can be identified. The only equilibrium candidate surviving is the one in Case 1.b.

Case 1.b Consider $X_n : \widetilde{s}_n=\varepsilon \rightarrow x_n>0, \widetilde{s}_n=-\varepsilon \rightarrow x_n<0, \widetilde{s}_n=\eta \rightarrow x_n>0$. Given $\underline{p}, \overline{p}$ set by M, for X_n in Case 1.b to be part of a possible equilibrium, it must be that M is of type $v>\overline{p}$, otherwise type $\widetilde{s}_n=\eta$ would deviate. The associated profits for M of type $v>\overline{p}$ are:

$$\pi(P, X_n, v)=\alpha w(\underline{p}, \overline{p}\text{: } v>\overline{p})+(1-\alpha)\overbrace{(\overline{p}-v)}^{<0}.$$

Case 2.b Consider $X_n : \widetilde{s}_n=\varepsilon \rightarrow x_n>0, \widetilde{s}_n=-\varepsilon \rightarrow x_n=0, \widetilde{s}_n=\eta \rightarrow x_n>0$. Given $\underline{p}, \overline{p}$, the profits for a market maker of type $v>\overline{p}$ when traders undertake X_n are smaller than the ones achieved in Case 1.b. We prove it by contradiction. Suppose that: $\pi(P, X_n, v)=\frac{\alpha}{2}(\overline{p} - v)+(1-\alpha)(\overline{p} - v) \geq \alpha w(\underline{p}, \overline{p} : v > \overline{p})+(1-\alpha)(\overline{p} - v) \therefore \underline{p} \geq v$. When $\underline{p} \geq v$ both type $\widetilde{s}=-\varepsilon$ and $\widetilde{s}_n=\eta$ deviate.

Case 3.b Consider $X_n : \widetilde{s}_n=\varepsilon \rightarrow x_n=0, \widetilde{s}_n=-\varepsilon \rightarrow x_n<0, \widetilde{s}_n=\eta \rightarrow x_n>0$. In this case M of type $v>\overline{p}$ achieves less than in Case 1.b. We prove it by contradiction. Suppose that: $\pi(P, X_n, v)=\frac{\alpha}{2}(v - \underline{p})+(1-\alpha)(\overline{p} - v) \geq \alpha w(\underline{p}, \overline{p} : v > \overline{p})+(1-\alpha)(\overline{p} - v) \therefore v \geq \overline{p}$. When $v \geq \overline{p}$, type $\widetilde{s}_n=\varepsilon$ deviates.

Case 4.b Consider $X_n : \widetilde{s}_n{=}\varepsilon \rightarrow x_n{=}0, \widetilde{s}_n{=}{-}\varepsilon \rightarrow x_n{=}0, \widetilde{s}_n{=}\eta \rightarrow x_n{>}0$. This function is not part of any equilibrium. Since $\pi(P, X_n, v){<}0$, M does better even when she sets an alternative pair of prices that makes every trader unwilling to trade.

Case 5.b Consider $X_n : \widetilde{s}_n{=}\varepsilon \rightarrow x_n{>}0, \widetilde{s}_n{=}{-}\varepsilon \rightarrow x_n{>}0, \widetilde{s}_n{=}\eta \rightarrow x_n{>}0$. When $v{>}\overline{p}$, M achieves less than in Case 1.b. This is not true when: $\pi(P, X_n, v){=}(\overline{p}{-}v) \geq \alpha w(\underline{p}, \overline{p} : v > \overline{p}){+}(1-\alpha)(\overline{p}-v) \therefore \frac{\overline{p}+\underline{p}}{2} \geq v$. We already proved that when $\frac{\overline{p}+\underline{p}}{2} \geq v$ type $\widetilde{s}{=}{-}\varepsilon$ deviates.

Case 6.b Consider $X_n : \widetilde{s}_n{=}\varepsilon \rightarrow x_n{<}0, \widetilde{s}_n{=}{-}\varepsilon \rightarrow x_n{<}0, \widetilde{s}_n{=}\eta \rightarrow x_n{>}0$. Again, M of type $v{>}\overline{p}$ gets less than in Case 1.b. By contradiction, suppose that $\pi(P, X_n, v){=}\ \alpha(v-\underline{p}){+}(1-\alpha)(\overline{p}-v) \geq \alpha w(\underline{p}, \overline{p} : v > \overline{p}){+}(1-\alpha)(\overline{p}-v) \therefore v \geq \frac{\overline{p}+\underline{p}}{2}$. From the uninformed traders' perspective, $E[\widetilde{v} \mid \underline{p}, \overline{p}]{=}E[\widetilde{v} \mid v \geq \frac{\overline{p}+\underline{p}}{2}]{=}\frac{\overline{p}+\underline{p}}{2}{+}\varphi$. It follows that type $\widetilde{s}{=}\varepsilon$ does not deviate if $\underline{p} - E[\widetilde{v} \mid \underline{p}, \overline{p}] \geq E[\widetilde{v} \mid \underline{p}, \overline{p}] - \overline{p}{+}\varepsilon \therefore -\frac{\varepsilon}{2} \geq \varphi$, which is false.

Cases from 1.c to 6.c Consider $X_n : \widetilde{s}_n{=}\eta \rightarrow x_n{<}0$. The six sub-cases are symmetric to the ones in which type $\widetilde{s}_n{=}\eta$ buys. The only equilibrium candidate surviving is the one in which $X_n : \widetilde{s}_n{=}\varepsilon \rightarrow x_n{>}0, \widetilde{s}_n{=}{-}\varepsilon \rightarrow x_n{<}0, \widetilde{s}_n{=}\eta \rightarrow x_n{<}0$ (Case 1.c). ■

Proof of Proposition 2. For the same profit-maximization argument presented in the case without informed speculative traders, M aiming to extract the maximum surplus from uninformed traders sets $\underline{p}{=}E[\widetilde{v} \mid \underline{p}, \overline{p}] - \varepsilon$ and $\overline{p}{=}E[\widetilde{v} \mid \underline{p}, \overline{p}]{+}\varepsilon$.

A pooling equilibrium with uninformed traders satisfying their needs might arise. At this equilibrium they believe $E[\widetilde{v} \mid \underline{p}, \overline{p}]{=}E[\widetilde{v}], \forall \underline{p}, \overline{p}$, and M sets $\underline{p}{=}E[\widetilde{v}] - \varepsilon$, $\overline{p}{=}E[\widetilde{v}]{+}\varepsilon$. The equilibrium exists for $f(\widetilde{v}) : \mathcal{V} \subset [-\frac{\varepsilon}{1-\alpha}{+}E[\widetilde{v}], E[\widetilde{v}]{+}\frac{\varepsilon}{1-\alpha}]$. In fact, when $\widetilde{v}{=}v{>}E[\widetilde{v}]{+}\varepsilon$ (Case 1.b), M suffers for the speculation by informed traders. She achieves non-negative profits when: $[\alpha w(\underline{p}, \overline{p}){+}(1-\alpha)(\overline{p}-v) \mid \underline{p}{=}E[\widetilde{v}] - \varepsilon, \overline{p}{=}E[\widetilde{v}]{+}\varepsilon] \geq 0 \therefore v \leq \frac{\varepsilon}{1-\alpha} + E[\widetilde{v}]$. Similarly, in case M turns out to be of type $\widetilde{v}{=}v{<}E[\widetilde{v}] - \varepsilon$, she achieves non-negative profits when: $[\alpha w(\underline{p}, \overline{p}){+}(1-\alpha)(v-\underline{p}) \mid \underline{p}{=}E[\widetilde{v}] - \varepsilon, \overline{p}{=}E[\widetilde{v}]{+}\varepsilon] \geq 0 \therefore E[\widetilde{v}] - \frac{\varepsilon}{1-\alpha} \leq v$.

There always exists a separating equilibrium with M of type $\widetilde{v}{=}v$ setting $\underline{p}{=}v-\varepsilon$ and $\overline{p}{=}v+\varepsilon$, type $\widetilde{s}_n{=}\mu$ not trading, and types $\widetilde{s}_n \neq \mu$ satisfying their liquidity needs.

A variety of partial-pooling equilibria with uninformed traders satisfying their liquidity needs might also exist. Their characteristics are analogous to the ones of partial-pooling equilibria presented in the case without type $\widetilde{s}_n{=}\mu$. Here however, because of the existence of informed speculators, it must be that $\mathcal{V}_. \subset [-\frac{\varepsilon}{1-\alpha}{+}E[\widetilde{v} \mid \underline{p}_., \overline{p}_.], E[\widetilde{v} \mid \underline{p}_., \overline{p}_.]{+}\frac{\varepsilon}{1-\alpha}]$.
In particular, consider M of type $v \in \mathcal{V}_i$ and $v \in \mathcal{V}_j$, where $\mathcal{V}_i \neq \mathcal{V}_j$. A partial-pooling equilibrium with liquidity traders observing $\underline{p}{=}\underline{p}_i, \overline{p}{=}\overline{p}_i$ or $\underline{p}{=}\underline{p}_j, \overline{p}{=}\overline{p}_j$ and satisfying their liquidity needs requires each type $v \in \mathcal{V}_i$ to (at least weakly) prefer the action $\underline{p}{=}\underline{p}_i, \overline{p}{=}\overline{p}_i$ to $\underline{p}{=}\underline{p}_j, \overline{p}{=}\overline{p}_j$, and each $v \in \mathcal{V}_j$ to (at least weakly) prefer the action $\underline{p}{=}\underline{p}_j, \overline{p}{=}\overline{p}_j$ to $\underline{p}{=}\underline{p}_i, \overline{p}{=}\overline{p}_i$. Suppose that $E[\widetilde{v} \mid \underline{p}_i, \overline{p}_i]{<}E[\widetilde{v} \mid \underline{p}_j, \overline{p}_j]$. We first study the case of $\overline{v}_i{<}\underline{v}_j$ [case (a)], then we consider the case of $\overline{v}_i \geq \underline{v}_j$ [case (b)], where $\overline{v}_i{=}\max v \in \mathcal{V}_i$. $\underline{v}_j{=}\min v \in \mathcal{V}_j$.

(a) Whenever $\overline{v}_i{<}\underline{v}_j$: (a.i) if $\overline{v}_i \leq E[\widetilde{v} \mid \underline{p}_i, \overline{p}_i]{+}\varepsilon{=}\overline{p}_i$ and $E[\widetilde{v} \mid \underline{p}_j, \overline{p}_j] - \varepsilon{=}\underline{p}_j \leq \underline{v}_j$, then no type $v \in \mathcal{V}_i$ (or $v \in \mathcal{V}_j$) has an incentive to deviate, setting $\underline{p}_j, \overline{p}_j$ (or $\underline{p}_i, \overline{p}_i$ resp.).[18] (a.ii) If $E[\widetilde{v} \mid \underline{p}_i, \overline{p}_i]{+}\varepsilon{=}\overline{p}_i{<}\overline{v}_i$ and $E[\widetilde{v} \mid \underline{p}_j, \overline{p}_j] - \varepsilon{=}\underline{p}_j \leq \underline{v}_j$, type $\overline{v}_i$ setting $\underline{p}_i, \overline{p}_i$ is earning less than $\alpha\varepsilon$ because of the informed speculator intervention. Type $\overline{v}_i$ does not set $\underline{p}_j, \overline{p}_j$ whenever $\overline{v}_i - \overline{p}_i{<}\underline{p}_j - \overline{v}_i \therefore \overline{v}_i{<}\frac{\overline{p}_i+\underline{p}_j}{2}$ (clearly, type $\underline{v}_j$ setting $\underline{p}_j, \overline{p}_j$ is earning $\alpha\varepsilon$: she has no incentive to

[18] This holds for any $v \in \mathcal{V}_i$. Both type $v : \underline{p}_i \leq v \leq \overline{p}_i$ and $v{<}\underline{p}_i$ setting $\underline{p}_i, \overline{p}_i$ earn $\alpha\varepsilon$ and less than $\alpha\varepsilon$ respectively. Since $\underline{p}_i{<}\underline{p}_j$ and $\overline{p}_i{<}\overline{p}_j$, they cannot do better when setting $\underline{p}_j, \overline{p}_j$ (a symmetric argument holds for any $v \in \mathcal{V}_j$).

set $\underline{p}_i,\overline{p}_i$). (a.iii) The case of $\overline{v}_i \leq E[\widetilde{v} \mid \underline{p}_i,\overline{p}_i]+\varepsilon=\overline{p}_i$ and $\underline{v}_j<E[\widetilde{v} \mid \underline{p}_j,\overline{p}_j]-\varepsilon=\underline{p}_j$ is symmetric to the previous sub-case. (a.iv) If $E[\widetilde{v} \mid \underline{p}_i,\overline{p}_i]+\varepsilon=\overline{p}_i<\overline{v}_i$ and $\underline{v}_j<E[\widetilde{v} \mid \underline{p}_j,\overline{p}_j]-\varepsilon=\underline{p}_j$, type $\overline{v}_i$ does not set $\underline{p}_j,\overline{p}_j$ whenever: $\pi(\langle\, \underline{p}_i,\overline{p}_i \,\rangle, X'_n,\overline{v}_i) \geq \pi(\langle\, \underline{p}_j,\overline{p}_j \,\rangle, X''_n,\overline{v}_i)=\pi(\langle\, \underline{p}_j,\overline{p}_j \,\rangle, X''_n,\underline{v}_j)$ $-(1-\alpha)(\underline{v}_j-\overline{v}_i)$, where the trading strategies $X'_n: \widetilde{s}_n=\varepsilon \rightarrow x_n>0, \widetilde{s}_n=-\varepsilon \rightarrow x_n<0, \widetilde{s}_n=\eta \rightarrow x_n>0$ and $X''_n: \widetilde{s}_n=\varepsilon \rightarrow x_n>0, \widetilde{s}_n=-\varepsilon \rightarrow x_n<0, \widetilde{s}_n=\eta \rightarrow x_n<0$ are sub-classes of the function X_n. Similarly, type $\underline{v}_j$ does not set $\underline{p}_i,\overline{p}_i$ whenever: $\pi(\langle\, \underline{p}_j,\overline{p}_j \,\rangle, X''_n,\underline{v}_j) \geq \pi(\langle\, \underline{p}_i,\overline{p}_i \,\rangle, X'_n,\underline{v}_j)$ $=\pi(\langle\, \underline{p}_i,\overline{p}_i \,\rangle, X'_n,\overline{v}_i)-(1-\alpha)(\underline{v}_j-\overline{v}_i)$. Combining these two conditions, we get that:

$$[\pi(\langle\, \underline{p}_j,\overline{p}_j \,\rangle, X''_n,\underline{v}_j)-(1-\alpha)(\underline{v}_j-\overline{v}_i)]\leq\pi(\langle\, \underline{p}_i,\overline{p}_i \,\rangle, X'_n,\overline{v}_i)\leq[\pi(\langle\, \underline{p}_j,\overline{p}_j \,\rangle, X''_n,\underline{v}_j)+(1-\alpha)(\underline{v}_j-\overline{v}_i)]$$

$$\therefore -(1-\alpha)(\underline{v}_j-\overline{v}_i) \leq[\pi(\langle\, \underline{p}_i,\overline{p}_i \,\rangle, X'_n,\overline{v}_i)-\pi(\langle\, \underline{p}_j,\overline{p}_j \,\rangle, X''_n,\underline{v}_j)]\leq (1-\alpha)(\underline{v}_j-\overline{v}_i).$$

It follows that, for type $\overline{v}_i$ and $\underline{v}_j$ to set $\underline{p}_i,\overline{p}_i$ and $\underline{p}_j,\overline{p}_j$ respectively, it must be that: $\mid \pi(\langle\, \underline{p}_j,\overline{p}_j \,\rangle, X''_n,\underline{v}_j)-\pi(\langle\, \underline{p}_i,\overline{p}_i \,\rangle, X'_n,\overline{v}_i) \mid \leq (1-\alpha)(\underline{v}_j-\overline{v}_i)$. Notice that, when $\underline{v}_j \simeq \overline{v}_i$, it is necessary that: $\pi(\langle\, \underline{p}_j,\overline{p}_j \,\rangle, X''_n,\underline{v}_j) \simeq \pi(\langle\, \underline{p}_i,\overline{p}_i \,\rangle, X'_n,\overline{v}_i)$.

(b) Whenever $\overline{v}_i \geq \underline{v}_j$: (b.i) if $\overline{v}_i \leq E[\widetilde{v} \mid \underline{p}_i,\overline{p}_i]+\varepsilon=\overline{p}_i$ and $E[\widetilde{v} \mid \underline{p}_j,\overline{p}_j]-\varepsilon=\underline{p}_j \leq \underline{v}_j$, similarly to case (a.i), no type $v \in \mathcal{V}_i$ (or $v \in \mathcal{V}_j$) has an incentive to set $\underline{p}_j,\overline{p}_j$ (or $\underline{p}_i,\overline{p}_i$ resp.). (b.ii) If $E[\widetilde{v} \mid \underline{p}_i,\overline{p}_i]+\varepsilon=\overline{p}_i<\overline{v}_i$, type $\overline{v}_i$ setting $\underline{p}_i,\overline{p}_i$ is earning less than $\alpha\varepsilon$. Since it must be that $\underline{p}_i<\underline{p}_j$, only three sub-cases arise; none of them is possible. In fact: (b.ii.I) when $\overline{p}_j<\overline{v}_i$, type $\overline{v}_i$ sets $\underline{p}_i,\overline{p}_i$ only if $\overline{v}_i-\overline{p}_i \leq \overline{v}_i-\overline{p}_j$: this however is verified only when $\overline{p}_i \geq \overline{p}_j$, a contradiction; (b.ii.II) when $\underline{p}_j \leq \overline{v}_i \leq \overline{p}_j$, type $\overline{v}_i$ setting $\underline{p}_j,\overline{p}_j$ earns $\alpha\varepsilon$ (more than what she earns when setting $\underline{p}_i,\overline{p}_i$); (b.ii.III) when $\overline{v}_i<\underline{p}_j$, type $\overline{v}_i$ sets $\underline{p}_i,\overline{p}_i$ when $\overline{v}_i-\overline{p}_i \leq \underline{p}_j-\overline{v}_i \therefore$ $\overline{v}_i \leq \frac{\overline{p}_i+\underline{p}_j}{2}$; however, when this is the case, type $\underline{v}_j$ sets $\underline{p}_i,\overline{p}_i$ as well. (b.iii) The case of $E[\widetilde{v} \mid \underline{p}_j,\overline{p}_j]-\varepsilon=\underline{p}_j>\underline{v}_j$ is symmetric to the one just considered.

Depending on the uninformed traders' initial beliefs, there might exist partial-pooling equilibria with M of type $v \notin \bigcup\mathcal{V}_{\cdot}$ setting a pair of prices that makes nobody (that is: neither uninformed nor informed traders) willing to trade. Partial-pooling equilibria with every type of M setting prices that make nobody trade are a possibility. In particular, when the pair of prices selected by each type of M is the same, we have the second class of pooling equilibria. The simplest example is the one of a $f(\widetilde{v}): \nexists v \in [-\frac{\varepsilon}{1-\alpha}+E[\widetilde{v}], E[\widetilde{v}]+\frac{\varepsilon}{1-\alpha}]$ and uninformed traders believing $E[\widetilde{v} \mid \underline{p},\overline{p}]=E[\widetilde{v}], \forall\underline{p},\overline{p}$.

Semi-pooling equilibria are also possible. When updating their beliefs, uninformed traders consider a probability distribution over actions for each type of M. In fact at least one type of M randomizes (achieving the same profits from each of the pairs she randomizes between). Consider semi-pooling equilibria with at least one type of M earning zero profits when randomizing. This type sets at least two different pairs of prices that make nobody willing to trade; at least one of the pairs is the same that another type of M is setting (since the price spread set is larger than 2ε, the latter M is earning zero as well). When randomizing, the number of pairs of prices that no other type of M is selecting can be more than one.

Consider semi-pooling equilibria with at least one type of M earning $\alpha\varepsilon$ per unit traded when randomizing. If this type $\widetilde{v}=v$ sets a pair that nobody else sets, this pair is equal to $\underline{p}=v-\varepsilon,\overline{p}=v+\varepsilon$. Each pair of prices she is selecting is such that $\underline{p} \leq v \leq \overline{p} \wedge \overline{p}-\underline{p}=2\varepsilon$. The other types of M setting the same pair of price do not necessarily earn $\alpha\varepsilon$, but a positive quantity less than $\alpha\varepsilon$ (these other types are relatively close to the type $\widetilde{v}=v$, so that the informed traders' speculation they suffer for is compensated by the surplus extracted from uninformed traders).

Consider semi-pooling equilibria in which at least one type of M earning positive profits less than $\alpha\varepsilon$ per unit traded is randomizing. Type $\widetilde{v}=v$ randomizes with some probability between only two pairs, $\underline{p}=\underline{p}',\overline{p}=\overline{p}'$ and $\underline{p}=\underline{p}'',\overline{p}=\overline{p}''$. In particular, $0<v-\overline{p}'=\underline{p}''-v<\frac{\alpha\varepsilon}{1-\alpha}$ (in fact, it must be that: $v-\overline{p}'<E[\widetilde{v}\mid \underline{p}',\overline{p}']+\frac{\varepsilon}{1-\alpha}-\{\ E[\widetilde{v}\mid \underline{p}',\overline{p}']+\varepsilon\ \}\ \therefore\ v-\overline{p}'<\frac{\varepsilon}{1-\alpha}-\varepsilon=\frac{\alpha\varepsilon}{1-\alpha}$). The other types of M selecting the same pair set by type $\widetilde{v}=v$ earn non-negative profits. Finally notice that, when $\mathcal{V}$ contains only two possible events, it is not necessarily the case that semi-pooling equilibria arise.

Concerning every equilibrium characterized above, all pairs of prices different from the prescribed ones represent an off-path strategy for M. For the same argument presented in Proposition 1, a profit maximizer M that wants uninformed traders to satisfy their liquidity needs sets a price spread equal to 2ε, and such that the uninformed traders' beliefs are correct in equilibrium. Starting from a situation in which uninformed traders are willing to satisfy their needs, and M is extracting $\alpha\varepsilon$ per unit traded from them, consider the case of informed traders speculating. Adjusting the prices to have the speculator not trading and at least one liquidity trader satisfying her needs is not a possibility. The only alternative to the case in which M extracts the maximum surplus from both types of uninformed traders is one in which prices are such that no agent in the market trades. ■

Proof of Proposition 3. Recall that in our framework the IC can be formalized as follows. Consider a PBE with M's equilibrium profits equal to $\pi^*(v)$. Step I - identification of $\widehat{\mathcal{V}}\left(\underline{p},\overline{p}\right)$. For each out-of-equilibrium pair $\underline{p}$,$\overline{p}$, let $\widehat{\mathcal{V}}\left(\underline{p},\overline{p}\right)$ be the set of all $v\in\mathcal{V}$ such that:

$$\pi^*(v) > \max_{x_n(\widetilde{s}_n=\varepsilon),x_n(\widetilde{s}_n=-\varepsilon)\in BR(\mathcal{V},\underline{p},\overline{p})} \pi(\underline{p},\overline{p},x_n(\widetilde{s}_n=\varepsilon),x_n(\widetilde{s}_n=-\varepsilon),x_n(\widetilde{s}_n=\mu),v).$$

Step II - failure condition. If for some out-of-equilibrium pair $\underline{p},\overline{p}$ there exists a $v'\in\mathcal{V}$ such that:

$$\pi^*(v') < \min_{x_n(\widetilde{s}_n=\varepsilon),x_n(\widetilde{s}_n=-\varepsilon)\in BR(\mathcal{V}\setminus\widehat{\mathcal{V}},\underline{p},\overline{p})} \pi(\underline{p},\overline{p},x_n(\widetilde{s}_n=\varepsilon),x_n(\widetilde{s}_n=-\varepsilon),x_n(\widetilde{s}_n=\mu),v'),$$

then the equilibrium fails the Intuitive Criterion. In particular, $BR(\Omega,\underline{p},\overline{p})$ is defined as follows:

$$BR(\Omega,\underline{p},\overline{p})\equiv\bigcup_{\kappa:\kappa(\Omega\mid\underline{p},\overline{p})=1} BR(\kappa,\underline{p},\overline{p}),$$

where $\Omega\subset\mathcal{V}$ is non-empty, and the set $BR(\kappa,\underline{p},\overline{p})$ contains the uninformed traders' best equilibrium responses, $x_n^{BR}(\kappa,\underline{p},\overline{p},\widetilde{s}_n=\varepsilon)$, $x_n^{BR}(\kappa,\underline{p},\overline{p},\widetilde{s}_n=-\varepsilon)$, to $\underline{p},\overline{p}$ for beliefs $\kappa(v\mid\underline{p},\overline{p})$ such that $\kappa(\Omega\mid\underline{p},\overline{p})=1$. For instance, with a discrete support Ω, every pair in $BR(\Omega,\underline{p},\overline{p})$ must be such that:

$$x_n^{BR}(\kappa,\underline{p},\overline{p},\widetilde{s}_n)=\arg\max_{x_n\in\{-1,0,1\}}\sum_{v\in\Omega}\kappa(v)\cdot U_n(\underline{p},\overline{p},x_n,\widetilde{s}_n,v).$$

Generalizing, for some beliefs associated to a specific out-of-equilibrium pair of prices $\underline{p},\overline{p}$, type $\widetilde{s}_n=\varepsilon$'s profits from buying equal $v_\kappa-\overline{p}+\varepsilon$, where v_κ is the expectation of $v\in\Omega$ given arbitrary posterior beliefs κ, and can take any value from $\underline{v}_\kappa=\min\Omega$ to $\overline{v}_\kappa=\max\Omega$ (notice conversely that $\mathcal{V}$ and $\widehat{\mathcal{V}}(\underline{p},\overline{p})$ are not necessarily full support). Her profits from selling equal $\underline{p}-v_\kappa$, while the ones from not trading equal zero. Analogously, type $\widetilde{s}_n=-\varepsilon$'s profits per unit traded equal $v_\kappa-\overline{p}$ if she buys, $\underline{p}-v_\kappa+\varepsilon$ if she sells, zero otherwise.

The following graph summarizes the best responses to any out-of-equilibrium pair $\underline{p},\overline{p}$ for beliefs $\kappa(v\mid\underline{p},\overline{p})$. In particular, for any $\underline{p},\overline{p},v_\kappa:\underline{p}<v_\kappa-\varepsilon\wedge\overline{p}>v_\kappa+\varepsilon$ (*Area 1*), the uninformed traders' best response is $X_n^1:\widetilde{s}_n=\cdot\ \rightarrow x_n=0$; for any $\underline{p},\overline{p},v_\kappa:\underline{p}<v_\kappa-\varepsilon\wedge\ v_\kappa<\overline{p}\leq v_\kappa+\varepsilon$

(*Area 2*), the best response is $X_n^2 : \widetilde{s}_n{=}\varepsilon \rightarrow x_n{>}0$; $\widetilde{s}_n{=}{-}\varepsilon \rightarrow x_n{=}0$; for any $\underline{p}, \overline{p}, v_\kappa : v_\kappa - \varepsilon \leq \underline{p}{<}v_\kappa \wedge \overline{p}{>}\ v_\kappa{+}\varepsilon$ (*Area 3*), the best response is $X_n^3 : \widetilde{s}_n{=}\varepsilon \rightarrow x_n{=}0$; $\widetilde{s}_n{=}{-}\varepsilon \rightarrow x_n{<}0$; for any $\underline{p}, \overline{p}, v_\kappa : \underline{p} \leq -\overline{p}{+}2v_\kappa - \varepsilon \wedge \overline{p} \leq v_\kappa$ (*Area 4*), the best response is $X_n^4 : \widetilde{s}_n{=}\cdot\ \rightarrow x_n{>}0$; for any $\underline{p}, \overline{p}, v_\kappa : v_\kappa - \varepsilon \leq \underline{p} \leq -\overline{p}{+}2v_\kappa{+}\varepsilon \wedge\ -\underline{p}{+}2v_\kappa - \varepsilon{<}\overline{p} \leq v_\kappa{+}\varepsilon$ (*Area 5*), the best response is $X_n^5 : \widetilde{s}_n{=}\varepsilon \rightarrow x_n{>}0$; $\widetilde{s}_n{=}{-}\varepsilon \rightarrow x_n{<}0$; for any $\underline{p}, \overline{p}, v_\kappa : v_\kappa \leq \underline{p} \wedge \overline{p}{>}{-}\underline{p}{+}2v_\kappa{+}\varepsilon$ (*Area 6*), the best response is $X_n^6 : \widetilde{s}_n{=}\cdot\ \rightarrow x_n{<}0$.

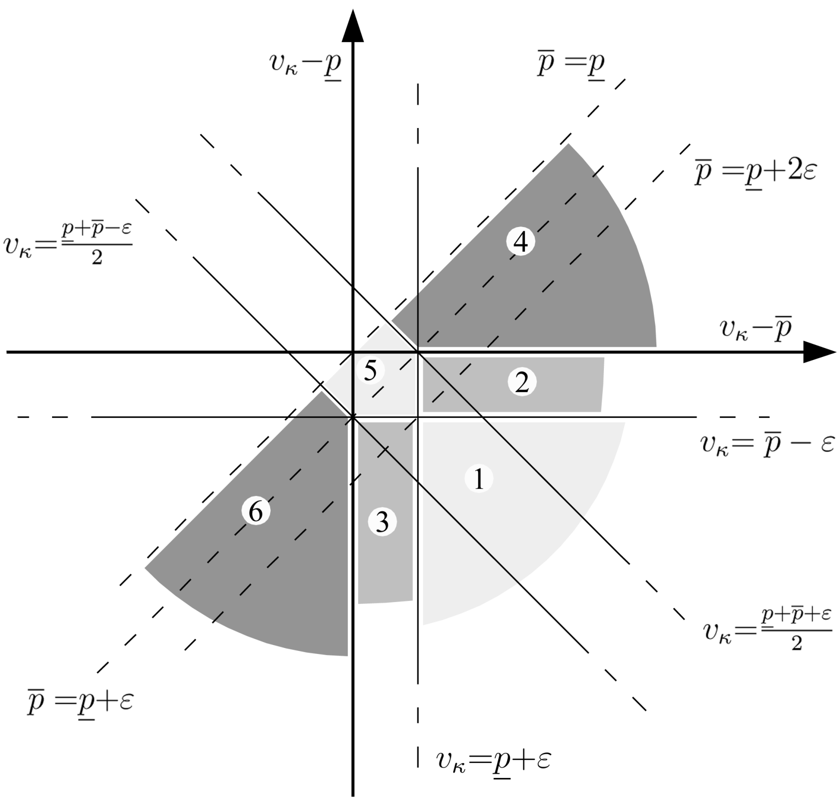

Figure 1. Best responses to any out-of-equilibrium pair $\underline{p}, \overline{p}$ for arbitrary beliefs $\kappa\left(v\,\middle|\underline{p}, \overline{p}\right)$.

Now we prove that, given a PBE such that there exists at least a type $v \in \mathcal{V}$ earning $\pi^*(v){<}\frac{\alpha\varepsilon}{2}$, this PBE always fails the IC.

Given a type $v_i \in \mathcal{V}$ earning in equilibrium less than $\frac{\alpha\varepsilon}{2} - \tau$, where $\tau \simeq 0 \wedge \tau \in \Re_{++}$, consider an out-of-equilibrium pair $\underline{p}, \overline{p} : \overline{p}{=}{-}\underline{p}{+}2v_i$. When $\overline{p} - \underline{p}{=}\varepsilon - \tau$, the set $BR(\mathcal{V}, \underline{p}, \overline{p})$ contains X_n^5 for sure. Regardless of this set also containing X_n^4 or X_n^6 or both, in Step I type v_i's out-of-equilibrium profits equal $\pi(\underline{p}, \overline{p} : \overline{p} - \underline{p}{=}\varepsilon - \tau, X_n^4 \wedge X_n^5 \wedge X_n^6, x_n(\widetilde{s}_n{=}\mu){=}0, v_i{=}\frac{\underline{p}+\overline{p}}{2})$ ${=}\frac{\alpha\varepsilon}{2} - \tau{>}\pi^*(v_i)$. Therefore $v_i \in \mathcal{V}\diagdown\widehat{\mathcal{V}}(\underline{p}, \overline{p})$, and the PBE fails the IC. In fact, regardless of the set $BR(\mathcal{V}\diagdown\widehat{\mathcal{V}}, \underline{p}, \overline{p})$ containing X_n^5 or $X_n^4 \wedge X_n^5$ or $X_n^5 \wedge X_n^6$ or $X_n^4 \wedge X_n^5 \wedge X_n^6$, in Step II type v_i's out-of-equilibrium profits equal $\frac{\alpha\varepsilon}{2} - \tau{>}\pi^*(v_i)$.

Unless further proprieties of a specific PBE are specified (that is, specifying neither the equilibrium payoff associated to any specific realization of $\widetilde{v}$, nor whether this realization has a positive probability to occur), no other condition for the IC failure to occur can be identified. To show this, let's define $v_l{=}\min v \in \mathcal{V}\diagdown\widehat{\mathcal{V}}(\underline{p}, \overline{p})$ and $v_h{=}\max v \in \mathcal{V}\diagdown\widehat{\mathcal{V}}(\underline{p}, \overline{p})$, and study how much a generic type v_j earns in Step II out-of-equilibrium in case $v_j \in \mathcal{V}\diagdown\widehat{\mathcal{V}}(\underline{p}, \overline{p})$. No attention in Step II is given any type $v \in \mathcal{V}\diagdown(v_l, v_h) \subset \widehat{\mathcal{V}}(\underline{p}, \overline{p})$; in fact, since in Step I she does not do better out-of-equilibrium than in equilibrium, in Step II the same result holds. Three relevant classes of out-of-equilibrium pair of prices can be identified: (i) $0 \leq \overline{p} - \underline{p}{<}\varepsilon$;

(ii) $\varepsilon \leq \overline{p} - \underline{p} < 2\varepsilon$; (iii) $2\varepsilon \leq \overline{p} - \underline{p}$.
(i.a) if $\overline{p} > -\underline{p} + 2v_h + \varepsilon$ (and therefore $\overline{p} > -\underline{p} + 2v_\kappa + \varepsilon, \forall \kappa$), then the set $BR(\mathcal{V} \diagdown \widehat{\mathcal{V}}, \underline{p}, \overline{p})$ contains only X_n^6 (uninformed clients sell at price $\underline{p}$). Thus, the lower the price $\underline{p}$ is, the greater type v_j's out-of-equilibrium profits in Step II are. Specifically, even in the extreme case in which $\underline{p} = \overline{p} - \varepsilon + \tau$, type $v_h = \frac{\underline{p} + \overline{p} - \varepsilon}{2} - \tau$ earns $\pi(\underline{p}, \overline{p} : \overline{p} - \underline{p} = \varepsilon - \tau, X_n^6, x_n(\widetilde{s}_n = \mu) < 0, v_h = \frac{\underline{p} + \overline{p} - \varepsilon}{2} - \tau) = -\tau$ out of equilibrium; when $\underline{p} > \overline{p} - \varepsilon + \tau$, she earns less; any type $v_j < \frac{\underline{p} + \overline{p} - \varepsilon}{2} - \tau$ earns less as well.
(i.b) if $\overline{p} < -\underline{p} + 2v_l - \varepsilon$ (and therefore $\overline{p} < -\underline{p} + 2v_\kappa - \varepsilon, \forall \kappa$), the solution is symmetric to the one in the sub-case above, and the conclusion is identical.
(i.c) if $-\underline{p} + 2v_l - \varepsilon \leq \overline{p} \leq -\underline{p} + 2v_h + \varepsilon$ (and therefore $\exists v_\kappa : \overline{p} \leq -\underline{p} + 2v_\kappa + \varepsilon \wedge \overline{p} \geq -\underline{p} + 2v_\kappa - \varepsilon$), then $BR(\mathcal{V} \diagdown \widehat{\mathcal{V}}, \underline{p}, \overline{p})$ contains X_n^5 for sure. When uninformed clients satisfy their liquidity needs, even in the extreme case in which $\overline{p} - \underline{p} = \varepsilon - \tau$ any type v_j earns an out-of-equilibrium profit in Step II not greater than $\frac{\alpha\varepsilon}{2} - \tau$.
(ii.a) if $v_h < \underline{p} \wedge v_h + \varepsilon < \overline{p}$ (and therefore $v_\kappa < \underline{p} \wedge v_\kappa + \varepsilon < \overline{p}, \forall \kappa$), then $BR(\mathcal{V} \diagdown \widehat{\mathcal{V}}, \underline{p}, \overline{p})$ contains only X_n^6. Since $v_\kappa < \underline{p}$, type v_j's out-of-equilibrium profits in Step II are negative, for all v_j.
(ii.b) if $\underline{p} < v_l - \varepsilon \wedge \overline{p} < v_l$, then $BR(\mathcal{V} \diagdown \widehat{\mathcal{V}}, \underline{p}, \overline{p})$ contains only X_n^4. This sub-case is symmetric to the one above, and leads to an identical conclusion.
(ii.c) if $\underline{p} \leq v_h < \underline{p} + \varepsilon$ (and therefore $v_\kappa - \varepsilon < \underline{p}, \forall \kappa \wedge \exists v_\kappa : \underline{p} \leq v_\kappa$), then $(\mathcal{V} \diagdown \widehat{\mathcal{V}}, \underline{p}, \overline{p})$ contains X_n^3 for sure. In Step II type $v_h = \underline{p} + \varepsilon - \tau$ earns $\frac{\alpha\varepsilon}{2} - \tau$ out of equilibrium; any type $v_j < \underline{p} + \varepsilon - \tau$ clearly earns less.
(ii.d) if $v_l \leq \overline{p} < v_l + \varepsilon$ (and therefore $\overline{p} < v_\kappa + \varepsilon, \forall \kappa \wedge \exists v_\kappa : \overline{p} \geq v_\kappa$), then $(\mathcal{V} \diagdown \widehat{\mathcal{V}}, \underline{p}, \overline{p})$ contains X_n^2 for sure. This sub-case is symmetric to the one above, and leads to an identical conclusion.
(ii.e) if $v_l + \varepsilon \leq \overline{p} \wedge \underline{p} \leq v_h - \varepsilon$ (and therefore $\exists v_\kappa : v_\kappa + \varepsilon \leq \overline{p} \wedge \underline{p} \leq v_\kappa - \varepsilon$), then $BR(\mathcal{V} \diagdown \widehat{\mathcal{V}}, \underline{p}, \overline{p})$ contains both X_n^2 and X_n^3 for sure. Type $v_j = \frac{\overline{p} - \underline{p}}{2}$ is the one that in Step II earns the most out of equilibrium. Specifically, when $\overline{p} - \underline{p} = 2\varepsilon - \tau$, she earns $\frac{\alpha\varepsilon}{2} - \tau$, otherwise less.
(ii.f) if $v_h - \varepsilon < \underline{p} \wedge \overline{p} < v_l + \varepsilon$ (and therefore $v_\kappa - \varepsilon < \underline{p} \wedge \overline{p} < v_\kappa + \varepsilon, \forall \kappa$), then $BR(\mathcal{V} \diagdown \widehat{\mathcal{V}}, \underline{p}, \overline{p})$ contains only X_n^5. Further conditions for the IC failure to occur might be identified; however the level of generality in this proof does not allow to assess whether in Step II every v_κ lies on Area 5; this is assessed in Proposition 4.
(iii.a) If $v_h < \underline{p}$ (and therefore $v_\kappa < \underline{p}, \forall \kappa$), then $BR(\mathcal{V} \diagdown \widehat{\mathcal{V}}, \underline{p}, \overline{p})$ contains only X_n^6. In Step II every type v_j earns negative out-of-equilibrium profits.
(iii.b) If $\overline{p} < v_l$, then $BR(\mathcal{V} \diagdown \widehat{\mathcal{V}}, \underline{p}, \overline{p})$ contains only X_n^4. Again, in Step II every type v_j earns negative out-of-equilibrium profits.
(iii.c) If $v_h - \varepsilon < \underline{p}$ (and therefore $v_\kappa - \varepsilon < \underline{p}, \forall \kappa$), then $BR(\mathcal{V} \diagdown \widehat{\mathcal{V}}, \underline{p}, \overline{p})$ contains X_n^3 for sure. In the extreme scenario in which $v_h = \underline{p} + \varepsilon - \tau$, her out-of-equilibrium profits equal $\frac{\alpha}{2}(\varepsilon - \tau)$; otherwise type v_h earns less. Clearly any other type belonging to $\mathcal{V} \diagdown \widehat{\mathcal{V}}(\underline{p}, \overline{p})$ earns less as well.
(iii.d) If $\overline{p} < v_l + \varepsilon$, then set $BR(\mathcal{V} \diagdown \widehat{\mathcal{V}}, \underline{p}, \overline{p})$ contains X_n^2 for sure. This sub-case is symmetric to the one above, and leads to an identical conclusion.
(iii.e) If $\underline{p} \leq v_h - \varepsilon \wedge v_l + \varepsilon \leq \overline{p}$ (and therefore $\exists v_\kappa : \underline{p} \leq v_\kappa - \varepsilon \wedge v_\kappa + \varepsilon \leq \overline{p}$), then $BR(\mathcal{V} \diagdown \widehat{\mathcal{V}}, \underline{p}, \overline{p})$ contains X_n^1 for sure. In Step II every v_j earns not more than zero out of equilibrium. ■

Derivation of Proposition 4. From Proposition 3 (sub-case ii.f), recall that a necessary condition for any PBE with dealers earning at least $\pi^*(v) = \alpha[\frac{\varepsilon}{2} + \varpi^*(v)]$, where $\varpi^*(\cdot) \in [0, \frac{\varepsilon}{2}]$, to fail the IC is that there exists an out-of-equilibrium price pair $\underline{p}, \overline{p} : \overline{p} = \underline{p} + \delta$ such that $BR(\mathcal{V} \diagdown \widehat{\mathcal{V}}, \underline{p}, \overline{p} : \overline{p} = \underline{p} + \delta)$ only contains X_n^5. Conditions for this to happen are listed below. If at the same time $\exists v : \overline{p} - \varepsilon < v < \underline{p} + \varepsilon \ \wedge \ \pi^*(v) < \pi(\underline{p}, \overline{p} : \overline{p} = \underline{p} + \delta, X_n^5, x_n(\widetilde{s}_n = \mu) = 0, v) = \frac{\alpha\delta}{2}$, it follows that at least a type $v \in \mathcal{V} \diagdown \widehat{\mathcal{V}}(\underline{p}, \overline{p} : \overline{p} = \underline{p} + \delta)$ does inevitably better than in equilibrium – the PBE fails the IC.

First notice the two scenarios below, in which, regardless of what α and the uninformed clients' beliefs are, the set $\mathcal{V}$ and the selection of a pair $\underline{p},\overline{p}:\overline{p}{=}\underline{p}{+}\delta$ do not allow X_n^5 to be the only reply belonging to $BR(\mathcal{V}\diagdown\widehat{\mathcal{V}},\underline{p},\overline{p}:\overline{p}{=}\underline{p}{+}\delta)$.

Scenario A $\quad \exists v:\underline{p}\leq v{<}\overline{p}-\varepsilon\wedge\overline{p}\leq\overline{v}$. Specifically, since $\overline{p}\leq\overline{v}$, in Step I it follows that $\exists v_\kappa:\overline{p}\leq v_\kappa$; this implies that $BR(\mathcal{V},\underline{p},\overline{p}:\overline{p}=\underline{p}+\delta)$ also contains X_n^4, and therefore type $v:\underline{p}\leq v{<}\overline{p}-\varepsilon$ earns more than how much she could ever achieve in equilibrium, being $\pi(\underline{p},\overline{p}:\overline{p}{=}\underline{p}{+}\delta,X_n^4,x_n(\widetilde{s}_n{=}\mu){=}0,\underline{p}\leq v{<}\overline{p}-\varepsilon){=}[\alpha(\overline{p}-v)\mid v{<}\overline{p}-\varepsilon]$ strictly greater than $\alpha\varepsilon$. Consequently, type $v:\underline{p}\leq v{<}\overline{p}-\varepsilon$ belongs to $\mathcal{V}\diagdown\widehat{\mathcal{V}}(\underline{p},\overline{p}:\overline{p}=\underline{p}+\delta)$. This means that $BR(\mathcal{V}\diagdown\widehat{\mathcal{V}},\underline{p},\overline{p}:\overline{p}=\underline{p}+\delta)$ also contains X_n^3.

Scenario B $\quad \exists v:\underline{p}{+}\varepsilon{<}v\leq\overline{p}\wedge\underline{v}\leq\underline{p}$. This case is symmetric to the one above, and the conclusion is identical.

Given a specific pair $\underline{p},\overline{p}:\overline{p}=\underline{p}+\delta$, if one of the following six conditions holds, then $BR(\mathcal{V}\diagdown\widehat{\mathcal{V}},\underline{p},\overline{p}:\overline{p}=\underline{p}{+}\delta)$ contains only X_n^5. Each condition below accounts for the remaining (relevant) scenarios others than the ones associated to *Scenario A* and *B*. In particular, the first requirement listed in each of the following conditions defines which expectations of $v\in\mathcal{V}$ given arbitrary posterior beliefs κ are reasonable, and consequently identifies what $BR(\mathcal{V},\underline{p},\overline{p}:\overline{p}=\underline{p}+\delta)$ contains. Because of what discussed in the beginning (and specified in this Proposition), it is implicit that in each of the following six scenarios at least one type $v:\overline{p}-\varepsilon{<}v{<}\underline{p}{+}\varepsilon$ also exists (this is the type that has to do inevitably better than in equilibrium), and therefore both $BR(\mathcal{V},\underline{p},\overline{p}:\overline{p}=\underline{p}+\delta)$ and $BR(\mathcal{V}\diagdown\widehat{\mathcal{V}},\underline{p},\overline{p}:\overline{p}=\underline{p}+\delta)$ always contain X_n^5. The remaining requirements guarantee that every type $v\leq\overline{p}-\varepsilon$ or $v\geq\underline{p}+\varepsilon$ cannot do better than in equilibrium by setting an out-of-equilibrium price pair $\underline{p},\overline{p}:\overline{p}=\underline{p}{+}\delta$, given that liquidity traders play any pair of strategies in $BR(\mathcal{V},\underline{p},\overline{p}:\overline{p}=\underline{p}{+}\delta)$; consequently, none of these types belongs to $\mathcal{V}\diagdown\widehat{\mathcal{V}}(\underline{p},\overline{p}:\overline{p}=\underline{p}+\delta)$.

Condition I: requirements R.I and R.a, R.b, R.c, R.d simultaneously hold, where:

$$\underline{v}{\leq}\ \underline{p}\leq\overline{p}\ {\leq}\overline{v}\ \ \wedge\ \ \nexists v:\underline{p}+\varepsilon<v\leq\overline{p}\vee\underline{p}\leq v<\overline{p}-\varepsilon, \tag{R.I}$$

$$\nexists v>\overline{p}:\frac{1-\alpha}{\alpha}{<}\ 1{+}\frac{\delta{-}\frac{\varepsilon}{2}{-}\varpi^*(v)}{v-\overline{p}}, \tag{R.a}$$

$$\nexists v<\underline{p}:\frac{1-\alpha}{\alpha}{<}\ 1{+}\frac{\delta-\frac{\varepsilon}{2}-\varpi^*(v)}{\underline{p}-v}, \tag{R.b}$$

$$\nexists v=\underline{p}+\varepsilon:\varpi^*(v)<\frac{\varepsilon}{2}, \tag{R.c}$$

$$\nexists v=\overline{p}-\varepsilon:\varpi^*(v)<\frac{\varepsilon}{2}. \tag{R.d}$$

Regardless of whether type $v{=}\underline{p}{+}\varepsilon$ and $v{=}\overline{p}-\varepsilon$ exist or not, $BR(\mathcal{V},\underline{p},\overline{p})$ contains X_n^2, X_n^3, X_n^4, X_n^5, and X_n^6.

In particular, requirement $R.a$ (or $R.b$) guarantees that, if type $v{>}\overline{p}$ (resp., $v{<}\underline{p}$) exists, then her equilibrium profits are not smaller than the maximum she could ever get out of equilibrium, which equals to $\pi(\underline{p},\overline{p}:\overline{p}{=}\underline{p}{+}\delta,X_n^6,x_n(\widetilde{s}_n{=}\mu){>}0,v{>}\overline{p}){=}\alpha(v-\underline{p}){+}(1-\alpha)(\overline{p}-v)$ (resp., $\pi(\underline{p},\overline{p}:\overline{p}{=}\underline{p}{+}\delta,X_n^4,x_n(\widetilde{s}_n{=}\mu){<}0,v{<}\underline{p}){=}\alpha(\overline{p}-v){+}(1-\alpha)(v-\underline{p})$).

Requirement $R.c$ (or $R.d$) ensures that, if type $v{=}\underline{p}{+}\varepsilon$ (resp., $v{=}\overline{p}-\varepsilon$) exists, in equilibrium this type does not achieve less than $\pi(\underline{p},\overline{p}:\overline{p}{=}\underline{p}{+}\delta,X_n^6,x_n(\widetilde{s}_n{=}\mu){=}0,v{=}\underline{p}{+}\varepsilon){=}\alpha\varepsilon$ (resp., $\pi(\underline{p},\overline{p}:\overline{p}{=}\underline{p}{+}\delta,X_n^4,x_n(\widetilde{s}_n{=}\mu){=}0,v{=}\overline{p}-\varepsilon){=}\alpha\varepsilon$), her maximum out-of-equilibrium profits.

Condition II: requirements R.II and R.e, R.f simultaneously hold, where:

$$\underline{p}{<}\underline{v}{\leq}\ \overline{v}\ {<}\overline{p}, \tag{R.II}$$

$$\nexists v : v \leq \overline{p} - \varepsilon \wedge \varpi^*(v) < \frac{\delta - \varepsilon}{2}, \tag{R.e}$$

$$\nexists v : \underline{p} + \varepsilon \leq v \wedge \varpi^*(v) < \frac{\delta - \varepsilon}{2}. \tag{R.f}$$

Requirement $R.II$ allows for the possibility of $BR(\mathcal{V}, \underline{p}, \overline{p})$ to contain X_n^2 and/or X_n^3 depending on the specific circumstances. This however does not change the highest out-of-equilibrium profits, which are associated to X_n^5.

Requirement $R.e$ (or $R.f$) ensures that every type $v \leq \overline{p} - \varepsilon$ (resp.,$v \geq \underline{p} + \varepsilon$) in equilibrium does not earn less than $\pi(\underline{p}, \overline{p} : \overline{p}{=}\underline{p}{+}\delta, X_n^5, x_n(\widetilde{s}_n{=}\mu){=}0, \underline{p}{<}v{<}\overline{p}){=}\frac{\alpha\delta}{2}$, which is the maximum she can get out of equilibrium.

Condition III: requirements R.III, and R.d, R.g, R.h simultaneously hold, where:

$$\underline{v} = \overline{p} - \varepsilon \wedge \overline{p} \leq \overline{v}, \tag{R.III}$$

$$\nexists v > \overline{p} : \alpha[\frac{\varepsilon}{2} + \varpi^*(v)] < \frac{\alpha}{2}(v - \underline{p}) + (1 - \alpha)(\underbrace{\overline{p} - v}_{<0}), \tag{R.g}$$

$$\nexists v : \underline{p} + \varepsilon \leq v \leq \overline{p} \wedge \varpi^*(v) < \frac{\delta - \varepsilon}{2}. \tag{R.h}$$

In particular, requirement $R.III$ guarantees that $BR(\mathcal{V}, \underline{p}, \overline{p})$ contains X_n^2, X_n^3, X_n^4, X_n^5. Although the set $BR(\mathcal{V}, \underline{p}, \overline{p})$ is smaller than the one associated to *Condition I*, the reply X_n^4 is still an option. Therefore requirement $R.d$ guarantees that out of equilibrium type $v = \overline{p} - \varepsilon$ does not earn more out of equilibrium.

Requirement $R.g$ (or $R.h$) guarantees that, if any type $v{>}\overline{p}$ (resp., $v : \underline{p}{+}\varepsilon \leq v \leq \overline{p}$) exists, the profits that this type earns in equilibrium are not lower than the highest she can achieve out of equilibrium, which equal to: $\pi(\underline{p}, \overline{p} : \overline{p}{=}\underline{p}{+}\delta, X_n^3, x_n(\widetilde{s}_n{=}\mu){>}0, v{>}\overline{p})$ (resp., $\pi(\underline{p}, \overline{p} : \overline{p}{=}\underline{p}{+}\delta, X_n^5, x_n(\widetilde{s}_n{=}\mu){=}0, \underline{p} + \varepsilon \leq v \leq \overline{p}){=}\alpha\frac{\delta}{2})$.

Condition IV: requirements R.IV and R.h, R.i simultaneously hold, where:

$$\overline{p} - \varepsilon < \underline{v} \wedge \overline{p} \leq \overline{v}, \tag{R.IV}$$

$$\nexists v > \overline{p} : \frac{\delta - \varepsilon}{2} - \varpi^*(v) > \frac{1 - \alpha}{\alpha}(v - \overline{p}). \tag{R.i}$$

Requirement $R.IV$ guarantees that $BR(\mathcal{V}, \underline{p}, \overline{p})$ contains X_n^2, X_n^4, and X_n^5.

As in *Condition III*, the set $BR(\mathcal{V}, \underline{p}, \overline{p})$ contains X_n^5, and therefore requirement $R.h$ still guarantees that, if any type $v : \underline{p}{+}\varepsilon \leq v \leq \overline{p}$ exists, she earns more than out of equilibrium.

Requirement $R.i$ ensures that there exists no type $v{>}\overline{p}$ that in equilibrium earns less than the highest out-of-equilibrium profits she can get, namely: $\pi(\underline{p}, \overline{p} : \overline{p}{=}\underline{p}{+}\delta, X_n^5, x_n(\widetilde{s}_n{=}\mu){>}0, v{>}\overline{p})$ $=\alpha\frac{\delta}{2}{+}(1 - \alpha)(\overline{p} - v)$.

Condition V: requirements R.V and R.c, R.l, R.m simultaneously hold, where:

$$\underline{v} \leq \underline{p} \wedge \overline{v} = \underline{p} + \varepsilon, \tag{R.V}$$

$$\nexists v < \underline{p} : \alpha[\frac{\varepsilon}{2} + \varpi^*(v)] < \frac{\alpha}{2}(\overline{p} - v) + (1 - \alpha)(\underbrace{v - \underline{p}}_{<0}), \tag{R.l}$$

$$\nexists v : \underline{p} \leq v \leq \overline{p} - \varepsilon \wedge \varpi^*(v) < \frac{\delta - \varepsilon}{2}. \tag{R.m}$$

The derivation of these requirements is symmetric to the one in *Condition III*.

Condition VI: requirements R.VI and R.m, R.n simultaneously hold, where:

$$\underline{v} < \underline{p} \wedge \overline{v} < \underline{p} + \varepsilon, \tag{R.VI}$$

$$\nexists v < \underline{p} : \frac{\delta-\varepsilon}{2} - \varpi^*(v) > \frac{1-\alpha}{\alpha}(\underline{p}-v). \tag{R.n}$$

The derivation of these requirements is symmetric to the one in *Condition IV.* ∎

Proof of Proposition 5. When $\widetilde{v}=v \in [E[\widetilde{v}]-\varepsilon, E[\widetilde{v}]+\varepsilon]$, M sets $\underline{p}=E[\widetilde{v}]-\varepsilon \wedge \overline{p}=E[\widetilde{v}]+\varepsilon$. In this way she extracts the maximum surplus from type $\widetilde{s}_n \neq \eta$, and does not loose from type $\widetilde{s}_n=\eta$.

When $\widetilde{v}=v \notin [E[\widetilde{v}]-\varepsilon, E[\widetilde{v}]+\varepsilon]$, M can either: (i) set $\underline{p}=E[\widetilde{v}]-\varepsilon \wedge \overline{p}=E[\widetilde{v}]+\varepsilon$; (ii) adjust $\underline{p}$ or $\overline{p}$, in such a way that type $\widetilde{s}_n=\eta$ does not trade any more. Suppose $\widetilde{v}=v<-\varepsilon+E[\widetilde{v}]$ (the argument is symmetric for $\widetilde{v}=v>E[\widetilde{v}]+\varepsilon$). In sub-case (i), we have that: $\widetilde{s}_n=\varepsilon \rightarrow x_n>0$, $\widetilde{s}_n=-\varepsilon \rightarrow x_n<0$, $\widetilde{s}_n=\eta \rightarrow x_n<0$. M earns $\alpha\beta(\overline{p}-v)+[\alpha(1-\beta)+(1-\alpha)](v-\underline{p})$. In sub-case (ii) the best thing for M is to set $\underline{p} \leq v<E[\widetilde{v}]-\varepsilon \wedge \overline{p}=E[\widetilde{v}]+\varepsilon$, so that type $\widetilde{s}_n=\eta$ does not trade any more. Notice however that even type $\widetilde{s}_n=-\varepsilon$ does not trade. M earns $\alpha\beta(\overline{p}-v)$, which is more than what she earns in sub-case (i). In fact: $\alpha\beta(\overline{p}-v)+[\alpha(1-\beta)+(1-\alpha)](v-\underline{p})<\alpha\beta(\overline{p}-v) \therefore 1>\alpha\beta$, which always holds. Finally, setting $\underline{p} \leq v< \overline{p}<E[\widetilde{v}]+\varepsilon$ is clearly dominated as well. ∎

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