Commitment in Private Equity Partnerships

Albert Banal-Estañol  
*Universitat Pompeu Fabra*

Filippo Ippolito*  
*Bocconi University*

albert.banaletanol@upf.edu  
filippo.ippolito@unibocconi.it

January 31, 2011

Abstract

In this paper we analyze the optimal degree of commitment in private equity partnerships. In the two-period framework of Laffont and Tirole (1987, 1990), we allow for an outside investment opportunity that becomes available to limited partners (LPs) in the second period. We show that the optimum contract requires the inclusion of special clauses to account for the possibility of an early exit of LPs from the contract at the end of the first period. In some circumstances, these clauses include default penalties to LPs for not honouring capital calls, while in others they require early termination provisions, such as no-fault divorce clauses. The model also predicts management fees that are proportional to the capital under management, a carried interest based on performance, and clawback provisions.

**JEL Classifications:** D82, D86, G23, G24, J33

**Keywords:** private equity partnership, commitment, carried interest, management fees, default penalties, no-fault divorce provision, clawback provision

---

*We are especially grateful to Alan Morrison, and to two anonymous referees for useful comments and feedback. We also thank seminar participants at Instituto de Empresa and Pompeu Fabra. All remaining errors are our own.

* Contact author. Address: Department of Finance, Bocconi University, Via Roentgen 1, 20136 Milan, Italy. Tel: +39 02 5836 ext 5918.
Commitment in Private Equity Partnerships

January 31, 2011

Abstract

In this paper we analyze the optimal degree of commitment in private equity partnerships. In the two-period framework of Laffont and Tirole (1987, 1990), we allow for an outside investment opportunity that becomes available to limited partners (LPs) in the second period. We show that the optimum contract requires the inclusion of special clauses to account for the possibility of an early exit of LPs from the contract at the end of the first period. In some circumstances, these clauses include default penalties to LPs for not honouring capital calls, while in others they require early termination provisions, such as no-fault divorce clauses. The model also predicts management fees that are proportional to the capital under management, a carried interest based on performance, and clawback provisions.

JEL Classifications: D82, D86, G23, G24, J33

Keywords: private equity partnership, commitment, carried interest, management fees, default penalties, no-fault divorce provision, clawback provision
1 Introduction

Limited partnerships constitute the standard vehicle for investment in private equity (PE). They are structured as closed-end funds with limited tradeability of shares and a fixed life, usually ten years. Capital is committed in full by the limited partners (LP) at the fund’s inception. However, only a small portion of this capital (usually between 5% to 15%) becomes immediately available for investment. The rest is supplied in installments over a several-year period. Although GPs and LPs could agree on a single up-front payment, this virtually never happens.

In this paper we question what it means for LPs to commit capital in a PE partnership. Does it mean that LPs are bound by an agreement that is extremely costly to break? What happens if they do not honour future capital calls? Can commitment be renegotiated? Historically, there have been few cases of defaults on capital calls that have ended in court ((Meerkatt and Liechtenstein 2009)). Perhaps, when choosing to default on an obligation to a fund, LPs give a lot of importance to issues like reputation and what breaking a contract would imply for their future investment opportunities. Or, perhaps partnership agreements allow for more flexibility than it may first appear. The recent financial crisis has offered several examples of well known institutions nearing default on their capital calls.\(^1\) Quite often LPs have been allowed to reduce their capital commitments, or to renegotiate the contractual terms of the agreement, so to avoid default ((Goldstein, Berger, Arora, Lillis, and Naylor 2009)).

A closer examination at partnership agreements reveals a large degree of het-

\(^1\)See Private Equity Insider, “Cash-Poor LPs Face Capital-Call Pressure”, 11 June 2008.
erogeneity in the contractual clauses that regulate the draw down of capital. On the one hand, a number of clauses against early termination or renegotiation are typically included in PE partnerships. The contractual agreement can for example include *default penalties* for LPs that default on a capital call. On the other, the stringency of these clauses may vary significantly from one contract to another. (Lerner, Hardymon, and Leamon 2005) show that penalty provisions for not honouring a capital call can range from very harsh (“reducing the defaulter’s account to zero”) to very mild (“defaulter is excused from making a contribution but retains the right to make other contributions in the future”).

The agreement may even include explicit early termination provisions, such as the *no-fault divorce* clause, which allows the LPs the right to terminate the partnership at any time of the life of the partnership at no cost. These clauses are very important as they are present in 44 percent of venture funds and in 60 percent of buyout funds ((Toll 1991)).

We provide a formal examination of the costs and benefits of commitment in private equity partnerships. On the one hand, a low degree of commitment allows LPs to terminate or renegotiate the contractual terms of the partnership agreement if GPs do not perform well enough. Having freed capital, LPs may then choose more profitable investment opportunities elsewhere. On the other hand, a high degree of commitment allows for more efficient contracting by reducing the information rents.

---

2 A recent example of the consequences of defaulting on a capital call is the lawsuit of CapGen Capital Advisors LLC against two of its LPs. CapGen is seeking payment of the outstanding capital contributions with interest and a court order compelling the LPs to make all future capital contributions (plus payment of the funds’ litigation expenses).

that GPs earn in equilibrium.\textsuperscript{4}

We provide a model which describes the interaction between LPs and GPs as a
dynamic principal-agent game characterized by adverse selection and moral hazard.
We consider three possible contracting strategies: full commitment, commitment
and renegotiation, and no commitment. With full commitment, contracts last two
periods and cannot be renegotiated. With commitment and renegotiation, contracts
last two period, but can be renegotiated at the beginning of the second period. With
no commitment, contracts last only one period and continuation occurs in the second
period only if the contracting parties wish to carry on.

We extend the two-period framework of Laffont and Tirole (1987, 1990) to in-
clude a stochastic reservation utility for the principal (the LP in our case), which is
modeled as an outside investment opportunity that becomes known only in the second
period. We show that due to the outside option, all types of contracts can be optimal
in some circumstances. Intuitively, commitment (with and without renegotiation)
dominates when the outside options have low value and the degree of information
asymmetry is large. On the contrary, one period contracts with no commitment are
preferable if the outside options in period two have a lot of value and there is little
asymmetry in the information available to LPs and GPs. We should then expect the
harshness of default penalties to be related to the value of outside options and to
asymmetric information. Lower outside options and higher information asymmetry
requires the inclusion of more severe default penalties that increase the cost of rene-

\textsuperscript{4}The principle applied here is that “in repeated-principal agent models, long-term contracts can
improve on short-term contracts only if they commit either principal or agent to a payoff in some
future circumstances lower than could be obtained from a short-term contract negotiated if that
circumstance occurs.” ((Malcomson and Spinnewyn 1998))
gotiation and ensure continuation. On the other hand, higher outside options and lower information asymmetry calls for a reduction in the cost of renegotiation. This can be achieved by including early termination provisions such as no-fault divorce clauses.

Furthermore, the optimum contracts predicted by the model resemble the contracts observed in reality in several other respects. We predict that GPs should be compensated with management fees that are proportional to the capital under management, and receive a carried interest based on the performance of the fund. They should also be subject to clawback provisions which require GPs to pay back some previously earned interest to LPs when performance is poor. More efficient GPs earn higher fixed fees than less efficient ones, on top of management fees and carried interest.

The model is, therefore, capable of providing a fairly accurate description of how GPs are compensated in the real world, and draws several untested empirical predictions about how the fee structure should vary for different types of GPs. It predicts that the carried interest and clawbacks should be on a deal-by-deal basis for less efficient GPs, while the they can be paid entirely at the end of the contract for more efficient GPs. The model also draws predictions on the evolution of management fees over the life of the contract.

In our analysis of long-term contracts we benefit primarily from the above cited work of Laffont and Tirole and from (Laffont and Martimort 2002). Indirectly, we borrow ideas about dynamic contracts from (Dewatripont, Jewitt, and Tirole 1999), (Holmström 1999), (Gibbons and Murphy 1992), (Lambert 1993), (Malcomson and
Spinnewyn 1998) and (Rey and Salanie 1990).\(^5\) (Axelson, Strömberg, and Weisbach 2009) (ASW) also explains similar features of investment in a PE fund. There are three main differences between this paper and ASW. The first is that ASW is a pure moral hazard model, while this is primarily an asymmetric information model which also accounts for moral hazard. The second one is that this paper looks at temporal capital commitment in PE (given that LPs funded an investment in period 1, will they also fund an investment in period 2?), while ASW are looking at why LPs commit the money to fund several investments (i.e. there is no temporal dimension, the investments could take place all at the same time). Third, our model draws predictions about early termination provisions and default penalties, and about the evolution of the fee structure over time, which are not discussed by ASW.

Also closely related to our study are (Gompers and Lerner 1999a) and (Lerner and Schoar 2004). Both papers examine how the compensation of a GP evolves from one fund to a successor fund. On the contrary, our analysis focuses on the relationship between compensation and incentives during the life of a fund and not across successor funds. By breaking the life of the fund into single investments, we are able to provide an explanation for the illiquidity of the securities held by LPs which is alternative to that of Lerner and Schoar. Among other literature on PE contracts it is worth mentioning (Casamatta 2003), (Cornelli and Yosha 2003), (Gompers 1996), (Gompers and Lerner 1999b), (Gompers 1995), (Hellmann 1998), (Kaplan and Strömberg 2003), (Kaplan and Strömberg 2004), (Sahlman 1990) and (Schmidt 2003).

\(^5\)See also (Bernardo, Cai, and Luo 2004) for an application of commitment to capital budgeting.
The rest of the paper is organized as follows: in Section 2 we present the basic structure of the model. In Section 3, 4 and 5, we examine respectively long-term contracts with commitment, with commitment and renegotiation, and short-term contracts (no commitment). In Section 6 and 7, we do an analytical and numerical comparison of the three types of contracts. In Section 8, we draw the empirical implications of the model. Finally, Section 9 concludes. The appendix contains the proofs. The web Appendix (available upon request) contains specifications used for the numerical simulation.

2 The model

2.1 Setup

A GP (the agent) has the possibility to invest capital on behalf of an LP (the principal) in two consecutive periods. In each period, an investment $k$ generates a high payoff $\hat{R}_h(k, \theta)$ with probability $\pi_e$ and a low payoff $\hat{R}_l(k, \theta)$ ($\leq \hat{R}_h(k, \theta)$) with probability $1 - \pi_e$. Investment is subject to moral hazard, as the probability of a high return $\pi_e$ increases with a privately observable effort, $e \in \{l, h\}$, exerted by the GP, i.e. $\pi_h = \pi_l + \Delta \pi \geq \pi_l$. The monetary value of the disutility of effort $\psi$ is assumed to be sufficiently low to ensure that providing effort is optimal.

Investment is also subject to adverse selection, as the returns on investment depend on the GP’s level of (in)efficiency, $\theta$, which is non-observable to the LP. For simplicity, assume that $\hat{R}_h(k, \theta) \equiv R_h(k) - \theta k$ and $\hat{R}_l(k, \theta) \equiv R_l(k) - \theta k$ where $R_h(k)$ and $R_l(k)$ are publicly observed and satisfy $R'_h(0), R'_l(0) > 1$, $R'_h(k), R'_l(k) > 0$.
and \( R^m_h(k), R^m_l(k) < 0 \) (ensuring the existence of a unique optimum investment) and 
\[ \theta \in \{ \theta, \overline{\theta} \} \text{ where } \overline{\theta} = \Delta \theta + \theta > \theta \text{ is only observed by the GP.} \]
A GP is of type \( \theta \) ("efficient") with a common knowledge probability \( \nu_1 \) and of type \( \overline{\theta} \) ("inefficient") with probability \( 1 - \nu_1 \). The GP’s marginal cost to manage capital is constant and therefore normalized to 0. Both LPs and GPs are risk neutral.

We assume that LPs have bargaining power when they stipulate an agreement with a GP. We believe this assumption to be more reasonable than the alternative — allocating bargaining power to GPs — because it reflects the historical shift in bargaining power from GPs to LPs observed in reality. This has been the result of the increasing use of gatekeepers and of the wider role played by institutional investors. Prime examples of strong LPs are CalPERS and the Oregon Public Employees’ Retirement Fund which are adopting standardized sets of principles for the structuring of PE partnerships, such as the ones defined by the Institutional Limited Partners Association (ILPA).

Between the two periods an outside investment opportunity \( I \geq 0 \) becomes available to the LP. Thus, her reservation utility changes from 0 in the first period to \( I \) in the second period. \( I \) is distributed with a density \( f(I) \) and a cumulative distribution function \( F(I) \) and becomes privately observable to the LP only at the

---

\(^6\)An alternative presentation of the same model is to say that returns are publicly observable, \( \tilde{R}_h(k, \theta) = R_h(k) \) and \( \tilde{R}_l(k, \theta) = R_l(k) \), and the GP has a cost to manage capital, \( C(k) = \theta k \), which depends on her ability \( \theta \), which is known to herself but unobservable to the LPs. We follow the specification provided in the main text as we find it more intuitive.

\(^7\)For more information on the issue of bargaining power see Albert J. Hudec "Negotiating Private Equity Fund Terms. The Shifting Balance of Power", Business Law Today, Volume 19, Number 5 May/June 2010; D. Peninon "The GP-LP Relationship: At the Heart of Private Equity." AltAssets, 22 January 2003; and ILPA Private Equity Principles (January 2011) downloadable from the ILPA website.
There are three contracting strategies. First, using “long-term contracts with commitment”, the two parties sign a binding long-term contract which cannot be renegotiated. Second, using “long-term contracts with commitment and renegotiation”, the two parties sign a long-term contract, which can be renegotiated if both parties agree to do so. Third, using a series of short-term contracts, the two parties have the option but not the obligation to continue in the agreement.

For each contracting strategy, the LP offers a take-it-or-leave-it menu of contracts to the GP, as the LP has bargaining power. For each period, a contract specifies the level of investment, \( k \), and the state dependent compensations \( t_h \) and \( t_l \) for the GP in exchange of repayments \( r_h \) and \( r_l \) to the LP, respectively for the high and low state realization of \( R \). In sum, for any investment level \( k \), the rents to GPs are \( t_h + \hat{R}_h - r_h = t_h + R_h(k) - \theta k - r_h \) in the case of high returns (and analogously in the case of low returns). Without loss of generality (transferring payments from \( t_h \) to \( r_h \)), we set \( r_h = R_h(k) \) and \( r_l = R_l(k) \). As a result, \( t_h \) and \( t_l \) represent the transfers gross of inefficiency losses.\(^9\)

The timing of contracting is summarized in Figure 1. At \( t_0 \), period one begins and the GP invest \( k \) for one period; at \( t_1 \) effort is chosen by the GP for the first period; at \( t_2 \) the payoff \( R \) is observed and the GP is compensated; at \( t_3 \) period two begins, \( I \) is observed and the game may terminate. In case of no termination

---

\(^8\)The way we model the outside investment resembles the "deepening investment" of (Holmström and Tirole 1998).

\(^9\)An alternative way to present the compensation structure is to set \( r_h = R_h(k) - \overline{\theta} k \) and \( r_l = R_l(k) - \overline{\theta} k \). Then the rents would be \( t_h + \Delta \theta k \) for a high type and \( t_h \) for a low type, respectively in case of high and low returns.
(continuation), a new investment is made ($k$ may be different from the first period). At $t_4$ the GP chooses effort for the second period. At $t_5$ the payoff $R$ of the second period is observed and the GP is compensated. The common discount factor between the two periods is $0 \leq \delta \leq 1$.

### 2.2 Moral hazard

A contract $(r_h, r_l, k)$ induces effort only when a GP’s moral hazard incentive constraint is satisfied. That is, when the compensation is higher when she exerts effort than when she does not, i.e.

$$
\pi_1 r_h + (1 - \pi_h) r_l - \theta k - \psi \geq \pi_l t_h + (1 - \pi_l) t_l - \theta k
$$

(1)
which simplifies to
\[ t_h - t_l \geq \frac{\psi}{\Delta \pi}. \] \hspace{1cm} (2)

That is, the minimum differential between high and low compensation should be \( \psi/\Delta \pi \). In an optimal contract that requires effort condition (2) must be satisfied. In what follows we denote the expected payment as \( t \equiv \pi_h t_h + (1 - \pi_h) t_l \) and the expected return as \( R(k) - \theta k \), where \( R(k) \equiv \pi_h R_h(k) + (1 - \pi_h) R_l(k) \).

3 Long-term contracts with full commitment

In this section, we assume that the two parties sign a binding long-term contract. Under full commitment, the LP and the manger do not have the option to renegotiate the terms of the contracts in the second period. Commitment prevents parties from behaving opportunistically ex post and thus promotes efficient conduct ex ante. However, it also prevents them from taking advantage of new opportunities. Given the non-contractibility of \( I \), commitment implies that the LP cannot exit at \( t_3 \).

The LP should optimally offer a menu of two long-term contracts specifying payments and investments in each period, \((t_1, t_2, k_1, k_2)\) and \((\overline{t}_1, \overline{t}_2, \overline{k}_1, \overline{k}_2)\), which will be taken respectively by the efficient and inefficient GP. The maximization of the LP is given by

\[
\max_{\{t_1, t_2, k_1, k_2, \overline{t}_1, \overline{t}_2, \overline{k}_1, \overline{k}_2\}} \nu_1 \left[ R(k_1) - t_1 \right] + (1 - \nu_1) \left[ R(k_1) - \overline{t}_1 \right] \\
\quad + \delta \left( \nu_1 \left[ R(k_2) - t_2 \right] + (1 - \nu_1) \left[ R(k_2) - \overline{t}_2 \right] \right)
\]
subject to the intertemporal incentive constraint of efficient GPs,

\[ t_1 - \theta k_1 + \delta (t_2 - \theta k_2) \geq \bar{t}_1 - \bar{\theta} \bar{k}_1 + \delta (\bar{t}_2 - \bar{\theta} \bar{k}_2), \tag{3} \]

the intertemporal participation constraint of an inefficient GP

\[ \bar{t}_1 - \bar{\theta} \bar{k}_1 + \delta (\bar{t}_2 - \bar{\theta} \bar{k}_2) - (1 + \delta) \psi \geq 0. \tag{4} \]

and the incentive constraint of an inefficient GP and the participation constraint of efficient GPs (which will be always satisfied). The solution is as follows:

**Proposition 1** The optimum menu of contracts with full commitment requires:

\[
\begin{align*}
t_1 + \delta t_2 &= (1 + \delta) \left( \theta k^{FB} + \Delta \theta \bar{k}^{SB} (\nu_1) + \psi \right) \quad k_1 = k_2 = k^{FB} \\
\bar{t}_1 + \delta \bar{t}_2 &= (1 + \delta) \left( \theta \bar{k}^{SB} (\nu_1) + \psi \right) \quad \bar{k}_1 = \bar{k}_2 = \bar{k}^{SB} (\nu_1)
\end{align*}
\]

where \( k^{FB} \) is the unique \( k \) that satisfies \( R'(k) = \theta \) and \( \bar{k}^{SB} (\nu) \) is the unique \( k \) that satisfies, for any \( \nu \),

\[ R'(k) = \bar{\theta} + \frac{\nu}{1 - \nu} \Delta \theta. \tag{5} \]

Under this menu of contracts, efficient GPs invest, in both periods, an efficient level, i.e. the amount that equates marginal revenues with marginal cost. The inefficient GP, however, invests less than the efficient level, \( \bar{k}^{SB} (\nu_1) < k^{FB} \), where \( k^{FB} \) is the unique \( k \) that satisfies \( R'(k) = \bar{\theta} \). Both contracts include a compensation for effort \( \psi \). The contract designed for an efficient GP also includes an adverse selection rent \( \Delta \theta \bar{k}^{SB} (\nu_1) \) in each period. As in the classic static problem, the LP
trades off efficiency and incentives. The LP sets the optimal $\bar{k}$ balancing efficiency, which would require $\bar{k} = \bar{k}^{FB}$ and rent minimization, which would require $\bar{k} = 0$. The capital committed to efficient GPs is always larger than that of inefficient GPs. This follows from the fact that $\bar{k}^{FB} \geq \bar{k}^{SB}(\nu)$. The surplus from implementing the first- and second-best levels of investment are $R^{FB} \equiv R(\bar{k}^{FB}) - \theta k^{FB}$, $\overline{R}^{FB} \equiv R(\bar{k}^{FB}) - \overline{\theta k}^{FB}$, and $\overline{R}^{SB}(\nu) \equiv R(\bar{k}^{SB}(\nu)) - \overline{\theta k}^{SB}(\nu)$.

4 Long-term contracts with commitment and renegotiation

Without full commitment long-term contracts are generally not sequentially optimal or renegotiation-proof. That is, in the process of implementing a long-term contract, the parties may be better off modifying the initial contract and negotiating a new one. In this section, we examine the case in which the two parties sign a long-term contract, which can be renegotiated at time $t_3$ if the parties agree to do so. Following Laffont and Tirole (1990), we are then in the world of renegotiation and commitment.

We proceed in three steps: first, we derive the sub-game perfect second-period contract. Second, we identify the highest $I$ below which continuation always occurs. Intuitively, for a high enough $I$ the LP wants to abandon the long-term contract and obtain $I$. Third, we derive the long-term optimum contract chosen at $t_0$. This contract accounts for possible renegotiations (sub-game perfection) and early exit.
4.1 Renegotiation-proof contracts

This section derives the conditions under which a long-term contract stipulated at \( t_0 \) is renegotiation-proof. The issue of renegotiation proofness in long-term contracts is relevant here, and not in the previous section, because here we do not have full commitment. Under full commitment, renegotiations are assumed away.

Suppose that, at the beginning of the second period, the LP updated belief's that the GP is efficient is \( \nu_2 \). In general full separation may not occur in the first period, which implies that at the beginning of the second period the LP cannot distinguish between types with certainty. Two cases must be considered for the second period: the LP wants 1) both types to invest, or 2) only the efficient type to invest.\(^{10}\)

Consider first the case in which the LP wants both types to invest in the second period. Let \( M_0 \) and \( \overline{M}_0 \) be the second-period rents of the efficient and inefficient GPs (not including the foregone first-period payment and the disutility of effort) specified by the initial contract binding the parties. Without loss of generality, we assume that \( \overline{M}_0 = 0 \), by adjusting if necessary the first-period payments. A second-period short-term renegotiation contract must provide efficient GPs with at least \( M_0 \). Again, the LP should offer a menu of two contracts, \((t_2, k_2), (\overline{t}_2, \overline{k}_2)\), which will

\(^{10}\)In Section 3, we implicitly assumed that the probability of finding an inefficient GP was not too small: for, above some cut-off level of \( \nu_1 \), the LP would choose not to let the inefficient GP invest at all. Here, however, the second-period beliefs of dealing with an efficient GP might be close to 1 even though the prior beliefs are not assumed to be so. However, we will show that along the equilibrium path either \( \nu_2 \leq \nu_1 \) (and then our assumption implies that both types of GPs might be kept) or \( \nu_2 = 1 \) (and then only the efficient GP is relevant). It can be shown that for any \( \nu_1 \) under some cut-off level, the equilibrium is as described in this paper.
be taken up by the efficient and inefficient GPs, and solve

$$\max_{\{t_2, k_2, \bar{t}_2, \bar{k}_2\}} \nu_2 [R (k_2) - t_2] + (1 - \nu_2) [R (\bar{k}_2) - \bar{t}_2]$$ \hspace{1cm} (6)

subject to the participation condition of an inefficient GP, the incentive compatibility condition of efficient GPs, and the renegotiation condition of efficient GPs, which are, respectively, $\bar{t}_2 - \bar{k}_2 \geq 0$

$$t_2 - \theta k_2 \geq \bar{t}_2 - \theta \bar{k}_2$$ \hspace{1cm} (7)

and

$$t_2 - \theta k_2 - \psi \geq M_0.$$ \hspace{1cm} (8)

The following propositions summarize under which conditions renegotiation will not take place (renegotiation proofness).

**Proposition 2** A long-term contract in which both types invest in the second period is renegotiation-proof iff $\Delta \theta k^S_B (\nu_2) \leq M_0 \leq \Delta \theta k^F_B$, where $M_0$ is the second-period adverse selection rent of efficient GPs. Moreover

(a) If $\Delta \theta k^S_B (\nu_2) \leq M_0 < \Delta \theta k^F_B$ then

$$t_2 = \theta k^F_B + M_0 + \psi \quad k_2 = k^F_B$$

$$\bar{t}_2 = \theta M_0 / \Delta \theta + \psi \quad \bar{k}_2 = M_0 / \Delta \theta.$$
(b) If $\Delta \theta \tilde{k}^{FB} \leq M_0 \leq \Delta \theta \bar{k}^{FB}$ then

\[
\begin{align*}
    t_2 &= \theta k^{FB} + \Delta \theta \tilde{k}^{FB} + \psi \quad k_2 = \bar{k}^{FB} \\
    \bar{t}_2 &= \theta \bar{k}^{FB} + \psi \quad \bar{k}_2 = \bar{k}^{FB}.
\end{align*}
\]

If $M_0 < \Delta \theta \tilde{k}^{SB} (\nu_2)$, efficient GPs want to mimic an inefficient GP. Therefore, she must be offered at least $\Delta \theta \tilde{k}^{SB} (\nu_2)$ in the second period.

This proposition tells us that the rents $M_0$ offered to efficient GPs in the long-term contract stipulated at $t_0$ are constrained by the renegotiations that can occur in the second period. In particular, $M_0$ cannot be too low. Contracts are in general not renegotiation-proof. The analogous of Proposition 2 when only an efficient GP is employed in the second period is as follows:

**Proposition 3** A long-term contract in which only the efficient types invest in the second period requires $t_2 = \theta k^{FB} + M_0 + \psi$ and $k_2 = \bar{k}^{FB}$, where $M_0$ is the second-period adverse selection rent of efficient GPs.

In this case, as the LP ignores the incentive constraint of efficient GPs, the renegotiation condition is not binding and $M_0$ is not subject to any constraint. In this case, contracts are always renegotiation-proof.

### 4.2 Exit options

In the absence of full commitment, a long-term contract not only can be renegotiated, but it can also be terminated at $t_3$. This happens if the outside investment
opportunities are sufficiently high. We now derive the maximum $I$ below which continuation occurs. We should again consider the two scenarios of Propositions 2 and 3. In both cases, at time $t_3$, after observing $I$ the LP may choose 1) to continue with the long-term contract as originally agreed, 2) to continue only with efficient GPs and exit with an inefficient GP, or 3) to exit regardless of what type of GP she is facing.

**Case 1: Continuation with both types** In Proposition 2, we showed that a renegotiation-proof contract should have $k_2 = k^{FB}$, $t_2 = \theta k^{FB} + M_0 + \psi$, and $\bar{t}_2 = \bar{\theta} k_2 + \psi$ with $k_2 = M_0/\Delta \theta$ (case (a)) or $\bar{k}^{FB}$ (case (b)). Therefore, when continuation occurs with both types, the expected profits are $\nu_2 \left( R^{FB} - M_0 \right) + (1 - \nu_2) \left( R(\bar{k}_2) - \bar{\theta} \bar{k}_2 \right) - \psi$.

**Case 2: Continuation only with efficient GPs** Suppose the LP wants to continue only with efficient GPs, then she will make an offer to terminate the contract that only an inefficient GP accepts. From Proposition 2 efficient GPs earn at least $M_0 + \psi$ in the second period, while the inefficient type earns only $\psi$. If the LP offers an exit fee equal to $\psi$, only an inefficient GP accepts it. Therefore, continuation occurs only if the type is efficient and the expected profits are $\nu_2 \left( R^{FB} - M_0 \right) + (1 - \nu_2) I - \psi$.

**Case 3: Exit regardless of GP** Suppose the LP wants to terminate the contract regardless of type. She makes an offer that both types accept. As the LP does not know with certainty the type she is facing, she must offer at least $M_0 + \psi$ to obtain
certain termination. In this case the expected profit of the LP is $I - M_0 - \psi$.

We can now identify under what conditions each of these continuation/exit strategies prevails.

**Lemma 4** In the long-term contract of Proposition 2, the LP continues with both types if $I \leq R(\overline{k}_2) - \overline{\sigma}k_2$; continues only with efficient GPs if $R(\overline{k}_2) - \overline{\sigma}k_2 < I < R^{FB} + M_0(1 - \nu_2)/\nu_2$; exits regardless of type if $I \geq R^{FB} + M_0(1 - \nu_2)/\nu_2$.

Following similar reasonings we obtain the following lemma.

**Lemma 5** In the long-term contract of Proposition 3, the LP continues if $I < R^{FB}$ and exits if $I \geq R^{FB}$.

The two lemmas identify the thresholds for $I$ above and below which different contracting strategies apply. In order to clarify, the first lemma applies to a long-term contract in which both types invest in the second period. This means that if the contract is not exited at $t_3$, continuation occurs with both types and Proposition 2 applies. In the second period there are then three options (continuation with both, with only the efficient type and exit with either type), which call for two exit thresholds, as reported in Lemma 1. If instead the contract stipulates that continuation occurs only if the type is efficient, then Proposition 3 applies. Then, there are only two options (continuation with the efficient type or exit), which call for one exit threshold only, as reported in Lemma 2. In sum, the long-term contract stipulated at $t_0$ affects the continuation options, the conditions for renegotiation-proofness, and the exit thresholds.
4.3 The optimal long-term contract without commitment

We now derive the optimum long-term contract stipulated at $t_0$. This contract is designed to maximize the expected profit of the LP over the two periods. The contract will in general separate efficient from inefficient GPs, and will account for the (endogenous) renegotiation-proofness conditions and exit thresholds. More precisely, the LP can offer a choice between two contracts, one chosen by efficient GPs, and the other chosen by an inefficient GP, and possibly also by efficient GPs. The ability to commit, despite the renegotiation-proofness condition, enables the LP to neglect an inefficient GP’s incentive constraint. The inefficient GP always tells the truth, while the efficient one may choose to lie and mimic the inefficient one. As a result, the optimal menu is going to consist of a contract $(t_{1a}, k_{1a}, t_{2a}, k_{2a})$, which will be taken by efficient GPs only, and a second contract $(t_{1b}, k_{1b}, t_{2b}, k_{2b})$, which might be taken by the inefficient type, or (possibly) by both. Denote as $x$ as the probability of an efficient GP telling the truth about her type. We use $\nu_2$ and $\overline{\nu}_2$ to indicate the probability of contracting with an efficient GP in the second period, respectively after the GP has chosen the first or the second contract. In this case we have $\nu_2 = 1$ and

\[
\overline{\nu}_2 = \frac{\nu_1(1 - x)}{1 - \nu_1 x}.
\]

Upon announcement of an efficient GP in the first period, the GP is fully identified as efficient, $\nu_2 = 1$. This implies that in the second period following the announcement of an efficient GP, the LP imposes FB contracts, $k_{2a} = k^{FB}$. Optimally, it should allocate no rents, as from Proposition 3. The LP sets $M_0 = 0$ and $t_{2a} = \theta k^{FB} + \psi$. 

19
Upon announcement of an inefficient GP in the first period, the GP is generally not fully identified. From Proposition 2, we have $\Delta \theta \kappa^{SB}(\nu_2) \leq M_0 \leq \Delta \theta \kappa^{FB}$. Notice that for $\Delta \theta \kappa^{FB} \leq M_0 \leq \Delta \theta \kappa^{FB}$ (i.e. case (b)), the optimum investment levels do not change, but the rents increase in $M_0$. It cannot be optimal then to choose $\Delta \theta \kappa^{FB} < M_0$, from which follows that an optimum contract requires $\Delta \theta \kappa^{SB}(\nu_2) \leq M_0 \leq \Delta \theta \kappa^{FB}$. From this we obtain that $k_{2b} = k^{FB}$, $\kappa_2 = M_0/\Delta \theta$, $t_{2b} = \theta k^{FB} + \Delta \theta \kappa_2 + \psi$ and $\bar{t}_2 = \bar{\theta} \kappa_2 + \psi$. We still need to determine $M_0$ or equivalently $\kappa_2$. Substituting $M_0$ and $\pi_2$ in Lemma 4, the LP continues with both types if $I \leq R^b \equiv R(\kappa_2) - \bar{\theta} \kappa_2$; continues only with an efficient GP if $R^b < I < R_1 \equiv R^{FB} + \frac{1-\nu_1}{\nu_1 (1-x)} \Delta \theta \kappa_2$; exits regardless of the type if $I \geq R_1$.

In the first period, following standard arguments, the investment level upon announcement of an efficient GP is set at $k_1 = k^{FB}$. Second, upon announcement of an inefficient GP in the first period, the LP offers no rents $t_1 = \bar{\theta} k_1 + \psi$. Third, an efficient GP is indifferent between telling the truth or lying because her intertemporal incentive compatibility condition is binding, i.e.

$$t_1 = \theta k^{FB} + \Delta \theta \kappa_1 + \delta \Delta \theta \kappa_2 + \psi.$$  

It only remains to determine the optimal $\kappa_1$, an inefficient GP’s second period investment $\kappa_2$ and the optimal $x$. The determination of the probability $x$ an efficient GP reveal her type (separates) is tackled in Section 7. For a given $x$, substituting all the other terms, we have that the LP should maximize the following problem subject
to the renegotiation proof condition (i.e. $\bar{k}_2 \geq \bar{k}^SB(v_2)$).

$$\max_{\{k_1,k_2\}} \nu_1 x \left[ R^{FB} - \Delta \theta \kappa_1 - \delta \Delta \theta \kappa_2 \right] + (1 - \nu_1 x) \left[ R(\kappa_1) - \bar{\kappa}_1 \right] - (1 + \delta) \psi +$$

$$\delta \left\{ \nu_1 x \left( \int_0^{R^{FB}} R^{FB} dF(I) + \int_0^{+\infty} I dF(I) \right) + \nu_1 (1 - x) \int_0^{R^{FB}} (R^{FB} - \Delta \theta \kappa_2) dF(I) \right. \right.$$

$$+ (1 - \nu_1) \int_0^{R^{FB}} R^{FB} dF(I) + (1 - \nu_1) \int_{R^{FB}}^{+\infty} I dF(I) + (1 - \nu_1 x) \int_{R^{FB}}^{+\infty} (I - \Delta \theta \kappa_2) dF(I) \right\}$$

The three terms in line 1 represent the expected profits from the first period and the effort compensations for both periods. The first term in line 2 describes the expected profits after the contract designed for efficient GPs, which depend on the realization of $I$. No exit fee is paid to terminate this contract. The second term in line 2 shows the expected profits from retaining an efficient that chooses the inefficient contract. The first term in line 3 describes the expected profits from retaining inefficient GPs. The second term represents the expected profits when an offer is made to retain the efficient type only, in which case no exit fee is paid to terminate the contract with the inefficient types. The last term identifies the expected profits when none of them is retained, in which case an exit fee $\Delta \theta \kappa_b$ is paid independently of the type.

The following proposition summarizes the terms of the optimum long-term contract, which is also depicted in Figure 2.

**Proposition 6** The optimal menu of long-term contracts consists of a contract $(t_1, \bar{k}_1, \bar{k}_2)$, which is chosen only by efficient GPs with probability $x$, and a contract $(\bar{t}_1, \bar{k}_1, \bar{k}_2, \bar{t}_2, \bar{k}_2)$, which is chosen by efficient GPs with probability $1 - x$ and by inefficient GPs with
probability one, where:

\[
\begin{align*}
\bar{t}_1 &= \theta \bar{k}^{FB} + \Delta \theta \bar{k}^{SB}(\nu_1 \bar{x}) + \delta \Delta \theta \bar{k}^{SB}(\bar{\nu}_2) + \psi \\
\bar{k}_1 &= \bar{k}^{FB} \\
\bar{t}_{2a} &= \theta \bar{k}^{FB} + \psi \\
\bar{k}_{2a} &= \bar{k}^{FB} \\
\bar{t}_{2b} &= \theta \bar{k}^{FB} + \Delta \theta \bar{k}^{SB}(\bar{\nu}_2) + \psi \\
\bar{k}_{2b} &= \bar{k}^{FB} \\
\bar{t}_2 &= \theta \bar{k}^{SB}(\nu_1 \bar{x}) + \psi \\
\bar{k}_2 &= \bar{k}^{SB}(\nu_1 \bar{x}) \\
\bar{t}_2 &= \theta \bar{k}^{SB}(\bar{\nu}_2) + \psi \\
\bar{k}_2 &= \bar{k}^{SB}(\bar{\nu}_2)
\end{align*}
\]

The proposition offers the following main findings: 1) an efficient GP is indifferent between the two contracts because they offer the same rents and investment levels; however, 2) rents to efficient GPs are paid at different times in the two contracts. With contract \(a\), rents to efficient GPs are paid all in the first period. With contract \(b\), a rent \(\Delta \theta \bar{k}^{SB}(\nu_1 \bar{x})\) is paid in the first period, and a rent \(\Delta \theta \bar{k}^{SB}(\bar{\nu}_2)\) is paid in the second period;\(^{11}\) 3) an efficient GP is always investing at first best, while an inefficient GP is distorted downwards; and importantly, 4) both contracts are independent of the distribution of \(I\). As the contract may terminate earlier than originally agreed, the LP may be tempted to lower second-period rents below what found in Proposition 2. However, this is not possible, because the conditions of Proposition 2 are binding, which implies that rents are already at the minimum that satisfies the renegotiation-proof condition and cannot be further reduced.

\(^{11}\)The first-period rent of contract \(b\) is obtained by mimicking the inefficient GP.
5 Short-term contracts (no commitment)

In this section we examine short-term contracts. They last only one period and at the beginning of the second period, the LP has the option to continue with a second contract. If continuation occurs, the new contract will be negotiated on the basis of the information available at $t_3$. Alternatively, the LP may choose to exit and receive her reservation utility $I$. In case of exit, the LP needs not pay an exit fee, as she is not bound by any commitment with the GP. Following Laffont and Tirole (1987), we start by solving the optimal contract in the second period, then derive the maximum $I$ below which continuation occurs and then we examine the optimum contract at $t_0$.

5.1 Second period contracts

Following first period contracting, let us again denote the updated belief that the GP is efficient as $\nu_2$. Then, the LP solves the same problem as in the second period of the renegotiation case, (6) subject again to the incentive compatibility constraint of an efficient GP and the participation constraint of an inefficient GP. In this case, however, the contract does not need to satisfy the renegotiation condition. As full separation may not occur during the first period, the solution to this maximization is the standard one-period contract based on the revised expectations $\nu_2$. As usual, in the second period the LP might decide to invest only with an efficient GPs, or with both types. If she decides to keep both types, she should offer a menu of two contracts, $(t_{2b}, k_{2b})$ and $(\bar{t}_2, \bar{k}_2)$, the former designed for efficient GPs and the other for inefficient ones. The LP optimally requires again first-best level of investment
for efficient GPs ($k_{2b} = k^{FB}$) and a downward distortion of the inefficient type ($\bar{k}_2 = \bar{k}^{SB}(\nu_2)$). The efficient GP receives $t_{2b} = \theta k^{FB} + \Delta \theta \bar{k}^{SB}(\nu_2) + \psi$, while an inefficient GP gets $\bar{t}_2 = \bar{\theta} \bar{k}^{SB}(\nu_2) + \psi$. Alternatively, the LP may choose to offer a contract only to efficient GPs, $(t_{2a}, k_{2a})$, in which case the offer is at first best with $k_{2a} = k^{FB}$ and $t_{2a} = \theta k^{FB} + \psi$. Summarizing we have that second period contracts are as in the following proposition.

**Proposition 7** If the LP chooses to keep both types, she offers a menu of contracts $(t_{2b}, k_{2b})$ and $(\bar{t}_2, \bar{k}_2)$ where:

$$
t_{2b} = \theta k^{FB} + \Delta \theta \bar{k}^{SB}(\nu_2) + \psi \quad k_{2b} = k^{FB}
$$
$$
\bar{t}_2 = \bar{\theta} \bar{k}^{SB}(\nu_2) + \psi \quad \bar{k}_2 = \bar{k}^{SB}(\nu_2)
$$

If instead the LP keeps only efficient GPs, she offers a single contract, with $t_{2a} = \theta k^{FB} + \psi$ and $k_{2a} = k^{FB}$.

We skip the proof of this proposition as it is based on standard arguments.

### 5.2 Exit options

Following the same logic of Section 4.2, we derive the maximum $I$ below which continuation occurs. After observing $I$, the LP can choose 1) to continue with the long-term contract as originally agreed, 2) to continue only with efficient GPs and exit with inefficient ones, or 3) to exit regardless of the type. The thresholds for $I$ are summarized in the following lemma.
Lemma 8 After a short-term contract, the LP continues with both types if $I \leq R_2(\nu_2) - \psi$; continues only with efficient GPs if $R_2(\nu_2) - \psi < I < R_{FB} - \psi$; and exits regardless of type if $I \geq R_{FB} - \psi$, where $R_2(\nu) \equiv \bar{R}^{SB}(\nu) - \frac{\nu}{1-\nu} \Delta \theta^{SB}(\nu)$.

From this lemma we conclude that in the second period efficient GPs will earn a rent only if $I < R_2(\nu_2) - \psi$ which is the case of continuation with both types. If $I > R_2(\nu_2) - \psi$, either there is continuation with only efficient GPs which implies first best and no rents. Or, there is no continuation, which also trivially implies no rents.

5.3 First-period contracts

In the first period, the LP should offer a choice between two contracts, $(t_{1b}, k_{1b})$ and $(\bar{t}_1, \bar{k}_1)$, or a pooling contract $(t_{1p}, k_{1p})$. As usual the efficient type may mimic and choose the contract designed for the bad type. However, in this case the reverse may also happen, in which case the incentive constraint of both types would be binding, with ensuing randomization of both types. We use again $\nu_2$ and $\bar{\nu}_2$ to indicate the probability of contracting with efficient GPs in the second period, respectively after the GP has chosen to tell the truth or lie in the first period. Denote $x$ as the probability that efficient GPs claim tells the truth and $y$ as the probability of an inefficient GP to lie. We have that

\[ \nu_2 = \frac{\nu_1 x}{\nu_1 x + (1 - \nu_1) y} \quad \text{and} \quad \bar{\nu}_2 = \frac{\nu_1 (1 - x)}{\nu_1 (1 - x) + (1 - \nu_1) (1 - y)}.\]
Substituting \( \nu_2 \) and \( \overline{\nu}_2 \) into Proposition 7 and Lemma 8 we obtain the optimal second-period contracts and exit thresholds. The optimal problem for the LP, described in the proof of the Proposition below) is similar to that of renegotiation. In this case, however, the incentive compatibility constraints of both types might be binding. Accounting for the optimum second-period contract, the intertemporal incentive constraint of efficient GPs is

\[
\tilde{t}_{1b} - \theta \overline{k}_{1b} + \delta F (R_2 (\nu_2) - \psi) \Delta \theta \overline{k}^{SB} (\nu_2) \geq \tilde{t}_1 - \theta \overline{k}_1 + \delta F (R_2 (\overline{\nu}_2) - \psi) \Delta \theta \overline{k}^{SB} (\overline{\nu}_2)
\]

As opposed to the case of commitment and renegotiation, efficient GPs do not always receive an exit fee. Furthermore, as continuation occurs only if \( I < R_2 (\nu_2) - \psi \), the expected rents paid to efficient GPs are lower than in the case of Laffont and Tirole in which there are no outside investment opportunities.

The intertemporal incentive constraint of an inefficient GP, accounting for the fact that in the second period she always receives a zero rent, depends solely on first-period payoffs, and is given by

\[
\tilde{t}_1 - \overline{\theta} \overline{k}_1 \geq \tilde{t}_{1b} - \overline{\theta} \overline{k}_{1b}. \quad (9)
\]

The participation constraint of an inefficient GP is always binding to minimize costs, which means that \( \tilde{t}_1 = \overline{\theta} \overline{k}_1 + \psi \). The following proposition summarizes the optimum first-period contracts.

**Proposition 9** The optimal menu of short-term contracts consists of:

(case 1) A contract \((\tilde{t}_{1b}, \overline{k}_{1b})\), which is taken by efficient GPs with probability \( x \) and
a contract \((\bar{t}_1, \bar{k}_1)\), which is taken by efficient GPs with probability \(1 - x\) and by an inefficient GP with probability one, where

\[
\bar{t}_{1b} = \theta \bar{k}^{FB} + \Delta \theta \bar{k}^{SB}(\nu_1 x) + \delta F (R_2(\bar{v}_2) - \psi) \Delta \theta \bar{k}^{SB}(\bar{v}_2) + \psi \quad \bar{k}_{1b} = \bar{k}^{FB} \\
\bar{t}_1 = \theta \bar{k}^{SB}(\nu_1 x) + \psi \quad \bar{k}_1 = \bar{k}^{SB}(\nu_1 x),
\]

This case holds only insofar as the intertemporal incentive constraint of an inefficient GP, \(\bar{k}^{FB} > \bar{k}^{SB}(\nu_1 x) + \delta F (R_2(\bar{v}_2(x)) - \psi) \bar{k}^{SB}(\bar{v}_2)\), is satisfied.

(case 2): A contract \((\bar{t}_{1b}, \bar{k}_{1b})\), which is taken by efficient GPs with probability \(x\) and an inefficient GP with probability \(y\), and a contract \((\bar{t}_1, \bar{k}_1)\), which is taken by efficient GPs with probability \(1 - x\) and by an inefficient GP with probability \(1 - y\) where

\[
\bar{t}_{1b} = \theta \bar{k}_1^*(x, y) + \Delta \theta \bar{k}_1^*(x, y) + \psi \quad \bar{k}_{1b} = \bar{k}_1^*(x, y) \\
\bar{t}_1 = \theta \bar{k}_1^*(x, y) + \psi \quad \bar{k}_1 = \bar{k}_1^*(x, y),
\]

for a uniquely defined \(\bar{k}_{1b}^*(x, y)\) and \(\bar{k}_1^*(x, y)\).

(case 3): A (pooling) contract \((\bar{t}_1^p, \bar{k}_1^p)\), which is taken by all GPs, where

\[
\bar{t}_1^p = \theta \bar{k}^{FB} + \psi \quad \bar{k}_1^p = \bar{k}^{FB}.
\]

6 Comparison between contracts

The purpose of this section is to show that full commitment is not always optimal. Contracting strategies cannot be compared in general analytically. As in Laffont and Tirole (1987, 1990), the optimal contract in the case of long-term with commitment
and short-term can only be found by maximizing numerically over \( x \) and \( y \) (see next section). However, for a given \( x \), we can compare the rents and exit options of the long-term contracts with short-term contracts in case 1.

**Lemma 10** For a given \( x \), rents are highest for commitment and renegotiation. The ranking between full commitment and short-term contracts (case 1) is unclear. For a given \( x \), the LP is more likely to exit with short-term contracts (case 1) than with commitment and renegotiation.

A comparison based on rents either favours full commitment (if adverse selection is severe) or short-term contracts (if outside investments matter a lot). A comparison based on exit, on the other hand, always favours short-term contracts because they have a lower cost of exit and effort is paid for only in case of continuation.

In order to compare the profitability of the various contracting strategies, for the rest of this section we assume that the optimal solution in both the commitment and renegotiation and short-term contracts involves full separation (i.e. \( x = 1 \), and therefore \( \nu_2 = 1 \) and \( \bar{\nu}_2 = 0 \)), and that case 1 of the short-term contracts is optimal. As in Laffont and Tirole (1990) (as we also show in the next section), this is optimal if \( \delta \) is small. Then, the second-period rents of efficient GPs in the case of full commitment, without commitment and short-term contracts are equal to \( \Delta \bar{k}^{SB}(\nu_1), \Delta \bar{k}^{FB}, F(\bar{R}^{FB} - \psi)\Delta \bar{k}^{FB} \). The first and the third are smaller than the second, but the first can be smaller or larger than the third depending on the distribution of the outside opportunities.
Suppose first that outside opportunities are low. Then, exit may not occur even with inefficient GPs, i.e. \( F(R^{FB} - \psi) = 1 \). The rents associated to full commitment are lower than without commitment, which in this case are equal to the rents of short-term contracts. As a result, we have the following proposition, which generalizes the results of Laffont and Tirole (1990).

\textbf{Proposition 11} Suppose that separation is optimal in all contracts \((x = 1)\). Long-term contract with commitment dominate commitment and renegotiation and short-term contracts if \( F(R^{FB} - \psi) = 1 \). Short-term contracts dominate full commitment and commitment and renegotiation if \( F(R^{FB}) = 0 \).

On the other end, suppose that outside opportunities are large, so that exit always happens, i.e. \( F(R^{FB} - \psi) = 0 \). The rents paid in the case of short-term contracts are lower.

7 Numerical Simulation

We now carry out numerical simulations to examine the choice between long- and short-term contracts for optimal values of \( x \) and \( y \). We assume the following specification for the revenue function, \( R = bk^a \), with \( 0 < a < 1 \) and \( b > 0 \). We also assume that investment opportunities follow a uniform distribution function between 0 and \( Z(\geq 0) \) so that \( f(I) = 1/Z \) and \( F(I) = I/Z \) for \( 0 \leq I \leq Z \).

Table 1 reports the results of the comparison between the three contracting strategies. We find all types of contracting strategies optimal in some circumstances. In broad terms, long-term contracts are optimal if the outside opportunities are small.
(Z small), whereas short-term contracts are optimal if the outside opportunities are large (Z large). If there are no outside opportunities (Z = 0, column 1), long-term contracts with commitment dominate other types of contracts, as in (Laffont and Tirole 1990). Instead, if Z = 1 short-term contracts almost always dominate because they offer more flexibility. Moreover, since the outside option will be taken very often (in around 75% of the cases in our simulations), the optimal short-term contract resembles the static solution, in which it is optimal to separate the types. This is true as long as b is not very large or a is small, in which case the outside opportunity is again relatively less important because investment levels are large.

As opposed to Laffont and Tirole’s setup, long-term contracts with full commitment are not always better than commitment and renegotiation. That is, it might be optimal not to use the ability to commit even if it is available. For example, when the prior of finding efficient GPs is small (ν₁ = 0.1) and the investment opportunities are small but positive (Z = 0.25), the LP prefers not to commit and take the outside investment opportunity, rather than deal with inefficient GPs.

By varying Z we can see the effect of δ on the optimal contract. If δ = 0, the model is isomorphic to the static case. If Z = 0.25, short-term contracts dominate only for δ small. The reason why this happens is that the trade-off between long- and short-term contracts is played on second-period rents, that are expressed in discounted terms. For a low δ, these rents almost disappear, thus taking away all the advantages of long-term contracts. The value of flexibility makes then short-term contracts dominant.

The severity of adverse selection Δθ also plays a role. If Z = 0, short-term
contracts with pooling (ST3) converge to full commitment, whereas the incentive constraints of the separating equilibrium (ST1, ST2) prevent this equilibrium from approaching the commitment welfare. As a result, with short-term contracts pooling is better than separation (still long-term contracts are even better than either).

If $\Delta \theta$ is small ($\bar{\theta} = 1.05$) and $Z = 0.5$, the LP prefers short-term contracts with pooling than any other type of contract. Long-term contracts here are dominated because the value of the exit option is high. Short-term with separation is dominated because there is little difference between efficient and inefficient GPs. Similarly, if the probability of finding efficient GPs is small ($v_1 = 0.1$), it might be optimal to pool rather than separate. Instead, if the probability of finding efficient GPs is very large ($v_1 = 0.1$), separation is always optimal and short-term contracts dominate.

The two parameters describing the profit function ($a$, $b$) make some of the previous effects stronger or weaker. If $a$ is large or $b$ is small, the investment levels are low and the outside option becomes more important and short-term contracts are better. Instead, if $a$ is small or $b$ is large, the investment levels are high and the outside option becomes less important and long-term contracts are better.

8 Empirical predictions

8.1 Commitment

The previous section bears precise implications for the optimality of long-term commitment. Intuitively, full commitment should be chosen when outside options are low, while short-term contracts are preferable for high values of the outside option.
Long term contracts without commitment dominate for low values of $v_1$ and an intermediate value of $Z$. What does this imply for our understanding of PE agreements? The answer depends on how one interprets the nature of these agreements. Following our discussion on default penalties and early termination provisions, PE partnership agreements can be interpreted alternatively as long-term contracts with and without commitment, or as short-term contracts.

**Prediction 1** *Severe default penalties apply when the value of outside opportunities is low. Medium default penalties apply to intermediate values of the outside opportunities and a low probability of efficient GPs. Weak default penalties and early termination provisions apply to high values of the outside opportunities.*

A test of this prediction requires a ranking of GPs according to efficiency. The ranking should probably account for historical performance, age, number of funds under management, as well as expenses.\(^{12}\) It also requires information about the severity of default penalties in each contract, and the presence of an early termination provision.\(^{13}\) Finally, the test requires an estimation of outside investment opportunities, which is probably the most challenging aspect of this test. One possibility is to start from the expected risk-adjusted performance of PE investments ((Gompers and Lerner 2000), (Kaplan and Schoar 2005), (Lerner, Schoar, and Wong 2005), (Ljungqvist and Richardson 2003a) and (Ljungqvist and Richardson 2003b)), and compare it to the performance of other asset classes. The endogeneity between contractual clauses and expected performance can be addressed with a differences-in-differences approach over the business cycle.

---

\(^{12}\)For a detailed list of the expenses born by GPs see for example Toll and Beltran (2010).  
\(^{13}\)See Table 1 of (Litvak 2004) for an example of how default penalties can be classified.
8.2 Fees

The model predicts three different sets of fees: 1) a fee that is proportional to the capital under management, 2) a fee that is linked to performance, 3) a fee \( \Delta \theta \bar{k} \) that depends on the efficiency of the GP. The proportional fee is given by \( \theta \bar{k} \) and \( \bar{k} \) for efficient and inefficient GPs respectively. As \( \bar{\theta} \geq \theta \) the fee of efficient GPs represents a smaller percentage of committed capital, than that paid to inefficient GPs. However, depending on the parameters, the absolute size of the fee may or may not be greater for efficient GPs than for inefficient ones.

The fee linked to performance is obtained by decomposing \( \psi \) into a compensation for the high state, \( t_h \), and for the low state, \( t_l \). Condition (2) may not be binding at the optimum, which means that potentially the LP may punish the GP with a very negative payment in the low state and a very positive payment in the high state. In a realistic situation, we envisage the GP to be paid \( \frac{\psi}{\Delta \pi} \) in the high state and \( -\frac{\pi_0 \psi}{(1-\pi_1)\Delta \pi} \) in the low state. Such compensation scheme satisfies (2) (which is now binding) and minimizes the punishment in the low state.

8.2.1 Management Fees

The model calls for fees that are linked to the size of the fund, the empirical counterparts of which are known as management fees. More precisely the model yields two predictions about management fees:

**Prediction 2** Management fees should be a percentage of \( k \). The percentage should be higher for inefficient than for efficient GPs.

The prediction of a linear relationship between management fees and \( k \) is cor-
roborated by the evidence that in practice fees are proportional to the size of the fund and tend to run in the 1.5 per cent to 2.5 per cent range of capital under management. The second prediction is supported by the findings of (Gompers and Lerner 1999a). Computing the size of a fund as the ratio of the capital invested in the fund to the total amount raised by all other funds, Gompers and Lerner identify four classes of size. Partnerships 1) with no previous funds, 2) with a ratio of 0 – 0.2%, 3) with a ratio of 0.2 – 0.7, 4) with a ratio greater than 0.7. They find that management fees for each of these classes are respectively, 18.8, 19.9, 18.2, 15.1, cumulative over the life of the fund. Their evidence provides support to the model prediction that larger funds receive lower management fees (per unit of capital) than smaller ones.

The model also predicts that:

**Prediction 3** *For full commitment the management fees can vary over time (Proposition 1). For commitment and renegotiation, the management fees must be constant over time (Proposition 4). For short-term contracts it can vary over time (Proposition 5).*

(Litvak 2009) shows that management fees patterns are subject to negotiation. There are five main patterns: 1) classic flat fee, constant percentage of committed capital; 2) flexible flat fee, time-varying percentage of committed capital; 3) fee based entirely on managed capital; 4) fee with a switch from committed to managed capital; 5) absolute dollar amount. The relationship between different types of GPs, length of contracts, commitment, and time variation of management fees remains untested.
8.2.2 Carried interest and clawback provisions

The model predicts a prize to GPs when returns are high, and a punishment when returns are low. We identify carried interests and clawback provisions as the empirical counterparts of the model predictions. Carried interest represents the variable component of a GPs compensation when returns are positive. It is normally expressed as a percentage of the total profits of the fund, with the industry norm being 20 per cent. Gompers and Lerner report that the share of profits allocated to the GP varies from 0.7% to 45%, but 81% of the funds are between 20% and 21%, inclusive.

The mechanics governing payment of the carried interest are set out in something called a distribution waterfall, which describes the sequence in which proceeds from the sale of portfolio companies are distributed between the general partner and the limited partners. The current market standard in the U.S. is a deal-by-deal approach, according to which the payment of carried interest is the manager-friendly deal-by-deal waterfall. The GP’s carried interest is paid out on a deal-by-deal basis as soon as the fund begins generating profitable exits from investments. This approach has two consequences. First, the early payment of carry takes money off of the table and is a drag on investor returns. Second, it creates the need to include complicated clawback provisions in the limited partnership agreement, along with an escrow account, in which a GP’s carry is held to secure future claw-back obligations.14

Clawback provisions require the GP to pay back profit distributions if losses subsequently arise from the sale of portfolio companies or from asset write-downs.

---

Typically, with a clawback provision realized portfolio losses and write-downs on unrealized investments are recovered before any distributions; there are significant carried interest escrows (30 percent or more); and there is joint and several liability of the fund’s management team and their family trusts for the clawback repayment obligation.\footnote{See also the discussion of clawback provisions in (Metrick and Yasuda 2010).} Clawbacks that are based on realized transactions during the life of the fund are known as deal-by-deal (or interim) clawbacks.

Our model calls for deal-by-deal carried interests and clawbacks in all cases, but that of full commitment for efficient GPs. For the latter case, the model has no precise prediction.

**Prediction 4** An optimum contract should generally include a deal-by-deal carried interest and clawback provision. Only for efficient GPs with long-term contracts and commitment, carried interest and clawback provisions may not be set on a deal-by-deal basis.

As for management fees, carried interest and clawbacks are subject to negotiation. Litvak (2009) shows that there are four main ways to set the carried interest which vary between deal-by-deal and all at once: 1) all interest to fund; 2) return first; 3) ceiling; 4) payback. As for deal-by-deal clawbacks, many LPs are currently proposing their introduction, but GPs are generally against them as there are problems with basing clawbacks on unrealized valuations.\footnote{Centre for Private Equity and Entrepreneurship, Tuck School of Business at Dartmouth, "Proceedings of the Limited Partnership Agreement Conference, 2004."} The relationship between different types of GPs, length of contracts, commitment, and types of carried interest and clawbacks remains untested.
8.2.3 Other fixed fees

Last but not least, the model predicts that efficient GPs should earn a fee for revealing information about their type, and that this fee should be proportional to the value of information \(\Delta \theta\) and to the investment level of inefficient GPs. There is no obvious empirical counterpart to the fee predicted by the model. However, the prediction of the model can be interpreted as saying that, on top of management fees and carried interest, better and more experienced GPs should be able to extract a higher compensation, e.g. in the form of transaction or monitoring fees, than younger less experienced GPs. Such compensation should be increasing in the relative market position of a GP within the PE industry. Intuitively, the model predicts that well known PE investors, such as KKR or Permira, can extract an overall compensation, net of the standard management fees and carry, that is higher than less known peers.

**Prediction 5** On top of management fees and carried interest, GPs earn fixed fees that are proportional to their efficiency.

Our prediction about the relationship between different types of GPs and fixed fees remains yet to be tested.

9 Conclusions

This paper offers both a theoretical and empirical contribution. On the theory side, it generalizes the model of Laffont and Tirole (1987, 1990) by allowing the reservation utility of the LP to vary stochastically over time. This new element introduces an important trade-off between long- and short-term contracts which was not present
in the original model of Laffont and Tirole. Without stochastic reservation utilities, long-term contracts with full commitment always dominate long-term contracts without full commitment (commitment and renegotiation), which in turn dominate short-term contracts. With stochastic reservation utilities, the ranking is not so clear anymore. If the reservation utility in the second period is expected to be low, then long-term contracts with full commitment still dominate the others. However, if the expectation is a bit higher, either long-term contracts without full commitment or short-term contracts may dominate. For high expectations, short-term contracts always dominate.

The empirical contribution of this paper consists in the application of the above model to the case of PE partnerships. A key feature of these partnerships is that they last for a long time and that capital is entirely committed at the beginning. The apparently natural interpretation of these partnerships is that they are long-term contracts with full commitment. However, we argue that one has to control for the severity of default penalties and for early termination provisions before concluding that the length of PE partnerships really implies commitment. Our model therefore calls for empirical tests in which the value of commitment is weighted against the flexibility of short-term contracting. The predictions of the model suggest that long-term contracts are the optimum choice for a situation in which adverse selection costs are high and exit options are low. Revealed preferences, then suggest that LPs are more concerned about reducing information costs in management selection, than in retaining the option to divert money to outside investment opportunities.

The model is also able to characterize PE contracts in detail. We find that
an optimum incentive-compatible compensation should include a state-dependent component which rewards a GP when returns are high and punishes her when returns are low. The empirical counterpart of this compensation scheme is a carried interest with a clawback clause. A GP’s compensation should also include a component which is proportional to the size of the fund; this fee is empirically known as management fee. On top of these fees, GPs should earn other fixed fees that are linked to their relative efficiency/experience.
References


Appendix

Proof of Proposition 1

Cost minimization requires that an inefficient GP is left at her reservation utility in both periods, which means that

$$t_1 + \delta t_2 = \overline{\theta k}_1 + \delta \overline{\theta k}_2 + (1 + \delta)\psi$$

Condition (3) is binding at the optimum. Using the (binding) participation constraints of an inefficient GP, (3) can be written as

$$t_1 = \theta k_1 - \delta (t_2 - \theta k_2) + \Delta \theta \overline{k}_1 + \delta \Delta \theta \overline{k}_2 + (1 + \delta)\psi.$$ 

Insert it into the objective function of the LP. Then, the maximization of the LP is given by

$$\max \left\{ \frac{\nu_1}{k_1} \right\} \left[R (k_1) - \theta k_1 - \Delta \theta \overline{k}_1 - \delta \Delta \theta \overline{k}_2 \right] + (1 - \nu_1) [R (\overline{k}_1) - \overline{\theta k}_1]$$

$$+ \delta \left[ \frac{\nu_1}{k_2} [R (k_2) - \theta k_2] + (1 - \nu_1) [R (\overline{k}_2) - \overline{\theta k}_2] \right] - (1 + \delta)\psi$$

and does not depend on $t_2$. The first order conditions yield $k_1 = k_2 = k^{FB}$ and $\overline{k}_1 = \overline{k}_2 = \overline{k}^{SB} (\nu_1)$, from which we obtain

$$t_1 + \delta t_2 = (1 + \delta) \left( \overline{\theta k}^{SB} (\nu_1) + \psi \right)$$

45
and
\[ t_1 + \delta t_2 = (1 + \delta) \left( \theta k_{FB} + \Delta \theta k_{SB}^S (\nu_1) + \psi \right) \]

One particular solution for an inefficient GP is \( t_1 = \bar{t}_2 = \overline{\theta k_{SB}^S} (\nu_1) + \psi \). Similarly, one particular solution for efficient GPs is \( t_1^{SB} = t_2^{SB} = \theta k_{FB} + \Delta \theta k_{SB}^S (\nu_1) + \psi \).

**Proof of Proposition 2**

Cost minimization by the LP requires \( \bar{t}_2 = \overline{\theta k_2} \). Thus, the Lagrangian can be written as

\[
\max_{\{t_2, k_2, \varphi_2\}} \nu_2 [R(k_2) - t_2] + (1 - \nu_2) [R(\bar{k}_2) - \overline{\theta k_2}] + \gamma_1 (t_2 - \theta k_2 - \Delta \theta \overline{k_2}) + \gamma_2 (t_2 - \theta k_2 - \psi - M_0),
\]

where \( \gamma_1 \) is associated with (7) and \( \gamma_2 \) is associated with (8). The first-order conditions w.r.t. \( t_2, k_2, \overline{k_2} \) are given by

\[ -\nu_2 + \gamma_1 + \gamma_2 = 0, \quad \text{(10)} \]

\[ \nu_2 R'(k_2) - \gamma_1 \theta - \gamma_2 \theta = 0, \quad \text{(11)} \]

and

\[ (1 - \nu_2) [R'(\bar{k}_2) - \overline{\theta}] - \gamma_1 \Delta \theta = 0. \quad \text{(12)} \]

Substituting (10) into (11), we have that \( R'(k_2) = \theta \) and therefore efficient GPs invest at the efficient level, \( k_2 = k_{FB} \). We now distinguish three cases, depending on
whether \( \gamma_1 \) and \( \gamma_2 \) are positive or zero. From (10), it cannot be that \( \gamma_1 = \gamma_2 = 0 \) unless \( \nu_2 = 0 \).

**Case 1: \( \gamma_1 > 0 \) ((7) is binding), \( \gamma_2 = 0 \) ((8) is slack)** Substituting \( \gamma_2 = 0 \) into (10) we have that \( \nu_2 = \gamma_1 \). Substituting into (12), we have that
\[
R'(k_2) = \overline{\theta} + \frac{\nu_2}{(1-\nu_2)} \Delta \theta
\]
and therefore \( \overline{k}_2 = \overline{k}^{SB}(\nu_2) \). From the participation constraint of inefficient GP binding we have that
\[
\overline{t}_2 = \overline{\theta}k^{SB}(\nu_2) + \psi.
\]
Finally, from (7), we have that
\[
t_2 = \theta k^{FB} + \Delta \theta \overline{k}^{SB}(\nu_2) + \psi.
\] (13)
Notice that this case is possible as long as (8), which can potentially be slack, is satisfied. That is, as long as,
\[
\Delta \theta \overline{k}^{SB}(\nu_2) > M_0.
\] (14)
But if this condition is slack then a renegotiation contract can be agreed upon. In order to have a renegotiation-proof contract, we need that \( M_0 = \Delta \theta \overline{k}^{SB}(\nu_2) \).

**Case 2: \( \gamma_1 > 0 \) ((7) is binding), \( \gamma_2 > 0 \) ((8) is binding)** Substituting \( \overline{t}_2 = \overline{\theta}k_2 + \psi \) and \( t_2 = \theta k^{FB} + M_0 + \psi \) ((8) is binding) into (7), we have that \( M_0 = \Delta \theta \overline{k}_2 \) from which we obtain \( \overline{t}_2 = \frac{\overline{\theta}M_0}{\Delta \theta} + \psi \). We now look for the conditions under which
this case applies. On the one hand, (12) can be written as

\[ \gamma_1 = (1 - \nu_2) \left[ R' \left( \overline{k}_2 \right) - \overline{\theta} \right] / \Delta \theta. \]

In order to have that \( \gamma_1 > 0 \) we need that

\[ R' \left( \overline{k}_2 \right) > \overline{\theta}, \]

i.e., that an inefficient GP invests less than at FB. On the other hand, substituting \( \gamma_1 \) into (11) we have that

\[ \gamma_2 = (\nu_2 \Delta \theta - (1 - \nu_2) \left[ R' \left( \overline{k}_2 \right) - \overline{\theta} \right]) / \Delta \theta. \]

This is positive as long as

\[ R' \left( \overline{k}_2 \right) < \overline{\theta} + \frac{\nu_2}{1 - \nu_2} \Delta \theta. \]

Hence, this case holds as long as

\[ \overline{\theta} < R' \left( \overline{k}_2 \right) < \overline{\theta} + \frac{\nu_2}{1 - \nu_2} \Delta \theta \]

or in other words, as long as

\[ R' \left( \overline{k}^{FB} \right) < R' \left( \overline{k}_2 \right) < R' \left( \overline{k}^{SB} (\nu_2) \right) \]

48
Given that $R'(\cdot)$ is decreasing in $\bar{k}$, this case applies when

$$\bar{k}^{SB}(\nu_2) < \bar{k}_2 < \bar{k}^{FB},$$

a condition that can be otherwise written as $\theta \bar{k}^{SB}(\nu_2) < M_0 < \Delta \theta \bar{k}^{FB}$.

**Case 3: $\gamma_1 = 0$ ((7) is slack), $\gamma_2 > 0$ ((8) is binding)** Substituting $\gamma_1 = 0$ into (10) we have that $\nu_2 = \gamma_2$ and substituting into (12) we have that $R'(\bar{k}_2) = \bar{\theta}$ and therefore $\bar{k}_2 = \bar{k}^{FB}$. As the participation constraint of an inefficient GP is binding,

$$\bar{t}_2 = \theta \bar{k}^{FB} + \psi$$

From (8) we have

$$t_2 = \theta \bar{k}^{FB} + M_0 + \psi. \quad (15)$$

This case is satisfied as long as (7) is satisfied, a condition that (using (15)) can be written as $M_0 > \Delta \theta \bar{k}^{FB}$. For large values of $M_0$ the incentive constraint of an inefficient GP may be violated. Therefore, we need to impose an upper limit to $M_0$. Carrying out the due substitutions, we have that the incentive constraint of an inefficient GP is satisfied if $M_0 \leq \Delta \theta \bar{k}^{FB}$.

**Summary of renegotiation-proof contracts** As we have shown, depending on the values of the parameters, the optimum second period renegotiation contract requires different thresholds for the reservation utility of efficient GPs, $M_0$. We can express the optimum second period contracts in the second period as in the proposi-
tion. From this we conclude that a long-term contract is renegotiation proof if and
only if \( \Delta \theta k^S_B (\nu_2) \leq M_0 \leq \Delta \theta k^F_B \).

**Proof of Proposition 3**

Cost minimization requires that the renegotiation-proof condition is binding, i.e. \( t_2 - \theta k - \psi - M_0 = 0 \). Then, in order to maximize \( R (k_2) - t_2 \), we set \( k_2 = k^{FB} \).

Therefore, \( t_2 = \theta k^{FB} + \psi + M_0 \).

**Proof of Lemma 4**

**Comparison between Case 3 and Case 2** The LP will prefer to exit in all cases rather than continuing with only efficient GPs iff

\[
I - M_0 \geq \nu_2 \left[ R^{FB} - M_0 \right] + (1 - \nu_2) I
\]

which simplifies to

\[
I \geq R^{FB} + \frac{1 - \nu_2}{\nu_2} M_0
\]

**Comparison between Case 2 and Case 1** The LP will prefer to continue only with efficient GPs rather than continue with both agents iff

\[
\nu_2 \left[ R^{FB} - M_0 \right] + (1 - \nu_2) I \geq \nu_2 \left[ R^{FB} - M_0 \right] + (1 - \nu_2) (R (\bar{k}_2) - \bar{\theta}k_2)
\]

which simplifies to

\[
I \geq R (\bar{k}_2) - \bar{\theta}k_2.
\]
Comparison between Case 3 and Case 1 The LP prefers to exit in all cases rather than continuation iff

\[ I - M_0 \geq \nu_2 \left[ R^{FB} - M_0 \right] + (1 - \nu_2) \left( R (\bar{k}_2) - \bar{\theta}k_2 \right) \]

which simplifies to

\[ I \geq M_0 + \nu_2 \left[ R^{FB} - M_0 \right] + (1 - \nu_2) \left( R (\bar{k}_2) - \bar{\theta}k_2 \right). \]

As \( R^{FB} \geq R (\bar{k}_2) - \bar{\theta}k_2 \), when the LP prefers exit to a contract only with efficient GPs at FB, she will also prefer exit to a contract with both types at SB. This implies that the latter comparison is irrelevant for the rest of the analysis.

In sum, the LP continues with both types if \( I \leq R (\bar{k}_2) - \bar{\theta}k_2 \); continues only if efficient GPs if \( R (\bar{k}_2) - \bar{\theta}k_2 < I < R^{FB} + \frac{1-\nu_1}{\nu_1(1-x)} \Delta \theta \bar{k}_2 \); exits regardless of types if \( I \geq R^{FB} + \frac{1-\nu_1}{\nu_1(1-x)} \Delta \theta \bar{k}_2 \).

Proof of Lemma 5

Consider the contracts of Proposition 3. The LP might now choose 1) to continue with efficient GPs, or 2) to exit.

Case 1: Continuation with the efficient type A renegotiation-proof contract has \( t_2 = \theta k^{FB} + M_0 + \psi \) and \( k_2 = k^{FB} \). Therefore, the payoff to the LP are

\[ R^{FB} - M_0 - \psi. \]
Case 2: Exit  To terminate the contract, the LP offers $M_0 + \psi$. In this case the expected profit of the LP is $I - M_0 - \psi$.

The LP prefers exit to continuation iff $I - M_0 \geq R^{FB} - M_0$ which simplifies to $I \geq R^{FB}$.

Proof of Proposition 4

For a given $x$, substituting all the other terms, the LP should maximize the following problem subject to the renegotiation proof condition (i.e. $k_2 \geq k^{SB}(\nu_2)$). We rewrite the maximization as follows

$$
\max_{\{k_1, k_2\}} \nu_1 x \left[ R^{FB} - \Delta \theta k_1 - \delta \Delta \theta k_2 \right] + (1 - \nu_1) \left[ R (k_1) - \overline{\theta} k_1 \right] - (1 + \delta) \psi 
+ \delta \nu_1 x \left[ \int_0^{R^{FB}} R^{FB} dF(I) + \int_{R^{FB}}^{+\infty} I dF(I) \right] 
+ \delta \int_0^{R_1} \left[ \nu_1 (1 - x) \left( R^{FB} - \Delta \theta k_2 \right) + (1 - \nu_1) R^{FB} \right] dF(I) 
+ \delta \int_{R_1}^{R^\phi} \left[ \nu_1 (1 - x) \left( I - \Delta \theta k_2 \right) + (1 - \nu_1) \left( I - \Delta \theta k_2 \right) \right] dF(I) 
+ \delta \int_{R^\phi}^{+\infty} \left[ \nu_1 (1 - x) \left( I - \Delta \theta k_2 \right) + (1 - \nu_1) \left( I - \Delta \theta k_2 \right) \right] dF(I)
$$

The first order condition with respect to $k_1$ gives $k_1 = \overline{k}^{SB}(\nu_1 x)$. To find the optimum level of $k_2$ observe that profits are higher in line 5 of the objective function than in line 4, and in line 4 than in line 3. The probability of line 5 occurring is decreasing in $k_2$. Profits in line 5 and in line 4 are decreasing in $k_2$. Profits in line 3 are increasing in $k_2$ up to $k_2 = \overline{k}^{SB}(\bar{\nu}_2)$ and decreasing after this point. Finally, profits in line
are decreasing in $k_2$. Thus, piecemeal maximization starting from line 3 requires $k_2 = \bar{k}_{SB}(\bar{v}_2)$, while in all other lines (1,4 and 5) it requires $k_2 = 0$. We then conclude that at the optimum it must be $\bar{k}_2 \leq \bar{k}_{SB}(\bar{v}_2)$, which implies that the LHS of the renegotiation-proof condition is binding. Therefore, the optimum renegotiation-proof contract requires $\bar{k}_2 = \bar{k}_{SB}(\bar{v}_2)$.

Proof of Lemma 8

**Case 1: Continuation with both types** The expected profits of the LP with continuation are given by

$$\nu_2 \left[ R^{FB} - \Delta \bar{k}^{SB}(\nu_2) \right] + (1 - \nu_2) R^{SB}(\nu_2) - \psi.$$  

**Case 2: Continuation only with efficient GPs** The expected profits of the LP with continuation only with efficient GPs are given by

$$\nu_2 \left[ R^{FB} - \psi \right] + (1 - \nu_2) I.$$  

**Case 3: Exit regardless of type** The expected profits of the LP with outright exit are given simply by $I$.

We now derive the conditions under which each of these continuation/exit strategies dominates.
Comparison between Case 3 and Case 2  The LP will prefer to exit rather than offering a contract only to efficient GPs iff

\[ I \geq \nu_2 \left( R^{FB} - \psi \right) + (1 - \nu_2)I \]

which simplifies to

\[ I \geq R^{FB} - \psi. \quad (16) \]

Comparison between Case 2 and Case 1  The LP will prefer to offer a contract to efficient GPs rather than offering a (separating) contract to both agents iff

\[ \nu_2 \left( R^{FB} - \psi \right) + (1 - \nu_2)I > \nu_2 \left( R^{FB} - \Delta \theta k^{SB} (\nu_2) - \psi \right) + (1 - \nu_2) \left[ R^{SB} (\nu_2) - \psi \right] \]

which simplifies to

\[ I > R_2 (\nu_2) - \psi. \]

Comparison between Case 3 and Case 1  In this case the condition for exit is

\[ I > \nu_2 \left[ R_1^{FB} - \Delta \theta k^{SB} (\nu_2) \right] + (1 - \nu_2) \bar{R}^{SB} (\nu_2) - \psi. \quad (17) \]

Observing that the RHS of (17) is smaller than the RHS of (16), we conclude that when (16) holds (17) also holds. Thus, we can ignore (17) in the rest of the analysis.

In sum, the LP offers a second best separating contract if \( I \leq R_2 (\nu_2) - \psi \), a
first best contract only to efficient GPs if \( R_2(\nu_2) - \psi < I < R^{FB} - \psi \), no contract if \( I \geq R^{FB} - \psi \).

**Proof of Proposition 6**

We analyze each of these cases in turn.

**Case 1: Incentive constraint of efficient GPs binding** A first possible case is that the incentive constraint of efficient GPs is binding and (9) is slack. This implies that an inefficient GP always declares her true type, while the efficient one is indifferent between declaring the truth or lying. Therefore, when a GP declares to be efficient, she is automatically identified with certainty as efficient (full separation).

More formally, this implies that \( \nu_2 = 1 \), i.e., when in the first period the GP declares to be efficient, in the second period only efficient types are observed. Under these conditions, efficient GPs’s second-period adverse selection rent is \( \Delta \theta k^{SB} (1) = 0 \). Using this result and the participation constraint of the inefficient GPs, the incentive constraint of efficient GPs then simplifies to

\[
t_{1b} = \theta k_{1b} + \Delta \theta k^1 + \delta \int_0^{R_2(\overline{\nu}_2) - \psi} \Delta \theta k^{SB} (\overline{\nu}_2) dF (I) + \psi
\]

Substituting for \( t_{1b} \) and \( \overline{t}_1 \), the maximization of the LP at \( t_0 \) is

\[
\max_{\{k_{1b}, \overline{k}_1\}} \nu_1 x \left[ R (k_{1b}) - \theta k_{1b} - \Delta \theta k^1 - \delta \int_0^{R_2(\overline{\nu}_2) - \psi} \Delta \theta k^{SB} (\overline{\nu}_2) dF (I) \right] \\
+ (1 - \nu_1 x) \left( R (\overline{k}_1) - \overline{\theta k}_1 \right) - \psi
\]
As only the first-period payoffs depend on $k_{1b}, k_1$, we have ignored second-period payoffs in this maximization problem. From the first order conditions of this maximization we obtain that the optimum investment levels are $k^{FB}$ and $k^{SB}(\nu_1 x)$. As the solution depends on $x$, a numerical maximization with respect to $x$ yields the optimum productions and transfers. Therefore, highlighting the dependence of $\bar{v}_2$ on $x$, the optimum (separating) contract requires 

$$\bar{t}_{1b} = \bar{\theta}k^{FB} + \Delta\bar{\theta}k^{SB}(\nu_1 x) + \delta \int_0^{R_2(\bar{v}_2(x)) - \psi} \Delta\bar{\theta}k^{SB}(\bar{v}_2(x)) dF(I) + \psi$$

and $k_{1b} = k^{FB}$ for efficient GPs; and $\bar{t}_1 = \bar{\theta}k^{SB}(\nu_1 x) + \psi$, $k_1 = k^{SB}(\nu_1 x)$ for an inefficient GP. This case applies as long as the incentive constraint of an inefficient GP is not binding. At the optimum, this constraint requires 

$$k^{FB} \geq k^{SB}(\nu_1 x) + \delta \int_0^{R_2(\bar{v}_2(x)) - \psi} k^{SB}(\bar{v}_2(x)) dF(I).$$

When this constraint is violated, Case 3 below applies.

Case 2.1: Incentive constraint of an inefficient GP is binding If in the first period a GP declares her type to be inefficient, then in the second period only inefficient types are observed, i.e., $\bar{v}_2 = 0$. This result follows from the fact that an efficient GP always declares her true type, so she will never say that she is an inefficient GP. On the contrary, if in the first period a GP declares her type as efficient, the updated belief about the distribution of efficient types in the second
period is

$$\nu_2 = \frac{\nu_1}{\nu_1 + y(1 - \nu_1)}$$

As the incentive and participation constraints of an inefficient GP are binding, we have that

$$t_{1b} = \bar{\sigma}k_1 + \psi \text{ and } \bar{t}_1 = \bar{\sigma}k_1 + \psi.$$  

The condition imposed by the incentive constraint of efficient GPs, rewritten here, requires

$$k_{1b} > k_1 + \delta \left( \int_0^{RFB} - \psi \bar{k}^{FB} dF (I) - \int_0^{R2(\nu_2(y)) - \psi} \bar{k}^{SB} (\nu_2) dF (I) \right)$$

Notice that $RFB > R2(\nu_2)$, then $\int_0^{RFB} - \psi \bar{k}^{FB} dF (I) > \int_0^{R2(\nu_2(y)) - \psi} \bar{k}^{SB} (\nu_2) dF (I)$ and also, 

$$\int_0^{RFB} - \psi \bar{k}^{FB} dF (I) - \int_0^{R2(\nu_2(y)) - \psi} \bar{k}^{SB} (\nu_2) dF (I) > 0$$

then we conclude that the incentive constraint of efficient GPs requires $k_{1b} > k_1$.

Noticing that second period profits do not depend on $k_{1b}, k_1$, the maximization then writes as

$$\max_{\{k_{1b}, k_1\}} (\nu_1 + (1 - \nu_1) y) \left[ R (k_{1b}) - \bar{\sigma}k_1 \right] + (1 - \nu_1) (1 - y) \left( R (k_1) - \bar{\sigma}k_1 \right) - \psi$$

The first order conditions of this maximization are

$$R' \left( k^* = \bar{k}^{FB} \right) = \bar{\sigma} = R' \left( k^* = \bar{k}^{FB} \right)$$

57
which implies $k^* = \bar{k}$ which violates the incentive constraint of efficient GPs. We conclude that at the optimum the incentive constraint of efficient GPs is always binding. Therefore, this case reduces to the next one (Case 3), in which the incentive constraints of both types are binding.

**Case 2: Incentive constraint of both types are binding** Both types are indifferent and may lie. In this case the updated belief of observing efficient GPs in the second period following a declaration of efficient GPs in period one is

$$\nu_2(x, y) = \frac{x \nu_1}{x \nu_1 + y(1 - \nu_1)}$$

And the updated belief of observing efficient GPs in the second period following a declaration of an inefficient GP in period one is

$$\nu_2(x, y) = \frac{(1 - x) \nu_1}{(1 - x) \nu_1 + (1 - y) (1 - \nu_1)}$$

As both incentive constraints are binding, from efficient GPs we have

$$t_{1b} - \theta k_{1b} + \delta \int_0^{R_2(\nu_2(x, y)) - \psi} \Delta \theta \bar{k}^{SB} (\nu_2) \, dF(I) = \bar{t}_1 - \theta \bar{k}_1 + \delta \int_0^{R_2(\nu_2(x, y)) - \psi} \Delta \theta \bar{k}^{SB} (\nu_2) \, dF(I)$$

while from the incentive constraint of an inefficient GP we have $\bar{t}_1 - \theta \bar{k}_1 = t_{1b} - \bar{t}_{1b}$. As usual the participation constraint of an inefficient GP is also binding, $\bar{t}_1 = \theta \bar{k}_1 + \psi$. Using the last two constraints we obtain $t_{1b} = \bar{t}_{1b} + \psi$. Plugging these values for $t_{1b}$
and \( \tilde{t}_1 \) into the incentive constraint of efficient GPs reduces to

\[
\tilde{\theta}k_1 - \theta k_1 + \delta \int_0^{R_2(\nu_2(x,y))-\psi} \Delta \tilde{\theta} \tilde{k}^{SB} (\nu_2) \, dF (I)
\]

\[
= \tilde{\theta}k_1 - \theta k_1 + \delta \int_0^{R_2(\nu_2(x,y))-\psi} \Delta \tilde{\theta} \tilde{k}^{SB} (\nu_2) \, dF (I)
\]

from which we obtain

\[
k_{1b} = k_1 + \delta \int_0^{R_2(\nu_2(x,y))-\psi} \tilde{k}^{SB} (\nu_2) \, dF (I)
\]

\[
- \delta \int_0^{R_2(\nu_2(x,y))-\psi} \tilde{k}^{SB} (\nu_2) \, dF (I)
\]

We then obtain a maximization in one variable

\[
\max_{\tilde{k}_1} \left( x\nu_1 + y(1 - \nu_1) \right) \left[ R \left( k_{1b}(\tilde{k}_1) \right) - \tilde{\theta} k_1 (\tilde{k}_1) \right]
\]

\[
+ (1 - x\nu_1 - y(1 - \nu_1)) \left[ R (\tilde{k}_1) - \tilde{\theta} \tilde{k}_1 \right] - \psi
\]

In the maximization we have ignored the second period, because it does not depend on \( \tilde{k}_1 \). Observe that the derivative of \( k_{1b}(\tilde{k}_1) \) equals one. The first order condition of the maximization then is

\[
(x\nu_1 + y(1 - \nu_1)) \left[ R' \left( k_{1b}(\tilde{k}_1) \right) - \tilde{\theta} \right] + ((1 - x) \nu_1 + (1 - y) (1 - \nu_1)) \left[ R' (\tilde{k}_1) - \tilde{\theta} \right] = 0
\]

A solution can be found via numerical optimization over \( x \) and \( y \).
**Case 3: Pooling**  The final case is that of pooling, in which in the first period the LP offers only one contract that is common for both types. The second period contract is the same as a standard one period separating contract, as no updating of beliefs has occurred, \( \nu_2 = \nu_1 \).

\[
\ell_2^{SB} = \theta k^{FB} + \Delta \theta \bar{k}^{SB} (\nu_1) + \psi \quad \text{and} \quad \ell_2^{SB} = \bar{\theta} k^{SB} (\nu_1) + \psi
\]

The maximization in the first period takes the following simple form

\[
\max_{k_1} R(k_1) - \bar{\theta} k_1 - \psi
\]

The first order condition requires \( R'(k^p = \bar{k}^{FB}) = \bar{\theta} \). The optimum contract for both types is then \( t^p_1 = \bar{k}^{FB} + \psi \), \( k_1 = \bar{k}^{FB} \).

**Proof of Lemma 10**

The rents of full commitment, without commitment and with short-term contracts (case 1) are, respectively, \( \Delta \theta \bar{k}^{SB} (\nu_1) + \delta \left[ \theta k^{FB} + \Delta \theta \bar{k}^{SB} (\nu_1) \right] \), \( \Delta \theta \bar{k}^{SB} (\nu_1 x) + \delta \left[ \theta k^{FB} + \Delta \theta \bar{k}^{SB} (\nu_1) \right] \), and

\[
\Delta \theta \bar{k}^{SB} (\nu_1 x) + \delta \left[ \theta k^{FB} + F \left[ R_2 (\bar{\nu}_2) - \psi \right] \Delta \theta \bar{k}^{SB} (\bar{\nu}_2) \right].
\]

The rents of full commitment are lower than without commitment as \( \nu_1 x < \nu_1 \), \( \bar{\nu}_2 < \nu_1 \) and \( \bar{k}^{SB} (\cdot) \) is decreasing. The rents of the short-term contracts are lower than those of commitment and renegotiation as \( F( R_2 (\bar{\nu}_2) - \psi) \leq 1 \). Those of full
commitment are lower than those of short-term contracts if $F(R_2(\overline{v}_2) - \psi) = 1$ but are greater if $F(R_2(\overline{v}_2) - \psi) = 0$.

In order to compare the likelihood of exit, notice that if an efficient GP has been identified in the first period, then the LP continues investing with the GP if $I < R^{FB}$. Suppose now that there has not been separation in the first stage. In the case of commitment and renegotiation, substituting in Lemma 4, the LP continues with both types if $I \leq R^{SB}(\overline{v}_2)$; continues only with efficient GPs if $R^{SB}(\overline{v}_2) < I < R_1$; exits regardless of type if $I \geq R_1$. In the case of short-term contracts, substituting in Lemma 8, the LP continues with both types if $I \leq (R_2(\overline{v}_2) - \psi)$; continues only with efficient GPs if $R_2(\overline{v}_2) - \psi < I < R^{FB} - \psi$; and exits regardless of type if $I \geq R^{FB} - \psi$. By definition $R^{SB}(\overline{v}_2) > R_2(\overline{v}_2) > R_2(\overline{v}_2) - \psi$. Therefore the LP continues more often with both types with commitment and renegotiation than with short-term contracts. Similarly, by definition $R_1 > R^{FB} > R^{FB}$. Then, the LP continues more often with the efficient type with commitment and renegotiation than with short-term contracts.

**Proof of Proposition 11**

In the case of long-term contracts, we have that substituting in the maximization problem of full commitment

$$\nu_1 \left[(1 + \delta) R^{FB} - (1 + \delta) \Delta \theta \overline{k}^{SB}(\nu_1)\right] + (1 - \nu_1)(1 + \delta) \overline{k}^{SB}(\nu_1) - (1 + \delta) \psi$$ (18)

Substituting $x = 1$ and $v_2 = 0$ and therefore $\overline{k}_1 = \overline{k}^{SB}(\nu_1)$ and $\overline{k}_2 = \overline{k}^{SB}(v_2) = \overline{k}^{FB}$ and $\overline{R}^b = \overline{R}^{FB}$ and $R_1 = +\infty$ in the profit function of the commitment and
renegotiation, we have

\[ \nu_1 \left[ R^{FB} - \Delta \theta k^{SB} (\nu_1) - \delta \Delta \theta k^{FB} \right] + (1 - \nu_1) R^{SB} (\nu_1) - (1 + \delta) \psi + \delta \nu_1 \left( \int_0^{R^{FB}} R^{FB} dF(I) + \int_{R^{FB}}^{+\infty} IdF(I) \right) + \delta (1 - \nu_1) \left( \int_0^{R^{FB}} R^{FB} dF(I) + \int_{R^{FB}}^{+\infty} IdF(I) \right) \]

and substituting the same optimal values of \( k \) and \( R_2 (\nu_2) = R^{FB} \) and \( R_2 (\nu_2) = -\infty \) into the profit function of the short-term contracts and in Lemma 3, we have

\[ \nu_1 \left[ R^{FB} - \Delta \theta k^{SB} (\nu_1) - \delta F(R^{FB} - \psi) \Delta \theta k^{FB} \right] + (1 - \nu_1) R^{SB} (\nu_1) - (1 + \delta) \psi + \delta \nu_1 \left( \int_0^{R^{FB}-\psi} R^{FB} dF(I) + \int_{R^{FB}-\psi}^{+\infty} IdF(I) \right) + \delta (1 - \nu_1) \left( \int_0^{R^{FB}-\psi} R^{FB} dF(I) + \int_{R^{FB}-\psi}^{+\infty} IdF(I) \right) \]

If \( F(R^{FB} - \psi) = 1 \), given that \( R^{FB} - \psi < R^{FB} < R^{FB} \), we have that the profits in the commitment and renegotiation and short-term contracts are the same and equal to

\[ \nu_1 \left[ (1 + \delta) R^{FB} - \Delta \theta k^{SB} (\nu_1) - \delta \Delta \theta k^{FB} \right] + (1 - \nu_1) \left[ R^{SB} (\nu_1) + \delta R^{FB} \right] - (1 + \delta) \psi. \]
Subtracting this case from the case of full commitment we have

\[
\delta \left\{ R_k^{SB} (\nu_1) - \frac{\nu_1}{1 - \nu_1} \Delta \theta \bar{k}^{SB} (\nu_1) - \left[ R_k^{FB} - \frac{\nu_1}{1 - \nu_1} \Delta \theta \bar{k}^{FB} \right] \right\} > 0
\]

given that the function

\[ R(k) - \bar{\theta}_k - \frac{\nu_1}{1 - \nu_1} \Delta \theta k \]

is maximized \( \bar{k}^{SB} (\nu_1) \).

If \( F(R_k^{FB}) = 0 \) then, given that \( \bar{R}_k^{FB} < R_k^{FB} \), the case of commitment and renegotiation is equal to

\[
\nu_1 \left[ R_k^{FB} - \Delta \theta \bar{k}^{SB} (\nu_1) - \delta \Delta \theta \bar{k}^{FB} \right] + (1 - \nu_1) \bar{R}_k^{SB} (\nu_1) - (1 + \delta) \psi + \delta \left\{ \int_0^{+\infty} IdF(I) \right\}
\]

whereas the case of short-term contracts, given that \( \bar{R}_k^{FB} - \psi < R_k^{FB} \) is equal to

\[
\nu_1 \left[ R_k^{FB} - \Delta \theta \bar{k}^{SB} (\nu_1) \right] + (1 - \nu_1) \bar{R}_k^{SB} (\nu_1) - (1 + \delta) \psi + \delta E(I)
\]

The profits in short-term contracts are higher. Subtracting this case from the case of full commitment we have

\[
\delta \left( \nu_1 \left[ R_k^{FB} - \Delta \theta \bar{k}^{SB} (\nu_1) \right] + (1 - \nu_1) \bar{R}_k^{SB} (\nu_1) - E(I) \right) < 0
\]

given that \( E(I) > \{ R_k^{FB} , \bar{R}_k^{SB} (\nu_1) \} \).
The first contract will be chosen by the efficient GP with probability $x$ and by inefficient GPs with probability $1-x$. This second long-term contract will be chosen by efficient GPs with probability $x$ and by inefficient GPs with probability $1-x$. The optimal sequence of short-term contracts in case 1, consist in a menu (long-term) and (short-term). In the first period, a second contract is chosen (both short- and long-term renegotiation contracts), which will be chosen by efficient managers with probability $x$ and by inefficient GPs with probability $1-x$. If the first of these two contracts is chosen, then the LP should offer a menu. If the second contract is chosen, then the LP should offer a menu.

\[
\frac{(x')^2 - x}{x - 1} = \frac{\epsilon}{\alpha - 1}
\]

\[
\hat{m} + (x')^2 - x = \frac{\epsilon}{\alpha - 1}
\]

\[
\hat{m} + (x')^2 - x = \frac{\epsilon}{\alpha - 1}
\]

\[
\hat{m} + (x')^2 - x = \frac{\epsilon}{\alpha - 1}
\]

\[
\hat{m} + (x')^2 - x = \frac{\epsilon}{\alpha - 1}
\]
Table 1: This table describes the ranking of the three contracting strategies. In the columns we take different values of $Z$, while in the rows, we take alternative specifications of each of the other parameters. In each block, we assume a base case of $a=0.5, b=1, \delta=0.8, \theta=1$, and $\nu=0$ and perform comparative statics with respect to one parameter at a time. Each cell reports the optimum values of $x$ and $y$ in each contract and the comparison between the profits associated with each contracting strategy. Long-term with full commitment (denoted as LTC) long-term with the possibility to renegotiate (LT), and short-term contracts in cases 1, 2, and 3 in Proposition 5 (ST1, ST2, and ST3, respectively). NP stands for not possible, and is used to identify the cases in which STI does not apply. The optimal contracting strategy in bold and red.