Endogenous Credit Ratings

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January 2012

Abstract

I study the role of credit ratings in crises by introducing a financial market in a coordination game with borrowers that must rollover their debts. The asset price aggregates dispersed private information acting as a public noisy signal. Credit rating agencies use this price to set their ratings. Moreover, agencies know that credit ratings influence lending decisions, thus affecting the creditworthiness of borrowers. I show that: (a) lenders overreact to changes in prices and credit ratings; (b) credit ratings are inaccurate during crises; (c) regulation relying on credit ratings should be redesigned to suspend their use in crises; (d) in the case of sovereign debt, an international financial institution helps prevent liquidity runs and reduce the negative effects of ratings; (e) transparency in financial markets makes credit ratings more volatile.

Keywords: Credit ratings, coordination risk, debt crises, nonlinearities.

JEL Classification Numbers: D82, G18, G24.
1 Introduction

I have often wondered what drives credit ratings. They are supposed to be independent opinion on the relative creditworthiness of an obligor. But the meaning of independent opinion is less than obvious in terms of the criteria and the methodology used in the construction of credit ratings. Credit rating agencies (CRAs) react to unfolding and impending events in financial markets. First, financial prices convey information that is valuable to determine the underlying quality of debt. Second, if downgrades contribute to lenders’s unwillingness to roll over debt claims, then the lowering of a credit score by a CRA may induce default because of the inability to sell new debt. Rating agencies thus face the problem of setting independent credit ratings when much of the relevant information is endogenous to financial markets and their actions affect the credit quality of issuers. These two features call into question the independence of external ratings.

Many claim that CRAs are as much following investor opinion as leading it. Following investor opinion is pertinent because perceptions about credit quality influence the lending decisions, thereby affecting the creditworthiness of borrowers that must roll over their debts. During times of crisis, individuals look frequently over their shoulders to learn about the actions and the opinions of others. Undoubtedly, CRAs monitor financial prices and economic indicators as these convey information about what market participants are doing and thinking.

But credit ratings themselves are closely watched by people making important economic decisions. If creditors rely on ratings to roll over their loans, credit ratings themselves affect the credit quality of borrowers. Besides, the big rating agencies have high public visibility and everybody knows that everybody else watches credit ratings. Credit ratings become reference points giving them a powerful coordination effect because they signal and influence what others are doing and thinking. In the case of sovereign debt, it is plausible that the lowering of a credit score by a CRA feeds a vicious circle and leads to self-fulfilling debt crises. Much, then, depends on the interaction between market participants and CRAs. Yet, little effort has been done to understand the channels of contagion linking credit ratings to credit quality.

What is missing from the literature is an understanding of the way credit ratings are formed in a marked-based financial system. This paper provides a theoretical model of credit ratings and their vulnerabilities. In the model, information is largely endogenous and the credit rating
is formed using private information known by the CRA and public information available in financial markets. This study draws on the theoretical model of defaultable debt by Morris and Shin (2004) and on the analysis of financial markets as endogenous sources of public information by Angeletos and Werning (2006). Consistent with these contributions, public information embodied in prices and credit ratings provides a reference point towards which the investors’ beliefs gravitate.\footnote{In game theory jargon, credit ratings have the capacity to become focal.} When issuing credit ratings, CRAs take into account the effects of ratings and prices on the creditworthiness of issuers.

There are two major areas in which the model contributes to the current debate on the reform of the credit rating system: the design of standards, laws and regulations that rely on credit ratings, and the effectiveness of external finance in the context of sovereign debt crises. As regards the first area, the Financial Stability Board, which coordinates the G20’s financial policies, has asked regulators and standard-setting bodies to reduce reliance on credit ratings in bank capital requirements and other forms of prudential oversight regulation, rules on investment-fund holdings, security regulations and rules, central bank operations and collateral eligibility standards. The Dodd-Frank Act on financial reform takes a firm approach regarding the use of credit ratings requiring their removal or replacement by appropriate alternatives. The European Union decided to bring CRAs into the regulatory net and the European Securities and Markets Authority will directly supervise them. Under one of the most contentious proposals being discussed, this institution would be able to ban sovereign credit ratings in "exceptional situations".

The model in this paper gives the conditions under which proposals to reform regulations and standards are effective and reasonable. In what I define as "crisis zone A", ratings turn out to be volatile and inaccurate. This feature justifies a flexible use of credit ratings in rules and regulations and, notably, it supports measures to suspend the use of credit ratings during episodes of financial instability. I also provide some clues about the role of prices, and in particular on the role of credit default swap (CDS) prices, in the design of regulation. I conclude that prices suffer from the same drawbacks as credit ratings.

As regards the second area, the analysis lends support to the hypothesis presented by Morris and Shin (2006) and Corsetti, Guimarães and Roubini (2006) that even a small amount of liquidity provision by official institutions can work to prevent a destructive liquidity run via
coordination of agent’s expectations. In what I define as "crisis zone B", sovereign credit ratings have the potential to precipitate the default of (illiquid but) solvent borrowers; in this zone a small amount of liquidity can shore up the issuer and prevent its default.

In this paper, I introduce a financial market in a coordination game with imperfect information. Individuals trade assets using their private information. As in Grossman and Stiglitz (1976), the (noisy) rational expectations equilibrium price aggregates dispersed private information. This price is an endogenous public signal. Once I allow for this specification, a new framework of analysis opens up, in which default probabilities are not independent of credit ratings and financial prices. This framework allows a full assessment of the response of credit ratings and default probabilities to changes in the fundamentals, financial prices and information disclosed by the CRAs. The analysis is related to a fast growing literature on the design of credit rating institutions, to which I contribute in a number of dimensions.

First, I study a rational expectations competitive equilibrium. Credit ratings incorporate new information - known only by the CRA - about the fundamentals and public information available in financial prices. Market participants recognize how credit ratings are formed and extract the new and relevant information produced by the CRA.

Second, there is a critical threshold for the value of the fundamentals below which the borrower is unable to roll over its debt and defaults. Conventional accounts obtain the default threshold for sovereign debt using some natural debt limit. This limit comes from assuming the risk-free status of government debt and finding the maximum debt that could be repaid under an optimal fiscal policy. Yet, in the short run, default depends mainly on the ability of the sovereign to coordinate creditors into rolling over their claims. In my model, the default threshold depends on natural debt limits and on the willingness of investors to roll over their credit. As a result, the default threshold is more stringent than in conventional models of sovereign debt because solvency and liquidity problems are not separable.

Third, there are crisis zones when economic fundamentals are near the default threshold. In a crisis zone a small deterioration in the fundamentals leads to a substantial reduction in debt roll over. The intuition is as follows. Imperfect information prevents investors from knowing the exact value taken by the fundamentals. When fundamentals are clearly above the default threshold, almost all investors agree on rolling over their credit; the opposite happens when fundamentals are clearly below the default threshold. When fundamentals are near the default
threshold, many investors are uncertain about whether others will roll over their claims or not and they will pay close attention to the information received in order to guess what others will do. A small deterioration of the fundamentals entails (slightly) negative information which is sufficient to induce many investors to refuse to roll over their claims with fear that others will do the same. This contributes to “credit cliff” situations, whereby small changes in the fundamentals induce considerable changes in creditworthiness. Such "overreaction" need not be based on irrational behavior on the part of investors.

Fourth, because agents want to coordinate their decisions they place too much weight on public information - financial prices and credit ratings. Public information serves largely as a focal point for the coordination of agent’s expectations. With agents overreacting to public information, small changes in prices or credit ratings have a large impact on investment decisions. Moreover, the weight given to public information is larger in the crisis zones thereby creating nonlinearities in the response of investors. The dilemma posed by the potential for overreaction to public information is well-known to government officials with high public visibility. Central bankers have developed specific communication skills, knowing how their public statements may disproportionately influence financial markets. Strikingly, no restrictions have been imposed so far on the communication strategies of CRAs, which have allowed them to unduly influence financial markets; CRAs insist their ratings are mere opinions, and as such should be protected by free speech.

Fifth, the degree of nonlinearities depends on the precision of public information. The weight given to public information increases with its precision, compelling agents to overreact (strongly) to precise public information. Again, this effect is larger in the crisis zones, thereby reinforcing the non linear effects. Since the impact of variations in credit ratings depends on the information structure, then credit ratings (and regulation relying on credit ratings) should reflect the specific features of each market. European authorities have decided that CRAs should clearly differentiate between ratings for structured finance instruments and ratings for other financial obligations.

The impact of public information is larger with more precise public information, and so is the impact of any noise in the public signals. For example, small drops in prices may be interpreted as a deterioration in credit quality and can reduce the willingness to roll over credit, thereby precipitating default of illiquid borrowers and creating instability in financial markets. This
issue is relevant from a practical perspective because it has implications regarding the recent regulatory trend towards more transparency in derivatives markets, I examine the sensitivity of outcomes to nonfundamental disturbances in prices and conclude that transparency is a double-edged instrument. Transparency may lead to more informative prices but it increases the sensitivity of debt default to nonfundamental shocks, bringing new challenges to the industry.

Sixth, I shed light into the claim, regarding periods of financial crisis, that CRAs are initially too slow to downgrade and subsequently downgrade faster and sharper than the worsening of the fundamentals would justify. I refocus the debate by showing that CRAs account for the coordination motive. This motive makes credit ratings very sensitive in the crisis zones and multi-notch downgrades may occur in response to small shocks to fundamentals. Moreover, if CRAs make small mistakes (for example when they collect information) then a small error may lead to a sharp and disproportionate downgrade. I also show that short term debt hinders coordination among lenders, leaving borrowers more vulnerable to financial crises.

I model the interaction between creditors and borrowers as a game with strategic complementarity and, like Manso (2011), I find feedback effects that amplify the impact of credit ratings. I analyze the role of CRAs using equilibrium selection in global games. Boot, Milbourn and Schmeits (2006) also see ratings as focal points, meaning that investors may rationally base their investment and pricing decisions on the rating, anticipating that sufficiently many will do the same. In doing so, credit ratings help fix the desired equilibrium in environments for which multiple equilibria would otherwise exist.

While my model explains why CRAs may opt for large adjustments in ratings, it does not explain the choice of quality of the information produced by CRAs. In this sense, the setting differs from recent models focusing on how incentive problems of financial intermediaries may reduce the quality of the information disclosed to investors. In those models ratings are biased due to reputational concerns or are inflated by the issuers’s ability to engage in "rating shopping" and possibly collude with rating agencies (Bolton, Freixas and Shapiro 2009). Finally, the issuance of ratings based on coarse information is seen by many not only as the source of pre-crisis mispricing of asset backed securities but also as the reason for the subsequent sequence of downgrades in the subprime crisis. The issuance of uninformative ratings is highlighted by Pagano and Volpin (2008) as a major inefficiency and suggests that there is a discrepancy between the private and the social benefits of transparency in debt issues.
According to those views, proposals to reform the credit rating industry should aim at guaranteeing more accurate information. This implies eliminating the conflicts of interest that CRAs face (Bolton, Freixas and Shapiro, 2007), reforming the issuer-pay model, minimizing the effects of barriers to entry in the industry and guaranteeing that reputational concerns do give incentives to reveal information (Mariano, 2008).

This paper specifically focus on sovereign ratings, given the most recent escalation in sovereign credit risk and the propensity for ratings to affect sovereign debt markets. Yet, the issues dealt with in the model are common to corporate and municipal credit ratings and this is why some of the examples presented belong to corporate issuers. I proceed by steps. The next Section presents the coordination problem when issuers must roll over their debts. It identifies the main drivers of the results by showing the nonlinear responses of credit ratings and default probabilities and how these responses depend on the precision of public information. The following Section incorporates a market for credit derivatives and examines the effects of nonfundamental volatility. Section 4 shows that a model with direct signals on the actions of investors is equivalent to a model with exogenous signals. Section 5 discusses the hardwiring of credit ratings into the regulatory framework, and Section 6 discuss welfare and policy related issues. Section 7 concludes.

2 The basic model with exogenous information

I assume that the sovereign government has an outstanding amount of one period debt equal to 1. The government can and is willing to repay an exogenous share $\theta$ of this debt while the remaining amount of debt $(1 - \theta)$ needs to be rolled over. Government debt is held by a continuum of short term creditors indexed by $i$ and uniformly distributed over the $[0, 1]$ interval. Each short term creditor individually decides whether or not to roll over his unit of debt and I define $a_i \in \{0, 1\}$ as individual investment. Let $A = \int_0^1 a_i \, di$ denote the aggregate level of investment.

I introduce strategic complementarity by assuming that the individual return to investment
depends on the aggregate level of investment. Accordingly, investors have utility

\[
   u(a_i, A, \theta, \psi) = \begin{cases} 
   R & \text{when } a_i = 1 \text{ and } \psi + A + \theta \geq 1 \\
   R - \Delta & \text{when } a_i = 1 \text{ and } \psi + A + \theta < 1 \\
   1 & \text{when } a_i = 0 
   \end{cases}
\]

where \( R \) and \( \Delta \) are constants with \( 0 < R - 1 < \Delta \leq R \). I give the following interpretation of the payoff function. Provided that the mass of investors \( A \) is large enough, the government is able to fulfil its promise and repays \( R - \Delta \) (where \( \Delta \) measures Loss Given Default, LGD); otherwise, the country is forced to default on its debt and repays zero. Alternative investment opportunities yield zero interest.

The random variable \( \theta \) measures the exogenous need for roll-over (the underlying fundamental) and gauges the ability of the sovereign to meet short-term obligations. I interpret \( \psi \) as the amount of exogenous funding that the government can guarantee. Factors determining this parameter are:

- The proportion of debt in the hands of institutional investors who have a long term horizon and follow "buy and hold strategies".
- Exogenous shocks in preferences of investors which affect the demand for sovereign bonds.
- Financial assistance from international agencies - for instance, obtained through stabilization programmes by the International Monetary Fund (IMF) or the European Stability Mechanism (ESM)\(^2\), the purchase of government bonds by the European Central Bank (ECB) and the purchase of government bonds by other countries - giving scope for international cooperation.

Parameter \( \psi \) captures the degree of strategic complementarity and measures the degree of vulnerability of the issuer; it is an indicator of how easy it is to coordinate investors. I confine my attention to the actions of short term creditors.

**Information and transparency.** The fundamental \( \theta \in R \) is not known at the time the investment decisions are made and short term creditors have heterogeneous beliefs about \( \theta \). The common prior about \( \theta \) is uniformly distributed on the real line. A sufficient statistic

\(^2\)The ESM is a permanent rescue funding programme that will succeed the temporary European Financial Stability Facility and European Financial Stabilization Mechanism. The ESM is due to be launched in mid-2013.
z summarizes the public information such that $z = \theta + \sigma_z \varepsilon$, where $\varepsilon$ is standard normal, independent of $\theta$ and common across agents. The private information of short term creditor $i$ is summarized by a sufficient statistic $x_i = \theta + \sigma_x \xi_i$, where $\xi_i$ is standard normal, independent of $\theta$ and independent and identically distributed across agents.

The information structure is parametrized by the standard deviations $\sigma_x$ and $\sigma_z$ or, equivalently, by $\alpha_x = \sigma^{-2}_x$ and $\alpha_z = \sigma^{-2}_z$, the precisions of private and public information. The posterior belief of agent $i$ about $\theta$ is normal with mean $E_i[\theta] = E[\theta|x_i, z] = x_i + \frac{\alpha_x}{\alpha_x + \alpha_z} x_i + \frac{\alpha_z}{\alpha_x + \alpha_z} z$ and variance $Var_i[\theta] = Var[\theta|x_i, z] = \frac{1}{\alpha_x + \alpha_z}$. Private information introduces heterogeneity in market expectations about the fundamental and may be read as heterogeneity in the filtering and interpretation of available information.

### 2.1 Equilibrium

A short term creditor finds it optimal to invest $a_i = 1$ if $E_i[\psi + A + \theta] \geq 1$, and $a_i = 0$ otherwise. I restrict attention to equilibrium with "switching strategies", in which case for every $z$ there exits $x^*(z)$ such that $a_i = 1$ if and only if $x_i \geq x^*(z)$. Aggregate investment is thus increasing in the economic fundamental $\theta$.

The equilibrium can be described by the threshold $\theta^*(z)$ below which there will be default in equilibrium because an insufficient number of short term creditors will choose to roll over their debt. When $\theta \geq \theta^*(z)$ there will be no default; let $p_i$ be the probability that agent $i$ attributes to $\theta \geq \theta^*(z)$. An agent finds it optimal to invest when the expected return from roll-over is larger than the payoff from the alternative, that is, when $p_i R + (1 - p_i) (R - \Delta) \geq 1$.

I restrict $\sigma^2_x \sqrt{2\pi} > \sigma_z$. This suffices for the equilibrium to be unique and amounts to saying that public information cannot be too precise, otherwise there are multiple equilibria. Let $\Phi$ and $\phi$ denote, respectively, the standard normal cumulative distribution function and the standard normal density distribution function.

**Proposition 1** (Morris and Shin 2004) There exists a unique equilibrium and $\theta^*(z)$ is implicitly determined by $\theta^* = \Phi_x \left[ \sqrt{\alpha_x + \alpha_z} \Phi^{-1} \left( 1 - \frac{R - 1}{\Delta} \right) + \alpha_z (\theta^* - z) \right] - \psi$. The default threshold $\theta^*$ is decreasing in the return $R$, increasing in the LGD $\Delta$, decreasing in the statistic for public information $z$, and decreasing in exogenous funding $\psi$.

**Proof.** Follows from Morris and Shin (2004). An agent invests if and only if $x \geq x^*$ where
\(x^*\) solves \(p_i R + (1 - p_i) (R - \Delta) = 1\) with \(p_i = 1 - \Phi \left( \frac{x^* - \frac{\alpha_x}{\alpha_x + \alpha_z} x^* - \frac{\alpha_z}{\alpha_x + \alpha_z} \theta^*}{\sqrt{\alpha_x + \alpha_z}} \right)\).

The mass of investors equals \(A = 1 - \Phi \left( \sqrt{\alpha_x} (x^* - \theta^*) \right)\). The sovereign defaults if and only if \(\theta < \theta^*\) where \(\theta^*\) solves \(\psi + A + \theta^* = 1\), or equivalently \(\theta^* = \Phi \left( \sqrt{\alpha_x} (x^* - \theta^*) \right) - \psi\). Solving for \(\theta^*\) yields \(\theta^* (z)\). The condition for uniqueness guarantees (with a slight abuse of notation) \(\frac{\partial \theta^*}{\partial R} < 0, \frac{\partial \theta^*}{\partial \Delta} > 0, \frac{\partial \theta^*}{\partial z} < 0\) and \(\frac{\partial \theta^*}{\partial \psi} < -1\).

The previous results account for a number of stylized facts. First, a high promised return \(R\) attracts short term creditors and reduces the probability of default while a high LGD has the opposite effect. Second, favorable public information, measured by \(z\), improves confidence regarding the capability of the sovereign to fulfil its obligations and favours debt roll-over. Additionally, since \(z = \theta + \sigma_z \varepsilon\), a stronger fundamental facilitates access to credit, which is a desirable property for any model of defaultable debt. Third, exogenous and long term funding, measured by \(\psi\), reduce the incidence of failure.

**Efficiency.** The efficient outcome is rolling over because the return from sovereign debt is higher than its alternative. Since \(A \leq 1\), the borrower defaults whenever \(\psi + \theta < 0\). In this sense, the sovereign is insolvent when \(\theta < -\psi\). If \(\theta \in [-\psi, \theta^*]\) there will be default in equilibrium, and this default would not occur if all investors were able to coordinate on rolling over the debt. Liquidation is inefficient but it is forced on the sovereign. I interpret \(\theta^*\) as a measure of inefficiency due to coordination failure.

### 2.2 Model with a credit rating agency

CRAs base their analysis on public information and on private information and confidential information which borrowers agree to share with them. A rating agency privately observes a signal \(\rho = \theta + \sigma_\rho \varepsilon_\rho\), where \(\varepsilon_\rho\) is standard normal, independent of \(\theta, \varepsilon,\) and \(\xi_i\), and the standard deviation \(\sigma_\rho\) is common knowledge. Signal \(\rho\) represents new available information not previously accessible to investors; variable \(\varepsilon_\rho\) is associated with errors in the agency signal. These errors may be interpreted as mistakes resulting from the process of collecting information and have the potential to mislead investors. The rating agency publicly announces its signal and short term creditors update their common prior accordingly. The new prior about \(\theta\) is normal with mean \(E[\theta | z, \rho] = \frac{\alpha_z}{\alpha_z + \alpha_\rho} z + \frac{\alpha_\rho}{\alpha_z + \alpha_\rho} \rho\) and precision \(\alpha_z + \alpha_\rho\) with \(\alpha_\rho = \sigma_\rho^{-2}\). I focus on the case in which there is a unique equilibrium, i.e. \(\sigma_z^2 \sigma_\rho^2 \sqrt{2 \pi} > \sigma_z (\sigma_z^2 + \sigma_\rho^2)\).
Proposition 2 (Carlson and Hale 2005) There exists a unique equilibrium and $\theta^* (z, \rho)$ is implicitly determined by $\theta^* = \Phi \left( \sigma_x \left( \sqrt{\alpha_x + \alpha_z + \alpha_p \Phi^{-1} (1 - \frac{\Delta}{\Delta x})} + (\alpha_z + \alpha_p) \left( \theta^* - \frac{\alpha_z}{\alpha_z + \alpha_p} z - \frac{\alpha_p}{\alpha_z + \alpha_p} \rho \right) \right) \right) - \psi$. The default threshold $\theta^*$ is decreasing in the signal $\rho$.

Proof. Follows from Carlson and Hale (2005). The uniqueness condition guarantees $\frac{\partial \theta^*}{\partial \rho} = -\frac{\sigma_{\rho} \Phi(\Phi^{-1}(\theta^* + \psi))}{1 - \sigma_{\rho} (\alpha_z + \alpha_p) \Phi^{-1}(\theta^* + \psi)} < 0$. □

The default threshold decreases with favorable information disclosed by the CRA but short term creditors are unable to identify the source of favorable information - a strong fundamental or a mistake in the information being disclosed (a positive shock $\varepsilon_{\rho}$).

Credit ratings. I map the probability that the sovereign is able to repay its debt into rating grades. The credit rating equals the probability that the actual value of $\theta$ lies above the threshold $\theta^*$, conditional on $z$ and $\rho$ - the information available to the CRA. Define $\hat{\rho}$ as the rating of sovereign debt such that $\hat{\rho} (z, \rho) = \Phi \left( \left( \frac{\alpha_z}{\alpha_z + \alpha_p} z + \frac{\alpha_p}{\alpha_z + \alpha_p} \rho - \theta^* \right) \sqrt{\alpha_x + \alpha_p} \right)$. Given $z$, variables $\hat{\rho}$ and $\rho$ have identical informational content in equilibrium so it is indifferent which is announced. Still, while $z$ and $\rho$ are exogenous variables, the rating is determined endogenously.

Rating actions. The response of credit ratings to shifts in the fundamental $\theta$ equals

$$\frac{\partial \hat{\rho}}{\partial \theta} = \sqrt{\alpha_x + \alpha_p} \Phi^{-1}(\hat{\rho}) + \frac{\phi(\Phi^{-1}(\hat{\rho})) \phi(\Phi^{-1}(\theta^* + \psi)) \sigma_x}{\sqrt{\alpha_x + \alpha_p} [1 - \phi(\Phi^{-1}(\theta^* + \psi)) \sigma_x (\alpha_z + \alpha_p)]} > 0 \quad (1)$$

where I use the result $\partial z/\partial \theta = \partial \rho/\partial \theta = 1$. The term $\sqrt{\alpha_x + \alpha_p} \Phi^{-1}(\hat{\rho})$ expresses the conventional intuition that favorable information reduces the probability of default. The term $\frac{\phi(\Phi^{-1}(\hat{\rho})) \phi(\Phi^{-1}(\theta^* + \psi)) \sigma_x}{\sqrt{\alpha_x + \alpha_p} [1 - \phi(\Phi^{-1}(\theta^* + \psi)) \sigma_x (\alpha_z + \alpha_p)]}$ is the novel feature. It arises because auspicious information favours coordination thereby reducing the default threshold $\theta^*$. Following Morris and Shin (2004), I call this term the "coordination effect" and it reinforces the conventional effect.

A fixed and stable base of investors (associated with a high $\psi$) facilitates debt roll-over and improves ratings (because $\partial \hat{\rho}/\partial \psi = \partial \rho/\partial \theta$). This effect comes about as a result of a shift in the default threshold. To the extent that individual investor $i$ has a choice between rolling over or withdrawing, we can regard investor $i$ as being a short-term claim holder while $\psi$ represents long term debt. The response of credit ratings suggests the hypothesis that issuers with a large

3S&P measures default risk in terms of default probability whereas Moody's ratings measure expected loss. Fitch uses probability of default for its issuer ratings and expected loss for its ratings of individual security issues. CRAs insist that they do not target their ratings to specific credit risk metrics but only to ordinal rankings of credit risk. Despite this claim, ratings are often used as though they map into specific credit-risk metrics, including the Basel Accord standardized approach.
proportion of short-term debt are more vulnerable to coordination problems and more fragile. This explains a significant review and change in sovereign risk methodologies which happened after the Asian crisis - when CRAs were widely criticized for failing to see the accumulation of risks that affected sovereign balance sheets. Today, the big rating agencies state that they closely monitor countries with a high proportion of short-term external debt.

Finally, parameter $\psi$ can be used to address explicitly a number of issues. If we interpret $\psi$ as exogenous shocks in investors's preferences, identical fundamentals may generate different ratings across borrowers. This parameter can also be used to explain the role played by contingent liabilities in the determination of credit ratings (with the materialization of large claims being represented by low values of $\psi$). This is potentially important given the role the extraordinary support to the banking sector played in the current financial crisis. No matter which interpretation we give, CRAs may adjust ratings without any changes in the fundamentals; they will reduce the credit score of a country whenever they perceive that $\psi$ is lower.

Nonlinearities and cliff effects. There is anecdotal evidence of rating failure. During the 1997 Asian crisis, CRAs were accused both of being initially too lenient with the East Asian sovereigns, and subsequently of downgrading more than the deterioration of fundamentals would warrant. Ferri, Liu and Stiglitz (1999) show that the drop in actual ratings was sharper than the predictions of a model of ratings based on economic fundamentals, suggesting that rating downgrades were larger than what economic fundamentals would justify. Similar anomalies have been documented by the IMF (1999) and Afonso, Gomes and Rother (2007). The CRAs in recent years have come under close scrutiny in the US, as regulators and lawmakers blamed them for feeding the mortgage bubble by awarding top grades to bonds backed by subprime mortgages, while in Europe the CRAs have been accused of being too slow initially to downgrade sovereigns and subsequently downgrading sovereign credits too aggressively (see, for example, European Commission 2010). Because credit ratings coordinate agent’s expectations, the relation between ratings, fundamentals and default is not linear. Nonlinearities create zones in which ratings (i) are inaccurate, and (ii) have a big influence on default probabilities and financial markets.

Consider $R-1 \leq \Delta/2$, that is, the LGD is large when compared with the net return. Let $\theta$ be the value of $\frac{\alpha_z}{\alpha_z + \alpha_p} z + \frac{\alpha_p}{\alpha_z + \alpha_p} \rho$ that maximizes expression (1). If $\theta \in \left[\theta^*, \theta^* + \frac{\sqrt{\alpha_z + \alpha_p + \alpha_p} \Phi^{-1}(1 - \frac{R-1}{\Delta})}{\alpha_z + \alpha_p} \right]$, where

Yet, recent empirical work has tried to reverse-engineer sovereign ratings from fundamental inputs and overall results indicate that short term external debt does not appear to be a significant factor in determining the level of credit ratings.
I call "crisis zone A" to the set of values of \( \frac{\alpha_x}{\alpha_z + \alpha_p} z + \frac{\alpha_p}{\alpha_z + \alpha_p} \rho \) that belong to the neighborhood of \( \vartheta \). In this zone, the response of credit ratings to shifts in \( \theta \) or \( \psi \) is large.\(^5\) For the special case \( R - 1 = \Delta/2 \), \( \vartheta = \theta^* \) which means that the credit rating is more sensitive when public information reveals that the fundamental is near the default threshold. When public information signals that the fundamental is near the default threshold, many investors are uncertain about whether others will roll over their claims or not and they will pay close attention to the information received in order to guess what others will do. A small deterioration of the fundamentals entails (slightly) negative information which is sufficient to induce many investors to refuse to roll over their claims with fear that others will do the same. Hence, the rating is sensitive to apparently innocuous shifts in the fundamental or in \( \psi \). To the extent that it is difficult to evaluate the nonlinearities, CRAs may attach higher weights to their qualitative judgement in "crisis zone A" in an attempt smooth credit ratings.\(^6\)

Figure 1 depicts the credit rating as a function of the fundamental, with the dashed line corresponding to a higher precision \( \alpha_z \) than the solid one (rewriting the credit rating as a function of \( \theta \), this function satisfies the single-crossing property). Studies relating credit ratings with historical default probabilities have shown that differences among the probabilities of default are minor among the highest ratings categories. Differences become significantly larger for the lowest rating categories. The reason for this non-proportional relation is evident from the shape of the function in Figure 1: for strong fundamentals, large shifts in \( \theta \) lead to small changes in the credit rating.

The model also predicts that large downgrades should be less frequent for strong fundamentals. Indeed, transition matrices among rating grades confirm that over the long term, higher rating grades are more stable than the lower rating categories. Figure 2 summarizes the distribution of large downgrades among the several rating grades for corporate issuers rated by Standard & Poor’s around the world in the past two decades, showing how low grades are less

\(^5\)If \( R > 1 + \frac{\Delta}{2} \) the interval becomes \( \left[ \theta^* + \frac{\sqrt{3\alpha_x + \alpha_z + \alpha_p}}{\alpha_z + \alpha_p} \Phi^{-1} \left( 1 - \frac{R - 1}{2} \right), \theta^* \right] \). When \( \theta \notin \left[ \theta^*, \theta^* + \frac{\sqrt{3\alpha_x + \alpha_z + \alpha_p}}{\alpha_z + \alpha_p} \Phi^{-1} \left( 1 - \frac{R - 1}{2} \right) \right] \), "crisis zone A" does not exist.

\(^6\)Various tests of quantitative, model-driven ratings indicate that qualitative judgment done by CRAs is indeed an important rating driver. CRAs acknowledge that their assessment relies both on quantitative and on qualitative analysis and accounts for characteristics that are difficult to measure objectively. Although there are a number of studies analyzing how fundamentals determine ratings, it less clear how qualitative assessment influences the actions of CRAs.
stable. Additionally, some suggest that the way CRAs try to smooth rating changes actually make them more prone to abrupt downgrades because these smoothing practices merely delay what is likely to be inevitable. The model in this paper suggests that CRAs’ attempts to avoid volatile ratings are likely to be successful for strong fundamentals and are likely to fail in "crisis zone A". Figure 2 is compatible with this hypothesis.

The impact of variations in the fundamental $\theta$ depends on the information structure, and therefore credit ratings should reflect the specific features of each market. During the subprime crisis, CRAs are generally seen to have performed reasonably well in the corporate bond market. It is in the structured finance segment in particular that ratings performance has come under severe criticism. Indeed, the Issing Committee (2008) argues that the CRAs were wrong to carry over a well-established methodology from bond markets to more complex, structured finance instruments. This critique supports the decision to give the European markets regulator the power to approve rating methods.

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7While downgrades are expected to some extent, a large number of them - in particular when they involve three or more notches at the same time or when the downgrading takes place within a short period after issuance or after another downgrade - are evidence of rating failure (see, for example, Bhatia 2002).
The response of the default threshold $\theta^* \text{ to new information released by the CRA is larger for } \Phi^{-1} (\theta^* + \psi) = 0 \Leftrightarrow \frac{\alpha_x}{\alpha_z + \alpha_x}z + \frac{\alpha_x}{\alpha_z + \alpha_x} \rho = \theta^* + \frac{\sqrt{\alpha_x + \alpha_z + \alpha_x}}{\alpha_z + \alpha_x} \Phi^{-1} \left( 1 - \frac{R - 1}{\Delta} \right)$. 

"Crisis zone B" is the set of values of $\frac{\alpha_x}{\alpha_z + \alpha_x}z + \frac{\alpha_x}{\alpha_z + \alpha_x} \rho$ that belong to the neighborhood of $\theta^* + \frac{\sqrt{\alpha_x + \alpha_z + \alpha_x}}{\alpha_z + \alpha_x} \Phi^{-1} \left( 1 - \frac{R - 1}{\Delta} \right)$; in this zone the issuer is vulnerable to information released by the CRA because short term creditors place too much weight on public information and the probability of default is largely determined by the beliefs of market participants. For the special case $R - 1 = \Delta / 2$, $\frac{\alpha_x}{\alpha_z + \alpha_x}z + \frac{\alpha_x}{\alpha_z + \alpha_x} \rho = \theta^*$ which means that the response of the default threshold is maximum when public information reveals that the fundamental is near the default threshold. Figure 3 illustrates the response of the default threshold to new information divulged by the CRA; the dashed line corresponding to a higher precision $\alpha_z$ than the solid one (again, I use the single-crossing property). As in Figure 1, the curvature of the relation becomes more pronounced when public information is more precise. In such circumstances, small variations in $\rho$ have a disproportionate impact on the willingness to invest because there is overreliance on the information released by the CRA. In "crisis zone B", noisy (but on average accurate) private information about the fundamentals of the borrowers is not so valuable; investors assign a lower weight to their private information relative to credit ratings, which lead to the default threshold becoming excessively dependent on ratings. Given that the impact of variations in $\rho$ depends on the information structure, then credit ratings should reflect the specific features
of each market. This feature supports the decision by European authorities that CRAs should discriminate between ratings for structured finance products and ratings for other financial obligations.

The nonlinearity in Figure 3 is also present in the relation between the default threshold $\theta^*$ and the fundamental $\theta$. In "crisis zone B", a small shift in the fundamental can easily lead to default. Results are similar regarding exogenous finance $\psi$, lending support to the hypothesis of "catalytic finance" by Morris and Shin (2006) and Corsetti, Guimarães and Roubini (2006). Liquidity provision by an official institution like the IMF, the ESM or the ECB can work to prevent a destructive run by moving the default threshold $\theta^*$ downwards. To the extent that these institutions do not have infinite resources, the results make clear that "catalytic finance" is not effective when the fundamental is too weak: as more and more individuals receive bad private signals, the unwillingness to invest will cause a crisis regardless of whether there is an intervention or not. Hence, assistance to countries with conditioned access to liquidity should be limited to the crisis zones. Additionally, the excess sensitivity of the default threshold $\theta^*$ suggests that the European markets regulator should be able to temporarily suspend the issuance of sovereign ratings for a country undergoing an international bailout programme.
For reasonable values of the parameters, "crisis zone B" lies above "crisis zone A" (because $\vartheta \leq \vartheta^* + \frac{\alpha_x + \alpha_z + \alpha_p}{\alpha_x + \alpha_p} \Phi^{-1} \left( 1 - \frac{R - 1}{\Delta} \right)$, see Figure 4), meaning that a sovereign facing a slow degradation of its balance sheet (for example, as a result of low economic growth) is likely to enter first "crisis zone B" before hitting "crisis zone A". This justifies preemptive action taken before ratings become too volatile. It also explains the fetish around the highest rating grades; as shown in Figure 1, for the highest rating categories, small differences in ratings indicate large differences in fundamentals. Hence government officials recognize how important it is to maintain a good rating in order to avoid the crisis zones.

3 Financial markets

The previous results presume that the precision of public information is independent of the precision of private information. This is unlikely to be the case when there are financial markets because prices aggregate private information. To investigate the role of prices, I introduce a financial market where agents trade a derivative security prior to playing the coordination game.

Because the return on the derivative depends on the underlying fundamental, the equilibrium price will convey information that is valuable in the coordination game. Figure 5 compares median CDS spreads for corporations and for sovereign issuers by rating grades and shows similarity of CDS spreads for borrowers of the same rating.8

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8 Some say that governments and investors may well be attaching too much importance to the AAA grade because a downgrade from AAA to AA means only a slight increase in default risk. Yet, the Treasury Secretary, Tim Geithner, claimed that the US would "never" lose its AAA credit rating and a countless number of government officials have promised to defend the AAA rating.

9 As of the credit crisis in 2007, CDS spreads have increased for all letter grades. This confirms that CRAs focus on the rank ordering of credit risk, instead of striving to maintain stable default rates for given rating
Setup. As before, the fundamental $\theta$ is withdrawn from an improper uniform distribution over the real line and each agent $i$ receives the exogenous private signal $x_i = \theta + \sigma_s \xi_i$. For tractability reasons I separate the investment coordination game from the derivatives market. Agents can be seen as interacting in two separate stages.

The first stage happens in the derivatives market: agents trade a risky asset with return $\theta$ at a price $p$. I adopt the CARA-normal framework introduced by Grossman and Stiglitz (1976). The utility of agent $i$ is $v(w_i) = -e^{-\gamma w_i}$ for $\gamma > 0$, where $w_i = w_0 + (\theta - p) k_i$ is the final wealth, $w_0$ is the initial endowment, and $k_i$ is investment in the asset. The supply of the asset is uncertain and not observed, given by $K^S(\varepsilon_s) = \sigma_s \varepsilon_s$, where $\sigma_s > 0$ and $\varepsilon_s$ is standard normal, independent of $\theta$ and $\xi_i$. This formulation means that the derivative security exists in zero net supply plus some noise - parametrized by $\sigma_s$ - which prevents a fully revealing equilibrium.

The second stage is essentially the same as the model with credit rating agency in the previous Section: short term creditors decide whether to invest or not; the sovereign government defaults if and only if $\psi + A + \theta < 1$ and the payoff from this stage is $u(a_i, A, \theta, \psi)$. The only difference is that short term creditors observe the price that cleared the financial market. The eventual default by the sovereign, the derivative’s return and the payoffs from both stages are
realized at the end of stage 2.

Individual derivative security demand is a function of \( x \) and \( p \), the realizations of the private and public signals, and the corresponding aggregate is a function of \( \theta \) and \( p \). The individual investment decision on sovereign debt is a function of \( x, \rho \) and \( p \) and the corresponding aggregate is a function of \( \theta, \rho \) and \( p \).

**Definition 1** An equilibrium is a price function, \( P(\theta, \varepsilon_s) \), individual strategies for investment in the derivative and in public debt, \( k(x, p) \) and \( a(x, \rho, p) \), and their corresponding aggregates, \( K(\theta, p) \) and \( A(\theta, \rho, p) \), such that:

\[
\begin{align*}
    k(x, p) &\in \arg \max_{k \in \mathbb{R}} E_i [v(w_0 + (\theta - p) k)] \\
    K(\theta, p) &= E[k(x, p) | \theta, p] \\
    K(\theta, P(\theta, \varepsilon_s)) &= K^S(\varepsilon_s)
\end{align*}
\]

and

\[
\begin{align*}
    a(x, \rho, p) &\in \arg \max_{a \in \{0, 1\}} E_i [u(a_i, A, \theta, \psi)] \\
    A(\theta, \rho, p) &= E[a(x, p) | \theta, \rho, p]
\end{align*}
\]

and the sovereign government defaults if and only if \( \psi + A(\theta, \rho, p) + \theta < 1 \).

The above conditions define a rational expectations competitive equilibrium for the first stage and a Bayesian equilibrium for stage 2. There is an important difference in the second stage with respect to the model in Section 2.2: the endogenous price \( p \) replaces the exogenous public signal \( z \).

**Equilibrium.** In the first stage, I guess a linear price function. Observing the price realization then is equivalent to observing a normally distributed signal with some precision \( \alpha_p = \sigma_p^{-2} \geq 0 \). The posterior of \( \theta \) conditional on \( x \) and \( p \) is normally distributed with mean \( \frac{\alpha_x}{\alpha_x + \alpha_p} x + \frac{\alpha_p}{\alpha_x + \alpha_p} p \) and precision \( \alpha_x + \alpha_p \). Individual asset demand is \( k(x, p) = \frac{\alpha_x}{\gamma} (x - p) \) and aggregate demand is \( K(\theta, p) = \frac{\alpha_x}{\gamma} (\theta - p) \). Market clearing implies \( P(\theta, \varepsilon_s) = \theta - \gamma \sigma_s \sigma_z^2 \varepsilon_s \) which verifies the initial guess with \( \sigma_p = \gamma \sigma_s \sigma_z^2 \). This result highlights the informative role of prices because the precision of public information improves with private information.
The second stage is equivalent to the benchmark model in the previous Section, with the price \( p \) playing the role of the public signal \( z \). Replace \( \sigma_z \) with \( \sigma_p \) and the uniqueness condition becomes \( \gamma^2 \sigma_x^2 \sigma_p^2 \sqrt{2\pi} > \gamma^2 \sigma_x^4 + \sigma_p^2 \).

**Proposition 3** (Angeletos and Werning 2006) There exists a unique equilibrium and \( \theta^\ast(p, \rho) \) is implicitly determined by

\[
\theta^\ast = \Phi \left( \sigma_x \left[ \sqrt{\alpha_p + \alpha_p \Phi^{-1}(1 - \frac{R_s}{\alpha_p})} + \alpha_p + \alpha_p \left( \theta^\ast - \frac{\alpha_p}{\alpha_p + \rho} p - \frac{\alpha_p}{\alpha_p + \rho} \rho \right) \right] \right) - \psi.
\]

The default threshold \( \theta^\ast \) is decreasing in the price \( p \).

**Proof.** Follows from Angeletos and Werning (2006). The uniqueness condition guarantees \( \frac{\partial \theta^\ast}{\partial p} < 0 \).

The model behaves in a similar way to the model with CRA in Section 2.2, with prices replacing the public signal \( z \); high prices signal a strong fundamental and favour debt rollover. It follows that the model of financial markets without CRA is the limit case in which the precision of the information disclosed by the agency is null \( (\alpha_\rho \rightarrow 0) \) and the market price of the derivative is the only source of public information.

### 3.1 Nonfundamental volatility

I examine the role of the information structure for nonfundamental volatility, that is, volatility conditional on \( \theta \). Specifically, I evaluate the sensitivity of the default threshold and credit ratings to shocks in prices \( (\varepsilon_s) \) and to shocks in the information disclosed by the CRA \( (\varepsilon_\rho) \). I interpret the former as noise in financial markets and the latter as mistakes done by the CRA.

I obtain two sets of results. First, there are zones in which the impact of the exogenous shocks \( \varepsilon_s \) and \( \varepsilon_\rho \) is larger and, second, less noise in public information increases volatility in credit ratings and instability in primary markets.

#### 3.1.1 Credit ratings

The CRA incorporates the information embedded in the financial price into the credit rating, so that \( \hat{\rho}(p, \rho) = \Phi \left( \left( \frac{\alpha_p}{\alpha_p + \rho} p + \frac{\alpha_p}{\alpha_p + \rho} \rho - \theta^\ast \right) \sqrt{\alpha_p + \alpha_p} \right) \). It follows that

\[
\frac{\partial \hat{\rho}}{\partial \varepsilon_s} = -\phi \left( \Phi^{-1}(\hat{\rho}) \right) \left( \frac{\alpha_p}{\alpha_p + \alpha_p} + \frac{\alpha_p}{\alpha_p + \rho} \Phi^{-1}(\theta^\ast + \psi) \right) \sigma_x \left( \frac{\alpha_p}{\alpha_p + \rho} + \alpha_p \right) < 0.
\]
A positive supply shock reduces the price of the derivative which the CRA interprets as a deterioration in credit quality. Noise in financial markets has two effects on the rating of sovereign debt. First, there is a direct effect because low prices signal low credit quality. Second, there is the "coordination effect" working through the default threshold. For completeness,
\[
\frac{\partial \hat{\rho}}{\partial \varepsilon} = \phi \left( \Phi^{-1} (\hat{\rho}) \right) \left( \frac{\alpha_p}{\alpha_p + \alpha_\rho} + \frac{\sqrt{\alpha_p \phi \left( \Phi^{-1} (\theta^* + \psi) \right) \sigma_x \sqrt{\alpha_p + \alpha_\rho}}}{1 - \phi \left( \Phi^{-1} (\theta^* + \psi) \right) \sigma_x (\alpha_p + \alpha_\rho)} \right) > 0.
\]

Again, the relation between the default threshold and the two types of noise is nonlinear.

3.1.2 Impact on sovereign default

In order to obtain a better understanding of the effect of noise on the default threshold, I perform the analysis for \( \theta = \theta^* \). The regime is abandoned if and only if \( \theta \leq \theta^* (p, \rho) \) where \( p = \theta - \sigma_p \varepsilon_s \) and \( \rho = \theta + \sigma_\rho \varepsilon_\rho \). Function \( \theta^* (p, \rho) \) is continuously decreasing in both arguments and \( p \) and \( \rho \) are continuously increasing in \( \theta \). Hence, the sovereign defaults if and only if \( \theta \leq \tilde{\theta} (\varepsilon_s, \varepsilon_\rho) \) where \( \tilde{\theta} (\varepsilon_s, \varepsilon_\rho) \) is the unique solution to
\[
\tilde{\theta} = \Phi \left( \sigma_x \left[ \sqrt{\alpha_x + \alpha_p + \alpha_\rho} \Phi^{-1} \left( 1 - \frac{R - 1}{\Delta} \right) + \left( \alpha_p + \alpha_\rho \right) \left( \tilde{\theta} - \frac{\alpha_p}{\alpha_p + \alpha_\rho} (\tilde{\theta} - \sigma_p \varepsilon_s) - \frac{\alpha_\rho}{\alpha_p + \alpha_\rho} (\tilde{\theta} + \sigma_\rho \varepsilon_\rho) \right) \right] \right) - \psi.
\]

Hence
\[
\frac{\partial \tilde{\theta}}{\partial \varepsilon_\rho} = -\phi \left( \Phi^{-1} \left( \tilde{\theta} + \psi \right) \right) \frac{\sigma_x}{\sigma_\rho},
\]
\[
\frac{\partial \tilde{\theta}}{\partial \varepsilon_s} = \phi \left( \Phi^{-1} \left( \tilde{\theta} + \psi \right) \right) \frac{\sigma_x}{\sigma_p} = \phi \left( \Phi^{-1} \left( \tilde{\theta} + \psi \right) \right) \frac{1}{\gamma \sigma_s \sigma_x}.
\]

The potential for real damage provoked by errors in credit ratings - as a result of noise \( \varepsilon_\rho \) - is immense in "crisis zone B" (with this zone being redefined as the set of values of \( \frac{\alpha_x + \alpha_p + \alpha_\rho}{\alpha_p + \alpha_\rho} \phi \left( \Phi^{-1} \left( \tilde{\theta} + \psi \right) \right) \frac{1}{\gamma \sigma_s \sigma_x} \)) that belong to the neighborhood of \( \theta^* + \sqrt{\alpha_x + \alpha_p + \alpha_\rho} \Phi^{-1} \left( 1 - \frac{R - 1}{\Delta} \right) \). This is a consequence of the feedback effects of credit ratings and the pivotal role played by CRAs; the use of credit ratings is so pervasive that market participants cannot ignore them even if they do not consider them reliable. The same can be said about the noise in prices; these too may increase the coordination motive and bring instability to financial markets.

21
Figure 6: Default threshold $\hat{\theta}$ as a function of $\varepsilon_s$. The dashed line corresponds to a lower $\sigma_s$.

A reduction in $\sigma_s$ or $\sigma_\rho$ increases the sensitivity of the equilibrium outcomes to the exogenous shocks $\varepsilon_s$ and $\varepsilon_\rho$, making primary markets more unstable. Figure 6 depicts function $\hat{\theta}(\varepsilon_s, \varepsilon_\rho)$ (keeping $\varepsilon_\rho$ fixed and using the single-crossing property) with the dashed line corresponding to lower standard deviation $\sigma_s$ than the solid one. This set of results calls for a reassessment of the current trend towards more transparency in derivatives markets as regulators move over-the-counter (OTC) contracts to exchanges (and clearing houses). By making prices more widely available to buyers of OTC derivatives, regulators will raise the coordination role of prices thereby creating new challenges for the industry.\textsuperscript{10}

## 4 Following investor opinion

In this Section, I examine situations where information originates within the coordination game itself: the CRA divulges a public signal about the aggregate level of investment in sovereign debt. Such feature seems relevant for thinking about CRAs because some authors argue that CRAs do not produce new information and follow investor opinion instead; somehow CRA are

\textsuperscript{10}The result that more transparency in public information can be damaging for welfare contrasts with some of the results on the social value of public information (Svensson, 2006). The difference is due to the specification of the payoff function and because I focus on the crisis zones.
able to observe (or anticipate) what investors do. Still, credit ratings are unique because they serve as focal points for the coordination of agent’s expectations.

The model is identical to the basic model of Section 2, except that the public signal $z$ is replaced with $y = \Phi^{-1}(1 - A) + \sigma_y \varepsilon_y$ where $\varepsilon_y$ is normally distributed and independent of $\theta$ and $\xi_t$ (see Dasgupta 2003, for details on this specification). The public signal $y$ is disclosed by the CRA and agents condition their investment decisions on this indicator of aggregate behavior.

**Definition 2** An equilibrium consists of an endogenous signal $y = Y(\theta, \varepsilon_y)$, an individual investment strategy $a(x, y)$ and aggregate investment $A(\theta, y)$, which satisfy:

$$a(x, y) \in \arg \max_{a \in \{0, 1\}} E_z[u(a, A(\theta, y), \theta, \psi)]$$

$$A(\theta, y) = E[a(x, p) | \theta, y]$$

$$y = \Phi^{-1}(1 - A(\theta, y)) + \sigma_y \varepsilon_y.$$  

In monotone equilibria an agent invests if and only if $x \geq x^*(y)$ and the sovereign defaults if and only if $\theta \leq \theta^*(y)$. Hence an equilibrium is identified with functions $x^*(y), \theta^*(y)$ and $y = Y(\theta, \varepsilon_y)$. The uniqueness condition becomes $\sigma_y^2 \sigma_x \sqrt{2\pi} > 1$.

**Proposition 4** (Angeletos and Werning 2006) There exists a unique equilibrium and $\theta^*(y)$ is determined by $\theta^* = \Phi\left(\frac{1}{\sqrt{\alpha_z}} \left(\frac{\alpha_z}{\alpha_z + \sigma_z \varepsilon_y} \Phi^{-1}(1 - \frac{B + 1}{\alpha_z}) + \frac{\sigma_y \varepsilon_y}{\alpha_z + \sigma_y \varepsilon_y} \sigma_x y\right)\right) - \psi$.

**Proof.** In a monotone equilibrium the mass of investors equals $A(y, \theta) = 1 - \Phi \left(\frac{1}{\sqrt{\sigma_x}} (x^*(y) - \theta)\right)$.

Using the definition of public signal, $y = \Phi^{-1}(1 - A) + \sigma_y \varepsilon_y = \frac{1}{\sigma_z} (x^*(y) - \theta) + \sigma_y \varepsilon_y$ and, therefore $x^*(y) - \sigma_z y = \theta - \sigma_x \sigma_y \varepsilon_y$. This expression can be seen as a function that relates $y$ and $z \equiv \theta - \sigma_x \sigma_y \varepsilon_y$. Given $y$, agents are able to infer $z$ where $\sigma_z = \sigma_x \sigma_y$. An agent invests if and only if $x \geq x^*(y)$ where $x^*(y)$ solves $R - \Phi \left(\sqrt{\alpha_x + \alpha_z} \left[\theta^* - \left(\frac{\alpha_z}{\alpha_x + \alpha_z} x^* + \frac{\alpha_x}{\alpha_x + \alpha_z} \sigma_x y\right)\right]\right) \Delta = 1$ with $\alpha_z = \frac{1}{\sqrt{\sigma_z}}$. This expression can be rewritten as

$$R - \left[1 - \Phi \left(\sqrt{\alpha_x + \alpha_z} \left[x^* - \theta^* - \frac{\alpha_z}{\alpha_x + \alpha_z} \sigma_x y\right]\right]\right] \Delta = 1.$$  

The sovereign defaults if and only if $\theta < \theta^*(y)$ where $\theta^*(y)$ solves $\psi + A(\theta, y) + \theta = 1$, or
equivalently
\[ \theta^* = \Phi \left( \frac{x^* - \theta^*}{\sigma_x} \right) - \psi. \]  
(3)

Substituting (2) into (3) I get \( \theta^* = \Phi \left( \frac{1}{\sigma_x} \left( \frac{1}{\sqrt{\alpha_x + \alpha_z}} \Phi^{-1} \left( 1 - \frac{R_1}{\Delta} \right) + \frac{\alpha_z}{\alpha_x + \alpha_z} \sigma_x y \right) \right) - \psi. \)

This expression yields a unique solution \( \theta^*(y) \) and substituting this solution into (3) I obtain the unique solution \( x^*(y) = \theta^* + \frac{1}{\sqrt{\alpha_x + \alpha_z}} \Phi^{-1} \left( 1 - \frac{R_1}{\Delta} \right) + \frac{\alpha_z}{\alpha_x + \alpha_z} \sigma_x y = \Phi \left( \frac{1}{\sigma_x} \left( \frac{1}{\sqrt{\alpha_x + \alpha_z}} \Phi^{-1} \left( 1 - \frac{R_1}{\Delta} \right) + \frac{\alpha_z}{\alpha_x + \alpha_z} \sigma_x y \right) \right) - \psi + \frac{1}{\sqrt{\alpha_x + \alpha_z}} \Phi^{-1} \left( 1 - \frac{R_1}{\Delta} \right) + \frac{\alpha_z}{\alpha_x + \alpha_z} \sigma_x y. \)

Finally, I confirm that \( F(y) = x^*(y) - \sigma_x y \) is indeed a function. Substituting \( x^*(y) \)
yields \( F(y) = \Phi \left( \frac{1}{\sigma_x} \left( \frac{1}{\sqrt{\alpha_x + \alpha_z}} \Phi^{-1} \left( 1 - \frac{R_1}{\Delta} \right) + \frac{\alpha_z}{\alpha_x + \alpha_z} \sigma_x y \right) \right) - \psi + \frac{1}{\sqrt{\alpha_x + \alpha_z}} \Phi^{-1} \left( 1 - \frac{R_1}{\Delta} \right) + \left( \frac{\alpha_z}{\alpha_x + \alpha_z} - 1 \right) \sigma_x y \) and computing the sign of \( F'(y) \) it is easy to show that \( F(y) \) is monotonic.

Just as in the benchmark model of Section 2.2, equilibrium depends on the credit rating. There is a fundamental difference, though, as the new information provided by the CRA is now endogenous.

**Financial markets.** As in the Section 3, a financial market opens in the first stage and reveals the price of the derivative. In this setup, it remains to characterize the public signal \( y \). Signal \( y \) comprises two types of information. First, it includes information embedded in market prices; were this the only type of information disclosed by the CRA, and \( y = \Phi^{-1} \left( 1 - A \right) + \frac{\alpha_z}{\sigma_x} \varepsilon_y \). This type of information is redundant since it conveys the same information as prices. The second type of information is new information, independent of the information contained in prices. Signal \( y \) is informative as long as it carries information beyond market prices. As a result, the noise in \( y \) - parametrized by \( \sigma_y \varepsilon_y \) - includes the noise in prices and the noise in the new information about the mass of investors \( A \). Let \( \alpha_2 \varepsilon \) be a measure of the precision in the new information provided by the CRA. I restrict \( \gamma^2 \sigma_2^2 \sigma_x^2 \sqrt{2\pi} \geq 1 + \alpha_2 \gamma^2 \sigma_2^2 \sigma_x^2 \).

**Proposition 5 (Equivalence Principle)** In a market-based financial system, the equilibrium obtained when the CRA follows investor opinion is equivalent to the equilibrium obtained when the CRA discloses a private signal \( \rho \).

**Proof.** As shown in the proof of Proposition 4, given \( y \), agents are able to infer the public signal \( z \equiv \theta - \sigma_x \gamma \varepsilon_y = \theta - \frac{1}{\alpha_2 \gamma \sigma_x} \varepsilon_y \). Variable \( z \) incorporates information about \( p \) and the new information produced by the CRA. Given that \( z \) and \( p \) are normally distributed, the new
information can be represented by a signal $\tilde{\rho} = \theta + \sigma_{\tilde{\rho}} \varepsilon_{\tilde{\rho}}$ with $\varepsilon_{\tilde{\rho}}$ independent of $\theta, \varepsilon_s$ and $\xi$, and with $\sigma_{\tilde{\rho}}^2 = \frac{1}{\alpha_{\tilde{\rho}}}$. Hence $z = \frac{\alpha_{\tilde{\rho}}}{\alpha_p + \alpha_{\tilde{\rho}}} \tilde{\rho} + \frac{\alpha_p}{\alpha_p + \alpha_{\tilde{\rho}}} \rho$ and, after the first stage, the new prior has precision $\alpha_p + \alpha_{\tilde{\rho}}$. The investment problem is equivalent to the second stage of the model of Section 3. Hence Proposition 3 holds with $\tilde{\rho}$ replacing $\rho$. ■

It follows that the results regarding nonfundamental volatility hold with this form of "herding". The main issue is whether CRAs improve the precision of the information provided by financial markets or if they just reproduce the information already contained in prices.

The standard deviation in the public signal $y$ is $\sigma_y = \frac{1}{\sigma_x} \sqrt{\frac{1}{\alpha_p + \alpha_{\tilde{\rho}}}$. As $\alpha_{\tilde{\rho}} \to 0$, the signal $y$ becomes uninformative and I obtain the equilibrium in financial markets without CRA. When $\alpha_{\tilde{\rho}} > 0$, signal $y$ is informative and $\sigma_y < \frac{\sigma_p}{\sigma_x}$, that is, the CRA improves the precision of the information included in prices.

5 Hardwiring

One key concern is whether rating downgrades destabilize financial markets because they are embedded in many regulations and private contracts. Prudential regulations typically allow for less capital or liquidity to be held against highly rated securities. Central banks use ratings to determine which assets can serve as collateral in their money market operations. Suitability standards, which discipline fund managers by restricting investments to assets with certain risk characteristics, are often based on rating thresholds. Credit ratings are used as triggers for collateral calls in margin agreements in financing transactions. In these ways ratings drive institutional demand and market liquidity.

The preceding Sections contain descriptions drawn from individual investor optimal behavior. This Section considers the mechanical use of credit ratings in investment decisions. To do this I reinterpret $\psi$ as the amount of debt in the hands of institutional investors who have a long term horizon and use ratings as "buy-sell triggers". As in Section 3, the price of the derivative security is determined before the CRA discloses $\rho$. Let $\psi$ depend on the price $p$ and the signal $\rho$ so that $\psi(p, \rho) = \Phi \left( p + \frac{\alpha_p}{\sigma_x} \rho \right)$; $\psi$ is increasing in the public signals $\rho$ and $p$ and takes values in the interval $(0, \overline{\psi})$ with $\overline{\psi}$ denoting the maximum amount of debt that can be rolled over using information on credit ratings. Following the same steps and using the same assumptions as in Section 3, I obtain the following result.
Proposition 6 (Hardwiring) There exists a unique equilibrium and \( \theta^* (p, \rho) \) is implicitly determined by \( \theta^* = \Phi \left( \sigma_x \left( \sqrt{\alpha_x + \alpha_p + \alpha_p \Phi^{-1} \left( 1 - \frac{R - 1}{\alpha} \right)} + (\alpha_p + \alpha_p) \left( \theta^* - \frac{\alpha_p}{\alpha_p + \alpha_p} p - \frac{\alpha_p}{\alpha_p + \alpha_p} \rho \right) \right) \right) - \Phi \left( \rho + \frac{\alpha_p}{\alpha_p} p \right) \). 

It could be argued that market practises, laws and regulations that hardwire buy or sell decisions to rating thresholds make the amount of debt held by institutional investors \( \psi \) vary with the credit rating \( \hat{\rho} \) and not with \( \rho \). In the current setup both representations are equivalent because \( \rho \) and \( \hat{\rho} \) have the same informational content once \( p \) is known. Represent the relationship between \( \psi \) and \( \hat{\rho} \) as a function \( \hat{\psi} (\hat{\rho}) \). The degree of hardwiring of investment decisions to credit ratings is given by the slope of this function, that is \( \hat{\psi}' (\hat{\rho}) = \frac{\partial \hat{\psi}}{\partial \rho} = \frac{\partial \hat{\psi}}{\partial p} > 0 \); lower ratings diminish the amount of debt in the hands of those investors who use ratings as "buy-sell triggers", which is a desirable property for any model of hardwiring.

Repeating the comparative-statics analysis performed earlier, reveals that hardwiring amplifies the response of the credit rating and the default threshold to shocks in prices \( (\varepsilon_s) \) and to shocks in the information disclosed by the CRA \( (\varepsilon_p) \). The additional effects can be measured by the following derivatives:

\[
\frac{\partial \hat{\psi}}{\partial \varepsilon_s} = -\phi \left( \Phi^{-1} (\hat{\rho}) \right) \left( \sqrt{\frac{\alpha_p}{\alpha_p + \alpha_p}} + \frac{\alpha_p \phi \left( \Phi^{-1} (\theta^* + \psi) \right) \sigma_x + \phi \left( \rho + \frac{\alpha_p}{\alpha_p} p \right) \frac{\alpha_p}{\alpha_p} \hat{\psi} \sigma_p}{1 - \phi \left( \Phi^{-1} (\theta^* + \psi) \right) \sigma_x (\alpha_p + \alpha_p)} \right) < 0,
\]

\[
\frac{\partial \hat{\psi}}{\partial \varepsilon_p} = \phi \left( \Phi^{-1} (\hat{\rho}) \right) \left( \sqrt{\frac{\alpha_p}{\alpha_p + \alpha_p}} + \frac{\alpha_p \phi \left( \Phi^{-1} (\theta^* + \psi) \right) \sigma_x + \phi \left( \rho + \frac{\alpha_p}{\alpha_p} p \right) \hat{\psi} \sigma_p}{1 - \phi \left( \Phi^{-1} (\theta^* + \psi) \right) \sigma_x (\alpha_p + \alpha_p)} \right) > 0,
\]

and, for sufficiently low \( \hat{\psi} \),

\[
\frac{\partial \hat{\theta}}{\partial \varepsilon_p} = -\phi \left( \Phi^{-1} (\hat{\theta} + 1) \right) \frac{\sigma_x}{\sigma_p} + \phi \left( \rho + \frac{\alpha_p}{\alpha_p} p \right) \frac{\alpha_p}{\alpha_p} \hat{\psi} \sigma_p > 0
\]

\[
\frac{\partial \hat{\theta}}{\partial \varepsilon_s} = \phi \left( \Phi^{-1} (\hat{\theta} + 1) \right) \frac{\sigma_x}{\sigma_p} + \phi \left( \rho + \frac{\alpha_p}{\alpha_p} p \right) \frac{\alpha_p}{\alpha_p} \hat{\psi} \sigma_p < 0
\]

Investors rely extensively on credit ratings, giving ratings a powerful coordination effect. This effect is exacerbated by market practises, laws and regulations that hardwire buy or sell decisions to rating thresholds. This hardwiring contributes significantly to market reliance on ratings, reinforcing their role as focal points. It is a cause of herding in market behavior,
because regulations effectively require or motivate large numbers of market participants to act in similar fashion, especially when downgrades cross into non investment grade categories. Omitting references to ratings in regulation would reduce their use as sell-triggers and could stabilize financial markets.

Unfortunately the same is true regarding regulations that rely on market-based indicators like financial prices. Some suggest replacing credit ratings with CDS premia and credit spreads, but these too may increase the coordination motive and bring instability to financial markets.

6 Welfare and policy implications

In order to keep the analysis tractable enough to investigate welfare effects and examine policy trade-offs, I restrict attention to the model with a CRA presented in Section 2.2 and I will be interested in the limiting case when the private signals of short term creditors become very precise. This corresponds to the case where \( \alpha_x \to \infty \). From its definition, the threshold \( \theta^* \) satisfies \( \theta^* = 1 - \frac{R-\Delta}{\Delta} - \psi \). The fact that the parameters are related in this way is a result of the payoff normalization, and should not be read literally. Of more importance is the observation that inefficiencies persist despite perfect coordination among short term creditors; all short term creditors invest if and only if \( \theta \geq \theta^* \). Since \( 1 - \frac{R-\Delta}{\Delta} > 0 \), \( \theta^* > -\psi \) so that there are states between \( -\psi \) and \( \theta^* \) at which the sovereign defaults even when it is fundamentally sound. Private signals reveal what the underlying state \( \theta \) is, but strategic uncertainty - uncertainty over the actions of other short term creditors – is not resolved.

Recall that the efficient outcome is rolling over. If the noise concerning \( \theta \) is very small, it is possible to augment the likelihood of roll-over by setting a high value for \( \psi \). But of course, achieving efficiency in the roll-over decision comes at a cost of increasing losses associated with investment \( \psi \) in case of default. When \( \theta > \theta^* \) social gains equal \( R(1 - \theta) \) and when \( \theta < \theta^* \) social gains equal \( (1 - \theta - \psi) + (R - \Delta) \psi \). These values illustrate a fundamental trade off in the model. On the one hand, when the fundamental is unambiguously high or low, it is better to have perfect coordination among investors and have \( \psi = 0 \). On the other hand, when \( \theta \) is near the default threshold, institutional investors have the capability to change the final outcome of the game; an increase in the mass of investment \( \psi \) can be welfare enhancing because it decreases the value of \( \theta^* \).
Disclosure of information. Short term creditors overreact to public information, and can magnify the damage done by any noise. The dilemma posed by the potential for overreaction to public information is well-known to government officials that have high public visibility. Central bankers have developed specific communication skills, knowing how their public statements may disproportionately influence financial markets. And yet, no restrictions have ever been imposed on the communication strategies of CRAs, which have allowed them to unduly influence financial markets.

There is scope for rethinking the disclosure policies of CRAs in terms of how much information should be disclosed, how it should be disclosed and when it should be disclosed. The results in this paper lend support to some of the reforms sought by the European Commission such as giving the European markets regulator the power to approve rating methods and to decide the appropriate timing for the publication of sovereign credit ratings, and in particular the power to suspend credit ratings of a country undergoing an international bailout program.

Europeans have also called for the creation of an independent European (eventually public) CRA, but it is likely that its ratings would become a focal point and substitute other credit opinions, exacerbating problems of overreliance on ratings and a lack of diversity in credit judgements. Even absent conflicts of interest, a rating agency may have an incentive to keep a rating higher than justified by the fundamental in recognition that the implications of a rating downgrade may be serious (particularly as the rating nears the investment-grade threshold). A public authority would expose itself to natural criticism if ratings proved unreliable. Anticipating the effects that a wrong grade would have on financial markets, the CRA would have little incentives to provide public information and it would delay downgrades for too long.

Extensions. So far the feedback effect from credit ratings to financial prices has been muted; prices were determined in the first stage and did not respond to subsequent developments in financial markets. Instead, $\rho$ can be disclosed before the derivative is traded, and the market price will reflect the private information as well as the information released by the CRA. The informativeness of the price would be improved, but the investment decisions on sovereign debt would be identical because agents would be able to disentangle the two sources of information. With a feedback effect of this kind, CRAs can drive a wedge between the fundamental and the market price. Moreover, prices are hardwired into the regulatory framework and financial contracts, creating further feedback loops; margin requirements often rely on prices to define
haircuts and higher haircuts lead agents to sell off their bonds. In this context, a small shift in the fundamental may have a disproportionate impact on the bond price in the secondary market.

I also maintained the cost of finance fixed for the sovereign. Relaxing this assumption would have two important consequences. First, one might think that when $\theta < \theta^*$ the sovereign will be tempted to raise $R$, thereby reducing the value for the default threshold. Arguably, this is what has been happening during the European sovereign debt crisis. Second, countries that are fundamentally solvent but have lost the confidence of bond investors can quickly go bust if their borrowing cost rise too fast. Corporate rating changes have a more immediate effect on borrowing costs due to the rating-based performance pricing in loan and bond contracts. Letting the sovereign adjust the return $R$ change would convey information about the fundamental $\theta$.

This paper’s results can be combined with the findings on CRAs’ objectives. Agencies may use the feedback effects and the endogeneity of rating information to influence markets to their benefit. For example, they may have and incentive to announce ratings that trigger market reactions that render their ratings even more precise ex post. Boot, Milbourn and Schmeits (2006) show that credit watch procedures condition issuers and may be used in this vein. Also, CRAs have an incentive to "rescue" borrowers in order to uphold their business relations with them (Mählmann, 2011).

7 Conclusion

With external ratings becoming reference points, ratings inform and at the same time influence the probability of default of borrowers who must roll over their debts. As CRAs take into account the feedback effects on credit quality, credit ratings become volatile and prone to cause financial instability in the crisis zones. I have described the nonlinear response of credit ratings and default probabilities to nonfundamental volatility - like noise in prices and flawed ratings. I have proved that the reduction of noise in public information increases instability in financial markets and credit ratings, bringing new challenges to the industry. I have also shown how important it is for a sovereign to maintain a extremely good rating so that it avoids the crisis zones.

For as long as ratings retain their widespread influence throughout the financial system there
seems to be a sound economic rationale for regulating CRAs, but it is one thing to identify the weaknesses of credit ratings, quite another to find solutions and alternative standards that are clearly better. The solution, it seems, is to mitigate the coordination motive and the feedback effects. The less embedded into deal-documentation ratings are, the lower the impact of credit ratings on investment decisions and the less focal credit ratings will be.\textsuperscript{11} Ending the contribution of credit ratings to financial instability calls for the reduction of the references to credit ratings in regulation and rules. Counter-intuitively, some industry leaders back these moves. Standard & Poor’s has publicly backed the LeMieux-Cantwell amendment which essentially removes the federal government’s seal of approval from rating agencies. Its president, Deven Sharma (2010), insists "we support removing investor rating requirements and believe the market - not government mandates - should decide the value of our work".

Unfortunately, finding alternatives to ratings is proving difficult. Several alternative approaches that remove references to ratings entirely have been considered in the debate, but no satisfactory substitutes have yet been identified. For example, market-based indicators, such as CDS premia and credit spreads, may increase the coordination motive and bring instability to financial markets. Even if the scope for hardwiring were reduced, it is likely that credit ratings would retain a significant influence in financial markets. Small and less-sophisticated investors that do not have the economies of scale to do their own credit analyses will continue to rely extensively on credit ratings and it is plausible that many institutional investors would be reluctant to do their own credit assessments and would continue to rely on ratings even if these were pulled from the regulations.

Credit ratings do their job well outside the crisis zones and there are no simple alternatives to their certification role. So I suggest a more flexible use of credit ratings for regulatory purposes and this paper provides an economic rationale behind some of the reforms being sought by the European Commission. Suspending the use of credit ratings as soon as they become volatile (an indicator that the borrower has entered a crisis zone) could help stabilizing financial markets by forcing agents to rely more on their own private information. Also, liquidity provision by official institutions is far more effective in the crisis zones, where credit ratings are not reliable and

\footnote{The concentration of the industry in a reduced number of big (and focal) rating agencies might have aggravated these problems in the past. Increasing their number may be part of the solution.}

\footnote{The conditions imposed by regulators for designating a CRA as an external credit assessment institution often require that the market already places substantial weight on the judgment of a rating agency. By giving the market a role in selecting rating agencies, regulators exacerbate the focal role of CRAs.}
can precipitate default; hence the CRAs' communication strategies should be monitored closely in the these zones. Michael Barnier, internal market commissioner, argued in favor of such restrictions "It is not the thermometer that causes the fever," he said. "But the thermometer has to work properly to ensure you do not exaggerate the fever."

References


