Abstract

The correlation between returns on US stocks and Treasury bonds has varied substantially over time. From being highly positive in the 1970-1980’s, correlations have been strongly negative throughout the 2000’s, and in particular during the recent financial crisis. I also document considerable time variation in the general relation between inflation and asset prices. I show that these observations line up remarkably well with the significant time variation in the relation between consumption growth and inflation going back to the 1930’s. While the 1970-1980’s was characterized by stagflation, I show that inflation switched to a procyclical state in the early 2000’s. I illustrate the link between cyclicity of inflation and asset prices using a long-run risk model in which persistent inflation shocks have real effects and affects both equity and bond risk premia. The sign of the market price of inflation risk reflects the cyclical nature of inflation. The paper shows the importance of accounting for changes in the cyclical nature of inflation and also provides a rational explanation for why the correlation between dividend yields and nominal yields, sometimes referred to as the Fed-model, changes over time. This stands in sharp contrast to the usual explanation of inflation illusion.
1 Introduction

The correlation between returns on US stocks and Treasury bonds has been close to zero on average in post-war data but has varied substantially over time as shown in Figure 1. From being highly positive in the 1970’s and 1980’s, correlations turned sharply negative in the early 2000’s, and were particularly low during the recent financial crisis. At the same time as the stock-bond correlation changed drastically, I document that several other relations between inflation, inflation uncertainty, and asset prices also changed sign. For example, the relation between nominal yields and inflation uncertainty switched to negative in the beginning of the 2000’s, in contrast to the conventional wisdom that a rise in inflation risk should raise bond risk premia and yields.

This paper suggests that a time-varying correlation between consumption growth and inflation is a likely explanation and that this variation needs to be incorporated into asset-pricing models. While several papers have explored the role of time-varying macroeconomic volatility for asset prices (e.g., Bansal and Yaron, 2004, and Lettau et al., 2008), the focus of this paper is on the time-varying covariance between growth and inflation. To date, very little has been said about the changing empirical relation between inflation and consumption growth and the asset-pricing implications stemming from such variation. The claimed contribution of this paper is to show that accounting for switches in the cyclical nature of inflation is critical for capturing movements in equity and bond risk premia and the effect of inflation on asset prices.

Figure 2 plots the correlation between stock and bond returns against the correlation between consumption growth and inflation for the period 1952-2009 where all correlations are computed for non-overlapping five-year periods. The graph suggests a negative relation between macro and asset correlations. The plot shows that inflation has become procyclical during the last ten years in contrast to the stagflationary period of the 1970’s and 1980’s. At the same time, the correlation between stock and bond returns switched sign in the early 2000’s and turned sharply negative. Figure 3 shows that the same pattern holds for the relation between nominal yields and dividend yields, sometimes referred to as the Fed-model. From being positively correlated throughout most of the sample period, dividend yields and nominal yields have been highly negatively correlated.

\footnote{The name appeared in the mid 1990’s following research reports at the Federal Reserve describing the relation.}
As nominal yields are closely associated to the level of inflation, the evidence suggest that the relation between equity valuations and inflation has changed considerably over time.

Taking a long-term perspective, Figure 4 plots annual consumption growth and inflation starting in 1930. Visual inspection suggests that the comovement between inflation and growth has varied considerably. In particular, the 1930’s experienced a strong positive comovement as The Great Depression in the early 1930’s was associated with low growth coupled with deflation. The positive correlation was further exacerbated by the strong rebound in growth and inflation starting in 1933. In contrast, the US economy underwent a significant stagflationary period in the 1970’s and early 1980’s. Furthermore, the figure shows that consumption growth and inflation have comoved positively again during the 2000’s.

Figure 5 computes 10-year correlations between growth and inflation and between dividend yields and inflation for the period 1930-2009. First, the graph quantifies the considerable variation in the cyclicity of inflation. From a correlation of 0.80 in the 1930’s, correlations reached −0.80 during the 1970’s and were close to 0.60 in the 2000’s. Second, the graph shows that the inverse relation between macro correlations and correlations between dividend yields and inflation, highlighted earlier, is robust when extending the time frame.

Not only asset correlations switched sign in the early 2000’s but also several other relations between inflation and asset prices. Figure 6 shows that price-dividend ratios have been negatively correlated with inflation uncertainty throughout the entire sample period. However, the correlation between price-dividend ratios and expected inflation turned highly positive in the beginning of the 2000’s. Figure 7 shows that while nominal interest rates have, unsurprisingly, been positively correlated with expected inflation, the relation between nominal interest rates and inflation uncertainty turned sharply negative in the early 2000’s. This contradicts the conventional wisdom that nominal yields and bond risk premia should move positively with inflation uncertainty. I also document in the paper that while inflation uncertainty predicted stock-bond return correlations positively up until 2000, the relation has been negative since then. Figure 8 shows that also correlations between expected growth and inflation and between industrial production and inflation turned positive in
the early 2000’s. Hence, time variation in the cyclicality of inflation is robust to how economic activity is measured.

To further investigate the relation between growth and inflation, I estimate a standard two-state Markov-switching (MS) model in which growth is regressed onto last periods growth and inflation
\[ \Delta c_{t+1} = \alpha_{s_{t+1}} + \rho_{0,s_{t+1}} \Delta c_t + \rho_{1,s_{t+1}} \pi_t + \epsilon_{t+1} \]
where \( \epsilon_{t+1} \sim N(0,\sigma^2) \) and where \( s_{t+1} \) is presumed to follow a two-state Markov chain with transition probabilities \( p_{ij} = P(s_{t+1} = j|s_t = i) \). The estimation results in Table 1 suggest one state in which inflation has a negative impact on future growth \((\rho_1 < 0)\) and one state in which the effect of inflation is positive \((\rho_1 > 0)\). The smoothed probabilities of being in the second, procyclical, state are plotted in Figure 9. The probabilities are virtually zero up until the early 2000’s after which they move to one. The probabilities exceed 0.50 in the third quarter of 2001 wherefore I treat 2001:3 as the point of regime switch.

So is there any plausible economic link between the cyclicality of inflation and the empirical observations described above? Yes. Asset-pricing theory suggests that nominal bonds are risky when inflation is countercyclical since their returns then are procyclical. In contrast, nominal bonds provide a hedge against bad times when inflation is procyclical. An increase in inflation risk should therefore raise bond risk premia and nominal yields in the first case while it should lower risk premia and yields in the second case. Expected returns on equity should load positively on inflation risk if inflation is bad for growth and leads to poor equity returns. The same also holds if inflation is positively associated with economic growth and stock returns since high inflation then coincides with low marginal utility of investors coupled with high returns. This suggests that risk premiums on equity should consistently load positive on inflation risk while bond risk premia can load negatively or positively depending on the cyclical state of inflation. This is shown to be consistent with data.

Two contrasting periods that exemplify this line of reasoning are the stagflation period of the 2

2 The setup follows Hamilton (1989,1994) among others. For simplicity, I let the variance term be constant across regimes. I have also estimated various MS-VAR specifications for growth and inflation allowing for correlated shocks, yielding similar conclusions. Also, using expected growth and inflation instead of realized values yields similar results. Additional estimation results are available upon request.

3 The limited number of observations in the procyclical state makes it hard to precisely estimate \( \rho_1 \) for that state. However the change in actual correlations, the change in estimated parameter values, and the change in smoothed probabilities all indicate that the relation between growth and inflation did change.
1970’s and the recent financial crisis in which inflation was highly procyclical as both economic growth and inflation dropped. While the 1970’s was characterized by poor stock returns and an increase in both yield levels and inflation uncertainty, the recent crisis experienced poor stock returns, a rise in inflation uncertainty, but a decrease in nominal yields and therefore positive bond returns. Figure 10 plots the evolution of inflation uncertainty over time. The graph shows the elevated levels of uncertainty throughout the 1970’s and early 1980’s and the sharp rise towards the end of the sample period which incorporates the recent financial crisis. These observations line up well with theory since returns on nominal bonds were highly procyclical in the 1970’s and therefore considered risky assets but in fact provided a hedge against inflation risk during the recent crisis since inflation was procyclical.

Identifying the structural source behind changes in the cyclicality of inflation is an important question. One possible underlying mechanism is the nature of demand and supply shocks affecting the economy. A large body of macroeconomic literature deals with identifying such shocks and linking it to the cyclical behavior of output, employment, and prices. However, a careful analysis along those lines are outside the scope of this paper. I focus instead on the direct consequences for asset prices stemming from such cyclical variations. It is also conceivable that time variation in the stance of monetary policy plays an important role. I leave these interesting questions for future work.

The model I calibrate builds on the long-run risk literature which makes use of recursive preferences, persistent macroeconomic shocks, and time-varying macroeconomic volatility. The setup follows Bansal and Yaron (2004) but in contrast to existing long-run risk models, the focus is on persistent long-run inflation shocks rather than shocks to consumption growth. The key mechanism is that inflation has real effects and affects the real pricing kernel directly. Inflation therefore has an impact on real asset prices and on both equity and bond risk premia. This feature makes expected inflation a state variable for price-dividend ratios, creating an immediate link between inflation and equity valuations. The market price of long-run inflation risk depends on whether inflation is positively or negatively associated with growth and can therefore switch sign. When inflation

\footnote{An incomplete list of relevant papers are: Mills (1927), Blanchard and Quah (1989), Kydland and Prescott (1990), Cooley and Ohanian (1991), and Stadler (1994)}
is bad news for the economy, as in periods of stagflation, both stocks and bonds are risky assets with respect to inflation risk which makes their risk premiums comove positively. When inflation is procyclical, as during the recent financial crisis, nominal bonds provide a hedge against inflation risk while stocks still are risky. In this case, risk premiums on bonds and equity comove negatively. Hence, risk premiums on equity and bonds both vary with inflation volatility in the model but the loading of bond risk premia on inflation uncertainty can be either positive or negative depending on the cyclical nature of inflation. I treat inflation volatility analogously to inflation uncertainty in this paper.

2 Related Literature

The relation between stock and bond returns has received great academic interest and is of central importance for e.g. asset allocation decisions. Early contributions focus on the unconditional correlation. Shiller and Beltratti (1992) fail to match the observed comovement using a present-value model. Campbell and Ammer (1993) decompose the variance of stock and bond returns and find offsetting effects from changes in real interest rates, excess returns, and expected inflation.

Recently, the focus has shifted towards understanding conditional correlations. Connolly et al. (2005) document a negative relation between stock market uncertainty and the stock-bond return correlation. Baele et al. (2010) find that macro factors have limited success in explaining the time-varying return correlation. Campbell et al. (2010) specify and estimate a reduced form model in which one of the latent state variables is the covariance between inflation and the real economy. They link this state variable to the covariance of stock and bond returns and describe how the covariance term impacts bond risk premia and the shape of the yield curve. David and Veronesi (2009) explore the role of learning about inflation and real earnings for the second moments of stock and bond returns. Investors in their model are uncertain about the current state of the economy and any perceived deviations from the “normal” state, either to a “good” or “bad” state, increases uncertainty and asset return volatilities. The valuation of bonds and stocks, however, react to the directional change in economic states. This create a V-shaped relation between return volatilities and asset valuations.
In contrast to these papers, I address asset correlations and the relation between inflation and asset prices using a consumption-based equilibrium model. This ties asset prices directly to fundamental macro factors such as consumption growth and inflation. I also explicitly focus on the empirical covariance between consumption growth and inflation and show that the cyclical behavior of inflation changes over time.

The correlation between US dividend yields and nominal interest rates has been positive throughout most of the post-war period and is commonly referred to as the Fed-model. This correlation has been considered puzzling since it implies that nominal interest rates, driven mainly by inflation, are correlated with a real variable. As is well known, dividend yields are negatively related to real cash-flow growth and positively related to expected returns. So in order to explain the positive correlation, inflation must be negatively related to real dividend growth rates and/or positively associated with expected returns.

Virtually all papers in the so-called Fed-model literature rule out a rational explanation for the link between inflation and dividend yields. Instead, inflation illusion has been advocated as the likely explanation, originally put forward by Modigliani and Cohn (1979).\(^5\) Inflation illusion suggests that investors are irrational and fail to adjust expected nominal dividend growth rates with changes in expected inflation but fully adjust nominal discount rates. Alternatively, one can view it as investors are discounting real cash flows with nominal interest rates, creating a positive correlation between inflation and dividend yields. What is rarely mentioned in the literature is that the correlation between dividend yields and nominal yields has been highly negative throughout the 2000’s. This observation questions inflation illusion as a viable explanation.

In a recent empirical paper, Bekaert and Engstrom (2010) argue that rational mechanisms are at work and ascribe the positive correlation to the large incidence of stagflation in US data. They show that the correlation between dividend yields and bond yields is mainly driven by a correlation between expected inflation and the equity risk premium as periods of high expected inflation are associated with periods of high risk aversion and high economic uncertainty. This piece of evidence suggests that any rational equilibrium model that would like to explain the Fed-model must contain

\(^5\) Among the papers that argue in favor of inflation illusion are Ritter and Warr (2002), Campbell and Vuolteenaho (2004), and Cohen et al. (2005).
a link between inflation and the equity risk premium. The model in this paper contains exactly that.

This paper is also related to the literature on whether stocks are a good or bad inflation hedge. Several articles have established a negative relation between inflation and stock returns. Fama and Schwert (1977) document that common stock returns are inversely related to expected inflation. Fama (1981) argues that a rise in expected output is expected to lead to a drop in inflation. Since stock prices rise in anticipation of higher output, a negative correlation between stock returns and inflation arises. Fama’s proxy hypothesis is distinct from this paper since I stress the link between inflation and risk premiums rather than inflation and cash flows.

This paper builds more generally on the literature of pricing stocks and bonds in equilibrium. Early contributions include Cox et al. (1985), Mehra and Prescott (1985), Campbell (1986), and Dunn and Singleton (1986). The recursive preferences of Epstein and Zin (1989) and Weil (1989) have been used extensively in the asset-pricing literature (e.g., Campbell, 1993, 1996, 1999, Duffie et al., 1997, and Restoy and Weil, 1998). Bansal and Yaron (2004) show that recursive preferences in conjunction with a time-varying first and second moment of consumption growth can explain the level of the equity risk premium and its variation over time. Piazzesi and Schneider (2006) make use of recursive preferences and show that the nominal yield curve slopes up if inflation is bad news for future consumption growth and explore the role of learning about inflation for interest rates. Hasseltoft (2011) estimates the long-run risk model using a simulation estimator and shows that it does well in explaining key features of both equity and bond markets. Bansal and Shaliastovich (2010) show that the long-run risk model can explain violations of both the expectation hypothesis in bond markets and uncovered interest rate parity in currency markets.\footnote{Other papers that model the term structure of interest rates in a long-run risk setting include Eraker (2008), Wu (2008), and Doh (2010).}

3 The Model

The introduction of the paper described the substantial time variation in macro and asset correlations and the switching relations between inflation, inflation uncertainty, and asset prices. Mo-
tivated by these empirical observations, this section presents a consumption-based representative-agent model which provides a rational explanation for these findings while at the same time matching standard asset-price moments.

The model builds on the so-called long-run risk literature which relies on Epstein and Zin (1989) and Weil (1989) recursive preferences, persistent macro shocks, and time-varying macroeconomic volatility. The original long-run risk model of Bansal and Yaron (2004) relies on persistent shocks to expected consumption growth which together with the preference specification produces a sizeable equity risk premia. Inflation plays no role in that model. In contrast, this paper focusses on long-run shocks to inflation and their effect on consumption growth and asset prices. The main feature of the model is that expected inflation and shocks to inflation impact future real economic growth. This specification is supported by data and allows inflation to have a direct impact on the real pricing kernel and therefore on both equity and bond risk premia.

The setup follows Bansal and Yaron (2004) but the introduction of inflation and the joint dynamics of growth and inflation represent the main distinctions from the original long-run risk model. I show that a range of novel asset-pricing implications arise by allowing inflation to have real effects in the sense that inflation affects consumption growth directly.

3.1 Macro Dynamics

Let $\Delta c_{t+1}, \pi_{t+1},$ and $\Delta d_{t+1}$ denote the logarithmic consumption growth, inflation, and dividend growth. Let $\mu_c, \mu_\pi,$ and $\mu_d$ denote the unconditional means and let $x_c$ and $x_\pi$ denote the time-
varying part of the conditional means. The following dynamics are then assumed:

\[
\begin{align*}
\Delta c_{t+1} &= \mu_c + x_{c,t} + \sigma_c \eta_{c,t+1} + \alpha \sigma_{\pi,t} \eta_{\pi,t+1}, \\
\pi_{t+1} &= \mu_\pi + x_{\pi,t} + \sigma_{\pi,t} \eta_{\pi,t+1}, \\
\Delta d_{t+1} &= \mu_d + \phi x_{c,t} + \varphi \sigma_{\eta_{d,t+1}}, \\
\end{align*}
\]

(1) \hspace{1cm} (2) \hspace{1cm} (3)

\[
\begin{pmatrix}
  x_{c,t+1} \\
  x_{\pi,t+1}
\end{pmatrix} = \begin{pmatrix}
  \beta_1 & \beta_2 \\
  0 & \beta_3
\end{pmatrix} \begin{pmatrix}
  x_{c,t} \\
  x_{\pi,t}
\end{pmatrix} + \begin{pmatrix}
  \delta_1 & \delta_2 \\
  \delta_3 & \delta_4
\end{pmatrix} \begin{pmatrix}
  \sigma_{\xi_{c,t+1}} \\
  \sigma_{\xi_{\pi,t+1}}
\end{pmatrix},
\]

(4)

\[
\sigma_{\pi,t+1}^2 = \sigma_\pi^2 + v_\pi \left( \sigma_{\pi,t}^2 - \sigma_\pi^2 \right) + \sigma_\nu \nu_{t+1},
\]

(5)

where all shocks are uncorrelated, i.i.d. normally distributed with a mean of zero and a variance of one. The \( \beta \) and \( \delta \) matrices govern the persistence of the shocks and their effect on the conditional means. Dividend growth is modeled as a function of expected consumption growth subject to a leverage parameter \( \phi \). Time-varying volatility of inflation is represented by \( \sigma_{\pi,t+1}^2 \) which follows an autoregressive process subject to a shock \( \nu_{t+1} \). \( \sigma_{\pi,t+1}^2 \) is the only source of time-variation in second moments and gives rise to time-varying risk premiums as shown in Section 5. Note that the typical long-run risk specification relies on time-varying volatility of consumption growth as opposed to inflation. Since the main focus of this paper is on inflation shocks, I have restricted time-variation in second moments to inflation only. Hence, the only channel of macroeconomic uncertainty is inflation uncertainty. The notion of heteroscedasticity in inflation is a well established empirical fact; early contributions include Engle (1982) and Bollerslev (1986). The state variables of the model are consequently \( x_{c,t}, x_{\pi,t}, \) and \( \sigma_{\pi,t}^2 \).

The conditional means of consumption growth and inflation are interdependent through \( \beta_2 \) which means that expected future growth depends on today’s expected inflation. \( \beta_2 \) is the key parameter and represents the main distinguishing feature with respect to other long-run risk specifications. More specifically, \( \beta_2 \) allows real asset prices and valuation ratios such as real bonds and price-dividend ratios, to be functions of expected inflation since the conditional mean of \( x_{c,t+1} \) depends on \( x_{\pi,t} \). \( \beta_2 \) creates a direct link between expected inflation and the real pricing kernel.
means that inflation affects risk premiums in the economy. Importantly, $\beta_2$ allows inflation to affect not only bond risk premia but also equity risk premia. The fact that the impact of inflation on future growth via $\beta_2$ is the key mechanism explains why I opted for the particular Markov-switching specification described in the introduction.

Examples of papers that also model inflation in a long-run risk setup is Bansal and Shaliastovich (2010) and Hasseltoft (2011). They both model inflation in order to price nominal bonds but inflation has no real effects, meaning inflation does not feed into consumption growth. Instead, persistent consumption shocks feed into inflation. This is enough to match the negative correlation between growth and inflation in data and to match interest-rate moments. However, I show in this paper that allowing inflation to directly impact consumption growth opens up a range of novel consequences for asset prices.

The specification provided above is the most parsimonious setup that is able to match both standard unconditional asset-price moments and the switching behavior between inflation and asset prices. In an earlier version of this paper, I considered a much richer specification where for example the entire variance-covariance matrix of growth and inflation varied over time. The insights of that specification were the same as here but the setup was less transparent. This paper attempts to streamline the model as much as possible while still providing an economic explanation for what we see in data.

### 3.2 Investor Preferences

The representative agent in the economy has Epstein and Zin (1989) and Weil (1989) recursive preferences:

$$U_t = \left\{ (1 - \delta)C_t^{\frac{1-\gamma}{1-\psi}} + \delta(E_t[U_{t+1}^{1-\gamma}])^\frac{\theta}{1-\gamma} \right\}^\frac{1}{1-\gamma}, \tag{6}$$

where $\theta = \frac{1-\gamma}{1-\psi}$, $\gamma \geq 0$ denotes the risk aversion coefficient and $\psi \geq 0$ the elasticity of intertemporal substitution (EIS). The discount factor is represented by $\delta$. This preference specification allows time preferences to be separated from risk preferences. This stands in contrast to time-separable expected utility in which the desire to smooth consumption over states and over time are interlinked. The agent prefers early (late) resolution of risk when the risk aversion is larger (smaller) than the
reciprocal of the EIS. A preference for early resolution and an EIS above one imply that $\theta < 1$. This specification nests the time-separable power utility model for $\gamma = \frac{1}{\psi}$ (i.e., $\theta = 1$).

The agent is subject to the following budget constraint $W_{t+1} = R_{c,t+1} (W_t - C_t)$ where the agent’s total wealth is denoted $W_t$, $W_t - C_t$ is the amount of wealth invested in asset markets and $R_{c,t+1}$ denotes the gross return on the agents total wealth portfolio. This asset delivers aggregate consumption as its dividends each period. Epstein and Zin (1989) show that this economy implies an Euler equation for asset return $R_{i,t+1}$ in the form of:

$$E_t[\delta^\theta G_{t+1}^{-\theta} R_{c,t+1}^{-(1-\theta)} R_{i,t+1}] = 1,$$

where $G_{t+1}$ denotes the aggregate gross growth rate of consumption and $M_{t+1}$ denotes the intertemporal marginal rate of substitution (IMRS). The logarithm of the IMRS can be written as:

$$m_{t+1} = \theta \ln (\delta) - \frac{\theta}{\psi} \Delta c_{t+1} - (1 - \theta) r_{c,t+1},$$

where $\ln R_{c,t+1} = r_{c,t+1}$ and $\ln G_{t+1} = \Delta c_{t+1}$. Note that the IMRS depends on both consumption growth and on the return from the total wealth portfolio. Recall that $\theta = 1$ under power utility, which brings us back to the standard time-separable IMRS.

### 3.3 Solving the model

Returns on the aggregate wealth portfolio and the market portfolio are approximated as in Campbell and Shiller (1988):

$$r_{c,t+1} = k_{c,0} + k_{c,1} p_{c,t+1} - p c_t + \Delta c_{t+1},$$

$$r_{m,t+1} = k_{d,0} + k_{d,1} p_{d,t+1} - p d_t + \Delta d_{t+1},$$
where \( pc_t \) and \( pd_t \) denote the log price-consumption ratio and the log price-dividend ratio. The constants \( k_c \) and \( k_d \) are functions of the average level of \( pc_t \) and \( pd_t \), denoted \( \bar{pc} \) and \( \bar{pd} \).

### 3.4 Solving for Equity

All asset prices and valuation ratios are conjectured to be functions of the time-varying conditional means of consumption growth and inflation plus the time-varying conditional variance of inflation. Starting with the log price-consumption ratio, it is conjectured to be a linear function of the state variables as follows:

\[
pc_t = A_{c,0} + A_{c,1}x_{c,t} + A_{c,2}x_{\pi,t} + A_{c,3}\sigma^2_{\pi,t}. \tag{11}
\]

Using the standard Euler equation together with the macro dynamics one can solve for the coefficients. Appendix A.1 explains how to solve for the \( A_c \)-coefficients and reports the expression for \( A_{c,0} \). The remaining coefficients are given by:

\[
A_{c,1} = \frac{1 - \frac{1}{\psi}}{1 - k_{c,1}\beta_1}, \tag{12}
\]

\[
A_{c,2} = \frac{k_{c,1}A_{c,1}\beta_2}{1 - k_{c,1}\beta_3}, \tag{13}
\]

\[
A_{c,3} = \frac{1}{2} \alpha^2 \left( \theta - \frac{\theta}{\psi} \right)^2 + \left( \theta k_{c,1}A_{c,1}\delta_2 + \theta k_{c,1}A_{c,2}\delta_4 \right)^2}{\theta(1 - k_{c,1}v_{\pi})}. \tag{14}
\]

The first coefficient \( A_{c,1} \) represents the loading of the price-consumption ratio onto expected consumption growth and is identical to the one in Bansal and Yaron (2004). An EIS greater than one produces a positive \( A_{c,1} \) which means asset valuations rise in response to higher expected economic growth. \( A_{c,2} \) is the distinguishing coefficient compared to existing long-run risk specifications. It represents the loading of the price-consumption ratio onto expected inflation. The key parameter determining its sign is \( \beta_2 \), i.e. how expected inflation affects future expected growth. An increase in inflation expectations will depress the price-consumption ratio \( (A_{c,2} < 0) \) when high inflation

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7Bansal et al. (2007a) show that the approximate analytical solutions for the returns are close to the numerical solutions and deliver similar model implications.

8Specifically, the constants are \( k_{c,1} = \frac{\exp(p_c)}{1 + \exp(p_c)} \) and \( k_{c,0} = \ln(1 + \exp(p_c)) - k_{c,1}p_c \) and similarly for the \( k_d \) coefficients.
signals low future growth, \( \beta_2 < 0 \), and the EIS is above one. That is, the relation between price-consumption ratios and expected inflation is negative (positive) when inflation expectations are countercyclical (procyclical). An EIS in excess of one implies that the intertemporal substitution effect dominates the wealth effect. When high inflation signals low future returns, agents sell risky assets which leads to lower valuation ratios. In the case of expected utility \((\frac{1}{\psi} = \gamma)\), a risk aversion coefficient above one instead implies that the wealth effect dominates which results in a positive value of \( A_{c,2} \) given that \( \beta_2 < 0 \). The relation between price-consumption ratios and inflation volatility is negative \((A_{c,3} < 0)\) provided the EIS is greater than one. Increasing the persistence of volatility shocks amplifies the effect.

The following expression represents innovations to the real pricing kernel, where the \( \lambda \)'s represent market prices of risk:

\[
\begin{align*}
m_{t+1} - E_t(m_{t+1}) &= -\lambda_{\eta_c} \sigma_c \eta_{c,t+1} - \lambda_{\varepsilon} \sigma_c \varepsilon_{c,t+1} - \lambda_{\nu} \sigma_\nu \nu_{t+1} - \lambda_{\eta_\pi} \sigma_\pi \eta_{\pi,t+1} - \lambda_{\varepsilon_\pi} \sigma_\pi \varepsilon_{\pi,t+1} \\
\lambda_{\eta_c} &= \gamma, \\
\lambda_{\varepsilon} &= (1 - \theta)(k_{c,1} A_{c,1} \delta_1 + k_{c,1} A_{c,2} \delta_3), \\
\lambda_{\nu} &= (1 - \theta) k_{c,1} A_{c,3}, \\
\lambda_{\eta_\pi} &= \gamma \alpha, \\
\lambda_{\varepsilon_\pi} &= (1 - \theta)(k_{c,1} A_{c,1} \delta_2 + k_{c,1} A_{c,2} \delta_4).
\end{align*}
\]

The three first sources of risk are the same as in Bansal and Yaron (2004), namely short run consumption risk, long-run consumption risk, and volatility risk. However, in contrast to existing long-run risk specifications, this setup allows for inflation shocks to be priced. The last two terms in (15) represent short-run and long-run inflation risk. For example, \( \lambda_{\varepsilon_\pi} < 0 \) implies that the representative agent dislikes positive shocks to expected inflation and therefore requires a positive risk premium on assets that perform badly in periods of high inflation. This represents an additional part of risk premiums in the economy compared to models in which only consumption shocks are priced. Recall that \( \theta = 1 \) under power utility, which means that long-run inflation risk is not priced and that the only sources of priced risk left are short-run consumption risk \( \lambda_{\eta_c} \) and short-run
inflation risk $\lambda_{\eta\pi}$.

The log price-dividend ratio is conjectured to be a linear function of the same three state variables as above:

$$pd_t = A_{d,0} + A_{d,1}x_{c,t} + A_{d,2}x_{\pi,t} + A_{d,3}\sigma_{\pi,t}^2.$$  \hfill (16)

Again, the coefficients are solved for using the standard Euler equation. Appendix A.2 describes the derivations and reports the expression for $A_{d,0}$. The remaining coefficients are given by:

$$A_{d,1} = \frac{\phi - \frac{1}{\psi}}{1 - k_{d,1}\beta_1}.$$  \hfill (17)

$$A_{d,2} = \frac{k_{d,1}A_{d,1}\beta_2}{1 - k_{d,1}\beta_3}.$$  \hfill (18)

$$A_{d,3} = \frac{(\theta - 1)A_{c,3}(k_{c,1}\nu_{\pi} - 1) + 0.5Y}{1 - k_{d,1}\nu_{\pi}}.$$  \hfill (19)

$$Y = \alpha^2 \left( \theta - 1 - \frac{\theta}{\psi} \right)^2 + ((\theta - 1)(k_{c,1}A_{c,1}\delta_2 + k_{c,1}A_{c,2}\delta_4) + k_{d,1}A_{d,1}\delta_2 + k_{d,1}A_{d,2}\delta_4)^2.$$  

Expected consumption growth and price-dividend ratios are positively associated for high values of the EIS meaning that $A_{d,1} > 0$. The expression for $A_{d,1}$ is identical to Bansal and Yaron (2004). The main focus of this paper is on $A_{d,2}$ which measures the relation between price-dividend ratios and expected inflation. $A_{d,2}$ is in general negative when the EIS is above one and when inflation is bad for future growth $\beta_2 < 0$. This means that high expected inflation depresses equity valuation ratios in periods of countercyclical inflation. As was shown in Figure 6 the relation between price-dividend ratios and expected inflation was negative up until 2000 but switched to positive in the early 2000’s. The model can accommodate this switch by allowing for a change in the sign of $\beta_2$, from negative to positive.

A rise in inflation uncertainty has a negative impact on price-dividend ratios ($A_{d,3} < 0$) provided a high value of the EIS. While the relation between price-dividend ratios and expected inflation can switch sign in the model, a rise in inflation uncertainty is always bad news for equity valuations. This is shown to be consistent with data and is elaborated further on in Section 5.2.
3.5 Solving for Real Bonds

The log price of a real bond with a maturity of \( n \) periods is conjectured to be function of the same state variables as before:

\[
q_{t,n} = D_{0,n} + D_{1,n}x_{c,t} + D_{2,n}x_{\pi,t} + D_{3,n}\sigma_{\pi,t}^2, \quad (20)
\]

Let \( y_{t,n} = -\frac{1}{n}q_{t,n} \) denote the \( n \)-period continuously compounded real yield. Then:

\[
y_{t,n} = -\frac{1}{n}(D_{0,n} + D_{1,n}x_{c,t} + D_{2,n}x_{\pi,t} + D_{3,n}\sigma_{\pi,t}^2), \quad (21)
\]

where the \( D \)-coefficients determine how yields respond to changes in expected consumption growth, inflation, and inflation volatility. Appendix A.3 shows how to solve for the coefficients and reports the expression for \( D_{0,n} \). The remaining coefficients are given by:

\[
D_{1,n} = -\frac{1}{\psi} + D_{1,n-1}\beta_1, \quad (22)
\]

\[
D_{2,n} = D_{1,n-1}\beta_2 + D_{2,n-1}\beta_3, \quad (23)
\]

\[
D_{3,n} = (\theta - 1)A_{c,3}(k_{c,1}v_{\pi} - 1) + D_{3,n-1}v_{\pi} + 0.5Y, \quad (24)
\]

\[
Y = \alpha^2 \left( \theta - 1 - \frac{\theta}{\psi} \right)^2 + ((\theta - 1)(k_{c,1}A_{c,1}\delta_2 + k_{c,1}A_{c,2}\delta_4) + D_{1,n-1}\delta_2 + D_{2,n-1}\delta_4)^2,
\]

where \( D_{i,0} \) equals zero for \( i = 1, 2, 3 \). For plausible parameter values, real yields increase in response to higher consumption growth \( (D_{1,n} < 0) \). Consumption shocks therefore generate countercyclical bond returns and contribute to negative risk premiums on real bonds. Real yields decrease in response to higher inflation when inflation is bad news for growth \( (D_{1,n-1}\beta_2 > 0) \), resulting in a positive \( D_{2,n} \) coefficient. In this case, inflation shocks also contribute to negative expected returns since they generate positive bond returns in bad inflationary times. This is consistent with earlier studies such as Fama and Gibbons (1982), Pennacchi (1991), and Boudoukh (1993). Ang et al. (2008) also document a negative relation between real rates and expected inflation but find the correlation to be positive for longer horizons. Note that a switch in the sign of \( \beta_2 \) to positive implies the opposite, namely that real yields move positively with inflation \( (D_{1,n-1}\beta_2 < 0) \). Hence,
the model can accommodate changes in the relation between real interest rates and inflation via
the $\beta_2$ parameter.

An increase in inflation uncertainty lowers real yields ($D_{3,n} > 0$) with long rates dropping
more than short rates. This occurs because inflation risk moves real bonds through a discount-
rate channel. When inflation is considered bad news for growth, inflation shocks lower real yields
as discussed above and therefore generates high bond returns in bad times. If inflation instead is
positively related to growth, inflation shocks raise real yields, generating poor bond returns in good
times. In both cases, inflation shocks contribute to a negative risk premium on real bonds. Hence,
a rise in inflation volatility is always associated with lower real yields regardless of economic state.

Negative expected excess returns on real bonds result in a downward sloping real yield curve.
This is supported by empirical evidence from UK-index linked bonds which have been trading since
the mid 1980’s (e.g., Evans, 1998, and Piazzesi and Schneider, 2006). Unfortunately, data for US
index-linked bonds only date back to 1997 but indicate a positively sloped yield curve. However,
the rather short sample period and the fact that the market was illiquid at the inception of trading
warrants some caution in interpreting the data.

### 3.6 Solving for Nominal Bonds

Nominal log bond prices are conjectured to be functions of the same state variables:

$$d_{t,n} = D_{0,n}^\delta + D_{1,n}^\delta x_{c,t} + D_{2,n}^\delta x_{\pi,t} + D_{3,n}^\delta \sigma_{\pi,t}^2.$$  \hspace{1cm} (25)

Let $y_{t,n} = -\frac{1}{n} d_{t,n}$ denote the $n$-period continuously compounded nominal yield. Then:

$$y_{t,n} = -\frac{1}{n} (D_{0,n}^\delta + D_{1,n}^\delta x_{c,t} + D_{2,n}^\delta x_{\pi,t} + D_{3,n}^\delta \sigma_{\pi,t}^2),$$  \hspace{1cm} (26)

where the $D^\delta$-coefficients determine how nominal yields respond to changes in expected consump-
tion growth, inflation, and inflation volatility. Solving for nominal log bond prices requires the use
of the nominal log pricing kernel which is determined by the difference between the real log pricing
kernel and the inflation rate:

\[ m_{t+1} = m_{t+1} - \pi_{t+1}. \]  

(27)

Appendix A.4 shows how to solve for the coefficients and reports the expression for \( D_{0,n}^S \). The remaining coefficients are given by:

\[
D_{1,n}^S = -\frac{1}{\psi} + D_{1,n-1}^S \beta_1,
\]

(28)

\[
D_{2,n}^S = D_{1,n-1}^S \beta_2 + D_{2,n-1}^S \beta_3 - 1,
\]

(29)

\[
D_{3,n}^S = (\theta - 1)A_{c,3}(k_{c,1}v_\pi - 1) + D_{3,n-1}^S v_\pi + 0.5Y,
\]

(30)

\[
Y = \left( \alpha \left( \theta - 1 - \frac{\theta}{\psi} \right) - 1 \right)^2 + \left( (\theta - 1)(k_{c,1}A_{c,1}\delta_2 + k_{c,1}A_{c,2}\delta_4) + D_{1,n-1}^S \delta_2 + D_{2,n-1}^S \delta_4 \right)^2,
\]

where \( D_{i,0}^S \) equals zero for \( i = 1, 2, 3 \). The response of nominal yields to changes in expected growth is the same as for real yields meaning that \( D_{1,n}^S < 0 \) for reasonable parameter values. This means that shocks to consumption growth contribute to negative risk premiums also for nominal bonds. As expected, nominal yields move positively with expected inflation implying a negative value of \( D_{2,n}^S \). This holds regardless of economic state and reflects a cash-flow effect on nominal bonds. Hence, while real yields may decrease or increase in response to inflation depending on the current cyclical state, nominal yields always rise with inflation.

The effect of inflation volatility on yields depends on whether inflation is counter or procyclical and reflects a discount-rate channel stemming from inflation risk. When inflation is negatively (positively) correlated with growth, higher inflation will raise yields and generate poor bond returns in bad (good) times. Hence, a rise in inflation risk can be associated with both higher or lower yields and bond risk premia depending on the cyclical state of inflation. This means that \( D_{3,n}^S \) is negative in the countercyclical state and positive in the procyclical state. I elaborate further on this in Section 5.
4 Data and Calibration of Model

4.1 Data

Quarterly aggregate US consumption data on nondurables and services is collected from the Bureau of Economic Analysis for the period 1952:2-2009:4. Real consumption growth and inflation are computed as in Piazzesi and Schneider (2006) using the price index that corresponds to the consumption data. The annual long-term data used in Figure 4 is computed in the same way. Value-weighted market returns (NYSE/AMEX) are retrieved from CRSP. Nominal interest rates are collected from the Fama-Bliss files in CRSP. Price-dividend ratios are formed by imputing dividends from monthly CRSP returns that includes and excludes dividends (e.g., Bansal et al., 2005). Quarterly dividends $D_t$ are formed by summing monthly dividends. Due to the strong seasonality of dividend payments, I use a four-quarter moving average of dividend payments, $\bar{D}_t = \frac{D_t + D_{t-1} + D_{t-2} + D_{t-3}}{4}$. Real dividend growth rates are found by taking the log first difference of $\bar{D}_t$ and deflating using the constructed inflation series. Data on industrial production is obtained from the Federal Reserve Bank of St. Louis.

4.2 Calibration

There are various way of parameterizing the model. One alternative is to formally estimate the model separately for each of the two subperiods. This is feasible for the countercyclical state which contains 197 observations but the number of observations in the procyclical state are only 34 which makes identification of the parameters somewhat problematic. I therefore choose to calibrate the model for the two subsamples. I first calibrate the entire set of parameters as to match key macro and asset-price moments for the countercyclical state covering the period 1952:2-2001:2. Having done so, I then re-calibrate only a subset of the parameters in order to match key moments between inflation and asset prices in the procyclical state. In order to keep matters as parsimonious as possible I do not change the entire set of parameters. I assume that the quarterly frequency of the model coincides with the decision interval of the agent. This means I abstract away from issues related to time-aggregation of consumption growth. The effect of time-aggregation does not affect
the qualitative results of the paper.

First consider the calibration for the countercyclical state 1952:2-2001:2, reported in Table 2. The mean parameters, \( \mu_c, \mu_\pi, \) and \( \mu_d \) are set equal to their sample values. The persistence of shocks to consumption growth, \( \beta_1 \), is set to 0.975 which translates to \( 0.975^{1/3} = 0.992 \) on a monthly frequency. This is in line with existing calibrated and estimated values in the long-run risk literature. The key parameter \( \beta_2 \) is set to \(-0.01\) which means inflation expectations have a negative impact on future growth expectations. This creates a negative correlation between growth and inflation and is in line with data. The negative value of \( \beta_2 \) is supported by evidence provided in Piazzesi and Schneider (2006) who estimate a state-space system for consumption growth and inflation. The persistence of inflation shocks is also calibrated to be high with \( \beta_3 \) being set equal to 0.96.

The next set of parameters refer to the \( \delta \) matrix which governs the size of long-run shocks to growth and inflation. The parameters governing shocks to expected growth, \( \delta_1 \) and \( \delta_2 \), are set equal to 0.12 and \(-0.10\) respectively which means that inflation shocks affect growth expectations negatively. \( \delta_3 \) and \( \delta_4 \) govern shocks to expected inflation and are calibrated to 0.25 and 0.65 respectively. The calibration implies that inflation shocks affect growth negatively but growth shocks have a positive impact on inflation. This is consistent with evidence provided in Piazzesi and Schneider (2006). The \( \delta_3 \) parameter is not critical for the qualitative results of the paper but helps to improve the fit of the model.

The persistence of volatility shocks \( v_\pi \) and their volatility \( \sigma_\nu \) are calibrated to standard values in the long-run risk literature, 0.98 and \( 1 \times 10^{-6} \) respectively. Dividend parameters and the preference parameters are also calibrated to standard values in the literature. The risk aversion is set to 10 and the EIS to 2. It is well-known that long-run risk models need an EIS above one in order to generate plausible asset pricing implications. The value of the EIS is subject to controversy. While for example Hall (1988), Campbell (1999), and Beeler and Campbell (2011) estimate the EIS to be close to zero, Attanasio and Weber (1993), Attanasio and Vissing-Jorgensen (2003), Chen et al. (2008), and Hasseltoft (2011) among others find the EIS to be above one.

\footnote{See for example Bansal et al., 2007a, Bansal et al., 2007b, and Hasseltoft (2011) who all estimate long-run risk models using simulation estimators, taking into account time-aggregation of consumption growth.}
Having calibrated the model, I simulate the model 150000 quarters and evaluate the implied macro and asset-price moments. Table 3 reports moments for macro data. All moments refer to the countercyclical period 1952:2-2001:2. The unconditional means are matched perfectly by construction. Volatility of the macroeconomic variables all lie close to their sample values with for example a standard deviation of 0.57% in the model for consumption growth compared to 0.49% in data. The persistence of macro data is matched almost perfectly where inflation is subject to substantial persistence with a first-order autocorrelation coefficient of 0.85 in data compared to 0.34 for consumption growth. I report the fourth-order autocorrelation coefficient for dividend growth since the moving-average procedure described above automatically induces positive autocorrelation for up to three lags.

Table 4 contains unconditional asset-price moments. The calibration generates model moments that are broadly in line with data. While the level of the equity risk premium and price-dividend ratios are more or less in line with data, the volatilities are lower than in data. The volatilities could be increased by for example allowing the volatility of consumption growth to vary or by relaxing other restrictions that are imposed in the model specification. This would, however, in most cases introduce additional state variables and increase the complexity of the model. Again, I have opted for a parsimonious model specification that still generates realistic asset-price moments. The level of nominal interest rates are matched well and the model generates a similar yield-curve slope as seen in data. The model-implied real yield curve is downward sloping as discussed in Section 3.5.

Table 5 reports various macro and asset correlations. This table reports moments for both of the economic states since the calibration for the procyclical state explicitly targets these moments. Overall, sample correlations for the countercyclical state are matched well by the model. The model-implied correlation between growth and inflation is -0.34 versus -0.40 in data while correlations between asset prices are somewhat higher in the model compared to data.

Next consider the calibration for the procyclical state 2001:3-2009:4. As described above, I choose to only re-calibrate a limited number of parameters in order to target key moments between inflation and growth and between inflation and asset prices. The parameters I change are $\beta_2, \delta_2, \delta_3, \text{ and } \alpha$. It is fully possible to change the entire set of parameters for the second state without
changing the qualitative results of the paper in any way. However the point here is to emphasize the impact of changing $\beta_2$. For the second state, I set $\beta_2 = 0.007$, $\delta_2 = 0.1$, $\delta_3 = 0$, and $\alpha = 0$.

The positive values of $\beta_2$ and $\delta_2$ produce a positive correlation between consumption growth and inflation of 0.33 which is close to the observed value of 0.23 as shown in Table 5. The fact that inflation is now procyclical in the model allows it to generate highly negative asset correlations as is observed in data. The difference in actual asset correlations between the two states is striking and suggests that something fundamental changed in the beginning of the 2000’s. Importantly, $\beta_2 > 0$ makes price-dividend ratios load positively on expected inflation and makes the market price of inflation risk switch sign to positive. The asset-pricing implications of this switch are elaborated extensively on in the next section of the paper.

5 Asset Pricing Implications

5.1 Nominal Bonds

Consider the innovation to nominal yields:

$$y_{t+1,n} - E_t(y_{t+1,n}) = -\frac{1}{n} \left[ \left( D_1^s \delta_1 + D_2^s \delta_3 \right) \sigma_{c,t+1} + \left( D_1^s \delta_2 + D_2^s \delta_4 \right) \sigma_{\pi,t} \right] + D_3^s \sigma_{\nu,t+1}. \tag{31}$$

The response of nominal rates to consumption shocks $\varepsilon_{c,t+1}$, depends on $D_1^s \delta_1$ which is negative and $D_2^s \delta_3$ which is negative in the countercyclical state and equals zero in the procyclical state. This implies that nominal yields increase in times of higher consumption growth. The second term in (31) determines how nominal rates move with shocks to expected inflation. The term $D_2^s \delta_4$ is highly negative for both states which means that nominal yields move positively with expected inflation. The calibration for the countercyclical state sets $\delta_2$ equal to $-0.10$ which somewhat mitigates the impact of inflation on yields since inflation shocks in this case lower future growth and therefore also yields. The calibration for the procyclical state assigns a positive value to $\delta_2$.

\[\text{For example, consumption growth exhibited high persistence and inflation low persistence throughout the period 2001:3-2009:4. In fact the first-order autocorrelation coefficient of consumption growth and inflation was 0.71 and 0.23 respectively during the procyclical period which is a notable difference compared to the countercyclical state. For the purpose of this paper, I choose to not elaborate further on these issues.}\]
which further amplifies the positive effect of inflation shocks on yields. Hence, yields increase in response to positive inflation shocks in both of the two economic states. This represents a cash-flow effect on nominal bonds.

The effect of volatility shocks on yields is positive and increasing with maturity in the countercyclical state. However the effect is reversed for the procyclical state, leading to lower yields as uncertainty increases. This arises since movements in $\sigma_{\pi,t}^2$ affects yields through a discount-rate channel. To see this more clearly, consider next risk premiums on nominal bonds.

First, positive shocks to expected growth means good times for the agent but low bond returns. As a result, shocks to consumption growth contributes to a negative risk premium. This holds for both the countercyclical and procyclical state. Second, higher inflation signals bad times ahead for the agent in the countercyclical state at the same time as bond returns are low. This results in procyclical bond returns and nominal bonds are therefore risky assets with respect to inflation shocks. In contrast, higher inflation in the procyclical state signals good times for the agent. Returns on nominal bonds are still affected negatively by an increase in inflation but returns are then countercyclical. Hence, nominal bonds in this case provide a hedge with respect to inflation risk.

To be more specific, let $h_{t+1,n}^s = q_{t+1,n-1}^s - q_{t,n}^s$ denote the one period log holding period return on a nominal bond with a maturity of $n$ periods. The risk premium can then be written as:

$$E_t(h_{t+1,n}^s - r_{f,t}) + \frac{1}{2} Var_t(h_{t+1,n}^s) = -\text{Cov}_t(m_{t+1,n}^s, h_{t+1,n}^s),$$

$$= A + B\sigma_{\pi,t}^2,$$

$$A = \lambda_c (\delta_1 D_{1,n-1} + \delta_3 D_{2,n-1}) \sigma_c^2 + \lambda_\nu D_{3,n-1}^s \sigma_\nu^2,$$

$$B = \lambda_c (\delta_2 D_{1,n-1} + \delta_4 D_{2,n-1}).$$

Time-varying volatility of inflation gives rise to a time-varying risk premium on nominal bonds. The B-coefficient is determined by the market price of long-run inflation risk times the response of bond prices to inflation shocks. Recall from (31) that nominal yields move positively with inflation across both states, meaning that $(\delta_2 D_{1,n-1} + \delta_4 D_{2,n-1}) < 0$. However, the market price of inflation
risk changes between the different states. For the period 1952:2-2001:2, investors dislike inflation shocks so $\lambda_\varepsilon$ is negative. Therefore risk premiums load positively on inflation volatility. However $\lambda_\varepsilon$ is positive in the procyclical state which means risk premiums move negatively with inflation uncertainty. Conventional wisdom often suggest that high uncertainty about inflation should result in higher risk premiums on nominal bonds. This only holds if inflation is bad for economic growth. If inflation instead is associated with high growth, high inflation uncertainty in fact lowers bond risk premiums. This paper suggests this was the case for the period 2001:3-2009:4.

During the stagflation period of the 1970's, investors disliked inflation shocks so bond risk premiums and yields should move positively with inflation risk according to the model. This is in line with data. The opposite was true during the recent crisis. A drop in both economic growth and inflation implied a procyclical inflation and therefore a positive price of inflation risk. A jump in inflation uncertainty should therefore lower expected returns on bonds and their yields. This is exactly what occurred during the financial crisis. Note that the effect of movements in inflation uncertainty on yields is distinct from changes in the level of inflation. An increase in the level of expected inflation represents a cash-flow effect on bonds and is always bad for bond returns and raises yields. This is implied by the model and is also what we observe in data.

Table 6 shows results from regressing the level of 5-year nominal rates onto expected inflation and inflation uncertainty. I run regressions for the full sample, for the countercyclical period, and for the procyclical period. Results in the left panel suggest that nominal rates in data load positively on both expected inflation and inflation uncertainty for the full sample period. The regression coefficients remain positive when the regression is run for the first sample period. However, the coefficient on inflation uncertainty switches sign in the procyclical state. While a higher level of inflation raises yields through a cash-flow channel, an increase in inflation uncertainty in fact lowers bond yields through its negative impact on expected returns. I run the same regressions inside the model and the results are reported in the right panel. The model accommodates the switching sign due to the change in the market price of inflation risk described above. Note that inflation uncertainty is measured differently in data and inside the model. Dispersion of inflation forecasts taken from Survey of Professional Forecasters are used for the sample regressions while simulated
inflation volatility $\sigma^2_{\pi,t}$ is used in the model. Therefore, the magnitude of the coefficients are not directly comparable. The important aspect here is that the coefficient for inflation uncertainty switches sign. These results indicate that it is critical for any asset-pricing model that prices nominal bonds to account for the changing cyclical nature of inflation.

5.2 Equity

Consider the innovation to log price-dividend ratios:

$$pd_{t+1} - E_t(pd_{t+1}) = (A_{d,1}\delta_1 + A_{d,2}\delta_3)\sigma_c\varepsilon_{c,t+1} + (A_{d,1}\delta_2 + A_{d,2}\delta_4)\sigma_{\pi,t}\varepsilon_{\pi,t+1} + A_{d,3}\sigma_{\nu_{t+1}}.$$  (33)

Given an EIS above one, $A_{d,1}$ is positive which means price-dividend ratios rise in response to higher expected consumption growth. The term $A_{d,2}\delta_3$ could in theory offset this effect. This happens if consumption shocks have a large positive effect on future inflation ($\delta_3 \gg 0$) together with a negative value of $A_{d,2}$. However this does not occur in any of the two calibrations and does not seem to be consistent with data. Price-dividend ratios therefore move positively with economic growth throughout the entire sample period.

The true focus of this paper is on the second term of (33). Recall that a negative relation between inflation expectations at time $t$ and future growth expectations ($\beta_2 < 0$) implies a negative relation between expected inflation and price-dividend ratios, i.e. $A_{d,2} < 0$. This is the case for the countercyclical state in which inflation shocks depress equity-valuation ratios. This effect is further amplified by the term $A_{d,1}\delta_2$ which is negative in the first state. Since inflation shocks in this case are bad news for growth, price-dividend ratios fall even further in response to inflation shocks.

As discussed in Section 3.4, the effect of inflation volatility on price-dividend ratios, $A_{d,3}$ is always negative. Intuitively, if positive inflation shocks are considered bad news by the agent and lead to low equity returns, inflation risk should give rise to a positive equity risk premium. If inflation instead represents good news for growth, stock returns will also be high. High returns in good times again implies that equity is risky with respect to inflation shocks. This suggests that risk premiums on equity should consistently load positive on inflation uncertainty.

To analyze this more closely, let $r_{m,t+1}$ denote the one period log market return. The equity
risk premium can then be written as:

\[
E_{t}(r_{m,t+1} - r_{f,t}) + \frac{1}{2} \text{Var}_{t}(r_{m,t+1}) = -\text{Cov}_{t}(m_{t+1}, r_{m,t+1}),
\]

(34)

\[
= A + B\sigma_{\pi,t}^2,
\]

A = \lambda_{\epsilon}(k_{d,1}A_{d,1}\delta_1 + k_{d,1}A_{d,2}\delta_3)\sigma_c^2 + \lambda_{\nu}k_{d,1}A_{d,3}\sigma_{\nu}^2,

B = \lambda_{\epsilon}(k_{d,1}A_{d,1}\delta_2 + k_{d,1}A_{d,2}\delta_4).

As with bonds, risk premiums on equity vary with inflation volatility. The B-coefficient is determined by the market price of long-run inflation risk times the impact of inflation shocks on real equity returns. In the countercyclical state, the agent dislikes inflation shocks so \(\lambda_{\epsilon}\) is negative. At the same time inflation is bad for stock returns which means \((k_{d,1}A_{d,1}\delta_2 + k_{d,1}A_{d,2}\delta_4)\) is also negative. Low returns in bad times implies that \(B > 0\) and the equity risk premium moves positively with inflation uncertainty.

Next consider the procyclical state. In this case, inflation is associated with good times so \(\lambda_{\epsilon}\) is positive. Now inflation is positively related to stock returns so the term \((k_{d,1}A_{d,1}\delta_2 + k_{d,1}A_{d,2}\delta_4)\) is also positive. High stock returns in good times means that B is positive also in this state. Note that this argument refers to inflation uncertainty and not to the level of inflation. In fact, in the procyclical state, the relation between price-dividend ratios and the level of expected inflation turns positive, i.e. \(A_{d,2} > 0\). Since dividend growth is a function of consumption growth, a positive impact of inflation on consumption growth in the model raises dividend growth and therefore also price-dividend ratios. However, an increase in the uncertainty about inflation raises expected returns and lowers equity-valuation ratios regardless of the cyclical state.

Table 7 shows results from regressing the log price-dividend ratio onto expected inflation and inflation uncertainty. Results show that price-dividend ratios in data load negatively on both expected inflation and inflation uncertainty for the full sample period. This is consistent with the voluminous literature that documents equity being a poor inflation hedge. Coefficients are similar for the countercyclical sample period. However, the coefficient on expected inflation switches sign in the procyclical state. The coefficient is not statistically significant but this depends on how
expected inflation is measured. Also, omitting inflation uncertainty from the regression tends to increase the significance.

The regressions results are consistent with Figure 6 which showed a significant positive shift in the correlation between price-dividend ratios and expected inflation in the early 2000’s. Consistent with the reasoning above, the coefficient on inflation uncertainty remains negative, reflecting the positive impact uncertainty has on expected returns. The same table shows that the model accommodates the switching sign due to a change in the $\beta_2$ parameter from negative to positive. Again, inflation uncertainty is measured differently in data and inside the model so the magnitude of the coefficients are not directly comparable.

It is noteworthy that expected inflation and inflation uncertainty explains such a large part of the variation in price-dividend ratios with an $R^2$ of 57% for the full sample period. To sum up, this section has shown that modeling the changing cyclicality of inflation is also of importance for pricing equity.

5.3 Asset Correlations

This section describes model implications for the correlation between stock and bond returns and for the correlation between dividend yields and nominal yields. Both of these correlations changed sign dramatically in the early 2000’s as was shown in Figures 2 and 3. I show below that the model can account for this switch by accounting for the changing cyclicality of inflation.

5.3.1 Stock and Bond Returns

Consider the analytical expression for the conditional covariance between stock and bond returns:

$$Cov_t[r_{m,t+1} + \pi_{t+1}, h_{t+1,n}] = A + B \sigma^2_{\pi,t},$$

$$A = \sigma^2_c(k_{d,1} \delta_1 A_{d,1} + k_{d,1} \delta_3 A_{d,2})(D^5_{1,n-1} \delta_1 + D^5_{2,n-1} \delta_3) + \sigma^2_v(k_{d,1} A_{d,3} D^5_{3,n-1}),$$

$$B = (k_{d,1} \delta_2 A_{d,1} + k_{d,1} \delta_4 A_{d,2})(D^5_{1,n-1} \delta_2 + D^5_{2,n-1} \delta_4).$$
The conditional covariance varies over time as inflation volatility changes. The two terms in the B-coefficient measure the impact of expected inflation shocks on price-dividend ratios and nominal bonds respectively. Based on the earlier discussion, we know that inflation shocks impact price-dividend ratios negatively in the countercyclical state and positively in the procyclical state. For nominal bonds we know that positive inflation shocks lower bond prices regardless of economic state. It is then evident from (35) that the conditional covariance moves positively with inflation uncertainty when inflation is bad for economic growth and negatively with inflation uncertainty when inflation is procyclical. The key parameter of the model that accommodates this switch is again $\beta_2$ which is negative in the first state and positive in the second state.

Table 8 shows results from predicting realized quarterly correlations between stock and bond returns in data using inflation uncertainty. The regression coefficient is positive for the full sample which means asset correlations move positively with uncertainty about inflation. This is consistent with stagflationary period of the 1970’s and early 1980’s in which asset correlations were highly positive at same time as inflation uncertainty was high. The positive relation between stock-bond return correlations and inflation uncertainty has also been reported elsewhere in the literature (e.g., David and Veronesi, 2009, and Viceira, 2010). However, the same table reports a statistically significant negative coefficient for the period 2001:3-2009:4. This switch in signs has to my knowledge not been noted in the literature. Hence, during this period, a rise in inflation uncertainty induced a negative comovement between stock and bond returns.

A prime example of such an episode is the recent financial crisis which saw an increase in inflation uncertainty together with a sharp drop in equity prices and nominal yields. The model is able to reproduce the switching coefficients as is evident from the last column in Table 8. Since I cannot construct quarterly asset correlations based on daily returns inside the model, I report instead the analytical value of the B-coefficient in expression (35). The model accommodates the switch in signs by accounting for the two different economic states.

In order to plot the conditional correlation between stock and bond returns implied by the model, I use (35) together with analytical expressions for the volatility of stock and bond returns.
and estimates of the conditional variance of inflation obtained from a GARCH(1,1) model. Figure plots the correlations. Consistent with data, the model implies highly positive correlations throughout the 1970’s and early 1980’s. As the so-called Great Moderation started, conditional correlations dropped together with inflation volatility. The correlations stay positive throughout the entire countercyclical period but lie close to zero at the end point, the second quarter of 2001. For the procyclical period I compute correlations using the second calibration. Since this period experienced highly negative correlations, a sharp drop occurs in the figure representing the regime switch. The last drop in the figure, from around −0.70 to −0.85 refers to the recent financial crisis. Hence, the model is able to generate realistic conditional correlations throughout the sample period relying on inflation volatility only. Since inflation volatility cannot be negative, the model accommodates the switch in correlations through a switch in the market price of inflation risk and therefore in the loading of bond risk premia on inflation uncertainty.

5.3.2 Dividend Yields and Nominal Yields

Since equity returns are closely related to changes in dividend yields and bond returns to changes in yields, the same mechanism can be used to explain the changing correlation between dividend yields and nominal yields. The existing literature has focussed on the highly positive correlation between these two variables measured from the mid 1960’s to the 2000’s. As mentioned earlier, this observation is often dubbed the Fed-model. However it is rarely mentioned in the literature that this correlation changed dramatically during the last 10 years. From a correlation of 0.28 during 1952:2-2001:2, the correlation was in fact -0.70 during 2001:3-2009:4. This is a striking change for an empirical relation that the existing literature has treated as positive.

Consider the expression for the conditional covariance between dividend yields and nominal yields and estimates of the conditional variance of inflation obtained from a GARCH(1,1) model. The variances of stock and bond returns are computed as follows: $\text{Var}_t(r_{m,t+1} + \pi_{t+1}) = \sigma^2_\ell[(k_{d,1}A_{d,1}\delta_1 + k_{d,1}A_{d,2}\delta_2)^2 + \varphi^2] + \sigma^2_\ell[(k_{d,1}A_{d,1}\delta_2 + k_{d,1}A_{d,2}\delta_2)^2 + 1] + \sigma^2_\ell(k_{d,1}A_{d,3})^2$ and $\text{Var}_t(h_{n+1,n}) = \sigma^2_\ell[D_{1,n-1}\delta_1 + D_{2,n-1}\delta_3]^2 + \sigma^2_\ell[D_{3,n-1}\delta_2 + D_{2,n-1}\delta_4]^2 + \sigma^2_\ell[D_{3,n-1}]^2$. 

[11] The variances of stock and bond returns are computed as follows: $\text{Var}_t(r_{m,t+1} + \pi_{t+1}) = \sigma^2_\ell[(k_{d,1}A_{d,1}\delta_1 + k_{d,1}A_{d,2}\delta_2)^2 + \varphi^2] + \sigma^2_\ell[(k_{d,1}A_{d,1}\delta_2 + k_{d,1}A_{d,2}\delta_2)^2 + 1] + \sigma^2_\ell(k_{d,1}A_{d,3})^2$ and $\text{Var}_t(h_{n+1,n}) = \sigma^2_\ell[D_{1,n-1}\delta_1 + D_{2,n-1}\delta_3]^2 + \sigma^2_\ell[D_{3,n-1}\delta_2 + D_{2,n-1}\delta_4]^2 + \sigma^2_\ell[D_{3,n-1}]^2$. 

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yields:
\[
\text{Cov}_t[dp_{t+1}, y_{t+1,n}] = A + B \sigma^2_{\pi,t},
\]
\[
A = \sigma^2_c (-\delta_1 A_{d,1} - \delta_3 A_{d,2}) \left( \frac{-D^S_{1,n} \delta_1 - D^S_{2,n} \delta_3}{n} \right) + \sigma^2_\nu \left( \frac{A_{d,3} D^S_{3,n}}{n} \right),
\]
\[
B = (-\delta_2 A_{d,1} - \delta_4 A_{d,2}) \left( \frac{-D^S_{1,n} \delta_2 - D^S_{2,n} \delta_4}{n} \right).
\]

As before, the conditional covariance between dividend yields and nominal yields vary with inflation volatility. The A and B-coefficients in (36) are very similar to the ones in (35) meaning the same intuition goes through as for the stock-bond correlation. Hence, the sign of the correlation between dividend yields and nominal yields depends on the cyclical nature of inflation.

6 Conclusion

The correlation between returns on US stocks and Treasury bonds and the relation between inflation and asset prices have varied substantially over time. For example, the 1970-1980’s was characterized by a highly positive correlation between stock and bond returns and a strong negative relation between inflation and price-dividend ratios. In contrast, the period 2000-2009 experienced the exact opposite with strongly negative asset correlations and a positive relation between inflation and equity valuations. I show that these observations line up remarkably well with the considerable time variation in the relation between consumption growth and inflation going back to the 1930’s. While the 1970-1980’s was characterized by stagflation, I show that inflation switched to a procyclical state in the early 2000’s.

I calibrate a long-run risk model that illustrates the connection between the cyclicality of inflation and the joint movements of bond and equity risk premia and inflation and asset prices. Persistent inflation shocks have real effects and affects both equity and bond risk premia. Equity and bond risk premia are both functions of inflation volatility in the model but the loading of bond risk premia on inflation uncertainty depends on the cyclicality of inflation and may therefore switch sign. The model suggests that both equity and nominal bonds are risky assets when inflation is
counter cyclical, leading to a positive comovement of asset risk premia in response to changes in inflation uncertainty. In contrast, nominal bonds provide a hedge against inflation risk when inflation is procyclical while equity is still risky. This implies that a rise in inflation uncertainty drive equity and bond risk premia in different directions, causing their returns to correlate negatively. An example of this would be the recent financial crisis in which uncertainty about future inflation increased markedly.

The model presented here is a first step in highlighting the importance of properly modeling the cyclicality of inflation. There are number of directions in which these insights can be taken. First, the model is silent on the underlying determinants of inflation cyclicality. What is the profound structural nature of shocks that characterize a procyclical or countercyclical state? What role does monetary policy play for understanding the joint dynamics of growth and inflation? Both our theoretical and empirical knowledge of this is limited. Second, more extensive regime-switching models for the joint dynamics of growth and inflation can be considered. I leave these issues for future research.
7 Appendix

Sections A.1 - A.4 solve the model using approximate analytical solutions.

A.1 The Price-Consumption Ratio

Coefficients governing the price-consumption ratio are derived using the logarithm of the intertemporal marginal rate of substitution, \( m_{t+1} = \theta \ln(\delta) - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1)r_{c,t+1} \), together with the macro dynamics and the approximation of the return on the consumption paying asset, \( r_{c,t+1} = k_{c,0} + k_{c,1}pc_{t+1} - pc + \Delta c_{t+1} \), where \( pc_t = A_{c,0} + A_{c,1}x_{c,t} + A_{c,2}x_{\pi,t} + A_{c,3}\sigma^2_{\pi,t} \). Coefficients \( k_{c,0} \) and \( k_{c,1} \) are defined as:

\[
\begin{align*}
    k_{c,1} &= \frac{\exp(pc)}{1 + \exp(pc)}; \\
    k_{c,0} &= \ln(1 + \exp(pc)) - k_{c,1}pc.
\end{align*}
\]

Consider the Euler equation for the consumption claim:

\[
E_t \left[ \exp \left( \theta \ln(\delta) - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1)r_{c,t+1} + r_{c,t+1} \right) \right] = 1.
\]

Due to the conditional normality of \( \Delta c \) and the state variables, and therefore also \( r_c \), the log Euler condition can be written as:

\[
E_t [m_{t+1} + r_{c,t+1}] + \frac{1}{2} Var_t [m_{t+1} + r_{c,t+1}] = 0.
\]

The conditional mean is given by:

\[
E_t [m_{t+1} + r_{c,t+1}] = \theta \ln(\delta) - \frac{\theta}{\psi} \mu_c + \theta(k_{c,0} + k_{c,1}(A_{c,0} + A_{c,3}\sigma^2_{\pi}(1 - v_\pi)) - A_{c,0} + \mu_c) + \\
x_{c,t} \left[ -\frac{\theta}{\psi} + \theta(k_{c,1}A_{c,1}\beta_1 - A_{c,1} + 1) \right] + \\
x_{\pi,t} \left[ \theta(k_{c,1}A_{c,1}\beta_2 + k_{c,1}A_{c,2}\beta_3 - A_{c,2}) \right] + \\
\sigma^2_{\pi,t} \left[ \theta A_{c,3}(k_{c,1}v_\pi - 1) \right],
\]

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and the conditional variance is given by:

\[
Var_t [m_{t+1} + r_{c,t+1}] = \sigma_c^2 X + \sigma_{x,t}^2 Y + (\theta k_{c,1} A_{c,3})^2 \sigma_{\psi}^2,
\]

\[
X = \left( \theta - \frac{\theta}{\psi} \right)^2 + (\theta k_{c,1} A_{c,1} \delta_1 + \theta k_{c,1} A_{c,2} \delta_3)^2,
\]

\[
Y = \alpha^2 \left( \theta - \frac{\theta}{\psi} \right)^2 + (\theta k_{c,1} A_{c,1} \delta_2 + \theta k_{c,1} A_{c,2} \delta_4)^2.
\]

Setting the conditional moments equal to zero and solving for the \(A_{c,0}\)-coefficients yields the following expression for \(A_{c,0}\):

\[
A_{c,0} = \frac{\theta \ln(\delta) - \frac{\theta}{\psi} \mu_c + \theta (k_{c,0} + k_{c,1} A_{c,3})^2 (1 - v_{\pi}) + \mu_c + 0.5(\theta^2 X + (\theta k_{c,1} A_{c,3})^2 \sigma_{\psi}^2)}{\theta (1 - k_{c,1})},
\]

where the X-coefficient is the same as in the conditional variance expression above. The remaining \(A_{c}\)-coefficients are given in the main text.

**A.2 The Price-Dividend Ratio**

Coefficients governing the price-dividend ratio are found in an analogous manner. The Euler condition for the market return, \(r_{m,t+1}\), is written as:

\[
E_t [m_{t+1} + r_{m,t+1}] + \frac{1}{2} Var_t [m_{t+1} + r_{m,t+1}] = 0,
\]

where \(r_{m,t+1} = k_{d,0} + k_{d,1} pd_{t+1} - pd_t + \Delta d_{t+1}\) and \(pd_t = A_{d,0} + A_{d,1} x_{c,t} + A_{d,2} x_{\pi,t} + A_{d,3} \sigma_{\pi,t}^2\). Using the macro dynamics, the conditional mean is given by:

\[
E_t [m_{t+1} + r_{m,t+1}] = \theta \ln(\delta) - \frac{\theta}{\psi} \mu_c + (\theta - 1)(k_{c,0} + k_{c,1} (A_{c,0} + A_{c,3} \sigma_{\pi}^2 (1 - v_{\pi})) - A_{c,0} + \mu_c) +
\]

\[
+ k_{d,0} + k_{d,1} (A_{d,0} + A_{d,3} \sigma_{\pi}^2 (1 - v_{\pi})) - A_{d,0} + \mu_d +
\]

\[
x_{c,t} \left[ -\frac{\theta}{\psi} + (\theta - 1)(k_{c,1} A_{c,1} \beta_1 - A_{c,1} + 1) + k_{d,1} A_{d,1} \beta_1 - A_{d,1} + \phi \right] +
\]

\[
x_{\pi,t} \left[ (\theta - 1)(k_{c,1} A_{c,1} \beta_2 + k_{c,1} A_{c,2} \beta_3 - A_{c,2}) + k_{d,1} A_{d,1} \beta_2 + k_{d,1} A_{d,2} \beta_3 - A_{d,2} \right] +
\]

\[
\sigma_{\pi,t}^2 [(\theta - 1)A_{c,3} (k_{c,1} v_{\pi} - 1) + A_{d,3} (k_{d,1} v_{\pi} - 1)].
\]
The conditional variance is given by:

\[
Var_t [m_{t+1} + r_{m,t+1}] = \sigma_c^2 X + \sigma_{\pi,t}^2 Y + ((\theta - 1)k_{c,1}A_{c,3} + k_{d,1}A_{d,3})^2 \sigma_{\nu}^2,
\]

\[
X = \left( \theta - 1 - \frac{\theta}{\psi} \right)^2 + ((\theta - 1)(k_{c,1}A_{c,1}\delta_1 + k_{c,1}A_{c,2}\delta_3) + k_{d,1}A_{d,1}\delta_1 + k_{d,1}A_{d,2}\delta_3)^2 + \varphi^2,
\]

\[
Y = \alpha^2 \left( \theta - 1 - \frac{\theta}{\psi} \right)^2 + ((\theta - 1)(k_{c,1}A_{c,1}\delta_2 + k_{c,1}A_{c,2}\delta_4) + k_{d,1}A_{d,1}\delta_2 + k_{d,1}A_{d,2}\delta_4)^2.
\]

Setting the conditional moments equal to zero and solving for the \( A_d \)-coefficients yields the following expression for \( A_{d,0} \):

\[
A_{d,0} = (1 - k_{d,1})^{-1} \left[ \theta \ln(\delta) - \frac{\theta}{\psi} \mu_c + (\theta - 1)(k_{c,0} + k_{c,1}(A_{c,0} + A_{c,3}\sigma_\pi^2(1 - \nu)) - A_{c,0} + \mu_c) + k_{d,0} + k_{d,1}A_{d,3}\sigma_\pi^2(1 - \nu) + \mu_d + 0.5(\sigma_c^2 X + ((\theta - 1)k_{c,1}A_{c,3} + k_{d,1}A_{d,3})^2 \sigma_{\nu}^2) \right],
\]

where the X-coefficient is the same as in the conditional variance expression above. The remaining \( A_d \)-coefficients are given in the main text.

### A.3 Real Bonds

Consider the Euler condition for real bonds:

\[
Q_{t,n} = E_t [M_{t+1}Q_{t+1,n-1}],
\]

where \( Q_{t,n} = \exp(D_{0,n} + D_{1,n}x_{c,t} + D_{2,n}x_{\pi,t} + D_{3,n}\sigma_{\pi,t}^2) \). Again, using the conditional lognormality of the state variables:

\[
q_{t,n} = E_t [m_{t+1} + q_{t+1,n-1}] + \frac{1}{2} Var_t [m_{t+1} + q_{t+1,n-1}].
\]
The conditional mean is given by:

\[ E_t [m_{t+1} + q_{t+1,n-1}] = \theta \ln(\delta) - \frac{\theta}{\psi} \mu_c + (\theta - 1)(k_{c,0} + k_{c,1}(A_{c,0} + A_{c,3}\sigma_\pi^2(1 - v_\pi)) - A_{c,0} + \mu_c) + D_{0,n-1} + D_{3,n-1}\sigma_\pi^2(1 - v_\pi) + \]

\[ x_{c,t} \left[ -\frac{\theta}{\psi} + (\theta - 1)(k_{c,1}A_{c,1}\beta_1 - A_{c,1} + 1) + D_{1,n-1}\beta_1 \right] + \]

\[ x_{\pi,t} \left[ ((\theta - 1)(k_{c,1}A_{c,1}\beta_2 + k_{c,1}A_{c,2}\beta_3 - A_{c,2}) + D_{1,n-1}\beta_2 + D_{2,n-1}\beta_3) + \]

\[ \sigma_\pi^2 \left[ ((\theta - 1)A_{c,3}(k_{c,1}v_\pi - 1) + D_{3,n-1}v_\pi) \right], \]

and the conditional variance by:

\[ \text{Var}_t [m_{t+1} + q_{t+1,n-1}] = \sigma_c^2 X + \sigma_\pi^2 Y + ((\theta - 1)k_{c,1}A_{c,3} + D_{3,n-1})^2 \sigma_\nu^2, \]

\[ X = \left( \theta - 1 - \frac{\theta}{\psi} \right)^2 + ((\theta - 1)(k_{c,1}A_{c,1}\delta_1 + k_{c,1}A_{c,2}\delta_3) + D_{1,n-1}\delta_1 + D_{2,n-1}\delta_3)^2, \]

\[ Y = \alpha^2 \left( \theta - 1 - \frac{\theta}{\psi} \right)^2 + ((\theta - 1)(k_{c,1}A_{c,1}\delta_2 + k_{c,1}A_{c,2}\delta_4) + D_{1,n-1}\delta_2 + D_{2,n-1}\delta_4)^2. \]

Matching the coefficients yields the following expression for \( D_{0,n} \):

\[ D_{0,n} = \theta \ln(\delta) - \frac{\theta}{\psi} \mu_c + (\theta - 1)(k_{c,0} + k_{c,1}(A_{c,0} + A_{c,3}\sigma_\pi^2(1 - v_\pi)) - A_{c,0} + \mu_c) + D_{0,n-1} + D_{3,n-1}\sigma_\pi^2(1 - v_\pi) + \]

\[ 0.5(\sigma_c^2 X + ((\theta - 1)k_{c,1}A_{c,3} + D_{3,n-1})^2 \sigma_\nu^2), \]

where the X-coefficient is the same as in the conditional variance expression above. The remaining \( D \)-coefficients are given in the main text. The coefficients are computed recursively using the fact that \( D_{i,0} = 0 \) for \( i = 0,1,2,3 \).
A.4 Nominal Bonds

The Euler condition for the real price of a nominal bond is:

\[
\frac{Q^S_{t,n}}{\Pi_t} = E_t \left[ M_{t+1} \frac{Q^S_{t+1,n-1}}{\Pi_{t+1}} \right],
\]

\[
Q^S_{t,n} = E_t \left[ M_{t+1} \frac{Q^S_{t+1,n-1}}{\Pi_{t+1}} \right],
\]

where the following is conjectured: \(Q^S_{t,n} = \exp(D^0_{0,n} + D^1_{1,n} x_{c,t} + D^2_{2,n} x_{\pi,t} + D^3_{3,n} \sigma^2_{\pi,t})\). Taking logs and again using the conditional lognormality yields:

\[
q^S_{t,n} = E_t \left[ m_{t+1} - \pi_{t+1} + q^S_{t+1,n-1} \right] + \frac{1}{2} \text{Var}_t \left[ m_{t+1} - \pi_{t+1} + q^S_{t+1,n-1} \right].
\]

The conditional mean is given by:

\[
E_t \left[ m_{t+1} - \pi_{t+1} + q^S_{t+1,n-1} \right] = \theta \ln(\delta) - \frac{\theta}{\psi} \mu_c + (\theta - 1)(k_{c,0} + k_{c,1}(A_{c,0} + A_{c,3} \sigma^2_{\pi}(1 - v_\pi)) - A_{c,0} + \mu_c) - \mu_\pi + D^5_{0,n-1} + D^5_{3,n-1} \sigma^2_{\pi}(1 - v_\pi) +
\]

\[
x_{c,t} \left[ -\frac{\theta}{\psi} + (\theta - 1)(k_{c,1} A_{c,1} \beta_1 - A_{c,1} + 1) + D^5_{1,n-1} \beta_1 \right] +
\]

\[
x_{\pi,t} \left[ (\theta - 1)(k_{c,1} A_{c,1} \beta_2 + k_{c,1} A_{c,2} \beta_3 - A_{c,2}) - 1 + D^5_{1,n-1} \beta_2 + D^5_{2,n-1} \beta_3 \right] +
\]

\[
\sigma^2_{\pi,t} \left[ (\theta - 1)A_{c,3}(k_{c,1} v_\pi - 1) + D^5_{3,n-1} v_\pi \right],
\]

and the conditional variance by:

\[
\text{Var}_t \left[ m_{t+1} - \pi_{t+1} + q^S_{t+1,n-1} \right] = \sigma^2_c X + \sigma^2_{\pi,t} Y + ((\theta - 1)k_{c,1} A_{c,3} + D^5_{3,n-1})^2 \sigma^2_{\psi},
\]

\[
X = \left( \theta - 1 - \frac{\theta}{\psi} \right)^2 + \left( (\theta - 1)(k_{c,1} A_{c,1} \delta_1 + k_{c,1} A_{c,2} \delta_3) + D^5_{1,n-1} \delta_1 + D^5_{2,n-1} \delta_3 \right)^2,
\]

\[
Y = \left( \alpha \left( \theta - 1 - \frac{\theta}{\psi} \right) - 1 \right)^2 + \left( (\theta - 1)(k_{c,1} A_{c,1} \delta_2 + k_{c,1} A_{c,2} \delta_4) + D^5_{1,n-1} \delta_2 + D^5_{2,n-1} \delta_4 \right)^2.
\]
Matching the coefficients yields the following expression for $D_{0,n}^\delta$:

$$D_{0,n} = \theta \ln(\delta) - \frac{\theta}{\psi^\theta} \mu_c + (\theta - 1)(k_{c,0} + k_{c,1}(A_{c,0} + A_{c,3}\sigma^2_\pi(1 - v_\pi)) - A_{c,0} + \mu_c) -$$

$$\mu_\pi + D_{0,n-1}^\delta + D_{3,n-1}^\delta \sigma^2_\pi(1 - v_\pi) +$$

$$0.5(\sigma^2_c X + ((\theta - 1)k_{c,1}A_{c,3} + D_{3,n-1}^\delta)^2 \sigma^2_\nu),$$

where the X-coefficient is the same as in the conditional variance expression above. The remaining $D^\delta$-coefficients are given in the main text. The coefficients are computed recursively using the fact that $D_{i,0}^\delta = 0$ for $i = 0, 1, 2, 3$. 

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References


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<th>Procyclical state</th>
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<tbody>
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<td>Estimate</td>
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This table presents results from estimating the Markov-switching model using maximum likelihood. The sample period is 1952:2 to 2009:4. The setup follows Hamilton (1989,1994) among others and is formulated as follows:

$$
\Delta c_{t+1} = \alpha_{s_{t+1}} + \rho_{0,s_{t+1}} \Delta c_t + \rho_{1,s_{t+1}} \pi_t + \epsilon_{t+1},
$$

$$
\epsilon_{t+1} \sim N(0, \sigma^2),
$$

where $s_{t+1}$ is presumed to follow a two-state Markov chain with transition probabilities $p_{ij} = P(s_{t+1} = j | s_t = i)$ and where $\sum_{j=1}^{N} p_{ij} = 1$ and $0 < p_{ij} < 1$. The intercept and the regression coefficients are assumed to follow the same regime process. The probability of ending up in tomorrow’s state $s_{t+1} = (0, 1)$ given today’s state $s_t = (0, 1)$ is governed by the transitional probability matrix:

$$
P = \begin{bmatrix}
p_{00} & p_{10} 
p_{01} & p_{11}
\end{bmatrix}.
$$
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</tr>
<tr>
<td>$\delta_3$</td>
<td>0.25</td>
<td>0.0</td>
</tr>
<tr>
<td>$\delta_4$</td>
<td>0.65</td>
<td>0.65</td>
</tr>
<tr>
<td>$\sigma_\pi$</td>
<td>0.0025</td>
<td>0.0025</td>
</tr>
<tr>
<td>$\nu_\pi$</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>$\sigma_\nu \times 10^{-6}$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>0.004</td>
<td>0.004</td>
</tr>
<tr>
<td>$\phi$</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>$\psi$</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.998</td>
<td>0.998</td>
</tr>
</tbody>
</table>

This table presents calibrated parameters for the two economic states. Parameters for the countercyclical state are calibrated as to match both standard macro and asset pricing moments as well as the various relations between stocks and bonds and between inflation and asset prices. For parsimonious reasons, only four parameters are re-calibrated in the procyclical state, $\beta_2, \delta_2, \delta_3,$ and $\alpha$. The second calibration targets a limited number of key moments between inflation and growth and between inflation and asset prices. The countercyclical state refers to 1952:2-2001:2 and the procyclical state to 2001:3-2009:4.
<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Data</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Consumption growth, ( \Delta c )</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.84</td>
<td>0.84</td>
<td>(0.05)</td>
</tr>
<tr>
<td>Std.Dev.</td>
<td>0.57</td>
<td>0.49</td>
<td>(0.03)</td>
</tr>
<tr>
<td>AC(1)</td>
<td>0.28</td>
<td>0.34</td>
<td>(0.06)</td>
</tr>
<tr>
<td><strong>Inflation, ( \pi )</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.95</td>
<td>0.95</td>
<td>(0.10)</td>
</tr>
<tr>
<td>Std.Dev.</td>
<td>0.76</td>
<td>0.65</td>
<td>(0.08)</td>
</tr>
<tr>
<td>AC(1)</td>
<td>0.84</td>
<td>0.85</td>
<td>(0.04)</td>
</tr>
<tr>
<td><strong>Dividend growth, ( \Delta d )</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.34</td>
<td>0.34</td>
<td>(0.16)</td>
</tr>
<tr>
<td>Std.Dev.</td>
<td>2.09</td>
<td>1.45</td>
<td>(0.12)</td>
</tr>
<tr>
<td>AC(4)</td>
<td>0.08</td>
<td>0.09</td>
<td>(0.12)</td>
</tr>
</tbody>
</table>

This table presents unconditional moments of macro data. Sample statistics refer to the countercyclical period 1952:2-2001:2. Model statistics are based on a simulation of 150000 quarters. AC(k) denotes the autocorrelation for k lags. Standard errors, denoted SE, are computed as in Newey West (1987), using four lags.
Table 4: Asset Price Moments

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E(r_{m} - r_{f})$</td>
<td>1.08</td>
<td>1.46</td>
</tr>
<tr>
<td>$\sigma(r_{m} - r_{f})$</td>
<td>4.08</td>
<td>7.86</td>
</tr>
<tr>
<td>$E(pd)$</td>
<td>3.16</td>
<td>3.40</td>
</tr>
<tr>
<td>$\sigma(pd)$</td>
<td>0.18</td>
<td>0.32</td>
</tr>
<tr>
<td>Nominal bonds</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E(y^{S}_{3m})$</td>
<td>4.98</td>
<td>5.45</td>
</tr>
<tr>
<td>$E(y^{S}_{5y})$</td>
<td>5.82</td>
<td>6.37</td>
</tr>
<tr>
<td>$\sigma(y^{S}_{3m})$</td>
<td>2.73</td>
<td>2.85</td>
</tr>
<tr>
<td>$\sigma(y^{S}_{5y})$</td>
<td>1.81</td>
<td>2.74</td>
</tr>
<tr>
<td>Real bonds</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E(y_{3m})$</td>
<td>1.14</td>
<td></td>
</tr>
<tr>
<td>$E(y_{5y})$</td>
<td>0.74</td>
<td></td>
</tr>
<tr>
<td>$\sigma(y_{3m})$</td>
<td>0.73</td>
<td></td>
</tr>
<tr>
<td>$\sigma(y_{5y})$</td>
<td>0.67</td>
<td></td>
</tr>
</tbody>
</table>

This table presents unconditional asset-price moments. Sample statistics refer to the countercyclical period 1952:2-2001:2. Model statistics are based on a simulation of 150000 quarters.

Table 5: Macro and Asset Correlations

<table>
<thead>
<tr>
<th></th>
<th>Countercyclical state</th>
<th>Procyclical state</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model</td>
<td>Data</td>
</tr>
<tr>
<td>$Corr(\Delta c, \pi)$</td>
<td>-0.34</td>
<td>-0.40</td>
</tr>
<tr>
<td>$Corr(dp, y_{5y}^{S})$</td>
<td>0.46</td>
<td>0.28</td>
</tr>
<tr>
<td>$Corr(r_{stock}, r_{bond})$</td>
<td>0.37</td>
<td>0.21</td>
</tr>
</tbody>
</table>

This table presents unconditional correlations of macro and asset-price data. $Corr(\Delta c, \pi)$ refers to the correlation between consumption growth and inflation, $Corr(dp, y_{5y}^{S})$ refers to the correlation between dividend yields and nominal yields, and $Corr(r_{stock}, r_{bond})$ refers to the correlation between stock and bond returns. Model statistics are based on a simulation of 150000 quarters. The countercyclical state refers to 1952:2-2001:2 and the procyclical state to 2001:3-2009:4.
### Table 6: Nominal Yields and Inflation

<table>
<thead>
<tr>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_\pi$</td>
<td>$\beta_\pi$</td>
</tr>
<tr>
<td>Full sample</td>
<td>0.49</td>
</tr>
<tr>
<td>Countercyclical state</td>
<td>0.18</td>
</tr>
<tr>
<td>Procyclical state</td>
<td>0.23</td>
</tr>
</tbody>
</table>

This table presents results from regressing 5-year nominal interest rates onto expected inflation ($\beta_\pi$) and inflation uncertainty ($\beta_{\sigma^2_\pi}$). Inflation uncertainty is measured as the dispersion of inflation forecasts from Survey of Professional Forecasters (SPF) for the sample regressions and measured as simulated inflation volatility $\sigma^2_{\pi,t}$ inside the model. Inflation forecasts are one-quarter ahead and all regressions are run contemporaneously. Standard errors are computed using Newey-West (1987) with 4 lags. Forecasts are taken from column PGDP3 in SPF using a cutoff of 3 standard deviations. The countercyclical state refers to 1968:4-2001:2 and the procyclical state to 2001:3-2009:4. The magnitude of the regression coefficients are not directly comparable since inflation uncertainty is measured differently inside the model and in data. Model statistics are generated by simulating the model 150000 quarters.

### Table 7: Price-Dividend Ratios and Inflation

<table>
<thead>
<tr>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_\pi$</td>
<td>$\beta_\pi$</td>
</tr>
<tr>
<td>Full sample</td>
<td>-0.04</td>
</tr>
<tr>
<td>Countercyclical state</td>
<td>-0.04</td>
</tr>
<tr>
<td>Procyclical state</td>
<td>0.02</td>
</tr>
</tbody>
</table>

This table presents results from regressing log price-dividend ratios onto expected inflation ($\beta_\pi$) and inflation uncertainty ($\beta_{\sigma^2_\pi}$). Inflation uncertainty is measured as the dispersion of inflation forecasts from Survey of Professional Forecasters (SPF) for the sample regressions and measured as simulated inflation volatility $\sigma^2_{\pi,t}$ inside the model. Inflation forecasts are one-quarter ahead and all regressions are run contemporaneously. Standard errors are computed using Newey-West (1987) with 4 lags. Forecasts are taken from column PGDP3 in SPF using a cutoff of 3 standard deviations. The countercyclical state refers to 1968:4-2001:2 and the procyclical state to 2001:3-2009:4. The magnitude of the regression coefficients are not directly comparable since inflation uncertainty is measured differently inside the model and in data. Model statistics are generated by simulating the model 150000 quarters.
Table 8: **Predicting Covariance of Stock and Bond Returns**

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta_{\sigma^2}$</td>
<td>t-stat</td>
</tr>
<tr>
<td>Full sample</td>
<td>1.72</td>
<td>3.80</td>
</tr>
<tr>
<td>Countercyclical state</td>
<td>1.12</td>
<td>3.36</td>
</tr>
<tr>
<td>Procyclical state</td>
<td>-4.84</td>
<td>-2.18</td>
</tr>
</tbody>
</table>

This table presents results from predicting quarterly covariances between returns on US stocks and Treasury bonds. Dependent variable is the realized quarterly covariance between stock and bond returns computed using daily returns. Independent variable is the dispersion of one-quarter inflation forecasts from Survey of Professional Forecasters (SPF). Model statistics refer to the analytical B-coefficient in Equation (35). Standard errors are computed using Newey-West (1987) with 4 lags. Inflation forecasts are taken from column PGDP3 in SPF using a cutoff of 3 standard deviations. The countercyclical state refers to 1968:4-2001:2 and the procyclical state to 2001:3-2009:4. The magnitude of the regression coefficients are not directly comparable since inflation uncertainty is measured differently inside the model and in data.

![Quarterly Correlations between Stock and Bond returns](image)

**Figure 1:** Quarterly correlations between returns on US stocks and 5-year Treasury bonds. Correlations are based on daily returns. The sample period is 1965:2-2009:4.
Figure 2: Correlation between quarterly consumption growth and quarterly inflation and between quarterly returns on US stocks and Treasury bonds. All correlations are computed for non-overlapping 5-year intervals over the period 1955-2009.

Figure 3: Correlation between quarterly consumption growth and quarterly inflation and between end-of-quarter dividend yields and 5-year nominal yields. All correlations are computed for non-overlapping 5-year intervals over the period 1955-2009.
Figure 4: Annual consumption growth and inflation 1930-2009.

Figure 5: Correlation between annual consumption growth and annual inflation and between end-of-year dividend yields and inflation. All correlations are computed for non-overlapping 10-year intervals over the period 1930-2009.
Figure 6: Correlation coefficients between log price-dividend ratios and expected inflation and between log price-dividend ratios and inflation uncertainty. All correlations refer to non-overlapping 5-year periods covering the period 1970-2009. Inflation uncertainty is measured as the dispersion of one-quarter ahead inflation forecasts from Survey of Professional Forecasters (SPF). Forecasts are taken from column PGDP3 in SPF using a cutoff of 3 standard deviations. Inflation expectations are created by projecting inflation onto lagged inflation, lagged consumption growth, lagged yield spread, and expected growth and inflation from SPF.
Figure 7: Correlation coefficients between the level of 5-year nominal interest rates and expected inflation and between the level of 5-year nominal interest rates and inflation uncertainty. All correlations refer to non-overlapping 5-year periods covering the period 1970-2009. Inflation uncertainty is measured as the dispersion of one-quarter ahead inflation forecasts from Survey of Professional Forecasters (SPF). Forecasts are taken from column PGDP3 in SPF using a cutoff of 3 standard deviations. Inflation expectations are created by projecting inflation onto lagged inflation, lagged consumption growth, lagged yield spread, and expected growth and inflation from SPF.
Figure 8: Correlations between expected consumption growth and inflation and between industrial production and inflation computed for non-overlapping 5-year intervals over the period 1955-2009. Expectations are created by projecting growth and inflation onto lagged inflation, lagged growth, and the lagged yield spread. Expected growth and inflation from Survey of Professional Forecasters are also included as predictive variables starting in 1968:4.

Figure 9: Smoothed probability of being in the procyclical inflation state. Probabilities are computed as in Hamilton (1994) and refer to the Markov-switching model described in Section 4.
Figure 10: Dispersion of one-quarter ahead inflation forecasts obtained from Survey of Professional Forecasters. Forecasts are taken from column PGDP3 in SPF using a cutoff of 3 standard deviations. Time period is 1968:4-2009:4.

![Dispersion of Inflation Forecasts](image)

Figure 11: Model-implied conditional correlation between stock and bond returns. Correlations are computed using Equation (35) for the covariances together with analytical expressions for the volatility of stock and bond returns. Proxies for $\sigma_{\pi,t}$ are obtained by fitting a GARCH(1,1) to inflation.

![Conditional Model Correlation between Stock and Bond Returns](image)