Consumer credit and payment cards

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Abstract

We consider debit and credit card networks. Our contribution is to introduce the role of consumer credit into these payment networks, and to assess the way this affects competition and equilibrium fees. We analyze a situation in which overdrafts are associated with current accounts and debit cards, and larger credit lines with ‘grace’ periods are associated with credit cards. If we just introduce credit cards, we find their merchant fees depend not only on the networks’ cost of funds and the probability of default, but also on the interest rates of overdrafts. Whilst debit card merchant fees do not depend on funding costs or default risk in a debit-card only world, this changes when they start to compete with credit cards. First, debit merchant acceptance increases with the default probability, even though merchant fees increase. Second, an increase in funding costs causes a surprising increase in debit merchant fees. Effectively, the bank offering the debit card benefits from consumers maintaining a positive current account balance, when they use their credit instead of their debit card. As a result, this complementarity may lead to relatively high debit card merchant fees as the bank discourages debit card acceptance at the margin.

Key Words: Payment pricing, Card competition, Consumer credit, Complementarity

JEL Codes: L11, G21, D53

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1 Introduction

Debit or credit? Every day, millions of consumers stand at store checkout counters and make a payment decision: whether to pay by debit or by credit card. Since the retail price at the checkout is generally the same either way, this decision looks pointless. It is not. Financial incentives, merchants’ interests, and available credit facilities do play an important role for consumer payment choice. Moreover, behind the scenes, billions of dollars are at stake.

In this paper, we study the pricing of payment cards. Since payment card networks are two-sided markets, we consider the optimal fees charged by the network to the consumer and to the merchant. Unlike most payment models, where consumer credit is not considered, our model is among the first to analyze payment network fees and competition by explicitly incorporating the different ways consumer credit is offered in debit and credit card networks. Specifically, we consider overdraft facilities and credit lines.

As is standard in the literature, we assume that part of the value of payment cards to consumers comes from the reduced need to hold cash. Specifically, both payment cards provide additional security over cash. They also enable merchants to avoid the cost of cash handling. We also consider the way in which cards help liquidity constrained consumers. If a payment card offers payment possibilities in extra states of the world, this is valuable to both consumers and merchants. The card fees, set by the payment network, then depend in part on the degree to which the network can extract surplus from consumers and merchants. The optimal combination of merchant fee and consumer fee is however determined by the need for the payment network to ‘internalize’ the network externalities from either side of the market (see Rochet and Tirole 2006 for discussion of two sided markets).

Debit and credit cards offer distinctly different credit possibilities for the consumer. A debit card enables its holders to make purchases and have these transactions directly and immediately charged to their current accounts. The consumer can access credit via her debit card as long as she has an overdraft facility on her current account. Typically, such credit faces immediate interest charges. By contrast, a credit card enables cardholders to make purchases up to a prearranged credit limit. Such credit is interest free for a limited ‘grace’ period, beyond which the consumer faces interest charges on any remaining negative balances. In short, debit and credit card networks operate different business models for supplying credit.
We show how the different models of credit affect equilibrium merchant and consumer fees, as well as the nature of competition between the two payment networks. We first consider cases in which only one card exists: in other words, we examine how debit and credit card networks set merchant and consumer fees monopolistically. We then consider a world in which the two networks compete for custom. First, when the credit card network behaves monopolistically, we find that default risk and funding cost are partly passed onto the merchant through the credit card merchant fee. Yet, debit card merchant fees do not share this feature in a debit-only world, as long as the only alternative to debit is cash. Debit merchant fees only depend on default risk and funding costs when we introduce competition with credit cards. In that case, however, we find that debit merchant acceptance actually increases with the probability of default, despite an increase in the merchant fee. This is because the credit card merchant fee responds more to the higher default risk, causing some merchants to switch from credit cards to debit cards.

Second, we find that monopolistic credit card fees also depend on overdraft interest rates, even though these are associated with the current account and therefore completely separate from the credit card network. The overdraft is an outside option for the consumer in one state of the world, so at the margin the credit line allows the consumer to ‘save’ on the costs of servicing the overdraft.

Third, when we turn to consider competition between a debit and a credit card model, we find that a degree of complementarity exists between debit and credit cards. If the consumer uses a credit card, she leaves a positive balance in her current account whilst using the ‘grace’ period on the credit card to make purchases. This means the bank that issues the debit card can earn interest on this positive balance. As a result, following an increase in the cost of funds, the debit card bank may increase merchant fees at the margin in order to decrease debit card acceptance in favor of credit cards; this means debit card merchant fees may approach monopolistic levels. When we consider welfare maximizing fees, we find that the competitive debit merchant fee is indeed high relative to the welfare-maximizing case. Whilst that has also been observed in other papers (Bolt and Schmiedel 2011), we have identified an extra wedge between the competitive and the socially optimal debit fee: this comes from the complementarity between the two business models.

The pricing of credit and debit cards is of particular relevance to policymakers and regul-
lators. Policymakers have focused on the level of interchange fees, paid by the acquiring bank to the issuing bank (see below for more details). Following discussions with the European Commission, Mastercard has recently agreed to reduce interchange fees on cross-border European card transactions. Similarly Visa Europe has agreed to reduce such fees for cross-border debit card payments. US policymakers have also proposed setting a cap on interchange fees for debit and prepaid cards.

In addition, our results may be relevant for the realization of the Single Euro Payments Area (SEPA). The broad aim of the SEPA project is to enable closer European financial integration, through enhancing harmonization in the means of payment, treating all payments in the euro area as domestic payments. With respect to payment cards, the SEPA framework has focused on the need to increase competition and efficiency between card networks. Our paper attempts to shed new light on what competition between debit and credit cards and access to funds imply for optimal payment pricing of payment cards.

Our paper can be seen in the context of existing literature both on payment cards and in the field of consumer finance. Various papers including Baxter (1983), Rochet and Tirole (2002, 2003), and Wright (2003, 2004) have analyzed payment cards and two sided markets, focusing on the optimal combination of the consumer and merchant fees (see Verdier 2010 for an overview). They highlight the fact that neither side of the market takes into account the positive externality to the other side from one’s own participation in the network. As a result, the network must find the optimal combination of fees to effectively ensure these externalities are internalized. For instance, there is no point setting an extremely high consumer fee and a low merchant fee, if this means no consumer will participate: in that case, the card is relatively worthless to merchants, irrespective of the fact they are paying a low fee. In a market where the network consists of two banks (the acquiring bank on the merchant’s side and the issuing bank on the consumer’s side), an interchange fee may be necessary to effectively enable one side to subsidize the other (see Rochet and Tirole 2006 for more details).

So far, no paper has explicitly studied the impact of overdraft facilities and access to credit on the pricing decisions for card payment networks. Chakravorti and To (2007) introduce a credit line into their model of credit cards, but do not consider periods beyond the ‘grace’ period and therefore do not consider the relevant interest charge for credit. Moreover, their paper lacks an analysis of competition between credit and debit cards. Our paper builds
on the modeling framework of Bolt and Chakravorti (2008) and, in particular, Bolt and Schmiedel (2011), but extends that work to consider consumer credit. In so doing, we attempt to bridge the gap between the payment card literature and that of consumer finance.

We structure the paper as follows. In section 2, we present the model, while in sections 3 and 4 we consider the optimal prices in a world just with a debit card, and subsequently a world just with a credit card. We refer to these as the monopolistic pricing models. In section 5, we consider optimal prices in a world where the credit and debit cards compete with each other. In Section 6 we discuss welfare implications, while in Section 7 we discuss possible extensions and robustness. We conclude in Section 8.

2 Model

The basic model closely follows that of Bolt and Chakravorti (2008) and Bolt and Schmiedel (2011). In our model, there are three types of agents: consumers, merchants and payment network providers. All agents are risk neutral. Banks are considered to play the role of payment network providers. As in Bolt and Chakravorti (2008) and Bolt and Schmiedel (2011), we use a three party network. In other words, we combine the issuing bank and the acquiring bank into a single network provider. This enables us to focus simply on the merchant and consumer fees, without also having to solve for an interchange fee. As an alternative to the three party network with merchants, consumers and the network provider, one can also consider a four party network. This would explicitly model the acquiring bank and the issuing bank, instead of combining the two into a single network provider. However, as Bolt (2006) discusses, the two models are equivalent if either the issuing bank or the acquiring bank are perfectly competitive. If this is the case, there is a close linear relationship between the optimal interchange fee, paid between the acquiring and issuing bank, and the consumer and merchant fees.

2.1 Consumers

Consumers are homogeneous and maximize linear utility. They obtain utility $v$ from consuming the single, homogenous good, which they would purchase from a merchant. Each consumer is matched with a single merchant near the beginning of period 1, after deciding whether to subscribe to a particular payment card. If she is able to make a purchase, she pays price $p$ and therefore obtains (net) utility from consumption equal to $v_0 = v - p$ where

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\( v_0 \geq 0 \). For simplicity we normalize \( p = 1 \). However, there is no guarantee that she will make a successful purchase at a given merchant.

The first potential friction in making a payment comes from liquidity. The consumer may or may not receive positive initial income. As a result, she may have insufficient funds available (through a combination of initial income and credit) to make the purchase. We discuss below the specific nature of her income shocks and available credit.

Second, the merchant may or may not accept the card to which the consumer has subscribed. If not, the consumer must rely on cash to make the payment, which itself faces a cost.\(^2\) We model the cost of cash in a reduced form way, by assuming she will be mugged with positive probability, \( 1 - \rho \), on her way to make the purchase; in that case, she will be unable to purchase and consume the good. This follows other papers which associate the cost of cash with theft, such as He, Huang and Wright (2005).

During period 2, night time, the consumer receives a second income shock. Income may arrive early at the beginning of period 2, or late at the end of period 2, or not at all. The only value of receiving period 2 income comes from the ability to pay back debt obligations from period 1. Before describing the credit options offered by each network, we consider the specific income shocks in more detail.

### 2.1.1 Income shocks and default

Period-1 (positive) income is given by \( x_1 \) and period-2 income by \( x_2 \). We assume that period-1 income is insufficient to cover the purchase, whilst period 2 income is greater than the price of the good. In other words,

\[
x_1 < 1 < x_2.
\]

At the beginning of period 1, the probability the consumer receives income \( x_1 \) is given by \( \delta \); otherwise she receives zero. In period 2, the probability she receives income early is given by \( \gamma_E \) and the probability she receives income late is given by \( \gamma_L \). With the remaining probability she receives no income in period 2: \( 1 - \gamma_E - \gamma_L \). Note that the probability she receives income in period 2 is completely independent of the period-1 income shock.

Given the independence between period-1 and period-2 income shocks, there are six pos-
Table 1: Income streams: timing and shocks

<table>
<thead>
<tr>
<th>Probabilities</th>
<th>Income</th>
<th>Total Income Received</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1</td>
<td>Period 2</td>
<td>$1$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>$\gamma_E$</td>
<td>$x_1$</td>
</tr>
<tr>
<td>$\gamma_L$</td>
<td>$x_1$</td>
<td>$x_2$</td>
</tr>
<tr>
<td>$1 - \gamma_E - \gamma_L$</td>
<td>$x_1$</td>
<td>$x_2$</td>
</tr>
<tr>
<td>$1 - \delta$</td>
<td>$\gamma_E$</td>
<td>0</td>
</tr>
<tr>
<td>$\gamma_L$</td>
<td>0</td>
<td>$x_2$</td>
</tr>
<tr>
<td>$1 - \gamma_E - \gamma_L$</td>
<td>0</td>
<td>$x_2$</td>
</tr>
</tbody>
</table>

Note: Period-1 income $x_1$ arrives with probability $\delta$, otherwise 0 with probability $1 - \delta$. Period-2 income $x_2$ arrives early (in $2_{\text{early}}$) with probability $\gamma_E$ and arrives late (in $2_{\text{late}}$) with probability $\gamma_L$. Default occurs with probability $1 - \gamma_E - \gamma_L$ when period-2 income $x_2$ does not arrive.

Possible outcomes in the game as a whole. We can summarize this by considering the total amount of income received by the end of period 2, gross of any outgoing payments. Income shocks and timing are captured by Table 1; the upper panel depicts the case of a positive period-1 income shock, the lower panel a zero period-1 income shock.

Regardless of period-1 income, the consumer must use credit for the purchase (since $x_1 < 1$). From Table 1, the consumer will default in two states, conditional on having purchased the good. Therefore, the ex ante probability of default, conditional on the consumer making a purchase, is given by $1 - \gamma_E - \gamma_L$.

Given the probabilities and income shocks described above, ex ante expected income is equal to:

$$E(I) = \delta x_1 + (\gamma_E + \gamma_L) x_2.$$ 

Since we assume consumers are ex ante solvent, conditional on making the purchase, this implies that $E(I) > 1$, or rearranging

$$1 - \delta x_1 < (\gamma_E + \gamma_L) x_2.$$ 

There are two distinct differences between credit and debit cards in our model. Both relate to the nature of credit offered in association with the two systems. Firstly, we assume that the consumer always has access to an overdraft associated with her current account, while the credit card offers a credit line to the consumer. If she holds a debit card, the
consumer can use her overdraft facility to make payments via this card. Whilst this debt will immediately accrue interest charges, the credit line of the credit card offers the consumer a free ‘grace’ period. In effect, the credit line associated with the credit card will not accrue interest charges until after the first period.

Secondly, we assume the credit line is larger than the overdraft facility, thus enabling the consumer to make payments in more states of the world. Specifically, we assume the overdraft limit is sufficient to cover the purchase if the consumer received period-1 income, but insufficient if there was no income received; as a result she will be unable to purchase the good. By contrast, the credit limit on the credit card is sufficient to cover the purchase, even if the consumer received no period-1 income. This captures the fact that both credit and debit cards are used alongside credit facilities, but the credit card enables payment in extra states, relative to the debit card.

2.2 Merchants

We assume merchants are heterogeneous. Specifically, we assume merchant $i$ receives profit $\pi(i)$ from a sale, where $\pi(i)$ is uniformly distributed between 0 and $p$ (where $p = 1$). As discussed in Bolt and Schmiedel (2011), this captures the idea that merchants vary in their profit margins due to differing production costs. If the merchant accepts a payment card and the consumer uses this to make a purchase, the merchant must pay a per-transaction merchant fee ($f$). Alternatively, if the consumer uses cash, the merchant will face a per transaction cost of cash handling ($h$). As is standard in the literature, we assume the no-surcharge rule holds: in other words, merchants are prohibited from setting a different price to cash-paying consumers as opposed to card-paying consumers.

2.3 Payment networks

We consider two different payment networks: a credit card network and a debit card network. Each network chooses per-transaction merchant fees $f^j \geq 0$, $j = C, D$ and consumer fixed fees $F^j$, $j = C, D$ to maximize profits. This mirrors what occurs in many countries; consumers generally pay fixed fees while merchants pay per-transaction fees.

We assume the network is a monopolist on the consumer side: in other words, the network will extract the maximum fixed fee that the consumer is willing to pay, for a given proportion
of merchant acceptance. Following that, the network will choose the merchant fee to maximize network profits.

The profits are a function of the fee revenues as well as the payment network costs. First, they bear a per-transaction processing cost \( c^C \geq c^D \geq 0 \). Second, they bear the expected costs of offering credit and bearing consumer default. It is to this issue we now turn in considering the pricing of credit.

2.3.1 Interest rates

As described above, regardless of period-1 income, the consumer must use credit to purchase the good. We assume the overdraft limit on the debit card (and by extension if the consumer pays by cash) is sufficient to cover the purchase if the consumer received period-1 income, but insufficient if there was no income received. If the consumer uses her overdraft, then she will be in debt by an amount \( m_d \equiv 1 - x_1 \). This debt will accrue interest at rate \( r_d \) from period 1, until she repays using period-2 income.

By contrast, we assume the credit limit on the credit card is sufficient to cover the purchase, even if the consumer received no period-1 income. If the consumer uses her credit card, she will not face any interest accrual for period 1. This is known as the ‘grace’ period. However, if she is unable to repay the debt using period-2 early income, she will face interest charges over period 2 at rate \( r_c \). As a result, she will not use any initial income to partially repay the debt until the beginning of period 2, since she doesn’t face interest charges until that period.\(^4\) Her expected debt in period 1 will be equal to \( p = 1 \), regardless of early income. Her expected debt in period 2, on which she accrues interest charges, amounts to \( m_c \equiv 1 - \delta x_1 \) (note that \( m_c > m_d \)).

We assume that debit and credit card loans are funded at the (exogenous) market interest

\(^3\)The Reserve Bank of Australia (2008) reported that the average cost of the payment functionality of the credit card was AUS$ 0.35 higher than a debit card based on a AUS$ 50 transaction size. Credit cards are more costly in terms of customer education, informational requirements regarding credit repayment, and advertisement costs, see Turján et al. (2011).

\(^4\)For simplicity, we assume she earns no interest in her current account: in which case, she is indifferent between keeping her initial income in her account or using it to pay off some of her credit card debt in period 1. However, we could reasonably suppose she earns \( \varepsilon \) interest in period 1; this would make her strictly prefer to maintain the balance without changing the results.
rate, $r$. Hence, for the bank providing the overdraft, the expected cost (including default) is\(^5\)

$$EC_o = m_d[r + (1 - \gamma_E)r + (1 - \gamma_E - \gamma_L)],$$

while the expected revenues, for given $r_d$, are

$$ER_O = r_d(\gamma_E + 2\gamma_L)m_d.$$

For the credit card provider, the expected cost (including default) is given by

$$EC_C = r + m_c[(1 - \gamma_E)r + (1 - \gamma_E - \gamma_L)],$$

while the expected revenues, for given $r_c$, are

$$ER_O = r_c\gamma_Lm_c.$$

Notice that there is no term for $\gamma_E$ in the credit card provider’s expected revenues because the first period credit will be free for the consumer.\(^6\)

It turns out our results for merchant fees and acceptance rates do not depend on the specific assumption we make regarding the way interest rates are set on credit lines ($r_c$). Moreover, most of our key results do not depend on the specific value of $r_d$, the overdraft interest rate. Intuitively, this is because the network is a monopolist on the consumer side, and effectively engaging in multi-part pricing, comprising of the fixed fee and the interest rate. If the network sets the consumer interest rate equal to the zero profit level (i.e. $ER_j(r_j) = EC_j$ for $j = C, D$), then the network will just extract the full consumer surplus through the fixed fee $F^j$. If the network alternatively sets a lower interest rate and makes a loss on the credit portion, it will extract the difference through the higher fixed fee which the consumer is willing to pay. In both cases, the merchant fee will be the same, since this is the result

\(^5\)Implicitly, we assume that the shocks are sufficiently correlated across consumers such that banks cannot rely on shocks canceling out by lending to a large enough group. Notice that we are also implicitly assuming the merchant will receive payment immediately after sale. So any credit extended to the consumer by the bank is extended in period 1, when the purchase takes place.

\(^6\)As discussed later, we assume that the overdraft cannot be used to pay off some of the credit card debt. Notice that if it could, then the bank supplying the overdraft would take on some of the consumer default risk otherwise borne by the credit card provider.
of network profit maximization, taking into account that the maximum consumer fee will
depend on the merchant fee: \( F^j(f^j) \) for \( j = C, D \).

We therefore proceed in the following way. We assume that the overdraft credit is set as a
zero Net Present Value (NPV) loan. One way to motivate this is to think of the credit part as
a competitive ‘after market’, implying consumers could substitute other loans such as store
credit for overdrafts. This assumption makes no difference to the debit merchant fee, for the
reasons given above. However, as we will see, the overdraft interest rate will affect the credit
card merchant fee. For simulation purposes, it is therefore helpful to have the assumption of
zero NPV in order to relate \( r_d \) to underlying parameters.\(^7\)

The per period simple interest rate \( r_d \) must therefore solve:

\[
r_d(\gamma E + 2\gamma L) = r + (1 - \gamma E)r + (1 - \gamma E - \gamma L).
\]

This gives \( r_d \) as follows:

\[
r_d = r_d(r, \gamma E; \gamma L) = \frac{(2 - \gamma E)r + (1 - \gamma E - \gamma L)}{\gamma E + 2\gamma L}.
\]

We can easily show that equilibrium \( r_d \) decreases with \( \gamma E \) and \( \gamma L \) and increases with \( r \).

In order to pin down \( r_c \), we assume the credit line of the credit card competes with the
same outside credit options as the overdraft, in the states with positive period 1 income when
the consumer could use either cash or credit card. We also assume that the network cannot
make \( r_c \) contingent on the state: so \( r_c \) is entirely pinned down by the outside credit options.\(^8\)

This is entirely without loss of generality, given the network extracts the full consumer surplus.
Moreover, credit line interest rates will have no affect on the debit network’s optimization,
so the specific way \( r_c \) is set makes no difference to our results.

From this assumption, the expected costs to the consumer must be the same for the credit

\(^7\)While we do not think the zero NPV assumption is necessarily an accurate description of reality, we feel
it is a helpful baseline to consider. We model interest revenue as simple interest so that the lender receives
revenue of \( 2r \) if the capital is left untouched over two periods. This keeps the notation simpler without
changing the qualitative results.

\(^8\)Since we assume the credit card adds value by enabling the consumer to make a purchase in one extra
state of the world, it must be that none of the outside credit alternatives are sufficiently large to help the
consumer when there is no period 1 income. Nevertheless, if the credit card network cannot charge different
interest rates for different marginal units of credit, and if \( r_c \) cannot be contingent on the state, the outside
credit options will still pin down \( r_c \).
line as the overdraft, conditional on having high initial income

\[ r_c \gamma_L = r_d (\gamma_E + 2\gamma_L). \]

This gives \( r_c \) as follows:

\[
r_c = r_c(r, \gamma_E, \gamma_L) = \frac{(2 - \gamma_E)r + (1 - \gamma_E - \gamma_L)}{\gamma_L}. \tag{2}
\]

If \( r_c \) is set as in this condition, the credit card network makes a loss on the credit in expectation. Effectively, it is subsidizing the consumer in period 1 by allowing the consumer to have a debt equal to 1 but to only pay interest in period 2 on the remaining portion of her debt \( 1 - x_1 \). To see that the credit card company makes a loss, note that:

\[
ER_C = r_c \gamma_L m_c = m_c[r + (1 - \gamma_E)r + (1 - \gamma_E - \gamma_L)] < EC_C.
\]

The loss will be captured by a cost term in the credit card provider’s profit function. This loss is equal to

\[
EC_C - ER_C = r(1 - m_c) = r \delta x_1.
\]

This directly shows the subsidy provided by the network given the consumer never has to pay interest on the \( x_1 \) part of her debt in period 1. The key point, however, is that our results do not change, whether the consumer is made to directly pay for the subsidy in higher interest rates, or whether the credit card provider bears the cost in the profit function. This is because the credit card provider extracts the full surplus from the consumer.\(^9\)

\(^9\)Note that \( r_c \) explodes when \( \gamma_L \) approaches zero. With credit cards, consumers that receive late income carry all the funding and default cost. When \( \gamma_L \) is small, only a few consumers carry this burden and so pay very high interest rates. In the extreme, if \( \gamma_L = 0 \), no consumer pays interest on its credit card loan (they receive grace or they default) and therefore the loan cannot be made NPV-zero. In this case, to recover cost, the burden must be shifted to merchants and consumers through higher payment fees. To avoid this exploding characteristic, we will mainly focus on distributions \((\gamma_E, \gamma_L)\) that are not too ‘skewed’. 

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2.4 Timeline

Figure 1 shows the timeline. In period 1, consumers choose whether to accept a card; they then get matched to a merchant and, hopefully, make a purchase using either cash (if they have not been mugged) or a payment card, if the merchant accepts it. In period 2, consumers either receive income (early or late) and repay any debts, or they default at the end of the game. Please note that the purchase and all previous actions take place near the beginning of period 1, so the consumer will be in debt for most of that period (possibly accruing interest). For artistic purposes, however, we have placed this near the end of the period.

3 Debit Card Only Model

In this section, the consumer can either rely solely on cash to make a purchase, or decide to hold a debit card. The overdraft facility works similarly for cash as for the debit card. Therefore, the only benefit to holding a debit card comes from the risk of getting mugged, and losing cash.
3.1 Consumer’s problem

The probability of getting mugged is \((1 - \rho)\). We denote by \(\alpha^D\) the proportion of merchants who accept the debit card and \(F^D\) is the consumer’s debit card fee. Recall that if debit cards and cash are the only payment instruments available to the consumer, she can only make a purchase if she receives a positive level of initial income; this occurs with probability \(\delta\).

With probability \((1 - \delta)\), there is no purchase, no consumption, but also no expected default loss. Observe that \(m_d \equiv 1 - x_1\) denotes the amount of debt when using the overdraft facility associated with the checking account.

The consumer will want to hold a debit card as long as:

\[
\delta \rho v_0 - \delta(\gamma_E + 2\gamma_L) r_d m_d \leq \delta \left( \alpha^D + \rho (1 - \alpha^D) \right) v_0 - \delta(\gamma_E + 2\gamma_L) r_d m_d - F^D.
\]

The left-hand side is the payoff from just holding cash; in this case, the consumer can purchase the good only if she receives high initial income and is not mugged. If she makes a payment (which occurs with probability \(\delta \rho\)), then she will have to pay interest on her overdraft of size \(m_d\). Note, however, if she gets mugged she will still have gone into her overdraft, having withdrawn 1, and thus will have to pay interest. In other words, we assume the consumer has no insurance against cash theft.

She will only have to pay interest in one period, if she receives an early second income shock (which occurs with probability \(\gamma_E\)). However, if she has to wait for positive income until the end of period 2, she will have to pay twice the amount of interest; this occurs with probability \(\gamma_L\).

On the right-hand side is the payoff from holding a debit card. The consumer can make a purchase with a debit card if she receives high initial income, and the merchant accepts the card. She can also rely on cash for the payment if the merchant does not accept the card (with probability \((1 - \alpha^D)\)), providing she is not mugged. Either way, the consumer must pay the debit card fee \(F^D\).\(^{10}\)

We continue to make the same assumptions about mugging. If a consumer is aware that

\[^{10}\text{There is a chance that period-2 income does not arrive at all so that the consumer cannot pay for the fixed fee. Hence, the default probability ‘artificially’ increases the consumer willingness-to-pay for the card. However, the payment network would discount the high fixed fee with the same probability. Mathematically, this effect cancels out.}\]
she cannot pay by debit card, she will withdraw cash equal to 1. At this point, she faces a risk of being mugged, in which case she loses the money, and thus must pay interest on the overdraft until she can repay. The participation constraint can be simplified as follows:

\[ F^D \leq \delta \alpha^D (1 - \rho) v_0. \]

Note that the debit card allows the consumer to pay in one extra state, which occurs with probability \( \delta \alpha^D (1 - \rho) \). For this reason, the surplus from buying the good \( v_0 \), is multiplied by this term.

3.2 Merchant’s problem

The merchant \( i \) receives profit \( \pi(i) \) from a sale, where \( \pi(i) \) is uniformly distributed between 0 and 1. His cost of handling cash is \( h \) whilst \( f^D \) is the merchant fee for accepting the debit card. His expected payoff from accepting cash is:

\[ Z_{\text{cash}}(i) = \delta \rho [\pi(i) - h], \]

and his expected payoff from accepting the debit card is:

\[ Z_D(i) = \delta [\pi(i) - f^D]. \]

Merchants accept debit cards only when

\[ Z_{\text{cash}}(i) \leq Z_D(i). \]

Since there is a level of profits \( \bar{\pi} \) above which merchants will accept debit cards, we can write the proportion of accepting merchants as follows:

\[ \alpha^D(f^D) = \Pr[\pi(i) \geq \bar{\pi}] = 1 - \bar{\pi} = 1 - \frac{(f^D - \rho h)}{1 - \rho}. \]

\[ ^{11} \text{Given } \pi \text{ can be as low as zero, we assume the outside option for the merchant instead of enabling the purchase is given by } -h. \text{ This normalization ensures that all merchants would prefer to accept cash than reject the purchase.} \]
3.3 Maximum consumer fee for debit cards

Using the function $\alpha^D(f^D)$, we can derive the maximum possible consumer fee as a function of $f^D$. This is obtained by finding the fee such that the consumer is indifferent between holding a debit card or solely relying on cash. It is given by:

$$F^D_{\text{max}}(f^D) = \delta (1 - f^D - \rho(1 - h)) v_0.$$

3.4 Debit card network

We make the standard assumption that the same bank operating the debit card network is the one to provide the consumer with a current account and associated overdraft facility. The Debit Card Bank (DCB) faces processing cost $c^D$ per debit card transaction. The DCB is also able to earn interest on a positive balance in the customer account; we assume the bank takes this interest rate $r$ as given. In addition, the bank charges interest rate $r_d$ on any overdraft.

The DCB’s payoff from issuing a debit card is:

$$\pi^{DCB} = F^D + \delta \alpha^D(f^D - c^D) + r[(1 - \delta)E_{x_2}^2 + \delta E_{x_2}(x_2 - m_d)].$$

The DCB receives the consumer fee regardless debit card usage. With probability $\delta \alpha^D$ the consumer will make a payment using the debit card, so the bank will receive the net per transaction payoff, which is a function of the merchant fee $f^D$.

In addition to the per transaction fee, the bank earns interest on a positive balance in the customer account. A positive balance may exist for two reasons. If she did not make a purchase, but receives early income in period 2, the balance will be $x_2$ throughout that period. Alternatively, if she did make a purchase (or was mugged), and receives early income in period 2, the balance will be $x_2 - m_d = x_1 + x_2 - 1$ throughout that period. These two cases correspond to the third and fourth terms in the DCB’s profit function.

Since the credit offered via the overdraft is priced perfectly competitively, the loan is zero NPV for the DCB. As a result, neither the revenues nor the costs from this loan show up in

\footnote{As discussed earlier, the cost of funds do not enter the profit function as we have assumed the loan is zero NPV. Our results for the debit merchant fee are entirely independent of the way in which $r_d$ is determined.}
the profit function.

The DCB sets the optimal merchant fee by maximizing its payoff with respect to $f^D$, subject to

$$F^D = F^D_{\text{max}}(f^D) \text{ and } \alpha^D = \alpha^D(f^D).$$

The optimal merchant fee is therefore:

$$f^D = \frac{1}{2}[c^D + 1 - \rho(1 - h)] - \frac{1}{2}(1 - \rho)v_0. \quad (3)$$

The merchant fee increases with the transaction cost faced by the bank and decreases with consumer surplus, $v_0$. When merchant extraction of consumer surplus is low, the debit card bank will set low merchant fees; this way, the acceptance rate will rise, thus increasing the value of the card to the consumer. As a result, the network can charge higher consumer fees. Note that the term $v_0$ is multiplied by $(1 - \rho)$, the probability of the state in which debit cards enable payment when cash cannot.\footnote{Observe that the optimal debit merchant fee in (3) is the same as in the model of Bolt and Schmiedel (2011) without overdraft facility. This holds because the overdraft facility works similarly for cash as for debit cards and so presents no value added.}

4 Credit Card Only Model

We now consider the case in which only a credit card is available to the consumer. We do, however, assume the consumer still has access to a current account, with an associated overdraft facility. The size of the overdraft facility is, once again, only sufficient to cover the desired overdraft in the positive initial income case. However, the credit line associated with the credit card enables the consumer to take out a larger loan. In this way, the credit card can enable payment in the zero period-1 income case. Moreover, as with debit cards, credit cards insure against theft.

4.1 Consumer’s problem

We denote by $\alpha^C$ the proportion of merchants who accepts the credit card and $F^C$ is the consumer’s credit card fee. Given merchant acceptance, recall that credit cards can be used
in all states of the world regardless of period-1 income. Observe that \( m_c = 1 - \delta x_1 \) denotes the average amount of debt when using the credit line associated with the credit card.

The consumer will want to hold a credit card as long as:

\[
\delta p v_0 - \delta (\gamma_E + 2\gamma_L) r d m_d \leq (\alpha^C + \delta p(1 - \alpha^C)) v_o - \alpha^C \gamma_L r_c m_c \\
- \delta (1 - \alpha^C)(\gamma_E + 2\gamma_L) r d m_d - F^C.
\]

If the consumer makes a payment with a credit card, she will have to pay interest on this credit line only if she needs to extend the credit for an extra period, having received no income at the end of period 1. If the merchant does not accept the card, and the consumer has to pay cash, she will then face the interest charges from the overdraft in each period, as previously discussed. This condition can be rearranged as follows:

\[
F^C \leq \alpha^C (1 - \delta p)v_o - \alpha^C \gamma_L r_c m_c + \delta \alpha^C (\gamma_E + 2\gamma_L) r d m_d.
\]

The consumer will never leave the house with cash if the merchant accepts a credit card. Indeed, given the way we have priced credit in the two models, the expected costs of servicing a credit line are the same as the equivalent costs associated with an overdraft, if the consumer has high initial income (it is only in this state where the consumer could use cash). This is because both are priced competitively. In other words,

\[
(\gamma_E + 2\gamma_L) r_d = \gamma_L r_c
\]

given, see (1)-(2),

\[
r + (1 - \gamma_E)r + (1 - \gamma_E - \gamma_L) = (2 - \gamma_E)r + (1 - \gamma_E - \gamma_L).
\]

Hence, since the consumer is indifferent regarding use of funds, she will certainly use her credit card so as to avoid mugging on her way to the store.

Of course, it was simply by assumption that the expected costs of servicing the credit line in the high income state were set the same as that of the overdraft. Yet, the credit card
network would not want to set the interest rate any higher, such that the consumer chose not to use the credit card. The higher interest rate would not directly affect the credit card network’s profits because higher interest revenues in the profit function would be offset by a lower maximum fixed fee (given multipart pricing). However, the network’s profits would indirectly reduce as the consumer uses her card for fewer transactions. As a result, the maximum fixed fee would be reduced. This would have a further effect on reducing network profits since there is no offsetting revenue term (given the network is not the same as the current account providing bank).

We further assume that if the credit line is taken down, the overdraft on the current account cannot be used to ‘pay off’ the credit line at the end of period 1. For instance, we assume the bank does not allow the overdraft to be used to pay off alternative debt; or at the very least, there exists a significant fixed cost to substituting overdraft debt for credit card debt.\textsuperscript{14}

4.2 Merchant’s problem

The merchant $i$ receives profit $\pi(i)$ from a sale, where $\pi(i)$ is uniformly distributed between 0 and 1. His cost of handling cash is $h$ whilst $f^C$ is the merchant fee for accepting the debit card. His expected payoff from accepting cash is:

\begin{equation}
Z_{\text{cash}}(i) = \delta \rho [\pi(i) - h],
\end{equation}

and his expected payoff from accepting the credit card is:

\begin{equation}
Z_c(i) = [\pi(i) - f^C].
\end{equation}

Merchants accept debit cards only when

\begin{equation}
Z_{\text{cash}}(i) \leq Z_c(i).
\end{equation}

\textsuperscript{14}In some European countries, the overdraft is ‘automatically’ used to pay off outstanding credit card obligations at the end of the month. Hence, these consumers do not face a credit card interest rate but rather an interest rate on overdraft. However, consumers in the U.S. do not typically use overdrafts to pay off credit card debt, even if there are significantly lower interest rates on the former. This is sometimes called the ‘credit card puzzle’, and may be attributed to a specific behavioral trait or economic friction, but that discussion is beyond the scope of this paper (see e.g., Gross and Souleles 2002; Telyukova and Wright 2008).
Since there is a level of profits $\bar{\pi}$ above which merchants will accept credit cards, we can write the proportion of accepting merchants as follows:

$$\alpha^C(f^C) = \Pr[\pi(i) \geq \bar{\pi}] = 1 - \frac{f^C - \delta \rho h}{1 - \delta \rho}.$$ 

This is different to the proportion associated with debit cards; the $\rho$ here is multiplied by $\delta$. This reflects the fact that the credit card allows for payment in both the high and low initial income states, unlike cash.

### 4.3 Maximum consumer fee for credit cards

Using the consumer’s participation constraint, as well as $\alpha^C$, we obtain the maximum consumer fee:

$$F^C_{\max}(f^C) = [1 - f^C - \delta \rho(1 - h)]v_0 - \frac{[1 - f^C - \delta \rho(1 - h)]}{(1 - \rho \delta)}[\gamma_Lr_c m_c - \delta (\gamma_E + 2 \gamma_L) r_d m_d].$$

Unlike the debit fee, the probability of high initial income $\delta$ does not pre-multiply both terms; unlike the debit card, the credit card does not restrict the consumer to purchase only in the high income state.$^{15}$

The second term above captures the expected costs of credit; however, it is a function of both the credit line and the overdraft on the current account. In states where the credit card enables payment that would be impossible with cash, the relevant term for the expected cost of credit is simply $\gamma_L r_c m_c$. However, in the case of high period-1 income (which occurs with probability $\delta$), the consumer could still use cash if she wished.$^{16}$ In this case, the relevant term is the difference between the cost of the credit line and the cost of the overdraft. It is this difference that captures the benefits (or otherwise) offered by the credit card.

$^{15}$Notice that the maximum consumer fee becomes negative if $v_0 = 0$. Whilst the debit consumer fee is zero in this case, the credit consumer fee is negative since consumers would be paying higher expected interest costs under the credit card, than they would under the overdraft.

$^{16}$Note of course that, if the consumer attempts to pay by cash, she will be mugged with probability $(1 - \rho)$. In this case, she still enters her overdraft, even though she has not successfully made a purchase.
Given our earlier assumptions regarding \( r_d \) and \( r_c \) this difference is positive:

\[
\gamma_L \rho mc - \delta (\gamma_E + 2\gamma_L)r_dm_d = [(2 - \gamma_E)r + (1 - \gamma_E - \gamma_L)](1 - \delta) > 0.
\]

It might seem initially counterintuitive that this difference is non-zero, given they are priced competitively relative to the high income state. However, the loan is priced, conditional on the consumer requiring the loan in each case. Yet, when the consumer, ex ante, considers the value of a credit card she takes into account expected costs of the overdraft and the credit line; these are unconditional expected costs, before she knows the value of initial income. Since the credit card enables payment in one extra state of the world, the unconditional expected costs of credit via the credit card are higher than via the overdraft facility. Notice that the difference is decreasing in \( \delta \). As the probability of period-1 income increases, so does the probability of being able to pay using the cash and the overdraft facility. This increases the expected cost of the overdraft relative to that of the credit line on the credit card.

Indeed, even without assuming the loans are priced as zero NPV, it is difficult to imagine that the expected costs of the credit line to the consumer would be lower than the overdraft. After all, the positive difference above does not actually capture full costs of the credit line to the lender, relative to the overdraft. For this, we would need to add the term \( \delta r x_1 \) to the difference above, capturing the free credit equal to \( x_1 \) in the high initial state as part of the credit line.

### 4.4 Credit card network’s problem

The Credit Card Network’s (CCN) payoff from issuing a credit card is:

\[
\pi^C = F^C + \alpha^C(f^C - c^C) - \alpha^C \delta r x_1.
\]

Note that the consumer still has a current account, and overdraft facility, but neither of these show up in the credit card network’s profit function. The final term reflects the expected loss on the credit line, as discussed in the section on interest rates. However, as discussed earlier, the profits would be equivalent if the network passed these costs onto the consumer, since the network extracts the full consumer surplus.

The network sets the optimal merchant fee by maximizing its payoff with respect to \( f^C \),
subject to

\[ F_C^{\text{max}} = F_C(f^C) \text{ and } \alpha_C = \alpha_C(f^C). \]

The optimal merchant fee for credit cards, as a function of \( r_c \), is therefore:

\[
f_C^* = \frac{1}{2} [c^C + 1 - \delta \rho(1-h)] + \frac{1}{2} \delta r x_1 + \frac{1}{2} \left[ (1 - \gamma_L)r + (1 - \gamma_E - \gamma_L) \right] m_c - \frac{1}{2} \delta (\gamma_E + 2 \gamma_L) r_d m_d - \frac{1}{2} (1 - \delta \rho) v_0, \quad (4a)
\]

where we substitute out \( r_c \). The fee is decreasing in the consumer’s expected costs of servicing the overdraft. The intuition is straightforward. The overdraft, even in the absence of a debit card, offers an outside option to consumers in one state. By choosing to pay by credit card, not cash, the consumer avoids the expected costs of servicing an overdraft; if these are high, then the benefit of holding a credit card is high. In this case, the network can extract a large fee from the consumer, and is therefore able to reduce the merchant fee. In turn, high funding costs \( (r) \) for the credit card network will dampen the consumer maximum fixed fee resulting in a higher merchant fee to restore the balance. This has interesting implications. Effectively, the credit card competes with the overdraft facility in the state where cash could be used. It shows that the interest rate charged can impact the acceptance ratio of credit cards. An increase in the costs of an overdraft can lead to higher acceptance of credit cards.

In the comparative statics that follow we continue with our earlier assumption that the overdraft is a zero NPV loan, in order to pin down \( r_d \). Recall, however, that our specific assumption about the credit line interest rate \( r_c \) will not affect the credit merchant fee.

The ‘total’ interest rate effect on merchant fees is then derived when we substitute \( r_d = r_d(r, \gamma_E, \gamma_L), r_c = r_c(r, \gamma_E, \gamma_L), m_d = 1 - x_1, \) and \( m_c = 1 - \delta x_1 \) in the optimal merchant fee \( f_C^* \). This yields:

\[
f_C^* = \frac{1}{2} [c^C + 1 - \delta \rho(1-h)] + \frac{1}{2} \delta r x_1 + \frac{1}{2} \left[ (2 - \gamma_E)r + (1 - \gamma_E - \gamma_L) \right] (1 - \delta) - \frac{1}{2} (1 - \delta \rho) v_0. \quad (4b)
\]

As with the consumer fee, the merchant fee is a function of the difference between the unconditional expected costs of servicing the credit line and the overdraft. It is also a function of
initial income, since the consumer effectively gets free credit equal to her high initial income in the ‘grace’ period, \(rx_1\). This equation also shows that higher funding rates \(r\) lead to higher merchant fees \(f^*_C\). Higher defaults \((1 - \gamma_E - \gamma_L)\) increase merchant fees as well. These effects make clear how merchants share the cost burden of credit card loans with consumers.

4.5 Comparison and comparative statics

In our model, the optimal debit card merchant fee \(f^*_D\) is not influenced by the funding cost or default risk. This derives from the fact that debit cards have no value added over cash regarding the use of the overdraft facility on the checking account. Debit cards only hedge against theft and that is why the probability of theft \(\rho\) plays an important role for the optimal merchant fee, as well as processing cost \(c^D\).

In contrast, funding cost and default risk do affect the merchant fee on credit cards. In effect, merchants pay their ‘fair’ share with respect to credit card debt. If the network can extract lower surplus from consumers through a lower consumer fee, they will require merchants to pay a higher fee to compensate. An increase in \(r\) leads to an overall increase of \(f^*_C\). In principle three effects are at play. One is because an increase in \(r\) leads to an increase in \(r_d\), and as discussed above, this increases the saving the consumer can make from avoiding the costs of servicing the overdraft. This has a negative effect on the merchant fee as the credit card network tries to increase acceptance \((\rho^C)\) to benefit from the higher extraction of surplus via the fixed consumer fee. The second is an opposing effect due to a lower consumer willingness-to-pay when credit card interest rates rise, making the credit card less acceptable to consumers and dampening the amount that the network can extract from consumers. The final effect is again a positive effect, coming from the subsidy provided to the consumer of free credit equal to \(x_1\). These latter two effects dominate and therefore the CCN must increase the merchant fee when the funding cost rises:

\[
\frac{\partial f^*_D}{\partial r} = 0 \quad \text{and} \quad \frac{\partial f^*_C}{\partial r} = \frac{1}{2}(2 - \gamma_E)(1 - \delta) + \frac{1}{2}\delta x_1 > 0.
\]

A higher probability of early period-2 income \(\gamma_E\) increases the value of a credit card to consumers because it makes enjoying the grace period more likely. This allows for a lower merchant fee, i.e.:
### Table 2: Comparison between debit and credit cards

<table>
<thead>
<tr>
<th></th>
<th>funding cost $r$</th>
<th>default $D$</th>
<th>early income $\gamma_E$</th>
<th>initial income $\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1%</td>
<td>3%</td>
<td>5%</td>
<td>10%</td>
</tr>
<tr>
<td></td>
<td>50%</td>
<td>55%</td>
<td>95%</td>
<td>99%</td>
</tr>
<tr>
<td>$f_D^*$</td>
<td>0.00675</td>
<td>0.00675</td>
<td>0.00675</td>
<td>0.00675</td>
</tr>
<tr>
<td>$\alpha_D^*$</td>
<td>0.42450</td>
<td>0.42450</td>
<td>0.42450</td>
<td>0.42450</td>
</tr>
<tr>
<td>$f_C^*$</td>
<td>0.01600</td>
<td>0.01863</td>
<td>0.01575</td>
<td>0.01600</td>
</tr>
<tr>
<td>$\alpha_C^*$</td>
<td>0.24510</td>
<td>0.11319</td>
<td>0.25767</td>
<td>0.24510</td>
</tr>
<tr>
<td>$r_d$</td>
<td>0.08846</td>
<td>0.11154</td>
<td>0.04643</td>
<td>0.08846</td>
</tr>
<tr>
<td>$r_c$</td>
<td>0.28750</td>
<td>0.36250</td>
<td>0.14444</td>
<td>0.28750</td>
</tr>
<tr>
<td>$c^D$</td>
<td>0.0025</td>
<td>0.0075</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c^C$</td>
<td>0.0075</td>
<td>0.0075</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h$</td>
<td>0.001</td>
<td>0.001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v_0$</td>
<td>0.99</td>
<td>0.99</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_1$</td>
<td>0.25</td>
<td>0.25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: We set: $c^D = 0.0025$, $c^C = 0.0075$, $h = 0.001$, $v_0 = 0$, and $\rho = 0.99$, $x_1 = 0.25$. Baseline parameters: $r = 0.01$, $\gamma_E = 0.50$, $\gamma_L = 0.40$ (but altered to $\gamma_L = 0.45$ for $D = 5\%$) and $\delta = 0.99$.

\[
\frac{\partial f_D^*}{\partial \gamma_E} = 0 \quad \text{and} \quad \frac{\partial f_C^*}{\partial \gamma_E} = -\frac{1}{2}(1 - \delta)(1 + r) < 0.
\]

Defining default $D = 1 - \gamma_E - \gamma_L$, and keeping $\gamma_E$ constant, it is easy to show that

\[
\frac{\partial f_D^*}{\partial D} = 0 \quad \text{and} \quad \frac{\partial f_C^*}{\partial D} = -\frac{\partial f_C^*}{\partial \gamma_L} = \frac{1}{2}(1 - \delta) > 0.
\]

That is, higher defaults lead to higher merchant fees. Once again, with higher default rates, the required interest rate on the credit line is higher; this reduces the maximum fee the network can charge consumers and so requires a higher fee from merchants.

For similar reasons, when the probability of receiving initial income rises then merchant fees go down

\[
\frac{\partial f_D^*}{\partial \delta} = 0 \quad \text{and} \quad \frac{\partial f_C^*}{\partial \delta} = -\frac{1}{2}((2 - \gamma_E - x_1)r + (1 - \gamma_E - \gamma_L) + \rho(1 - h - v_0)) < 0,
\]

for sufficiently small $v_0$ and $h$. As $\delta$ increases, the unconditional expected cost of the credit line decreases relative to the overdraft (since there is an increase in the probability of being able to use the overdraft). Effectively, then, the credit card becomes relatively more valuable to consumers. Since the network can extract a high fee from consumers, it will set a low merchant fee in order to maximize the network size. If the merchant fee is low, more merchants will accept the card and thus the card will become attractive to more consumers.

Table 2 illustrates the results. As we can see from the table, the debit merchant fee and

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17 The calibration of the parameters broadly corresponds to average interchange fees levels currently observed.
merchant acceptance rate do not depend on funding costs \( r \), on the probability of default \( D \) or on the probability of different income shocks, \( \gamma_E \) or \( \delta \). However, the credit merchant fee increases with a higher cost of funding and a higher probability of default. We also see how the debit and the credit interest rates increase with the probability of default, and the funding cost, given they are priced in a competitive aftermarket.

Notice how the credit merchant fee decreases with a higher probability of period-1 income \( \delta \): the unconditional expected cost of servicing an overdraft increases, as the probability of using it increases. This decreases the relative cost to the consumer of using the credit card (as the overdraft, the outside option, increases in expected cost). As a result, the network can charge higher fees to the consumer and lower fees to the merchant. However, in these numerical examples, there is actually lower merchant acceptance of the credit card following an increase in \( \delta \): this is despite a lower merchant fee. There are actually two opposing effects at work in determining the effect of \( \delta \) on \( \alpha^C \); although a lower merchant fee increases the attractiveness of the card, the fact that \( \delta \) increases the merchant’s expected payoff from accepting only cash means that, at the margin, a given merchant is less willing to accept the card. For certain parameter values, such as those above, the latter effect dominates.\(^{18}\)

5 Competition between Debit and Credit Cards

In this section, we examine competition between debit and credit cards.\(^{19}\) We analyze the case in which the consumer multihomes and the merchant singlehomes (see discussion in section 7 on this issue). We also follow the preceding model and assume that the overdraft cannot be used to pay off the credit line in period 2. In other words, the consumer is committed to using the credit facility associated with the card she used for payment.

5.1 Consumers’ participation

In what follows, \( \alpha^i \) is the proportion of merchants who accept card \( i \), where \( i = C, D \). In addition, \( \alpha \) denotes the proportion of merchants who hold either a debit or a credit card.

\(^{18}\)The comparative static for \( \alpha^C \) with respect to \( \delta \) is complex, so we omit it here. It is available from the authors on request.

\(^{19}\)See e.g. Armstrong (2006) and Guthrie and Wright (2007) for a comprehensive analysis of competition in two-sided markets.
Under the assumption of singlehoming merchants, this implies $\alpha = \alpha^D + \alpha^C$.

The consumer will hold both cards if:

$$\delta\rho v_0 - \delta(\gamma_E + 2\gamma_L)r_dm_d \leq \left(\delta[(1-\alpha)\rho + \alpha] + (1-\delta)\alpha^C\right)v_o - \alpha^C\gamma_Lr_cm_c - \delta(1-\alpha)(\gamma_E + 2\gamma_L)r_dm_d - \delta\alpha^D(\gamma_E + 2\gamma_L)r_dm_d - F_T,$$

where $F_T$ denotes the maximum total fee that the consumer is prepared to pay to hold both a debit and a credit card. For the reasons discussed earlier we continue to assume that the indifferent consumer will use a credit card, rather than use the overdraft.

We can rearrange to find the maximum total consumer fee, as a function of merchant acceptance:

$$F_T = [(1-\rho)\alpha + (1-\delta)\alpha^C]v_o + \delta\alpha^C(\gamma_E + 2\gamma_L)r_dm_d - \alpha^C\gamma_Lr_cm_c$$

$$= [(1-\rho)\alpha + (1-\delta)\alpha^C]v_o - \alpha^C(1-\delta)[(2-\gamma_E)r + (1-\gamma_E - \gamma_L)].$$

Note that individual contributions to the total fee will satisfy\textsuperscript{20}

$$F_T = F_T^D + F_T^C,$$

where

$$F_T^D = \delta\alpha^D(1-\rho)v_o,$$

and

$$F_T^C = \alpha^C(1-\delta)\rho v_o - \alpha^C(1-\delta)[(2-\gamma_E)r + (1-\gamma_E - \gamma_L)].$$

\textsuperscript{20}This breakout is derived by observing that merchants only accept one type of payment card (see discussion in Bolt and Schmiedel 2011).
5.2 Merchants’ acceptance

We assume that merchants singlehome; if they accept a card at all, it is either a debit or a credit card. In equilibrium, only merchants with high profit margins accept credit cards, since these are more costly: intermediate merchants accept debit cards, and low-end merchants accept cash.\textsuperscript{21} Using the expected payoffs above, we can find the profit level above which merchants are prepared to accept debit cards $\bar{\pi}_d$ and likewise the profit level above which they will accept credit cards $\bar{\pi}_{dc}$:

$$\bar{\pi}_d(f_d) = \frac{f_D - \rho h}{1 - \rho} \quad \text{and} \quad \bar{\pi}_{dc}(f_d, f_c) = \frac{f_C - \delta f_D}{1 - \delta}.$$ 

This gives us the following acceptances:

$$\alpha(f_d) = 1 - \bar{\pi}_d(f_D) \quad \text{and} \quad \alpha^C(f_d, f_c) = 1 - \bar{\pi}_{dc}(f_D, f_C),$$

where debit card acceptance is:

$$\alpha^D = \alpha(f_D) - \alpha^C(f_D, f^C).$$

5.3 Networks’ optimization

The CCN and the DCB engage in Bertrand competition.

5.3.1 Debit card network

The DCB, issuing the debit card, maximizes its profit function, with respect to to $f_D$, subject to:

$$F_D = F_{T_{\text{max}}}(f_D) \quad \text{and} \quad \alpha^C = \alpha^C(f_D, f^C).$$

However, its profit function is slightly altered from that of the debit-only world:

\textsuperscript{21}Debit cards will have lower merchant fees in equilibrium, otherwise they would not be accepted by merchants nor supplied by banks.
\[
\pi^{DCB} = F^D + \delta \alpha^D (f^D - c^D) \\
\quad + (1 - \alpha^C) r [(1 - \delta) \gamma_E x_2 + \delta \gamma_E (x_2 - m_d)] \\
\quad + \alpha^C r [\delta x_1 + \gamma_E (x_2 - m_c)].
\]

As in the no competition case, the bank can earn interest on positive balances, even in the absence of a credit card. However, the presence of the credit card affects both the frequency and size of the consumer’s positive balance. This has positive and negative effects on the DCB’s profit function. When the consumer pays by credit card, the DCB benefits from the delayed deduction of funds from the current account. Any funds remain in the current account for the duration of period 1, until the end of the ‘grace’ credit period. During this time, the DCB can earn interest on any positive balance, at market interest rate \( r \). However, the credit card also enables the consumer to make a purchase in more states of the world. As a result, the size of the positive balance following early income will be smaller in period 2, relative to the no credit card case. This can be seen by rearranging the above profit function:

\[
\pi^{DCB} = F^D + \delta \alpha^D (f^D - c^D) + r [(1 - \delta) \gamma_E x_2 + \delta \gamma_E (x_2 - m_d)] \\
\quad + \alpha^C r [\delta x_1 - (1 - \delta) \gamma_E].
\]

The last line captures this trade-off. It reflects an interesting case: if expected period-1 income \( \delta x_1 \) is sufficiently large, the DCB’s profit function will increase with any increase in the proportion of merchants accepting the credit card. This is important. Although the credit and debit networks are in competition, there is also this element of complementarity between the debit card and the credit card. However, if the reverse holds, this complementarity will not exist.

The tradeoff continues to play a role when we solve for the optimal debit card merchant fee.
\[ f^D(f^C) = \frac{1}{2}c^D + \frac{(1-\delta)\rho h + (1-\rho)f^C}{1-\rho\delta} + \frac{1}{2}\frac{(1-\rho)}{1-\delta}\rho r[\delta x_1 - (1-\delta)\gamma_E] - \frac{1}{2}(1-\rho)v_0. \quad (5) \]

For a given \( f^C \), the optimal merchant fee in the debit network is increasing in the market interest rate, as long as \( \delta \) is sufficiently high such that \( \delta x_1 > (1-\delta)\gamma_E \). At the margin, if the DCB expects to earn a large amount on positive balances in period 1, it will set a high merchant fee to discourage debit acceptance in favor of credit cards.

### 5.3.2 Credit card network

The profit function of the CCN remains unchanged, relative to the no competition case. That is:

\[ \pi^{CCN} = F^C + \alpha^C(f^C - c^C) - \delta\alpha r x_1. \]

It now maximizes this profit function, with respect to \( f_d \), subject to

\[ F_T^C = F_T^{\max}(f^C, f^D) \quad \text{and} \quad \alpha^C = \alpha^C(f^D, f^C). \]

The optimal merchant fee for credit cards, having substituted in \( r_d \), is therefore:

\[ f^C(f^D) = \frac{1}{2}[c^C + 1 - \delta(1-f^D)] + \frac{1}{2}\delta r x_1 + \frac{1}{2}(1-\delta)[(2-\gamma_E)r + (1-\gamma_E-\gamma_L)] - \frac{1}{2}(1-\delta)\rho v_0. \quad (6) \]

This is similar to the merchant fee in the credit-only model. The major difference is that the fee is a function of the debit merchant fee \( f^D \), rather than the merchant’s cost of cash, \( h \).

The unique equilibrium merchant fees \( (f^*_D, f^*_C) \) are found from the intersection of the two best response functions, \( f^D(f^C) \) and \( f^C(f^D) \) (see appendix).

### 5.4 Comparison and comparative statics

Table 3 compares competitive and monopolistic card fees for two different default levels \((D = 5\% \text{ vs. } D = 10\%)\) and funding cost levels \((r = 1\% \text{ vs. } r = 3\%)\).
First notice how an increase in default risk affects interest rates on debit and credit cards. As observed before, monopolistic debit card fees are not affected by default risk changes. The value of debit cards is driven solely by security concerns as they generate no advantage over cash with respect to the use of the overdraft facility. However, competitive debit card merchant fees are affected by default risk movements. Notice that higher default leads to higher competitive debit card merchant fees but to higher debit card acceptance as well. (We show the comparative static in the appendix.) Total card acceptance decreases however. Intuitively, higher default increases the credit card merchant fee, allowing the competing debit merchant fee to rise as well. Although this has a negative effect on merchant acceptance of debit cards, this effect is smaller than the reduction in acceptance of credit cards. Since the merchants who no longer accept credit cards will switch to debit cards, this results in an overall increase in debit card acceptance.

Second, we observe higher competitive merchant fees when the funding cost increases. Note that debit card merchant fees, which were not affected by funding costs in the monopolistic case, may rise considerably in the competition case. They may even exceed monopolistic levels. However, this is not primarily due to relaxation of competitive pressure given the rise in credit merchant fees—in fact, the latter rises by a lower proportion compared with the
debit merchant fee. The effect is coming from the complementarity between debit and credit cards. The bank can benefit from a positive balance in the current account while the consumer enjoys the ‘grace period’; the returns on the positive balance increase with \( r \) and so the bank substantially increases the debit merchant fee, to discourage debit card usage. As a result, debit card acceptance \( \alpha^D \) actually decreases. Despite this compensating effect, credit card acceptance decreases considerably as a result of higher funding cost.

This demonstrates our key result. Although competitive pressures generally reduce fees for both cards, they also lead to an element of complementarity between debit and credit cards when the two cards offer different credit possibilities. As a result, debit card fees may actually be relatively high, despite competition from credit cards.

6 Welfare

We now turn to consider welfare maximizing fees. In effect, we will derive the fee in each case such that the optimal proportion of merchants are induced to accept the card. These results closely mirror those in Bolt and Schmiedel (2011), but we review them here as a means of comparing them with our new results in earlier sections. We only review the cash-only world, and derive the welfare in an environment with both cards. (We leave the debit-only and credit-only environments to the appendix.)

When we consider social welfare, interest rates and fees are merely transfers between agents. As a result, the complementarity effect which operates in the context of competition will be irrelevant for considering social welfare. However, welfare will be a function of the probability of default since this represents deadweight loss. Notice this explicitly enters the welfare function, unlike in the case of private optimization, when default only featured through the interest rates. In effect, the benefit from cards accrues from the extra surplus \( v_0 \) but part of the cost comes from the possibility the consumer cannot repay the loan.

6.1 Cash-only economy

As a baseline case, the welfare in a cash-only economy is given as follows:

\[
W^{\text{cash}} = \delta \rho v_0 + \delta \rho \left( \frac{1}{2} - h \right) - \delta (1 - \gamma_E - \gamma_L) m_d.
\]
Notice that there is still a positive probability of default in a world without payment cards; this is because the consumer can use her overdraft facility to withdraw cash for payment. Regardless of whether she is mugged, she will then default on repaying this debt if she receives no period-2 income.

6.2 Credit and Debit Cards

When both cards are present, welfare $W^{DC}$ is given by:

$$W^{DC} = \delta(1-\alpha)\rho + \alpha v_o + (1-\delta)\alpha^C v_o$$
$$+ \alpha C \left( \frac{2-\alpha^C}{2} \right) + \delta \alpha^D \left( \frac{2-\alpha^D}{2} \right) + \delta \rho (1-\alpha) \left( \frac{1-\alpha}{2} - h \right)$$
$$- \alpha^C c^C - \delta \alpha^D c^D - \alpha^C (1 - \gamma_E - \gamma_L) m_c - \delta (1 - \alpha^C)(1 - \gamma_E - \gamma_L)m_d.$$  

The first line captures the expected benefit of the purchase to the consumer. The second line represents the expected benefit to merchants, using the fact that merchants are distributed uniformly on the interval 0 to 1. The final terms capture the deadweight costs of debit cards; the expected transaction costs and the expected costs of default.

The welfare maximizing fees are given by the following

$$\arg \max_{f^D, f^C} W^{DC}$$

s.t. $\alpha = 1 - \frac{f^D - \rho h}{1-\rho}$, $\alpha^C(f_d, f_c) = 1 - \frac{f^C - \delta f^D}{1-\delta}$

and $\alpha^D = \alpha(f^D) - \alpha^C(f^D, f^C)$.

This yields the following two best response functions:

$$f^D_{opt}(f^C_{opt}) = c^D + \frac{f^C(1 - \rho) - c^C(1 - \rho) - (1 - \delta)(1 - \rho)(1 - \gamma_E - \gamma_L)}{1 - \delta \rho},$$

$$f^C_{opt}(f^D_{opt}) = c^C - c^D \delta - v_o(1 - \delta) + (1 - \delta)(1 - \gamma_E - \gamma_L) + \delta f^D.$$
These equations can be solved to find the unique welfare maximizing fees:

\[ f_{opt}^D = c^D - (1 - \rho)v_o, \]  
(6)

\[ f_{opt}^C = c^C + (1 - \delta)(1 - \gamma_E - \gamma_L) - (1 - \delta \rho)v_o. \]  
(7)

Using these fees, we can obtain the welfare maximizing proportion of merchants accepting each type of card. The total card acceptance is given by

\[ \alpha_{opt} = (1 + v_o) - \frac{c^D - \rho h}{1 - \rho}, \]  
(8a)

while credit card acceptance is given by

\[ \alpha_{opt}^C = (1 + v_o) - \frac{\Delta c}{1 - \delta} - c^D - (1 - \gamma_E - \gamma_L), \]  
(8b)

where \( \Delta c = c^C - c^D \) denotes the cost differential. This means debit card acceptance is given by

\[ \alpha_{opt}^D = \alpha_{opt} - \alpha_{opt}^C = \frac{\Delta c}{1 - \delta} - \frac{\rho(c^D - h)}{1 - \rho} + (1 - \gamma_E - \gamma_L). \]  
(8c)

These equations show that default risk and cost differentials are major drivers for relative merchant acceptance of debit versus credit cards. Whereas the complementarity between cards was relevant in the privately competitive framework, this does not affect the socially optimal fees as discussed above. As a result, the debit merchant fee is no longer a function of expected costs of default and is identical to the socially optimal fee in the debit-only world. Nevertheless, the proportion of merchants accepting debit cards relative to credit cards still increases with the probability of default.

The final table compares monopolistic and competitive merchant fees, with the socially optimal fees, at default levels \((D)\) of 10\%. This table confirms the results of Bolt and Schmiedel (2011), that competitive fees are still large relative to the socially optimal level.
Table 4: Comparison between Monopoly, Competition and Social Optimality.

<table>
<thead>
<tr>
<th></th>
<th>Cash</th>
<th>Debit Only Monopoly</th>
<th>Debit Only Social</th>
<th>Credit Only Monopoly</th>
<th>Credit Only Social</th>
<th>Debit and Credit Competition Monopoly</th>
<th>Debit and Credit Social</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f^D )</td>
<td>0.00675</td>
<td>0.00250</td>
<td></td>
<td></td>
<td></td>
<td>0.00544</td>
<td>0.00250</td>
</tr>
<tr>
<td>( \alpha^D )</td>
<td>0.42450</td>
<td>0.84900</td>
<td></td>
<td></td>
<td></td>
<td>0.34222</td>
<td>0.45150</td>
</tr>
<tr>
<td>( f^C )</td>
<td></td>
<td>0.01600</td>
<td>0.00850</td>
<td></td>
<td></td>
<td>0.01325</td>
<td>0.00850</td>
</tr>
<tr>
<td>( \alpha^C )</td>
<td></td>
<td>0.24510</td>
<td>0.62212</td>
<td></td>
<td></td>
<td>0.21294</td>
<td>0.39750</td>
</tr>
<tr>
<td>Welfare</td>
<td>0.41482</td>
<td>0.41750</td>
<td>0.41839</td>
<td>0.41726</td>
<td>0.41867</td>
<td>0.41858</td>
<td>0.41918</td>
</tr>
</tbody>
</table>

Note: We set: \( c^D = 0.0025 \), \( c^C = 0.0075 \), \( h = 0.001 \), \( v_0 = 0 \), \( p = 0.99 \), \( \gamma_E = 0.5 \), \( \gamma_L = 0.4 \), \( \delta = 0.99 \), \( r = 0.01 \), and \( x_1 = 0.25 \).

However, there is an extra reason in this paper for the large wedge between competitive and socially optimal debit fees; as discussed above, competitive debit fees are inefficiently high in part due to the complementarity effect between debit and credit cards. Note, also, that for high probabilities of period-1 income (\( \delta \) high) there is very little extra benefit to a credit card, relative to a debit card. As a result, the proportion of debit card acceptance increases relative to credit card acceptance as \( \delta \) increases.

As competitive payment card fees do not necessarily reach their socially efficient levels, regulatory intervention regarding merchant fees may still be necessary to raise total surplus. The question arises whether lower merchant fees will drive up consumer fixed fees, the so-called “waterbed” effect.\(^\text{22}\) In our model under full consumer homogeneity, lower merchant fees would induce higher merchant acceptance and therefore a higher willingness-to-pay by consumers. Consumer surplus will always be fully extracted. Under consumer heterogeneity, some consumers will stop using their payment cards because of higher fixed fees and total card usage may go down as a result (see the next section on robustness of our homogeneity assumption).

7 Robustness

We make certain simplifying assumptions regarding consumer and merchant type. First, consumers are homogeneous and merchants are heterogeneous in our model. Second, we assume that consumers multihome while merchants singlehome. It is worth bearing in mind, \(^{22}\)Genakos and Valetti (2011) analyze the impact of regulatory intervention to cut termination rates of calls from fixed lines to mobile phones. They find evidence of the waterbed effect in the telecom market and their results suggest that although regulation reduced termination rates by about 10%, this also led to a 5% increase in mobile retail prices.
however, that our key results do not qualitatively depend on these assumptions.

First of all, note that if both sides were homogeneous, there would be no element of competition required, leading to a trivial equilibrium. If instead we considered heterogeneous consumers, this would lead some of them to accept the card and others to reject; or, in the case of two cards, some consumers would accept both cards, while others just accepted one, or none at all. If merchants were homogeneous we would be back considering an ‘all-or-nothing’ corner solution, whilst if they were heterogeneous as well as consumers, this would lead to separation on both sides of the market.

In any case, however, these alternative assumptions would not change the element of complementarity between debit and credit cards. Nor does it change the fact that credit cards will incorporate default costs in a more direct way than debit cards. In the model above, we found that debit card merchant fees were relatively high in competition, due to the complementarity effect. This would not disappear if card networks were forced to compete either on different sides of the market, or on both. Whilst greater competition might lower fees all round, the complementarity effect would still leave debit card fees relatively high.

At this point, we should also note the assumption we have made about use of overdrafts and credit lines. We have assumed that consumers are willing and able to use their overdrafts via their debit cards. Moreover we have assumed that they cannot (or will not) use their overdraft to partially pay off their credit line.

In reality, we actually observe many different practices, some the result of cultural or behavioral characteristics. European consumers differ from US consumers regarding their credit card use. European consumers are less likely to use their card for credit purposes, and are more likely to simply rely on the payment facility of the card. In some parts of Europe, consumers are able to use overdrafts with debit cards, while in others they are not (or do not choose to do so).

Sometimes, checking account balances are used to pay back outstanding credit card payments. Instead of revolving the credit card debt and paying interest rate $r_c$, consumers may now draw upon their overdraft facility for repayment and pay interest rate $r_d$. Although the interest effects in our model will be somewhat mitigated, the credit line channel will still affect payment fees, since $m_c - m_d > 0$.

Related to this observation is the fact that few European consumers pay interest on their
credit card debt. Those loans are repaid at the end of the ‘grace’ period, or not at all; in other words, consumers default. These consumers use their card as 'charge cards'. This implies that credit card loans cannot be zero NPV. In this case the cost of funds burden must be shifted explicitly towards merchants and consumers in the form of higher payment fees.

No model can hope to capture all different types of observed behavior. Nevertheless, we have taken a key step in highlighting the important role of credit in payment card competition, and in so doing explored a hitherto ignored element of complementarity.

8 Conclusions

In this model we examine the role of consumer credit in both debit and credit card networks. We allow for the fact that the consumer will always have access to a current account, with an associated overdraft facility. This account is provided by the bank which would issue an associated debit card.

In the ‘credit card only’ world, the credit card effectively competes with the overdraft facility in the state where cash could be used. As a result, higher expected costs of servicing an overdraft will allow the credit card network to increase the consumer fee and lower the merchant fixed fee; this will increase the acceptance ratio of credit cards among merchants.

Our model also shows that cost of funds and default risk affect debit cards and credit cards in different ways. Specifically, in a ‘debit card only’ world, these factors have no effect on the merchant fee, while they do affect credit card merchant fees. In a competitive situation, these cost factors drive both cards, but credit card merchant fees are more affected than debit card merchant fees. Debit merchant acceptance actually increases with the probability of default, despite an increase in the debit merchant fee, since some merchants switch from credit cards to debit cards.

The debit merchant fee also depends on funding costs in the context of competition. However, as a result, the debit card fee may be increased to discourage debit card acceptance at the margin. Effectively, we find there is a degree of complementarity, as well as competition, between the two networks. The bank providing the debit card and current account actually benefits from consumers using credit cards, if they have positive initial income. In effect the bank benefits from the ‘free credit’ period offered to the consumer by the credit card.
network, as the bank can earn interest on the balance that remains in the current account during this period. If the probability of initial income is high, therefore, this complementarity incentivises the bank to increase the merchant debit card fee and reduce merchant acceptance of the debit card.

These results help to inform current debates about the pricing of debit and credit card fees. Recent discussion has focused on whether there should be differential interchange fees for debit and credit cards. Although we do not explicitly model the interchange fee, it will be closely related to the merchant fee—high interchange fees are passed onto the merchants in the form of high merchant fees. We therefore shed new light on how to understand the different drivers at work in affecting debit and credit card fees.
Appendix 1: Derivation of Competitive Merchant Fees and Comparative Statics

Note: All algebraic expressions and numerical results in our paper are verified using Mathematica, version 8, and program files are available upon request.

The intersection of (upward-sloping) reaction functions \( f^C(f^D) \) and \( f^D(f^C) \) yields \((f^*_D, f^*_C)\), where

\[
\begin{align*}
    f^*_D &= \frac{1}{4 - 3\rho \delta - \delta} (ab[1 - \gamma_E - \gamma_L] + [ab(2 - 3\gamma_E) + 3b\delta x_1]r + [bc^C + 2cc^D + 2aph + ab] - 3bcv_0), \\
    f^*_C &= \frac{1}{4 - 3\rho \delta - \delta} (2ac[1 - \gamma_E - \gamma_L] + [2ac(2 - \gamma_E) - ab\delta \gamma_E + (b\delta + 2c)\delta x_1]r + [c(2c^C + \delta c^D) + 2a\rho \delta h + 2ac] - c(b\delta + 2c)v_0),
\end{align*}
\]

where:

\[
a = 1 - \delta, \quad b = 1 - \rho, \quad \text{and} \quad c = 1 - \rho \delta. \quad \text{Note that} \quad 4 - 3\rho \delta - \delta = 4c - b\delta > 0.
\]

For the partial derivative wrt default \( D \), we find:

\[
\begin{align*}
    \frac{\partial f^*_D}{\partial D} &= -\frac{\partial f^*_D}{\partial \gamma_L} = \frac{ab}{4 - 3\rho \delta - \delta} > 0, \\
    \frac{\partial f^*_C}{\partial D} &= -\frac{\partial f^*_C}{\partial \gamma_L} = \frac{2ac}{4 - 3\rho \delta - \delta} > 0.
\end{align*}
\]

It easy to show that \( \partial f^*_C/\partial D > \partial f^*_D/\partial D \). Furthermore, by combining our results for merchant fees with the conditions for merchant acceptance, we can show that debit card merchant acceptance is increasing in the default rate:

\[
\frac{\partial a^*_D}{\partial D} = \frac{c}{4 - 3\rho \delta - \delta} > 0.
\]

For funding cost \( r \) we find

\[
\frac{\partial f^*_D}{\partial r} = \frac{ab(2 - 3\gamma_E) + 3b\delta x_1}{4 - 3\rho \delta - \delta} > 0,
\]

for sufficiently large average period-1 income \( \delta x_1 \) if \( \gamma_E \geq 2/3 \), and:

\[
\frac{\partial f^*_C}{\partial r} = \frac{2ac(2 - \gamma_E) - ab\delta \gamma_E + (b\delta + 2c)\delta x_1}{4 - 3\rho \delta - \delta} > 0,
\]

since \( c > b \) and \( 2 - \gamma_E > 1 \). For the probability of early period-2 income \( \gamma_E \), we find:

\[
\begin{align*}
    \frac{\partial f^*_D}{\partial \gamma_E} &= -\frac{ab(1 + 3r)}{4 - 3\rho \delta - \delta} < 0, \\
    \frac{\partial f^*_C}{\partial \gamma_E} &= -\frac{2ac(1 + r) + ab\delta r}{4 - 3\rho \delta - \delta} < 0.
\end{align*}
\]
Finally, defining $x^e = \delta x_1$, we find (keeping $\delta$ constant):

$$\frac{\partial f_D^{**}}{\partial x^e} = \frac{3br}{4 - 3\rho\delta - \delta} > 0 \quad \text{and} \quad \frac{\partial f_C^{**}}{\partial x^e} = \frac{(b\delta + 2c)r}{4 - 3\rho\delta - \delta} > 0.$$ 

### Appendix 2: Welfare Analysis

**Debit-only economy**  
In a debit-only world, welfare is given by $W^{\text{debit}}$ where

$$W^{\text{debit}} = \delta[\alpha^D + \rho(1 - \alpha^D)]v_0$$

$$+ \delta \left\{ \alpha^D \left(\frac{2 - \alpha^D}{2}\right) + \rho(1 - \alpha^D) \left(\frac{1 - \alpha^D}{2} - h\right) \right\}$$

$$- \delta \alpha^D c^D - \delta(1 - \gamma_E - \gamma_L)m_d.$$ 

The welfare maximizing fee is given as follows:

$$\arg \max_{f_D} W^{\text{debit}}$$

$$s.t. \quad \alpha^D = 1 - \frac{f_D - \rho h}{1 - \rho}.$$ 

Although the expected cost of default enters the welfare measure, it has no effect on the optimal fee because the probability of default does not depend on $\alpha^D$, the probability of debit card usage. This follows from our assumption that the overdraft, rather than the debit card itself, provides the consumer with sufficient means to make the purchase in the high income state. The benefit of the debit card only comes from greater security over cash.

The optimal proportion of merchant acceptance is

$$\alpha^D_{\text{opt}} = 1 + v_0 - \frac{c^D}{1 - \rho} + \frac{\rho h}{1 - \rho},$$

which means the optimal fee is

$$f^D_{\text{opt}} = c^D - v_0(1 - \rho).$$

Intuitively, the merchant fee increases in the network cost $c^D$, but decreases with the social benefit from debit cards. This benefit comes from the additional consumer surplus $v_0$ that can be obtained in states where cash is insecure. Note the merchant fee is the same as the welfare maximizing debit fee in Bolt and Schmiedel (2011).

**Credit-only economy**  
In a credit-only world, the welfare is $W^{\text{credit}}$ where:
\[ W_{\text{credit}} = [\alpha^C + \delta \rho (1 - \alpha^C)]v_o \]
\[ + \alpha^C \left( \frac{2 - \alpha^C}{2} \right) + \delta \rho (1 - \alpha^C) \left( \frac{1 - \alpha^C}{2} - h \right) \]
\[ - \alpha^C c^C - \alpha^C (1 - \gamma_E - \gamma_L)m_c - \delta (1 - \alpha_c)(1 - \gamma_E - \gamma_L)m_d. \]

The optimal fee is given by

\[
\arg \max_{f^C} W_{\text{credit}} \\
\text{s.t. } \alpha^C = 1 - \frac{f^C - \delta h}{1 - \delta \rho}.
\]

In contrast to the debit only economy, the probability of default in the credit-only economy is a function of \( \alpha^C \), the proportion of merchants accepting the card. The socially optimal proportion is given by

\[
\alpha_{\text{opt}}^C = v_o + 1 - \frac{c^C - \rho \delta h + (1 - \gamma_E - \gamma_L)(1 - \delta)}{1 - \delta \rho},
\]

which is clearly decreasing in the expected cost of default. By extension, the optimal fee is

\[
f_{\text{opt}}^C = c^C - (1 - \delta \rho)v_o + (1 - \delta)(1 - \gamma_E - \gamma_L).
\]
References


