The Dynamics of Arbitrage: Evidence from the Yen Forward Markets

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Abstract:

Covered interest parity arbitrage maintains the pricing between financial products traded in financial markets with different currency denominations and time periods. Market frictions cause the parity price to oscillate within a trading band, which varies over time and maturity. We investigate the drivers behind the arbitrage dynamics using a three regime Bivariate Threshold AutoRegressive (BTAR) model where the bivariate pair is the implied and actual forward exchange rates and the threshold value is the difference between the two. When studying different maturities of the US dollar-Japanese yen rates, one state represents times when US dollar borrowers have a comparative advantage, one state represents times when Japanese yen borrowers have an advantage, and a third state represents white noise around the theoretical exchange rate. The profit associated with exploiting an arbitrage varies with maturity as well as state: the largest profit arises when US dollar borrowers have the advantage, while the largest variance in profit occurs when Japanese yen borrowers have the advantage.

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1. Introduction

Covered interest parity (CIP) arbitrage ensures that the difference between spot and forward exchange rates is based upon the interest rate differential between the home and the foreign currency. This relationship remains a cornerstone of modern international finance and security pricing. However, an extensive literature has observed systematic deviations from CIP due to irregularities in the institutional features present in either the home, or foreign, financial market caused by with-holding taxes, capital controls and trading costs, which subsequently affect market liquidity and the extent of possible trading. One observed consequence of these market frictions is that the observed market price oscillates within a trading band around the parity price (e.g. Eaton and Turnovsky, 1984; Taylor, 1989; Strobel, 2001; Peel and Taylor, 2002)[1].

Price deviations from the parity price -both within and outside the trading band- create advantages for a select group of financial market participants. These participants, through their credit quality or country of residence have cheaper access to domestic money market or deposit funds, providing them with a comparative advantage in one segment of the money, or foreign exchange markets. Poitras (1988) and then Popper (1993), show how exploiting these opportunities favors those with the ability to borrow US dollars (US$). For example, participants may borrow cheap US$ for three months, sell these in the spot market against a foreign currency, while simultaneously buying them back in the three month forward market. The future value of the foreign currency funds is then matched with cash-flow required under the three month forward contract resulting in a profit, where the cheaper the US$ and the higher the return on the foreign funds, the greater the profit to the arbitrager.

Taylor (1999) amongst others, observes how the maturity preferences of some market participants may cause persistent deviations in price for longer dated financial assets (Fletcher and Taylor,1996; Popper, 1993). These findings suggest that while the process of price

[1] Note that there is an alternate literature which explores the uncovered interest parity relationship, especially so in the context of financial market integration and perceived success of short-term anomalies such as the returns generated by carry-trades –borrowing in low yielding currencies, such as yen, to invest in high yielding foreign currency assets (e.g. Isard, 2006; Juvenal and Taylor, 2008; Fukuda, 2010; Levent, 2010).

adjustment within the trading band clearly displays complex dynamics, in practice we know very little about those market features that cause prices to deviate from the equilibrium price.

Our objective is to investigate the dynamics of price movements around the arbitrage relation. We use technology, which allows the temporal nature of the price adjustment process to be more carefully monitored and investigated than earlier studies. Importantly, by investigating this phenomenon using daily data instead of the weekly or monthly series utilized by many researchers, we are better able to observe the complex dynamics that underpin arbitrage while also enabling the institutional features that affect price movements to be carefully identified. Specifically, we apply recent innovations in threshold dynamic modeling to those originally used by Taylor (1989) and Balke and Wohar (1998) in their investigation of the pricing band around the CIP arbitrage relation. We investigate this relation using a 25-year dataset of daily spot and 3 and 6 month forward U.S. dollar–Japanese yen (US$/¥) prices and matching maturities in LIBOR markets. These prices are used to estimate implied forward rates based on interest rate differentials, from January 1983 through to April 2008[2]. The implied forward rates are then matched with actual forward rates quoted in the market enabling estimation of the deviation from CIP condition, or the CIP spread. Analysis using daily prices over the 25-year period allows identification of the temporal nature of the CIP spread, while highlighting the factors that cause changes in the direction of arbitrage, and the maturity preferences of financial participants.

The US$/¥ has particular economic appeal: The limited foreign bank access to the Japanese financial system provides Japanese banks with a unique home currency advantage to the third largest financial market behind the euro and the US dollar. The advantage that Japanese banks enjoy at home, reflected in very cheap deposit rates, could offset the comparative advantage that US based financial institutions enjoy in their domestic deposit and securities markets. The combined actions of these groups of institutions should presuppose a more dynamic and complex two-way CIP relation than previously observed in other markets. The US dollar-Japanese yen spot and forward exchange rates are known to have complex dynamics (Elliott and Ito, 1999) and are the most liquid after the US dollar-euro (BIS, 2010).

[2] Our sample does overlap with Baba and Packer (2009) who investigate CIP deviations on the euro-dollar in the (September 2007- September 2008 period). However, their study only touches upon the extreme pricing events that occurred later in 2008 and early 2009. We avoid the period of the recent system-wide financial crisis (September 2008-2009) where market volatility violates the underlying assumptions of the BTAR model.

The investigation of the dynamics of the arbitrage relationship between the US$/¥ spot, forward and money markets is undertaken using a Bivariate Threshold AutoRegressive (BTAR) model. In this case the bivariate pair is the implied and actual forward rates and the threshold value is the difference between the two. The class of threshold autoregressive (TAR) models (Tong, 1983; Tsay, 1989; Tsay 1998) has been widely employed in the literature to explain nonlinear phenomena observed in other economic and financial time series. For example, Chapell, Padmore and Ellis (1995) applied threshold modeling to understand the pre-euro French Franc and Deutsche mark exchange rate, Chan and Cheung (2005) observed three regimes in Australian interest rate markets markets, Chiang and Wang (2008) investigated complex dynamics in various Asian index futures, while De Gooijer and Vidiella-i-Anguera (2003) applied these techniques to understanding inflation dynamics. This paper fits in with those empirical studies that apply TAR models to financial decisions triggered by a threshold or control variable, such as arbitrage in the presence of transaction costs (e.g. Dwyer, Locke and Yu, 1996) and market interventions by regulators (Yadav, Pope and Paudyal, 1994).

While this is not the first study to apply these techniques to CIP arbitrage conditions (e.g. Balke and Wohar, 1998; Peel and Taylor, 2002), we build upon these earlier works by using the regimes identified by the BTAR analysis as the basis for further analysis of the economic context of arbitrage. The use of this model confirms the presence of three distinct regimes, where one state represents times when USD borrowers have a comparative advantage; one state represents times when JPY borrowers have an advantage, and a third state represents white noise around the theoretical exchange rate.

In the context of this paper, there is a CIP spread, which can be measured in terms of basis points, between the known forward rate and an implied rate based upon interest rate differentials. Under these circumstances BTAR modeling has three main advantages: First, this model provides an exact measure of the economic incentive for a portfolio investor to arbitrage two financial instruments- the actual forward and the implied forward of equivalent maturity. This measure, termed a “threshold” or “critical” value in the BTAR model, may also be interpreted as the hidden cost necessary for financial market participants to shift the arbitrage between investing/borrowing in US dollars, and the reverse in Yen.

Second, threshold values can also be used to anticipate the change in the direction of the arbitrage. This will allow traders to be more cautious in managing risk and help policymakers and central banks fine tune monetary policies. Third, the threshold value can also be expressed in terms of exchange rate "basis points"; a number that can be easily understood and interpreted by financial markets. This is quite different from the information provided by other related models, such as Markov Switching Models (Hamilton, 1996).

The paper is structured into the following sections: Next, the CIP relation is discussed and a simple CIP relation is derived; then the Yen-dollar exchange and interest rate data and the BTAR model are explained. Section four, provides the results and finally section five, provides some concluding comments.

2. The Covered Interest Parity Relation

The interaction between spot and forward exchange rates and the underlying forward margins and interest rates, are relationships that have been commonly tested in financial economics. Of the relationships in cash-based foreign exchange and money markets covered interest parity has been investigated empirically for the past thirty years[3]. Recent improvements in the method of trading from broker driven over-the-counter markets to mostly electronic trading via dealing systems such as Reuters D2000 and the EBS trading platform have significantly affected the arbitrage spreads that had been available (see Payne (2003) for a discussion). Importantly for this study, the usage of these systems by market participants has increased in recent years so that they now dominate trading practice. Prices from these trading platforms now also feed into other pricing systems such as those for calculating derivative contracts such as Forward Rate Agreements (FRA) and interest rate swaps. These are the likely reasons that CIP arbitrage has become less frequent and the economic spreads have been reduced[4].

[3] Other CIP studies to those already mentioned include: Kia (1996) in short-dated cash markets and Moosa and Bhatti (1996) under Fisherian expectations.
[4] Attention shall later be drawn to Figure 3, which plots the frequency of the USD-JPY CIP relation for three and six month maturities. Importantly, note the declining frequency of the BTAR regimes one and three since the widespread introduction of integrated trading and pricing technology.

Frenkel and Levich (1981) and Popper (1993), begin by expressing the CIP relation between the spot (e_s) and forward (e_f) exchange rates and the underlying interest rates over a specific maturity (m). Using their notation

$$(1 + i_m) = e_{sm}/e_{fm}(1+i_m^*) \qquad (1)$$

where i_m and i_m^* are the respective domestic and foreign interest rates on securities with the same maturity as the forward rate. We follow Batten and Szilagyi (2010) and note that the spot exchange rate (t_s) requires cash settlement two working days from its trade date (t_0) and is expressed as one unit of domestic, or home, currency in terms of a specific amount of foreign currency (e_s). The forward exchange rate (e_f) is expressed the same way as the spot rate, is also observable at t_0, but instead requires cash settlement at a future date t_f. The maturity of these contracts (the number of days between t_0 and t_f) is commonly expressed by market convention in weeks and months from spot. In addition, two sets of interest rates i_m for the home currency rate and i_m^* for the foreign interest rate, represent either the cost, or investment return, from either borrowing, or lending, for the *m* period.

Importantly, Equation (1) assumes that financial market participants have access to domestic and foreign currency funding and asymmetric information due to customer order imbalances in forward foreign exchange markets are eliminated quickly. However, the effect of capital market segmentation (Blenman, 1991) and market illiquidity (Moore and Roche, 2001, 2002) may result in a forward price band around an equilibrium price.

Since CIP arbitrage ensures that equilibrium prices in forward currency markets are maintained based upon interest rate differentials. Therefore from (1) the interest rate differential

$$(i_m^* - i_m)/ (1+i_m^*) = (e_{fm} - e_{sm}) / e_{sm}. \qquad (2)$$

In practice, this mathematically simple calculation requires consideration of the different money market bases (either a 360, 365 or actual number of days –to accommodate leap years) that exists by convention in different financial markets (e.g. the US dollar operates on a 360 day year whereas the euro operates on a 365 day year). Also, arbitrage requires undertaking actual cash flow in all currency positions, which may add to the transaction costs and impose boundaries around the equilibrium price which create episodes of market inefficiency (Crowder, 1995). To some extent these costs may be avoided or reduced using derivatives such as options (Ghosh and Ghosh, 2005), although the Basle II capital adequacy guidelines now impose a capital charge on both off as well as on-balance sheet transactions. Thus, transaction costs apply to all financial intermediaries undertaking arbitrage of this type, although comparative advantages in some or all segments of the transaction due to differences in scale and scope economies, and home currency advantages may mitigate their effects.

Our aim is to investigate the deviations from equilibrium (δ) between the implied forward rate based upon the interest rate differentials (e_f*) and the actual forward rate e_f that is quoted at t_0 in the foreign exchange market to time dependence and related factors. These deviations will be investigated using the BTAR techniques previously described. In this way we do not specifically consider the transaction band associated with two-way quotes (due to the bid-ask spread) in foreign exchange markets and the associated algebra (see Balke and Wohar, 1998), given that it is now well-known that participants quote market established spreads of 5 or 10 basis points depending on the market. Therefore, from equation (1) the implied forward rate based upon interest rate differentials should be equivalent to the observable forward rate

$$e_{fm}^* = e_{sm}(1+i_m^*) / 1(1 + i_m) \equiv e_{fm} \qquad (3)$$

with the deviation from equilibrium (δ_m) being simply the difference between the actual and implied forward rate for a specific maturity

$$e_{fm} - e_{fm}^* = \delta_m \qquad (4)$$

What is of interest in this paper is the behavior over time of the residuals δ_m. These could be expected to be random, stationary and possess *i.i.d.* $N(0, \sigma^2)$ properties since arbitrage should cause the deviations to revert to an equilibrium around zero (or close to zero if there is a trading band) over time (Abeysekera and Turtle, 1995 and Turtle and Abeysekara, 1996).

Empirical evidence, however, does not support such a claim. In an earlier study Cosandier and Lang (1981) find the distribution of the arbitrage margins to be non-normal, while Taylor (1987) and Blenman (1991) find a no-arbitrage band within which deviations are random, outside of which deviations revert to the edge of the band. A number of authors observe that the degree of deviation over time is both time varying and also a function of the maturity of the arbitrage investigated. Thus, while Taylor (1999) finds evidence of a maturity effect in shorter dated bill markets, Fletcher and Taylor (1996) and Popper (1993) find evidence of persistent deviations in longer maturities. Later, Poitras (1988) when investigating the CIP relationship on the US$ - Canadian dollar noted that the presence of the arbitrage boundaries (the likely consequence of transaction costs and market segmentation) would affect the residual distribution and recommended the use of an autoregressive model to correct for residual persistence and identify any permanent components. These issues are developed further in the next section.

3. Data and Method

3.1 Data

London interbank spot and forward foreign exchange midrates on the US$/¥ and Euromarket yen and US dollar LIBOR interest rates with 3 and 6 month maturities were chosen to investigate the CIP relationship on the US$/¥. All prices were at the daily close of trading. Initially, series from the January 1, 1983 to April 23, 2008 were downloaded from Datastream. Due to some incomplete series for the forward and money market rates, the starting date of the series was made the October 11, 1983, for a total of 6,398 daily observations. As noted earlier we end the sample before the main effects of the financial crisis occurred later in September 2008, although our sample does overlap with the "onset of turmoil" investigated by Baba and Packer (2009). Our understanding is that this is the longest sample period yet tested for CIP. Implied forward rates, for a specific maturity (m) based upon US ($) and yen (¥) interest rates were calculated based upon Equations (3, 4) and implied consistent with Taylor's (1989) discussion:

$$\delta_m = e_{fm} - e_{sm}\,(1+i^{¥}_m) / 1(1 + i^{\$}_m) \qquad (5)$$

(Insert Figure 1 about here)

In earlier work Batten and Szilagyi (2010) illustrate the process of actually executing a CIP arbitrage, which is illustrated as Figure 1. From this Figure begin by either buying or selling US$ spot against yen (top left and right hand corners of the Figure). This results in either a positive or negative spot cash flow in US$ and the reverse cash flow in yen, which then must either be invested or borrowed. In practice, the bid-ask spread, representing the market offer (for you to borrow) and bid (for you to lend) is commonly 1/8th of a percent on Euromarket deposits. Initially, we simply use midrates – and these are what are reported in the subsequent Tables – although simply adding or subtracting 1/16th of a percent to the midrate can recreate the underlying bid and offer rates. In the case of the spot and forward exchange rates, while midrates are also used for the reported calculations, the 5 basis point bid-ask spread typically required by currency traders can also be accommodated by adding or subtracting a 2.5 basis point spread from the midrate.

The resulting cash flows in spot markets sum to zero, with the remaining spot cash flows occurring at the future date being the maturity of the loan or the borrowing. The implied forward rate can easily be derived by dividing the future yen cash flow with the future US$ cash flow. This implied rate might then be compared with the actual forward market rate (bottom left and right hand corners of the Figure). If markets are in perfect equilibrium (and no transaction costs) then the difference (δ_m) should be zero. If δ_m is positive, that is $e_{fm} > e_{fm}$ * then an arbitrage can be executed which requires selling e_{fm} and buying e_{fm} *. The long e_{fm} * position can be created by buying US dollars spot against yen, lending US$ and then borrowing yen. The alternative, when the borrower has the advantage in US dollar money markets is also true; if δ_m is negative, that is $e_{fm} < e_{fm}$ * then an arbitrage can be executed, which requires buying e_{fm} and selling e_{fm} *. The short e_{fm} * position can be created by selling US dollars spot against yen, borrowing US$ and then lending yen. Segmentation in interest rate markets due to credit constraints might prevent access to one particular market. For example, Poitras (1988) noted that in the US-Canadian dollar market limited access to US dollar borrowing ensured that CIP arbitrage tended to be one way, favoring those with the ability to borrow in US interest rate markets.

3.2 Econometric Specification of Bivariate TAR models

We consider a bivariate time series along the lines of Tsay (1998) and Chan and Cheung (2005) where $Z_t = (z_{1t}, z_{2t})'$ with z_{1t} = the market quoted forward price and z_{2t} = an implied forward rate based on CIP relationships. Therefore a k-regime BTAR ($d; p_1, \ldots, p_k$) model is defined as

$$\mathbf{Z}_t = \begin{cases} \boldsymbol{\omega}_0^{(1)} + \sum_{j=1}^{p_1} \boldsymbol{\Phi}_j^{(1)} \mathbf{Z}_{t-j} + \mathbf{a}_t^{(1)}, \text{ if } y_{t-d} \le r_1 \\ \boldsymbol{\omega}_0^{(2)} + \sum_{j=1}^{p_2} \boldsymbol{\Phi}_j^{(2)} \mathbf{Z}_{t-j} + \mathbf{a}_t^{(2)}, \text{ if } r_1 < y_{t-d} \le r_2 \\ \vdots \qquad \vdots \qquad \vdots \qquad \qquad \vdots \\ \boldsymbol{\omega}_0^{(k)} + \sum_{j=1}^{p_k} \boldsymbol{\Phi}_j^{(k)} \mathbf{Z}_{t-j} + \mathbf{a}_t^{(k)}, \text{ if } r_{k-1} < y_{t-d} \end{cases}, \qquad (6)$$

where k is the number of regimes in the model, d is the delay parameter, p_i is the autoregressive order in the ith regime of the model, $\boldsymbol{\omega}_0^{(i)}$ are (2 x 1)-dimensional constant vectors and $\boldsymbol{\Phi}_j^{(i)}$ are (2 x 2)-dimensional matrix parameters for $i = 1, \ldots, k$. The threshold parameters satisfy (the constraint) $-\infty = r_0 < r_1 < r_2 < \ldots < r_{k-1} < r_k = \infty$. The innovational vectors in the ith regime satisfy $\mathbf{a}_t^{(i)} = \boldsymbol{\Sigma}_i^{1/2}\mathbf{e}_t$, where $\boldsymbol{\Sigma}_i^{1/2}$ are symmetric positive definite matrices and $\{\mathbf{e}_t\}$ is a sequence of serially uncorrelated normal random vectors with a mean of $\mathbf{0}$ and a covariance matrix $\mathbf{I}$, which is the (2 x 2)-dimensional identity matrix. The threshold variable y_{t-d} is assumed to be stationary and depends on the observable past history of $\mathbf{Z}_{t-d}$. For example, we set $y_{t-d} = \boldsymbol{\eta}' \mathbf{Z}_{t-d}$, where $\boldsymbol{\eta}$ is a pre-specified (2 x 1) dimensional vector. When $\boldsymbol{\eta} = (1, 0)'$, the threshold variable is simply $y_{t-d} = z_{1,t-d}$. When $\boldsymbol{\eta} = (½, ½)'$, the threshold variable is the average of the two elements in $\mathbf{Z}_{t-d}$.

3.3 Testing for non-linearity

Given $p = \max\{p_1, \ldots, p_k\}$ and $d \le p$, we observe the bivariate vector time series $\{Z_1 \ldots, Z_n\}$. It should be noted that the threshold variable y_{t-d} in (1) can only assume values in $Y = \{y_{p+1-d}, \ldots, y_{n-d}\}$. Let (i) be the time index of the ith smallest observation in Y. Tsay (1998) considers the multivariate generalisation of the ordered regression arrangement. Rolling ordered bivariate autoregressions in the form

$$\begin{pmatrix} \mathbf{Z}'_{(1)+d} \\ \mathbf{Z}'_{(2)+d} \\ \vdots \\ \mathbf{Z}'_{(j)+d} \end{pmatrix} = \begin{pmatrix} 1 & \mathbf{Z}'_{(1)+d-1} & \cdots & \mathbf{Z}'_{(1)+d-p} \\ 1 & \mathbf{Z}'_{(2)+d-1} & \cdots & \mathbf{Z}'_{(2)+d-p} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \mathbf{Z}'_{(j)+d-1} & \cdots & \mathbf{Z}'_{(j)+d-p} \end{pmatrix} \text{x} \begin{pmatrix} \boldsymbol{\omega}'_0 \\ \boldsymbol{\Phi}'_1 \\ \vdots \\ \boldsymbol{\Phi}'_p \end{pmatrix} + \begin{pmatrix} \mathbf{a}'_{(1)+d} \\ \mathbf{a}'_{(2)+d} \\ \vdots \\ \mathbf{a}'_{(j)+d} \end{pmatrix} \qquad (7)$$

can be arranged successively, where j = m, m+1,…, n-p and m is the number of start-up observations in the ordered autoregression. Tsay (1998) suggests a range of m (between $3\sqrt{n}$ and $5\sqrt{n}$). Different values of m can be used to investigate the sensitivity of the modeling results with respect to the choice. It should be noted that the ordered autoregressions are sorted by the variable y_{t-d}, which is the regime indicator in the BTAR model.

Let $\hat{\boldsymbol{\varepsilon}}_{(m+1)+d}$ denote the one-step-ahead standardised predictive residual from the least-squares fitted multivariate regression for $j = m$. This enables a direct computational formula for $\hat{\boldsymbol{\varepsilon}}_{(m+1)+d}$, which can also be easily obtained from many commonly used statistical software packages (e.g., Timm and Mieczkowski, 1997). Analogous to the univariate case, if the underlying model is a linear autoregressive process, then the predictive residuals are white noise, and they are uncorrelated with the regressor $X'_t = \{1, Z'_{t-1}, Z'_{t-2}, \ldots, Z'_{t-p}\}$.

Importantly in this study, if Z_t follows a threshold process, then the predictive residuals are correlated with the regressor, which allows specification for the multivariate regression

$$\hat{\boldsymbol{\varepsilon}}'_{(l)+d} = \mathbf{X}'_{(l)+d}\,\mathbf{B} + \mathbf{w}'_{(l)+d} \tag{8}$$

for $l = m + 1, \ldots, n\text{-}p$, where B is the matrix regression parameter and $w'_{(l)+d}$ is the matrix of the residuals. The problem of testing non-linearity is then transformed into the testing of the hypothesis H_0: B = 0 in this regression. Tsay (1998) employs the test statistic

$$C(d) = (n - p - m - kp - 1)\ \text{x}\ \left\{\ln|\mathbf{S}_0| - \ln|\mathbf{S}_1|\right\}, \tag{9}$$

where $|\mathbf{S}|$ denotes the determinant of the matrix S, and

$$\mathbf{S}_0 = \frac{1}{n-p-m}\sum_{l=m-1}^{n-p} \hat{\boldsymbol{\varepsilon}}_{(l)+d}\,\hat{\boldsymbol{\varepsilon}}'_{(l)+d}$$

$$\mathbf{S}_1 = \frac{1}{n-p-m}\sum_{l=m-1}^{n-p} \hat{\mathbf{w}}_{(l)+d}\,\hat{\mathbf{w}}'_{(l)+d}\ ,$$

where $\hat{\mathbf{w}}$ is the least-squares residual of regression (2). Under the null hypothesis that Z_t is linear, then *C(d)* is asymptotically a chi-squared random variable with ($4p + 2$) degrees of freedom.

Alternatively, one might consider the bivariate threshold-type of nonlinearity test of Hansen and Seo (2002). Their method is to test the class of linear bivariate vector error correction models versus the class of two-regime threshold cointegration processes. Hansen and Seo (2002) also derive a SupLM test for the presence of a threshold.

3.4 Model specification, estimation and diagnostic checking

To perform the *C(d)* test for non-linearity in (3), both values of p and d must be given. In practice, we can select p by the partial autoregression matrix (PAM) of Z_t. Tiao and Box (1981) define the PAM at lag l, which is denoted by $P(l)$, to be the last matrix coefficient when the data are fitted to a vector autoregressive process of order l. This is a direct extension of the definition of Box and Jenkins (1976) of the partial autocorrelation function for a univariate time series. The partial autoregression matrices $P(l)$ of a linear vector $AR(p)$ process are zero for $l > p$. This 'cut-off' property provides very useful information for the identification of the order p. In practice, we select $p=p^*$ by testing the null hypothesis that the PAM matrices are zero matrices beyond lag p^* using a likelihood ratio test. We shall illustrate this method in detail in Section 4. Once p is selected, d is chosen, such that it gives the most significant *C(d)* statistic.

In univariate TAR modeling, we use various scatter plots to specify the number of regimes k and the threshold parameters (that is, the r values). Unfortunately, these plots are not applicable to high-dimensional multivariate TAR analysis. Following Tong (1983), we use Akaike's information (AIC) to search for these parameters. Given p, d, k and $R_k = \{r_1,\ldots, r_{k-1}\}$, the full-length ordered bivariate autoregression can be divided into different regimes. For the jth regime of data, we have a general model of the form $Z_j = A_j\Phi^{(j)} + a_j$, where

$$\mathbf{Z}_j = (\mathbf{Z}'_{(\pi_{j-1}+1)+d}, \mathbf{Z}'_{(\pi_{j-1}+2)+d}, \ldots, \mathbf{Z}'_{(\pi_j)+d})'$$
$$\mathbf{\Phi}^{(j)} = (\boldsymbol{\omega}'_0, \mathbf{\Phi}'^{(j)}_1, \ldots, \mathbf{\Phi}'^{(j)}_p)'$$
$$\mathbf{a}_j = (\mathbf{a}'_{(\pi_{j-1}+1)+d}, \mathbf{a}'_{(\pi_{j-1}+2)+d}, \ldots.\mathbf{a}'_{(\pi_j)+d})' \quad ,$$

$$\mathbf{A}_j = \begin{pmatrix} 1 & \mathbf{Z}'_{(\pi_{j-1}+1)+d-1} & \cdots & \mathbf{Z}'_{(\pi_{j-1}+1)+d-p} \\ 1 & \mathbf{Z}'_{(\pi_{j-1}+2)+d-1} & \cdots & \mathbf{Z}'_{(\pi_{j-1}+2)+d-p} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \mathbf{Z}'_{(\pi_j)+d-1} & \cdots & \mathbf{Z}'_{(\pi_j)+d-p} \end{pmatrix}$$

and π_j is the largest value of (j), such that $\{r_{j-1} < z_{(j)} \le r_j\}$ for $j = 1, \ldots, k - 1$. We define $\pi_0 = 0$ and $\pi_k = n - p$. The number of observations in the jth regime is $n_j = \pi_j - \pi_{j-1}$. The least-squares estimate $\Phi^{(j)}$ can be obtained by the ordinary multivariate least-squares method:

$$\hat{\mathbf{\Phi}}^{(j)} = (\mathbf{A}'_j \mathbf{A}_j)^{-1}(\mathbf{A}'_j \mathbf{Z}_j).$$

The residual variance-covariance matrix of the jth regime can be obtained by

$$\hat{\sum}_j = \frac{1}{n_j} \sum_{t=1}^{n_j} \{\hat{\mathbf{a}}_{(\pi_{j-1}+t)+d} \hat{\mathbf{a}}'_{(\pi_{j-1}+t)+d}\}.$$

The AIC of the bivariate fitted TAR model in (1) is then defined as

$$AIC(p,d,k,R_k) = \sum_{j=1}^{k} \{n_j \ln\left|\hat{\sum}_j\right| + 2k(kp+1)\}. \tag{10}$$

Given p and d, we can search for the parameters k and R_k by minimising the AIC. Due to the computational complexity and the possible interpretations of the final model, we restrict k to be a small number, such as 2 or 3. For the threshold parameters R_k, we divide the data into subgroups according to the empirical percentiles of y_{t-d} and use the AIC to select the r values. Finally, the AIC is used to refine the AR order ($p_k \le p$) in each regime. To guard against the incorrect specification of the model, a detailed diagnostic analysis of the residuals is required. This includes an examination of the plots of the standardised residuals and the sample cross-correlation (SCC) matrices of the residuals (Tiao and Box, 1981).

(Insert Figure 2 about here)

4. Results

To distinguish the threshold variables from the earlier discussion on the CIP based forward and spot rates, consider the threshold variable z_t, where *JXmf* = the Japanese Yen forward price for maturity X = 3 and 6 months, and *JXmI* = the Japanese Yen Implied forward price based on CIP (Equation 3), for X = 3 and 6 months. Then allow the following first differences of these variables where $y_{1t} = ln(JXmf)_t - ln(JXmf)_{t-1}$; $y_{2t} = ln(JXmI)_t - ln(JXmI)_{t-1}$; then $z_t = ln(JXmf)_t - ln(JXmI)_t$. Examples of these variables are provided in Figure 2, for the 6 month case. The time varying properties of these series are clearly seen in Figure 2.

(Insert Table 1 about here)

With 6,398 effective observations, the descriptive statistics of the data series used in the BTAR analysis are first computed and presented in Table 1. Typical of existing studies of financial series all the return series (3 and 6 month) are characterized by significant negative skewness and leptokurtosis. It is also important to determine whether the implied series contains a unit root, or are nonstationary, since this will directly affect the regressions. We rely upon conventional Augmented Dickey-Fuller tests to investigate this issue and the results (not reported in the Table) are consistent with the two estimated return series (y_{1t}, y_{2t}) and the threshold variable (z_t) being characterized as *I(1)* stationary, which is consistent with much previous research in financial markets (e.g., Chan and Chung, 2005).

(Insert Table 2 and 3 about here)

We begin the analysis by presenting the results for the 3 and 6-month data set. We first examine the partial autoregression matrices (PAM) of the observed bivariate vector time series. The SCC and PAM matrices are complex when the dimension of the vector is increased, with crowded figures often making recognition of patterns difficult. To alleviate this problem, Tiao and Box (1981) suggest summarizing these matrices using indicator symbols + or – and ·, where + denotes a value greater than twice the estimated standard error, – denotes a value less than twice the estimated standard error, and · denotes an insignificant value based on the above criteria. As an example of the procedure that was followed the indicator matrices for the 6-month PAM are provided in Table 2. There is a clear "cut-off" pattern of the PAM matrices after l = 9 that suggests $p = 9$ for the *C(d)* test for nonlinearity. To confirm this finding we perform the *C(d)* test for $p = 9$, $d \leq$ p and $m = 320$. The results for the 6-month data are given in Table 3. The results clearly reject the linear hypothesis. The test statistics also suggest using the delay parameter d = 2.

(Insert Table 4 about here)

In practice, a two-regime or three-regime BTAR model is often adequate, i.e., $k = 2$ or 3. Given p, d, and k, we use a grid search method and select the thresholds by minimizing the AIC values that are defined in (7). Table 4 provides this procedure for the 6-month data, with the AIV values in the right hand column. First, consider a two-regime ($k = 2$) BTAR models for the data where the upper panel shows the selected threshold values under different combinations of ($p; d$) when $k = 2$. It indicates that the minimum AIC is -152220.18 when $p = 9$; $d = 2$ and ^*r1* = -0.0011.The lower panel provides the results for three-regime models (k = 3). When considering two and three-regime models, the overall minimum AIC is -152926.55 when $k = 3$; $p = 9$; $d = 2$ and ^r1 = -0.001067 and ^r2 = 0.000250. It should be noted that we had previously specified $p = 9$ and d = 2 from Tables 2 and 3. The results in Table 4 for the 6-month data further support the specification of $p = 9$ and $d = 2$ for the BTAR models (The results for the 3-month data are not provided).

(Insert Table 5 and 6 about here)

Next, we further refine each model by allowing different AR orders for different regimes. In the case of the 6-month model the AIC selects (*p1; p2; p3*) = (9; 7; 3). The final minimized AIC value is -152872.24. Least squares estimation results of the specified 6-month data model are given in Table 5 with the final fitted model provided in Table 6. Table 7 repeats the estimation results for the 3-month data, with Table 8 providing the fitted model. The indicator matrices for the residual sample cross-correlations and the residual PAM are examined, and they do not show any model inadequacy.

(Insert Table 6 and 7 about here)

5. Discussion

Using the bivariate threshold autoregressive modeling framework shown in Tables 5 and 7, three regimes can be identified for the dynamic structure of the 3-month and the 6-month CIP series. For the 6-month series the first regime exists when $z_{t-2} \leq -0.001067$, the second exists between -0.00167 and 0.000250 $\{-0.001067 < z_{t-2} \leq 0.000250\}$. The third regime exists when $z_{t-2} > 0.000250$. In term of frequencies, the first regime occupies about ten percent of the time (626 out of 6,396 observations) while the second regime consists of eighty four percent of the sample period (5,383 out of 6,396 observations). The third regime occurs most infrequently and occupies six percent of the time (385 out of 3,572 observations).

There are also three regimes for the 3-month series: The first regime exists when $z_{t-2} \leq -0.00042271$ (which is a smaller negative number than this regime in 6-month series), the second exists between -0.00042271 and 0.00016762 $\{-0.00042271 < z_{t-2} \leq 0.0001672\}$. The third regime exists when $z_{t-2} > 0.000016762$. In term of frequencies, the first regime for the three month series occupies about twenty percent of the times (1310 out of 6,395 observations) while the second regime consists of seventy four percent of the sample period (4,760 out of 6,395 observations). Like the 6-month series, the third regime occurs most infrequently and occupies five percent of the time (325 out of 3,572 observations). These findings are all consistent with arbitrage related transaction bands.

To provide additional economic meaning to the interpretation of the three regimes we also conduct one-way Analysis of Variance (ANOVA) tests on the relationship between the three regimes identified from the BTAR model and δ_m from Equation (4). This comparison has the added advantage of being readily understood in an economic sense given that δ_m is in exchange rate basis points.

Beginning with the 6-month series, the average δ_m for the three regimes is -0.122 (regime 1 $\sigma = 0.185$), -0.046 (regime 2 $\sigma = 0.137$) and -0.011 (regime 3 $\sigma = 0.188$). The *F-statistic* of difference in the means is 92.7 ($p = 0.000$). In this case regime two is characterized by less negative returns and much lower variance than regime one and lower variance than regime three. For the 3-month series, the average δ_m for the three regimes is -0.055 (regime 1 $\sigma = 0.149$), -0.026 (regime 2 $\sigma = 0.111$) and -0.037 (regime 3 $\sigma = 0.227$). The *F-statistic* of difference in the means is 26.5 ($p = 0.000$). Note that in both the three and six month case, regime one offers the greatest potential profit to arbitrageurs. For the 3-month case regime two is characterized by less negative returns and much lower variance than regime one and three, while regime three is characterized by higher variance than either regime one or three. These results suggest that regime variance as well as potential returns may be a driving factor in leaving a potential arbitrage unexploited.

Insert Figure 3 about here)

One final point worthy of mention is the irregularity of the occurrences of the regimes over time. Figure 3 graphs the frequency of regime one and three occurrences over the period from 1982-2008. The irregularity in annual occurrences is evident from significance differences in the annual frequency of the three-regimes in the six and three month maturities (χ^2 test = 995.0, with DF =50, *p*-value = 0.000; and χ^2 test = 1087.85, DF =50, *p*-value = 0.000 respectively). The Figure clearly shows the time-varying nature of the regime frequency with the greatest frequency during the Asian Crisis period of 1998-1998. This result is also consistent with Taylor's (1989) prediction that there would be divergence from CIP equilibrium during periods of market turbulence.

6. Conclusion

We examine the dynamics of the Covered Interest Parity (CIP) relationship between the actual and implied forward price using Bivariate Threshold AutoRegressive (BTAR) modeling for different maturities in the US dollar-Japanese yen forward market. This approach estimates a threshold variable, which is easily expressed in exchange rate basis points, that can changes from negative to positive depending on the direction of the CIP arbitrage. The study highlights the dynamic structure of CIP arbitrage and offers four major findings:

First, three regimes are identified for the 3 and 6-month series, which also coincide with significant differences in frequency, with regime two the most frequent in both cases. This regime is also characterized by low variance. The presence of these regimes is consistent with existing theories of the presence of a trading band of white noise around a parity price. Second, while arbitrage appears to be bidirectional the lower frequency and lower negative values present in regime one, is also consistent with previous studies that highlight the advantage that those with access to US dollar borrowings have in exploiting arbitrage in international markets: the lower spreads are only available to those who can sell US dollars spot and borrow US dollars to achieve hedged yen that can then be invested. These positions also offer the prospect of the greatest economic profit from arbitrage. Third, the results confirm the presence of a time-varying transaction band around the parity price that varies with the maturity of the forward contract.

Finally, the variance of the average price differs within the three regimes, with regime three being the most volatile. Thus volatility of arbitrage (the difficulty of securing hedged positions immediately) in regime three likely affects the ability of those with yen funds that would like to undertake reverse arbitrage positions to those holding US dollars. We speculate that this may be due simply to time zone differences: deep US dollar Euromarkets are not open until the end of Japanese trading, whereas London based yen Euromarkets are open during morning trading in both currency and money markets in the United States.

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Table 1: Descriptive Statistics of the BTAR Variables

Variable	Mean	Standard Deviation	Minimum	Maximum	Skewness	Kurtosis
zt_12 month	-0.00082	0.00145	-0.01966	0.016297	-0.16	28.2
y1_12 month	-0.00013	0.006689	-0.0552	0.034961	-0.55	4.76
y2_12 month	-0.00013	0.006673	-0.05415	0.037678	-0.5	4.51
zt_6 month	-0.00041	0.001086	-0.01032	0.01715	2.06	48.72
y1-6 month	-0.00013	0.006638	-0.05468	0.034206	-0.54	4.74
y2-6 month	-0.00013	0.006631	-0.05518	0.034814	-0.53	4.81
zt-3 month	-0.00025	0.000982	-0.01245	0.016082	1.24	52.68
y1-3 month	-0.00013	0.006622	-0.05511	0.034229	-0.55	4.78
y2-3 month	-0.00013	0.006616	-0.05643	0.034621	-0.53	4.74

The threshold variable z_t, where *JXmf* = the Japanese Yen forward price for maturity X = 3 and 6 months, and *JXmI* = the Japanese Yen Implied forward price based on CIP (Equation 3), for X = 3 and 6 months. Then allowing for first differences of these variables where $y_{1t} = ln(JXmf)_t - ln(JXmf)_{t-1}$; $y_{2t} = ln(JXmI)_t - ln(JXmI)_{t-1}$; *then* $z_t = ln(JXmf)_t - ln(JXmI)_{t-1}$

Table 2. Indicator Matrices for the PAM for the 6-month Data

Lag (l)	1	2	3	4	5	6	7
	$\begin{pmatrix} - & + \\ - & + \end{pmatrix}$	$\begin{pmatrix} - & + \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} - & + \\ + & \cdot \end{pmatrix}$	$\begin{pmatrix} - & + \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} - & + \\ \cdot & \cdot \end{pmatrix}$
Lag (l)	8	9	10	11	12	13	14
	$\begin{pmatrix} - & + \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ + & - \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$	$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$

Table 3. Tests for Non-Linearity for the 6-Month Data

d	1	2	3	4	5
$C(d)$	117.49	134.64	108.89	117.43	73.06
d	6	7	8	9	
$C(d)$	76.34	65.27	62.88	65.45	

*The critical value for the test is $\chi^2_{0.95,38} = 53.4$.

Table 4: Selection of k, p, d and Threshold Values for the 6-month Data

k	p	d	$\hat{r}_1$	$\hat{r}_2$	AIC
***** Two-Regime Models *****					
2	1	1	0.000100		-150741.73
2	2	1	0.000100		-151103.74
2	2	2	-0.001100		-151366.26
2	3	1	0.000100		-151272.63
2	3	2	-0.001100		-151603.19
2	3	3	0.000100		-151222.49
2	4	1	-0.001000		-151422.23
2	4	2	-0.001100		-151797.35
2	4	3	0.000100		-151370.18
2	4	4	-0.000900		-151504.91
2	5	1	-0.001000		-151524.26
2	5	2	-0.001000		-151871.66
2	5	3	0.000100		-151415.27
2	5	4	-0.000900		-151563.97
2	5	5	-0.000400		-151492.76
2	6	1	-0.001000		-151906.81
2	6	2	-0.001000		-152084.00
2	6	3	0.000100		-151768.81
2	6	4	-0.000900		-151964.87
2	6	5	-0.000400		-151933.76
2	7	1	-0.001000		-151973.39
2	7	2	-0.001000		-152149.84
2	7	3	0.000100		-151818.86
2	7	4	-0.000900		-152040.95
2	7	5	-0.000400		-152013.13
2	8	1	-0.001100		-151997.25
2	8	2	-0.001000		-152180.04
2	8	3	0.000100		-151844.99
2	8	4	-0.000900		-152081.02
2	8	5	-0.000400		-152056.22
2	9	1	-0.000600		-152036.26
2	9	2	-0.001100		-152220.18
2	9	3	-0.001000		-151889.21
2	9	4	-0.000900		-152123.27
2	9	5	-0.000400		-152100.54
***** Three-Regime Models *****					
3	9	1	-0.001465	0.000674	-152904.29
3	9	2	-0.001067	0.000250	-152926.55
3	9	3	-0.001003	0.000642	-152360.97

Table 5: Estimation Results for the 6-month Data

The estimated coefficients $\widehat{\mathbf{\Phi}}_j^{(k)}$

(a) The first regime ($k = 1, p_1 = 9$), $n_1 = 627$ when $z_{t-2} \leq -0.001067$

Lag(j)	0	1	2	3	4	5
	$\begin{pmatrix} 0.00 \\ 0.00 \end{pmatrix}$	$\begin{pmatrix} -0.36^* & 0.43^* \\ 0.44^* & -0.37 \end{pmatrix}$	$\begin{pmatrix} -0.31 & 0.28 \\ 0.43 & -0.47 \end{pmatrix}$	$\begin{pmatrix} -0.41 & 0.41 \\ 0.21 & -0.20 \end{pmatrix}$	$\begin{pmatrix} -0.62^* & 0.58^* \\ -0.12 & 0.09 \end{pmatrix}$	$\begin{pmatrix} -0.58 & 0.63^* \\ -0.21 & 0.25 \end{pmatrix}$

Lag(j)	6	7	8	9
	$\begin{pmatrix} -0.49 & 0.39 \\ -0.17 & 0.06 \end{pmatrix}$	$\begin{pmatrix} -0.07 & 0.07 \\ 0.21 & -0.22 \end{pmatrix}$	$\begin{pmatrix} 0.00 & 0.00 \\ 0.29 & -0.29 \end{pmatrix}$	$\begin{pmatrix} 0.59^* & -0.57^* \\ 0.70^* & -0.68^* \end{pmatrix}$

(b) The second regime ($k = 2, p_2 = 7$), $n_2 = 5376$ when $-0.001067 < z_{t-2} \leq 0.000250$

Lag(j)	0	1	2	3	4	5
	$\begin{pmatrix} 0.00 \\ 0.00 \end{pmatrix}$	$\begin{pmatrix} -0.85^* & 0.86^* \\ 0.01 & 0.01 \end{pmatrix}$	$\begin{pmatrix} -0.09 & 0.11 \\ 0.08 & -0.06 \end{pmatrix}$	$\begin{pmatrix} -0.20 & 0.20 \\ -0.06 & 0.06 \end{pmatrix}$	$\begin{pmatrix} -0.11 & 0.12 \\ 0.00 & 0.00 \end{pmatrix}$	$\begin{pmatrix} -0.24 & 0.22 \\ -0.18 & 0.16 \end{pmatrix}$

Lag(j)	6	7
	$\begin{pmatrix} -0.28^* & 0.29^* \\ -0.23 & 0.24^* \end{pmatrix}$	$\begin{pmatrix} -0.23^* & 0.25^* \\ -0.20^* & 0.22^* \end{pmatrix}$

(c) The third regime ($k = 3, p_3 = 3$), $n_3 = 385$ when $z_{t-2} > 0.000250$

Lag(j)	0	1	2	3
	$\begin{pmatrix} 0.00 \\ 0.00 \end{pmatrix}$	$\begin{pmatrix} -0.89^* & 0.95^* \\ -0.34 & 0.39^* \end{pmatrix}$	$\begin{pmatrix} -0.61^* & 0.62^* \\ -0.09 & 0.14 \end{pmatrix}$	$\begin{pmatrix} -0.40^* & 0.46^* \\ -0.16 & 0.20 \end{pmatrix}$

An asterisk indicates that the estimate is significant at the 5% level.

Table 6: The Fitted Model for the 6-month Data.

For the Regime 1 ($n_1 = 627$): when $z_{t-2} \leq -0.001067$

$$\begin{aligned} y_{1,t} &= \underset{(0.21)}{-0.36 y_{1,t-1}} \underset{(0.30)}{- 0.62 y_{1,t-4}} \underset{(0.23)}{+ 0.59 y_{1,t-9}} \underset{(0.21)}{+ 0.43 y_{2,t-1}} \underset{(0.29)}{+ 0.58 y_{2,t-4}} \underset{(0.30)}{+ 0.63 y_{2,t-5}} \underset{(0.22)}{- 0.57 y_{2,t-9}} + \varepsilon_{1,t}^{(1)} \\ y_{2,t} &= \underset{(0.21)}{0.44 y_{1,t-1}} \underset{(0.23)}{+ 0.70 y_{1,t-9}} \underset{(0.22)}{- 0.68 y_{2,t-9}} + \varepsilon_{2,t}^{(1)} \end{aligned}$$

The above two equations are interacted via the residual variance-covariance matrix:

$$\begin{pmatrix} 0.000056 & 0.000057 \\ 0.000057 & 0.000061 \end{pmatrix}$$

For the Regime 2 ($n_2 = 5376$): when $-0.001067 < z_{t-2} \leq 0.000250$

$$\begin{aligned} y_{1,t} &= \underset{(0.10)}{-0.85 y_{1,t-1}} \underset{(0.12)}{- 0.28 y_{1,t-6}} \underset{(0.09)}{- 0.23 y_{1,t-7}} \underset{(0.10)}{+ 0.86 y_{2,t-1}} \underset{(0.12)}{+ 0.29 y_{2,t-6}} \underset{(0.09)}{+ 0.25 y_{2,t-7}} + \varepsilon_{1,t}^{(2)} \\ y_{2,t} &= \underset{(0.09)}{-0.20 y_{1,t-7}} \underset{(0.12)}{+ 0.24 y_{2,t-6}} \underset{(0.12)}{+ 0.22 y_{2,t-7}} + \varepsilon_{2,t}^{(1)} \end{aligned}$$

The above two equations are interacted via the residual variance-covariance matrix:

$$\begin{pmatrix} 0.000042 & 0.000042 \\ 0.000042 & 0.000042 \end{pmatrix}$$

For the Regime 3 ($n_3 = 385$): when $z_{t-2} > 0.000250$

$$\begin{aligned} y_{1,t} &= \underset{(0.17)}{-0.89 y_{1,t-1}} \underset{(0.19)}{- 0.61 y_{1,t-2}} \underset{(0.19)}{- 0.40 y_{1,t-3}} \underset{(0.17)}{+ 0.95 y_{2,t-1}} \underset{(0.19)}{+ 0.62 y_{2,t-2}} \underset{(0.19)}{+ 0.46 y_{2,t-3}} + \varepsilon_{1,t}^{(2)} \\ y_{2,t} &= \underset{(0.17)}{0.39 y_{2,t-1}} + \varepsilon_{2,t}^{(1)} \end{aligned}$$

The above two equations are interacted via the residual variance-covariance matrix:

$$\begin{pmatrix} 0.000034 & 0.000033 \\ 0.000033 & 0.000035 \end{pmatrix}$$

Table 7: Estimation Results for the 3-month Data

The estimated coefficients $\widehat{\mathbf{\Phi}}_j^{(k)}$

(a) The first regime ($k = 1, p_1 = 9$), $n_1 = 1309$ when $z_{t-2} \leq -0.00042271$

Lag(j)	0	1	2	3	4	5
	$\begin{pmatrix} 0.00 \\ 0.00 \end{pmatrix}$	$\begin{pmatrix} -0.76^* & 0.82^* \\ 0.13 & -0.08 \end{pmatrix}$	$\begin{pmatrix} -0.45^* & 0.40 \\ 0.40 & -0.44^* \end{pmatrix}$	$\begin{pmatrix} -0.62^* & 0.64^* \\ 0.14 & -0.12 \end{pmatrix}$	$\begin{pmatrix} -0.70^* & 0.66^* \\ -0.05 & 0.01 \end{pmatrix}$	$\begin{pmatrix} -0.93^* & 0.91^* \\ -0.41 & 0.39 \end{pmatrix}$
Lag(j)		6	7	8	9	
		$\begin{pmatrix} -1.03^* & 0.99^* \\ -0.64^* & 0.60^* \end{pmatrix}$	$\begin{pmatrix} -0.41 & 0.41 \\ -0.10 & 0.10 \end{pmatrix}$	$\begin{pmatrix} -0.43 & 0.45 \\ -0.19 & 0.21 \end{pmatrix}$	$\begin{pmatrix} -0.05 & 0.01 \\ 0.12 & -0.16 \end{pmatrix}$	

(b) The second regime ($k = 2, p_2 = 7$), $n_2 = 4754$ when $-0.00042271 < z_{t-2} \leq 0.00016762$

Lag(j)	0	1	2	3	4	5
	$\begin{pmatrix} 0.00 \\ 0.00 \end{pmatrix}$	$\begin{pmatrix} -1.02^* & 1.04^* \\ -0.11 & 0.12 \end{pmatrix}$	$\begin{pmatrix} -0.02 & 0.06 \\ -0.01 & 0.05 \end{pmatrix}$	$\begin{pmatrix} -0.09 & 0.09 \\ -0.07 & 0.06 \end{pmatrix}$	$\begin{pmatrix} 0.04 & -0.03 \\ 0.09 & -0.08 \end{pmatrix}$	$\begin{pmatrix} -0.11 & 0.10 \\ -0.08 & 0.08 \end{pmatrix}$
Lag(j)		6	7			
		$\begin{pmatrix} -0.16 & 0.17 \\ -0.13 & 0.13 \end{pmatrix}$	$\begin{pmatrix} -0.27^* & 0.29^* \\ -0.27^* & 0.29^* \end{pmatrix}$			

(c) The third regime ($k = 3, p_3 = 3$), $n_3 = 325$ when $z_{t-2} > 0.00016762$

Lag(j)	0	1	2	3
	$\begin{pmatrix} 0.00 \\ 0.00 \end{pmatrix}$	$\begin{pmatrix} -0.77^* & 0.87^* \\ -0.24 & 0.34 \end{pmatrix}$	$\begin{pmatrix} -0.58^* & 0.58^* \\ -0.09 & 0.12 \end{pmatrix}$	$\begin{pmatrix} -0.31 & 0.32 \\ -0.10 & 0.09 \end{pmatrix}$

An asterisk indicates that the estimate is significant at the 5% level.

Table 8: The Fitted Model for the 3-month Data.

For the Regime 1 ($n_1 = 1309$): when $z_{t-2} \leq -0.00042271$

$$\begin{aligned} y_{1,t} &= \underset{(0.17)}{-0.76}y_{1,t-1} - \underset{(0.22)}{0.45}y_{1,t-2} - \underset{(0.25)}{0.62}y_{1,t-3} - \underset{(0.28)}{0.70}y_{1,t-4} - \underset{(0.29)}{0.93}y_{1,t-5} - \underset{(0.28)}{1.03}y_{1,t-6} + \underset{(0.17)}{0.82}y_{2,t-1} \\ &\quad + \underset{(0.25)}{0.64}y_{2,t-3} + \underset{(0.28)}{0.66}y_{2,t-4} + \underset{(0.29)}{0.91}y_{2,t-5} + \underset{(0.28)}{0.99}y_{2,t-6} + \varepsilon_{1,t}^{(1)} \\ y_{2,t} &= \underset{(0.28)}{-0.64}y_{1,t-6} - \underset{(0.22)}{0.44}y_{2,t-2} + \underset{(0.28)}{0.60}y_{2,t-6} + \varepsilon_{2,t}^{(1)} \end{aligned}$$

The above two equations are interacted via the residual variance-covariance matrix:

$$\begin{pmatrix} 0.000054 & 0.000053 \\ 0.000053 & 0.000054 \end{pmatrix}$$

For the Regime 2 ($n_2 = 4757$): when when $-0.00042271 < z_{t-2} \leq 0.00016762$

$$\begin{aligned} y_{1,t} &= \underset{(0.11)}{-1.02}y_{1,t-1} + \underset{(0.11)}{1.04}y_{2,t-1} - \underset{(0.10)}{0.27}y_{1,t-7} + \underset{(0.10)}{0.29}y_{2,t-7} + \varepsilon_{1,t}^{(2)} \\ y_{2,t} &= \underset{(0.10)}{-0.27}y_{1,t-7} + \underset{(0.10)}{0.29}y_{2,t-7} + \varepsilon_{2,t}^{(1)} \end{aligned}$$

The above two equations are interacted via the residual variance-covariance matrix:

$$\begin{pmatrix} 0.000040 & 0.000040 \\ 0.000040 & 0.000041 \end{pmatrix}$$

For the Regime 3 ($n_3 = 325$): when $z_{t-2} > 0.00016762$

$$\begin{aligned} y_{1,t} &= \underset{(0.19)}{-0.77}y_{1,t-1} - \underset{(0.19)}{0.58}y_{1,t-2} + \underset{(0.22)}{0.87}y_{2,t-1} + \underset{(0.21)}{0.58}y_{2,t-2} + \varepsilon_{1,t}^{(2)} \\ y_{2,t} &= \varepsilon_{2,t}^{(1)} \end{aligned}$$

Figure 1: The mechanics of Covered Interest Parity (CIP) arbitrage using the US$-Yen spot, forward and Euro-interest rates

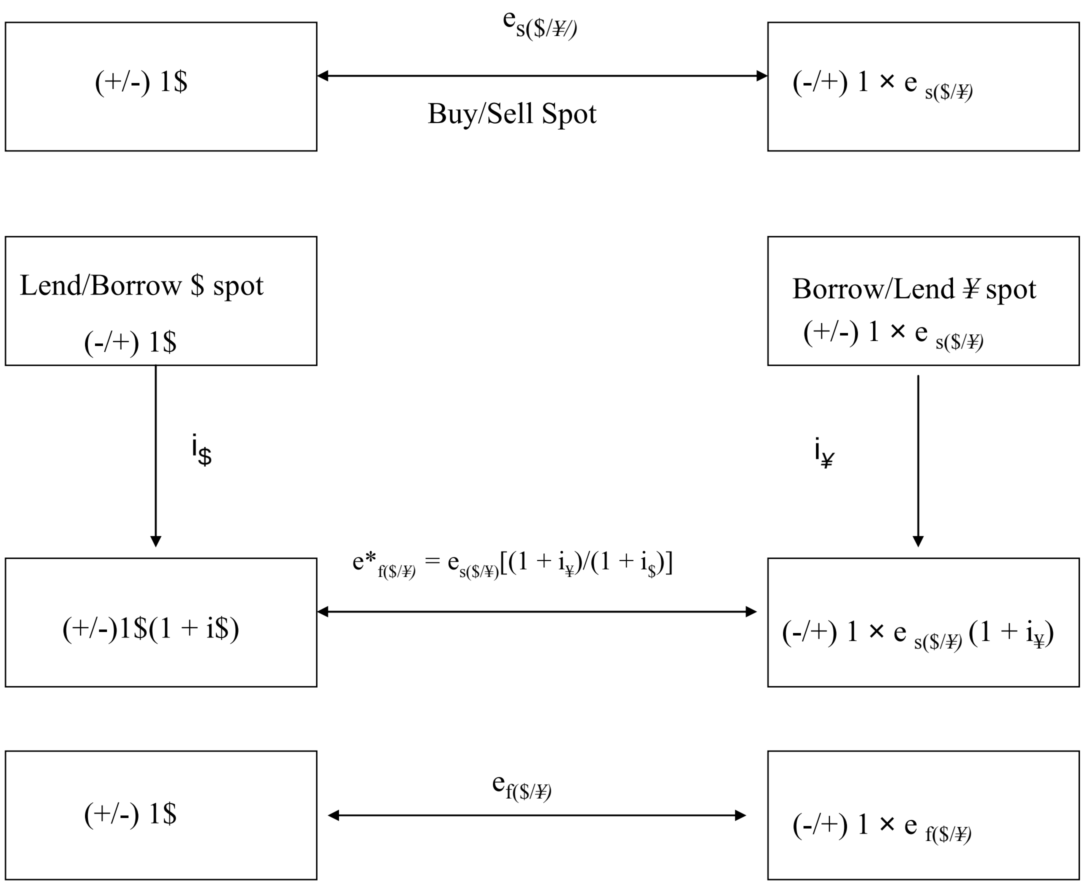
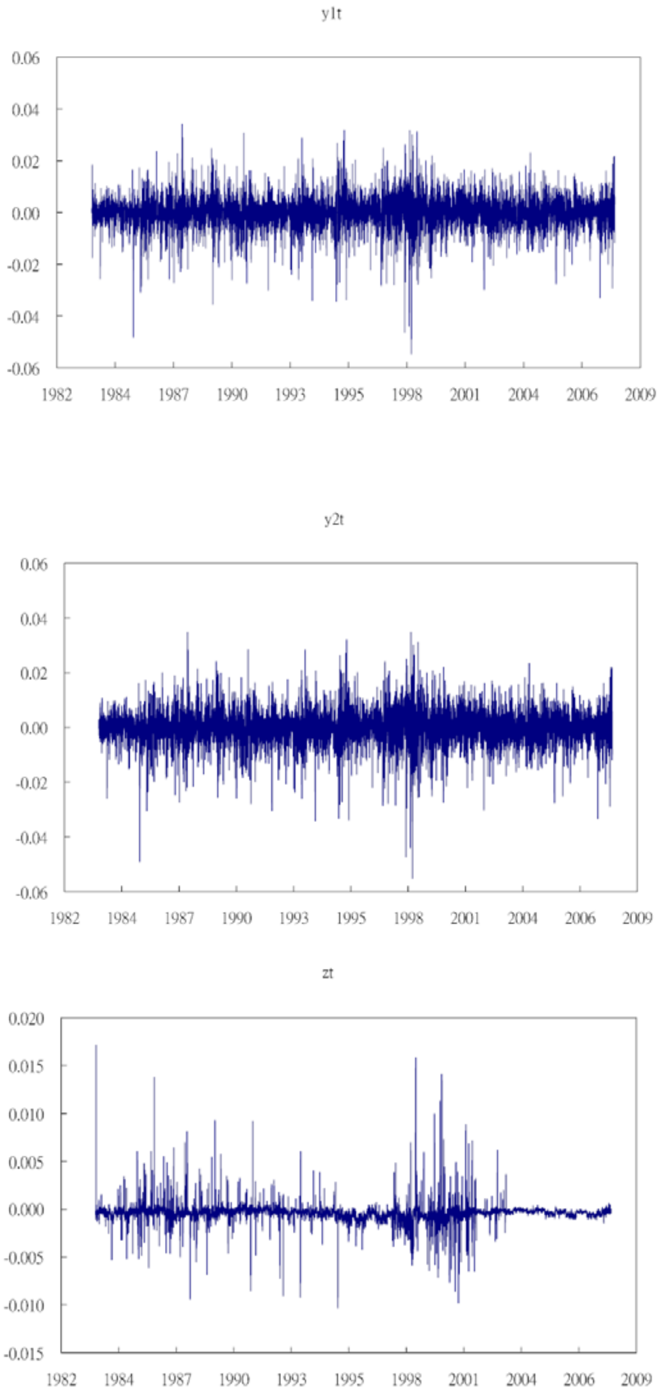


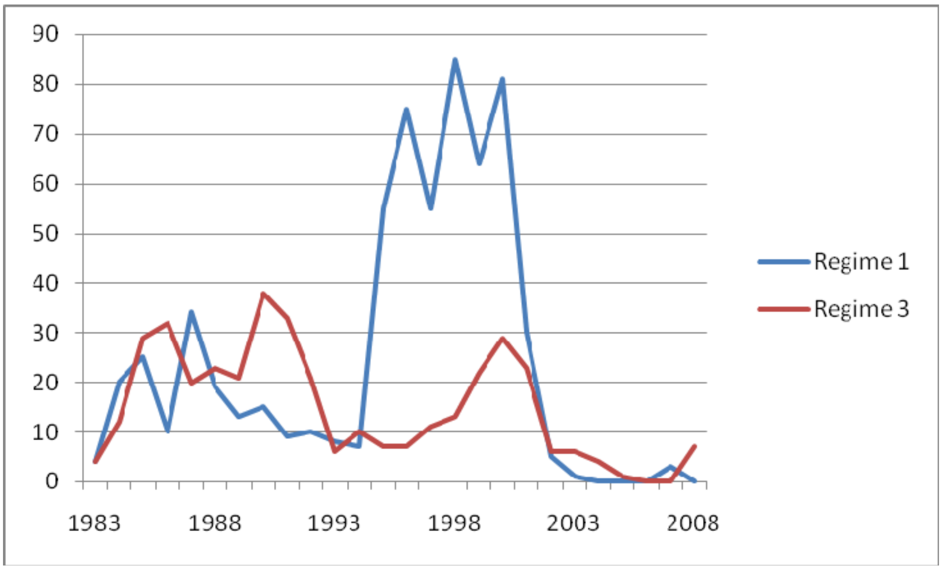
Figure 2: Plots of First Differences of the Forward, Implied Forward and Threshold Variable for the 6-month US$/¥.



Note: $y_{1t} = ln(JXmf)_t - ln(JXmf)_{t-1}$; $y_{2t} = ln(JXmI)_t - ln(JXmI)_{t-1}$; $z_t = ln(JXmf)_t - ln(JXmI)_t$.

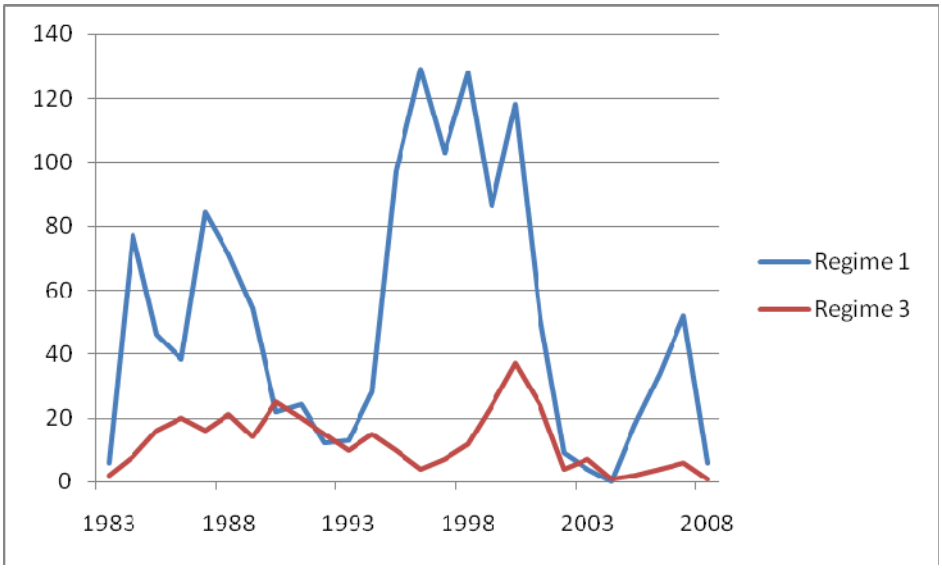
Figure 3: Frequency of the BTAR Regime One and Regime Three for the Period 1982-2008

6-Month US$/¥



Note a χ^2 test of Annual Differences in the Frequency of the Three-Regimes is 995.0, DF =50, p-value = 0.000

3-Month US$/¥



Note a χ^2 test of Annual Differences in the Frequency of the Three Regimes is 1087.85, DF =50, p-value = 0.000