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ARBITRAGE PRICING THEORY IN INTERNATIONAL
MARKETS¹

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ABSTRACT

In this paper we study the impact of multiple pre-specified sources of risk in the return of three non-overlapping groups of countries through an Arbitrage Pricing Theory (APT) model. The groups are composed of emerging and developed markets. Two strategies are used to choose two set of risk factors. The first one is to use macroeconomic variables, often cited by the relevant literature, such as the world excess return, exchange rates, the change in the TED spread and the variation in the oil price. The second strategy is to extract the factors from a principal component analysis, designated as statistical factors. The first important result from our work is the great resemblance between the first statistical factor and the world excess return. We estimate our APT model using two statistical methodologies: Iterated Nonlinear Seemingly Unrelated Regression (ITNLSUR) by McElroy and Burmeister (1988) and the Generalized Method Moments (GMM) by Hansen (1982). The results from both methods are very similar. In general, in the model with macroeconomic variables, only the world excess of return is priced with a premium varying from 4.4% to 6.3% per year and, in the model with statistical variables, only the first statistical factor is priced with a premium varying from 6.2% to 8.5% per year.

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1. INTRODUCTION

In this work we develop a study about the common sources of risk that can impact the changes in equity return of several different countries. A database from Morgan Stanley Capital International containing 24 developed markets, 16 emerging markets and 4 frontier markets from December 1992 to December 2009 is available. This is an originally approach because, unlike previous related works, we are dealing with developed, emerging and frontier countries together.

In order to account for multiple sources of risk, an empirical analysis of the Arbitrage Pricing Theory (APT) model developed by Ross (1976) will be performed. The Iterated Nonlinear Seemingly Unrelated Regression by McElroy and Burmeister (1988) and the Generalized Method of Moments by Hansen (1982) will be the econometric methodologies applied to calculate the average risk premiums of the global sources of risk. Both methods are strongly consistent and asymptotically normal even in the absence of normal errors and they overcome the problems presented by the usual Two Step Procedure by Fama and Macbeth(1973). The Generalized Method of Moments still presents the advantage of comprising the available information in the estimation process.

The main difficulty associated with the APT model is that the theory doesn't say anything about which risk factors should be included. In a global framework that contains countries with so many disparities, it's even harder to imagine what the common sources of risk are. To better address this problem, two sets of variables were treated as potential risk measures. The first set is composed of macroeconomic variables frequently cited by the relevant literature such as the excess return of the world portfolio, changes in the exchange rates, variation in the spread between Eurodollar deposit tax and U.S. Treasury bill (TED spread) and changes in the Oil Price. The second group of potential risk measures was obtained in a more unusual way. Following Campbell, Lo and MacKinlay (1997), for the 44 countries equity returns in our database, a Principal Component Analysis were performed allowing us to compose five portfolios pointing the directions of greatest variability of the original return data. That means, the first portfolio is in the direction of higher variance, the

second portfolio, in the direction of second higher variance and so on. These portfolios are regarded as sources of risk and denominated statistical risk factors.

First of all, we investigate the relation among statistical and macroeconomic sources of risk. We discovered a great similarity between the factor extracted from the first principal component and the world excess return. Second, for a robustness analysis, we divided the countries into three groups with similar geographic distribution and calculated the risk premiums for the statistical and macroeconomic factors separately. In general, in the model with macroeconomic variables, only the world excess of return is priced with a premium varying from 4.4% to 6.3% per year and, in the model with statistical variables, only the first statistical factor is priced with a premium varying from 6.2% to 8.5% per year. Other variables can present significant risk premiums, but the results are sensible to the group and method of estimation considered. However, the inclusion of more variables tends to reduce the average pricing error.

This paper is organized as follows. Section 2 review the related literature, Section 3 presents the model and econometric methods, Section 4 describes the countries equity return data, Section 5 explain the choice for the potential risk measures, Section 6 presents our empirical results and in Section 7 we highlight our conclusions.

2. LITERATURE REVIEW

Stephen Ross (1976) derived rigorously the Arbitrage Pricing Theory model (APT), whose starting premises are that markets are competitive and that individuals homogeneously believe that the return of all assets in the economy are driven by a linear structure of k risk factors.

The APT model represented an answer to criticisms suffered by the popular Capital Asset Pricing Model (CAPM), of Sharpe (1964), Lintner (1965) and Treynor (1961). CAPM establishes a linear relation between the excess assets' return and a single risk factor – the excess return on the market portfolio. It assumes that all assets can be held by an individual investor. Although it can be considered a particular case of APT, the theoretical construction of CAPM requires normality of returns or quadratic utility function, what isn't always easy to justify. Besides, it can be proved that any mean-variance portfolio satisfies exactly the CAPM equation. So, testing the CAPM is equivalent to testing the mean-variance efficiency of the market portfolio. However, the true set of all investment opportunities would include everything with worth. There are some assets, human capital for example, that are non-tradable. Nevertheless, transaction costs and market frictions can preclude individuals from owning the portfolio of all marketable assets. Those facts originated the famous Roll's critique (1977), which states that CAPM isn't empirically testable as the true market portfolio can't be observed and is substituted by its proxy. The market portfolio proxy isn't necessarily mean-variance efficient, even if the real market is and the contrary is also true.

In opposition to CAPM, APT allows for multiples risk factors, accounting for various sources of non diversifiable risks. The market portfolio doesn't have any special importance and can be or not included as a risk factor. It's not necessary to assume any hypothesis related to the returns' distribution or the individuals' utility function. The model proposed by Ross, however, doesn't specify which the risk factors are. Several empirical works focused on the attempt to determine them through two different strands: using pre-specified observed macroeconomic factors or assuming that, a priori, the factors were unknown.

For equities from the United States economy, the empirical work of Roll and Ross (1980) adopted the second strand. The authors used a statistical technique denominated factor analysis to extract the risk factors and estimate the sensitivity's coefficients. They conclude that at least three factors were important for pricing the assets. A clear interpretation for those risk factors isn't available, though. Also, an investigation about the return's individual variance revealed that, although expected returns are highly correlated with their respective variance, the variance itself doesn't add any explanatory power to the factors previously estimated in the APT.

Chen, Roll and Ross (1986) used macroeconomic variables to estimate an APT applying the two-pass methodology from Fama and Macbeth (1973). Based on Financial theories they choose the following variables: the spread between long and short run interest rate, expected and unexpected inflation, industrial production, the spread between high and low grade bonds, market portfolio, aggregated consumption and oil price. However, only the first four variables were found to be significantly priced.

Still working with data from U.S. economy, McElroy and Burmeister (1988) employed a new methodology to estimate an APT with macroeconomic variables. The Iterated Nonlinear Seemingly Unrelated Regression (ITNLSUR), which will be further discussed in the Section 3, presents several advantages over factor analysis and the Fama and Macbeth two-pass procedure. ITNLSUR overcomes the econometric problems of previous methodologies such as loss of efficiency, non uniqueness of the second step and unrobustness of the estimate if the errors are not normally distributed. Estimators obtained from ITNLSUR are strongly consistent and asymptotically normal, despite the distribution of the errors. The five macroeconomic factors adopted by McElroy and Burmeister were the spread between 20 years government and corporate bonds portfolios, the excess return of 20 years government bond portfolios over the one month Treasury bill, an unexpected deflation series, an expected growth in sales and the S&P 500 index. Although significant risk prices were found to all of them, the authors warning that there isn't justification for which or how many factors to use and nothing suggests the existence of just one set of variables with important role in asset pricing.

The APT model was also expanded to an international framework and this application is the one that will be used throughout this work. Solnik (1983) provides

an analysis of the model developed by Ross (1976) when investors from different countries are considered. The author argues that the models of international asset pricing used until that moment were controversial due to different hypothesis for the utility function and sources of uncertainty. International Arbitrage Pricing Theory (IAPT) is an alternative, since it isn't based in any hypothesis about the utility function and only requires perfect capital market. The article shows that (1) every riskless portfolio will be riskless to any foreigner investor and (2) if the linear factor model is believed to hold in one given currency, it must also be valid in any arbitrarily currency chosen as numeraire.

Ikeda (1991) discuss the introduction of foreign exchange risk when adapting the APT model developed for closed economies for an international framework. The author concludes that if the return generation process is specified in a numeraire currency, the foreign exchange risk is automatically diversified away. Previous works of Solnik (1974), Stulz (1981) and Adler and Dumas (1983), however, stated that, under deviation from purchasing power parity, the foreign exchange rate must be priced.

Ferson and Harvey (1993) applied a multifactor model to study the cross section difference in the returns of sixteen OECD countries plus Singapore/Malaysia and Hong Kong. Several factors are included in an unconditional version of seemingly unrelated regression model and estimated by Hansen's (1982) Generalized Method of Moments. The authors came to the conclusion that world market beta alone doesn't explain much of the difference among returns and that explanation power is added by a multifactor model. Besides that, significant risk premiums are encountered for the world return and for the trade-weighted U.S. dollar price of the currencies of 10 industrialized countries (G-10 index).

The empirical work of Harvey, Solnik and Zhou (2002) also applies the Generalized Method of Moments and uses return data from sixteen OECD countries plus Singapore/Malaysia and Hong Kong. They are interested, however, not only in explaining cross section differences but also in understanding the time variation in international assets return. They specify an information set to construct a conditional model with factors not pre-specified. The author don't reject that at least two factors would be necessary to explain the conditional variance of the returns. The first factor

is similar to the global market portfolio and the second factor would be related to foreign exchange risk.

3. MODEL

We propose that countries' equity returns are driven by multiple risk factors and follow a multifactor Arbitrage Pricing Theory (APT) model. The main assumptions of APT, formulated by Ross (1976), is that the difference between actual and expected returns on all assets are linearly related to a finite number of risk factors and the number of assets in the economy is large relative to the number of factors. Then, if there are n assets and k risk factors, with $n > k$, the model can be written as:

$$R_i(t) = E_t[R_i(t)] + \sum_{j=1}^K \beta_{ij} f_j(t) + \varepsilon_i(t) \quad (1)$$

$$i=1, \dots, n, \quad t=1, \dots, T,$$

where $E_t[R_i(t)]$ is the expected return of country i conditional to the information available in t ; β_{ij} is the sensitivity of asset i to $f_j(t)$, the j th risk factor realization on time t ; $\varepsilon_i(t)$ is the idiosyncratic risk independent of the k risk factors.

Under restriction of no asymptotic arbitrage and some regularity conditions, the Arbitrage Pricing Theorem states that the expected return is approximated by the relation in equation:

$$E_t[R_i(t)] = \lambda_0(t) + \sum_{j=1}^K \beta_{ij} \lambda_j(t), \quad (2)$$

Where, $\lambda_j(t)$ is the premium obtained by an investor for assuming the risk factor j . If there is a risk free asset in the economy and its return is known at time t , then $\beta_{ij} = 0$, to all j , and $\lambda_0(t)$ can be regarded as the risk free return.

No asymptotic arbitrage condition is necessary instead of simple non arbitrage condition because each asset return has an idiosyncratic risk. If an asymptotic arbitrage opportunity exists, then as n gets larger the idiosyncratic risk can be diversified away and it is possible to create a portfolio of the n risk asset that demands zero net investment and deliver close to a riskless return.

The usual way to estimate this model is using the two stage procedure proposed by Fama and MacBeth(1973). At first the b_{ij} 's are estimated. If the factors are assumed as unknown, one can use factor analysis to extract the b_{ij} 's. In the second stage, the b_{ij} 's are treated as data in order to estimate the risk prices. To

attenuate the error related to the second stage, the assets are grouped into portfolios. Some of the econometric problems associated to this methodology are loss of efficiency, non uniqueness of the second step and unrobustness of the estimate if the errors are not normally distributed.

3.1.ITNLSUR

McElroy and Bumeister (1988) suggested an alternative method to estimate the risk sensitivities, b_{ij} 's, and the risk prices, λ_j 's, simultaneously. First of all, for purposes of estimation, they assumed that $\lambda_1, \dots, \lambda_j$ do not vary over time. Second they substituted (2) into (1) and obtained:

$$R_i(t) - \lambda_0(t) = \sum_{j=1}^K \beta_{ij} (\lambda_j + f_j(t)) + \varepsilon_i(t) \quad (3)$$

$$i=1, \dots, n, \quad t=1, \dots, T,$$

$\lambda_0(t)$, as already mentioned, will be assumed as the risk-free rate. The factors, $f_j, j = 1, \dots, k$, are mean 0. If a chosen factor doesn't have zero mean, f_j will be the risk factor less it's mean.

To estimate the NK β 's and the K λ 's they use a iterated nonlinear seemingly unrelated regression method. It must be assumed that

$$E_t[\varepsilon_i(t)] = 0, \quad E_t[\varepsilon_i(t)\varepsilon_j(s)] = \begin{matrix} \sigma_{ij} & t = s \\ 0, & t \neq s \end{matrix}, \quad E_t[\varepsilon_i(t)|f_j(s)] = 0,$$

Then rewrite the system in matrix form in terms of excess return.

$$\rho_i = [\lambda' \otimes \mathbf{1}_T + \mathbf{F}] \beta_i + \varepsilon_i = \mathbf{X}(\lambda) \beta_i + \varepsilon_i,$$

$$i = 1, \dots, N$$

Where ρ_i is a Tx1 vector of excess return, λ is a Kx1 vector of risk premiums, $\mathbf{1}_T$ is a Tx1 vector of ones, \mathbf{F} is a TxK matrix of the factors and β_i is a Kx1 vector of sensitivities.

And Stacking the N equations,

$$\begin{pmatrix} \rho_1 \\ \vdots \\ \rho_N \end{pmatrix} = \begin{bmatrix} \mathbf{X}(\lambda) & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \mathbf{X}(\lambda) \end{bmatrix} \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_N \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_N \end{pmatrix}$$

Or, in matrix notation, $\rho = [I_N \otimes \mathbf{X}(\lambda)] \beta + \varepsilon$

The NLSUR method follows three steps. At first step λ is not identifiable, so $\lambda' \beta_i$ is replaced by an intercept $\alpha_i, i = 1, \dots, N$, and one estimates $\theta_i = (\alpha_i, \beta_i)$ by OLS.

This step is very similar to the first step of Fama and Macbeth(1973). However, we are not interested in the inference of β 's itself, but in obtaining the residuals. In the second step, the residuals are used to estimate the covariance matrix $\hat{\Sigma} = T^{-1}(e_i'e_j)$. Finally, (λ, β) are taken as the parameter that minimizes the following quadratic expression:

$$Q(\lambda, \beta, \hat{\Sigma}) = \{\rho - [I_N \otimes X(\lambda)]\beta\}' (\hat{\Sigma}^{-1} \otimes I_T) \{\rho - [I_N \otimes X(\lambda)]\beta\},$$

The third step can be iterated until convergence is reached. The residuals, obtained by substitution over the last (λ, β) estimated, are used to update the covariance matrix and, iteratively, we obtain (λ, β) from the minimization of Q. The ITNLSUR estimators are strongly consistent and asymptotically normal, despite the distribution of the errors. If the errors are normally distributed, then these estimators are also maximum likelihood estimators.

The deficiency associated with ITNLSUR is that it only account for heteroskedasticity errors and does not allow the existence of autocorrelation. However, under the efficient market hypothesis, only unexpected events aren't incorporated to the price and so the errors should be serially uncorrelated.

3.2. GMM

The Generalized Method of Moments, by Hansen (1982), has a clear advantage over the ITNLSUR, as it allows the use of all the available information in the estimation process. Starting again from equation (3), we will employ this method to estimate β_{ij} and $\lambda_j, i = 1, \dots, N, J = 1, \dots, k$. As in the ITNLSUR, the GMM doesn't rely on any assumption about the data distribution. It's rather based on the specification of moment conditions.

Considering equation (3), $\theta^* = (\lambda^*, \beta^*)$ as the real population parameter and $g(\cdot)$ a mx1 vector of real functions, the population moment conditions are:

$$\begin{aligned} E[g(w_t, \theta)] &= E[g(\rho(t), f(t), Z(t), \theta^*)] = E[(\rho(t) - \beta^*(\lambda^* + f(t))) \otimes Z(t)] \\ &= [0] \end{aligned} \quad (4)$$

Where, $\rho(t)$ is a Nx1 vector of excess returns, β^* is NxK matrix of sensitivities of asset i to factor j, λ^* is a Kx1 vector of the risk premias, $f(t)$ is a Kx1 vector of the k factors' realization, $Z(t)$ is a vector of instruments that contains a constant, $f(t)$ and the variables that represent the available information set.

The sample counterpart of this moment condition is:

$$g_T(w, \theta) = T^{-1} \sum_{t=1}^T g(w_t, \theta), \quad (5)$$

The GMM estimator is defined as:

$$\hat{\theta}^{GMM} = \underset{\theta}{\operatorname{argmin}} J_T(\theta) = g_T(w, \theta)' W_T g_T(w, \theta) \quad (6)$$

Where W_T is a mxm positive semidefinite matrix that efficiently weights the moments. In order to make the estimation of the model possible, the number of moments should be equal or higher than the number of parameters.

Hansen (1982) showed that efficient estimators are obtained with W_T equals the inverse of moment's long run covariance matrix. As this matrix isn't known, it must also be estimated. Among the ways to solve for that, we chose the iterated GMM. In this approach, we start with \widehat{W}_T equals to the identity matrix and solve for the parameters. Next, using a consistent method, the parameters from the first step are used in the estimation of the covariance matrix. These two steps are repeated until convergence is reached.

Unlike ITNLSUR, depending on the choice for \widehat{W}_T estimation method, GMM allows dealing with heteroskedastic, contemporaneous correlated and serially autocorrelated errors. However, because of the efficient market's assumption and the GMM's poor performance in small sample, autocorrelation isn't going to be treated here. Since we will work with a not too extensive time series, an incorrect arbitrary selection of the number of significant lags can introduce a lot of noise in our estimation. White's covariance matrix is used to construct the weighting matrix robust to heteroskedasticity and contemporaneous correlation of unknown form in the following way:

$$\widehat{\Omega}_w = \frac{1}{T - N} \sum_{t=1}^T g(w_t, \theta) g(w_t, \theta)', \quad (7)$$

3.2.1. Test of Overidentifying Restrictions

Since the number of moments exceeds the number of parameters, the estimated moments won't all equal zero. We can perform an overidentification test,

introduced by Hansen (1982), to evaluate if the moments are sufficiently close to zero. The J test refers to the objective function, presented in equation (6), that we intend to minimize and is defined as:

$$J = T g_T(w, \theta)' W_T g_T(w, \theta) \sim \chi_{m-k}^2, \quad (8)$$

J statistic has a chisquared distribution with degrees of freedom equal to the number of moments in excess to the number of parameters.

The rejection of the overidentification statistic denotes an incorrect specification of the model itself, as it isn't possible to make all the moments conditions sufficiently close to zero. The inclusion of an additional moment, without the rejection of J, indicates that this moment is useful in the estimation of the parameters.

Two problems are associated with this test, though. First, the rejection of the test doesn't give any clue on how is the model mis-specified. Second, there are models with a great number of moments and the inclusion of redundant moments can result in biased or inconsistent estimators. A consensus on the quantity of moments doesn't exist.

4. DATA

4.1. Countries' Equity Index

The equity indices for all the countries considered here are calculated by Morgan Stanley Capital International (MSCI). These indices are measured in US dollar, monthly, with dividends reinvestments in excess of the 30 day Treasury Bill, assumed as the proxy for the risk free asset.

MSCI indices are designed to represent the investable opportunity set for international investor. The methodology does not vary across country and the following characteristics favor its composition according to diversification principles: doesn't have controlled and controllers in the same portfolio to avoid double count; the composition is free float adjusted market capitalization weighted; one sector can't overcome more than 30% of the portfolio composition.

Unlike most researches in this area, we will consider not only developed markets, but also emergent and frontier markets. All the equity indices available by MSCI from December 1992 to December 2009 will be used. This results in 24 developed markets, 16 emerging markets and 4 frontier markets⁵. The three non-overlapping categories of country classification - frontier, emergent or developed - are held by MSCI, following criteria of economic development, size and liquidity and market accessibility. The classification is annually revised.

MSCI also available a value weighted equity index of 24 developed markets and 21 emergent markets, called MSCI All Country World Index. We will use this index as the market portfolio and this is better described in Section 5.1.

4.2. Descriptive Statics

Table 1 presents annualized mean, annualized standard deviation and autocorrelation of the logarithm return for each country and world portfolio. The significant autocorrelations are marked with an asterisk. It's Interesting to note that,

⁵ The 24 developed markets that we will use are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hong-Kong, Ireland, Israel, Italy, Japan, Netherland, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, United Kingdom and United States. The 19 emerging markets are Brazil, Chile, China, Colombia, India, Indonesia, Korea, Malaysia, Mexico, Peru, Philippines, Poland, South Africa, Taiwan, Thailand and Turkey. The 4 frontier markets are Argentina, Jordan, Pakistan and Sri Lanka.

while standard deviations for the developed vary from 15% to 35%, the standard deviation of the emerging vary from 19% to 57%. Brazil has the highest mean return (15.98%) with standard deviation of 41.06%. Turkey has the highest standard deviation (56.68%) with return of 10.78%. United States have the smallest standard deviation (15.37%) with a 4.16% return. Five countries - Ireland, Japan, China, Philippines and Thailand - have negative mean return in the analyzed period.

At APPENDIX 1 we show the correlation matrix for the equity index and the world portfolio. Most of the countries presents high correlation with the world return which is justified by the way this portfolio is constructed. The correlation among developed markets is, usually, higher than the correlation among emerging and between emerging and developed markets.

Table 1: Descriptive Statistics

Index	Mean	Standard Deviation	ρ_1	ρ_2	ρ_3	ρ_4	ρ_{12}	ρ_{24}
WORLD	3.87%	15.80%	0.185 *	0.012	0.094	0.118 *	0.048	0.026
Developed								
AUSTRALIA	8.33%	20.80%	0.105	0.051	0.118	0.056	-0.033	0.043
AUSTRIA	1.91%	25.58%	0.282 *	0.177 *	0.086	0.145 *	0.048	0.006
BELGIUM	3.71%	22.91%	0.325 *	0.032	0.025	0.225 *	0.043	0.013
CANADA	8.09%	21.47%	0.167 *	0.058	0.035	0.051	-0.084	-0.051
DENMARK	8.79%	20.05%	0.110	-0.003	0.123	0.087	-0.051	0.017
FINLAND	12.52%	34.49%	0.191 *	-0.075	0.062	0.006	0.040	0.160
FRANCE	5.24%	19.98%	0.132 *	-0.053	0.036	0.116	0.075	0.091
GERMANY	5.47%	23.19%	0.075	0.037	0.000	0.084	0.117	0.064
GREECE	4.90%	31.10%	0.141	0.025	0.039	0.129	-0.019	0.015
HONG KONG	5.54%	27.61%	0.114	0.036	-0.056	-0.060	-0.149	0.003
IRELAND	-0.21%	22.41%	0.292 *	0.161 *	0.184 *	0.132 *	0.078	-0.023
ISRAEL	4.49%	25.00%	0.082	0.002	0.005	-0.006	-0.062	-0.081
ITALY	5.19%	23.61%	0.055	-0.075	0.063	0.173 *	0.145	0.075
JAPAN	-2.42%	20.27%	0.181 *	-0.047	0.148 *	0.041	0.000	-0.107
NETHERLANDS	6.20%	20.74%	0.099	0.008	0.015	0.140 *	0.098	0.108
NEW ZEALAND	4.78%	23.31%	0.010	-0.008	0.231 *	0.097	0.060	0.060
NORWAY	8.53%	27.56%	0.175 *	0.030	0.106	-0.032	-0.118	-0.047
PORTUGAL	6.01%	22.41%	0.137 *	-0.002	0.055	0.030	-0.034	-0.049
SINGAPORE	4.01%	27.50%	0.105	0.111	-0.017	0.083	0.029	-0.031
SPAIN	10.43%	22.95%	0.083	-0.068	0.057	0.084	-0.005	-0.058
SWEDEN	9.52%	27.13%	0.101	-0.018	0.181 *	0.080	0.002	0.076
SWITZERLAND	7.49%	16.88%	0.146 *	-0.053	0.056	0.028	0.019	0.109
UNITED KINGD	4.00%	15.59%	0.242 *	0.100	0.075	0.191 *	0.044	0.027
USA	4.16%	15.37%	0.120 *	-0.013	0.125 *	0.091	0.097	0.032
Emerging								
BRAZIL	15.98%	41.06%	0.075	0.056	-0.065	0.026	0.037	-0.053
CHILE	7.55%	24.70%	0.120 *	-0.001	-0.076	0.175 *	0.023	-0.033
CHINA	-3.72%	37.57%	0.115 *	0.097	-0.100	-0.082	-0.076	-0.075
COLOMBIA	12.59%	33.35%	0.204 *	-0.025	-0.026	0.082	0.038	0.096
INDIA	7.27%	31.60%	0.134 *	0.136 *	-0.033	0.070	0.003	0.093
INDONESIA	3.21%	48.54%	0.214 *	-0.086	0.027	0.174 *	-0.122	0.019
KOREA	4.35%	40.02%	0.094	0.004	0.087	-0.037	-0.047	0.017
MALAYSIA	1.82%	31.72%	0.200 *	0.277 *	-0.011	0.026	-0.005	0.032
MEXICO	6.35%	32.74%	0.111	0.062	0.055	-0.042	-0.028	0.006
PERU	14.31%	33.92%	-0.006	0.002	0.007	0.078	-0.069	-0.072
PHILIPPINE	-2.30%	32.71%	0.168 *	0.082	-0.033	0.008	0.116 *	-0.091
POLAND	11.54%	46.71%	0.100	0.033	0.119	-0.002	-0.066	-0.030
SOUTH AFRICA	8.64%	28.75%	0.039	-0.036	-0.008	-0.015	0.018	-0.142
TAIWAN	1.97%	31.91%	0.076	0.121 *	-0.071	-0.046	-0.024	0.075
THAILAND	-2.59%	42.09%	0.011	0.157 *	-0.014	-0.084	0.086	-0.113
TURKEY	10.78%	56.68%	0.048	-0.012	0.087	-0.046	0.002	-0.020
Frontier								
ARGENTINA	3.31%	40.03%	0.074	0.074	0.010	0.067	-0.069	0.025
JORDAN	2.34%	19.62%	0.262 *	0.114	0.119 *	0.124 *	-0.029	0.078
PAKISTAN	1.38%	41.54%	0.049	0.037	-0.050	0.071	-0.019	0.078
SRI LANKA	2.60%	36.07%	0.102	0.132 *	-0.010	0.076	0.002	0.017

5. POTENTIAL RISK MEASURES

It's not an easy task to imagine which global risk factors can influence the return variation of different countries. In the attempt to do so, we will try two approaches. The first one is to apply the same macroeconomic factors suggested by earlier works, which were only made using more restrictive and more similar countries. The second approach, more unusual, involves constructing the factors from principal component analysis.

5.1. Macroeconomic Variables as Risk Factors

Here we expose the macroeconomic variables that will be used throughout this research as risk factors that driven the movement on the countries' equity returns. All of them are available from January 1993 to December 2009 and measured monthly. The choice of each variable is reasoned in the relevant literature.

i. Market Portfolio Return

The market portfolio in our case is a world portfolio. MSCI available the All Country World Index, a market value weighted equity index of 24 developed markets and 21 emergent markets⁶. Notice that the four frontier markets included in this research - Argentina, Jordan, Pakistan and Sri Lanka - don't participate in the composition of the World Index. The considerations about the calculation's methodology are the same that were made for the countries equity index. The return in the world index in excess of the one month Treasury bill will be the macroeconomic variable adopted.

In many asset pricing models, the market portfolio is included as a potential risk measure. The Capital Asset Pricing Model (CAPM) developed by Sharpe (1964), Lintner (1965) and Treynor (1961) can be understood as particular case of the Arbitrage Pricing Model in which the risk premia of each asset is only related to the excess return of the market portfolio. To the american market, Fama and French (1993) created a three factor model, including a market portfolio with significant risk premia. In an international framework, Harvey (1991) don't reject an unconditional

⁶ The 24 developed markets are the same described in the previous footnote. The 21 emerging market are those described in the previous footnote plus Chez Republic, Egypt, Hungary, Morocco, Russia.

version of CAPM. Ferson and Harvey (1993,1994) infer a significant risk premia to the world portfolio in the presence of multiple factors.

ii. Foreign Exchange Index

The Federal Reserve Board calculates two non-overlapping weighted indexes of real exchange rates. The weights are based on the trading volume with the United States⁷. The Major Currencies Index⁸ encompasses seven currencies that are largely traded outside their internal markets. The Other Important Trading Partners Index (OITP Index)⁹ is composed by 19 currencies, essentially from Asian and Latin America emerging countries. The logarithm variation of those indices will be included as macroeconomic variables for the reasons exposed in the next paragraphs.

Following the models of Solnik(1974), Stulz(1981) and Adler and Dumas (1983), under deviations from purchasing power parity, the foreign exchange risk must be priced. In Adler and Dumas (1983), returns in a reference currency are driven not only by the covariance with the market portfolio return but also by the covariance with the inflations' variation, in the reference currency, of all countries under consideration. Inflation in the reference currency can be decomposed into local inflation plus the variation in the nominal exchange rate. If the local inflation is stable, then inflation in the reference currency can be approximated by the variation in the nominal exchange rate. However, as our sample contains emerging and frontier markets, considering local inflation stable isn't reasonable. So, we follow the suggestion of Carrieri, Errunza, Majerbi (2004) that, if the inflation in the reference currency is stable, a better approximation would be the real exchange rate. This can be better understood in the formulation bellow.

Let US Dollar be our reference currency and e_{it}^R be the real exchange rate in (US dollar \$)/(currency of country I \$).

$$e_{it}^R = e_{it} \times \frac{P_{it}}{P_t}$$

⁷ The methodology of the Index calculation is detailed in "Index of Foreign Exchange Value of the Dollar", Federal Reserve Bulletin, winter 2005.

⁸ The included currencies are from Euro area countries, plus Canada, Japan, United Kingdom, Switzerland, Australia and Sweden.

⁹ The other important partners are China, Mexico, South Korea, Taiwan, Malaysia, Singapore, Brazil, Thailand, India, Philippines, Israel, Indonesia, Russia, Saudi Arabia, Chile, Argentina, Colombia and Venezuela.

Where, e_{it} is the nominal exchange rate, P_t is the price level in the United States and P_{it} is the price level of country i.

The inflation of country i in the US dollar reference, $\pi_{it}^{US\$}$, is

$$\pi_{it}^{US\$} = \Delta \ln(e_{it} \times P_{it}) = \Delta \ln(e_{it}^R \times P_t) = \Delta \ln(e_{it}^R) + \pi_{US\$t},$$

and $\pi_{US\$t}$ is the inflation in the US.

So, if the inflation in the reference currency is reasonably stable, we can approximate inflation of country i in the reference currency by the variation in the real exchange rate.

The 44 countries of our base, using US as reference, would demand the inclusion of 43 real exchange rates. Empirically, this is very complex to implement. The aggregated indices provided by the Federal Reserve give the model tractability. The real OITP and Major indices formulation is

$$I_t^R = I_{t-1}^R \times \prod_{i=1}^{N(t)} \left(\frac{x_{it}^R}{x_{it-1}^R} \right)^{w_{it}}$$

Where I_{t-1}^R is the index real value in t-1, w_{it} is the weight of currency i in t, $N(t)$ is the number of currencies that composes the index in t and x_{it}^R is the American dollar price in terms of the foreign currency from country i in (currency of country i \$)/(US Dollar \$) at time t. Notice that $x_{it}^R = 1/e_{it}^r$. So, for Major and OITP indices, we will use $\ln \left(\frac{I_t^R}{I_{t-1}^R} \right) = \sum_{i=1}^{N(t)} w_{it} \Delta \ln(e_{it}^R)$ as the macroeconomic measure that approximates aggregated inflation for a group of countries.

iii. TED Spread

Another macroeconomic variable included in our study is the change in the spread between the 90 days Eurodollar deposit Tax, represented by LIBOR, and the 90 day U.S. Treasury bill yield. This measure is known as TED spread and can be considered an indicator of global risk credit. LIBOR is the tax offered for commercial banks' loans, while the U.S. treasury is the proxy for the risk free. Changes in the spread would reflect alterations in the risk of nonpayment of interbank loans.

iv. Oil Price

The monthly variation in the oil price in U.S. dollar per barrel (FMI/IFS), in excess of the one month Treasury bill, will be the last macroeconomic variable included in our study. This factor is suggested in Chen, Roll and Ross (1986) for the American market. The authors' conclusion, however, is that the risk premia for the variation in the oil price isn't significant for two of the three analyzed periods. Wayne and Ferson (1994) don't find a significant price error for the variation in the oil price in a study that only considered developed countries.

5.2. Risk Factors From Principal Component Analysis

5.2.1. Principal component analysis

In our work we are interested in the common factors that have impact in the return's movement of several different countries. In the attempt to identify these factors we will apply to our sample of 44 countries a Principal Component Analysis.

This technique consists in rewriting the sample in order to explain its variance-covariance structure. Algebraically, we will be rotating the original data through a new set of orthogonal axes. These axes represent the directions of greater variability and are designated Principal Components. The first Principal Component accumulates the higher variance, the second Principal Component, the second higher variance and so on.

To describe the total system variability it would be necessary as many Components as present variables in the system. In our case, the 44 country equity returns would require the use of 44 Components. In general, however, a small set of Principal Components accounts for a substantial part of this variability. This allows working with reduced dimensionality.

The Principal Component Analysis doesn't rely on any hypothesis about the variables' joint distribution and is only based on the covariance structure of the data. This technique is frequently mistaken with Factor Analysis. Despite of the resemblances presented in both methodologies, Factor Analysis imposes questionable restrictions on the data.

More explicitly, the procedure for obtaining the Principal Components is like the following. Let $R' = [R_1, R_2, \dots, R_n]$ be a vector of the n variables that integrate the system (the 44 country equity returns) such that $Var(R) = \Sigma$. The first Principal Component will be the linear combination $Y_1 = c_1'R$ that maximizes $Var(c_1'R)$ such

that $c_1'c_1 = 1$. The second Principal Component will be the linear combination $Y_2 = c_2'R$ that maximizes $Var(c_2'R)$ such that $c_2'c_2 = 1$ and $Cov(c_1'R, c_2'R) = 0$. Successively, the i th Principal Component will be obtained as the linear combination $Y_i = c_i'R$ that maximizes $Var(c_i'R)$ such that $c_i'c_i = 1$ and $Cov(c_i'R, c_j'R) = 0$, for $j < i$.

Actually, is very simple to determine the Principal Components based on the following property. Let Σ be the covariance matrix of $R' = [R_1, R_2, \dots, R_n]$. And let $(\lambda_1, e_1), (\lambda_2, e_2), \dots, (\lambda_n, e_n)$ be the pairs of eigenvalues-eigenvectors of Σ , such that $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n \geq 0$. So, the i th Principal Component is given by:

$$Y_i = e_i'R, \quad i = 1, 2, \dots, n$$

In this case,

$$Var(Y_i) = e_i'\Sigma e_i = \lambda_i, \quad i = 1, 2, \dots, n$$

$$Cov(Y_i, Y_j) = 0, \quad i \neq j$$

Other important property is that the sum of the variances of the original data is equal the sum of the eigenvalues. Thus,

$$\sigma_{11} + \sigma_{22} + \dots + \sigma_{nn} = \sum_{i=1}^n Var(R_i) = \lambda_1 + \lambda_2 + \dots + \lambda_n = \sum_{i=1}^n Var(Y_i).$$

And the proportion of the total population's variance due to i th Component is:

$$\frac{\lambda_i}{\lambda_1 + \lambda_2 + \dots + \lambda_n}, \quad i = 1, 2, \dots, n$$

Application

The described methodology will be applied to the countries' equity returns in excess of the one-month Treasury bill. Using the notation of the previous section to our data, $R' = [R_1, R_2, \dots, R_{44}]$ is a matrix 44 by 203, representing the returns of the 44 countries from January 1993 to December 2009. $\Sigma = Var(R)$ is a 44 by 44 matrix of covariance with the pairs of eigenvalues-eigenvectors $(\lambda_1, e_1), (\lambda_2, e_2), \dots, (\lambda_{44}, e_{44})$, such that $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_{44} \geq 0$. Finally, the Principal Components, $Y_i = e_i'R$, are vectors of size $t=203$, $i = 1, \dots, 44$.

The first step is to determine how many Components are necessary to describe a reasonable amount of the sample's variability and reduce the dimensionality. This question doesn't have a closed answer. However, an analysis based on the magnitude of the eigenvalues and on the proportion of the explained

variability can support this decision. Figure 1 shows the graph of the ordered eigenvalue's magnitudes. Compared to the value of the first eigenvalue (0.150) around the fifth one the difference between successive eigenvalues is already reduced to near 0.02 or less, and the magnitudes itself are relatively close to zero. Figure 2 shows that the first Component alone explains around 44% of the variability of the data and that around 65% is accumulated by the first five Components. Additional Components marginally contribute with very small variance's proportion. Considering what was exposed, five Components will be adopted as describing a fair amount of the sample's variability.

Scree Plot (Ordered Eigenvalues)

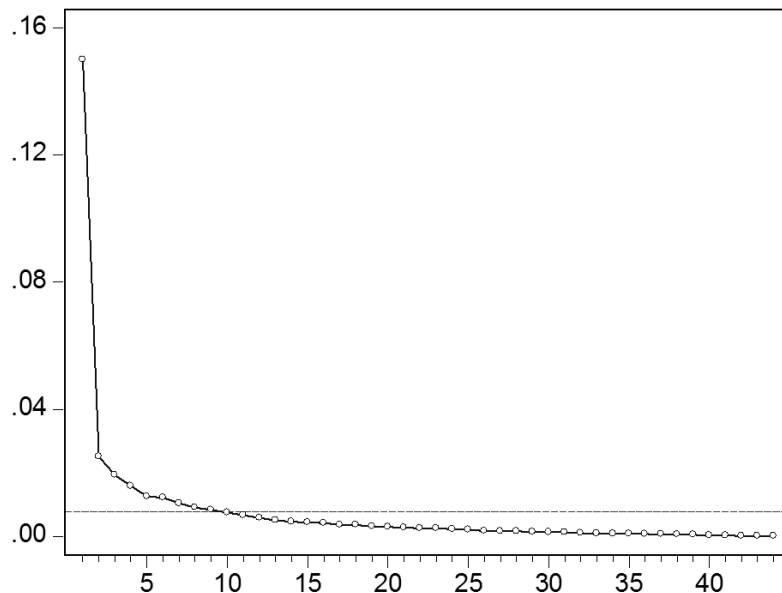


Figure 1: Ordered Eigenvalues

Eigenvalue Cumulative Proportion

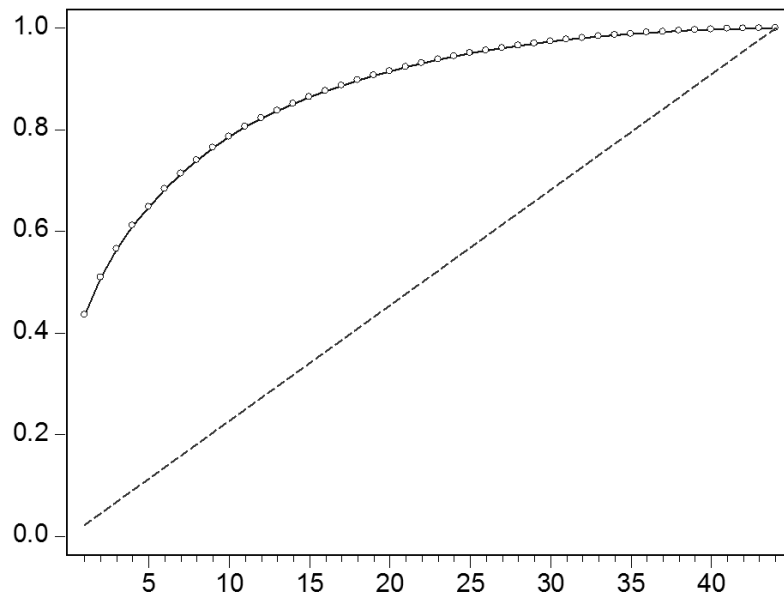


Figure 2: Cumulative Variance Proportion

The first five eigenvectors corresponding to the fifth's largest eigenvalues and the correlation among the respective Principal Component and the return of each country are presented in Table 2. The first Principal Component is an almost equally weighted index of the countries' equity returns. The correlation of this Component with each country, with the exception of Jordan, Pakistan and Sri-Lanka, is above 50%. From the second to the fifth Component, due to signal alternation, we have a relation of contrast among countries. For example, the second Component, considering only the countries with more than 10% of correlation, detaches two distinct groups. The first one is composed by the countries from Occidental Europe, plus United States, Israel, Brazil and Turkey. The second group encompasses the emerging markets (with the exception of Brazil) plus the Asians countries.

Table 2: First Five Eigenvectors and their Correlation with the Countries` Returns

R	e1	$\rho(e1,R)$	e2	$\rho(e2,R)$	e3	$\rho(e3,R)$	e4	$\rho(e4,R)$	e5	$\rho(e5,R)$
Developed										
AUSTRALIA	0.1301	0.8393	0.0048	0.0126	0.0619	0.1433	0.0321	0.0675	-0.0196	-0.0366
AUSTRIA	0.1385	0.7266	0.0473	0.1016	0.1377	0.2593	0.0891	0.1521	0.0033	0.0050
BELGIUM	0.1183	0.6930	0.0906	0.2172	0.1858	0.3906	0.0636	0.1213	0.0197	0.0335
CANADA	0.1334	0.8340	0.0367	0.0940	0.0333	0.0746	0.0121	0.0246	-0.0552	-0.1001
DENMARK	0.1069	0.7156	0.0872	0.2390	0.1181	0.2837	0.0567	0.1236	0.0147	0.0284
FINLAND	0.1564	0.6088	0.2414	0.3847	0.0960	0.1340	-0.0204	-0.0258	0.1655	0.1867
FRANCE	0.1183	0.7945	0.1164	0.3203	0.1167	0.2814	-0.0083	-0.0181	0.0138	0.0270
GERMANY	0.1346	0.7787	0.1223	0.2898	0.1288	0.2674	-0.0139	-0.0262	0.0278	0.0466
GREECE	0.1536	0.6631	0.1327	0.2345	0.0876	0.1357	0.1242	0.1745	0.1553	0.1944
HONG KONG	0.1548	0.7524	-0.1188	-0.2364	0.0219	0.0382	-0.0776	-0.1227	-0.0782	-0.1102
IRELAND	0.1102	0.6597	0.0927	0.2274	0.1175	0.2525	0.0739	0.1440	0.0323	0.0561
ITALY	0.1171	0.6654	0.1320	0.3073	0.1218	0.2485	0.0471	0.0872	0.0510	0.0841
ISRAEL	0.1039	0.5578	0.1298	0.2855	-0.0397	-0.0765	-0.0116	-0.0203	-0.0399	-0.0622
JAPAN	0.0877	0.5808	0.0068	0.0183	0.0897	0.2132	-0.0120	-0.0258	0.0495	0.0950
NETHERLANDS	0.1234	0.7987	0.0942	0.2496	0.1323	0.3072	0.0127	0.0267	0.0297	0.0557
NEW ZEALAND	0.1292	0.7439	-0.0069	-0.0163	0.0709	0.1465	-0.0265	-0.0498	0.0709	0.1183
NORWAY	0.1633	0.7955	0.0866	0.1726	0.1285	0.2246	0.0962	0.1526	-0.0405	-0.0572
PORTUGAL	0.1123	0.6725	0.1062	0.2603	0.1314	0.2825	0.0628	0.1224	0.0755	0.1311
SINGAPORE	0.1672	0.8163	-0.1346	-0.2690	0.0009	0.0016	-0.0613	-0.0973	-0.0325	-0.0459
SPAIN	0.1333	0.7796	0.0985	0.2358	0.1366	0.2867	0.0180	0.0343	-0.0293	-0.0497
SWEDEN	0.1577	0.7800	0.1504	0.3047	0.1015	0.1801	-0.0255	-0.0410	0.0598	0.0857
SWITZERLAND	0.0846	0.6724	0.0645	0.2098	0.1218	0.3476	-0.0057	-0.0148	0.0539	0.1243
UNITED KINGDOM	0.0924	0.7952	0.0677	0.2385	0.0605	0.1870	0.0185	0.0520	0.0118	0.0295
USA	0.0901	0.7867	0.0538	0.1923	0.0540	0.1692	-0.0276	-0.0784	-0.0001	-0.0004
Emerging										
BRAZIL	0.2310	0.7552	0.0794	0.1063	-0.1241	-0.1456	0.2194	0.2335	-0.2394	-0.2269
CHILE	0.1359	0.7387	-0.0458	-0.1018	-0.0620	-0.1209	0.0407	0.0719	-0.0921	-0.1452
CHINA	0.1762	0.6295	-0.1862	-0.2723	-0.0708	-0.0908	-0.0904	-0.1051	-0.3158	-0.3271
COLOMBIA	0.1298	0.5224	-0.0191	-0.0315	-0.1284	-0.1855	0.0696	0.0911	0.0750	0.0874
INDIA	0.1489	0.6322	-0.0068	-0.0119	-0.0809	-0.1233	0.1155	0.1596	0.1237	0.1523
INDONESIA	0.2396	0.6623	-0.4119	-0.4663	0.0645	0.0640	-0.1511	-0.1360	0.2128	0.1705
KOREA	0.1888	0.6332	-0.1359	-0.1867	0.0988	0.1189	-0.1117	-0.1220	0.3201	0.3113
MALAYSIA	0.1412	0.5973	-0.2438	-0.4224	-0.0273	-0.0415	-0.1176	-0.1619	0.0627	0.0769
MEXICO	0.1805	0.7398	0.0234	0.0393	-0.0670	-0.0986	0.0939	0.1253	-0.2012	-0.2390
PERU	0.1612	0.6377	-0.0299	-0.0484	-0.0424	-0.0601	0.1828	0.2355	-0.2612	-0.2996
PHILIPPINE	0.1507	0.6183	-0.2737	-0.4598	0.0197	0.0290	-0.1362	-0.1819	-0.0399	-0.0475
POLAND	0.2266	0.6509	0.1083	0.1274	0.0479	0.0494	0.0988	0.0924	0.1932	0.1610
SOUTH AFRICA	0.1625	0.7586	-0.0461	-0.0881	0.0265	0.0445	-0.0092	-0.0140	-0.0634	-0.0858
TAIWAN	0.1545	0.6497	-0.1173	-0.2020	-0.0293	-0.0442	-0.0564	-0.0772	-0.1340	-0.1634
THAILAND	0.2139	0.6820	-0.3823	-0.4991	0.0230	0.0263	-0.1480	-0.1536	0.0673	0.0622
TURKEY	0.2583	0.6115	0.4186	0.4059	-0.5508	-0.4681	-0.5995	-0.4621	0.0547	0.0376
Frontier										
ARGENTINA	0.1916	0.6424	0.0191	0.0262	-0.1050	-0.1263	0.1361	0.1485	-0.4971	-0.4832
JORDAN	0.0438	0.2994	0.0097	0.0272	0.0113	0.0278	0.0529	0.1178	0.1029	0.2040
PAKISTAN	0.0826	0.2668	-0.0938	-0.1240	-0.5571	-0.6459	0.4312	0.4535	0.3465	0.3245
SRI LANKA	0.1043	0.3881	-0.0615	-0.0937	-0.1930	-0.2578	0.3820	0.4627	0.1220	0.1316

5.2.1. Statistical risk factors from principal component analysis

The Principal Components can be understood as information that summarizes the covariance structure of the data. However, it's not clear how to interpret this information. Following Campbell, Lo and Mackinlay (1997), we can normalize the eigenvectors so that their sum is equal to one. That means, let $w_i = e_i / (\sum_{j=1}^{44} e_{ij})$, $i=1,2,3,4,5$, and the normalized Components will be $Y_i^* = \left(\frac{e_i}{\sum_{j=1}^{44} e_{ij}} \right)' R = w_i' R$, $i=1,2,3,4,5$. As $\sum_{j=1}^{44} w_{ij} = 1$, the normalized Components are now portfolios of the countries equity indices whose weights add up to one. From the geometric point of view, this normalization causes a distortion of the Components, but doesn't alter its direction, preserving all the correlation relations contained in the original components.

The countries' portfolios formed by the normalized Principal Components still point the directions of greater variability of the original return data. For that reason, we are going to regard these portfolios as risk factors to be included in our model. They will be denominated statistical factors, $f_i, i = 1, 2, 3, 4, 5$, to distinguish them from the macroeconomic factors. However, before presenting the model's empirical results, we will investigate the relation, if any exists, between statistical and macroeconomic factors.

5.3. Relation Between Statistical and Macroeconomic Factors

First of all, in

Table3 we present the descriptive statistics for the five statistical factors, f_1, f_2, f_3, f_4 e f_5 , and for the five macroeconomic factors, excess return of the world portfolio (world), logarithm variation on the Major Index (major), logarithm variation on the OITP Index (oitp), change in the spread between 90 day Eurodollar tax and the 90 day US Treasury bill yield (ted) and excess change in the oil price (oil). The mean and standard deviation are annualized.

In Table4, we show the correlation matrix for statistical and macroeconomic factors. By construction, the statistical factors have between them zero correlation. The first statistical factor has 90% correlation with the excess return of the market portfolio and 53% correlation with the change in the oitp index. The 90% correlation really stands out. This means that the first Principal Component - the one with higher

capacity to explain the covariance structure of the returns (around 44%) - have a strong relationship with the market portfolio.

In

Table5, the linear projections of the statistical factors on the macroeconomic factors and a constant are presented, with robust standard deviation in brackets. The excess return of the world portfolio and the change in the OITP index are significant in the regressions with dependent variables f_1, f_2, f_3 . The change in the Major index is significant only in the regression of f_3 . The change in the TED spread and the excess variation on the oil price are 10% significant, respectively, on the regression of f_5 and f_4 . It's interesting to note the R^2 of 0.85 on the first regression. For the following regressions the R^2 is considerably lower.

What we can obtain from this analysis, is the great similarity between the first statistical factor and the world market portfolio. Remind that the statistical factors are just a rescaling of the Principal Components and that they preserve all the correlations relation of the latest. The world market portfolio is a weighted average of the countries returns based on the market value. The first Principal Component is also a weighted average of the countries portfolio, but the weights are obtained so that the First Principal Component points the direction of greater variability. This finding gives a justification for the importance of the world market portfolio in explaining the covariance structure of the countries returns.

Table3: Descriptive Statisticals for Macroeconomic and Statistical Factors

	Mean	Standard Deviation	ρ_1	ρ_2	ρ_3	ρ_4	ρ_{12}	ρ_{24}
f1	5.89%	21.22%	0.256 *	0.093	0.068	0.082	-0.005	0.000
f2	55.80%	179.49%	0.234 *	0.108	0.096	-0.037	-0.067	-0.002
f3	2.40%	102.97%	-0.038	0.011	0.057	-0.037	-0.048	-0.013
f4	8.89%	53.43%	-0.026	0.103	-0.011	0.069	0.027	-0.010
f5	1.02%	103.78%	0.040	-0.170 *	0.027	0.018	-0.082	-0.008
world	3.87%	15.80%	0.185 *	0.012	0.093	0.116	0.046	0.023
major	0.26%	5.67%	0.326 *	-0.030	-0.047	0.096	-0.099	-0.027
oitp	0.12%	4.39%	0.290 *	0.023	-0.085	-0.074	-0.008	0.010
ted	0.00%	0.24%	-0.556 *	0.077	0.030	-0.106	0.256 *	0.134 *
oil	5.00%	29.18%	0.244 *	0.111	0.068	-0.039	-0.041	-0.030

Table4: Correlation Among Factors

		<i>Correlation</i>								
	f1	f2	f3	f4	f5	world	major	oitp	ted	oil
f1	1.000									
f2	0.000	1.000								
f3	0.000	0.000	1.000							
f4	0.000	0.000	0.000	1.000						
f5	0.000	0.000	0.000	0.000	1.000					
world	0.904	0.188	0.209	-0.024	0.022	1.000				
major	0.349	-0.080	0.210	-0.057	0.008	0.326	1.000			
oitp	0.526	-0.207	-0.018	-0.046	0.085	0.386	0.331	1.000		
ted	0.087	-0.013	0.029	-0.024	0.125	0.066	-0.053	0.054	1.000	
oil	0.152	-0.071	0.024	0.144	-0.103	0.110	0.234	0.137	-0.007	1.000

Table5: Linear Projection of the Statistical Factors on the Macroeconomic Factors

		Linear Projection							
		$f_i = \beta_0 + \beta_1 \text{world} + \beta_2 \text{major} + \beta_3 \text{oitp} + \beta_4 \text{ted} + \beta_5 \text{oil} + e_i, i=1, 2, 3, 4, 5.$							
	c	world	major	oitp	ted	oil		R^2/\bar{R}^2	
f1	0.001 (0.002)	1.098 (0.052) ***	0.033 (0.010)	0.977 (0.161) ***	1.945 (2.359)	0.023 (0.020)		0.855 0.851	
f2	0.037 (0.034)	3.833 (0.851) ***	-2.519 (2.336)	-12.394 (3.606) ***	-17.791 (52.878)	-0.294 (0.455)		0.136 0.114	
f3	-0.002 (0.021)	1.343 (0.552) **	3.720 (1.676) **	-3.837 (1.793) **	14.886 (30.265)	-0.086 (0.323)		0.090 0.067	
f4	0.006 (0.011)	0.007 (0.327)	-0.805 (0.806)	-0.497 (1.368)	-5.511 (17.777)	0.310 (0.170) *		0.032 0.007	
f5	0.002 (0.021)	-0.098 (0.547)	0.279 (1.348)	2.246 (2.663)	51.833 (28.683) *	-0.416 (0.308)		0.035 0.011	

*** 1% significant, **5% significant, *10% significant.

6. EMPIRICAL RESULTS

For a robustness analysis we selected three non overlapping groups of ten countries – each containing 6 developed markets and the remainder four countries from emerging or frontier markets. Their composition, though arbitrarily, was chosen so that the three groups¹⁰ have similar geographic distribution and the same proportion of developed and emerging countries.

For each group an APT model using, separately, statistical and macroeconomic factors was estimated by both methodologies, ITNLSUR and GMM. In the GMM estimation we used as instruments the current risk factors and the lagged macroeconomic variables representing the available information for the investors.

A usual way to evaluate Asset Pricing Models, which will be applied here, is by their absolute pricing error. This measure is obtained by the absolute value of the difference between the expected return, given by equation 2, and the mean return. For the GMM estimation we can also use the J-statistic to test the overidentifying restriction.

6.1. The Model with Macroeconomic Variables as Risk Factors

Using macroeconomic variables, we tested the significance of the risk premiums for three different models. The option of which macroeconomic variable to include in each model was due to the importance of the variable in the relevant literature.

To start, Model 1 is a CAPM and has only the excess world return as risk factor. Model two includes also the exchange risk factors. Finally, Model 3 comprises all macroeconomic variables suggested earlier in section 5.1.

In Table 6 to Table 11 we show the estimated risk prices for each group by ITNLSUR and GMM and in Table 12 to Table 14 we present the absolute pricing error.

¹⁰ The first group is Argentina, Australia, Austria, Canada, China, Finland, Germany, Pakistan, Portugal and Taiwan. The second group is Belgium, Brazil, France, Hong Kong, Indonesia, Korea, Spain, Sri Lanka, Sweden and USA. The third group is Chile, Denmark, India, Italy, Japan, Malaysia, Mexico, New Zealand, Switzerland and United Kingdom.

One important result is that in the CAPM (Model 1), the world excess return is always priced. The premium varies from 0.4% to 0.5% per month, or 4.4% to 6.3% per year, and is significant at 1% in all groups, independent of the estimation method. The inclusion of other potential risk factors, however, diminishes the significance of the world excess return.

The estimation of group one using ITNLSUR presents a significant risk premium for the oitp foreign exchange risk index. Nevertheless, this is an exception and no other macroeconomic variable has a significant risk price besides the world excess return.

When we include all macroeconomic variables (Model 3), the results are very sensible to the group and estimation method. No pattern is observed. In some cases the world excess return is still priced while in others the significance is lost.

The absolute pricing error, unfortunately, doesn't show a clear tendency to guide us in the choice of the best model. In general, it seems to decline with the inclusion of more variables. However, the model with only three factors (Model 2) reaches the minimum pricing error for group one, using both methods, and for group two, when estimated by GMM.

For the GMM estimation we also performed the overidentifying restrictions test. In all cases, the J-statistic (not reported here) indicates that we can't reject the null hypothesis that the overidentifying restrictions equal zero. Consequently, we can't reject the specification of model 1, 2 and 3 in any of the groups.

Table 6: ITNLSUR with Macroeconomic Factors for Group One

		ITNLSUR Group One				
Model	Factors	λ_{world}	λ_{oitp}	Premiums		
				λ_{major}	λ_{ted}	λ_{oil}
1	world	0.005 *** 0.001				
2	world,oitp, major	0.006 *** 0.001	-0.005 * 0.003	-0.001 0.004		
3	world,oitp, major, ted,oil	0.006 *** 0.001	-0.005 * 0.003	-0.001 0.004	0.000 0.000	0.002 0.015

Table 7: GMM with Macroeconomic Factors for Group One

GMM Group One						
Model	Factors	λ_{world}	λ_{oitp}	Premiums		
				λ_{major}	λ_{ted}	λ_{oil}
1	world	0.005 *** 0.001				
2	world,oitp, major	0.005 *** 0.001	-0.001 0.002	0.003 0.003		
3	world,oitp, major, ted,oil	0.137 1.221	0.056 0.542	0.290 2.622	-0.024 0.218	1.414 12.665

Table 8: ITNLSUR with Macroeconomic Factors for Group Two

ITNLSUR Group Two						
Model	Factors	λ_{world}	λ_{oitp}	Premiums		
				λ_{major}	λ_{ted}	λ_{oil}
1	world	0.004 *** 0.001				
2	world,oitp, major	0.004 *** 0.001	0.000 0.002	0.000 0.003		
3	world,oitp, major, ted,oil	-0.002 0.012	0.005 0.011	-0.017 0.032	0.000 0.001	-0.268 0.489

Table 9: GMM with Macroeconomic Factors for Group Two

GMM Group Two						
Model	Factors	λ_{world}	λ_{oitp}	Premiums		
				λ_{major}	λ_{ted}	λ_{oil}
1	world	0.004 *** 0.001				
2	world,oitp, major	0.004 *** 0.001	0.001 0.001	-0.001 0.002		
3	world,oitp, major, ted,oil	0.005 *** 0.001	0.002 0.001	0.002 0.002	0.000 0.000	0.016 0.015

Table 10: ITNLSUR with Macroeconomic Factors for Group Three

ITNLSUR Group Three						
Model	Factors	Premiums				
		λ_{world}	λ_{oitp}	λ_{major}	λ_{ted}	λ_{oil}
1	world	0.004 *** 0.001				
2	world,oitp, major	0.004 ** 0.001	0.001 0.002	-0.001 0.005		
3	world,oitp, major, ted,oil	-0.0003 0.00469	0.0039 0.0046	0.02279 0.02173	-0.0007 0.00067	-0.0311 0.0421

Table 11: GMM with Macroeconomic Factors for Group Three

GMM Group Three						
Model	Factors	Premiums				
		λ_{world}	λ_{oitp}	λ_{major}	λ_{ted}	λ_{oil}
1	world	0.004 *** 0.001				
2	world,oitp, major	0.003 *** 0.001	0.000 0.001	0.003 0.003		
3	world,oitp, major, ted,oil	0.001 0.002	0.005 0.003	0.011 0.007	0.000 0.000	-0.020 0.019

Table 12: Absolute Pricing Error for ITNLSUR and GMM for Group One with macroeconomic factors

Model	ITNLSUR			GMM		
	1	2	3	1	2	3
Group One						
Argentina	0.393%	0.060%	0.097%	0.432%	0.467%	1.106%
Australia	0.143%	0.053%	0.028%	0.184%	0.218%	0.136%
Austria	0.414%	0.085%	0.118%	0.330%	0.391%	0.102%
Canada	0.085%	0.109%	0.084%	0.086%	0.130%	0.542%
China	0.908%	0.067%	0.065%	0.938%	0.463%	0.048%
Finland	0.268%	0.151%	0.204%	0.303%	0.388%	0.007%
Germany	0.175%	0.157%	0.131%	0.191%	0.180%	0.184%
Pakistan	0.053%	0.528%	0.480%	0.207%	0.237%	0.481%
Portugal	0.018%	0.048%	0.086%	0.051%	0.088%	0.014%
Taiwan	0.409%	0.069%	0.071%	0.448%	0.010%	0.321%
Average	0.287%	0.133%	0.136%	0.317%	0.257%	0.294%

Table 13: Absolute Pricing Error for ITNLSUR and GMM for Group Two with macroeconomic factors

Model	ITNLSUR			GMM		
	1	2	3	1	2	3
Group Two						
Belgium	0.151%	0.092%	0.050%	0.125%	0.044%	0.046%
Brazil	0.597%	0.535%	0.129%	0.523%	0.187%	0.041%
France	0.029%	0.012%	0.024%	0.030%	0.054%	0.019%
Hong kong	0.033%	0.015%	0.014%	0.082%	0.074%	0.236%
Indonesia	0.389%	0.195%	0.092%	0.531%	0.493%	0.786%
Korea	0.255%	0.204%	0.030%	0.231%	0.341%	0.414%
Spain	0.192%	0.150%	0.547%	0.173%	0.215%	0.310%
Sri Lanka	0.039%	0.027%	0.072%	0.016%	0.057%	0.358%
Sweden	0.196%	0.191%	0.000%	0.230%	0.128%	0.233%
USA	0.034%	0.055%	0.023%	0.037%	0.062%	0.016%
Average	0.191%	0.148%	0.098%	0.198%	0.166%	0.246%

Table 14: Absolute Pricing Error for ITNLSUR and GMM for Group Three with macroeconomic factors

Model	ITNLSUR			GMM		
	1	2	3	1	2	3
Group Three						
Chile	0.283%	0.235%	0.032%	0.272%	0.447%	0.062%
Denmark	0.385%	0.394%	0.008%	0.385%	0.320%	0.073%
India	0.231%	0.218%	0.110%	0.180%	0.333%	0.071%
Italy	0.049%	0.056%	0.109%	0.041%	0.017%	0.163%
Japan	0.513%	0.474%	0.073%	0.551%	0.523%	0.030%
Malaysia	0.171%	0.228%	0.076%	0.231%	0.032%	0.409%
Mexico	0.029%	0.061%	0.192%	0.006%	0.330%	0.347%
New Zealand	0.033%	0.071%	0.019%	0.045%	0.051%	0.207%
Switzerland	0.333%	0.348%	0.002%	0.354%	0.256%	0.024%
United Kingdom	0.022%	0.019%	0.025%	0.035%	0.021%	0.044%
Average	0.205%	0.210%	0.064%	0.210%	0.233%	0.143%

6.2. The Model with Statistical Variables as Risk Factors

Using the statistical factors extracted from Principal Components, we created five distinct models and tested the significance of the risk premiums. The first model is composed only of the first statistical factor. The second one covers the first and second statistical factors and so on. We ordered the inclusion of the factors according to its capacity of explaining the covariance structure of the data.

In

Table 15 to Table 20 we show the estimated risk prices for each group by ITNLSUR and GMM and in Table 21 to Table 23 we present the absolute pricing error.

In the model with just one factor (Model 1), for the three groups, the first statistical factor has always a significant risk price. This premium varies from 0.5% to 0.7% per month or 6.2% to 8.5% per year. It is similar to the premium obtained in section 6.1, for the model that considers only the world return as a risk factor. For Model 2 to Model 5, the inclusion of more statistical factors can diminish the significance of the first factor.

Other factors, depending on the model and the methodology of estimation, present significant risk premiums. However, the results aren't robust and no pattern is observed across groups.

The absolute pricing error, in general, shows a tendency to decline with the inclusion of more factors. However, this tendency isn't straight and there are cases where the inclusion of one more factor results in a small increase of the absolute pricing error.

As for the overidentifying restriction test, the calculation of the J-statistic (not reported here), usually indicates that we can't reject our model specification. However, there is one exception. In group two, using the model with three statistical factors (Model 3), we reject the spare restrictions at a 10% significance level.

Table 15: ITNLSUR with Statistical Factors for Group One

		ITNLSUR Group One				
Model	Factors	λ_1	λ_2	Premiums		λ_5
				λ_3	λ_4	
1	f1	0.005 *** 0.001				
2	f1, f2	0.004 *** 0.001	0.060 0.043			
3	f1, f2, f3	0.004 *** 0.001	0.058 0.050	0.003 0.020		
4	f1, f2, f3, f4	0.004 ** 0.002	0.057 0.054	0.003 0.031	0.001 0.021	
5	f1, f2, f3, f4, f5	0.005 ** 0.002	0.040 0.058	0.000 0.031	-0.008 0.024	0.025 0.024

Table 16: GMM with Statistical Factors for Group One

		GMM Group One				
Model	Factors	λ_1	λ_2	Premiums		
				λ_3	λ_4	λ_5
1	f1	0.005 *** 0.001				
2	f1, f2	0.005 *** 0.001	0.012 0.038			
3	f1, f2, f3	0.004 *** 0.001	0.010 0.050	0.017 0.019		
4	f1, f2, f3, f4	0.001 0.002	-0.112 0.071	0.108 *** 0.036	0.060 *** 0.023	
5	f1, f2, f3, f4, f5	0.003 * 0.002	-0.040 0.067	0.052 0.037	0.024 0.023	0.008 0.020

Table 17: ITNLSUR with Statistical Factors for Group Two

		ITNLSUR Group Two				
Model	Factors	λ_1	λ_2	Premiums		
				λ_3	λ_4	λ_5
1	f1	0.006 *** 0.001				
2	f1, f2	0.006 *** 0.001	0.043 * 0.025			
3	f1, f2, f3	0.006 *** 0.001	0.048 * 0.027	-0.016 0.026		
4	f1, f2, f3, f4	0.006 *** 0.001	0.047 0.034	-0.015 0.032	0.001 0.016	
5	f1, f2, f3, f4, f5	0.006 *** 0.001	0.036 0.037	-0.008 0.034	0.001 0.016	-0.027 0.031

Table 18: GMM with Statistical Factors for Group Two

		GMM Group Two				
Model	Factors	λ_1	λ_2	Premiums		
				λ_3	λ_4	λ_5
1	f1	0.007 *** 0.001				
2	f1, f2	0.007 *** 0.001	0.026 0.020			
3	f1, f2, f3	0.007 *** 0.001	0.058 *** 0.020	-0.043 ** 0.020		
4	f1, f2, f3, f4	0.007 *** 0.001	0.052 ** 0.026	-0.037 0.024	0.003 0.012	
5	f1, f2, f3, f4, f5	0.006 *** 0.001	0.032 0.034	-0.026 0.028	0.002 0.013	-0.023 0.029

Table 19: ITNLSUR with Statistical Factors for Group Three

		ITNLSUR Group Three				
Model	Factors	λ_1	λ_2	Premiums		
				λ_3	λ_4	λ_5
1	f1	0.005 *** 0.001				
2	f1, f2	0.005 *** 0.001	0.048 0.041			
3	f1, f2, f3	0.005 *** 0.001	0.042 0.051	0.007 0.036		
4	f1, f2, f3, f4	0.000 0.004	-0.245 0.224	0.156 0.122	0.160 0.119	
5	f1, f2, f3, f4, f5	0.000 0.004	-0.250 0.227	0.164 0.130	0.157 0.118	-0.018 0.072

Table 20: GMM with Statistical Factors for Group Three

GMM						
Group Three						
Model	Factors	Premiums				
		λ_1	λ_2	λ_3	λ_4	λ_5
1	f1	0.006 *** 0.001				
2	f1, f2	0.006 *** 0.001	0.012 0.035			
3	f1, f2, f3	0.005 *** 0.001	-0.027 0.040	0.057 ** 0.027		
4	f1, f2, f3, f4	0.000 0.004	-0.181 * 0.099	0.162 ** 0.072	0.110 * 0.066	
5	f1, f2, f3, f4, f5	0.002 0.003	-0.118 0.072	0.127 ** 0.052	0.068 0.044	0.004 0.036

Table 21: Absolute Pricing Error for the ITNLSUR and GMM for Group One with statistical factors

Model	ITNLSUR					GMM				
	1	2	3	4	5	1	2	3	4	5
Group One										
Argentina	0.328%	0.296%	0.281%	0.281%	0.137%	0.366%	0.313%	0.170%	0.173%	0.064%
Australia	0.281%	0.311%	0.305%	0.304%	0.268%	0.299%	0.358%	0.328%	0.067%	0.254%
Austria	0.277%	0.315%	0.327%	0.334%	0.318%	0.244%	0.233%	0.275%	0.304%	0.368%
Canada	0.250%	0.224%	0.224%	0.226%	0.219%	0.245%	0.283%	0.279%	0.239%	0.308%
China	0.861%	0.457%	0.462%	0.455%	0.450%	0.863%	0.726%	0.665%	0.316%	0.626%
Finland	0.545%	0.152%	0.160%	0.167%	0.034%	0.580%	0.566%	0.480%	0.337%	0.564%
Germany	0.030%	0.148%	0.154%	0.152%	0.193%	0.016%	0.005%	0.060%	0.145%	0.025%
Pakistan	0.145%	0.053%	0.115%	0.108%	0.093%	0.229%	0.117%	0.340%	0.074%	0.184%
Portugal	0.145%	0.013%	0.020%	0.024%	0.049%	0.142%	0.128%	0.051%	0.148%	0.087%
Taiwan	0.322%	0.056%	0.061%	0.056%	0.139%	0.324%	0.203%	0.131%	0.455%	0.108%
Average	0.319%	0.203%	0.211%	0.211%	0.190%	0.331%	0.293%	0.278%	0.226%	0.259%

Table 22: Absolute Pricing Error for the ITNLSUR and GMM for Group Two with statistical factors

Model	ITNLSUR					GMM				
	1	2	3	4	5	1	2	3	4	5
Group Two										
Belgium	0.136%	0.253%	0.153%	0.162%	0.176%	0.172%	0.274%	0.067%	0.009%	0.058%
Brazil	0.455%	0.347%	0.188%	0.183%	0.021%	0.254%	0.164%	0.232%	0.147%	0.097%
France	0.010%	0.161%	0.118%	0.116%	0.104%	0.065%	0.172%	0.014%	0.007%	0.011%
Hong kon	0.121%	0.030%	0.030%	0.035%	0.085%	0.144%	0.172%	0.191%	0.096%	0.150%
Indonesia	0.631%	0.101%	0.042%	0.041%	0.020%	0.758%	0.459%	0.362%	0.203%	0.009%
Korea	0.346%	0.172%	0.119%	0.115%	0.126%	0.384%	0.405%	0.266%	0.144%	0.090%
Spain	0.197%	0.328%	0.272%	0.274%	0.318%	0.264%	0.372%	0.228%	0.188%	0.187%
Sri Lanka	0.175%	0.097%	0.254%	0.276%	0.104%	0.210%	0.197%	0.431%	0.537%	0.380%
Sweden	0.197%	0.000%	0.017%	0.023%	0.096%	0.125%	0.007%	0.221%	0.231%	0.297%
USA	0.007%	0.064%	0.051%	0.048%	0.049%	0.042%	0.077%	0.046%	0.027%	0.019%
Average	0.228%	0.155%	0.124%	0.127%	0.110%	0.242%	0.230%	0.206%	0.159%	0.130%

Table 23: Absolute Pricing Error for the ITNLSUR and GMM for Group Three with statistical factors

Model	ITNLSUR					GMM				
	1	2	3	4	5	1	2	3	4	5
Group Three										
Chile	0.159%	0.233%	0.254%	0.078%	0.065%	0.142%	0.153%	0.341%	0.224%	0.357%
Denmark	0.361%	0.239%	0.222%	0.084%	0.088%	0.285%	0.255%	0.070%	0.075%	0.111%
India	0.092%	0.111%	0.144%	0.143%	0.088%	0.066%	0.062%	0.313%	0.227%	0.235%
Italy	0.029%	0.154%	0.165%	0.031%	0.024%	0.024%	0.072%	0.127%	0.092%	0.027%
Japan	0.500%	0.500%	0.523%	0.253%	0.251%	0.575%	0.581%	0.675%	0.445%	0.567%
Malaysia	0.332%	0.032%	0.010%	0.018%	0.021%	0.323%	0.261%	0.398%	0.086%	0.140%
Mexico	0.092%	0.113%	0.077%	0.014%	0.047%	0.121%	0.142%	0.275%	0.077%	0.304%
New Zeal:	0.046%	0.027%	0.043%	0.067%	0.075%	0.054%	0.046%	0.159%	0.268%	0.303%
Switzerla:	0.330%	0.240%	0.217%	0.114%	0.114%	0.334%	0.316%	0.106%	0.019%	0.011%
United Kir	0.014%	0.077%	0.080%	0.061%	0.060%	0.048%	0.069%	0.091%	0.033%	0.006%
Average	0.196%	0.173%	0.174%	0.086%	0.083%	0.197%	0.196%	0.255%	0.155%	0.206%

7. CONCLUSIONS

We developed an empirically analysis about the common sources of risk driven changes in equity returns of three non- overlapping groups of countries. Since each group was composed of very heterogeneous countries in relation to economic development, size, liquidity and market accessibility, two strategies were adopted in the attempt to encounter the potential sources of risk. In the first one, macroeconomic variables often cited in the relevant literature were used. In the second strategies, the risk factors were the portfolios – denominated statistical factors - constructed from a Principal Component Analysis using all 44 countries equity index available by MSCI.

The first result that draws the attention is the great resemblance between the first statistical factor and the world excess of return. The first statistical factor points the direction of greatest variability of the system containing the time series returns of the 44 markets. The world excess return is a market value weighted equity index of 24 developed markets and 21 emergent markets. They have a correlation of over 90%, their mean and standard deviation are of the same magnitude and, in a regression of the first statistical factor against all the macroeconomic factors, the coefficient of the world excess return is significant at 1%.

We use the statistical and macroeconomic variables separately as sources of risk factors in APT models with different number of factors for each of the three groups. Two methods of estimation were applied: GMM and ITNLSUR. For the macroeconomic factors, in the CAPM (Model 1), the world excess return is priced in all groups, independent of the estimation method. As for the statistical risk factors, in the model with just one factor (Model 1), for the three groups, the first statistical factor has always a significant risk price. A significant risk premium is observed for other factors, but the results are sensible to the group and method of estimation chosen. However, the inclusion of more factors tends to reduce the absolute pricing error.

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9. APPENDIX 1

	WORLD	AUSTRIA	AUSTRI	BEL	CAN	DEU	FIN	FRA	GER	GRE	HK	IRE	ISR	ITA	JAP	NET	NEW	NOR	POR	SIN	SPA	SWE	SWI	
AUSTRA	1.000	0.821	1.000																					
AUSTRI	0.702	0.677	1.000																					
BEL	0.754	0.638	0.770	1.000																				
CAN	0.850	0.759	0.652	0.600	1.000																			
DEN	0.753	0.591	0.692	0.745	0.683	1.000																		
FIN	0.695	0.546	0.390	0.430	0.581	1.000																		
FRA	0.873	0.710	0.682	0.785	0.726	0.730	0.627	1.000																
GER	0.848	0.688	0.667	0.745	0.685	0.734	0.615	0.889	1.000															
GRE	0.620	0.527	0.630	0.637	0.515	0.590	0.434	0.644	0.644	1.000														
HK	0.671	0.641	0.488	0.447	0.660	0.493	0.409	0.538	0.535	0.398	1.000													
IRE	0.735	0.640	0.655	0.730	0.592	0.636	0.514	0.651	0.663	0.519	0.427	1.000												
ISR	0.564	0.441	0.281	0.372	0.557	0.476	0.467	0.530	0.522	0.377	0.395	0.384	1.000											
ITA	0.711	0.572	0.596	0.640	0.577	0.661	0.545	0.746	0.709	0.581	0.390	0.545	0.468	1.000										
JAP	0.679	0.602	0.458	0.398	0.531	0.444	0.446	0.493	0.414	0.331	0.436	0.444	0.247	0.433	1.000									
NET	0.875	0.705	0.728	0.846	0.697	0.775	0.593	0.885	0.874	0.622	0.571	0.727	0.494	0.706	0.513	1.000								
NEW	0.687	0.761	0.624	0.550	0.599	0.510	0.474	0.602	0.566	0.520	0.553	0.530	0.296	0.506	0.522	0.624	1.000							
NOR	0.778	0.740	0.734	0.726	0.751	0.746	0.531	0.731	0.695	0.599	0.544	0.647	0.464	0.616	0.506	0.754	0.626	1.000						
POR	0.668	0.537	0.621	0.675	0.573	0.669	0.506	0.739	0.697	0.613	0.438	0.565	0.426	0.638	0.367	0.706	0.521	0.615	1.000					
SIN	0.719	0.676	0.539	0.524	0.658	0.509	0.404	0.583	0.576	0.466	0.782	0.455	0.413	0.463	0.469	0.613	0.632	0.612	0.433	1.000				
SPA	0.812	0.691	0.625	0.690	0.649	0.697	0.540	0.810	0.782	0.635	0.542	0.629	0.489	0.723	0.477	0.779	0.601	0.675	0.751	0.576	1.000			
SWE	0.823	0.692	0.553	0.623	0.699	0.697	0.695	0.805	0.808	0.561	0.557	0.592	0.610	0.669	0.491	0.791	0.614	0.691	0.658	0.576	0.751	1.000		
SWI	0.749	0.589	0.645	0.696	0.568	0.654	0.490	0.743	0.710	0.544	0.452	0.605	0.343	0.580	0.477	0.785	0.588	0.618	0.643	0.486	0.690	0.647	1.000	
UK	0.874	0.737	0.723	0.786	0.732	0.729	0.604	0.824	0.767	0.581	0.608	0.743	0.480	0.630	0.520	0.846	0.612	0.756	0.643	0.632	0.747	0.721	0.726	
USA	0.930	0.692	0.567	0.666	0.791	0.655	0.646	0.768	0.777	0.530	0.588	0.687	0.549	0.594	0.484	0.778	0.567	0.646	0.562	0.633	0.718	0.728	0.656	
BRA	0.665	0.617	0.518	0.492	0.643	0.548	0.448	0.562	0.579	0.434	0.518	0.482	0.436	0.502	0.396	0.594	0.464	0.607	0.507	0.537	0.584	0.554	0.472	
CHL	0.610	0.538	0.447	0.467	0.584	0.502	0.343	0.475	0.480	0.488	0.539	0.433	0.407	0.425	0.358	0.516	0.515	0.592	0.415	0.623	0.546	0.517	0.435	
CHN	0.504	0.531	0.406	0.319	0.541	0.356	0.284	0.391	0.406	0.292	0.674	0.292	0.297	0.223	0.253	0.410	0.375	0.453	0.265	0.636	0.402	0.369	0.274	
COL	0.392	0.391	0.443	0.358	0.356	0.371	0.174	0.305	0.359	0.367	0.301	0.355	0.230	0.361	0.231	0.391	0.405	0.405	0.289	0.391	0.355	0.343	0.311	
INDI	0.518	0.509	0.459	0.411	0.513	0.438	0.332	0.470	0.445	0.480	0.425	0.337	0.378	0.438	0.348	0.472	0.400	0.497	0.497	0.492	0.438	0.473	0.350	
KOR	0.509	0.483	0.455	0.401	0.493	0.385	0.231	0.412	0.397	0.357	0.554	0.306	0.271	0.336	0.364	0.437	0.518	0.445	0.387	0.664	0.423	0.403	0.430	
INDO	0.580	0.569	0.373	0.387	0.472	0.396	0.412	0.439	0.418	0.405	0.441	0.409	0.270	0.373	0.548	0.439	0.504	0.435	0.342	0.492	0.444	0.466	0.407	
MAL	0.447	0.396	0.359	0.267	0.441	0.321	0.243	0.374	0.392	0.300	0.584	0.282	0.235	0.269	0.287	0.408	0.439	0.345	0.295	0.628	0.362	0.398	0.306	
MEX	0.670	0.606	0.474	0.445	0.619	0.461	0.447	0.553	0.522	0.419	0.549	0.467	0.470	0.452	0.401	0.531	0.499	0.549	0.413	0.602	0.594	0.520	0.423	
PER	0.517	0.613	0.476	0.400	0.580	0.369	0.265	0.433	0.417	0.393	0.366	0.334	0.310	0.392	0.400	0.425	0.488	0.531	0.408	0.487	0.501	0.441	0.362	
PHI	0.480	0.496	0.409	0.338	0.470	0.323	0.217	0.368	0.381	0.306	0.585	0.276	0.253	0.277	0.292	0.393	0.487	0.386	0.326	0.648	0.393	0.360	0.365	
POL	0.563	0.530	0.418	0.374	0.503	0.425	0.471	0.492	0.441	0.468	0.399	0.309	0.408	0.377	0.377	0.466	0.491	0.522	0.495	0.467	0.523	0.525	0.464	
POL	0.670	0.705	0.593	0.500	0.668	0.498	0.402	0.577	0.569	0.502	0.590	0.467	0.352	0.450	0.528	0.557	0.588	0.632	0.467	0.642	0.529	0.535	0.490	
AS	0.563	0.550	0.401	0.378	0.524	0.369	0.289	0.496	0.493	0.331	0.612	0.424	0.319	0.366	0.340	0.470	0.441	0.466	0.327	0.584	0.432	0.469	0.354	
TAI	0.550	0.614	0.414	0.365	0.518	0.377	0.273	0.392	0.424	0.294	0.615	0.367	0.208	0.332	0.424	0.437	0.546	0.427	0.338	0.672	0.434	0.390	0.357	
THA	0.534	0.421	0.342	0.314	0.496	0.356	0.434	0.496	0.473	0.391	0.377	0.324	0.462	0.378	0.441	0.426	0.397	0.351	0.428	0.424	0.509	0.366	0.366	
THA	0.523	0.527	0.444	0.384	0.540	0.448	0.307	0.460	0.397	0.376	0.458	0.417	0.403	0.402	0.324	0.412	0.406	0.517	0.343	0.495	0.505	0.443	0.311	
ARG	0.269	0.288	0.351	0.314	0.244	0.304	0.149	0.197	0.216	0.297	0.163	0.265	0.121	0.238	0.209	0.214	0.303	0.292	0.174	0.205	0.196	0.163	0.236	
JOR	0.126	0.186	0.089	-0.011	0.166	0.081	0.125	0.048	0.079	0.169	0.166	0.125	0.145	0.085	0.014	0.074	0.111	0.103	0.065	0.218	0.020	0.130	-0.025	
PAK	0.266	0.294	0.300	0.254	0.315	0.238	0.185	0.215	0.154	0.345	0.251	0.236	0.231	0.162	0.213	0.222	0.279	0.308	0.250	0.313	0.201	0.172	0.196	
SL																								

	UK	USA	BRA	CHIL	CHIN	COL	INDI	INDO	KOR	MAL	MEX	PER	PHI	POL	AS	TAI	THA	TUR	ARG	JOR	PAK	SL	
WORLD																							
AUSTRA																							
AUSTRI																							
BEL																							
CAN																							
DEN																							
FIN																							
FRA																							
GER																							
GRE																							
HK																							
IRE																							
ISR																							
ITA																							
JAP																							
NET																							
NEW																							
NOR																							
POR																							
SIN																							
SPA																							
SWE																							
SWI																							
UK	1.000																						
USA	0.775	1.000																					
BRA	0.583	0.587	1.000																				
CHIL	0.519	0.553	0.639	1.000																			
CHIN	0.449	0.478	0.469	0.504	1.000																		
COL	0.379	0.322	0.406	0.431	0.277	1.000																	
INDI	0.426	0.430	0.480	0.528	0.417	0.387	1.000																
INDO	0.412	0.435	0.424	0.508	0.410	0.391	0.428	1.000															
KOR	0.439	0.496	0.383	0.434	0.356	0.322	0.393	0.362	0.599	0.377	1.000												
MAL	0.371	0.352	0.324	0.462	0.474	0.303	0.362	0.599	0.377	1.000													
MEX	0.559	0.621	0.639	0.569	0.458	0.346	0.428	0.428	0.401	0.363	1.000												
PER	0.405	0.382	0.605	0.542	0.405	0.389	0.387	0.403	0.331	0.337	0.555	1.000											
PHI	0.368	0.436	0.358	0.510	0.499	0.296	0.340	0.595	0.371	0.588	0.442	0.355	1.000										
POL	0.468	0.479	0.471	0.401	0.354	0.266	0.397	0.355	0.400	0.373	0.530	0.385	0.343	1.000									
AS	0.574	0.549	0.540	0.563	0.564	0.349	0.454	0.436	0.513	0.424	0.560	0.553	0.506	0.476	1.000								
TAI	0.446	0.493	0.478	0.514	0.555	0.319	0.432	0.381	0.472	0.511	0.449	0.406	0.467	0.333	0.515	1.000							
THA	0.442	0.488	0.423	0.513	0.505	0.307	0.357	0.583	0.614	0.567	0.452	0.387	0.670	0.356	0.614	0.533	1.000						
TUR	0.480	0.496	0.455	0.434	0.331	0.360	0.369	0.281	0.313	0.281	0.436	0.303	0.264	0.366	0.415	0.338	0.276	1.000					
ARG	0.475	0.445	0.553	0.541	0.406	0.332	0.329	0.349	0.304	0.327	0.614	0.514	0.359	0.383	0.451	0.429	0.404	0.382	1.000				
JOR	0.262	0.207	0.197	0.241	0.047	0.238	0.226	0.226	0.133	0.178	0.204	0.204	0.133	0.166	0.200	0.124	0.197	0.170	0.114	1.000			
PAK	0.166	0.103	0.295	0.222	0.134	0.249	0.299	0.136	0.123	0.194	0.221	0.206	0.126	0.183	0.174	0.202	0.232	0.190	0.155	0.097	1.000		
SL	0.283	0.194	0.333	0.367	0.251	0.238	0.342	0.262	0.233	0.238	0.321	0.238	0.210	0.196	0.255	0.169	0.199	0.166	0.295	0.251	0.331	1.000	