Modelling the Dependence Structure between Australian Equity and Real Estate Markets –

a Conditional Copula Approach

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Abstract

We apply conditional copula models to investigate the dependence structure between returns of Australian equity markets and Real Estate Investment Trusts (REITS). The dependence between these assets has a significant impact on the diversification potential and risk for a portfolio of multiple assets and is therefore of great interest to portfolio managers and investors. We observe significant correlations and tail dependence between the considered series indicating a limited diversification potential of investments in REITS in Australia. Conducting a backtesting Value-at-Risk analysis, we also find that ignoring the complex dependence structure could lead to a significant underestimation of the actual risk.

Keywords*: REITS, Dependence Structure, Copula Models, Goodness-of-Fit Tests, Risk Analysis

1. Introduction

All over the world, the market for Real Estate Investment Trusts (REITS) has shown substantial growth rates within the last decades. REITS were originally a tax design for corporations investing in real estate assets in order to reduce or eliminate the corporate income tax. In return, REITS are required to distribute 90% of their income, which may be taxable, into the hands of the investors. Overall, the REIT structure was designed to provide a similar vehicle for investment in real estate as mutual funds provide for investment in stocks. Since investments in real estate assets are quite different to those in equity, the correlation between REITS and equity markets and therefore their potential for diversification in a portfolio of multiple assets has traditionally been of great interest, see e.g. Brueggeman et al (1984), Clayton and MacKinnon (2001) or Chen et al (2004). For example, Hudson et al (2003) analyzing the reasons for considering real estate investments, suggest the reduction of overall risks of the portfolio as the most prominent one. This paper contributes to the literature by investigating the relationship between REITS and equity returns in the Australian market using an approach that combines GARCH models with different copula functions for the dependence structure. Our analysis focuses on the dependence structure between these assets from a diversification and risk management perspective for a portfolio of multiple assets and is therefore of great interest to investors or portfolio managers.

Australia has long been a nation with a strong interest for property investments such that the sector has shown substantial growth rates throughout the last decades and has played a major role in domestic financial markets. Australian Real Estate Investment Trusts (AREITS) are a unitized portfolio of property assets, listed on the Australian stock exchange which allows investors to purchase a share in a diversified and professionally managed portfolio of real estate. Currently, the sector represents more than 5% of the total market capitalization of the S&P/ASX 200 Index. The market is often classified by offering four different types of REITS, see e.g. Davidson et al (2003): (i) equity trusts where the assets are invested in ownerships claims to various types of properties, (ii) mortgage trusts where the assets are invested in claims where interest is the main source of income like for example mortgages, (iii) hybrid trusts that invest in both equity and mortgages and (iv) specialized trusts that invest for example in development and construction or are involved in sale and lease-back arrangements.

As pointed out e.g. by Hartzell et al (1999) REITS returns are typically lower than returns of both

small and large capitalization stocks. On the other hand, the literature also reports that returns from investement in REITS are less volatile than those of equity investments, see e.g. Mueller et al (1994); Ghosh et al (1996); Clayton and MacKinnon (2001) just to name a few. This is not really surprising, since for AREITS by regulation 95% of the income must be paid out as dividend such that a lower volatility of the returns could be expected, see e.g. Tien and Sze (2000).

Another focus is usually set on investigating the correlation between returns from investments in REITS and stock markets, see e.g. Brueggeman et al (1984), Chen and Peiser (1999), Clayton and MacKinnon (2001), Hudson et al (2003) or Chen et al (2004). As mentioned above, such an analysis helps to determine diversification effects of REITS in portfolios of multiple assets. While earlier studies on the topic have shown significant negative correlations between REITS and other assets (Brueggeman et al, 1984), more recent studies often report weak but positive correlations between investments in REITS and shares (Chen and Peiser, 1999; Hartzell et al, 1999; Clayton and MacKinnon, 2001) that still provide some potential for diversification. Chen et al (2004) examine the economic significance of including REITS into an investment portfolio, and show that the mean-variance frontier can be augmented and the investment opportunity set can be enlarged. However, due to changes in the correlation structure, it may be difficult to determine diversification effects and constructing the Markowitz optimal portfolio through time. Glascock et al (2000) find time-varying correlations and suggest that since structural changes in the early 1990s REITs behave more like investment in stocks and less like bonds. Inspired by Engle (2002) and his work on Dynamic Conditional Correlation (DCC) models, also the changing nature of the dependence between REITS and other investments has been investigated. Huang and Zhong (2006) apply Engle's model for portfolio constructing with REITS, and the authors find that the DCC model outperforms other correlation structures such as rolling, historical and constant correlations. Cotter and Stevenson (2006) use a VAR-GARCH model to study the daily REIT volatility in the US market, and the same methodology is applied in Zhou and Bao (2009) for the examination of cross-correlation between types of properties indices in Hong Kong. The studies find strong evidence of multivariate volatilities both in the US and Hong Kong market. Finally, Knight et al (2005) comparing UK and global public real estate stocks with equivalent general equity market returns find some tail dependence, in particular in the lower tail. They conclude that real estate and common equity stocks are more closely related when markets produce highly negative returns.

Recently, there has been some criticism towards the assumptions underlying the DCC model, in particular with respect to the assumptions of multivariate normality for the joint distribution of asset returns and the use of a covariance matrix as the natural measure of dependence between the assets. As shown in various studies, see e.g. Cherubini and Luciano (2001), Jondeau and Rockinger (2006), Junker et al (2006), Luciano and Marena (2003) or McNeil (2003), the use of correlation does not appropriately describe the dependence structure between financial assets and could lead to inadequate measurement of the risk. The authors suggest the application of copula methods for modelling the dependence structure of the asset returns in order to overcome this problem. With respect to analysing the dependence structure between different financial assets, the methodology of copulas as alternative to the DCC model has the advantage that it doesn't require the assumptions of joint normality for the distributions. Instead it allows joining arbitrary marginal distributions into their multivariate distribution allowing for a wide range of dependence structures by using different copulas.

In this paper we apply copula models in order to investigate the dependence structure between returns of AREITS and the Australian stock market, represented by the All Ordinaries Index (AOI). Hereby, we contribute to the literature in several dimensions. To our best knowledge this is a pioneer study on investigating the nonlinear relationship between AREITS and returns of the Australian stock market. Further, to our knowledge this is a pioneer study on applying and testing different copula models in real estate markets. Finally, we provide a risk analysis comparing copula models to alternative approaches including the standard multivariate normal approach and a bivariate GARCH BEKK model with respect to risk quantification for a portfolio that combines investments in real estate and stock markets.

The remainder of the paper is set up as follows. Section 2 provides a review of different copula models as well as the GARCH BEKK model. Section 3 describes the data while Section 4 provides an empirical analysis of the considered models to a time series consisting of an Australian REIT and equity index. We further conduct a risk analysis for an exemplary portfolio consisting of investments in both Australian equity and real estate markets. Section 4 concludes and provides suggestions for future work.

2 Dynamic Correlation and Copula Models

This section provides a brief review of the approaches that will be used in the empirical analysis to

examine the dependence structure between the returns of ASX–REITS and the Australian All Ordinaries Index (AOI). First we provide a brief overview of copula functions and their application to dependence modelling between random variables. Then we review multivariate GARCH models as an alternative way to model the time-varying dependence structure between time series.

2.1 Copula Functions and Estimation

A copula is a function that combines marginal distributions to form a joint multivariate distribution. The concept was initially introduced by Sklar (1959), but has only gained high popularity in modelling financial or economic variables in the last decade. For an introduction to copulas see e.g. Nelsen (1999) or Joe (1997), for applications to various issues in financial economics and econometrics, see Cherubini et al. (2004), McNeil et al. (2005), Frey and McNeil (2003) and Hull and White (2004) just to name a few. As shown by Cherubini and Luciano (2001), Jondeau and Rockinger (2006), Junker et al (2006) or Luciano and Marena (2003), the use of correlation usually does not appropriately describe the dependence structure between financial assets and could lead to inadequate measurement of the risk. Longin and Solnick (2001) and Ang and Chen (2002) empirically show that generally asset returns are more highly correlated during volatile markets and during market downturns. Dowd (2004) suggests that the strength of the copula framework comes from its feature that it does not have any assumptions on the joint distributions among the financial assets in a portfolio. Patton (2006) and Jondeau and Rockinger (2006) illustrate how copulas can be applied not directly to the observed return series but for example to vectors of innovations after fitting univariate GARCH models to the individual return series. Overall, the use of copulas offers the advantage that the nature of dependence can be modelled in a more general setting than using linear dependence only that is explained by correlation. It also provides a technique to decompose a multivariate joint distribution into marginal distributions and an appropriate functional form for the dependence between the asset returns.

In the following we will briefly summarize the basic ideas and properties of copulas, for a definition of copulas we refer e.g. to Sklar (1959) or Nelsen (2006). Let $(X_1, X_2, .., X_d)$ be continuous random variables with distribution functions $F_i(x_i) = \Pr(X_i \leq x_i)$, i=1,..,d. Following Sklar (1959), there exists a unique function C such that:

$$\Pr(X_1 \le x_1, X_2 \le x_2, ..., X_d \le x_d) = C(F_1(X_1), F_2(X_2), ..., F_d(X_d)).$$

Further setting $F_i(X_i) = U_i$, the function $C(u_1, ..., u_d) = \Pr(U_1 \le u_1, ..., U_d \le u_d)$ is the distribution of

$(U_1, U_2, ..., U_d) = (F_1(x_1), ..., F_d(x_d))$ whose margins are uniform on [0,1]. This function C is called a

copula and denotes a joint cumulative density function (CDF) of the d U~ [0; 1] distribution functions.

Another way to express this is that a copula maps uniform distributions U~ [0; 1] into one joint

distribution. The copula framework can be generalized for any collection of marginal distributions

and joint distributions. In our application we will only consider the bivariate case with a function

C(u,v) such that,

$$C(u,v) = C[F(x), G(y)].$$

Then the function C(u,v) is defined as a copula function which relates the marginal distribution

functions F(x) and G(y) into their joint probability distribution. Moreover, if the marginal

distributions F(x) and G(y) are continuous, the copula function C(x,y) is unique, see Sklar (1959) and

the copula is an indicator of the dependence between the variables X and Y. The literature reports is a

wide range of different copulas, see e.g. Joe (1997) or Nelsen (2006) for an overview of the most

common parametric families of copulas. In the following we will limit ourselves to a description of a

number of families of copulas that will be used later on in the empirical analysis. These families of

copulas commonly used in finance include the Gaussian copula, the Student t-copula, the Clayton and

Gumbel copula.

The probably most intensively used copulas in financial applications are the ellipitical Gaussian and

Student t copula. The Gaussian copula is constructed using the multivariate normal distribution and

can be denoted by

$$C^N(u_1, ..., u_d) = \Phi_\Sigma(\Phi^{-1}(u_1), ..., \Phi^{-1}(u_d))$$

Hereby, Φ^{-1} denotes the inverse of the standard normal cumulative distribution function and Φ_Σ the

standard multivariate Normal distribution with correlation matrix Σ. The multivariate normal copula

correlates the random variables rather near the mean and not in the tails. Therefore, it fails to

incorporate tail dependence what can often be observed in financial data. To also add more

dependence in the tails, alternatively, the Student t-copula can be used. The Student t-copula is denoted by

$$C_v^t(u_1,...,u_d) = t_{v,\Sigma}(t_v^{-1}(u_1),...,t_v^{-1}(u_d))$$

where t_v^{-1} denotes the inverse of the Student t cumulative distribution function with v degrees of freedom and $t_{v,\Sigma}$ the multivariate Student t distribution with v degrees of freedom and correlation matrix Σ. While the concepts of a multivariate Gaussian and Student t copula seem to be quite similar, for the Student t copula we need to identify the degrees of freedom parameter. Depending on the degrees of freedom parameter, the Student t copula can also incorporate tail dependence. Hereby, low values of the parameter v indicate strong tail dependence.

Both the Gaussian and Student t copula are symmetric. However, often financial variables are observed to exhibit tail-dependence in only one of the tails, either the upper right or lower left edge of the data. For example, tail-dependence in the lower left tail indicates that the two variables show simultaneous extreme negative returns while when returns of one of the variables are highly positive this may not affect the other financial variable that much. To model asymmetric tail-dependence, so-called Archimedean copulas can be used, see e.g. Cherubini et al (2004). Two of the most prominent members of the family of Archimedean copulas are the Clayton and Gumbel copula that will be briefly described in the following.

The Clayton copula is an asymmetric Archimedean copula, exhibiting greater dependence in the negative lower tail than in the positive upper one. The multivariate Clayton copula can be denoted by:

$$C(u_1,...,u_d) = \left[\sum_{i=1}^{d} u_i^{-\theta} - d + 1 \right]^{-\frac{1}{\theta}} \quad with \quad \theta > 0$$

For the Clayton copula, the parameter θ is used to measure the degree of dependence. The Gumbel copula, on the other hand, exhibits greater dependence in the upper right tail. The multivariate Gumbel copula is given by:

$$C(u_1,...,u_d) = \exp\left\{ -\left[\sum_{i=1}^{d}(-\ln u_i)^\phi \right]^{-\frac{1}{\phi}} \right\} \quad with \quad \phi > 1$$

Similar to the Clayton copula, a parameter ϕ is used to measure the dependence. For further properties and examples of elliptical and Archimedean copulas and the on the construction of such copulas by using generator functions, we refer to Nelsen (2006) and Cherubini et al (2004).

Copulas offer various alternatives to the correlation coefficient as it comes to modeling the dependence structure. Often Kendall's tau is used to measure the dependence structure when e.g. employing Archimedean copulas like the Clayton or Gumbel copulas as well as the elliptical Gaussian and Student t copulas. Kendall's tau τ is a rank-based measure of dependence that provides consistent estimation of the true underlying copula as it is shown for example in Deheuvels (1979). The use of Kendall's tau can easily be motivated for the bivariate case. Assume that we have observations of two financial variables (X_s, Y_s), s=1,…n, for example the return series of two indices or financial assets. Instead of using the returns to measure the dependence, to calculate Kendall's tau usually the ranks or the empirical probability integral transforms (u_i, v_i) with $u_s = \hat{F}(x_s)$ and $v_s = \hat{G}(y_s)$ are used. To compute τ we then draw a line to connect two pairs (u_s, v_s) and (u_t, v_t). If the slope of this line is positive, we say the pair is concordant and count +1. If the slope of the line is negative, we say the pair is discordant and count −1. This process is repeated for all choices of distinct pairs (u_s, v_s) and (u_t, v_t). Overall, there are $m = n(n − 1)/2$ such choices. Kendall's τ is then simply the sum of all concordant minus discordant pairs or the sum of +1s and −1s, divided by m. Values of τ range from −1 to +1, in the case of independence τ will be equal to 0, see e.g. Nelsen (1999). In some applications as an alternative to Kendall's tau also Spearman's rank correlation coefficient rho is used. For comparison of these two measures that emphasize different aspects of the dependence, see e.g. (Capéraà and Genest, 1993).

In the bivariate case, based on the estimated value of τ the corresponding dependence parameters for the Gaussian, Student t, Clayton and Gumbel copula can be calculated as a function of τ. As pointed out by Genest and Rémillard (2008) under weak regularity conditions on the copula family, this yields a consistent estimator of the dependence parameter. In the bivariate case, the estimated dependence parameters then completely specifies the Gaussian, Student t, Clayton and Gumbel copula are completely specified. Due to its simplicity in comparison to other estimation techniques, copula estimation via rank transformation and Kendall's tau is often applied in practical applications.

Unfortunately, it is limited to a bivariate setting because it makes inference on the dependence structure of the multivariate model from a chosen dependence coefficient. For our application this will be sufficient, but we refer to Cherubini et al (2004) for estimation of copula dependence parameters in a multivariate setting.

Note that in the empirical analysis we will apply the copula framework not directly to the observed returns but to the vectors of innovations after fitting univariate GARCH models to the individual return series of ASX–REITS and the Australian All Ordinaries Index (AOI). This approach has been suggested and successfully applied e.g. by Patton (2006) or Jondeau and Rockinger (2006). For the specification of the GARCH models for the univariate series we refer to section 3.

2.2 Multivariate GARCH Models

As an alternative approach to the use of copula models we suggest to capture the different regimes of volatility and correlation between the returns of the considered series using multivariate GARCH models. The literature suggests a variety of models including the VECH model (Bollerslev et al, 1988), the diagonal VECH model or the so-called Baba-Engle-Kraft-Kroner (BEKK) model defined in Engle and Kroner (1995) or Kroner and Ng (1998). One of the advantages of these models is that they allow for a high degree of flexibility in the estimation of the time varying covariance structure between the considered variables. In our paper, we decided to apply the class of BEKK model as it overcomes two issues that make the application of VECH or diagonal VECH models difficult: the problem of over-parameterisation that is typically associated with VECH models as well as the problem of not guaranteeing a positive semi-definite covariance matrix related to the diagonal VECH model. Following Engle and Kroner (1995) the multivariate GARCH BEKK model can be represented by the following equation:

$$H_{i,j,t} = WW' + A' \, e_{i,t-1} e_{j,t-1} A + B' \, H_{i,j,t-1} \, B \qquad i,j = [ii,jj,ij]'$$

Hereby W, A, B are 3*3 coefficient matrices, referring to the dynamic relationship of the variance and covariance in the bivariate asset case. The positive definiteness of the matrices is ensured by using WW', AA' and BB' in above equation. Often, a further simplified version of the BEKK model in which A and B are diagonal matrices is used in empirical applications. This model is then referred

to the as the 'diagonal BEKK' model and only requires the estimation of the main diagonal of the coefficient matrices W, A, B. Futhermore, often in the diagonal BEKK model for the covariance terms the restrictions $A(3,3)=A(1,1)*A(2,2)$ and $B(3,3)=B(1,1)*B(2,2)$ are imposed such that in the bivariate case only seven model parameters need to be estimated. In our empirical analysis, we decided to apply this version of the diagonal BEKK model.

3. The Data

In this section we investigate the dependence structure between returns from Australian Real Estate Investment Trusts and the Australian All Ordinaries Index (AOI). With respect to Australian REITS we will consider monthly log-returns from the ASX 200 A-REIT index, which is an index comprising approximately 70 listed Australian property trusts representing various types of properties under management. The considered time period ranges from January 1980 to April 2009. The data for the considered time period was obtained from DataStream and returns are calculated based on monthly observations of the two indices, with a total number of 351 observations for each series. For our analysis we consider log-returns that are calculated as $r_t=\ln(P_{t+1}/P_t)$ from the original price series. In the following we will refer to the return series of the All Ordinaries Index as AOI and to the return series of ASX 200 A-REIT index as AREITS. Table 1 provides descriptive statistics for the logreturns of the two series.

Name	Mean	Median	Max	Min	StDev	Skew	Kurtosis
AREITS	0.08%	0.24%	9.02%	-29.16%	4.29%	-2.1916	14.37
AOI	0.59%	1.06%	21.47%	-47.82%	5.40%	-2.1614	22.24

Table 1: Summary statistics of log differenced AREITS and AOI for the sample period from January 1980 to April 2009.

For the considered time period, the mean and standard deviation of AREITS is 0.08% and 4.29%. Obviously, both the average return and the standard deviation are smaller than the comparable figures for AOI yielding a mean of 0.58% and a standard deviation of 5.40%. This is in line with the findings of other studies that generally report lower returns and standard deviations for REITS than for both small and large capitalization stocks (Hartzell et al, 1999; Mueller et al, 1994). Also for Australia,

AREITS on average offer lower returns but are slightly less volatile than investments in a stock index. The table further indicates that both return series exhibit skewness and excess kurtosis. In particular for AOI a number of highly negative returns could be observed. The highest loss occurred was 47% for the AOI in November 1987, while the maximum loss for AREITS was 29%.

In a first step, we test for stationarity in the returns what is a required condition for the validity of fitting a time-series model to the data. We apply the standard Augmented Dickey-Fuller (ADF) test for unit roots (Dickey and Fuller, 1979) to the returns of AOI and AREITS. Given the results in Table 2, we clearly reject the null hypothesis of a unit root for both series and conclude that the return series AOI and AREITS are stationary.

ADF test	Test Statistic(Tau)	Asymptotic p-value
AREITS	-4.76621	2.254e-006
AOI	-7.52149	8.346e-013

Table 2: Results for ADF unit root tests for return series AREITS and AOI. The table provides the value for the test statistic tau as well as the corresponding p-value.

We also test for normality of the return series using the Jarque-Bera test. The test statistic has an asymptotic chi-square distribution with two degrees of freedom and can be tested against the null hypothesis that the data is normally distributed. We obtain a test statistic of 2171 for AREITS and 5874 for AOI such that both are significantly greater than the corresponding critical level of the chi-square distribution 5.99. Hence we can reject the null hypothesis that the return series AREITS and AOI follow a normal distribution and conclude that a standard multivariate normal approach is not really suited to model the joint dynamics of the return series.

4. Empirical Results

4.1 Time Series Models

The results of the previous section suggest that the return series of AOI and AREITS exhibit non-normality and heteroskedasticity. Generally, before applying the copula framework in order to capture the dependence structure between the series an appropriate specification for the marginal distributions is required. Therefore, the models for the univariate time series should take into account the characteristics of the individual return series. Therefore, to capture the non-normality and the indicated different regimes of volatility, we decided to fit AR(1)-GARCH(1,1) models to the data:

$$X_{i,t}=c_i+a_iX_{i,t-1}+e_{i,t;}$$

$$e_{i,t}= Z_{i,t}h^{0.5}{}_{i,t}$$

$$h_{i,et}=w_i+b_{1i}e^2{}_{i,t-1}+b_{2i}h_{i,t-1}$$

Since both of the return series exhibit skewness and excess kurtosis we further relax the assumption of a normal distribution for the conditional distribution of the standardized innovations and test different model specifications including the Gaussian, Student t and Generalized Error Distribution (GED), see e.g. Newey and Steigerwald (1997). Note that for the cases when using a Student t and GED for the conditional distribution of the standardized innovations, the parameters of the AR-GARCH model are estimated using quasi-maximum likelihood. Among the tested models we find that in particular the AR-GARCH model with a GED distribution and tail parameter $r=1.221$ performs well for the standardized innovations of AOI. For AREITS the model using a Student t distribution with $v=7$ degrees of freedom yields the best fit for the standardized innovation series. Estimation results for both marginal series are provided in Table 3 while a plot of the original return series and estimated conditional variance based on the AR-GARCH models is provided in Figure 1.

AREITS:	Coefficient	Std.error	P-value
c	0.00082	0.002278	0.750
a	0.13000	0.053110	0.020
w	0.000199	9.83E-05	0.043
b_1	0.298785	0.095148	0.002
b_2	0.599135	0.119817	0.000
AOI:			
c	0.00592	0.002840	0.050
a	0.08000	0.05248	0.210
w	0.000269	0.000190	0.156
b_1	0.119152	0.061481	0.053
b2	0.778642	0.105870	0.000

Table 3: Parameter estimates of AR(1)-GARCH (1,1) model for AREITS with a Student t distribution (v=7) *for the standardized innovations and AR(1)-GARCH (1,1) for AOI with a GED distribution (r=1.221) for the standardized innovations.*

Figure 2 provides a kernel density plot of the standardized innovation indicating that the time series of innovations still exhibits some skewness and excess kurtosis. In the following section, we will now continue to model the dependence structure between the conditional distributions of the standardized innovations using different copula models.
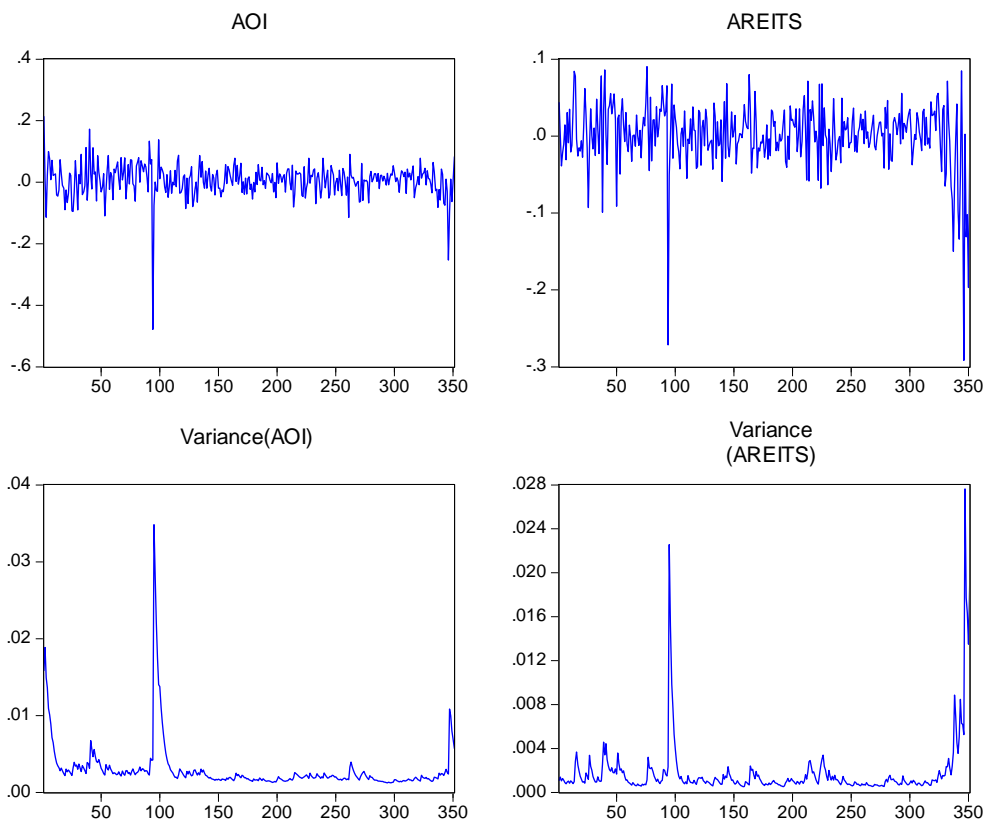
Figure 1: The time series plot for the returns (upper panel) and estimated conditional variance (lower panel) based on the estimated GARCH models for the return series AOI and AREITS.
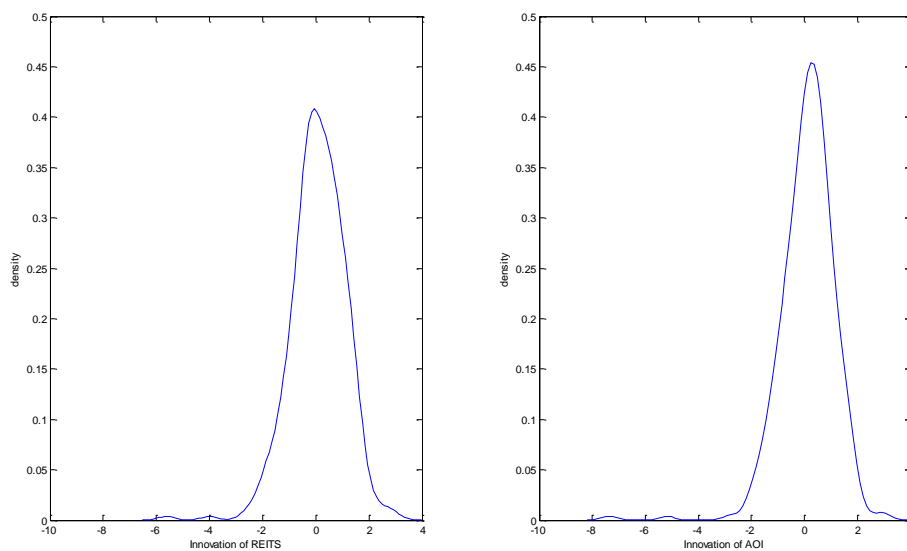

Figure 2: Kernel smoothed density plot for standardized innovation series for AREITS (left panel), and AOI (right panel). The density plot indicate that both series are not normally distributed.

4.2 Modelling the Dependence Structure

As described in Section 2, a possible way to derive the dependence structure between two time series via a copula is to examine the dependence between the rank transforms of the series. This has the advantage that the possibly unknown marginal distribution is not required, since the empirical marginal cdf can be used. While one could also model the dependence between the ranks for the original return series, due to the heteroscedastic behaviour of the return series, we suggest a conditional approach that models the dependence structure of the standardized innovations after applying AR-GARCH model to the univariate series, see e.g. Patton (2006) , Jondeau and Rockinger (2006) and Grégoire et al (2008). To determine Kendall's tau for the two series, in a first step the rank transformations of the standardized innovations are calculated. Figure 3 provides bivariate scatter plots of the standardized innovation series and the corresponding rank transformations. The rank transforms can then be used to calculate Kendall's tau and infer the corresponding dependence parameters for the Gaussian, Student t, Clayton and Gumbel copula.
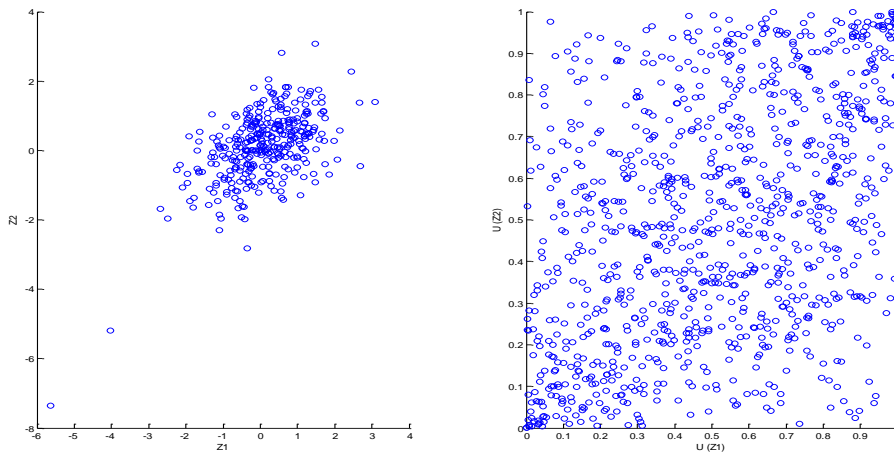

Figure 3: Scatter plot of innovations (z1,z2) from the estimated GARCH models for AREITS and AOI (left panel) and corresponding rank transformations of the innovations (right panel).

Based on the determined rank transforms, the estimate for Kendall's tau yields $\hat{\tau}=0.327$, with standard error at 0.0358, corresponding to a p-value of 0.000 such that the null hypothesis $\tau=0$ can be rejected at any confidence level. Thus, Kendall's tau is significantly different from indicating a significant positive relationship between the rank transformations of the innovations. Note that these results somehow contradict earlier studies by Brueggeman et al (1984), Chen and Peiser (1999), Hartzell et al (1999) or Clayton and MacKinnon (2001) reporting significant negative or only weak

positive correlations between REITS and other assets, in particular equity investments. Therefore, for the Australian market the diversification potential of investments in REITS might be less significant than for other markets.

Clayton copula	$\hat{\theta}_{clayton} = \dfrac{2 \cdot \hat{\tau}}{1 - \hat{\tau}}$ for $\hat{\tau} > 0$	0.9727
Gumbel copula	$\hat{\phi}_{gumbel} = \dfrac{1}{1 - \hat{\tau}}$ for $\hat{\tau} > 0$	1.4864
Gaussian Copula	$\hat{\rho} = \sin(\dfrac{\pi \cdot \hat{\tau}}{2})$	0.4916
Student t copula ($v=27.2842$)	$\hat{\rho} = \sin(\dfrac{\pi \cdot \hat{\tau}}{2})$	0.4916

Table 4: Estimated dependence parameters for the different copula families based on the estimated Kendall's tau. $\hat{\tau}=0.327$.

Copulas	LOGL	AIC	SIC	HQIC
Clayton copula	2.34×10^{21}	-96.40	-92.55	-94.87
Gumbel copula	3.21×10^{15}	-69.40	-65.55	-67.87
Gaussian copula	1.13×10^{21}	-94.95	-91.10	-93.43
Student t copula	1.42×10^{21}	-93.38	-85.69	-90.34

Table 5: Results for loglikelihood andparsimony model selection criteria Akaike Information Criterion (AIC), Schwarz Information Criterion (SIC) and Hannan-Quinn Information Criterion (HQIC).

Using the given relationship between Kendall's tau and the copula dependence parameters for the Clayton, Gumbel, Gaussian and Student t copula in Table 1 we then calculate the corresponding parameter estimates that are reported in Table 4. Note that for the Gaussian, Clayton and Gumbel copula are completely specified by the dependence parameter, for the Student t copula we also need to estimate the degrees of freedom parameter v. To do this, we apply the so-called inference for the margins (IFM) method, see e.g. Cherubini et al (2004). The method yields an estimate of $v = 27.2842$ for the considered rank series. In a next step we want to investigate which of the considered copulas is most appropriate for the dependence structure between the innovation series.

While the estimation of the dependence parameter for each copula function is easy to implement, the decision which of the considered copulas provides the best fit to the actual dependence structure of the data is often not that straightforward. In a first step we investigate the fit of the different copulas based on the loglikelihood as well as parsimonious model selection criteria like the Akaike Information Criterion (AIC), Schwarz Information Criterion (SIC) and Hannan-Quinn Information Criterion (HQIC), see e.g. Greene (2003). Results are reported in Table 5. We find that these criteria unambiguously support the Clayton copula as providing the best fit to the data. However, Berg and Bakken (2006) point out that Akaika type and related information criteria are usually not able to provide enough understanding about the power of the decision rule employed. Therefore, Genest et al (2006, 2009) suggest so-called 'blanket' goodness-of-fit tests that are able to reject or fail to reject a parametric copula based on bootstrapped P value and are usually preferred in empirical applications.

For selecting the most appropriate among various copulas, in these goodness-of-fit tests usually the distance between the estimated and the so-called empirical copula is examined. To derive the empirical copula, the empirical marginal distributions are used. Let $(X_{1i}, \ldots, X_{ni})$ be n independent observations of the random variable X_i with empirical marginal cdf $\hat{F}_i(x)$, $i = 1, \ldots d$. Then the empirical probability integral transforms u_{ji} can be denoted by:

$$u_{ji} = \hat{F}_i(x_{ji}) \quad i = 1, ..d, \quad j = 1, ..n$$

.

For the vector $\mathbf{u} = (u_1, \ldots, u_d)$, using the marginal cdf's, the empirical copula is given by

$$\hat{C}^{emp}(u) = \frac{1}{n+1} \sum_{j=1}^{n} I(\hat{F}_1(X_{j1}) \le u_1, \ldots, \hat{F}_d(X_{jd}) \le u_d) = \frac{1}{n+1} \sum_{j=1}^{n} I(U_1 \le u_1, \ldots, U_d \le u_d)$$

Thus, the empirical copula is the observed frequency of $P(U_1 \le u_1, \ldots, U_d \le u_d)$. Genest et al (2009) provide various options for copula goodness-of-fit tests, including tests based on ranks, probability integral transforms and Rosenblatt's transform. They also investigate different implementations of the tests using the Cramér-Van Mises, Kolmogorov-Smirnov and Anderson-Darling statistic to measure the difference between the estimates and the empirical copula. They report in particular good results for the Cramér-Von Mises statistic that will also be implemented in this study. For various alternative tests, we refer to Berg and Bakken (2006) or Genest et al (2009). As mentioned before for the goodness-of-fit tests, the null hypothesis is that the examined copula provides an appropriate fit to

the data is examined. Then for the suggested approach the test procedure for investigating whether the dependence structure of a multivariate distribution is well-represented by a specific parametric family of copulas can be summarized as follows:

1. Based on the vectors of rank observations $(U_1, \ldots, U_n)$ and the estimated Kendall's tau for the empirical data, the corresponding dependence parameters for the copula families can be determined. Then the values $\hat{C}^{emp}(u)$ and $C_\theta(u)$ for the empirical and the estimated family of copulas can be calculated.

2. Using the Cramér-Von Mises statistic, the distance between the empirical and estimated copula is calculated by $S_n = \sum_{i=1}^{n}[\hat{C}^{emp}(U_i) - C_\theta(U_i)]^2$.

3. Then for some large integer N, the following steps are repeated:

   a) Generate a random sample from $C_\theta(u)$ and compute the associated rank vectors $(U_1^*, \ldots, U_n^*)$ as well as the empirical copula $\hat{C}^{emp*}(u)$.

   b) Estimate Kendall's tau τ^* for the generated random sample and estimate the parametric copula $C_\theta^*(u)$.

   c) Determine $S_n^* = \sum_{i=1}^{n}[\hat{C}^{emp*}(U_i^*) - C_\theta^*(U_i^*)]^2$ for the generated sample.

4. From the N bootstrap samples, an approximate p-value (which measures the goodness-of-fit of the copula) can be calculated as the fraction of simulations with $S_n^* > S_n$. If the considered copula provides a good fit to the actual dependence structure of the data, we should expect to get high p values, while for a copula providing a bad fit to the actual data, we will expect the p-value to be low. In this case, depending on the level of confidence, the hypothesis that the dependence structure of the bivariate distribution is well-represented by a specific parametric family of copulas is rejected.

5.

For the four considered copula families, the results are reported in Table 6. We obtain the smallest distance $S_n = 0.273$ for the Clayton copula indicating that it provides the best fit to the dependence

structure between the standardized innovation series. However, the distances for the Gaussian and Student t copula are only slightly higher with $S_n = 0.277$ and $S_n = 0.278$.

Copula	S_n	p-value
Clayton	0.0273	0.083
Gumbel	0.0694	0.000
Normal	0.0277	0.066
Student t with (v=28)	0.0278	0.069

Table 6: Cramer-Van-Mises statistic for distance between estimated and empirical copula as well as p-values based on bootstrap goodness-of-fit tests for the different copula families.

For the goodness-of-fit tests, we generate N=10,000 random samples from $C_\theta(u)$, compute the associated rank vectors $(U_1^*, \ldots, U_n^*)$, and calculate the approximate p-value as the fraction of simulations with $S_n^* > S_n$. The p-value provides a measure of how much evidence we have against the null hypothesis of an appropriate fit of the copula to the data. We find that the null hypothesis of an appropriate fit of the Gumbel copula is rejected at all levels, the corresponding p-value is 0.000. The highest p-value is observed for the Clayton copula is 0.083 such that the null hypothesis is not rejected at the 1% or 5% significance level. On the other hand, for the Gaussian and Student t copula we obtain p-values of 0.066 and 0.069, so also for these copulas the null hypothesis of an appropriate fit cannot be rejected at the 1% and 5% significance level.

Figure 4 provides pdf and cdf plots of the estimated Clayton copula for the considered data. The figure illustrates the tail dependence in the lower left tail exhibited by the Clayton copula.

Overall, the Clayton copula provides the best fit according to the loglikelihood as well as for the considered parsimony model selection criteria. Further, it yields the smallest distance between fitted and empirical copula, and the highest p-value for the conducted goodness-of-fit test. So it provides the best fit to the dependence structure between the standardized residuals after fitting an AR-GARCH model to the individual return series. This is an indication in particular for lower left tail dependence between the two series. However, also the symmetric Student t and Gaussian copula provide an appropriate fit to the dependence structure and perform only slightly worse with respect to the considered criteria.
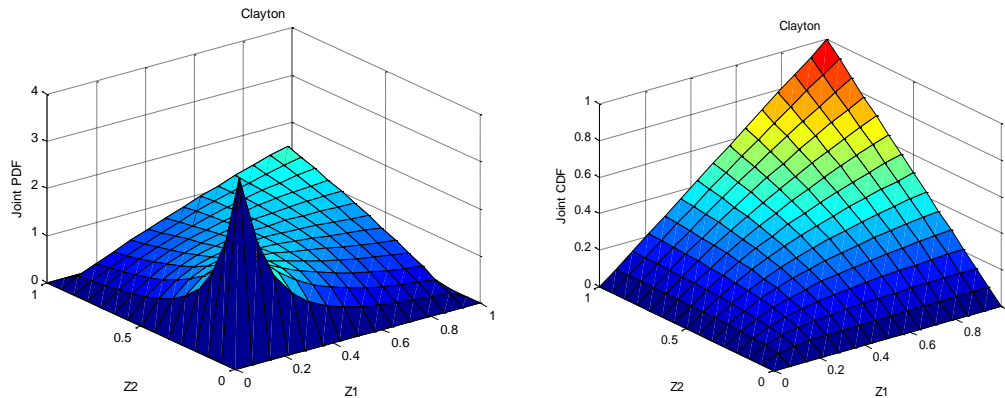
Figure 4: Plot of the estimated probability density function (left panel) and cumulative distribution function(right panel) for the Clayton copula.

Overall, we find a significant dependence structure between the returns of the considered AREIT index and the AOI. These findings somehow contradict earlier studies by e.g. Brueggeman et al (1984), Chen and Peiser (1999), Hartzell et al (1999) or Clayton and MacKinnon (2001) reporting significant negative or only weak positive correlations between REITS and equity investments. Our results are more in line with a more recent study by Knight et al (2005) who report some dependence in the lower left for the relationship between UK and global public real estate stocks with equivalent general equity market returns. The superior fit of the Clayton copula in our empirical analysis suggests that real estate and common equity stocks are more closely related when markets produce highly negative returns. Therefore, for the Australian market the diversification potential of investments in REITS might be less significant than for other markets. An explanation for this might also be the overvaluation of Australian real estate markets during the considered time period that were at least partially driven by wealth effects and portfolio shocks from Australian equity markets as pointed out by Frye et al (2010). This on the one hand explains the existing significant positive relationship between real estate and equity returns and can also be considered as a reason for joint highly negative returns when both markets simultaneously return to their market fundamental levels. Such a strong dependence in the lower left tail can be adequately modeled by the Clayton copula.

4.3 Results for the bivariate GARCH model

As mentioned above this study aims to compare different approaches to modelling the dynamic dependence structure of returns from Australian REITS and the All Ordinaries Index. Therefore, we

also applied the previously described diagonal GARCH BEKK model to the original AREITS and AOI return series. Note that hereby, for the mean equation a simple AR(1) process $X_{i,t}=c_i+a_iX_{i,t-1}+e_{i,t}$ was applied. Further, as mentioned in Section 2.2, for the variance equation A a diagonal BEKK model is applied such that for the variance equation $H_{i,j,t}=WW'+A' e_{i,t-1}e_{j,t-1}A+B' H_{i,j,t-1} B$, only the coefficients on the main diagonal of the coefficient matrix were estimated and for the covariance terms the restrictions $A(3,3)=A(1,1)*A(2,2)$ are $B(3,3)=B(1,1)*B(2,2)$ were imposed. The estimation of the model was conducted using Eviews using maximum likelihood estimation and results for the model parameters are provided in Table 7.

	Coefficients	std.error	Z-stat	p-value
c1	0.002557	0.001678	1.5238	0.1276
c2	0.004279	0.002154	1.986919	0.0469
a1	-0.068083	0.002154	-1.27796	0.2013
a2	-0.000855	0.049915	-0.01713	0.9863
W(1,1)	0.000149	6.84E-05	2.180742	0.0292
W(2,2)	7.27E-05	3.28E-05	2.212662	0.0269
W(3,3)	0.000120	4.77E-05	2.526315	0.0115
A(1,1)	0.370196	0.032271	11.47165	0.0000
A(2,2)	0.520696	0.054348	9.580808	0.0000
B(1,1)	0.906939	0.020647	43.92541	0.0000
B(2,2)	0.831442	0.036600	22.71724	0.0000

Table 7: Parameter estimates for diagonal BEKK model with imposed restrictions for the covariance terms A(3,3)=A(1,1)*A(2,2) and B(3,3)= B(1,1)*B(2,2) by maximum likelihood estimation.
We find that all model parameters in BEKK model are significant at the 5% level and show the expected signs.

4.4 Risk Analysis

After fitting appropriate time series models to the bivariate series, in the following we conduct a risk analysis for a portfolio consisting of the above-mentioned Australian real estate and equity indices. Hereby, in particular the suggested approach using copula functions and the considered GARCH BEKK model are compared to a standard multivariate normal approach that is usually used for risk quantification. In particular we will look at the performance of the models with respect to the adequate quantification of Value-at-Risk.

The Value-at-Risk (VaR) method was first suggested as a standard measure of risk in the 1990s, see e.g. JP Morgan (1996). Since then, it has become the probably most popular tool for internal capital

allocations, determining regulatory capital and reporting in risk management of financial institutions. Generally, VaR can be written as:

$$VaR(a)= F_p(x)>a$$

where F_p is the probability distribution of the portfolio returns X, measured against some threshold probability level a, which usually refers to a probability of e.g. 0.1%, 1% or 5%. Thus, for example the one-day $VaR_{99\%}$ can be interpreted in a way that we are 99% confident that the loss of the portfolio will not exceed $VaR_{99\%}$ within one day. Traditionally, financial returns are assumed to follow a normal distribution, such that a standard variance-covariance approach can be used. In this case, the dependence structure between the different assets in the portfolio is then completely described by a correlation matrix.

It is important to point out that an inadequate estimation of VaR can lead to serious problems incurred from e.g. the underestimation of risk and inadequate capital allocations. Problems with a static variance-covariance approach might include the following issues: (i) non-normality of the marginal distributions, (ii) misspecifying the actual dependence structure by the use of correlation as the only measure of dependence and (iii) ignoring the dynamic dependence structure between the considered return series. In the following, we therefore illustrate how the applied copula and GARCH BEKK approach may be used to appropriately describe the dependence structure and risk for a portfolio consisting of investments in equity and REITs.

Recall that in order to deal with the heteroscedastic behaviour of the return series, initially univariate or bivariate GARCH models were fitted to the return series. As illustrated in Figure 1, both standardized innovation series still exhibit some skewness and excess kurtosis. Therefore, to appropriately capture the risk instead of using a normal distribution for the innovation series we use a non-parametric estimate for the distribution of the marginal innovation series. Hereby, a Gaussian kernel was applied in order to determine the nonparametric estimate of the CDF. For modelling the dependence structure between the standardized innovation series, the copula functions estimated in section 4.2 can be applied. Due to superior fit of the Clayton copula in comparison to the other investigated copulas, in the following we will provide results only for the model using the Clayton copula. Results for the Gaussian, Student t and Gumbel copula are available upon request to the authors. In order to determine VaR figures for a portfolio, we require not only a forecast for the expected return but for the whole distribution of the portfolio returns. In the following we will briefly

describe the necessary steps to determine distributional forecasts for a portfolio consisting of investments in the All Ordinaries Index and ASX 200 A-REIT index. For a more detailed description we refer to e.g. Frees and Valdez (1998), Genest and MacKay (1986), Lee (1993) or Marshall and Olkin (1988). In a first step, we simulate pairs of bivariate uniformly distributed random variables $(u_{1,t}, u_{2,t})$ from a Clayton copula with dependence parameter $\theta = 0.9727$.
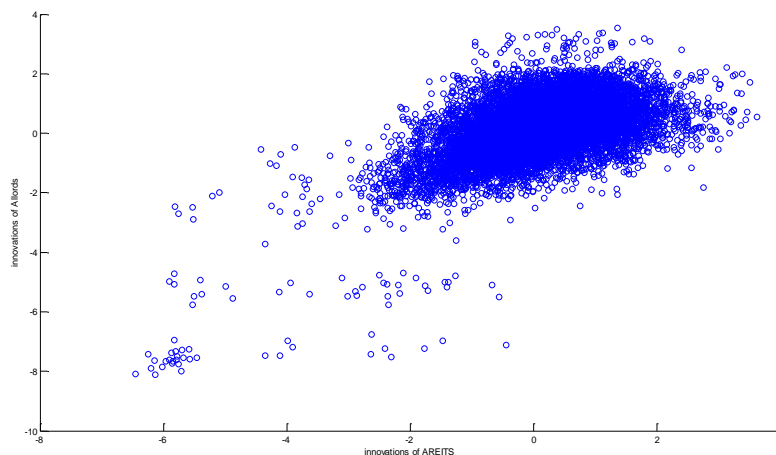

Figure 5: 10000 simulated standardized innovations based on probability integral transform of uniformly distributed random variables ($u_{1,t}$, $u_{2,t}$) from a Clayton copula with dependence parameter $\theta = 0.9727$.

The obtained random numbers are then plugged into the inverse of the nonparametric CDF for the standardized residuals such that by setting $Z_{i,t} = F^{-1}(u_{i,t})$, i=1,2 a pair of dependent innovations are obtained. Figure 5 shows an exemplary bivariate plot for 10000 simulated standardized innovations using the Clayton copula. The simulated innovations in combination with the estimated AR-GARCH model

$$X_{i,t} = c_i + a_i X_{i,t-1} + Z_{i,t}\, h^{0.5}_{i,t,}, \text{ for i=1,2.}$$

for each of the marginal series can then be used in order to determine a distributional forecast of the returns. Hereby, $X_{i,t-1}$ denotes the most recent return observation, c_i and a_i are parameters of the estimated AR(1) process, $Z_{i,t}$ the simulated innovation for series i, $h^{0.5}_{i,t}$ the estimated conditional standard deviation for period t from the AR-GARCH model. Then, using the asset weights for the two investments, a probability distribution of portfolio returns for the next period t+1 can be determined.

Using the 0.1%, 1%, 5% and 10% quantile of this distribution, we can calculate the corresponding 99.9%, 99% and 95% Value-at-Risk figures for each period. For an exemplary portfolio we choose the weights as 17% for investment in AREITS and 83% for investments in the AOI. Note that these weights can be determined using modern portfolio theory by choosing the portfolio on the efficient frontier yielding the best expected return to volatility combination with respect to the Sharpe ratio. Of course, alternative portfolio weights could be applied and were also tested in the empirical analysis. A similar procedure is conducted to obtain forecasts of the portfolio return distribution for the estimated GARCH BEKK model. In this case the Cholesky decomposition of the correlation matrix for time t can be used to simulate dependent random variables for the standardized innovations, see e.g. Cherubini et al (2004). Then using the corresponding weights for the investments in AREITS and AOI and the estimated parameters of the bivariate GARCH model it is straightforward to determine a probability distribution of the portfolio returns for forthcoming periods that can be used to calculate the corresponding VaR figures.

Figure 6 and 7 provide a plot of the actual returns for the exemplary portfolio and the 99% VaR forecasts for the conditional copula and the GARCH BEKK model. From a first glance, we find that both models provide similar results with respect to capturing the dynamics of the portfolio return quite well. Results on the number of exceedances for determined 90%, 95% and 99%-VaR figures are provided in Table 8. Hereby, three different models are considered: (i) a standard (static) variance-covariance approach that estimates the return distribution of the portfolio based on the observed individual returns and correlation between the return series, (ii) the described framework using AR-GARCH models for the marginal series in combination with a Clayton copula to model the dependence structure between the standardized residuals, (iii) the described GARCH-BEKK model that provides dynamic estimates of the variances and covariance between the two series. We compare the number of actually observed exceedances to the expected ones in order to evaluate the model performance with respect to an appropriate quantification of the risk, see e.g. Christoffersen (1998). We find that the static variance-covariance approach underestimates the risk at all confidence levels such that we observe significantly more exceedances of the estimated VaR than expected. On the other hand, the number of exceedances is significantly reduced for the two dynamic models in comparison to that of the static one. Both models yield approximately the expected number of exceedances for the 90% and 95% VaR. For the 99.9% VaR level the more conservative copula

approach including tail dependence seems to provide a more appropriate quantification of the risk yielding only one exceedance while there are two exceedances for the GARCH-BEKK model.

MODEL	95% level	99% level	99.9% level
Static Bivariate Normal	23/ (17.55)	7 / (3.51)	2 / (0.35)
Dynamic Clayton copula	15/ (17.55)	3/ (3.51)	1 / (0.35)
Diagonal BEKK model	18/ (17.55)	4 / (3.51)	2/ (0.35)

Table 8: Actual and expected number (in brackets) of exceedances at 95%, 99% and 99.9% confidence levels for static bivariate normal, diagonal BEKK and Student t copula models.

Conducting robustness tests for various alternative portfolio weights yields quite similar results both for the static variance-covariance approach and the considered dynamic models. For all considered combinations of investment weights the static approach clearly underestimates the actual risk for the portfolio, while the considered dynamic models lead to a more appropriate risk quantification. Further, in the extreme tail - 99.9% VaR – the copula framework provides the most conservative risk figures. Overall, our backtesting study suggests that the dynamic models clearly outperform a static variance-covariance approach. Reasons for this are that the dynamic approaches can take into account important features of the series like heteroscedasticity, non-normality of the returns or standardized residuals, changes in the correlation structure through time and tail dependence.
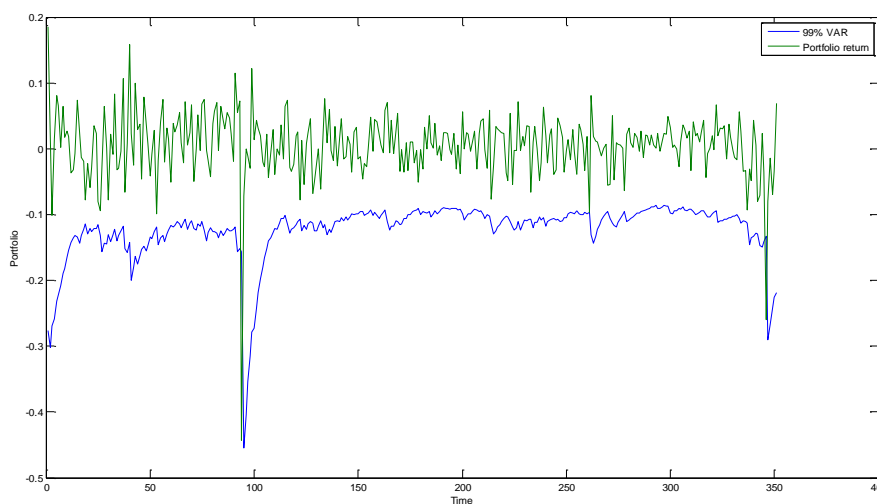

Figure 6: Actual returns and estimated 99% VAR based on a dependence structure modeled using the Clayton copula for a portfolio with weights 17% in AREITS and 83% in AOI.
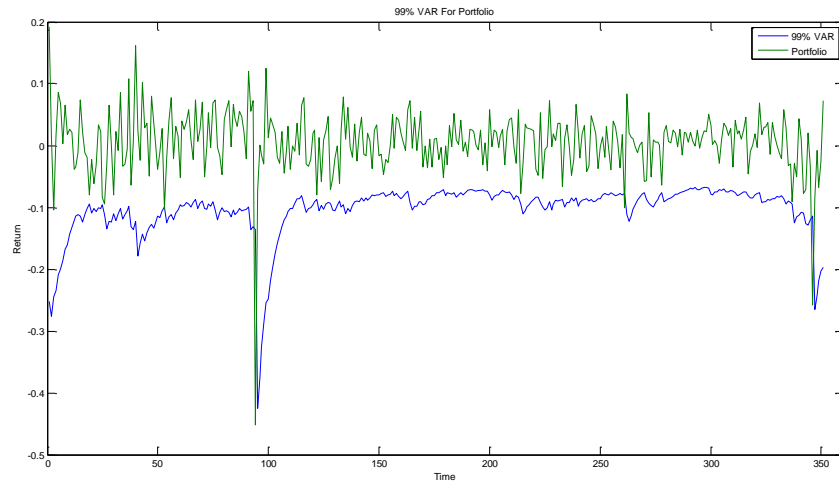
Figure 7: Actual returns and estimated 99% VAR for a portfolio with 17% in AREITS and 83% in AOI based on the estimated diagonal GARCH BEKK model.

5. Conclusion

This paper provides an investigation of the dependence structure between monthly returns from Australian Real Estate Investment Trusts (AREITS) and the All Ordinaries Index (AOI) for the time period 1980 to 2009. Australia has long been a nation with a great interest in property investments such that in the last decades also investments in REITS have shown substantial growth rates. The literature argues that generally real estate investments enable investors to further diversify their portfolio. In this study we apply univariate and multivariate GARCH models in combination with different copula models including the Clayton, Gumbel, Gaussian and Student t copula in order to investigate the dependence structure between Australian real estate and equity returns. To our best knowledge this is one of the first studies to apply and test conditional copula models in these markets. We find significant correlations between the returns of an Australian REIT index and the AOI. These findings somehow contradict earlier studies by e.g. Brueggeman et al (1984), Hartzell et al (1999) or Clayton and MacKinnon (2001) reporting significant negative or only weak positive correlations between REITS and equity investments.

With respect to the dependence structure, we apply different multivariate GARCH and copula models. For the latter we also apply goodness-of-fit tests in order to determine which of the copula functions best describes the dependence structure between the series. Overall, the Clayton copula provides the best fit to the dependence structure between the series with respect to various considered model selection criteria.Thus, we find that the return series exhibit dependence particularly in the lower left

tail suggesting that Australian real estate and common equity stocks are more closely related when markets produce highly negative returns. Our results are more in line with a study by Knight et al (2005) who report some tail dependence, particularly in the lower tail, when investigating the relationship between UK and global public real estate stocks with equivalent general equity market returns. Therefore, also for the Australian market the diversification or hedging potential of investments in REITS might be limited. An explanation for this could be that the overvaluation of Australian real estate markets during several years of the considered time period was also driven by wealth effects and portfolio shocks from Australian equity markets, see e.g. Frye et al (2010). This explains not only the positive relationship between real estate and equity returns but also joint extreme negative returns when both markets simultaneously return to their market fundamental levels as during the financial crisis.

We also provide a risk analysis for the different models with respect to the quantification of the risk for a portfolio combining investments in real estate and equity markets. Our results show that copula functions provide a powerful tool for modelling the dependence structure between financial assets. We also find that dynamic models clearly provide a more appropriate measurement of the risk in a portfolio. Both using univariate AR-GARCH models in combination with copula functions for the dependence structure between the standardized residuals as well as a bivariate GARCH BEKK model provide an adequate quantification of the risk for a portfolio of investments in REITS and equity. Our findings further suggest that ignoring heteroscedasticity of the marginal series and the complex dependence structure in favor of a simple static multivariate normal model leads to a severe underestimation of the actual risk.

Extensions of the conducted work could examine the impacts of the detected dependence structure on optimal portfolio construction, e.g. by including asymmetric or tail dependence in the asset allocation decision, see e.g. Patton (2004) or Hatherley and Alcock (2007). Furthermore, since the analysis so far is only based on a bivariate setting, future research should extend the analysis to the multivariate case including various other asset classes next to real estate and equity returns.

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