Panel Stochastic Dominance test and Panel Informational Efficiency LR Test: an Application to UK Covered Warrants Market Efficiency

Christian de Peretti
Laboratory of Actuarial and Financial Sciences (LSAF, EA2429)
Institute of Financial and Actuarial Sciences
University Claude Bernard Lyon 1, France.
Email: christian.de-peretti@sfr.fr

Chia-Ying Chan
Department of Finance, College of Management
Yuan Ze University, Taiwan.
Email: sherrychan@saturn.yzu.edu.tw

Carole Siani
Research Center in Knowledge Engineering (ERIC, EA3083)
University Claude Bernard Lyon 1, France
Email: carole.siani@univ-lyon1.fr

Wing-Keung Wong
Department of Economics
Hong Kong Baptist University
Kowloon Tong, Hong Kong

Tel: (+852) 3411-7542
Fax: (+852) 3411-5580
awong@hkbu.edu.hk

Keywords: Covered warrants, market efficiency, panel stochastic dominance, panel information efficiency likelihood ratio test, bootstrap.

JEL Classification: C14, G11, G14, G15.

Abstract

The contributions of this paper is to propose a new panel stochastic dominance (SD) test- PDD test, the asymptotic properties are derived, which extends Davidson and Duclos (DD) SD test to a panel context, no such inference test exists in the previous literature, and this paper would be the pioneer. This paper further applies this new Panel DD test (the PDD test) to examine the dominance relationship between the UK covered warrants and their underlying shares. With this unique data set, the existence of arbitrage opportunity and the confirmation of market efficient hypothesis are hoping to be addressed.

The PDD test also contributes to settle one of the demerits while working with financial derivatives time series: that the standard individual tests for Stochastic Dominance in time series are unsatisfactory in terms of power when the sample size is too small, and typically the financial derivatives have a limited life, in particular, stock options and covered warrants. This is because the pairwise SD tests are nonparametric, and nonparametric tests require large sample size, in this case, the individual tests for financial derivative time series may not distinguish between the null and the alternative hypotheses for each series, and lead to retain the null hypothesis, even if the alternative is true. Hence the PDD test would improve the power of individual SD tests: a panel test gathers all the information of all the series, and then increases the power compared to its corresponding individual test. It is one of the main motivations for building panel tests in the literature.

This paper also extends the classical likelihood ratio (LR) information efficiency test to a panel framework to get more powerful new tests. A bootstrap methodology is developed to correct the size distortion of the LR test.

The empirical analyses reveal that neither covered warrants nor the underlying shares stochastically dominate the other, indicating the nonexistence of potential arbitrage gains in either wealth or utility, which implies the market efficiency. Our findings show that UK covered warrants returns efficiently reflect the return information of the underlying shares.

1. Introduction

There are several contributions in this study, first, a new a panel stochastic dominance (SD) test-PDD test, is proposed, extending Davidson and Duclos (DD) SD test to a panel context, there is no such study in the previous literature, and this paper would be the pioneer. This paper further applies this new Panel DD test (the PDD test) to examine the dominance relationship between the UK covered warrants and their underlying shares. With this unique data set, the existence of arbitrage opportunity and the confirmation of market efficient hypothesis are hoping to be addressed.

The idea of developing a new panel stochastic dominance test arises from the need of examining the efficiency between derivatives and their underlying. In practice, (individual) stochastic dominance tests are used to assess the efficiency of financial markets (Wong et al. (2007, 2008)). However, the economic question that is addressed in the pairwise sample sets cannot be answered globally, hence in this project, a generalized panel stochastic dominance test is firstly been presented in the literature. This new panel stochastic dominance test, later will be referred as "PDD" test, is aimed to extend the individual SD test to a panel framework.

The PDD test also contributes to settle one of the demerits while working with financial derivatives time series: that the standard individual tests for Stochastic Dominance in time series are unsatisfactory in terms of power when the sample size is too small, and typically the financial derivatives have a limited life, in particular, stock options and covered warrants. This is because the pairewise SD tests are nonparametric, and nonparametric tests require large sample size, in this case, the individual tests for financial derivative time series may not distinguish between the null and the alternative hypotheses for each series, and lead to retain the null hypothesis, even if the alternative is true. Hence the PDD test would improve the power of individual SD tests: a panel test gathers all the information of all the series, and then increases the power compared to its corresponding individual test. It is one of the main motivations for building panel tests in the literature. We shall expect the power of the panel PDD test is much greater than its corresponding individual SD test.

In this paper, using the likelihood framework, a testing procedure based on averaging individual SD test statistics for panels will be conducted. In particular, this paper first proposes a test which based on the average of Davidson and Duclos (DD) (2000) statistics is computed for each group in the panel, which is referred as the PDD test. Under very general settings, we would expect this statistic to converge in probability to a standard normal variate sequentially with $T \rightarrow \infty$, followed by $N \rightarrow \infty$. A diagonal convergence result with T and $N \rightarrow \infty$ while $N / T \rightarrow \delta$, δ being a finite nonnegative constant, is also conjectured.[1] Another contributions in this study is a new a panel likelihood ratio (PLR) information efficiency (SD) test- PLR test, extending the classical likelihood ratio (LR) information efficiency test to a panel context. Again, the asymptotic distribution of the test statistic is derived. However, the LR statistic distribution suffers from a large size distortion and has to be corrected. We propose a bootstrap methodology to correct the size distortion of the LR tests.

Furthermore, this paper attempts to conducting an empirical examination on UK covered warrants and the underlying shares by applying the PDD and PLRtest, to assess the stochastic dominance relation between the derivatives and their underlying shares, hence lead to a discussion over the efficiency and informative role of the financial derivatives- UK covered warrants.

The rest of the paper is organized as follows. Section 2 provides a brief review of the previous studies which applied the individual SD tests over difference issues; Section 3 provides a description of the framework for panel, and derives the PDD test. The cases of cross sectional dependence and panel bootstrap likelihood ratio test are also discussed. Section 4 presents results of the Monte Carlo simulation on the performance of the new procedures for a variety of experimental setups. A financial application of the PDD test to a panel of covered warrants and the underlying shares in UK is addressed in Section 5. Finally Section 6 some concluding remarks is drawn.

2. Brief review of the previous studies which applied the individual SD tests over difference issues and interest to extend them to panel

2.1 Application Fields of Stochastic Dominance Tests

Stochastic Dominance has been originally developed in the traditional expected utility framework. Behavioral studies have shown that the expected utility framework may not always provide a good description of human behavior under uncertainty. A SD criterion called Prospect Stochastic Dominance (PSD) has been developed to determine preference for all Prospect Theory individuals (Levy and Wiener (1998); Levy and

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[1] T is the sample period, and N the number of cross sections.

Levy (2004)). SD offers two advantages: It requires no assumptions regarding the normality of return distributions, and it imposes few restrictions on investors' risk-return tradeoff preference. Previous literatures have applied the individual SD tests over difference issues.

Jarrow (1986), Falk and Levy (1989), and recently some studies by Wong et al. (2008,2009) apply the SD test to determine the market efficiency, market rationality, and arbitrage opportunity by examining the whole distribution of returns, without the need of identifying a risk index or a specific model. In the conventional theories of market efficiency if one is able to earn abnormal return in normal circumstance, the market is considered to be inefficient or irrational. Further, Al-Khazali *et al.* (2008) used SD to detect temporal predictability of returns in the Athens Stock Exchange (ASE): they find a strong “day” effect and rather weak “week” and “January” effects. De Giorgi (2005) first proposed the “reward risk portfolio selection” model, differ from the Markowitz (1952), an axiomatic definition of reward and risk measures based on a few basic principles, including consistency with second-order stochastic dominance is arisen. De Giorgi and Post (2008) further derived the reward-risk Capital Asset Pricing Model (CAPM) analogously to the classical mean-variance CAPM.

The stochastic dominance criteria were also applied in the evaluation of the portfolio performance; using the SD criteria, Annaert *et al.* (2009) evaluates the performance of different proportion of portfolio insurance techniques based on a block-bootstrap simulation, and the impact of changing the rebalancing frequency and level of capital protection is assessed. Ringuest *et al.* (2000) apply the conditional SD in R&D portfolio selection. Post (2003) developed a test for stochastic dominance efficiency, and Knight and Satchell (2008) even developed an infinite order stochastic dominance test and applied it to the U.S. and U.K. stock markets, and to the income data of Anderson (1996). Falk and Levy (1989) provide a compelling argument that the observed abnormal returns may be due to omitted variables, a market proxy effect, or other specification errors in implementing the traditional event study methodology, hence they propose a “cross-sectional SD” method as an alternative to traditional event studies. Larsen and Resnick (1999) improve the cross-sectional SD method by proposing a bootstrap method of statistical testing. Abhyankary, *et al.* (2005) use the idea of stochastic dominance to study the long-run post-merger stock performance of UK acquisition. Performance is compared by using the entire distribution of returns rather than only the mean as in traditional event studies. Abhyankary *et al.* (2008) also study the relative performance of value versus growth strategies from the perspective of stochastic dominance by using the Barrett and Donald (2003) test. Leshno and Levy

(2002) proposed the "almost stochastic dominance" (ASD), which can be used to compare the preference between stocks and bonds for the long run. The Almost Stochastic Dominance criterion has been developed to solve no First degree Stochastic Dominance (FSD) problem. The Almost Stochastic Dominance criterion has been recently employed in various studies (Levy (2006), Levy *et al.* (2004), Benitez *et al.* (2006), and Gasbarro *et al.* (2007)). Finally, a combination of DEA with SD criteria is proposed by Kuosmanen (2007) and by Lozano and Gutiérrez (2008). In their paper, six distinct DEA-like linear programming models are proposed for computing relative efficiency scores consistent with second-order stochastic dominance (SSD). The aim is that, being SSD efficient, the obtained target portfolio should be an optimal benchmark for any rational risk-averse investor.

2.2 Interest of Extending SD Tests to a Panel Framework

In the above section, we could see that the SD criteria have been applied to a very large variety of economic and financial questions. Stimulated by the facts SD test has became an important technique in finance literature, an extension of the classical SD tests to a panel framework is essential and brings a great contribution to the existing body of research.

A generalized PDD model permits us to get a rigorous test on the significance level basis, which has a much greater power than its corresponding individual test. It is very useful when the time sample size is small, for instance, financial data with finite life like the options and covered warrants: a panel PDD test gathers all the information of all the series, hence increases the power compared to its corresponding individual test.

3.1 Calculation of the Implied Stock Price

To conduct the SD test and the likelihood ratio efficiency test to assess the covered warrants prices and the underlying share prices, transformation of the covered warrants prices to equilibrium prices for their underlying shares is required. For this purpose, we apply the option-implied share price model proposed by Manaster and Rendleman (1982).

The implied stock price, S_{it}*, and implied standard deviations, σ_{it}* for each pair *i* at time *t* can be calculated as:

$$(S_{it}^*, \sigma_{it}^*) = \underset{S_{it}, \sigma_{it}}{Arg\min} \sum_{j=1}^{N_{it}} [\ W^j - W^j(\ S_{it}, \sigma_{it})\ ]^2, \qquad (1)$$

where *j* denotes the covered warrant number. The solution to Equation (1) minimizes the sum of the squared deviations between the observed and theoretical covered warrant prices, where i is the number of implied share price pairs, S_{it} is the market share price, W^j is the observed covered warrant market prices, and $W^j(S_{it} , \sigma_{it})$ is the calculated theoretical covered warrants price. N_{it} represents the number of covered warrants used to compute the i^{th} implied price series at time t, where $j \geqq 2$, since at least two warrants are needed to generate the argument minimum (Arg min).

3.2 Panel Stochastic Dominance Test- PDD test

In this section, the new panel SD (PDD) test is presented by extending the Davidson and Duclos (DD) test into a panel framework.

Consider a sample of N cross sections (e.g. stocks) observed over T time periods. Suppose that investors attempt to choose between two risky assets, X_i and Y_i, i=1, …, N. In this paper, X_i corresponds to the return of a covered warrant with a cumulative density function or simple called distribution F_i, and Y_i corresponds to the return of corresponding underlying share with a distribution G_i. i is the index of the stock.

The null hypothesis of non SD for all i=1,…,N. For s=1, 2, and 3, the null hypothesis of SD then becomes:

H_0’: $D_{US}{}^{S}{}_{,i}= D_{DER}{}^{S}{}_{,i}$, for all i=1,…N,

H_A’: $D_{US}{}^{S}{}_{,i} \neq D_{DER}{}^{S}{}_{,i}$, for some i=1, …, N_0,
and $D_{US}{}^{S}{}_{,i}= D_{DER}{}^{S}{}_{,i}$, for i= N_0+1, …, N,

H_{A1}’: $DER_i \succ_S US_i$, for some i=1, …, N_1,
and $D_{US}{}^{S}{}_{,i}= D_{DER}{}^{S}{}_{,i}$, for i= N_1+1, …, N,

H_{A2}’: $US_i \succ_S DER_i$, for some i=1, …, N_2,
and $D_{US}{}^{S}{}_{,i}= D_{DER}{}^{S}{}_{,i}$, for i= N_2+1, …, N.

$D_{US}{}^{S}{}_{i}$ represents cumulative density distribution of the underlying shares, and $D_{DER}{}^{S}{}_{,i}$ represents cumulative density distribution of the covered warrants, i represents different dominants order. US_i represents the underlying shares, $D_{DER}{}^{S}{}_{,i}$ represents the covered warrants.

This formulation of the alternative hypothesis allows for some (but not all) of the individual pairs of series to have SD under the alternative hypothesis. Formally, following Im *et al.* (2003), we assume the following assertion.

In this section the proposed PDD test can be used to test the hypothesis that the series distributions are equal against the alternative of a proportion of them are being stochastically dominated, the series are independent across sections.

Assumption 1:

Under the alternative hypothesis the fraction of the individual processes that are stationary is non-zero, namely

$$\lim_{N\to\infty}\frac{N_0}{N}=\delta,\quad 0<\delta\leq 1.$$

This condition is necessary for the consistency of the PDD test.
For s=1, 2, and 3, the number of the order, consider the DD statistic for the pair number i (Davidson and Duclos, 2000):

$$T_i^S(x)=\frac{\hat{D}_{DERi}^s(x)-\hat{D}_{USi}^s(x)}{\sqrt{\hat{V}_i^S(x)}}, \tag{2}$$

where $\hat{V}_i^S(x)=\hat{V}_{DERi}^s(x)+\hat{V}_{USi}^s(x)-2\hat{V}_{DERi,USi}^s(x)$, for testing the equality of $D_{USi}^s(x)$ and $D_{DERi}^s(x)$.

Case of Fixed T, N→∞

In this section, *T* is fixed. For this purpose the following assumption is made:

Assumption 2:

Let the joint population moments of order 2*s*-2 of U*Si* and *DERi* be finite for all *i*=1, …, N.

Assumption 3:

Let T be large enough so that $T_i^S(x)$ has finite heterogeneous variances σ_i^2 for all i, ..., N.

A PDD test based on the average of individual DD statistics is the central focus. Therefore, for a fixed T (the focus of this section), the following average statistic is considered:

$$\overline{T}^S(x,N) = \frac{1}{N}\sum_{i=1}^{N}\left(T_i^S(x)\right)^2 . \qquad (3)$$

Since deriving theoretical results is very difficult in the case of finite sample size, Davidson and Duclos (2000) do not provide any results about the moments of the DD statistic. It should be noted that even if bootstrap techniques are applied, it should be checked that at least the two first moments of the distribution exist. Hence in this project the two first moments of the DD distribution are computed using Monte Carlo experiments.

Theorem 1

Under Assumptions 1, 2, and 3, the individual statistics $T_i^S(x)$, $i = 1, \ldots, N$, are independent with finite second order moments. Therefore by Lindberg-Levy central limit theorem under the null hypothesis and as $N \to \infty$ the standardized $\overline{T}^S(x,N)$ statistic

$$Z^s(x,N) = \frac{\overline{T}^S(x,N) - E\left(\overline{T}^S(x,N)\right)}{\sqrt{V\left(\overline{T}^S(x,N)\right)}} = \frac{\overline{T}^S(x,N) - 1}{\sqrt{2N}} \xrightarrow[N\to\infty]{} N(0,1).$$

Proof:

A more general case where the sample sizes T differ across groups can be considered. Let us denote T_i the sample size of pair number i.

Assumption 3':

Let T_i be large enough so that $T_i^S(x)$ has finite heterogeneous third moment for all i, …, N.

Theorem 2

Under Assumptions 1, 2, and 3', the individual statistics $T_i^S(x)$, i = 1, …, N, are identically but not independently distributed across i with finite third order moments. Therefore by Lyapunov central limit theorem under the null hypothesis and as $N\to\infty$ the standardized $\overline{T}^S(x,N)$ statistic

$$Z^s(x,N) \xrightarrow[N\to\infty]{} N(0,1).$$

Proof:

The sample size T for which the third moment of $T_i^S(x)$ is finite has to be determined.

Case of T→∞, fixed N

For $T\to\infty$, as it is proved by Davidson and Duclos (2000), the DD statistic converges to the standard normal distribution.

Theorem 3

Under Assumptions 1 and 2, under the null hypothesis, and as $T\to\infty$,

$$N \cdot \overline{T}^S(x,N) \xrightarrow[T\to\infty]{} \chi^2(N).$$

Proof

The proof is straightforward.

Case of T→∞ first, N→∞ second

Theorem 4

Under Assumptions 1 and 2, under the null hypothesis, and as $T\to\infty$ first, and as $N\to\infty$ second, the standardized $\overline{T}^S(x,N)$ statistic

$$Z^s(x,N) \xrightarrow[T\to\infty,\quad N\to\infty]{} N(0,1).$$

Testing at a particular point x in the panel framework

We assume that N is large enough for permitting to use the normal asymptotic approximation. If not, we assume that T is large enough for permitting to use the χ^2 asymptotic approximation, and the same reasoning holds using the critical values of the χ^2 distribution instead of the normal distribution critical values. The test statistic presented above permits to test for the following hypotheses at point x:

$$\begin{aligned}
&H_0'(x): D^s_{USi}(x) = D^s_{DERi}(x), \text{ for all pairs i,}\\
&H_A'(x): D^s_{USi}(x) \neq D^s_{DERi}(x), \text{ for at least one pair i,}\\
&H_{A1}'(x): D^s_{USi}(x) > D^s_{DERi}(x), \text{ for a proportion of pairs i,}\\
&\quad \text{and } D^s_{USi}(x) = D^s_{DERi}(x), \text{ for all the other pairs,}\\
&H_{A2}'(x): D^s_{USi}(x) < D^s_{DERi}(x), \text{ for a proportion of pairs i,}\\
&\quad \text{and } D^s_{USi}(x) = D^s_{DERi}(x), \text{ for all the other pairs.}
\end{aligned}$$

Let n(α) denotes the 1-α percentile of the normal distribution. We adopt the following decision rules:

1. If Z(x,N)≤ n(α), accept $H_0(x)$',
2. if Z(x,N)> n(α), accept $H_A(x)$'.

In the case where H_A' is accepted, the test statistic does not permit to distinguish between H_{A1}' and H_{A2}'. Consequently, the individual $T_i^S(x)$ statistics are examined:

3. If $T_i^S(x) < M^N_{\infty,\alpha}$ for all i and $T_i^S(x_i) < -M^N_{\infty,\alpha}$ for some i accept $H_{A1}(x)'$,
4. if $T_i^S(x_i) > -M^N_{\infty,\alpha}$ for all i and $T_i^S(x_i) > M^N_{\infty,\alpha}$ for some i accept $H_{A2}(x)'$.

In should be noted that under the hypothesis of independence of the pairs, a large value for N does not violate the independence assumption required by the SMM distribution. Consequently, there is no restriction on the value of N. This will be wrong under cross sectional dependence, even for small value for N.

Testing for stochastic dominance in the panel framework

We are faced to a similar problem (but not exactly the same) as for the non-panel framework: to test for panel stochastic dominance, H_0' has to be examined for the full support of each pair of series. Empirically, H_0' can be tested for a pre-designed finite number of values of x with procedure proposed by Bishop, Formby and Thistle (1992). Following Bishop, Formby and Thistle, we consider fixed values $x_1, x_2, \ldots, x_M$ and use their corresponding statistics $Z^S(x_k,N)$ for $k = 1, 2, \ldots, M$ to test the hypotheses H_0', H_A',H_{A1}', and H_{A2}' using the studentized maximum modulus distribution with M and infinite degrees of freedom, denoted by $M^M_{\infty,\alpha}$. We denote the 1-α percentile of M^M_{∞} by $M^M_{\infty,\alpha}$ and adopt the following decision rules:

1. If $|Z(x_k,N)| \le M^M_{\infty,\alpha}$, for k=1, …, M, accept H_0',

2. if $Z(x_k,N) > M^M_{\infty,\alpha}$, for some k, and $Z(x_k,N) < -M^M_{\infty,\alpha}$, for some other k, accept H_A'.

Again, in the case where H_A' is accepted, we examine the individual $T_i^S(x_k)$ statistics:

3. If $T_i^S(x_k) < M^{N\times M}_{\infty,\alpha}$ for all i and k, and $T_i^S(x_i) < -M^{N\times M}_{\infty,\alpha}$ for some i and k, accept H_{A1}',

4. if $T_i^S(x_i) > -M^{N\times M}_{\infty,\alpha}$ for all i and k, and $T_i^S(x_i) > M^{N\times M}_{\infty,\alpha}$ for some i and k, accept H_{A2}'.

It is important to note that the degree of freedom of the SMM is now equal to $N \times M$ to account for joint test size.

Following these properties, we can now propose a panel test methodology:

$\underset{k\le K}{Max}\left|Z^s(x_k, N)\right| > M^K_{\infty,\alpha/2}$ rejects the null hypothesis.

3.3 Another Panel Stochastic Dominance Test- PDD2 test

Another way which can be used here is to base the panel test statistic on the average of the max individual statistics. It permits to test directly the panel framework form global individual test statistics, without considering the individual test statistics for each value for x in the distributions of the assets. The same assumptions in subsection above are supposed.

The maximum value of the statistics over the whole distribution, for only one pair, can be considered:

$$\tilde{T}_i^s(K) = \underset{x}{Max}\left|T_i^s(x)\right|,$$

and then the panel average statistic:

$$\tilde{T}^s(N,K) = \frac{1}{N}\sum_{i=1}^{N}\tilde{T}_i^s(K)\,. \qquad (*)$$

Theorem 5

Under Assumptions 1, 2, and 3, the individual statistics $\tilde{T}_i^S(K)$, i = 1, …,N, are independent with finite second order moments. Therefore by Lindberg-Levy central limit theorem under the null hypothesis and as $N\to\infty$ the standardized $\tilde{T}^S(N,K)$ statistic

$$\tilde{Z}^s(N,K) = \frac{\tilde{T}^S(N,K) - E\left(\tilde{T}^S(N,K)\right)}{\sqrt{V\left(\tilde{T}^S(N,K)\right)}} \xrightarrow[N\to\infty]{} N(0,1).$$

The expectation and the variance of these statistics are the expectation and the variance of the corresponding studentized maximum modulus distribution. In their paper, Stoline and Ury (1979) provide the critical values for the studentized maximum modulus distribution, but not its moments. To compute our panel statistic, the two first moments are required. Using Monte Carlo simulation, we computed the first four moments. The third and fourth moments give an idea to the rate of convergence to the N(0,1) distribution of our statistic. These moments are provided in Table 1.

Table 1: First four moments of the $\widetilde{T}_i^s(K) = \underset{x}{Max}\left|T_i^s(x)\right|$ statistic

This table provides, using Monte Carlo simulations, the mean, variance, standard deviation, skewness, and kurtosis of the $\widetilde{T}_i^s(K) = \underset{x}{Max}\left|T_i^s(x)\right|$ statistic with parameter *K*, and infinite degree of freedom. 1,000,000 of simulation were run.

Parameter K	Mean	Variance	Standard deviation	Skewness	Kurtosis
1	0.796	0.3630	0.6025	0.9879	3.819
2	1.127	0.3661	0.605	0.7173	3.456
3	1.329	0.3469	0.589	0.6196	3.389
4	1.467	0.3265	0.5714	0.5838	3.392
5	1.572	0.3108	0.5575	0.5867	3.504
6	1.654	0.2972	0.5452	0.5691	3.477
7	1.723	0.2853	0.5341	0.5614	3.456
8	1.787	0.2782	0.5274	0.5609	3.476
9	1.833	0.2679	0.5175	0.5692	3.491
10	1.881	0.2639	0.5137	0.5779	3.531
12	1.957	0.2502	0.5002	0.5740	3.512
14	2.023	0.2423	0.4922	0.5851	3.547
16	2.077	0.2334	0.4831	0.6037	3.617
18	2.125	0.2291	0.4787	0.6086	3.617
20	2.166	0.2230	0.4723	0.6230	3.695
24	2.236	0.2140	0.4626	0.6410	3.720
28	2.297	0.2083	0.4564	0.6235	3.631
32	2.348	0.2011	0.4485	0.6347	3.675
36	2.389	0.1949	0.4415	0.6360	3.671
40	2.429	0.1898	0.4356	0.6364	3.697
44	2.464	0.1872	0.4326	0.6607	3.757
48	2.492	0.1837	0.4286	0.6667	3.799
52	2.524	0.1812	0.4257	0.6840	3.825
56	2.548	0.1777	0.4216	0.6810	3.811
60	2.575	0.1787	0.4228	0.6877	3.840
70	2.627	0.1703	0.4127	0.6873	3.832
80	2.669	0.1664	0.4080	0.7016	3.906
90	2.711	0.1626	0.4032	0.7035	3.864
100	2.748	0.1619	0.4024	0.7186	3.879

In case of rejection of the null hypothesis, it can be useful to distinguish between H_{A1}, and H_{A2}. We then propose two addition test statistics: in Equation (*), $\tilde{T}_i^s = \underset{x}{Max} \left| T_i^s(x) \right|$ is replaced by $\tilde{T}_i^s = \underset{x}{Max}\, T_i^s(x)$ or $\tilde{T}_i^s = \underset{x}{Min}\, T_i^s(x)$. Theorem 4 applies to both these new statistics. We also need the moments of the statistics. They are provided in Table 2 and Table 3.

Table 2: First four moments of the $\tilde{T}_i^s = \underset{x}{Max}\, T_i^s(x)$ statistic

This table provides, using Monte Carlo simulations, the mean, variance, standard deviation, skewness, and kurtosis of the $\tilde{T}_i^s = \underset{x}{Max}\, T_i^s(x)$ statistic with parameter *K*, and infinite degree of freedom. 1,000,000 of simulation were run.

Parameter K	Mean	Variance	Standard deviation	Skewness	Kurtosis
1	0.004626	0.997	0.9985	-0.003055	2.992
2	0.561	0.6838	0.8269	0.1408	3.113
3	0.8472	0.5634	0.7506	0.2089	3.13
4	1.032	0.493	0.7021	0.2761	3.187
5	1.166	0.4505	0.6712	0.312	3.24
6	1.267	0.4172	0.6459	0.3422	3.251
7	1.353	0.3914	0.6256	0.3528	3.254
8	1.424	0.3741	0.6116	0.3826	3.306
9	1.484	0.3569	0.5974	0.3934	3.34
10	1.539	0.3452	0.5875	0.4272	3.359
12	1.629	0.3216	0.5671	0.432	3.37
14	1.702	0.3084	0.5553	0.4583	3.363
16	1.766	0.2933	0.5416	0.4709	3.412
18	1.823	0.2862	0.535	0.4992	3.492
20	1.865	0.2742	0.5236	0.5098	3.498
24	1.947	0.2625	0.5123	0.5401	3.558
28	2.014	0.2512	0.5012	0.5382	3.51
32	2.071	0.2431	0.4931	0.5503	3.567
36	2.118	0.233	0.4827	0.5685	3.561
40	2.162	0.226	0.4754	0.5603	3.57
44	2.201	0.2224	0.4716	0.5714	3.562
48	2.233	0.2164	0.4652	0.6	3.675
52	2.265	0.2136	0.4622	0.6021	3.652
56	2.291	0.2094	0.4576	0.599	3.625
60	2.321	0.208	0.4561	0.6327	3.725
70	2.378	0.1988	0.4459	0.626	3.685
80	2.422	0.1918	0.438	0.6553	3.831
90	2.469	0.1863	0.4316	0.6473	3.782
100	2.508	0.1865	0.4318	0.6647	3.782

Table 3: First four moments of the $\tilde{T}_i^s = \underset{x}{Min}\, T_i^s(x)$ statistic

This table provides, using Monte Carlo simulations, the mean, variance, standard deviation, skewness, and kurtosis of the $\tilde{T}_i^s = \underset{x}{Min}\, T_i^s(x)$ statistic with parameter *K*, and infinite degree of freedom. 1,000,000 of simulation were run.

Parameter K	Mean	Variance	Standard deviation	Skewness	Kurtosis
1	0.004626	0.997	0.9985	-0.003055	2.992
2	-0.564	0.6798	0.8245	-0.1521	3.061
3	-0.8468	0.5622	0.7498	-0.2305	3.116
4	-1.028	0.4953	0.7038	-0.2578	3.132
5	-1.163	0.4467	0.6683	-0.3049	3.239
6	-1.266	0.4144	0.6438	-0.3338	3.24
7	-1.353	0.3898	0.6243	-0.3552	3.256
8	-1.428	0.3753	0.6126	-0.3707	3.256
9	-1.484	0.3544	0.5954	-0.3861	3.277
10	-1.54	0.3436	0.5862	-0.4086	3.333
12	-1.628	0.3241	0.5693	-0.4282	3.32
14	-1.703	0.3076	0.5546	-0.4563	3.401
16	-1.767	0.2937	0.542	-0.4802	3.45
18	-1.82	0.283	0.532	-0.4768	3.434
20	-1.871	0.2761	0.5255	-0.5115	3.516
24	-1.946	0.2597	0.5096	-0.5238	3.543
28	-2.014	0.2527	0.5027	-0.5366	3.51
32	-2.069	0.241	0.4909	-0.5518	3.533
36	-2.116	0.2338	0.4835	-0.5566	3.543
40	-2.16	0.2271	0.4766	-0.5659	3.562
44	-2.197	0.2213	0.4704	-0.6009	3.679
48	-2.229	0.2164	0.4652	-0.5921	3.637
52	-2.265	0.2136	0.4622	-0.6009	3.695
56	-2.291	0.2091	0.4573	-0.6101	3.705
60	-2.321	0.2083	0.4564	-0.6159	3.688
70	-2.377	0.1988	0.4459	-0.6234	3.717
80	-2.427	0.1935	0.4399	-0.6279	3.687
90	-2.471	0.1889	0.4346	-0.6409	3.724
100	-2.509	0.1858	0.431	-0.6599	3.757

3.3 SD Test for Heterogeneous Panels with Cross Sectional Dependence

Sometimes the series of the panel are not necessarily on the same period; consequently, it is not required to take into account for cross sectional dependence between the pairs. Nevertheless, a methodology that accounts for heterogeneous panels with cross sectional dependence may be useful in the context of same period cylindrical panel data, where heterogeneous cross section dependence tends to be important in most empirical applications.

The model

We employ the Pesaran (2007) panel framework that enables us to account for heterogeneous cross section dependence. Suppose the underlying share prices S_t and the warrant prices W_t are modeled as follows:

$$S_t = \alpha_S + \beta_S S_{t-1} + u_{S,t} \qquad t=1,\ldots T,$$

$$W_t = \alpha_W + \beta_W W_{t-1} + u_{W,t}, \qquad t=1,\ldots T,$$

where the initial value, S_0 and W_0, are given, and the error term, u, has the one-factor structure:

$$u_{S,t} = \gamma_S f_t + \varepsilon_{S,t}$$

$$u_{W,t} = \gamma_W f_t + \varepsilon_{W,t}$$

$$\varepsilon_{S,t} \sim \text{i.i.d.}(0,\sigma_S^2)$$

$$\varepsilon_{W,t} \sim \text{i.i.d.}(0,\sigma_W^2) \text{ independent of } \varepsilon_{S,t}$$

in which f_t is the unobserved common effect, and ε is the individual-specific (idiosyncratic) error.

Panel Neural Test accounting for Cross Section Dependence

It should be recalled that the properties of the previous panel stochastic dominance test are based on the assumption that the error terms are not cross-correlated. When this assumption is violated, the derived distributions for these test statistics are no longer valid: they suffer from nuisance parameter problems. The distributions of the test statistics are not the same as before and are not known. For the DD individual tests in our PDD test are correlated and hence our average PDD statistic does not have the stated variance in its (asymptotic) normal distribution. Even if cross sectional dependence is accounted for, the distribution under the null is not Gaussian: see Pesaran (2007) and Cerrato *et al.* (2007).

One way out of the problem of cross-correlated errors is proposed by Maddala and Wu

(1999). They propose to use the bootstrap method to get the empirical distributions of the test statistics to make inferences. The bootstrap method for univariate time series is well developed. See Li and Maddala (1996) for a good introduction. Meanwhile the bootstrap method for panel data is in its infancy. Let us consider the model $y = x\,\beta + u$. Generally speaking, if the null hypothesis is H0: $\beta = \beta_0$, the bootstrap method suggests generating bootstrap sample y^* as $y^* = x\,\beta_0 + u^*$, where u^* is the bootstrap sample from $u^0 = y - x\,\beta$. Since we have panel data here, we should also take care of special problems arising from the serial correlation.

In our case, we get the bootstrap sample of the error term u^0 from:

$$\Delta\, y_{i,t} = u_{i,t}^0$$

since yi,t is W or S and has a unit root. Since there are cross-correlations among $u_{i,t}^0$, Li and Maddala (1996) cannot resample $u_{i,t}^0$ directly. They propose resampling $u_{i,t}^0$ with the cross-section index fixed, i.e. instead of resampling $u_{i,t}^0$ they resample $u_t^0 = (u_{W,t}^0, u_{S,t}^0)$ to get u^*. In this way, they can preserve the cross-correlation structure of the error term.

3.4 The Informational Efficiency Test -The Panel Bootstrap Likelihood Ratio Test

In this section, the new panel bootstrap LR (PLR) efficiency test is presented.[2] We consider the same sample as for the panel stochastic dominance test, as well as the same assumptions. The null hypothesis is efficiency for all *i*=1,…,N.

It should be noted that if a panel LR statistic is constructed by averaging the individual LR statistics, the panel statistic will suffer from size distortion, and a new panel bootstrap procedure has to be developed. However, the bootstrap individual P values can also be used to construct a panel statistic. First, it is worth to note that under the null hypothesis, the bootstrap P value is asymptotically distributed as a uniform variable over [0,1]. (The asymptotic P value is also asymptotically distributed as a uniform variable over [0,1], but suffers from a large distortion from this distribution.) Then a panel test statistic can be constructed as follows:

$$\bar{p} = \frac{1}{N}\sum_{i=1}^{N} p_i$$, where p_i is the bootstrap P value for the pair number *i*,

$$Z(N) = \frac{\bar{p} - E(\bar{p})}{\sigma(\bar{p})}$$, the standardized statistic.

[2] Regards LR bootstrap test, please refer to Xu and Taylor (1995) and Claessen and Mittnik (2002)

Under the null hypothesis and since the p_i are supposed to be independent, we have:

E($\bar{p}$)= $\frac{1}{N}\sum_{i=1}^{N} E(p_i)$ =E(p_i)=0.5,

V($\bar{p}$)= $\frac{1}{N^2}\sum_{i=1}^{N} V(p_i) = \frac{1}{N}V(p_i) = \frac{1}{12\cdot N}$,

thus $\sigma(\bar{p}) = \sqrt{1/12\cdot N}$.

Theorem 6

Under the previous assumptions, as N→+∞, the following standardized statistic:

$$Z(N)= \frac{\bar{p}-0.5}{\sqrt{1/12\cdot N}} \sim N(0,1) \text{ under the null hypothesis.}$$

This bootstrap procedure can also be accommodated to cross sectional dependence.

4. Performance of the Panel Stochastic Dominance tests: Monte Carlo Experiments

In this section we use Monte Carlo experiments to examine small sample properties of the alternative panel stochastic dominance tests. We compare the performance of the panel test with the performance of the individual version of the tests, to examine the power gain. The number of Monte Carlo replications S is set equal to 1000. The DD test is computed on a grid of values for the possible observations $x : x_1, \ldots, x_I$. In other words, the individual DD test is computed at certain values of the observation distribution. I is the number of values in the grid. We use the following rule to fix I: $I=\left\lfloor \sqrt{\min(T1,T2)} \right\rfloor$, which is the integer part of the square root of the sample size.

4.1 Simulation under the null hypothesis

The observations are generated under the null hypothesis that there is no stochastic dominance. The size of an inference test of hypothesis is the probability to reject the null hypothesis at a certain significance level, say 5%. If the test is accurate, its size should be equal to the significance level. If there is no size distortion, the size is equal to the significance level. Here we choose 0.05 (i.e. 5%). Consequently, a test that performs correctly on the size basis should present a size close to 0.05.

4.1.1 First set of experiments, with respect to the sample size

Simulated samples are generated from Gaussian distribution with same mean (here 0) and same variance (here 1). The number of cross sections *N*, and the sample sizes *T*1 and *T*2 of the samples which will be compared are varying. The results are presented in Table 4.

Table 4: Size of the tests with respect to the sample size and the number of cross section using Gaussian distribution

No. Cross Sections	Sample		Individual DD test			Panel DD test		
N	T1	T2	Max $\|T_1^1(x)\|$	Max $\|T_1^2(x)\|$	Max $\|T_1^3(x)\|$	$\tilde{T}^1(N,K)$	$\tilde{T}^2(N,K)$	$\tilde{T}^3(N,K)$
2	20	20	0.032	0.029	0.021	0.055	0.033	0.026
2	50	50	0.036	0.013	0.012	0.042	0.017	0.012
2	100	100	0.029	0.019	0.012	0.043	0.028	0.017
2	200	200	0.025	0.012	0.008	0.036	0.016	0.01
2	500	500	0.036	0.011	0.009	0.039	0.013	0.01
5	20	20	0.065	0.034	0.02	0.065	0.038	0.026
5	50	50	0.03	0.021	0.015	0.036	0.028	0.018
5	100	100	0.027	0.013	0.006	0.039	0.019	0.009
5	200	200	0.027	0.006	0.005	0.035	0.01	0.006
5	500	500	0.027	0.009	0.004	0.037	0.013	0.005
10	20	20	0.047	0.038	0.03	0.048	0.046	0.032
10	50	50	0.037	0.021	0.016	0.043	0.025	0.018
10	100	100	0.028	0.011	0.008	0.034	0.015	0.01
10	200	200	0.022	0.008	0.006	0.034	0.011	0.008
10	500	500	0.02	0.006	0.003	0.025	0.007	0.004
20	20	20	0.052	0.035	0.022	0.055	0.042	0.026
20	50	50	0.029	0.017	0.013	0.035	0.021	0.017
20	100	100	0.034	0.014	0.006	0.042	0.016	0.008
20	200	200	0.023	0.014	0.01	0.029	0.019	0.014
20	500	500	0.026	0.008	0.006	0.031	0.012	0.008
50	20	20	0.042	0.027	0.018	0.051	0.032	0.024
50	50	50	0.035	0.015	0.014	0.047	0.016	0.014
50	100	100	0.018	0.009	0.008	0.036	0.014	0.012
50	200	200	0.02	0.008	0.007	0.029	0.011	0.008
50	500	500	0.028	0.009	0.004	0.03	0.009	0.004

It can be observed that all the tests are conservative less than 0.05 (5%), and the panel tests presents better size properties than the corresponding individual DD tests since the panel tests sizes are closer to 5% than the individual tests sizes in different order.

4.1.2 Second set of experiments, with another sample distribution using Student distribution

We have chosen the Gaussian distribution for the observations, but many other distributions can be chosen to assess the performance of the tests. Consequently, we run a second set of experiment using Student distribution instead of Gaussian ones to check that the tests perform correctly for other distribution than the Gaussian one. The results are presented in Table 5.

Table 5: Size of the tests for Student distributed observations

			Individual DD test			Panel DD test		
N	T1	T2	Max $\lvert T_1^1(x)\rvert$	Max $\lvert T_1^2(x)\rvert$	Max $\lvert T_1^3(x)\rvert$	$\tilde{T}^1(N,K)$	$\tilde{T}^2(N,K)$	$\tilde{T}^3(N,K)$
2	20	20	0.01	0.022	0.014	0.027	0.026	0.018
2	50	50	0.023	0.014	0.006	0.025	0.016	0.006
2	100	100	0.009	0.008	0.003	0.016	0.01	0.004
2	200	200	0.008	0.008	0.003	0.013	0.011	0.003
2	500	500	0.008	0.005	0.002	0.009	0.006	0.002
5	20	20	0.031	0.024	0.015	0.038	0.027	0.016
5	50	50	0.02	0.011	0.007	0.021	0.016	0.01
5	100	100	0.015	0.008	0.003	0.02	0.008	0.004
5	200	200	0.009	0.003	0.002	0.016	0.004	0.002
5	500	500	0.003	0.002	0.001	0.009	0.002	0.002
10	20	20	0.021	0.014	0.01	0.029	0.017	0.01
10	50	50	0.015	0.01	0.009	0.021	0.013	0.009
10	100	100	0.013	0.003	0.002	0.019	0.005	0.003
10	200	200	0.007	0.008	0.006	0.011	0.009	0.006
10	500	500	0.009	0.005	0.004	0.012	0.006	0.005
20	20	20	0.024	0.017	0.013	0.024	0.018	0.014
20	50	50	0.016	0.01	0.007	0.019	0.013	0.009
20	100	100	0.013	0.005	0.004	0.015	0.008	0.005
20	200	200	0.007	0.004	0.001	0.013	0.008	0.002
20	500	500	0.009	0.004	0.002	0.012	0.006	0.002
50	20	20	0.032	0.014	0.01	0.042	0.016	0.01
50	50	50	0.021	0.015	0.008	0.024	0.017	0.01
50	100	100	0.012	0.003	0	0.014	0.003	0
50	200	200	0.008	0.003	0.003	0.011	0.005	0.004
50	500	500	0.014	0.007	0.004	0.018	0.008	0.006

4.2 Simulation under the alternative hypothesis

The observations are now generated under the alternative hypothesis that there is stochastic dominance. The power of a test is the probability to accept the alternative when it is true. If a test performs correctly, its power should be close to one.

4.2.1 First set of experiments, with respect to the sample size

In this set of experiments we make the sample size T1 and T2 varying, as well as the number of cross sections N. This time, the distributions are different: N(0,1) for the first one, N(1.4,1) for the second one. The results are presented in Table 6.

Table 6: Power of the tests with respect to the sample size and the number of cross section using Gaussian distribution

			Individual DD test			Panel DD test		
N	T1	T2	Max $\lvert T_1^1(x)\rvert$	Max $\lvert T_1^2(x)\rvert$	Max $\lvert T_1^3(x)\rvert$	$\tilde{T}^1(N,K)$	$\tilde{T}^2(N,K)$	$\tilde{T}^3(N,K)$
2	20	20	0.156	0.148	0.111	0.169	0.159	0.123
2	50	50	0.297	0.297	0.237	0.338	0.329	0.261
2	100	100	0.559	0.571	0.494	0.597	0.602	0.522
2	200	200	0.883	0.877	0.824	0.904	0.9	0.854
2	500	500	1	1	1	1	1	1
5	20	20	0.117	0.149	0.11	0.167	0.175	0.132
5	50	50	0.28	0.291	0.239	0.308	0.306	0.258
5	100	100	0.561	0.567	0.497	0.6	0.603	0.528
5	200	200	0.865	0.893	0.858	0.887	0.905	0.87
5	500	500	0.999	0.998	0.997	0.999	0.999	0.997
10	20	20	0.111	0.142	0.105	0.151	0.159	0.119
10	50	50	0.281	0.291	0.23	0.314	0.309	0.252
10	100	100	0.558	0.586	0.517	0.605	0.62	0.549
10	200	200	0.875	0.87	0.831	0.899	0.883	0.842
10	500	500	1	1	0.999	1	1	0.999
20	20	20	0.134	0.121	0.094	0.157	0.134	0.107
20	50	50	0.301	0.283	0.223	0.335	0.308	0.249
20	100	100	0.545	0.56	0.484	0.587	0.587	0.508
20	200	200	0.883	0.904	0.859	0.899	0.913	0.879
20	500	500	0.999	0.999	0.998	1	0.999	0.998
50	20	20	0.128	0.147	0.118	0.18	0.174	0.136
50	50	50	0.28	0.287	0.231	0.311	0.319	0.255
50	100	100	0.586	0.597	0.515	0.618	0.623	0.548
50	200	200	0.876	0.887	0.844	0.889	0.895	0.857
50	500	500	1	1	1	1	1	1

We can observe that our panel tests display a better power than the corresponding individual DD tests.

We run another set of experiment using student distribution instead of Gaussian ones. The results are presented in Table 7.

Table 7: Power of the tests for Student distributed observations

			Individual DD test			**Panel DD test**		
N	T1	T2	Max $\lvert T_1^1(x)\rvert$	Max $\lvert T_1^2(x)\rvert$	Max $\lvert T_1^3(x)\rvert$	$\tilde{T}^1(N,K)$	$\tilde{T}^2(N,K)$	$\tilde{T}^3(N,K)$
2	20	20	0.089	0.092	0.066	0.094	0.1	0.075
2	50	50	0.194	0.153	0.087	0.213	0.166	0.097
2	100	100	0.305	0.237	0.147	0.337	0.262	0.167
2	200	200	0.582	0.443	0.279	0.611	0.473	0.302
2	500	500	0.831	0.766	0.535	0.841	0.779	0.558
5	20	20	0.074	0.099	0.069	0.094	0.12	0.083
5	50	50	0.171	0.154	0.094	0.198	0.179	0.108
5	100	100	0.296	0.244	0.16	0.321	0.271	0.172
5	200	200	0.584	0.429	0.268	0.608	0.467	0.289
5	500	500	0.836	0.761	0.522	0.844	0.777	0.547
10	20	20	0.082	0.109	0.077	0.109	0.13	0.092
10	50	50	0.18	0.156	0.089	0.198	0.179	0.111
10	100	100	0.341	0.27	0.172	0.365	0.302	0.199
10	200	200	0.562	0.45	0.282	0.577	0.469	0.302
10	500	500	0.85	0.768	0.521	0.862	0.777	0.544
20	20	20	0.086	0.088	0.066	0.095	0.107	0.074
20	50	50	0.158	0.137	0.087	0.189	0.166	0.099
20	100	100	0.311	0.246	0.165	0.338	0.278	0.182
20	200	200	0.603	0.438	0.264	0.632	0.476	0.293
20	500	500	0.831	0.77	0.55	0.835	0.784	0.571
50	20	20	0.087	0.107	0.074	0.102	0.119	0.081
50	50	50	0.163	0.136	0.083	0.195	0.152	0.103
50	100	100	0.323	0.238	0.139	0.356	0.254	0.158
50	200	200	0.58	0.445	0.279	0.594	0.459	0.291
50	500	500	0.85	0.757	0.506	0.864	0.778	0.535

4.2.2 Second set of experiments, with respect to the moments

We fixT1 and T2 to 100, and N=10, and we make the means μ_1 and μ_2 and the standard deviations σ_1^2 and σ_2 of both the distributions varying: $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$. The results are presented in Table 8.

Table 8: Power of the tests with respect to the means and the standard deviations

Mean 1	Mean 2	s.d. 1	s.d. 2	Individual DD test			Panel DD test		
μ_1	μ_2	σ_1	σ_2	Max $\lvert T_1^1(x)\rvert$	Max $\lvert T_1^2(x)\rvert$	Max $\lvert T_1^3(x)\rvert$	$\tilde{T}^1(N,K)$	$\tilde{T}^2(N,K)$	$\tilde{T}^3(N,K)$
0	0.2	1	1.2	0.238	0.07	0.029	0.275	0.083	0.034
0	0.2	1	1.4	0.523	0.123	0.048	0.556	0.15	0.062
0	0.2	1	1.6	0.762	0.261	0.193	0.805	0.304	0.222
0	0.2	1	1.8	0.935	0.522	0.4	0.95	0.566	0.441
0	0.2	1	2	0.98	0.717	0.561	0.99	0.782	0.645
0	0.4	1	1.2	0.612	0.421	0.258	0.649	0.455	0.282
0	0.4	1	1.4	0.756	0.332	0.113	0.784	0.36	0.127
0	0.4	1	1.6	0.902	0.319	0.124	0.929	0.367	0.147
0	0.4	1	1.8	0.977	0.456	0.221	0.983	0.505	0.253
0	0.4	1	2	0.996	0.646	0.382	0.997	0.694	0.434
0	0.6	1	1.2	0.912	0.857	0.704	0.923	0.868	0.723
0	0.6	1	1.4	0.946	0.746	0.47	0.97	0.776	0.516
0	0.6	1	1.6	0.981	0.665	0.284	0.99	0.7	0.312
0	0.6	1	1.8	0.993	0.63	0.258	0.996	0.679	0.289
0	0.6	1	2	0.998	0.698	0.285	1	0.759	0.348
0	0.8	1	1.2	0.993	0.992	0.967	0.993	0.993	0.972
0	0.8	1	1.4	0.995	0.964	0.875	0.995	0.967	0.881
0	0.8	1	1.6	0.998	0.925	0.693	0.999	0.943	0.716
0	0.8	1	1.8	0.999	0.893	0.53	1	0.918	0.561
0	0.8	1	2	0.999	0.862	0.419	0.999	0.886	0.465
0	1	1	1.2	1	1	1	1	1	1
0	1	1	1.4	1	1	0.993	1	1	0.994
0	1	1	1.6	1	0.993	0.931	1	0.994	0.942
0	1	1	1.8	1	0.973	0.811	1	0.981	0.822
0	1	1	2	1	0.964	0.729	1	0.974	0.763

Again, we can observe that our panel tests display a better power (more close to 1) than the corresponding individual DD tests.

4.3 Robustness with respect to the size of the grid

The DD test is computed on a grid of values for x : $x_1, \dots, x_I$. I is the number of values in the grid. If I is too large, the hypothesis of independence between the DD statistics $T^j(x)$ is wrong. This is used to compute the distribution of the statistic under the null hypothesis. This distribution is the studentized maximum modulus distribution. However, if I is too small, the DD test won't be powerful enough.

In this subsection, we examine the size and power of the tests when *I* is varying. We fixT1 and T2 to 100, and N=10, and we make *I* varying: 10% of the sample size *T*, 20%, 30%… 100%.

Under the null hypothesis, simulated samples are generated from Gaussian distribution with same mean (here 0) and same variance (here 1). The results are presented in Table 9. We recall that the tests should display a size close to 0.05.

Table 9: Size of the tests with respect to the size of the grid

No.Grid	Individual DD test			Panel DD test		
	Max $\|T_1^1(x)\|$	Max $\|T_1^2(x)\|$	Max $\|T_1^3(x)\|$	$\tilde{T}^1(N,K)$	$\tilde{T}^2(N,K)$	$\tilde{T}^3(N,K)$
10	0.023	0.016	0.009	0.041	0.022	0.011
20	0.03	0.014	0.007	0.039	0.014	0.009
30	0.022	0.005	0.002	0.028	0.006	0.003
40	0.023	0.006	0.004	0.028	0.009	0.004
50	0.015	0.004	0.001	0.017	0.005	0.002
60	0.025	0.006	0.002	0.026	0.007	0.003
70	0.014	0.001	0	0.02	0.001	0
80	0.01	0.001	0	0.01	0.001	0
90	0.016	0.002	0.002	0.018	0.005	0.002
100	0.01	0.002	0	0.011	0.002	0.001

It can be observed that increasing the size of the grid is associated with decreasing power of the tests, leading to a strong undersize. Our rule to fix the size of the grid leads to the least decrease of the size, and then can be retained as a correct rule.

Under the alternative hypothesis, the distributions are different: N(0,1) for the first one, N(1.4,1) for the second one. The results are presented in Table 10. We recall that the tests should display a power close to 1.

Table 10: Power of the tests with respect to the size of the grid Student distributed observations

No.Grid	Individual DD test			Panel DD test		
	Max $\|T_1^1(x)\|$	Max $\|T_1^2(x)\|$	Max $\|T_1^3(x)\|$	$\tilde{T}^1(N,K)$	$\tilde{T}^2(N,K)$	$\tilde{T}^3(N,K)$
10	0.546	0.569	0.494	0.591	0.592	0.52
20	0.539	0.474	0.402	0.562	0.501	0.441
30	0.499	0.424	0.35	0.536	0.445	0.377

40	0.473	0.388	0.316	0.506	0.415	0.341
50	0.464	0.376	0.312	0.5	0.402	0.33
60	0.508	0.372	0.299	0.529	0.412	0.324
70	0.477	0.374	0.308	0.507	0.398	0.325
80	0.46	0.34	0.266	0.498	0.363	0.29
90	0.441	0.314	0.238	0.477	0.348	0.264
100	0.476	0.337	0.269	0.504	0.351	0.29

We can also see that the power decreases if the size of the grid is too large. Again, our choice for the rule to fix the grid size seems robust.

5. Empirical Results and Analysis

5.1. Panel stochastic dominance tests results

Table 11 provides the results dealing with the panel stochastic dominance results. The stochastic dominance orders presented are 1, 2, and 3. We use the max and min statistics to distinguish between both the alternative hypothesis H_{A1} and H_{A2}.

Table 11: Panel stochastic dominance tests results

Statistic	T1	T1	T2	T2	T3	T3
Type	Max	Min	Max	Min	Max	Min
$\widetilde{T}_i^s(K)$	2.06422535	-2.10983099	1.00488732	-1.11652113	0.6805493	-1.00630986
$\widetilde{Z}^s(N,K)$	-0.70368838	0.59916801	-3.07941843	2.82681963	-3.80679682	3.07398551

The associated unilateral 5% critical values for the *Z* statistic are - 1.645 and 1.645 (the normal ones). From these results, it is clear that there is no stochastic dominance of order one: the max statistics is not smaller than -1.645, and the min statistics is not greater than 1.645. We do not reject the null hypothesis that $H_0: D_s^1(x_k) = D_w^1(x_k)$, for all x_k, k=1,…,I, and for all pairs (S_i,X_i), i=1,…,N. This suggests that there is no arbitrage opportunity between covered warrants and their underlying stocks.

Conversely, for the order two and three, the null hypothesis is rejected. However none of the asset (*S* or *W*) stochastically dominates the other one at the order two and three. At the order two, this suggests that there is no abnormal profit obtained by switching from *S* to *W* (or the reverse); and switching would not allow risk-averse investors to have preference and increase their expected utility. In this scenario, there maybe no arbitrage opportunity and the market is also efficient if all investors are risk averse.

At the order three, which assumes that all investors' utility functions exhibit non-satiation, risk aversion, and decreasing absolute risk aversion (DARA), it means that the market is efficient if investors are associated with risk aversion and DARA. One would not make an expected utility by switching from S to W, even though switching would allow risk-averse DARA investors to increase their expected utility.

5.2 Panel Informational Efficiency Test Result

The value of the panel bootstrap LR test statistic using a GED distribution for the data is presented in Table 12 with other comparative specifications.

Table 12: Panel Informational Efficiency Test Result

Test	Asymptotic		Bootstrap	
Error terms distribution	Gaussian	GED	Gaussian	GED
bar p	0.00	0.11	0.18	0.38
Z	-15.29	-7.76	-9.36	-3.74

The left 5% critical value of a normal distribution is equal to -1.65. All the Z statistics are smaller than this critical value, rejecting the null hypothesis of efficiency. However, the first three statistics are very small, much more than the statistic in the case of GED-bootstrap tests, suggesting a misspecification of the first three models. The mean of the p values in the case of the GED-bootstrap tests is not so small: 0.38 (under the null hypothesis, it should be close to 0.5). The null hypothesis may be rejected for two reasons:

1. the GED-bootstrap test is not perfectly specified and reject the null hypothesis whereas it is true,
2. for some pairs (S_i,W_i) the algorithm to optimize[3] the log-likelihood of the model has not converged, and provides some extreme values for the statistics, and then for the p values (which is too close to 0). This problem is often encountered when optimizing GARCH-type log-likelihood functions.

[3] The software Eviews ® was used to optimize the log-likelihood.

6. Conclusions

This paper contributes to be the first attempt to propose two new panel tests for testing market efficiency, one is a panel stochastic dominance test (PDD test), and a panel informational efficiency LR test (PLR test). We further propose a bootstrap correction for the size distortion of the informational efficiency LR test. Six theoretical theorems are carefully developed and are examined by applying both Monte Carlo simulations and empirical application.

Monte Carlo experiments show that our panel stochastic dominance test has good size and power properties, better than the properties of the corresponding individual tests. The panel tests have less size distortion and are more powerful than their corresponding individual tests. This permits us to examine market efficiency for UK covered warrant with more powerful tests. Empirical results shows that investors perceive the same utility investing in UK covered warrants and their underlying shares, implying there is no arbitrage opportunity. The asymptotic panel LR tests also do not confirm the market efficiency in a first stage. However, they present some technical problems: when correcting these tests using bootstrap methodology and by correcting the error term distribution using a GED distribution instead of the Gaussian one, the results are much more encouraging. Our theoretical derivation and empirical results contributes largely to the literature and also provide importation intuition to the market participants.

References:

Abhyankar, A., Ho, K. Y., Zhao, H., 2005. Long-run post-merger stock performance of UK acquiring firms: a stochastic dominance perspective. Applied Financial Economics. 15, 679-690.

Abhyankar, A., Ho, K. Y., Zhao, H., 2008. Value versus growth: stochastic dominance criteria. Quantitative Finance. 8, 693-704.

Al-Khazali, O. M., Koumanakos, E. P., Pyun, C. S., 2008. Calendar anomaly in the Greek stock market: Stochastic dominance analysis. International Review of Financial Analysis. 17, 461-474.

Anderson, G., 1996. Nonparametric tests of stochastic dominance in income distributions. Econometrica. 64, 1183-1193.

Annaerta, J., Osselaerb, S. V., Verstraetec, B., 2009. Performance evaluation of portfolio insurance strategies using stochastic dominance criteria. Journal of Banking and Finance. 33, 272-280.

Barrett, G.F., Donald, S.G., 2003. Consistent tests for stochastic dominance. Econometrica. 71, 71-104.

Benítez, P. C., Kuosmanen, T., Olschewski, R., van Kooten., G. C., 2006. Conservation payments under risk: a stochastic dominance approach. American Journal of Agricultural Economics. 88, 1-15.

Bishop, J.A., Formby, J.P., Thistle, P.D., 1992. Convergence of the South and non-South income distributions. American Economic Review. 82, 262-272.

Black. F., 1975. Fact and fantasy in the use of options. Financial Analysts Journal. 31, 36-41.

Cerrato, M., de Peretti, C., Larsson, R.and Sarantis, N., 2007, A nonlinear panel unit root test under cross section dependence. Working paper, Centre for International Capital Markets, London Metropolitan Business School, London Metropolitan University.

Claessen, H., Mittnik, S., 2002. Forecasting stock market volatility and the informational efficiency of the DAX-index options market. European Journal of Finance. 8, 302-321.

Davidson, R., Duclos, J. Y., 2000. Statistical inference for stochastic dominance and for the measurement of poverty and inequality. Econometrica. 68, 1435-1464.

De Giorgi, E., 2005. Reward-risk portfolio selection and stochastic dominance. Journal of Banking and Finance. 29, 895-926.

De Giorgi, E., Post, T., 2008. Second-Order Stochastic Dominance, Reward-Risk Portfolio Selection, and the CAPM. Journal of Financial and Quantitative Analysis. 43, 525-546.

Falk, H., Levy, H., 1989. Market reaction to quarterly earnings' announcements: A stochastic dominance based test of market efficiency. Management Science. 35, 425-446.
Gasbarro, D., Wong, W. K., Zumwalt, J. K., 2007. Stochastic dominance analysis of iShares. European Journal of Finance. 13, 89-101.
Im, K.S., Pesaran, M.H., Shin, Y., 2003. Testing for unit roots in heterogeneous panels. Journal of Econometrics 115, 53-74.
Jarrow, R., 1986. The relationship between arbitrage and first order stochastic dominance. Journal of Finance. 41, 915-921.
Knight, J., Satchell, S., 2008. Testing for infinite order stochastic dominance with applications to finance, risk and income inequality. Journal of Economics and Finance. 32, 35-46.
Kuosmanen, T., 2007. Performance measurement and best-practice benchmarking of mutual funds: combining stochastic dominance criteria with data envelopment analysis. Journal of Productivity Analysis. 28, 71-86.
Larsen, G.A., Resnick, B.G., 1999. A performance comparison between cross-sectional stochastic dominance and traditional event study methodologies. Review of Quantitative Finance and Accounting 12, 103–112.
Leshno, M., Levy, H., 2002. Preferred by "All" and preferred by "Most" decision makers: Almost stochastic dominance. Management Science. 48, 1074-1085.
Levy, H., Levy, M., 2004. Prospect theory and mean-variance analysis. Review of Financial Study. 17, 1015-1041.
Levy, H., Wiener, Z., 1998. Stochastic dominance and prospect dominance with subjective weighting functions. Journal of Risk Uncertain. 16, 147–163.
Levy, H., 2006. Stochastic dominance: investment decision making under uncertainty. Springer Publishers.
Lozano, S., Gutiérrez, E., 2008. Data envelopment analysis of mutual funds based on second-order stochastic dominance. European Journal of Operational Research. 189, 230-244.
Li, H., Maddala, G. S., 1996. Bootstrapping time series models. Econometric Reviews. 15, 115-195.
Maddala, G. S., Wu, S., 1999, A comparative study of unit root tests with panel data and a new simple test. Oxford Bulletin of Economics and Statistics, 61, 631-665.
Manaster, S., Rendleman, J., 1982. Option prices as predictors of equilibrium stock prices. Journal of Finance. 37, 1043-1057.
Markowitz, H.M., 1952. Portfolio selection. Journal of Finance. 7, 77-91.
Pesaran, M. H., 2007, A simple panel unit root test in the presence of cross section dependence. Journal of Applied Econometrics. 22, 265-312.

Post, T., 2003. Empirical tests for stochastic dominance efficiency. The Journal of Finance. 58, 1905-1932.

Ringuest, J. L., Graves, S. B., 2000. A sampling-based method for generating nondominated solutions in stochastic MOMP problems. European Journal of Operational Research. 126, 651-661.

Stoline, M.R., Ury, H.K., 1979. Tables of the studentized maximum modulus distribution and an application to multiple comparisons among means. Technometrics. 21, 87-93.

Wong, W.K., 2007. Stochastic Dominance and Mean-Variance Measures of Profit and Loss for Business Planning and Investment. European Journal of Operational Research. 182, 829–843.

Wong, W. K., Chan, R., 2008. Markowitz and prospect stochastic dominances. Annals of Finance. 4, 105-129.

Wong, W.K., Ma, C. 2008. Preferences over Meyer's Location-Scale Family. Economic Theory. 37, 119-146.

Wong, W. K., Phoon, K. F., Lean, H. H., 2008. Stochastic dominance analysis of Asian hedge funds. Pacific-Basin Finance Journal. 16, 204-223.

Xu, X., Taylor, S. J., 1995. Conditional volatility and information efficiency of the PHXL currency options market. Journal of Banking and Finance. 19, 803-21.

Appendix. Summary Statistics of the Sample Set

UK list	Type	Listing	Issue price	UK list	Type	Listing	Issue price
Ang. Amer	call	2005/4/12	13.3	M&S	call	2006/7/31	67.75
	call	2005/4/12	7.8		call	2006/7/31	23.35
	put	2005/4/12	5.35	National grid	call	2006/7/31	6.7
Ang. Amer	call	2006/7/31	41.2		call	2006/7/31	2.54
	call	2006/7/31	27.15	Partygaming	call	2006/7/31	33.15
	put	2006/7/31	20.9		call	2006/7/31	14.85
Ang Amer	put	2006/10/31	21.7	Pearson	call	2005/4/8	60.65
	call	2006/10/31	16.3		call	2005/4/8	17
Antofagasta	call	2006/7/31	72.15		put	2005/4/8	27.6
	call	2006/7/31	53.75	Prudential	call	2005/4/8	7.8
	put	2006/7/31	52.25		call	2005/4/8	3.13
Arm.hdg	call	2006/7/31	23.35		put	2005/4/8	1.67
	call	2006/7/31	12.35	Prudential	call	2006/2/24	6.5
AstraZeneca	call	2005/12/1	28.28		call	2006/2/24	2.86
	call	2005/12/1	13.42	Prudential	call	2006/7/31	7.2
	put	2005/12/1	11.17		call	2006/7/31	3.26
AstraZeneca	call	2006/2/24	25.63	Rbos	call	2006/7/31	15.65
	call	2006/2/24	10.36		call	2006/7/31	5.6
	put	2006/2/24	19.37		put	2006/7/31	9.2
AstraZeneca	call	2006/7/31	16.4	Reuters	call	2005/4/8	46.93
	call	2006/7/31	38.7		call	2005/4/8	23.83
	put	2006/7/31	19.2		put	2005/4/8	27.73
Aviva	call	2005/4/11	6.7	Reuters	call	2006/2/24	8.14
	call	2005/4/11	1.83		call	2006/2/24	3.77
	put	2005/4/11	2.38		put	2006/2/24	70.36
B sky	call	2005/4/11	5.7	Reuters	call	2006/7/31	56.15
	call	2005/4/11	1.48		call	2006/7/31	30.15
	put	2005/4/11	2.75		put	2006/7/31	22.65
BA	call	2006/7/31	70.25	Rio Tinto	call	2006/7/31	41.05
	call	2006/7/31	26.95		call	2006/7/31	21.45
	put	2006/7/31	32		put	2006/7/31	29.95
Bae	call	2005/4/11	28.9	Rio Tinto	call	2005/12/1	22.5
	call	2005/4/11	7.3		call	2005/12/1	10.82
	put	2005/4/11	10.05		put	2005/12/1	14.35
Barclays	put	2005/4/11	2.9	Rio Tinto	call	2006/2/2	28.93
	call	2005/4/11	3.82		call	2006/2/2	15.77
	call	2005/4/11	1.73		put	2006/2/2	30.75
Barclays	call	2005/12/1	37.34	Rolls Royce	call	2006/7/31	67.45
	call	2005/12/1	19.78		call	2006/7/31	30.25
	put	2005/12/1	44.82	Saga group	call	2006/10/3	23.15
Barclays	call	2006/2/24	58.61		call	2006/10/3	12.65
	call	2006/2/24	22.77		put	2006/10/3	23.35
	put	2006/2/24	53.31	Shire	call	2006/7/31	15.45
Barclays	call	2006/7/31	93.45		call	2006/7/31	7.55
	call	2006/7/31	40.85	Smith	call	2006/7/31	9.98
	put	2006/7/31	26.15		call	2006/7/31	4.26
Batob	call	2005/4/11	9.35	Std.cht	call	2006/2/23	15.95
	call	2005/4/11	3.69		call	2006/2/23	5.02
	put	2005/4/11	2.97		put	2006/2/23	10.92
Batob	call	2006/7/31	19.75	Std.cht	call	2006/7/31	19.45
	call	2006/7/31	5.05		call	2006/7/31	6.6
BG	call	2006/7/31	96.7	Tesco	call	2005/4/11	29.45
	call	2006/7/31	35.6		call	2005/4/11	6.53
BP	call	2005/4/11	40.25		put	2005/4/11	9.48
	call	2005/4/11	18.1	Vodafone	call	2005/4/11	9.48
	put	2005/4/11	27.4		call	2005/4/11	2.37
BP	call	2005/12/1	72.12		put	2005/4/11	3.82
	call	2005/12/1	22.94	Vodafone	call	2006/2/2	13.29
	put	2005/12/1	22.5		call	2006/2/2	7.25
BP	call	2006/2/23	51.11	Vodafone	call	2005/12/13	5.33
	call	2006/2/23	17.42		call	2005/12/13	1.63
	put	2006/2/23	54.26		put	2005/12/13	9.53
BP	call	2006/7/31	40.15	Vodafone	call	2006/8/	10.75
	call	2006/7/31	12.95		call	2006/8/	3.77
	put	2006/7/31	39.75		put	2006/8/	7.48
B.sky	call	2006/7/31	7.5	William Hill	call	2006/7/31	5.35
	call	2006/7/31	1.35		call	2006/7/31	1.76
BT	call	2005/4/11	14.4	Wpp	call	2006/7/31	6.85
	call	2005/4/11	6.3		call	2006/7/31	2.61
	put	2005/4/11	11.9	Hsbc	call	2006/3/1	3.82

BT	call	2006/7/31	25.85		call	2006/3/1	1.66
	call	2006/7/31	7.1		put	2006/3/1	5.79
Cable & wireless	call	2006/7/31	16.25	Hsbc	call	2006/7/31	7.05
	call	2006/7/31	7.45		call	2006/7/31	1.91
Centrica	call	2005/9/14	14.87		put	2006/7/31	3.52
	call	2005/9/14	25.13	Itv	call	2006/10/3	10.45
Diageo	call	2005/4/8	6.28		call	2006/10/3	5.45
	call	2005/4/8	1.82		put	2006/10/3	10.65
	put	2005/4/8	2.69	Land secs	call	2006/2/3	22.75
Glxsk	call	2006/7/31	8.4		call	2006/2/3	12.85
	call	2006/7/31	2.88	Legal&Gen	call	2005/4/11	11.18
	put	2006/7/31	9.5		call	2005/4/11	2.44
Hbos	call	2005/4/8	7.58		put	2005/4/11	5.43
	call	2005/4/8	2.73	Llds tsb	call	2006/2/23	38.06
	put	2005/4/8	3.26		call	2006/2/23	14.6
Hbos	call	2006/2/24	9.72		put	2006/2/23	68.68
	call	2006/2/24	2.37	Llds tsb	call	2006/7/31	44.55
	put	2006/2/24	6.34		call	2006/7/31	13.75
Hbos	call	2006/7/31	10.05		put	2006/7/31	39.55
	call	2006/7/31	2.4	Logiccmg	call	2006/7/31	25.65
Hsbc	call	2005/4/7	4		call	2006/7/31	11.35
	call	2005/4/7	2.12	M&S	call	2005/4/8	18.33
	put	2005/4/7	4.36		call	2005/4/8	4.84
					put	2005/4/8	24.93