Analysis of the Impact of Contagion Flow on High Yield Bond Portfolio

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Abstract

Portfolios are constructed to increase returns and manage risk. In high-risk investment strategies, central measures of risk must be complemented with tail measures of risk. An unanticipated event impacting securities of one firm can contagiously affect those of other firms through a contagion flow process. The connections between firms due to a variety of factors can spread the contagion, and potentially impact firms in a network. This can adversely affect the level of tail risk in an investment strategy, especially when a number of these connected firms are included in a portfolio. A model is developed for flow of contagion between firms which will define, characterize and calibrate a contagions impact on default risk of a portfolio of debt instruments. The model assesses the impact of network structure underlying contagion flow and evaluates the contagion related excess risk in a portfolio of high-yield debt instruments.

1 Introduction

A portfolio of high yield bonds has characteristics that will appeal to certain investors over other investment opportunities. Returns are usually greater than government bonds, but with increased risk. This risk should be less than that of the related companies' stocks and the volatility should also be lower. To judge the worth/risk of a bond it is important to have a good understanding of probability of default of the particular bond. When constructing a bond portfolio, it is not only important to understand the default risk of individual bonds, but also how these default risks are correlated between various bonds.

When developing a high-yield bond portfolio strategy, the degree of default risk is a prime determinant of portfolio returns. According to Mr. Don Cassidy, senior analyst for Lipper, on average 5 percent of these bonds default. Due to this high default rate, it is extremely important that selected bonds do not have higher than anticipated levels of correlated defaults. These default rates also display high variability from year to year, increasing the challenge of accurately predicting defaults. Altman and Bana (2004) analyzed data from defaults occurring between 1971 and 2003. There results showed default rates ranged from .158 to 12.795 percent per year, with a weighted average default rate of 5.453 percent. In addition, during the time period covered by the study, the amount of money lost through default has increased substantially with a record par value default of 96.858 billion dollars in 2002. The following figure shows default rates during the 1971 to 2007 time period.

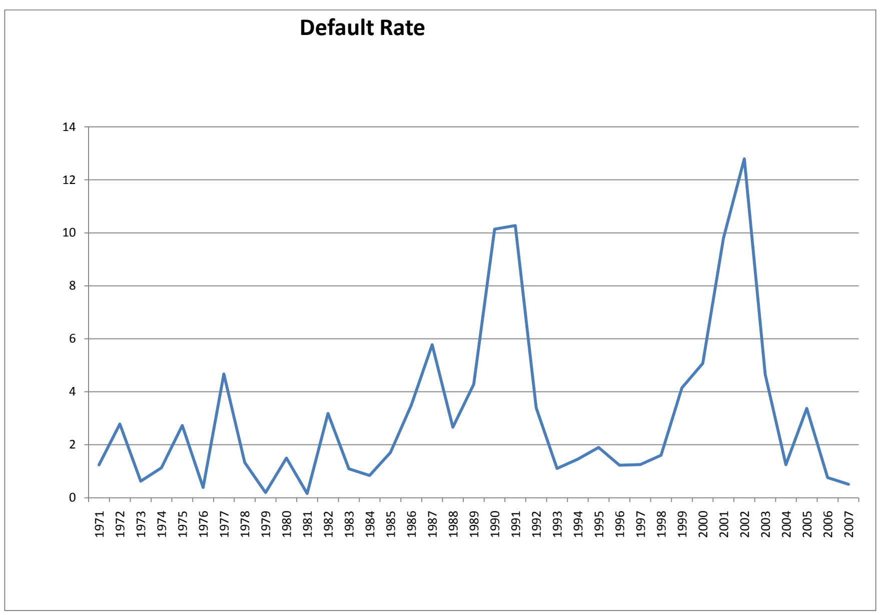


Figure 1: Default Rate

In addition to default variability, bond ratings are a main source used to assess and manage risk. The latest financial crisis, involving Mortgage Backed Securities, has called into question this rating system and how well it predicts risk. This issue has also been raised in fallen angels, discussed by Altman and Bana(2004). Fallen angels are companies whose bond ratings are downgraded from investment to speculative grade, subsequent to the actual drop in the bond's price, showing that ratings often are not an accurate indicator of risk.

To pick the best portfolio of high yield bonds a better understanding of the associated risk must be modeled. Macroeconomic, sector and idiosyncratic risks are evaluated when constructing a portfolio and have been extensively studied. On the other hand, contagion has also been studied in detail, but with limited attention given to high-yield bond investments and how contagion flow impacts defaults within a portfolio.

In a bond portfolio, contagion can arise due to an unanticipated event that impacts one bond, and then spreads to other bonds for reasons not fully understood or expected. This can result in increased spreads for the bonds, and ultimately bond defaults. Therefore, contagion flow can increase clustering of bond defaults. Much of this clustering is not predicted using current models. According to Altman and Bana (2004) in 2002 thirty six percent of bankruptcies were telecom-related companies. The relationship of these firms in a given sector is straightforward; however why a particular sector is impacted may be less clear. Conversely, 24 percent of bankruptcies from that same year were related to alleged fraud. When the initial fraud was revealed, all companies taking part in similar fraudulent activities were at increased risk. This risk was not understood until the initial event occurred and the connections between the firms became apparent.

This paper adds to the knowledge of the impact of a contagion by showing how actual firm information can be used to determine a network structure. This information is combined with a factor model which results in defaults and lower tail risk evident in historical data. Contagion is separated into several variables and the effect of changes to these variables is evaluated. Finally the impact of contagion on a portfolio of bonds is evaluated.

The paper is organized as follows: section 2 will review the current financial contagion and bond default research. Section 3 presents an extension of default risk modeling using a structural firm value model incorporating the arrival and flow of contagion, impacting the default characteristics. The 4th section describes the calibration and simulation methodology for the model along with an overview of the data utilized. Section 5 presents results and analysis of the impact of contagion flow through a network structure on individual firms and portfolio value. Concluding remarks are made in Section 6.

2 Literature Review

Three firms in the same sector will tend to move together. However, can that movement be more substantial than what is due to macroeconomic and sector correlations? The following chart presents the stock performance of three firms. The event that caused the drop in stock price of the three firms was a surprise announcement that the FDA had denied approval for a new drug. As stated by the headline "Amylin shares crater after FDA rejects Bydureon application." Amylin was the focus of the most negative publicity. However this negative impact also spread to two other firms: Alkermes and Eli Lilly. All three firms were connected by a joint venture.

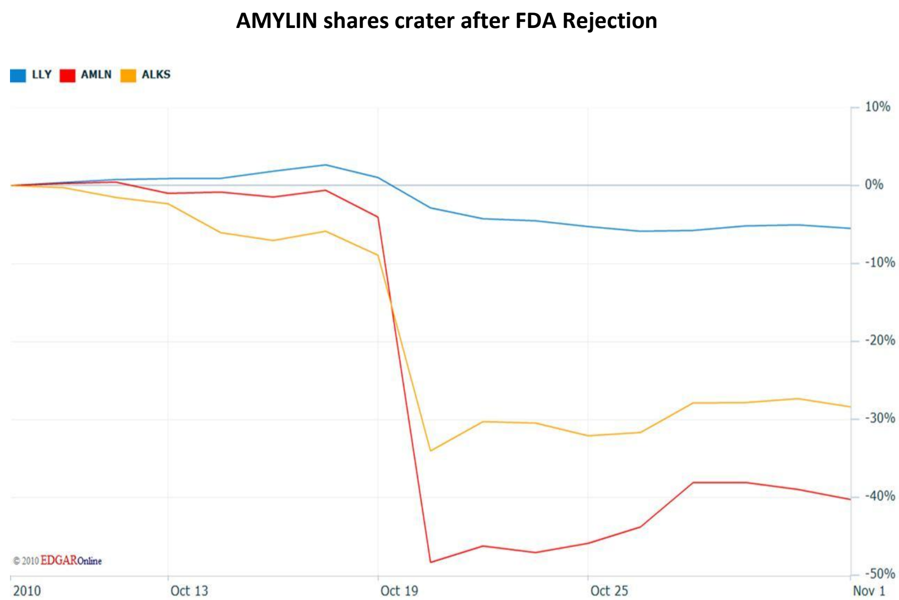


Figure 2: Stock Performance

This is an example of how a exogenous contagious event can spread from one firm to another. Without an understanding of these connections and how they can result in the spread of a negative impact, a portfolio that contains all three of these firms is more risky than originally anticipated.

Clustering of defaults and bankruptcies tends to be more substantial than impacts of macroeconomic and sector movements are able to explain. Connections like those in the Amlyin example offer an explanation of how an exogenous event can spread to other firms resulting in a contagion. Various models have been developed to predict future firm values. Current research supports the idea that there are more intricate relationships between firms than considered by many models. A better understanding of what causes clustering of drops in firm value is needed to prevent exposure to unanticipated levels of risk. This is especially critical when considering a portfolio of high yield bonds as the amount of defaults varies and has been increasing.

Historically, there are many examples of unanticipated events affecting an entity and then spreading to similar entities for reasons not anticipated or fully understood at the time of the incident. When the entity is a firm, a bank, or even a country these incidents can impact their stability and ability to survive. The term contagion is used to describe the spread of a negative event. Various researchers use slightly different definitions of what constitutes a contagion. Fabozzi and Focardi (2004) define contagion as sudden and unexplained increase in the correlation level. Egloff, Leippold, and Vanini (2007) say that through micro-structural channels the credit deterioration of a counterparty triggers the credit deterioration of other counterparties. Along these lines this paper defines contagion as the co-movement in asset values not explained by common fundamentals. This co-movement is caused by an unanticipated event that impacts one bond and then spreads to other bonds due to a connection between the firms resulting in increased defaults, leading to heavier tail loss distributions.

Limit of Existing models

Many models used to predict defaults fail to include contagion. When these models are tested empirically results are often disappointing. Accounting models used to predict bankruptcy have been studied to determine their accuracy in predicting default on high yield bonds. Bryan, Marchesini, and Perdue (2004) looked at four such models, two models based on accounting ratios and two based on cash flows, to determine if they could be used to predict the bond defaults. The results were barely better than flipping a coin, leading them to develop a model with a greater variety of variables and indicating that there are other influences that are not being picked up in the current models.

Models of defaults are often based on the double stochastic assumption, stating that if conditioned on risk factors, all firms default intensities are independent poisson arrivals with conditional deterministic intensity paths. These models only consider observable factors, therefore not allowing for contagion. Das et al (2007) explained how an understanding of corporate defaults is important when putting together a portfolio of corporate debt instruments; especially the clustering of these defaults. If defaults cluster more than anticipated by various models, risk to a portfolio's value increases. The authors analyze default and corporate data from 1979 to 2004 to test the double stochastic assumption. Results fail to support this assumption, leading them to conclude that better models for default correlations are needed, and speculating that contagion could cause this increased clustering.

Other researchers such as Hull and White (2008) have developed ways to model this increased default correlation. Market data shows that as default environment worsens, correlation increases. To model this they use a hazard rate for a firm that is based on a deterministic process subject to periodic impulses. They use these impulses to create increased default correlations – the larger the impulse the more firms are impacted. In order to fit the model to the term structure of CDS spreads, an increase in impulse is required. This model is a simplification since all firms are impacted at the same time by the same size shock, and no specific network structures are involved. It does, however, show that an impulse or exogenous contagion is required for increased default correlations to fit market data.

Previous research leads us to believe there are more intricate relationships between firms than considered by many models. A better understanding of what is causing this default clustering is needed to prevent exposure to a greater amount of risk than expected. Network structures are used as a way to explain this default clustering. The spread of a negative impact through these networks is often referred to as contagion. Some of the first models of contagion pertained to financial institutions; specifically, banking.

Contagion: Banks and Countries

Due to the severe global impact a shock and the resulting contagion can have on the banking system, the implications has been well studied. In the seminal work by Allen and Gale (2000), instead of viewing contagion as a random event, they show that contagion can be driven by real shocks and linkages between entities. They use a network of banks and the impact of a liquidity crisis to see how this can result in a contagion being spread through a network. The size of the liquidity shock and the structure that connects the banks determines how far the contagion spreads and its impact. Cases are presented where the contagion can either dissipate or cause total network failure. Research into the impact of contagions include Pericoli and Sbracia (2003), spreading between countries, and Kordres and Pritsker (2002) across markets. The credit crisis of 2007-2010 continues to show how shocks can spread between linked nations and banks. Recently, Stiglitz (2010) argued that integration can result in contagions more easily spreading between countries and that these negative impacts can be greater than the positive impacts. Martinez-Jaramillo, etc (2010) model systemic risk as two components – a random shock and then a network that allows the transmission which represents the contagion spreading.

Network Structures

Bank and country connections that spread shocks are easier to see and understand. Work has also been done to develop models showing that these types of connections exist between firms. Many of the models are reduced form or stylized models focusing on the resulting loss distribution versus the impact on firm value. Increases in the tail sections of a loss distribution are often taken as an indicator of contagion. Models have been developed to show how a contagion, consisting of an exogenous shock and a network structure, allow the contagion to spread and result in the heavy tailed loss distributions seen in empirical data. They, however, often do little to understand these connections. Davis and Lo (2001) developed a model for contagion in a bond portfolio. They use Bernoulli random variables to model default distribution and also the probability of this default spreading to other bonds in the portfolio. Showing how the probability of the default spreading increases the risk of the portfolio. In this model, the spread of contagion is controlled by an infection probability and is limited to a given sector. No specifics are given about the structure that allows the contagion to spread.

Random and directed graphs have also been used to represent the network structure between firms that allow a contagion to spread. Fabozzi and Focardi (2004) proposed a highly idealized static model for determining the credit loss distribution based on exogenous factors and contagion. They use a random graph model to form a network between firms. These random connections are not based on any particular characteristic of a given firm. Concluding that these network connections impact the loss distribution, giving results that more closely fit empirical data, especially in the tail areas. Addition of a parameter to represent these connections is suggested. No attempt is made to understand the business relationships that can cause these linkages or the impact of individual firm characteristics. Egloff, Leippold,and Vanini (2007) also used directed graphs, but included empirical links and more realistic assumptions. They proposed a dynamic, structural, stylistic model of contagion. The model considers macro and microstructure dependencies. These microstructure dependencies represent the links between debtors allowing the contagion to spread among the debtors, impacting the tails of the loss distribution.

Schellhorn and Cossin developed a structural model with their network structure based on a queueing model. Random and cyclical network structures were considered. Other stylized models of contagion have been proposed by Giesecke and Weber (2004), Kraft and Steffensen (2009) and Horst (2007). The above models show how a contagion can be spread through a network and impact credit loss, but little is done to use actual firm data to develop and understand these networks.

Firm Connections

The previous research shows the importance of connections between firms, but a better understanding of what actually causes these connections is required to help in portfolio selection. Hertzel, Li, Officer, Rodgers (2008) show a possible source of these connections. They present data to support the spreading of contagion through supply chain relationships, from one firm to rivals and suppliers of the firm. Historical data for approximately 250 firms that have filed for bankruptcy and their customers and suppliers are used to test the hypothesis. Daily abnormal returns are calculated for these firms. Negative daily abnormal returns on the filing day or the firm distress day are taken to indicate the contagion is spreading. It also shows that the impact on the suppliers is more severe if the industry of the firm has been impacted by the contagion. This contagion can also be spread beyond reliant suppliers and major customers to their industries.

Since there are many possible sources of relationships between firms, multiple firm characteristics can be used to determine connections. As with this paper's model, Egloff, Leippold, and Vanini (2007) used firm specific knowledge to determine the firm connections, focusing on weights determined by business volume of debtors with counter parties or predefined structures. Selecting from 1 of 4, based on the "quality of portfolio," the rating of the firm is determined. To increase the granularity, this paper looks at actual firms data to reflect the quality of the firm. The goal is to create a model of contagion that has increased similarities to the way a contagion is actually seen, having an external event that spreads to other firms through a connection. Based on this, the model is developed as a structural model providing a more intuitive interpretation focused on value of a firm.

Firm Value to determine defaults

Determining defaults based on firm value is similar to the model outlined by Uhrig-Homburg (2005) where firm defaults are due to either liabilities exceeding a firms assets or a firm having insufficient cash flow to meet its obligations. The authors build on Merton's approach and add the impacts of default and bankruptcy. The Merton model is limited to only the individual firm and therefore does not address the spread to other firms. The firm value is determined based on unleveraged firm asset value, value of future payments, tax benefits, bankruptcy costs and equity issuance costs. This paper's model instead considers the following factors that impact firm value: Macroeconomic, interest rate, sector and firm specific factors. A contagion piece is then added to the model which includes connections between firms.

3 Firm Value Model with Contagion

Previous research has shown that contagion added to a model helps explain default clustering. This paper proposes a structural model for firm value that incorporates various factors including contagion. In this section equations are developed to reflect how contagion can impact one firm and then spread to other firms. The characteristics that define the contagion are the size of an external contagion, the speed of its' spreading and the network that allow it to spread to other firms. Statistical measures are developed to quantify the impact of the contagion at the firm and portfolio level. The goal is a model that matches the heavy tails seen in empirical data and offers increased understanding of the dynamics of contagion and its impact at the firm and portfolio level.

3.1 Outline of the Model

To evaluate the probability of default, the change in a firm's value due to various factors will be simulated. When a firm's value falls below a simulated debt level, it is indicative of financial distress and increased likelihood of a firm defaulting on its debt. By analyzing the firm's value, it is possible to consider the default event and the impact on the lower tail of the firm value distribution. Firm value evolution is more detailed than focusing on rating class changes, since changes to a firm's rating class are rare and can often lag the fundamental change in firm value. Seeking to go beyond the common macro and micro-factors considered in many models, this paper includes a structure resulting in contagion flowing between firms.

The following are factors impacting firm value included in the model.

1. Macro Economic conditions:

As stated in Altman and Bana(2004), there is a tie between economic performance and defaults. Common belief is that defaults increase at the beginning of a recession and peak when a recession ends, or shortly thereafter. In more recent recessions, however, defaults started to increase before the recession begins. Due to the relationship between Macroeconomic conditions and defaults, it is important to include a variable for economic outlook in modeling the firm value.

2. Interest Rates and Credit Spreads:

The cost of borrowing has an impact on the return on assets of most firms. Since interest costs vary and impact the rate of return for a firm, the cost of borrowing is an important factor to include. The rate for the lowest end of the investment grade is used .

3. Sector Factors:

A portfolio is often made up of a group of companies from one sector. This can be desirable because of an investors' views that this sector will out perform or be more stable than the market in general. Companies in the same sector often move together. As a result, an index of the selected sector can help predict movements of firms in that sector.

4. Firm specific variables:

Numerous firm specific factors influence a firm's value. As situations change, this information becomes available in the markets, and these changes are reflected in the price of a firm's stock. Therefore, return on a firms stock is used as an indicator of changes in the health of a firm.

5. Contagion factors:

A contagion that impacts a firm's value isn't represented by the above factors. Consider a group of firms each having issued a bond. These firms have business linkages such as a supply chains relationships. These links can impact default correlations but not be reflected in the portfolio risk assessment. Businesses with a supply chain linkage may not be in the same industry or sector. If one firm is impacted by an unanticipated event, this can pass to firm two by firm one's inability to pay its obligations to firm two, due to a liquidity crisis caused by the original event. This continues throughout the network of connected firms due to cascading liquidity issues resulting in a contagious event.

The model separates the impact of a contagion into five factors. The first two focus on the exogenous contagion represent by the arrival rate and amplitude of the initial impact of the contagion on one firm. The next three focus on how it then spreads to other firms. There is a network structure that allows the contagion to spread to other firms, examples include: supply chain relationships, reliance on the same bank, geographic locations (proximity to each other), same audit firm, common investment firm ownership or funding sources, alliances, and common directors. The remaining two factors determine the speed and amount of decay of the contagion as it travels through the network.

3.2 Firm Valuation Evolution Model: Non-Contagion

Formulas for evolution of firm value, not including contagion, which are based on the structural models of Merton(1974) are presented in this section. Factors are chosen that impact a firm's return on assets including: macroeconomic, interest rates, sector and firm specific factors. In addition, the impact of contagion due to a network structure will be added to the firms value's rate of return. The contagion formulas will be described in the following section. Default occurs when a firm's value falls below its simulated default boundary (debt level).

Definition of Terms:

i : Bond i $\in [1, n]$, n Number of Bonds in portfolio

V_i : Firm value of the firm issuing i^{th} Bond

$r_v^i(t)$: Rate of Return on Asset for a Firm

r : Rate of return for individual factor

σ : Volatility of rates of return for individual factor

NC : Net contagion - see next section

Variable Subscripts:

m : Market index

s : Industry sector

c : Firm specific

Baa : Interest rate for a Baa Bond

Changes to firm value come from two sources, the rate of return on the assets for a firm and contagion.

$$\frac{dV_i}{V_i} = r_v^i(t)dt + dNC_i(t), \tag{1}$$

The return on asset for a given bond is dependent on factors related to macroeconomic, interest rates, sector, and firm specific issues. The βs represent the factor loadings.

$$r_v^i(t) = \beta_{0i} + \beta_{Baai} r_{Baa}(t) + \beta_{mi} r_m(t) + \beta_{si} r_s(t) + \beta_{ci} r_{ci}(t) \tag{2}$$

The following equations reflect returns determined by changes in the macroeconomic, sector and firm specific factors. The macroeconomic effect is contained in two equations: the impact of the market and interest rates. Each factor is represented by a mean reversion equation with rate of convergence to the mean(γ), long term mean(μ) and a standard Wiener process (W).

$$dr_{Baa} = \gamma_{Baa}(\mu_{Baa} - r_{Baa})dt + \sigma_{Baa} dW_{ft} \tag{3}$$

$$dr_m = \gamma_m(\mu_m - r_m)dt + \sigma_m dW_{mt} \tag{4}$$

$$dr_s = \gamma_s(\mu_s - r_s)dt + \sigma_s dW_{st} \tag{5}$$

$$dr_c = \gamma_c(\mu_c - r_c)dt + \sigma_c dW_{it} \tag{6}$$

With a basic model defined for future values of a firm's return on asset, contagion is added, a factor often overlooked in models.

3.3 Contagion Model

An exogenous event impacting an individual firm becomes a contagion when it spreads to other firms due to connections between firms. In the following section this process is separated into five factors representing the evolution of the contagion flow. Amplitude and arrival rate describe the exogenous contagion. Connections between firms define how it is able to spread. Additional variables describe the speed and amount of decay of the contagion as it spreads through this network of firms.

3.3.1 Contagion Model Equations

The net contagion piece of the model consists of two parts: the exogenous contagion arriving at this node and the contagion from a neighbors' node that arrive at this node after a time delay and having partially decayed.

i : Bond i $\in [1, n]$, n Number of Bonds in portfolio

R_i : Set of Neighbors of Bond i

$NC_i(t)$: Net Contagion at a Bond (Node) i

$C_i(t)$: Exogenous Contagion at time t at Bond (Node) i

$\tau_{ji}(t)$: Stochastic travel time for Contagion from Bond (Node) j to Bond (Node) i

$\lambda_{ji}(t)$: Stochastic decay rate for Contagion from Bond (Node) j to Bond (Node) i

$A_i(t)$: Stochastic amplitude of the exogenous contagion arriving at Bond (Node) i

$N_i(t)$: Contagion arrival process, Poisson Process with rate $\mu_i(t)$

For an $i \in I$, we define the Net Contagion for the Bond i as follows,

$$NC_i(t) = C_i(t) + \sum_{j \in R_i} NC_j(t - \tau_{ji}(t)) \exp(-\lambda_{ji}(t)(\tau_{ji}(t))), \tag{7}$$

where $dC_i(t) = A_i(t)dN_i(t)$. NC_j represents the contagion at all neighbors (those connected to i) that will arrive at Bond (Node) i in time τ_{ji} as it exponentially decays in intensity at rate of λ_{ji}. Therefore, τ captures the fact that between two nodes, contagion can travel at different speeds, and λ allows the possibility that contagion intensity doesn't remain the same as contagion travels. These two parameters are designed to capture the complexity of contagion flow characteristics.

3.3.2 Contagion Variables

The contagion model will be separated into a variable that represents contagion and can be calibrated based on historical or firm specific data. Similar to Martinez-Jaramillo, etc (2010) work on contagion in a network of banks, this paper focuses on two main components: a random shock and a network that allows the contagion to spread. The network is represented by three parts: the existence of a connection between firms, the speed and decay that the contagion experiences as it travels along this connection. The following summarizes these variables: the arrival rate and amplitude of the contagion, the network estimation,and the speed and decay of the contagion traveling from one firm to another firm.

Exogenous Contagion: Arrival Rate and Amplitude

A contagion arriving at a firm is an unexpected and rare event. Arrival rates reflect how often a contagion, on average, will arrive at a firm over a given time period. Characteristics of the triggering event will impact the strength of the contagion. The amplitude of the triggering event is used to reflect the strength of the arriving exogenous contagion.

Contagion Spread: Connections, Decay Rate and Travel Time

Kannan, Kohler-Geib (2009) consider contagion spreading from one country to another and propose an "uncertainty channel of contagion," showing that a surprise crisis, unlike an anticipated crisis, increases the likelihood of a contagion spreading. When something unexpected happens, people trust their information less and react in ways more likely to contribute to the contagion spreading. Additional support for the existence of other contagion channels is provided, including overexposed fund investor, trade links and common creditors. The same ideas can be applied to firms. With firm-pair connections a negative surprise at one firm can travel to another firm due to linkages between the two firms. As the spread of information, in general, depends on the length and quality of the network it travels on, the spread of a contagion will also depend on the strength of the linkage between the firms and the length of time it takes to transmit the information contained in the contagion event.

Firms can be linked in a network structure due to relationships or characteristics they have in common. These specific firm-pair values represent the unique way that two firms are linked or connected to each other. The variables used to define the impact of these firm pair connections are the decay rate and travel time. The decay rate represents the strength of the connection. The travel time represents the speed of the transmission from one firm to another. These variables determine the size and speed an exogenous contagion will spread from one firm to other firms.

Connections

Lui(2009) used business relationships to develop a flexible default correlation structure to be used to optimize a portfolio of corporate bonds. Our model also focuses on actual firm relationships to determine connections between firms. A list of possible connections between firms has been developed based on historical events where linkages have allowed contagion to spread from one firm to another. This can include joint venture involvement, shared distribution channel and supply chains, shared directors, and financial channels: including banking relationships, investors and institutional ownership. Based on this list of possible areas for connections, information on a group of firms has been collected. Where there are matches between firms on the above items, a connection is assumed.

Similar to the random graphs used by Focardi and Fabozzi(2004) and a directed graph model used by Egloff, Leippold,and Vanini (2004), firms will be vertices, and the edges will represent connections between firms. These edges will be directed. A_i will represent an exogenous contagion arriving at a firm. $\lambda\tau$ will represent the flow from one firm to another.

Examples of Stylized Connections

Firms can have multiple connections to other firms with the direction determined by the nature of the connection. As illustrated in figure 3, Firm i is impacted by one other firm's events and an exogenous contagion. Firm i affects firm j, which spreads contagion to two other firms.

Figure 4 shows examples of stylized network connections. Numerous larger structures can be created from these basic structures.

In a serial network, which is the top example in the figure 4, three firms are connected allowing the contagion to spread from firm 1 to firm 2 and from firm 2 to firm 3. A exogenous contagion hits firm one and spread to firm 2 and then from firm 2 to firm 3. The types of connections between 1 and 2 and 2 and 3 can be unrelated.

Five firms connected allowing the contagion to spread from firm 1 to firms 2, 3, 4 and 5 in a star

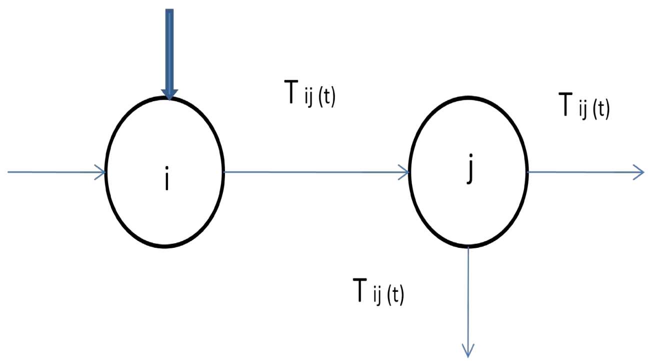


Figure 3: Networks

network is presented in the bottom of figure 4. This is a situation where a contagion impacts firm one with connections to multiple firms which allows the impact to spread. An example scenario is firm 1 is a supplier to firms 2, 3, 4, and 5.

Decay Rate and Travel Time

Connections are ranked by the speed of information traveling on these channels (travel time) and the strength of the relationship between the two firms (decay rate). These are further defined as:

Travel Time: τ The speed contagion travels from one firm to another firm.

Range : (0, infinity), Integer values. The larger the number the slower moving and therefore the more the contagion will decay.

Examples: 1: fast moving, 2-4: medium, 5-10: slow

Decay Rate: λ The strength of the linkage:

strong link = long (slow) decay = small lambda, λ = 0 - less than 1

weak link = short (fast) decay = large lambda, λ = 1 -10

The amount of the contagion that propagates depends on the amplitude, travel time and decay rate. The amplitude of the exogenous contagion is multiplied by exp(- τ * λ) to determine the size of the propagated contagion. The values that exp(-τ * λ) take will range from [0, 1], with 1 being no decay, and 0 being total decay. Since this is exponential, when τ * λ is greater than 4, less than 2 percent of the contagion will propagate. At 1, approximately 37 percent will propagate. The closer to 0, the closer to total propagation of the exogenous contagion.

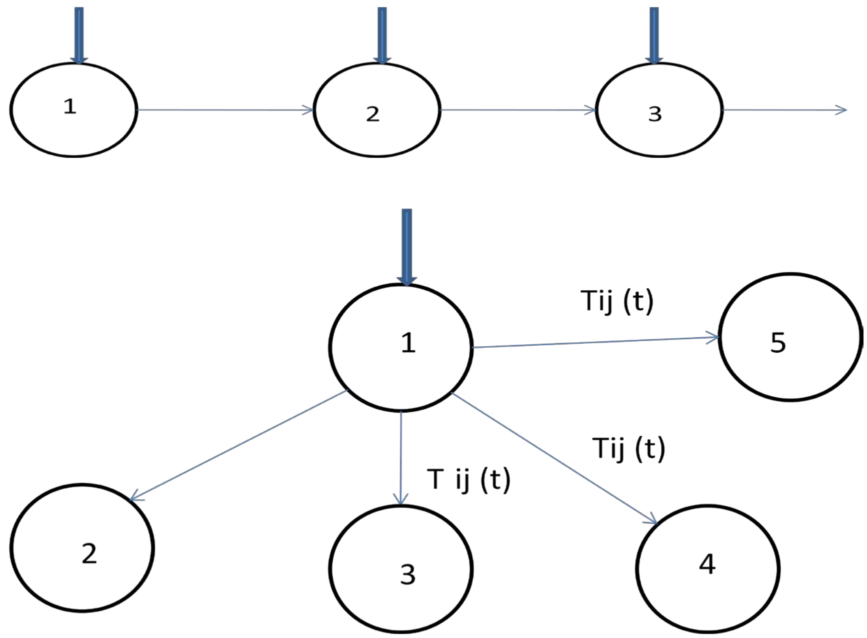


Figure 4: Serial and Star Network

3.4 Statistical Measurements of Contagion Impact

Measurements illustrating the increased defaults and risk of default, due to contagion and the network connections which allows it to spread, are necessary to quantify the impact. The following describes these measurements which are calculated per firm per run of the simulation. The means of these values for all runs of the simulation are then calculated for use in the analysis. Measures are separated by firm and portfolio.

Definition of Terms:

$FV_i(t)$: Firm Value

$D_i(t)$: Debt of Firm

$DR_i(t)$: Default rate

$SV_i(t)$: Semi Variance

$COV_i(t)$: Coefficient of Variation

ρ_{ij} : Correlation coefficient

PR_{cov} : Portfolio Risk measure based on Coefficient of Variation

w_i : Investment weight on Firm i Bond

ρ : Correlation Coefficient

Firm Value Measurements

The goal of the firm value measurements is to show the impact of a contagion due to network on various firm measurements. Values for each measurement are calculated for both the impact due to an exogenous contagion assuming no network structure and with a network structure. The percent increase due to the network is then calculated as shown by the following equation. $VC_i(t)$ is the variable value due to contagion with a network, $VNC_i(t)$ is the variable value due to contagion no network, $PV_i(t)$ is the percent change in the variable.

$$PV_i(t) = \frac{VC_i(t) - VNC_i(t)}{VNC_i(t)}, \tag{8}$$

Percent change is calculated for the following variables:

1. Firm value.

2. Defaults.

   The percentage of firms where: Firm Value - Simulated Debt Level <= 0

3. Variables representing the number of firms close to default,therefore becoming increasingly risky. This is often seen as a heavy tailed distribution. The following measurements quantify this exposure.

   (a) Semi-variance.

   Variance for data to the left of the mean.

   (b) Coefficient of Variation.

   Data dispersion is an important indicator or risk This measurement is based on the square root of semi-variance which is then normalized for differences in the mean firm values.

$$COV_i(t) = \frac{\sqrt{SV_i(t)}}{MC_i(t)} \tag{9}$$

   (c) Distance to default.

   Distance to default, Firm Value/ Debt Level, is calculated for firms in the lower tailed 1, 5, 10, and 20 percent.

Determination of Optimal Portfolio

When determining weights for the bonds in a portfolio, the goal is minimization of the portfolio risk while controlling for return. The following describes the risk and return measurements used in portfolio selection.

Risk Measurement

The objective function reflecting minimization of portfolio risk is based on the Markowitz model. The coefficient of variation ($COV_i(t)$) is the proxy for a firm's standard deviation. Semi-variance is used since default risk focuses on variation of the lower tail. This is then converted to a standard deviation and combined with the mean firm value to calculate the coefficient of variation for a firm, allowing for the normalization due to the large variations in the size of firm values. The following equation represents a portfolio measure of risk based off of the coefficient of variation.

$$PR_{cov} = \Sigma\omega_i^2 COV_i^2 + \Sigma\Sigma\omega_i\omega_j COV_i COV_j \rho_{ij}, \tag{10}$$

The value used for coefficient of variation is the mean value from the simulations.

The correlation coefficient (ρ) is based on stock prices for the firms from Jan 1, 2005 to Dec 31, 2009.

Portfolio return to debt holders

Return on Asset (ROA) for a firm, as described by equation 2, will go either to the debt or the equity holders. To assure the firm is able to continue to pay off its' debt, we control for sufficient returns going to the debt holder. Beta times the rate of return of the firm's equity is used as a proxy for the part of returns going to the equity holders, since stock prices will fluctuate based on shareholders views of the returns they are getting for their investment. Therefore the following equation will be used to represent the returns going to the debt holders.

$$r_{vd}^i(t) = \beta_0 + \beta_{Baai} r_{Baa}(t) + \beta_{mi} r_m(t) + \beta_{si} r_s(t) \tag{11}$$

Then for a given portfolio a set level of return will be required.

Portfolio Level Measurements

Based on the portfolios determined by the above measurements, the following impacts on the portfolio due to contagion will be considered:

1. Shift to the efficient frontier.

   Based on the above optimization, the efficient frontier for portfolios with both no contagion and contagion will be compared. In addition, various optimal portfolios in the no contagion environment will be evaluated in the contagion environment.

2. Portfolio increased risk measures due to network structure.

   To evaluate the impact of contagion and various network structures on a portfolio, the percent change to the risk factors will be calculated for the portfolio. This will show how the network between firms results in the percent change in various measurements of the risk contained in the portfolio. These will be a weighted average of the firm value measurements described by equation 8. The risk variables considered are the portfolio weighted average default, distance to default and coefficient of variation. PPV represent the portfolio percent change in these variable.

$$PPV = \Sigma\omega_i(\frac{VC_i(t) - VNC_i(t)}{VNC_i(t)}), \tag{12}$$

4 Calibration, Data Sources, and Simulation

Calibration of the model is critical for the results to be useful. Historical data is used to calibrate the non-contagion piece of the model and a meta model to represent the network structure between firms which allows an exogenous contagion to spread to connected firms. The following section describes this calibration process, lists the sources of the data, and describes the simulation of the model. The topics are separated by non contagion and contagion factors.

4.1 Calibration: Non Contagion Factors

Models containing macro and micro-factors, are calibrated using historical data. The following types of data are used to represent the factors in the firm value model.

- Interest Rate: Baa bonds
- Macro Economic: Nasdaq
- Sector: Nasdaq Biotech
- Firm: Stock Price.

Mean Reversion Equations

Mean reversion equations are used to simulate the future values of factors that will impact the rates of return for each company. A process similar to that outlined by Smith(2010) for estimation and simulation of Mean-Reverting Ornstein-Uhlenbeck Processes is used. Four years of data (Jan 1, 2006 until Dec 31, 2009), taken at 10 day intervals, to represent bi-monthly intervals, is used to calibrate the mean, standard deviation and revision rate for the factors affecting the return on assets of the firms. MLE's are used to arrive at these values. Results are then used to simulate one year of bimonthly values for each of the factors.

See Appendix B for the results of the calibration

Return on Assets Equation

Quarterly data (Jan 2006 - Dec 2009) is collected for the Return on Assets of each firm and for factor that impacts the ROA. Using Matlab's Regression function with ROA as the dependent variable provides the betas for equation 2 for each firm.

See Appendix C for the results of the calibration

4.2 Calibration: Contagion Factors

Use of historical data to calibrate the non contagion pieces of the model is straightforward. Calibrating the contagion piece of a model, however, presents unique issues. It is difficult to draw out the impact of a contagion event in order to calibrate the model. Due to this issue, researchers have used stylized structures to simulate the spread of a contagion. The disadvantage of this approach is the inability to relate to actual firm events and relationships. As illustrated by the contagion model of Egloff, Leippold and Vanini (2007), firm specific knowledge is a better alternative for calibration. Empirical data from historical incidences of a contagion flowing between firms is analyzed. Abnormal returns will be evaluated to show the presences of a contagion and to help determine sizes of the amplitude of the impact, travel time and decay rate.

4.2.1 Exogenous Contagion Arrival Rate

The arrival of the exogenous contagion is represented as a poisson process, time between arrivals of an exogenous contagion is exponential with mu equal to 1/(arrival rate per time period). The model assumes on average one arrival per year. The time step, delta t, will vary depending on the number of periods selected.

4.2.2 Exogenous Contagion Amplitude and Propagation

January 11, 2011 ClickSoftware announced their failure to make revenue expectations, resulting in the company stock dropping 15.2 percent that day. Two reasons were sighted in the press release. One related to company hiring, which can be expected and understood. The other, an exogenous event originating at another firm, was the bankruptcy of one of ClickSoftware's customers. An event, starting at one firm and then spreading to another firm can be viewed as a contagious event difficult to predict and often with severe effects. Consideration of historical events helps obtain a better understanding of the impact of an exogenous contagion and its spread to connected firms. Change in a firm's stock price will be used as an indicator for the impact on firm value. Percent drop in the stock price of the firm, where an exogenous event occurs, represents the amplitude. Drop in stock price for the connected firms is used to calibrate the amount of contagion that has propagated.

When studying the impact of a firm-specific event, abnormal returns are used to help quantify the share value impact. Brown and Wagner (1985) examined the use of daily stock returns for event studies and found results well specified under a variety of conditions. Regression is used to predict expected returns for a day and then these are compared to the actual returns. Focus is on the exogenous event that starts the contagion, the time elapsed and the size of the impact as it spreads to other firms. Combining results with those of Chaney and Philipich (2002) and Hertzel, etc (2008) this paper quantifies the size of the contagion as it spreads through the network.

Firms Linked by use of the same Auditor

Chaney and Philipich (2002) calculated abnormal returns to evaluate the impact of Arthur Andersen's audit failure at Enron on other clients of the auditors. They showed that, three days following the admission by Arthur Anderson to shredding documents, their other clients experienced significant negative market reaction. Both Waste Management and Peregrine had particularly large drops within one month of the event. Chaney and Philipich (2002) found clients of the Houston office, which was most directly linked to the Enron incident experienced abnormal returns of -3.96 % three days after the incident. The abnormal returns for all Andersen clients on this date was -1.63%.

In addition to this one day impact, a number of the former clients of Arthur Anderson, such as World-Com, Inc. and Peregrine followed Enron into bankruptcy court within a year. Before these events occurred it would have been difficult to imagine these very different firms being linked together by the bankruptcy of their auditor. To consider the impact on individual firms we will look at the abnormal return of two firms following the admission by Arthur Anderson to shredding documents on January 10, 2002, followed on January 11th with Enron being de-listed. This is the period represented in figure 5. Table 4-1 reflects the abnormal returns for theindividual firms:

Table 4-1: Contagion due to Auditor: Incident 1

Firm	Days after Incident	Abnormal Return
World Com	14	-45.5%
Waste Management (WM)	14	-12.4%

Contagion continued to spread through these connected firms even after Enron was gone. Approximately six months later, World Com was trading under one dollar and close to being de-listed and the abnormal return for two connected firms, presented in Table 4-2, take further drops. The figure 6 represents this period.

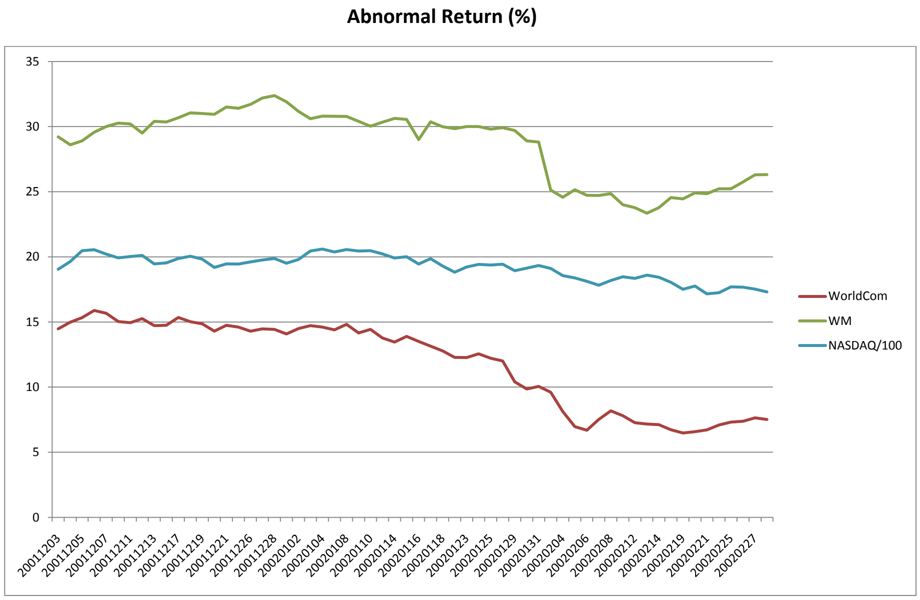


Figure 5: WorldCom and Waste Management

Table 4-2: Contagion due to Auditor: Incident 2		
Firm	Days after Incident	Abnormal Return (approximation)
Peregrine (PPHM)	Approximately 1 month	-43%
Waste Management (WM)	Approximately 1 month	-20%

In both of the above cases it is hard to quantify the size of the initial impact since the events were set off by the delisting of a firm, leading to almost total loss in stock value. The average abnormal return for these four firms connected to Arthur Andersen is 30.23% .

Firms Linked by Supply Chain Relationships

Hertzel, etc (2008) also looked at abnormal returns and applied them to supply chain customers linked to distressed firms. They focused on the filing day and a distress day, defined as the pre-bankruptcy date where a firm experienced the largest loss of shareholder wealth, with the majority having larger abnormal returns on distress day than on filing day. For the firms declaring bankruptcy, distress day abnormal returns average -26%. Abnormal returns of 1-3% over a five day period centered on a distress day were reported for the firms linked through the supply chain.

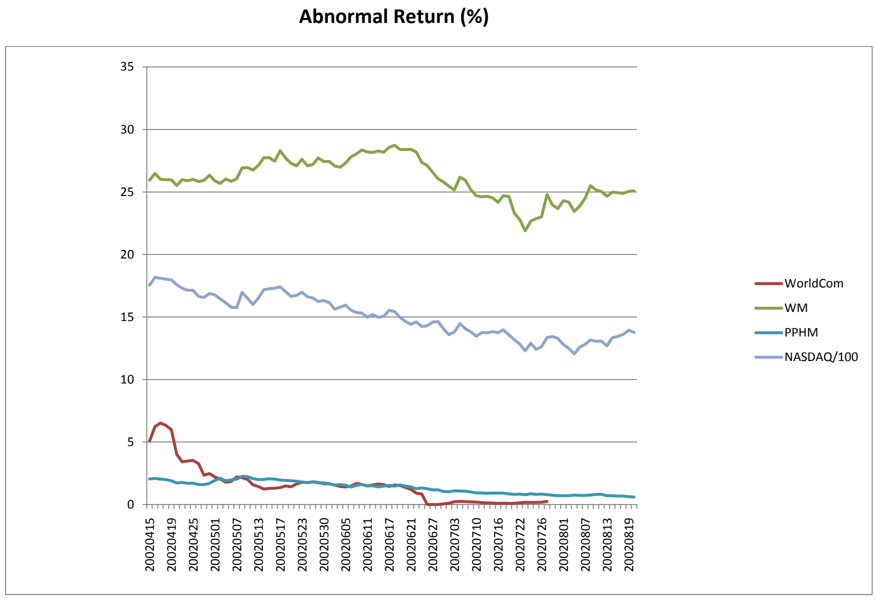


Figure 6: WorldCom, Waste Management, and Peregrine

The U.S. auto industry in 2008-2009 shows how the spread of a contagion can be particularly debilitating when the group of firms affected is already fragile. In 2009, twenty seven automotive suppliers filed for Chapter 11 bankruptcy, with estimated trade credit recoveries of less the 2 percent predominating. While the economy played a part in this, the connections between these firms increased the severity of the situation and allowed the contagion to spread. In a survey reported in November 2008, 12 percent of executives of industry suppliers said they would likely or definitely close if General Motors declared bankruptcy.

The figure 7 shows the impact of these events on the stock prices and the percent change in the stock price. Reinforcing how these firm values moved together.

Next consider the correlations between these firms. The movement of these firms values together dramatically increased in 2008 to mid 2009 versus the year earlier. This indicates that forces outside of the normal macro and micro economic effects were connecting these firms values together and causing increased correlations as shown in figure 8.

Firms Linked by a Joint Venture

Firms connected through joint ventures can have a particularly strong impact on each other when an issue arises in the product that forms the basis of their joint venture. As illustrated earlier, when an

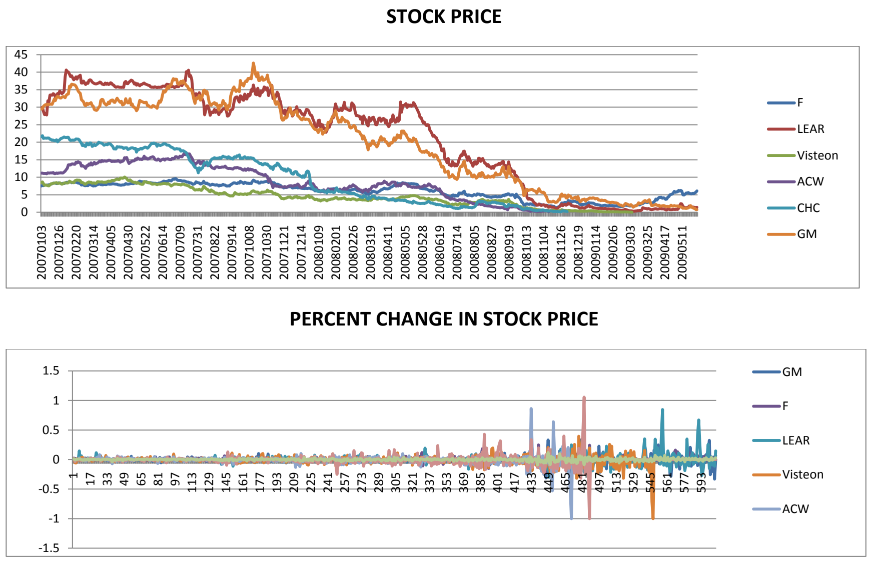


Figure 7: Supply Chain Relations

unexpected FDA announcement effected the product that linked AMLN, ALKS and LLY all the firms were impacted. On the day of the announcement AMLN experienced and abnormal return of -44.34%. Three business days later, ALKS had its highest abnormal return of -12.884 %. Due to the size of Lilly it wasn't impacted as much or as quickly, four days after the incident it had its largest abnormal return of -2.049%.

Summary

The table 4-3 compares the size of the contagion and it spread based on the type of connection.

Table 4-3: Averages by type of connection

Type of Connection	Size of Initial Impact	Speed of Transmission	Impact after decay
Audit Firm: Average		3 Days	-1.63-3.96%
Audit Firm: Individual	-75% est	14 Days - 1 month	-12.4-45.5%
Supply Chain: Average	-26%	5 Days	-1-3%
Joint Venture	-44.34 %	3-4 Days	-2-13%

Based on averages from Table 4-3, the values for decay rate and associated values for τ and λ are determined. These values will be used to help estimate the strength of connections between firms based

CORRELATIONS

2007	GM	F	LEAR	Visteon	ACW	CHC
GM	1					
F	0.825633	1				
LEAR	0.431418	0.610756	1			
Visteon	0.145345	0.425107	0.770073	1		
ACW	0.3421	0.599643	0.652504	0.736679	1	
CHC	0.17992	0.403821	0.618491	0.852404	0.608266	1
NASDAQ/100	0.436568	0.221437	-0.18232	-0.58323	-0.28057	-0.60598

2008 - June 2009

	GM	F	LEAR	Visteon	ACW	CHC
GM	1					
F	0.793016	1				
LEAR	0.974741	0.855075	1			
Visteon	0.902428	0.952972	0.956887	1		
ACW	0.866245	0.872486	0.930773	0.806936	1	
CHC	0.920991	0.679071	0.823286	0.744743	0.677623	1
NASDAQ/100	0.854019	0.860346	0.908287	0.959636	0.69394	0.642259

Figure 8: Correlation Comparisons

on the type of connections that exist.

Table 4-4: τ time λ by type of connection

Type of Connection	Size of Initial Impact	Impact after decay	Decay Percent	τ time λ
Audit Firm: Individual	-75 %	-28.95%	38.6%	.95
Supply Chain: Average	-26%	-2.0%	8%	2.56
Joint Venture	-44.34 %	-7.5%	17%	1.77

4.2.3 Network

Firm specific information is used to determine if connections between firms exist. This involves collecting information about the individual firms such as joint ventures involvement, distribution channel, supply chain partners, directors, banking relationships, investors and institutional ownership. When there are matches between firms on the above items, a connection is assumed. A complete list and description of possible connections is given in appendix A.

4.3 Data Sources

The following lists the sources for the data collected.

Non Contagion Factors

Data sources used for calibration and time intervals available for the data are given in Table 4-5.

Table 4-5: Data Sources Non Contagion Factors

Factor	Variable	Source	Equation	delta time
Macro Economic	NASDAQ Index	NASDAQ	4	Daily
Interest Rate	Baa Bonds	Compustat	3	Daily
Sector Return	NASDAQ sector	NASDAQ	5	Daily
Firm	ROE - stock Price	Compustat	6	Daily

Contagion Factors

Firm specific data was collected from the following sources: Compustat, 10-K and 10-Q reports and the Nasdaq website.

4.4 Simulation

The following section describes the process of simulating the firm value. All of the modules are written in Matlab.

Return on Asset

Future firm values are determined by simulating changes to return on assets for individual firms. Simulations are performed for each of the factors that impact the firm value using the mean reverting equations. For Interest rates, a yearly rate of return is calculated, it is then converted to a quarterly value. For all other factors, a new rolling quarterly rate of return is calculated for each 10 day period. These rates of returns are used with the betas from the calibration section to arrive at a simulated firm return on asset which is divided by 6 to convert to a bi-monthly rate. Based on this, a new firm value is calculated.

Default Boundary-Debt Level

As described in Leland(02) various methods can be used to determine the default boundary in a structural model. Similar to the Longstaff-Schwartz Model, the value of debt of the firm will be used. Then to project future debt levels, regression is performed using yearly debt data for 2000-2009 as the dependent variable with time as the independent variable.

Contagion Factors

Exogenous contagions are simulated, then based on the network structure determined by the firm data, the impact of contagion for each bond in the portfolio is determined. It is then combined with the firm value simulation.

4.4.1 Stylized Characteristics of Contagion Propagation in Serial Network

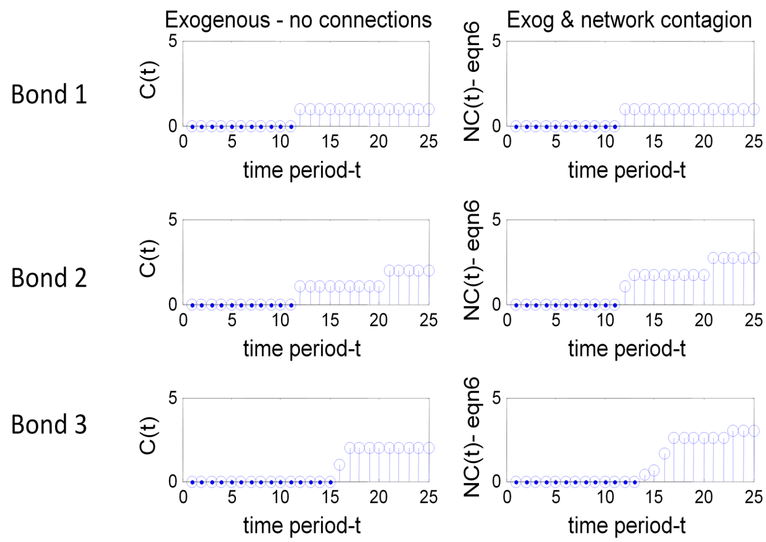


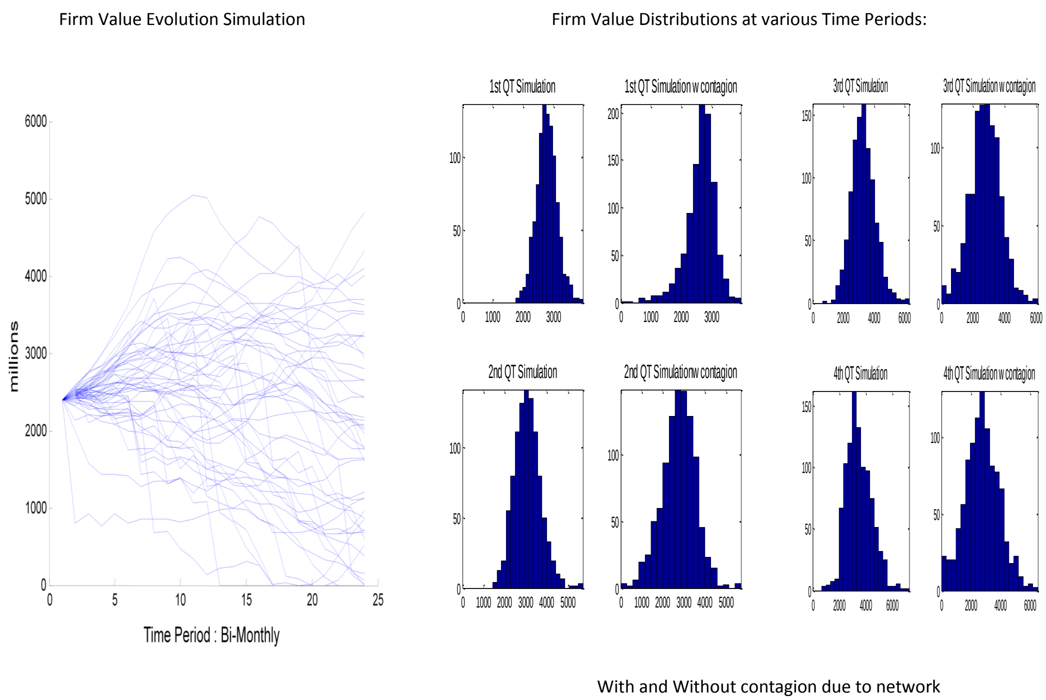
Figure 9: Contagion Impacting Three Bonds

Figure 9 represents an exogenous contagion impacting three firms and then spreading to other firms through their connections. The charts on the left show the exogenous contagion that impacts each firm. The right side charts show the exogenous contagion plus any contagion that has spread to a firm due to the network connections. As an example, an external problem impacts firm one. Once investors are aware of the problem, they re-evaluate the prospect of other firms due to their belief that the firms are similar and could have the same problem. The contagion spreads to firm two in one time period and then from two to three in two time periods.

4.4.2 Firm Value with Contagion: Simulation Results

Output for 50 firm value evolution simulations, for one firm, is presented in the following graphs. The first set shows how the firm value evolves over time due to the changes in the various factors and contagion.

Next is the distributions of a firm's value by quarter. On the left are the charts showing the values with out the contagion and on the right showing those with firm values impacted by contagion spread due to the network connections with the resulting heavy tailed distribution.



5 Results:Impact of Contagion on Firm Value and Portfolio Selection

Understanding default and lower tail risk is critical when setting up a portfolio of bonds. Contagion will impact firm value, but does the diversification that results from creating a portfolio of bonds help limit the impact? A portfolio based on actual firms is analyzed to see the importance of considering contagion when creating a portfolio. Of the variables that describe a contagion are certain more critical? Simulation are run, varying these contagion variables, to gauge their impact on defaults and lower tail risk at a firm and portfolio level.

5.1 Three Bond Portfolio Simulation

Three biotechnology firms are selected to form a portfolio with a sector-specific investment strategy, based on investor belief that this sector will out perform the market. Does this choice make the investment more susceptible to a contagion and can the risk be quantified? The firms selected are: Celegene (CELG), Cephalon (CEPH), and Akorn (AKRX).

Researching these firms revealed various connections, which are listed in Table 5-1. Two firms are connected by the same investor and two firms are connected by the use of the same distributor. Based on the idea that an investor can usually impact a firm more severely than a distributor due to funding sources being critical to a firms survival, λ is chosen to decay slower when firms are connected by the same investor. On the other hand distributor are more likely to be involved in day to day operations, therefore the travel time of information is shorter. Since an investment firm and an audit firm can have similar impacts, τ time λ in the institutional investor case is set at 1.0. This compares to a value of .95 from the Arthur Andersen case of connections due to the same audit firm. For the same distributor, τ time λ is set to a value of .7. This has a slower decay rate due to the sever impacts we saw among the automotive suppliers in the calibration section.

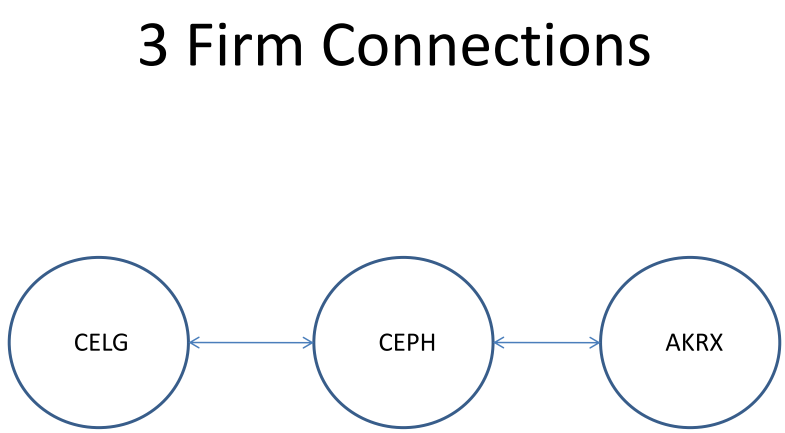


Figure 10: Network

Table 5-1: Contagion Variables Defined			
	CELG	CEPH	AKRX
CELG		Same Inst Inv: WAM τ = 2, λ =.5	
CEPH	Same Inst Inv: WAM tau = 2, λ =.5		Same Dist τ = 1, λ =.7
AKRX		Same Dist τ = 1, λ =.7	

5.1.1 Firm Results

The measurement from the simulations of the three bond portfolio, contained in Table 5-2, include changes to firm value, defaults, distance to default, and coefficient of variation. Since we are interested in the impact of the network structure, values show the percent change that occurs due to the firm connections existing. In other words, it compares what the values would be from the exogenous impact to those that would occur due to this exogenous impact plus the spread due to the network structure.

Table 5-2: Average percent changes due to Network				
Results	Mean	Coefficient of Variation	Default	Distance to Default (10 percent)
CELG	-0.08793	0.07765	0.57055	-0.06587
CEPH	-0.22935	0.31224	3.30603	-0.08299
AKRX	-0.06715	0.05207	0.8	-0.01198

A network structure negatively impacts the firm value not only by increasing defaults, but by bringing the firm closer to the possibility of default. Defaults increased for all firms in the portfolio due to the spread of the contagion. In addition, when the contagion didn't cause a default, it negatively affected the firms through drops in the mean firm values and in distance to defaults. The drop in firm value shows that the spread of the contagion is having a negative impact and distance to default indicates that these events are moving the firms closer to a default position. The coefficient of variation increased showing that variability, also an indicator for risk, increased. Illustrating how actual firm data and a network structure can create the increased default correlations and heavy tails seen in empirical data.

These results also show how the number and size of the connections impact the firms. CEPH is the most negatively impacted of the three firms in all four categories. This is due to the connections it has to both of the other firms. Indicating that not all firms will be impacted to the same extent by contagion, with some being susceptible to a more severe negative impact. Unless one is aware of these connections, a firm risk can be higher than anticipated.

5.1.2 Optimal Portfolio Results

Having seen how individual firms are at increased risk due to contagion, the impact on the optimal portfolio will be considered. Based on simulation results, the efficient frontier is plotted for both portfolios, with and without contagion, using the measures of risk and return as described by equations 10 and 11 in section 3.4. Figure 11 illustrates the impact on the efficient frontier from contagion. The curve shifts down and to the right, showing and increase in risk and decreased return. In addition, points are plotted that represent portfolios that were optimal in the no contagion optimization in the contagion environment. The majority of these portfolio are sub optimal when contagion is considered. The portfolios that are optimal in both cases consist of only the one bond with the highest return. This would be an unlikely investment choice since there is no diversification and also, unlike equities, with a bond higher return on debt is of less value since it won't increase the return on a bond. The risk measures represent the dispersion of risk for the portfolio of firms. For a high yield bond portfolio's risk is more of an issue since controlling the amount of variation in the firm value is increasingly important to reduce the risk of default. Since these risk measures are normalized and diversified over various firms, we would want lower values.

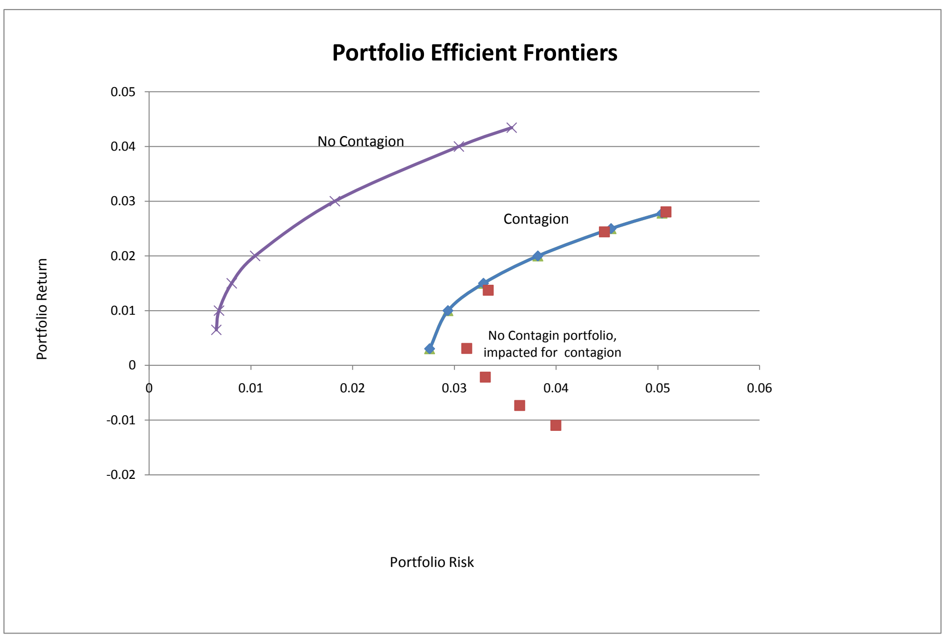


Figure 11: Efficient Frontier Three Bond Portfolio

The presence of a contagion shifted the make up of the portfolios dramatically. When controlling for a return of .01, in the no contagion portfolio CEPH was weighted .796. In the contagion portfolio, CEPH's weight dropped to .280. This was the firm most negatively impacted by the contagion in the firm value section. Showing the increased risk to this firm due to the contagion's impact.

The table 5-3 shows the difference in the weights for a portfolio at a return of .01, depending on whether or not contagion is considered.

Table 5-3: Portfolio Weights

Portfolio Return .01	CELG	CEPH	AKRX	Portfolio Variance
No Contagion Portfolio	0.039314387	0.796335304	0.164349308	0.006863
Contagion Portfolio	0.160092375	0.280605514	0.559303112	0.02937

It is not possible to create a portfolio which avoids exogenous contagions or connections to other firms. Therefore a better understanding of the impact of the contagion variables is required to assist in the portfolio selection process. In the following section the model is applied to stylized firm data with focus placed on how changes in these variables impact the firm and portfolio value.

5.2 Study of contagion variables

To truly understand portfolio risk, an improved understanding of what characteristics make a portfolio more susceptible to contagion is critical. Simulations are run modifying the contagion variables. One set of firm data is used for the mean reverting part of the model, allowing for control of the firm specific factors and placing focus on the impact of the network variables. The variables analyzed are: amplitude of exogenous contagion, decay rate times travel time, number of firms connected, and shape of network connections.

The number of bonds in the portfolio are varied to study the contagion impact as the size of portfolio is increased. Various network structures are considered, representing the possible ways bonds can be connected to each other with in the portfolio. All of the connections are one directional. In the serial network, bond A connects to B, B to C, etc. continuing for the total number of connections. As an example, firms 1 and 2 could be linked by a joint venture, as in the earlier Amylin example. Firm 2 could then be linked to another firm through the use of the same lending institution. When firm 2 experiences problems, the lender could see this as a sign of larger problems and decreases credit to firm 3, spreading the impact to this firm. The loop connection is the same as the serial except that the first and last nodes are also connected. The star connection starts with one node and all other bonds are connected to this central node, as seem in the earlier example of Arthur Andersen. The auditor being the central node and the firms they audited being the spokes.

5.2.1 Three and Five Bond Portfolio

Focusing on three and five bond portfolios, the variables considered are network connections of serial, star or loop, an amplitude of .1 or .3., and tau time lambda set to .25 or .5. Values for the average only consider those firms to which contagions can spread to. All the outputs are in terms of percent changes to the values due to the network structure. They are the comparisons to the what the values would be with only the exogenous impact and those that occur with the exogenous impact plus the network structure.

Table 5-4 contains results for 3 bond simulations. Tables 5-5 and 5-6 focus on the impact of the amplitude and decay rate (tau * lambda) for various 5 bond networks.

The type of structure does influence the impact of contagion as seen in Table 5-4 a firm in a serial network is more negatively impacted by a contagion than one that is in a star network. This is expected since in a serial network a contagion has a greater likelihood of impacting a firm that has another firm to spread the contagion to. With a star network, the impact on all firms would require a exogenous contagion hitting the main firm. This would have a extreme impact, but the chances of this happening are rare. This increased negative impact on a serial network is of increased concern in portfolio selection since in the business environment serial connections will are harder to recognize and more likely to go undetected. The network with the greatest negative impact, however, is the loop. Having a closed network, when a contagion hits any firm in the network it will then dissipate within the structure, increasing the impact to the total portfolio.

Table 5-4: Three bond Portfolio

Average percent changes due to network

Results 3 bonds	Serial	Serial	Star	Star	Loop	Loop
Amplitude	0.1	0.3	0.1	0.3	0.1	0.3
Tau*Lambda	.25	.25	.25	.25	.25	.25
Mean 4QT	-0.14736	-0.36383	-0.10827	-0.25603	-0.34371	-0.60845
Coef of Variation 4QT	0.16856	-0.15484	0.10215	-0.08666	0.42928	-0.33319
Increase in Default percent	4.27989	0.88861	1.07895	0.52921	10.33406	2.33672
Decrease in DtoD 10 percent	-0.05535	-0.19907	-0.03917	-0.13378	-0.13922	-0.36934

In tables 5-4, 5-5, and 5-6, the results of the coefficient of variation are misleading at times. As the mean and number of firms defaulting increases, it appears that the data is less dispersed. This has more to do with the mean decreasing, and therefore the range also decreasing, than the values being less dispersed.

Tables 5-4 and 5-5 reflect the initially puzzling results that as the amplitude increases the percentage change in default decreases. With a higher amplitude, however, more failures are a result of the exogenous contagion itself, therefore seeing a smaller percent increase. When the amplitude is lower the initial defaults will be lower from the less severe exogenous contagion, therefore as it spreads the percentage increase in defaults will be larger. Comparing tables 5-5 and 5-6, we see that as τ time λ increases, which results in a faster decay of the contagion, the increase in default is smaller, but the amplitude impact is more inline with expectations. A possible explanation is that the contagion is dissipating more quickly and therefore the size of the initial impact actually has a more expected impact. These results support the conclusion that information about networks between firms can be more important than focusing on the size of a possible future event when one wants to have a complete understanding of risk.

As expected, results in all three tables show that the higher the amplitude of the exogenous contagion, the greater the decrease in the distance to default. When considering tables 5-5 and 5-6, the impact of a slower decay rate has a greater impact decreasing distance to default on the serial network than on the star network. With a decay rate of 28 percent faster, an amplitude of .1, distance to default dropped by almost 60 percent for a serial connection, versus 35 percent for a star connections. The same type of results can be seen for the amplitude of .3. This indicates again that the serial connection is more susceptible to the impact of a contagion than the star connection.

Table 5-5: 5 Bond Portfolio

Average percent changes due to network

Results 5 bonds	Serial	Serial	Star	Star
Amplitude	0.1	0.3	0.1	0.3
Tau*Lambda	.25	.25	.25	.25
Mean 4QT	-0.21345	-0.48305	-0.11423	-0.2863
Coef of Variation 4QT	0.22104	-0.17227	0.15255	0.08458
Increase in Default percent	1.722	1.45212	1.15581	0.807776
Decrease in DtoD 10 percent	-0.07989	-0.27112	-0.04248	-0.16032

Table 5-6: 5 Bond Portfolio

Average percent changes due to network

Results 5 bonds	Serial	Serial	Star	Star
Amplitude	0.1	0.3	0.1	0.3
Tau*Lambda	.5	.5	.5	.5
Mean 4QT	-0.14787	-0.37041	-0.08962	-0.26052
Coef of Variation 4QT	0.12405	-0.08098	0.09574	0.06175
Increase in Default percent	0.75047	0.76884	0.47675	0.62092
Decrease in DtoD 10 percent	-0.05001	-0.17974	-0.03138	-0.12943

At the portfolio level of particular concern is the observation that the size of the network not only increases risk due to greater number of firms being impacted but also the risk to the individual firms, and therefore the overall portfolio can be impacted twice. When comparing the results in tables 5-3 and 5-4, the averages for the 3 bond versus 5 bond networks for the mean and distance to default, the average values show increased risk for the larger networks.

5.2.2 Ten Bond Portfolio: Complex Network Structures

Since a portfolio of bonds is at risk for containing various structures, the following are results for a network of 10 firms with a combination of structures, including star, loop and serial. In tables 5-7 we compare two similar structures, they both have a star network of 5 firms. The difference, in the first results the remaining 5 firms connected serially off one of the spokes of the star. In the second result these firms form a loop of 4 firms that are connected to one of the spokes of the star through another firms. All the outputs are in terms of percent changes to the values due to the network structure, comparing what the values would be with only the exogenous impact and those that occur with the exogenous impact plus the network structure.

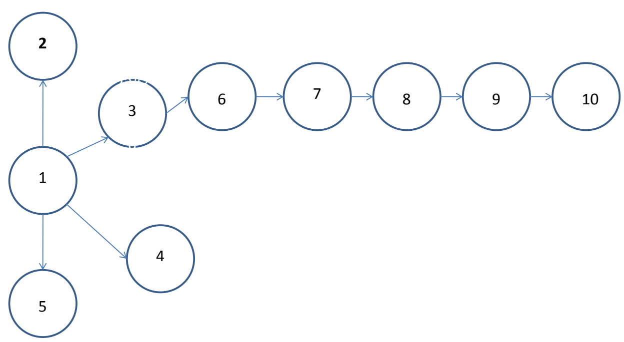


Figure 12: Star and Serial Network

Star and Loop Network

The first two sets of results in Table 5-7, show the negative impact of changing the part of the structure from serial to a loop connection. This is adding only one additional connection but results in more defaults and greater lower tail risk. To further understand the difference, in Table 8 the firm averages are separated by type of connection. When looking at the output for the individual firms, those in the star formation have the smallest negative impact and are very similar regardless of how the remaining firms are connected.

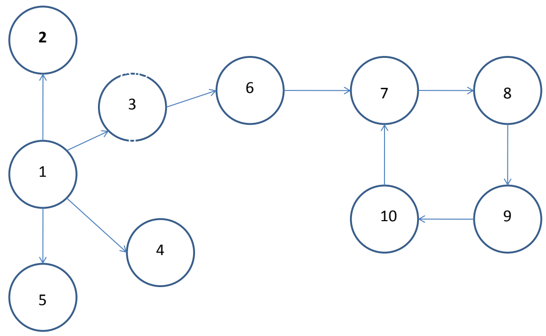


Figure 13: Star and Loop Network

Firms in the serial connection are more negatively impacted than those in the star formation, however as seen in our earlier results, those in the loop structure are the most severely impacted.

Table 5-7: 10 Bond Portfolio				
Results 10 bonds	Star and Serial	Star and Loop	Star and Loop	Star and Loop
Amplitude	0.1	0.1	0.3	0.1
Tau*Lambda	.25	.25	.25	.5
Mean 4QT	-0.20336	-0.3038	-0.55308	-0.18425
Coef of Variation 4QT	0.24489	0.27679	-0.32306	0.17062
Increase in Default percent	2.79096	7.3327	2.1182	1.5596
Decrease in DtoD 10 percent	-0.07519	-0.12679	-0.34906	-0.06292

Table 5-8: Average by Connection Type			
Results 10 bonds	Average Star Firm	Average Serial Firm	Average Loop Firm
Mean 4QT	-0.11004	-0.24785	-0.48857
Coef of Variation 4QT	0.14557	0.29077	0.38842
Increase in Default percent	1.21442	3.71156	14.66025
Decrease in DtoD 10 percent	-0.04135	-0.09105	-0.21107

5.3 Portfolio Results

Having seen the impact on firm values due to changes in the contagion variables, we consider the impact to the efficient frontier of a portfolio due to various network structures. Based on the optimization described in section 3.4, a portfolio is created from a selection of five firms. Simulations were performed connecting these firm with various network structures: star, serial, and loop. Figures 14 compare the impact on the efficient frontiers, with and without considering contagion for each network structure. In addition, points are plotted that represent the weights of optimal portfolios from the no contagion optimization in the contagion environment. The majority of these portfolio are sub optimal when contagion is considered. The only portfolio that remain optimal are the ones at the high risk return end of the efficient frontier. These points however represent a portfolio of one bond, the one with the highest return from the five firms, which is an unlikely investment strategy.

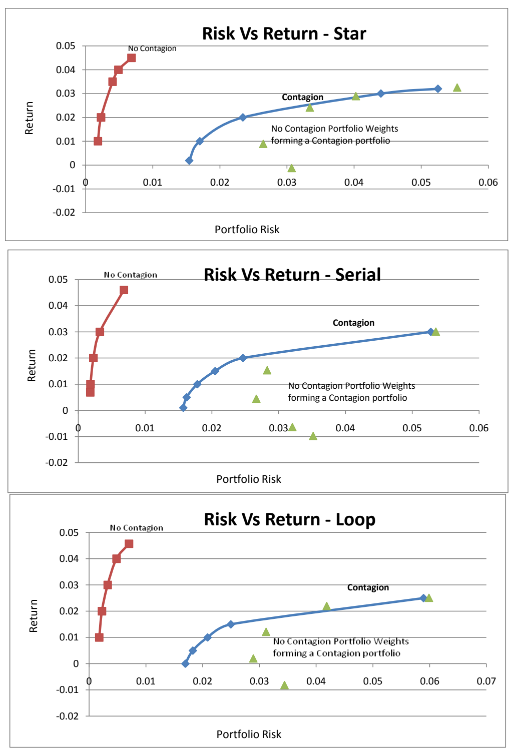


Figure 14: Efficient Frontier - Star, Serial, and Loop Networks

These charts show that without contagion the simulated efficient frontiers are the same. The slope is much steeper for the no contagion curves versus those including contagion, showing that with contagion the same increase in return will result in a much greater increase in risk. All the contagion efficient frontiers

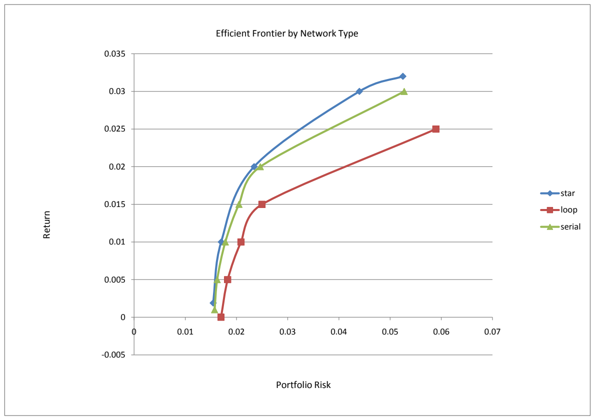


Figure 15: Efficient Frontier

shift to the right and down compared to the no contagion curves.

As illustrated in Figure 15, the structure of the firm connections also impacts the resulting efficient frontier. Similar to the analysis of the firm value measurements, when considering a portfolio the structure with the most negative impact is the loop, followed by the serial and last the star. Therefore a group of the same firms hit by the same contagion will be more severely impacted if they are connected in a loop structure than if they are connected in a star structure. This is illustrated in table 5-9 which compares risk values versus return for the various structures. At the rate of return of .03 it was no longer possible to generate a feasible solution for the loop formation. In addition, the higher the required level of return, the larger the separation of risk based on the connection type.

Table 5-9: 5 Firm Portfolio Risk and Return by Connection Type			
Return	Star: Risk	Serial: Risk	Loop: Risk
.01	0.017014	0.017811	0.020879
.02	0.023432	0.024646	0.033917
.03	0.043998	0.052766	not feasible

Next we consider the changes to the weights of the firms that make up the various portfolios. Table 5-10 presents the portfolio weights for a return of .01 based on a no contagion portfolio and then contagion portfolios for the following network structures: Star, Loop, and Serial.

Each of the network contagion (NWC)portfolio's are more diversified than the no contagion portfolio. This may be due to the risk of all firms increasing. Whether this is transferable to a larger portfolio is difficult to extrapolate.

We will now consider the changes to individual firm weights for the various portfolios. Firm 1's percent increases in all NWC portfolios. Specifically in the serial and star connection this is due to the one directional connections used in the simulation. In these networks firm 1 transmits contagion to other firms, but doesn't receive from other firms. Its' risk then decreases relative to the other firms in the portfolio when a contagion is considered, making it more attractive to include in the portfolio.

Firm 2 and 4 are the major components of the no contagion portfolio. The interesting result is that in the NWC portfolios, firm 2's percent increases and firm 4's decreases substantially. Both firms are low in risk compared to the other firms in the no contagion simulations. Firm 4's distance to default was lower than firm 2's which appears to be a reflection on the health of the firm. Even though at first glance they both seemed relatively low risk, firm 2 was actually in better health and this came out when contagion was added to the simulation. Since firm 4 had a lower distance to default, it would be more susceptible to contagion risk and therefore its' percentages dropped.

Firm 5, the only firm negatively correlated with all other firms, increases to approximately 30% for all NWC portfolios. The loop portfolio, the greatest risk portfolio based on the firm value measurements, is the portfolio with the greatest percentage of firm 5. As the firms are increasing in risk, the diversification through a negatively correlated firm appears to have increased benefits

Table 5-10: 5 Firm Portfolio Weights: By network connections

.01 Return	Firm 1	Firm 2	Firm 3	Firm 4	Firm 5
Aver No Contagion	0	0.303273386	0	0.577951375	0.118775239
Serial Connection	0.124506075	0.460891695	0.033812498	0.084477253	0.296312479
Loop Connection	0.082172699	0.522922973	0	0.068422349	0.326481979
Star Connection	0.11230063	0.429114261	0.047859413	0.113507557	0.297218138

In addition to the increased risk for a given level of return shown by the shift in the efficient frontier, other risk measures of the portfolios also increase. Portfolio weights from the no contagion optimal portfolio with a return of .02 are applied to the percent increase in risk measurements due to the contagion spread through network structures as described by equation 12 in section 3.4. These results are in Table 5-11. This shows by network type how the risk measurements increased when a network structure is introduced to these portfolios. Again we also see that the amount of increase depends on the type of structure and that the loop structure is the most increased risk.

Table 5-11: 5 Firm Portfolio Risk Measurements: percent increase due to contagion network			
Measurement	Star	Serial	Loop
Distance to Default (.1)	-0.079408871	-0.116919634	-0.158484713
Coefficient of Variation	0.025252372	0.026103274	0.043758445

6 Concluding Remarks

Much research has been done to improve understanding of the causes of increased clustering in defaults which is of particular concern in high yield bond portfolio selection. Often this focuses on contagion, most models, however, lack the use of actual firm data and therefore the ability to increase the understanding of how the contagion process actually works between firms. We see that these firm connections can makes an unanticipated event worse and have sever ramifications at the portfolio level. Building on research such as Egloff, Leippold, and Vanini,(2007) we show that actual firm data can be used to show impacts on the defaults and tail behavior of firm values. We see that the connections can result in differing impacts for firms depending on the type of connections and the firm's position in the contagion flow network. This assessment can be gainfully utilized to develop a more robust investment strategy involving the debt issued by the firms.

Analysis of the variables defining the model evaluates their impacts due to types of structures, the strengths of the contagion and the strength of the connections provides insight on the how the portfolio strategy might need to shape up in order to control the tail risk of the portfolio. Firms connected in a loop structure are the most susceptible to the impacts of contagion. An even greater concern since this type of firms connection will usually be the most difficult to uncover.

When considering a portfolio, the selection of an optimal portfolio is affected when adding contagion to the model. The efficient frontiers are impacted negatively for all three structures considered and the type of connection determines the severity of the impact.

With a better understanding of the impact of a connections between firms on the spread of a contagion, the aim of future research will be to make further advancements in applications to the portfolio optimization problem. Using this information to select a portfolio of bonds that will have less exposure to the negative impacts of contagion.

7 Appendix A: Firm-Pairs Connections, Decay, and Travel Time

The following are attributes for sets of Firm-Pairs. All values will be same for both directions of the connections unless otherwise mentioned. Travel times and decay rates are determined based on characteristic of each attribute and also considering how the attributes compare to each other. The following attributes are separated by the strength of the linkages. It is set that in the case of no linkages, $\tau * \lambda$ will equal 10 which will result in no transmission. Depending on the types of Firm-Pair connections, $\tau * \lambda$ will be set according to the following table.

7.0.1 Joint Venture

A problem that impacts a firm can spread to another firm through a joint venture. This will impact the firm-pairs that participate in a joint venture.

7.0.2 Distributors:

One firm can negatively impact their distributor or a distributor can run into some unrelated issue. The distributor can spread the problem to other firms. This will Impact firm-pairs with the same distributors.

7.0.3 Suppliers:

A problem at a key supplier can impact all the firms that use this supplier. This will impact firm-pairs that use the same supplier.

7.0.4 Lending Institutions:

Problems at one firm that impact its relationship at a bank can result in the bank tightening credit for other firms it views as similar. This can lead to the credit issues with firms that use the same bank.

7.0.5 Sales Region (Geographic):

Due to geographically proximity, a contagion could spread among firms due to their location. This will impact firm-pairs located in the same region

7.0.6 Same Institutional Ownership of common stock:

Sharing an investment firm between firm-pairs can result in a contagion spreading, impacting firm-pairs that have the same investors.

7.0.7 Audit Firm:

If practices of an accounting firm are called into question at one firm, the fear of a similar problem at another firm with the same auditor can cause a contagion to spread. As in the example of Enron and their auditor Arthur Anderson. This impacts firm-pairs with the same audit firm.

7.0.8 Shared Director:

When a person is a director on multiple boards of firms in the portfolio, it is possible for them to spread information or panic between firms. This will impact firm-pairs who have a director that sits on both of their boards.

7.1 Table for Firm-Pair Tau and Lambda Values:

The following table compares the firm-pair attributes and separates each into one of three categories for both speed of decay and speed of transmission. The lower numbers represent a slower decay or a faster transmission. Either leads to more of the contagion spreading to connected firms.

7.1.1 Values:

τ : 1 (Fast) = 1 : 2 (Medium) = 2 : 3(Slow) = 3

λ: 1 (Fast) = .5 : 2 (Medium) = .7 : 3(Slow) = 1.0

Firm Pair Attribute	Speed of Decay	Speed of Transmission
Joint Venture	1	1
Distribution	2	1
Suppliers	2	1
Lending Institutions	1	2
Sales Region	3	3
Institutional Ownership of Common Stock	1	2
Audit Firms	3	3
Shared Director	3	2

8 Appendix B: Mean Reversion Calibration

The following is the results of the Mean Reversion Calibration

time length	Delta time	Factor	μ	γ	σ
4 years	10 day	Baa Interest	0.06969	1.165059522	0.011510443
4 years	10 day	LT Interest	0.022634	4.553292506	0.008333346
4 years	10 day	Sector	792.1908	3.049868092	153.4902079
4 years	10 day	NASDAQ	2219.504	0.631341561	391.3251931
4 years	10 day	spread	0.014635	0.469385556	0.006995279
25573.2	10 day	CELG	54.18427	3.62897671	24.43641446
4676.319	10 day	CEPH	67.66029	3.590322264	21.37126195
161.7981	10 day	AKN	2.945095	0.43173099	2.317565001

(Above Values are starting firm values)

9 Appendix C: ROA Calibration

The following is the results of the ROA calibration:

Firms	Beta0	Baa Interest rate	Nasdaq	Biotech	Company	rsquared
CELG	-0.10435	6.293483	0.389054	0.509935	-0.31811	0.461583
CEPH	-0.05732	3.61051	0.111774	-0.14547	0.117535	0.121228
AKRX	-0.1315	10.39525	0.448623	-0.51068	0.012447	0.499131

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