THE LIMITS OF MARKET DISCIPLINE: PROPRIETARY TRADING AND AGGREGATE RISK†

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Abstract. This paper studies the impact of external claim-holders heterogeneity on entrepreneurial risk-taking. We show that risk-taking by entrepreneurs and demand for risky securities by risk-neutral traders (e.g. fund managers or proprietary traders) are mutually reinforcing. Market discipline is the mechanism by which the adjustment of cost of capital to the level of risk feeds into managerial incentives and deters excessive risk-taking. Larger risk-neutral traders (relative to risk-averse investors) not only undermine market discipline and lead to more risk-taking, but they also benefit more from the upside of risk and become relatively larger if the project pays out, which leads to even more risk-taking in the next period. The model explains documented features of the business cycle: bubble-like asset prices (procyclical run-up in prices and procyclical underpricing of risk), shorter cycles and countercyclical leverage of the non-financial sector. We extend the model to allow risk-neutral traders to raise external financing and show that the endogenous procylical leverage of risk-neutral traders can lead to additional aggregate volatility.

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> *"We have also seen the emergence of a whole range of intermediaries, whose size and appetite for risk may expand over the cycle. Not only can these intermediaries accentuate real fluctuations, they can also leave themselves exposed to certain small probability risks that their own collective behaviour makes more likely. As a result, under some conditions, economies may be more exposed to financial-sector-induced turmoil than in the past."* (Rajan, 2006).

Ivashina and Sun (2011) document that the volume of syndicated leverage loans grew from \$310bn in 2000 to \$690bn in 2007. Similarly, in 2006, the issuance of non-agency mortgage-backed securities reached almost \$920bn, a fourfold increase from 2001 in which it was \$220bn (see Figure 1).[1] These securities turned out ex-post to be very risky and underpriced, and arguably led to an increase in systemic risk and caused the financial crisis of 2007-2009. What were the incentives of issuers of these risky projects? In this paper, we study the hypothesis that the demand for risky securities drove the increase in issuance. Ivashina and Sun (2011) show that the share of institutional investors (e.g e.g. CDO, hedge funds, mutual funds, pension funds and insurance companies) financing the syndicated leverage loans grew from 16% to 62%. At the aggregate level, Figure 2 shows that a proxy for the share of assets owned by proprietary traders increased prior to financial crises in 1987, 1994, 1998 and 2007. What role did proprietary traders play in fostering risk-taking? What is the role of market discipline in preventing such run-ups in risk-taking? Are there externalities that lead to a misallocation of capital? What is the impact of monetary or prudential policy and regulation on risk-taking?

In this paper we study the role of external claim-holders heterogeneity on entrepreneurial risk-taking incentives. Market discipline is the mechanism by which the adjustment of the cost of capital to risk feeds into managerial incentives and deters risk-taking (Flannery, 2001; Hellwig, 2006). First, we show how the risk-sensitivity of the capital (and therefore the efficiency of market discipline) is positively correlated with aggregate risk-aversion. Second,

[1] Other events: in 2000, dot-com stocks; in 1997, emerging market stocks.

Figure 1.

Global CDO. *Souces*: Sifma.

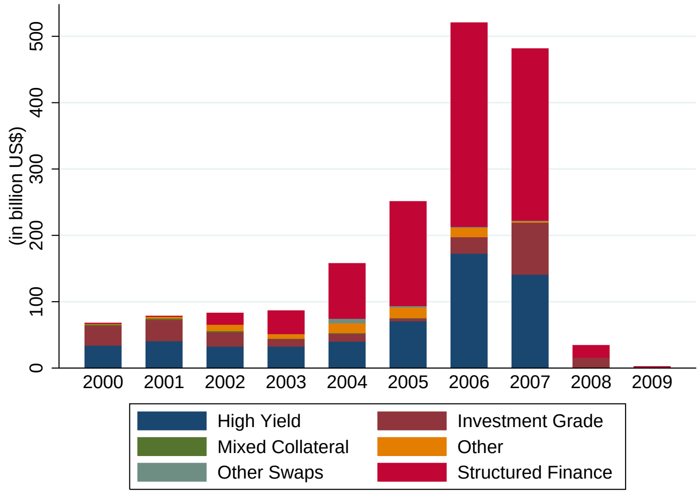


Figure 2.

Share of broker/dealer assets in total US assets. While the true proprietary trading firms (hedge funds) are merged with households statistics in the Flow of Funds data, data on broker /dealers provide an indication of the overall proprietary trading behavior (e.g. Lehman Brothers generated 58% of revenues from proprietary trading in 2006, Gandel, 2010). In the mid 1980s, the deregulation of the Savings and Loans sector allowed mortgage bonds trading which benefited broker/dealers until the S&L crisis; the Mexican crisis (1994) handed Goldman Sachs heavy losses; broker/dealers were less exposed to the internet bubble than the general public (their share of assets shrunk until the crash in 2000); in the mid 2000s, broker/dealer balance-sheets grew into real estate and credit derivatives until 2007. *Sources*: Flow of Funds (Federal Reserve).

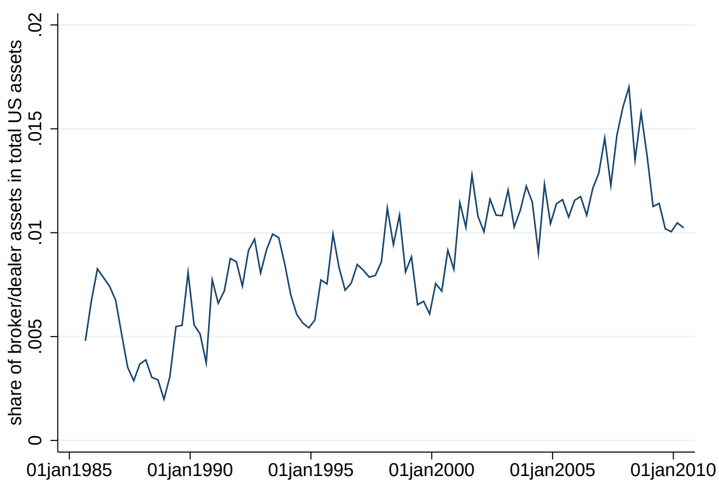


FIGURE 3.

The allocation of capital by heterogeneous investors.

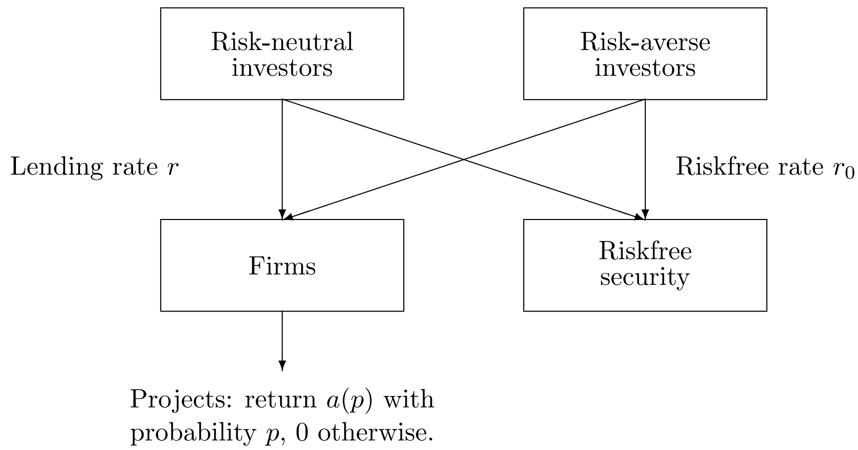
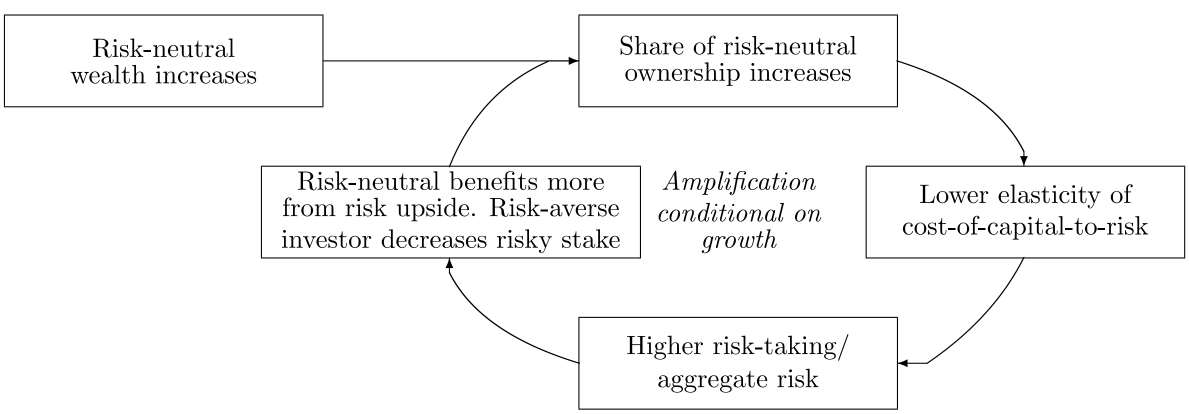


FIGURE 4.

Risk-taking amplification mechanism. This figure illustrates the amplification of risk. Conditional on the project paying out, the share of risk-neutral ownership of the firm increases: larger risk-neutral fund managers undermine market discipline and lead to more risk-taking, and if the project pays out, their wealth grows more than that of risk-averse investors, leading to more risk-taking in the next period.



we show that risk-taking by entrepreneurs and demand for risky securities by risk-neutral traders (e.g. fund managers or proprietary traders) are mutually reinforcing. Larger risk-neutral traders (relative to risk-averse investors) not only undermine market discipline and lead to more risk-taking, but risk-neutral traders also benefit more from the upside (than risk-averse investors) because risky securities represent a larger share of their portfolios. The model explains documented features of the business cycle: bubble-like asset prices (procyclical run-up in prices and procyclical underpricing of risk) and countercyclical leverage of the non-financial sector.[2] Third, we study the effect of policy on risk-taking. We show that a tightening of monetary policy can induce (rather than deter) risk-taking.

The setup is as follows. There are two infinitely-lived agents, a risk-averse investor and a risk-neutral trader; they provide funds to a penniless entrepreneur who lives for one period and has access to a set of projects with different risk characteristics. The model endogenizes the risk-taking decisions of entrepreneurs as in Blum (2002); Allen and Gale (2004a); Boyd and De Nicolo (2005); Martinez-Miera and Repullo (2009). All securities are correctly risk-adjusted (ie. there are no mispricing or behavioral types). The main contribution of this paper is to highlight a risk amplification mechanism in this context (Figure 4). If the relative wealth of risk-neutral traders is small, their ownership share of the firm is small too, and the elasticity of the cost of capital to risk is high, inducing the entrepreneur to choose safer projects. Since this project represents a higher share of the portfolio of the risk-neutral investor (than the risk-averse investor), the risk-neutral investor benefits more from the upside. If the project pays off, the ownership share by risk-neutral traders increases and the elasticity of the cost of capital decreases inducing the entrepreneur to choose a riskier project than before (leading the risk-averse investor to rebalance his portfolio into the riskfree

[2] In this paper we focus on long-only investors. Abreu and Brunnermeier (2003) study boom/bust dynamics with exogenous price divergence from fundamentals. In this paper, the "divergence" from fundatementals is endogenous and the consequence of demand for assets from risk-averse investors and risk-neutral fund managers in a production economy with an equilibrium supply of risky security.

asset). As long as the project generates positive returns, the stake of the risk-neutral traders increases and aggregate risk increases.[3]

The distortions due to the monopolistic supply of securities by the entrepreneurs, that generate by this amplification mechanism in aggregate risk, are assessed through a welfare measure that weighs the utility of the risk-averse and risk-neutral investor and the profit of the entrepreneur. For the risk-averse agent, there is always too much risk and for the entrepreneur, there is a the appropriate level (since he chooses the level of risk). The risk-neutral traders are pivotal in determining whether there is too much risk from a welfare point of view. More precisely, we show that at high levels of risk, the markups are excessively high and risk-neutral traders would benefit from lower risk, lower markups and the market discipline exercised by risk-averse investors.

We extend the setup to endogenize the leverage of risk-neutral traders. More precisely, leverage increases to compensate for lower expected returns. In particular, we show that for high aggregate risk, procyclical leverage generate additional volatility beyond the feedback loops betwene risk-taking by entrepreneurs and demand for risky securities by risk-neutral traders.

This paper is organized as follows. The next section describes the related literature. Section II introduces the model and characterizes the equilibrium portfolio decisions of investors and the allocation of capital. Section III shows that the allocation of capital generates an externality and discusses the policy and regulatory implications. In Section IV, the financing of the risk-neutral traders is endogenized and we show that it can generate a second amplification mechanism.

[3]The risk-neutral and risk-averse investors have the same effective discount factor so that no agent dominates asymptotically the economy, the asymptotic cross-sectional distribution of wealth is non-degenerate and the state variable (the relative wealth of risk-neutral arbitrageurs) is a random-walk.

I. Related literature

I.1. **Corporate governance with heterogeneous claim-holders.** A large literature on corporate governance studies how the ownership structure affects the investment decisions of a firm (Hellwig, 2006). In Bolton and Von Thadden (1998), the higher the stake of external investors into the firm, the easier it is to provide such incentives, but this large stake can generate liquidity costs. In fact, when small and large external investors agree on corporate decisions, small shareholders can free ride on the actions of the large ones. In this paper, the different investors disagree on the level of risk (chosen by the entrepreneur) because of differences in risk-aversion.[4] Blum (2002) presents of model of bank risk-taking with homogeneous external investors. He analyzes the ability to commit to a level of risk at the financing stage affects aggregate risk. Boot and Schmeits (2000) analyze risk-taking in a conglomerate model with an imperfect monitoring technology.

Closely related is a theory of supply effects in which investor demand is not driven by fundamentals affect corporate decisions (Baker, 2009). Shleifer and Vishny (2010) present a model in which banks cater to investor sentiment and invest in securitized loans when asset prices are high, accepting the risk of having to liquidate the portfolios at below fundamental values in bad times. Bolton, Scheinkman, and Xiong (2006) show that when investors have heterogeneous beliefs, optimal compensation contracts emphasize short-term stock performance to induce managers to pursue actions which increase the speculative component in the stock. Empirically, Ivashina and Sun (2011) associate the decrease in CLO spreads to an increase in institutional demand. Cheng, Hong, and Scheinkman (2010) interpret the correlation between compensation of top executives and risk-taking as a result of shareholders with short horizons incentivizing managers to take short-term risk.

In this paper, we abstract from informational asymmetries commitment issues and limited liability and analyzes how the distribution of wealth across heterogeneous investors affect risk-taking managerial incentives. If the largest potential shareholder is very risk-averse,

[4]See Dittmar and Thakor (2007) for a theory of equity financing based on disagreement.

the entrepreneur will have an incentive to choose a safe project to raise as much capital as possible from him. However, when risk-neutral traders are large, the elasticity of the cost of capital is low and the project's risk chosen by the entrepreneur is high.

I.2. **Asset pricing with heterogeneous investors.** The asset pricing properties of models in which agents have different risk-aversion parameters carries over to this model (Dumas, 1989; Wang, 1996; Longstaff and Wang, 2008; Weinbaum, 2009). In particular, the portfolio of risk-tolerant agents is biased towards the risky security so that adverse shocks to risky securities affect it more and reduce its contribution to aggregate risk aversion. Such logic explains why there is a negative correlation between risky return and the risk premium. One contribution of this paper is to endogenize the supply of securities. In particular, when the risk premium is low, this is precisely when entrepreneurs want to issue risky securities, which links the issuance of risky securities to the aggregate risk-aversion and risk premium.

I.3. **Aggregate risk.** This argument in Acemoglu and Zilibotti (1997) is that growth and wealth accumulation decrease the volatility of the economy. In this paper, we focus on the 2007-2009 financial turmoil that centered on developed economies by showing how the accumulation of wealth in certain parts of the financial sector might be destabilizing. A large literature has looked at the consequences of technological waves on the real economy and the financial sector (Jovanovic and Rousseau, 2003). The uncertainty associated to the new technology or financial innovations generates an amplification of the business cycle through learning with booms and busts (Barbarino and Jovanovic, 2007; Hobijn and Jovanovic, 2001; Biais, Rochet, and Wolley, 2009).

Another view is that booms and busts are not caused primarily by exogenous technology shocks, but by the demand for risky securities from risk-tolerant intermediaries. The emergence of new entities such as private equity and hedge funds as very large players on financial markets might have increased not only the risk-bearing capacity of the economy but also the issuance of risky securities.[5] Allen and Gale (2000); Challe and Ragot (2010) study the role of limited liability in inducing endogenous bubbles. Kiyotaki and Moore (1997); Fostel and Geanakoplos (2008) describe collateral-driven cycles [6] Empirically, tracking the balance sheet quantities of investors with different risk-aversion (and in particular the assets held by these agents) is important in determining the price of risk and the level of risk-shifting as in Adrian and Shin (2010, forthcoming). In Xiong (2001), arbitrageurs's wealth matters for the risk premium because arbitrageurs (ie. convergence traders) can absorb some of the fluctuations created by noise traders by trading against them (and occasionally amplifying the fluctuations when the wealth shocks are large enough).

I.4. **The impact of policy.** A key theme of the policy literature is whether prudential or monetary policy can supplement market discipline. In this environment, we study the role of interest-rate policy and of prudential regulation. Monetary policy in this model operates through a credit channel as in Champonnois (2010). A tightening of monetary policy (with an increase of the riskfree rate) induces the risk-averse investor to rebalance its portfolio towards the riskfree security and therefore undermines the market discipline by increasing the importance of the risk-tolerant trader for the decisions of the entrepreneur.[7]

If a goal of policy is to smooth fluctuations (Bordo and Jeanne, 2002), then it needs to induce a stable share of risk-tolerant ownership in the firm. For example, a money-supply neutral policy (with a countercyclical interest-rate policy) lowers the riskfree rate as the size of the trader increases relative to the risk-averse investor) and incentivizes the risk-averse investor not to rebalance its portfolio as the size of the risk-neutral trader increases.

[5]See also Philippon (2008) for theory and evidence on the equilibrium between the demand and supply of financial services.

[6]See also He and Krishnamurthy (2008, 2010); Brunnermeier and Pedersen (2009) for the role of varying margins)

[7]This is related to Landier, Sraer, and Thesmar (2010) who show that a monetary tightening in 2004 led the mortgage lender Century Corporation to offer riskier loans. See also Adrian and Shin (2010, forthcoming) who describe a risk-shifting channel of monetary policy.

Equivalently, prudential regulation with procyclical liquidity requirement could prevent the trader from taking over the ownership of the firm.

II. Baseline model

II.1. **Utility functions and technology.** There are two infinitely-lived agents, a risk-averse investor with log-utility and a risk-neutral trader. The risk-averse investor maximizes

$$U = \max \sum \beta^t \log c_t.$$

Assumption 1. *With probability δ, the risk-neutral trader is forced to consume at least $1-\theta$ of the funds wealth in a given period.*

Under this constraint the risk-neutral fund-manager maximizes

$$U^* = \max \sum \beta^t c_t^*.$$

The two investors consume a single good and have access to two investment opportunities. They can invest in a riskfree security return r_0.

Assumption 2. *Liquidity requirements force the trader to put at least a share ω in the riskfree security.*

The investor and the trader can invest in a risky project that pays r with probability p or zero otherwise. The project is chosen and run by a penniless risk-neutral entrepreneur, who lives for one-period.[8] More specifically, he chooses among constant-return projects in which the productivity $a(p)$ of a given project is a function of the probability p. In what follows, we call p, the probability of growth: if the project repays, the aggregate wealth grows and otherwise, it shrinks.

[8]The entrepreneur can be interpreted as a mortgage originator as in Landier, Sraer, and Thesmar (2010). More generally, he represents the "sell-side" in the financial sector while the investors represent the "buy side".

The technology is such that projects with a higher probability of generating revenues have a lower productivity: the higher the productivity, the lower the probability of success and the more rapidly the probability of success falls ($a(p)$ is decreasing and convex). We make the stronger assumption:

Assumption 3. *The elasticity of productivity a to growth probability p $\frac{-pa'(p)}{a(p)}$ is positive and increasing in p.*

The risk and return associated with this productivity function can be described introducing

$$\frac{-p_{\min}a'(p_{\min})}{a(p_{\min})} = 1.$$

We show below that in equilibrium, $p \geq p_{\min}$ and, given that $\frac{-pa'(p)}{a(p)}$ is increasing in p, we have $\frac{-pa'(p)}{a(p)} \geq 1$ for any $p \geq p_{\min}$. The risk associated to the technology is decreasing in the probability p in the sense that the variance $p(1-p)a(p)^2$ decreases in p

$$\frac{\partial \log[p(1-p)a(p)^2]}{\partial \log p} = 1 - \frac{p}{1-p} - \frac{2pa'(p)}{a(p)} \leq -\frac{1}{1-p} \leq 0.$$

We will say that aggregate risk increases when the growth probability p decreases.

The timing is as follows in a given period. The entrepreneur chooses a project with probability p and productivity $a(p)$ and proposes a rate r to external investors. Given the return r and risk p of the project, the risk-averse investor and the trader make their portfolio decisions. The project generates revenues or not and external claimants are repaid. When the risk-averse investor and the risk-neutral trader makes their portfolio decisions, they take the returns as given, which gives the entrepreneur some market power in making the investment decision.[9]

An important consequence of the timing is that external claimants cannot condition their financing on the risk-profile of the project. But ex-ante, they decide how much financing to provide if they think the project chosen is going to be too risky, which provides some market discipline.

[9]Since the entrepreneur is a monopolistic supplier of securities, the equilibrium allocation is different from the planner's allocation (Dumas, 1989; Wang, 1996).

Assumption 4. *The (effective) discount rates of the risk-averse and risk-neutral traders are equal*

$$1 - \beta = \delta(1 - \theta) \tag{2.1}$$

We show below the risk-averse agent consumes a fraction $1 - \beta$ of its wealth (because of the log-utility) while the risk-neutral investor consumes $1 - \theta$ of its wealth with probability δ.[10]

Assumption 5. *The liquidity requirement is such that the trader is always more exposed to the risky asset than the risk-averse investor (for any z and any r_0)*

Lemma 4 provides below a simple sufficient condition for Assumption ass: capital requirement.

II.2. **Supply of capital.** The consumption and portfolio decisions are considerably simplified by the risk-preference specifications (log utility and risk-neutral utility) and Assumptions 1 and 2 on consumption and liquidity requirements.

Lemma 1 (Consumption decisions)**.** *The risk-averse investor consumes a constant fraction $(1 - \beta)$ of its wealth at the beginning of the period. The risk-neutral trader consumes the exogenous fraction $1 - \theta$ when it is forced to (with probability δ) and otherwise does not consume.*

The log-preferences of the risk-averse investor yield a familiar interior solution in the sense that he consumes a constant fraction of his wealth. In the absence of constraints, the risk-neutral investor would never consume and the constraint of Assumption 1 is always binding. If $\langle p_t, r_t, y_t, x_t \rangle$ are the probability of growth, the return of the risky project, the savings of the risk-averse investor and the risk-neutral trader, we have

[10]A generalization of this formula can be found in the "almost surely stationarity" condition in Bhamra and Uppal (2010). The expression is simpler thanks to the assumption on the utility functions.

Lemma 2 (Portfolio decisions)**.** *If $p_t r_t \geq r_0$, the risk-averse investors puts $K(r_t, p_t) = p_t y_t - \frac{r_0 y_t(1-p_t)}{r_t - r_0}$ into the risky project and $y_t - K(r_t, p_t)$ into the riskfree security. The risk-neutral trader invests a fraction $1 - \omega$ of his wealth into the risky project and ω into the riskfree security.*

In the absence of constraints, the risk-neutral investor would invest all his wealth into the risky security and Assumption 2 is always binding. Note that $K(r, p)$ is increasing concave in r and increasing in p. The dynamics are

$$y_{t+1} = \begin{cases} \beta p_t r_t y_t & \text{w.p } p_t \\ \beta \frac{y_t(1-p_t) r_0 r_t}{r_t - r_0} & \text{w.p } 1 - p_t \end{cases} \quad ; \quad x_{t+1} = \begin{cases} x_t(\omega r_0 + (1-\omega) r_t) & \text{w.p } p_t(1-\delta) \\ \theta x_t(\omega r_0 + (1-\omega) r_t) & \text{w.p } p_t \delta \\ x_t \omega r_0 & \text{w.p } (1-p_t)(1-\delta) \\ \theta x_t \omega r_0 & \text{w.p } (1-p_t)(1-\delta) \end{cases}$$

If we introduce the relative wealth of the trader $z_t = \frac{x_t}{y_t}$, we have

$$\frac{z_{t+1}}{z_t} = \begin{cases} \left(\frac{\omega r_0 + (1-\omega) r_t}{\beta p r_t}\right) & \text{w.p } p_t(1-\delta) \\ \theta \left(\frac{\omega r_0 + (1-\omega) r_t}{\beta p r_t}\right) & \text{w.p } p_t \delta \\ \left(\frac{\omega (r_t - r_0)}{\beta (1-p) r_t}\right) & \text{w.p } (1-p_t)(1-\delta) \\ \theta \left(\frac{\omega (r_t - r_0)}{\beta (1-p) r_t}\right) & \text{w.p } (1-p_t)\delta \end{cases} \tag{2.2}$$

Lemma 3. *Under Assumption 4, the state variable z_t is a random walk: for all $i \geq 1$, $E_t(z_{t+i}) = z_t$.*

Using Equation (2.2), it is easy to show that $E(z_{t+1}) = \frac{z_t[\theta\delta + (1-\delta)]}{\beta} = 1$. The important consequence of Assumption 4 and Lemma 3 is that asymptotically, neither the risk-averse investor nor the risk-neutral investor dominate the economy with their wealth.

In the case of a positive output by the project, the state variable z will grow if the risk-neutral investor is more exposed to the risky security than the risk-averse agent. Since the portfolio choice of the risk-neutral investor is constrained by the liquidity requirement ω, this happens if

$$1 - \omega \geq \frac{pr - r_0}{r - r_0} \tag{2.3}$$

We now choose ω small enough so that this condition is generally satisfied. Introduce $p_{\max}$ defined as $\epsilon(p_{\max}) = 2$ and $p_{\min}$ is the growth probability when $pr = r_0$. $p_{\max}$ is the probability of growth if the risk-neutral traders are infinitely small ($z = 0$) and the riskfree rate r_0 is zero. We make the following assumption on the liquidity requirement ω:

Lemma 4. *The liquidity requirement is such that the trader is always more exposed to the risky asset than the risk-averse investor (for any z and any r_0) if*

$$1 - \omega \geq p_{\max}$$

Assumption 5 imposes that the maximum portfolio share in the risky security for the risk-averse agent (for $z = 0$) is always below that of the risk-neutral trader. A consequence of this assumption is that the state variable z increases with good returns from the risky technology and decreases with bad returns and this leads to procyclical risk.

II.3. **Demand for capital.** In this section only, we omit the subscript t. The profit of the entrepreneur is $p[a(p) - r][x(1 - \omega) + K(r, p)]$ and the first-order conditions of in the return r and the probability of growth p are

$$\frac{a - r}{r} = \frac{1}{\frac{\partial \log(x(1-\omega)+K(r,p))}{\partial \log r}} \tag{2.4}$$

$$-\frac{pa'(p)}{a - r} = 1 + \frac{\partial \log[x(1 - \omega) + K(r, p)]}{\partial \log p} \tag{2.5}$$

The first equation is the usual relation between markups and the elasticity of capital supply in monopolistic models: the markup of the entrepreneur is smaller when the supply of capital is more elastic in the return r. The second equation relates the elasticity of productivity and to market discipline and the elasticity of capital supply relative to risk: the productivity elasticity (relative to risk) is convex and risk is higher if there is less market discipline (that is, if the supply of capital is more elastic relative to risk). The entrepreneur increases risk until the decrease in the supply of capital has a higher cost than the benefit of an increase in productivity. Under the condition $\frac{a-r_0}{r_0} \geq \frac{y(1-p)}{x(1-\omega)+py}$, Equation (2.4) implies that $\frac{a-r}{a-r_0} = \frac{x(1-\omega)+k}{x(1-\omega)+py} = 1 - \sqrt{\frac{r_0 y(1-p)}{(a-r_0)(x(1-\omega)+py)}}$.

The equilibrium depends on one unique state variable $z = \frac{x}{y}$ defined as the ratio of the wealths of risk-neutral trader and risk-averse investor. The following Lemma characterizes the equilibrium aggregate risk.

Lemma 5. *With $\epsilon(p) = -\frac{pa'}{a-r_0}$, the equilibrium probability of growth $p = P(z)$ is decreasing in the trader's relative wealth z*

$$\epsilon(p) \;=\; 1 + \frac{p}{z(1-\omega)+p} + \sqrt{\frac{r_0(1-p)}{(a-r_0)(z(1-\omega)+p)}}\left(\frac{p}{1-p} - 1\right) \qquad (2.6)$$

Lemma 5, in conjunction with Assumption 5, delivers the main result of this paper: when the relative wealth z increases, the probability of growth p decreases and the productivity $a(p)$ increases and conditional on the risky security generating revenues, the relative increases in the next period leading to an even lower probability of growth in the next period.

Corollary 1. *The share of the portfolio of the risk-averse agent in the risky security, the leverage of the entrepreneur $\frac{a}{a-r}$ and the risk-adjusted return $\frac{r}{a}$ are decreasing in z.*

Over the business cycle, when the probability p is the lowest and the productivity $a(p)$ is the highest, the return r is very high (measured relative to productivity a) and the leverage of the firm is also very large, leading to "bubble-like" features of the economy before a crash.

Corollary 2 (Market discipline)**.** *The elasticity of the supply of capital $\frac{\partial \log[x+K(r,p)]}{\partial \log p}$ and the elasticity of the cost to risk $-\frac{\partial \log(r-r_0)}{\partial \log p}$ are positive, decreasing in the trader's relative wealth z.*

Equation (2.6) defines $p = P(z)$ and the inverse function $z = Z(p)$ can be solved in closed form

$$Z(p) = \frac{p}{1-\omega}\left[\left(\frac{\sqrt{\frac{r_0(2p-1)^2}{4(a-r_0)(1-p)p} + \epsilon(p) - 1} + \sqrt{\frac{r_0(2p-1)^2}{4(a-r_0)(1-p)p}}}{\epsilon(p)-1}\right)^2 - 1\right].$$

Lemma 6 (Expected return)**.** *The expected return pr is in general increasing in z for small z and decreasing in z for large z and we have*

$$pr = \frac{r_0}{2} + \frac{\sqrt{r_0\left[r_0 + 4(1-p)a(p)p\left(-\frac{pa'(p)}{a(p)} - 1\right)\right]}}{2}$$

where $p = P(z)$.

The function $(1-p)a(p)$ is decreasing in p while $p\left(-\frac{pa'(p)}{a(p)} - 1\right)$ is increasing in p (Assumption 3). For high p (close to 1), the first effect dominates while for low p (close to 0), the second effect dominates.

II.4. **Special case: productivity linear in risk.**

Assumption 6. *The productivity is linear in the probability of growth p: $p \in [1/2, 1]$, $a = 1 - hp$.*[11]

Lemma 7. *The equilibrium risk $P(z)$ is decreasing in z (and r_0) and is determined by*

$$\frac{hp}{1 - hp - r_0} = 1 + \frac{p}{p + z(1-\omega)} + \sqrt{\frac{r_0(2p-1)^2}{(1-hp-r_0)(1-p)(p+z(1-\omega))}} \qquad (2.7)$$

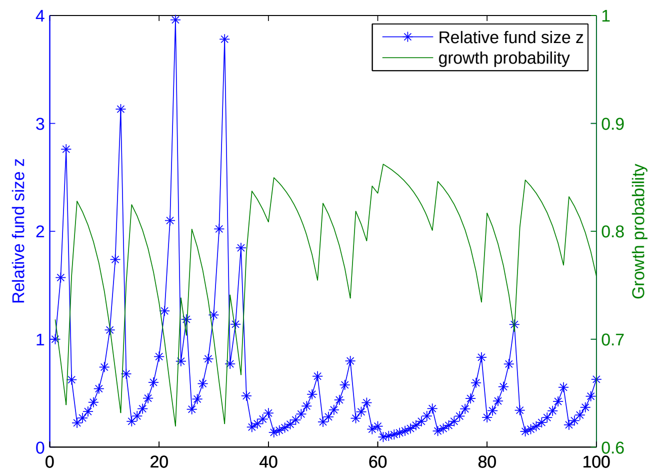
Moreover, $p_{\min} = \frac{1}{2h}$ and $p_{\max} = \frac{2}{3h}$.

Assumption 5 is now equivalent to $1 - \omega \geq \frac{2}{3h}$. The condition that the investment of the risk-averse agent in the risky project is positive ($pr \geq r_0$) implies $p \geq p_{\min}$ or equivalently $z \leq z_{\max} = \frac{\frac{1}{4h} - r_0}{r_0(2h-1)(1-\omega)}$. Figure 5 shows the negative correlation between the relative size of risk-neutral traders z and the probability of growth p. Figure 6 shows as functions of the relative fund size z, the growth probability p, the risk-adjusted return $\frac{r}{a}$ and the private sector leverage $\frac{a}{a-r}$, the share of the risk-averse agent's portfolio is the risky project and the elasticities of the supply of capital and the cost of capital to risk.

[11] This is without loss of generality. If $a = A(1 - hp)$, we can renormalize a and r_0 by A.

FIGURE 5.

Relative fund size z and growth probability p. This figures shows the negative correlation between the relative fund size z and the probability of growth p: the relative size of the risk-neutral trader z increases, the probability of growth p decreases and aggregate risk increases. Parameter values: $r_0/A = .1$, $h = .8$, $\beta = .8$, $\delta = .5$, $\theta = .6$, $\omega = 1/6$.



Lemma 8. *The expected return pr is maximal for $p = \frac{2h+1+\sqrt{(2h+1)^2-6h}}{6h}$ with*

$$pr = \frac{r_0}{2} + \frac{\sqrt{r_0[r_0(2p-1)^2 - 4p(1-p)(1-2hp-r_0)]}}{2}$$

Figure 7 shows the hump shaped function for the expected return pr as a function of z. For low relative wealth z, a lower growth probability p leads to higher productivity a and therefore higher return r (resp. expected return pr). For high relative wealth z, the lower growth probability p decreases the expected return pr.

III. Welfare and policy

In this section, we study normative measure to determined whether aggregate risk is "too high" and we evaluate the instruments to adjust the level of risk. Specifically, we ask whether welfare (measured as a weighted sum of the utilities of the risk-averse investor, risk-neutral trader and entrepreneur) decreases when the equilibrium probability of growth p is increase to $p + dp$.

Figure 6.

Properties of the equilibrium: growth probability, risk-adjusted return, share of risk in portfolios, and elasticities. Parameter values: $r_0/A = .1$, $h = .8$, $\beta = .8$, $\delta = .5$, $\theta = .6$, $\omega = 1/6$.

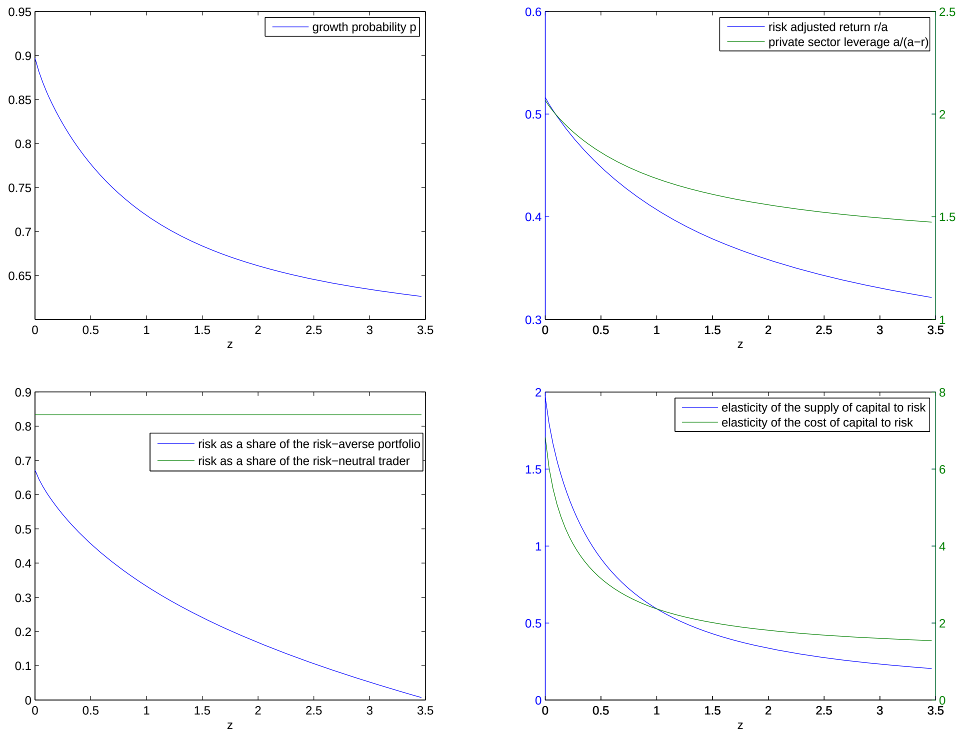
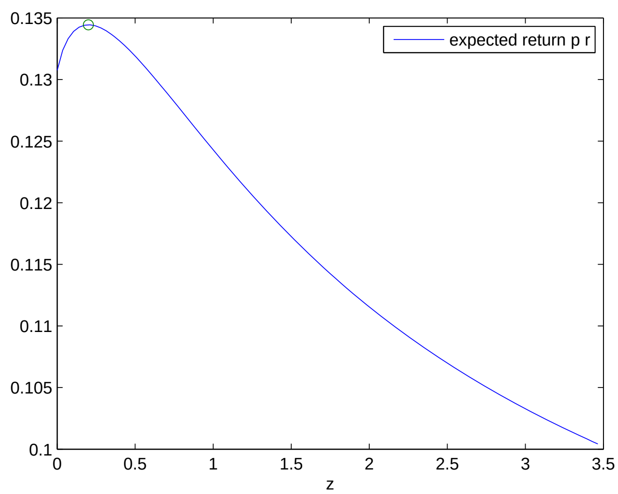


Figure 7.

Properties of the equilibrium: expected return. Parameter values: $r_0/A = .1$, $h = .8$, $\beta = .8$, $\delta = .5$, $\theta = .6$, $\omega = 1/6$.



III.1. **Welfare.** The per-period expected utility of the risk-averse agent (measured as the expected consumption in the next period) conditional on a relative wealth $z = z_0$ is

$$E(u|z = z_0) = \beta \log(c) = \beta \left[\log((1-\beta)y) + p\log(pr) + (1-p)\log\left(\frac{(1-p)r_0 r}{r - r_0}\right)\right],$$

where p and r depend in equilibrium on z. Similarly, the utility of the risk-neutral trader conditional on $z = z_0$ is

$$E(u^*|z) = \beta\delta c^* = \beta\delta(1-\theta)x\left(pr(1-\omega) + \omega r_0\right)$$

The expected profit of entrepreneur is $E(\pi|z = z_0) = p(a-r)(x(1-\omega)+k)$. Similarly $E(U|z)$ and $E(U^*|z)$ are the present-discounted utilities of the risk-averse and risk-neutral agents conditional on z. Since the relative fund wealth z is a random walk, for any horizon t, the expectation at time $t = 0$ of z_t is the current of the state variable z_0: $E_0(z_t) = z_0$ and the following Lemma:

Lemma 9. *The present discounted utilities are approximately*

$$E_0(U|z = z_0) \approx \frac{E(u|z = z_0)}{1-\beta}; \qquad E_0(U^*|z = z_0) \approx \frac{E(u^*|z = z_0)}{1-\beta}$$

The approximation is exact if $u(z)$ and $u^*(z)$ were linear.

Assumption 7. *The objective of a planner is the weighted sum of the utility of the risk-neutral and risk-averse investors and the entrepreneur*

$$W = \lambda_{\text{risk-averse}} E(U|z = z_0) + \lambda_{\text{risk-neutral}} E(U^*|z = z_0) + \lambda_{\text{entrepreneur}} \pi$$

Corollary 3 (Welfare)**.** *The utility of the risk-averse agent is increasing in the probability of growth p. The utility of the risk-neutral trader is increasing (resp. decreasing) in the probability of growth p for low p (resp. for high p).*

Corollary 4. *The planner is more likely too assess that there is too much risk (in the sense that* $\frac{\partial W}{\partial p} < 0$*) if the welfare weight of the risk-averse agent* $\lambda_{\text{risk-averse}}$ *is high (relative to* $\lambda_{\text{risk-neutral}}$*) and if the risk is high (so that* $\frac{\partial u^*}{\partial p} < 0$*).*

First, since the entrepreneur chooses the level of risk, any marginal change in risk generates only a second-order loss and so its utility does not matter for the considerations of the planner. Second, the risk-averse investor always considers that there is too much risk. If he is important enough (as measured by $\lambda_{\text{risk-averse}}$), then the planner will have an incentive to decrease risk. The utility of the risk-neutral traders is decreasing in p because as risk increases, the markup charged by the entrepreneur increases too which leads to a lower utility.

III.2. **Policy.** As in Champonnois (2010), we study the role of interest-rate policy in affecting the allocation of capital. More precisely, interest-rate policy operates through a credit channel of monetary policy in which higher interest rates lead to a rebalancing of the portfolio of agents.

Corollary 5. *An increase of riskfree rate r_0 by the monetary authority increases aggregate risk in the economy.*

In this model, a higher riskfree rate leads the risk-averse investor to have a lower share of the ownership of the project which, in turn, undermine market discipline and induces the entrepreneur to choose riskier projects. Empirically, Landier, Sraer, and Thesmar (2010) show that a sudden increase in risk-shifting at New Century's lending behavior followed the sharp monetary policy tightening implemented by the Fed in the spring of 2004. However, the interpretation of Landier, Sraer, and Thesmar is that New Century was financing projects with a high beta on its own survival. An alternative interpretation is that New Century was catering to risk-neutral traders, since risk-averse investors were rebalancing their portfolio into riskfree securities following the rate increase.

Corollary 6. *An increase in the liquidity buffers for the risk-neutral trader ω leads to a decrease aggregate risk.*

A change in the portfolio allocation of the risk-neutral trader through higher liquidity buffers reduces the buying pressure on the issuance of risky securities and in turn, improves market discipline and lowers aggregate risk.

IV. Extension: endogenous leverage of risk-neutral traders

In this section, we introduce leverage for the risk-neutral traders as a way for them to generate better returns in the face of lower profitability as in Khandani and Lo (2007). We assume that not all risk-neutral agents are allowed to trade because of limited participation (Allen and Gale, 2004b; He and Krishnamurthy, 2008, 2010): the risk-neutral active traders raise funds from the non-active risk-neutral agents. The probability λ of becoming an active trader in a given period is constant and identical across risk-neutral agents. For managing the money on behalf on the inactive risk-neutral agents, the active traders charge a spread ξr so that for a leverage L, the return of active traders is

$$r_{\text{active}} = r(1+L) - r(1-\xi)L = r(1+\xi L).$$

The capital of inactive agents not lent to active traders is invested in the riskfree security at the riskfree rate r_0. For a wealth x of risk-neutral (active and inactive) agents at the beginning of the period, $\lambda(1+L)$ is invested in the riskfree security and $1-\lambda(1+L)$ in the riskfree security.[12]

Assumption 8. *The risk-neutral traders require an expected return above the expected return in the market: for* $\theta \leq 1$*,*

$$pr_{\text{active}} = (pr)^{\theta}.$$

We can rewrite the leverage L as a function of the expected return pr

$$L = \frac{(pr)^{\theta-1} - 1}{\xi},$$

[12]In all applications, we focus on $\lambda(1+L) < 1$.

where $(1-\theta)$ measures the elasticity of a decrease in leverage to an increase in expected returns: when the expected return pr decreases, the leverage increases and more so for lower θ. The share of risk-neutral capital ω invested in the riskfree security is then

$$\omega = 1 - \lambda\left(1 + \frac{(pr)^{\theta-1} - 1}{\xi}\right)$$

While in previous sections, we assumed that ω was an exogenous liquidity requirements, limited participation and the leverage of active traders provides an alternative interpretation of the aggregate asset allocation of risk-neutral agents. When $\theta = 1$, the leverage is constant and $\omega = 1 - \lambda$. When $\theta < 1$, the share of the riskfree asset allocation decreases when pr decreases because the limited participation constraint is loosened through a higher leverage. The equilibrium growth probability $p = P(z)$ can determined explicitly through the inverse function $z = Z(p)$, where

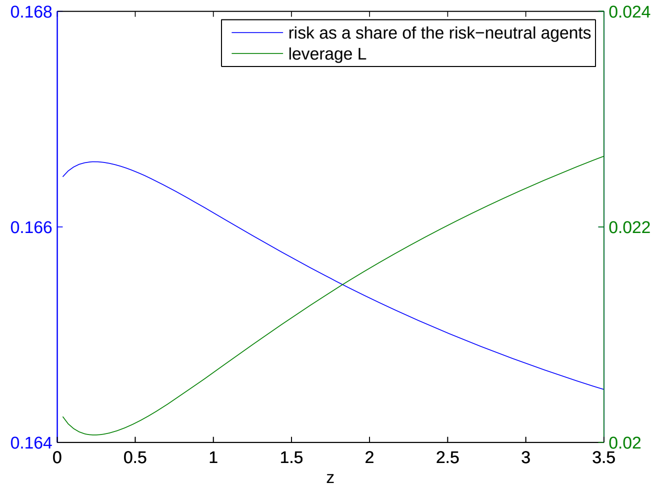
$$Z(p) = \frac{p\xi}{\lambda}\frac{\left[\left(\frac{\sqrt{\frac{r_0(2p-1)^2}{4(a-r_0)(1-p)p}+\epsilon(p)-1}+\sqrt{\frac{r_0(2p-1)^2}{4(a-r_0)(1-p)p}}}{\epsilon(p)-1}\right)^2 - 1\right]}{\left(\frac{r_0}{2} + \frac{\sqrt{r_0[r_0(2p-1)^2+4p(1-p)(a-r_0)(\epsilon(p)-1)]}}{2}\right)^{\theta-1} - (1-\xi)}.$$

Lemma 10. *When the expected returns decrease as the risk-neutral relative wealth z increases $\left(\frac{\partial(pr)}{\partial z} < 0\right)$, the time-varying leverage generates an additional amplification mechanism.*

From Lemma 6, the expected return is decreasing for high risk-neutral relative wealth z, so that we have a second amplification mechanism: in the decreasing region for pr in z, an increase in the relative wealth z leads to lower expected return, so higher leverage and higher risk-neutral capital invested in the risky project and induces a higher aggregate risk. Figure 8 shows the procyclical leverage of the risk-neutral agents (for high z) and the countercylical leverage of the non-financial sector (entrepreneurs). Such procyclical leverage for the risk-neutral agents is related to the model in He and Krishnamurthy (2010) and the empirical findings in Adrian and Shin (2010) and for the banking sector, in Ang, Gorovyy, and van Inwegen (2011).

Figure 8.

Leverage of the financial sector (risk-neutral agents) and the non-financial sector (entrepreneurs). Parameter values: $r_0/A = .1$, $h = .8$, $\beta = .8$, $\delta = .5$, $\theta = .6$, $\lambda = .817$, $\xi = .01$, $\theta = 1 - .0001$.



However the mechanism for the procyclical leverage for risk-neutral traders is different from the reason in He and Krishnamurthy (2010). He and Krishnamurthy present a model in which negative news on long-term risky assets (e.g. MBS) increase leverage because the value of assets reacts more rapidly that the value of liabilities. In the present paper, there are only one-period securities and risk-neutral traders increase leverage to compensate for a lower profitability at the peak of the business cycle.

Capital requirements on risk-neutral traders prevents the additional volatility from time-varying leverage. More precisely, constant capital requirements (leading to a constant leverage) eliminates the additional volatility generated by time-varying leverage. Moreover counter-cyclical leverage addresses some of the underlying causes for the increase in aggregate volatility due to the feedback loops between risk-taking by entrepreneurs and demand for risky securities by risk-neutral traders.

V. Conclusion

This paper introduces a model in which the trading of heterogeneous investors is destabilizing over the business cycle. During a period of economic expansion, as the relative wealth of risk-neutral traders increases, they induce higher risk-taking by entrepreneurs, eventually leading to so risky projects that a crash is likely. As these risk-neutral traders are more exposed to the risky projects than risk-averse investors, their relative wealth decreases more with a crash, leading to some Schumpetarian cleansing in which only safer projects are financed. And as these risk-neutral traders rebuild their wealth, the aggregate risk in the economy also increase leading to another boom-bust period.

This model has several policy implications. First, interest-rate policy is unable to affect aggregate risk-taking behavior. As the return on the risk-free security – the instrument of the central bank – increases, the risk-neutral investor rebalances his portfolio into the riskfree security and leads the risk-neutral trader to hold a larger share of the firm, which in turn encourages entrepreneurial risk-taking. In this sense, a tighter monetary policy undermines market discipline because it affects more the portfolio decision of the risk-averse investor than those of the risk-neutral trader. Second, liquidity requirements regulation is better at affecting aggregate risk-taking behavior. But to be effective, capital requirements should vary over the business cycle with the aim of keeping constant the relative share of fund ownership of the firm.

VI. Appendix: Proofs

Proof of Lemma 2. The value function of an infinitely-lived agent with log utility is $V(y) = a + b\log(y)$. The first-order condition ∂k is then

$$\frac{p}{r_0 k_0 + rk} = \frac{1}{ry} \quad \Leftrightarrow \quad k = py - \frac{r_0(y-k)}{r}$$

and using $k_0 = y - k$, we find the supply of capital.

Proof of Lemma 3. The expected value of next period state variable is

$$\begin{aligned}
\frac{\beta E(z_{t+1})}{z_t} &= P(z_t)(1-\delta)\frac{(\omega r_0 + (1-\omega)R(z))}{P(z_t)R(z_t)} + P(z_t)\delta\frac{\theta(\omega r_0 + (1-\omega)R(z))}{P(z_t)R(z_t)} + ... \\
& ... + (1-P(z_t))(1-\delta)\frac{\omega r_0}{(1-P(z_t))\frac{R(z_t)r_0}{R(z_t)-r_0}} + (1-P(z_t))(1-\delta)\frac{\theta\omega r_0}{(1-P(z_t))\frac{R(z_t)r_0}{R(z_t)-r_0}} \\
&= (1-\delta)\frac{(\omega r_0 + (1-\omega)R(z))}{R(z_t)} + \delta\frac{\theta(\omega r_0 + (1-\omega)R(z))}{R(z_t)} + (1-\delta)\frac{\omega(R(z_t)-r_0)}{R(z_t)} + \delta\frac{\theta\omega(R(z_t)-r_0)}{R(z_t)} \\
&= (1-\delta) + \delta\theta
\end{aligned}$$

Proof of Lemma 5. The first-order conditions are

$$\begin{aligned}
\partial r: &\qquad \frac{(a-r_0)r_0y(1-p)}{(r-r_0)^2} = x+py \\
\partial p: &\qquad (a-r)(x+k) + py\left(\frac{r}{r-r_0}\right)(a-r) = -pa'(x+k)
\end{aligned}$$

where $k = K(r,p) = py - \frac{(1-p)r_0y}{r-r_0}$. The condition ∂p can be written as

$$\begin{aligned}
\epsilon(p) &= 1 - \sqrt{\frac{r_0y(1-p)}{(a-r_0)(x+py)}} + \frac{py}{x+py}\left(1 + \frac{r_0}{(a-r_0)\sqrt{\frac{r_0y(1-p)}{(a-r_0)(x+py)}}}\right) \\
&= 1 + \frac{py}{x+py} + \sqrt{\frac{r_0y(1-p)}{(a-r_0)(x+py)}}\left(\frac{p}{1-p} - 1\right)
\end{aligned}$$

Two remarks

- When $r_0 = 0$, then the profit of the entrepreneur is $\pi = p(x+py)a$. A necessary condition for the risk choice p to be decreasing in $z = x/y$ is that the elasticity $-\frac{pa'(p)}{a(p)}$ is increasing.
  - The first-order condition is $\frac{-pa'(p)}{a} = 1 + \frac{p}{p+z}$. The right-hand side is increasing in p, decreasing in z. The solution $p(z)$ is decreasing in z if the left-hand side crosses the right-hand side from below.
- The condition $pr \geq r_0$ is satisfied if $\epsilon(p) \geq \frac{a}{a-r_0} \Leftrightarrow -\frac{pa'}{a} > 1$. This last condition imposes that the expected productivity pa is always decreasing. Rewriting with $z = \frac{x}{y}$, we get

$$(\epsilon(p) - 1)(z+p) - \sqrt{z+p}(2p-1)\sqrt{\frac{r_0}{(1-p)(a-r_0)}} - p = 0$$

  There exists a positive z solution to this equation if and only if

$$2 + (2p-1)\sqrt{\frac{r_0}{p(1-p)(a-r_0)}} \geq \theta \tag{6.1}$$

  If this condition is satisfied,

$$\begin{aligned}
\sqrt{z+p} &= \frac{(2p-1)\sqrt{\frac{r_0}{(1-p)(a-r_0)}} + \sqrt{(2p-1)^2\frac{r_0y}{(1-p)(a-r_0)} + 4p(\theta-1)}}{2(\theta-1)} \\
\sqrt{\frac{r_0y(1-p)}{(a-r_0)(x+py)}} &= \sqrt{\frac{r_0(1-p)}{(a-r_0)}}\left(\frac{\sqrt{(2p-1)^2\frac{r_0}{(1-p)(a-r_0)} + 4p(\theta-1)} - (2p-1)\sqrt{\frac{r_0}{(1-p)(a-r_0)}}}{2p}\right) \\
\frac{pr}{r_0} - 1 &= \left(\frac{\sqrt{(2p-1)^2 + \frac{4(1-p)p(\theta-1)(a-r_0)}{r_0}} - 1}{2}\right) \geq 0
\end{aligned}$$

Proof of Corollary 1. We first study the share of the risk-averse agent in the risky security. The weight is $\frac{pr-r_0}{r-r_0}$ so that

$$\begin{aligned}
\frac{\partial\left(\frac{pr-r_0}{r-r_0}\right)}{\partial \log p} &= \frac{pr}{r-r_0}+\frac{r_0(1-p)}{r-r_0}\frac{\partial \log(r-r_0)}{\partial \log p}\\
&= \frac{(1-p)r_0}{r-r_0}\left[\frac{p}{1-p}\left(1+\frac{(a-r_0)\sqrt{\frac{r_0(1-p)}{(a-r_0)(p+z)}}}{r_0}\right)-\frac{1}{2(1-p)}-\frac{p}{p+z}-\frac{1}{2}\sqrt{\frac{r_0(1-p)}{(a-r_0)(p+z)}}\left(\frac{p}{1-p}-1\right)\right]\\
&= \frac{(1-p)r_0}{r-r_0}\left[\frac{1}{2}\left(\frac{p}{1-p}-1\right)\left(1-\sqrt{\frac{r_0(1-p)}{(a-r_0)(p+z)}}\right)+\frac{p}{p+z}\left(\frac{1}{\frac{1}{\sqrt{\frac{r_0(1-p)}{(a-r_0)(p+z)}}}}-1\right)\right]\geq 0
\end{aligned}$$

Since $\frac{pr-r_0}{r-r_0}=p-(p+z)\sqrt{\frac{r_0(1-p)}{(a-r_0)(p+z)}}$, the share of the risk-averse agent in the risky asset is decreasing in r_0. It is also easy to show that $\frac{\partial\left(\frac{pr-r_0}{r-r_0}\right)}{\partial z}<0$ and since p is decreasing in z and r_0, the share of the risk-averse agent in the risky security $\frac{pr-r_0}{r-r_0}$ is decreasing in the relative wealth if the risk-neutral trader z and r_0.

We then study the leverage of the entrepreneur $\frac{a}{a-r}$.

Proof. Using $r=r_0\left(1-\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\right)+a\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}$, we have

$$\begin{aligned}
\frac{\partial \log(r/a)}{\partial \log p} &= \left(\frac{a-r_0}{ar}\right)\left(\theta r+a\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\frac{\partial \log(r-r_0)}{\partial \log p}\right)\\
&= \left(\frac{a-r_0}{ar}\right)\left[\theta r_0\left(1-\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\right)+a\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\frac{\partial \log\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}}{\partial \log p}\right]\\
&= \left(\frac{a-r_0}{ar}\right)\left(1-\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\right)\left[\theta r_0-a\frac{1}{2}\left(\frac{p}{1-p}-1\right)\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\right]\\
&= \left(\frac{r_0(a-r_0)}{ar}\right)\left(1-\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\right)\left[1+\frac{p}{p+z}-\frac{1}{2}\left(\frac{p}{1-p}-1\right)\left(\frac{1-p}{p+z}-\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\right)\right]\\
&= \left(\frac{r_0(a-r_0)}{ar}\right)\left(1-\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\right)\left[1+\frac{1}{2(p+z)}+\frac{\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}}{2}\left(\frac{p}{1-p}-1\right)\right]>0
\end{aligned}$$

□

Proof of Corollary 2. We have

$$\begin{aligned}
\frac{\partial \log\left(\sqrt{\frac{r_0 y(1-p)}{(a-r_0)(x+py)}}\right)}{\partial \log p} &= \frac{1}{2}\left(-\frac{pa'(p)}{a-r_0}-\frac{p}{1-p}-\frac{p}{p+z}\right)\\
&= \frac{1}{2}\left[1+\frac{p}{p+z}+\sqrt{\frac{r_0 y(1-p)}{(a-r_0)(x+py)}}\left(\frac{p}{1-p}-1\right)-\frac{p}{1-p}-\frac{p}{p+z}\right]\\
&= -\frac{1}{2}\left(1-\sqrt{\frac{r_0 y(1-p)}{(a-r_0)(x+py)}}\right)\left(\frac{p}{1-p}-1\right)<0
\end{aligned}$$

Since $\frac{x+k}{x+py} = 1 - \sqrt{\frac{r_0 y(1-p)}{(a-r_0)(x+py)}}$, the elasticity of the supply of capital is positive and decreasing in z

$$\frac{\partial \log(x+k)}{\partial \log p} = \frac{p}{p+z} + \frac{1}{2}\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\left(\frac{p}{1-p} - 1\right) > 0$$

and the elasticity of the cost of capital to risk is positive and decreasing in z

$$-\frac{\partial \log(r-r_0)}{\partial \log p} = \frac{1}{2(1-p)} + \frac{p}{z+p} + \frac{1}{2}\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\left(\frac{p}{1-p} - 1\right) > 0$$

Moreover

$$\begin{aligned}\frac{\partial^2 \log(x+k)}{\partial \log p^2} &= \frac{pz}{(p+z)^2} + \frac{p^2}{2(1-p)^2}\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}} - ...\\ &\quad ... - \frac{1}{4}\left(\frac{p}{1-p} - 1\right)^2\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\left(1 - \sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\right)\\ &= \frac{pz}{(p+z)^2} + \sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\underbrace{\left[\frac{p^2}{2(1-p)^2} - \frac{1}{4}\left(\frac{p}{1-p} - 1\right)^2\right]}_{>0} + \frac{1}{4}\left(\frac{p}{1-p} - 1\right)^2\frac{r_0(1-p)}{(a-r_0)(z+p)}\end{aligned}$$

and

$$\begin{aligned}-\frac{\partial^2 \log(r-r_0)}{\partial \log p^2} &= \frac{p}{2(1-p)^2} + \frac{pz}{(z+p)^2} + \frac{p}{2(1-p)^2}\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\\ &\quad -\frac{1}{4}\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\left(1 - \sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\right)\left(\frac{p}{1-p} - 1\right)^2 > 0\end{aligned}$$

since the term in $\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}$ is $\frac{2p-(2p-1)^2}{4(1-p)^2} > 0$ whenever $p > \frac{1}{2}$.

Proof of Lemma 9. For a path of state variables $[z_1, z_2, ..., z_t, ...]$, the probability of growth and interest rates are $[p_1, p_2, ...]$ and $[r_1, r_2, ...]$ and the utility of the agents are

$$\begin{aligned}U - \frac{\log[(1-\beta)y]}{1-\beta} &= \sum_{t=1}^{\infty}\beta^t\left[p_t\log(p_t r_t) + (1-p_t)\log\left(\frac{(1-p_t)r_0 r_t}{r_t - r_0}\right)\right]\\ \frac{U^*}{\delta(1-\theta)x} &= \sum_{t=1}^{\infty}\beta^t\left[p_t r_t(1-\omega) + \omega r_0\right]\end{aligned}$$

The expectation of the per-period utility in the second period given that $z = z_1$ is $f(z_1) = E(u_2|z = z_1)$. A Taylor decomposition around z_0 yields

$$f(z_1) \approx f(z_0) + (z_1 - z_0)f'(z_0) + \frac{(z_1 - z_0)^2}{2}f''(z_0)$$

Taking the expectations in time 0, we have

$$E(u_2|z = z_0) = f(z_0) = E(u|z = z_0) + \frac{E[(z_1 - z_0)^2|z = z_0]}{2}f''(z_0)$$

Ignoring the last term, we have $E(u_2|z = z_0) = E(u|z = z_0)$. The same intuition holds for any $t \geq 1$.

Proof of Corollary 3. The utility of the risk-averse agent is $\tilde{u} = p\log(pr) + (1-p)\log\left[\frac{(1-p)r_0 r}{r-r_0}\right]$. Moreover we have

$$\frac{\partial \log(r-r_0)}{\partial \log p} = -\frac{1}{2}\left[\frac{2p}{p+z} + \frac{1}{1-p} + \sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\left(\frac{p}{1-p} - 1\right)\right] \leq 0$$

This implies

$$\begin{aligned}
\frac{\partial \tilde{u}}{\partial p} &= \log\left[\frac{p(r-r_0)}{(1-p)r_0}\right] + \frac{\partial r}{\partial p}\left(\frac{1}{r} - \frac{1-p}{r-r_0}\right) \\
&= \log\left[1 + \frac{pr-r_0}{(1-p)r_0}\right] + \left(\frac{pr-r_0}{pr}\right)\frac{\partial \log(r-r_0)}{\partial \log p} \\
&\geq \left(\frac{pr-r_0}{pr}\right)\left\{\frac{pr}{(1-p)r_0} - \frac{1}{2}\left[\frac{2p}{p+z} + \frac{1}{1-p} + \sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\left(\frac{p}{1-p} - 1\right)\right]\right\} \\
&= \left(\frac{pr-r_0}{pr}\right)\left\{\frac{p}{1-p} + \frac{p(a-r_0)}{(1-p)r_0}\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}} - \frac{1}{2}\left[\frac{2p}{p+z} + \frac{1}{1-p} + \sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\left(\frac{p}{1-p} - 1\right)\right]\right\} \\
&= \left(\frac{pr-r_0}{pr}\right)\left[\frac{1}{2}\left(\frac{p}{1-p} - 1\right)\left(1 - \sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\right) + \frac{p}{p+z}\left(\frac{1}{\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}} - 1\right)\right] \geq 0
\end{aligned}$$

The utility of the risk-neutral trader is $\delta(1-\theta)x[pr(1-\omega) + \omega r_0]$ and we have

$$\begin{aligned}
\frac{\partial \log(pr)}{\partial \log p} &= 1 + \left(\frac{r-r_0}{r}\right)\frac{\partial \log(r-r_0)}{\partial \log p} \\
&= \left(\frac{r-r_0}{2r}\right)\left[2 + \frac{2(p+z)}{1-p}\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}} - \frac{2p}{p+z} - \frac{1}{1-p} - \sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\left(\frac{p}{p-1} - 1\right)\right] \\
&= \left(\frac{r-r_0}{2r}\right)\left[\frac{2z+1}{1-p}\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}} + \frac{2z}{p+z} - \frac{1}{1-p}\right]
\end{aligned}$$

The condition $pr \geq r_0$ (risk-averse investor investing in the firm) implies

$$\frac{p}{p+z} \geq \sqrt{\frac{r_0(1-p)}{(a-r_0)(p+z)}}$$

The term in bracket is decreasing in p (since in particular $\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}$ is decreasing in p), so that for high risk (low p), a lower risk increases the expected return, while for low risk (high p), an increase in risk leads to higher expected return.

Note: when $z = 0$, then $\frac{\partial \log(pr)}{\partial \log p} < 0$ and when $z = +\infty$, $\frac{\partial \log(pr)}{\partial \log p} > 0$.

Proof of Lemma 6. We use the second-order polynomial equation in $\sqrt{\frac{p}{z(1-\omega)+p}}$

$$\left(\sqrt{\frac{p}{z(1-\omega)+p}}\right)^2 + \sqrt{\frac{p}{z(1-\omega)+p}}\sqrt{\frac{r_0(2p-1)^2}{(a-r_0)(1-p)p}} - (\epsilon(p) - 1) = 0$$

or

$$\sqrt{\frac{p}{z(1-\omega)+p}} = \sqrt{\frac{r_0(2p-1)^2}{4(a-r_0)(1-p)p}+\epsilon(p)-1} - \sqrt{\frac{r_0(2p-1)^2}{4(a-r_0)(1-p)p}}$$

Using the expression in $\frac{p}{p+z(1-\omega)}$, we have

$$\begin{aligned} pr &= pr_0 + \sqrt{r_0 p(1-p)(a-r_0)}\sqrt{\frac{p}{z(1-\omega)+p}} \\ &= pr_0 + \sqrt{r_0 p(1-p)(a-r_0)}\left(\sqrt{\frac{r_0(2p-1)^2}{4(a-r_0)(1-p)p}+\epsilon(p)-1} - \sqrt{\frac{r_0(2p-1)^2}{4(a-r_0)(1-p)p}}\right) \\ &= \frac{r_0}{2} + \sqrt{r_0(a-r_0)\left[\frac{r_0(2p-1)^2}{4(a-r_0)} + p(1-p)(\epsilon(p)-1)\right]} \end{aligned}$$

Since $p = P(z)$ is decreasing in z, then pr depends on z only through p.

References

Abreu, D., and M. Brunnermeier (2003): "Bubbles and Crashes," *Econometrica*, 71, 173–2004.

Acemoglu, D., and F. Zilibotti (1997): "Was Prometheus Unbound by Chance? Risk, Diversification, and Growth," *Journal of Political Economy*, 105, 709–751.

Adrian, T., and H. Shin (2010): "Liquidity and Leverage," *Journal of Financial Intermediation*, 19, 418–437.

——— (2010, forthcoming): "Financial Intermediaries and Monetary Economics," *Handbook of Monetary Economics*.

Allen, F., and D. Gale (2000): "Bubbles and Crisis," *Economic Journal*, 110, 236–255.

——— (2004a): "Competition and Stability," *Journal of Money, Credit and Banking*, 36, 453–480.

——— (2004b): "Financial Intermediaries and Markets," *Econometrica*, 72, 1023–1062.

Ang, A., S. Gorovyy, and G. van Inwegen (2011): "Hedge Fund Leverage," .

Baker, M. (2009): "Capital Market Driven Corporate Finance," *Annual Review of Financial Economics*, 1, 181–205.

Barbarino, A., and B. Jovanovic (2007): "Shakeouts and Market Crashes," *International Economic Review*, 48, 385–420.

Bhamra, H., and R. Uppal (2010): "Asset Prices and Heterogeneity in Preferences and Beliefs," .

Biais, B., J. Rochet, and P. Wolley (2009): "The Lifecycle of the Financial Sector and Other Speculative Industries," .

Blum, J. (2002): "Subordinated debt, market discipline, and banks'risk taking," *Journal of Banking and Finance*, 26, 1427–1441.

Bolton, P., J. Scheinkman, and W. Xiong (2006): "Executive Compensation and Short-Termist Behavior in Speculative Markets," *Review of Economic Studies*, 73, 577–610.

Bolton, P., and E.-L. Von Thadden (1998): "Blocks, Liquidity, and Corporate Control," *Journal of Finance*, 53, 1–25.

Boot, A., and A. Schmeits (2000): "Market Discipline and Incentive Problems in Conglomerate Firms with Application to Banking," *Journal of Financial Intermediation*, 9, 240–273.

Bordo, M., and O. Jeanne (2002): "Monetary Policy and Asset Prices: Does Benign Neglect Make Sense," *International Finance*, 5, 139–164.

Boyd, J., and G. De Nicolo (2005): "The Theory of Bank Risk Taking and Competition Revisited," *Journal of Finance*, 60, 1329–1343.

Brunnermeier, M., and L. Pedersen (2009): "Market Liquidity and Funding Liquidity," *Review of Financial Studies*, 22, 2201–2238.

Challe, E., and X. Ragot (2010): "Bubbles and Self-fulfilling Crises," .

Champonnois, S. (2010): "Bank Competition and Economic Stability: the Role of Monetary Policy," mimeo, UCSD.

Cheng, I., H. Hong, and J. Scheinkman (2010): "Yesterday's Heroes: Compensation and Creative Risk-Taking," .

Dittmar, A., and A. Thakor (2007): "Why Do Firms Issue Equity?," *Journal of Finance*, 62, 1–52.

Dumas, B. (1989): "Two-Person Dynamic Equilibrium in the Capital Market," *Review of Financial Studies*, 2, 157–188.

Flannery, M. (2001): "The Faces of Market Discipline," *Journal of Financial Services Research*, 20, 107–119.

Fostel, A., and J. Geanakoplos (2008): "Leverage Cycles and the Anxious Economy," *American Economic Review*, 98, 1211–1244.

Gandel, S. (2010): "Is Proprietary Trading too Wild for Wall Street," *Time Magazine*, Feb 5.

He, Z., and A. Krishnamurthy (2008): "A Model of Capital and Crisis," *NBER Working Paper No. 14366*.

——— (2010): "Intermediary Asset Pricing," .

Hellwig, M. (2006): "Market Discipline, Information Processing, and Corporate Governance," *GESY Discussion Paper N 155*.

Hobijn, B., and B. Jovanovic (2001): "The Information-Technology Revolution and the Stock Market," *American Economic Review*, 91, 1203–1220.

Ivashina, V., and Z. Sun (2011): "Institutional Demand Pressure and the Cost of Corporate Loans," *Journal of Financial Economics*, 99, 500–522.

Jovanovic, B., and P. Rousseau (2003): "Two Technological Revolutions," *EEA Papers and Proceedings*.

Khandani, A., and A. Lo (2007): "What Happened to the Quants in August 2007," *Journal of Investment Management*, 5, 5–54.

Kiyotaki, N., and J. Moore (1997): "Credit Cycles," *Journal of Political Economy*, 105, 211–248.

Landier, A., D. Sraer, and D. Thesmar (2010): "Going for Broke: New Century Financial Corporation, 2004-2006," .

Longstaff, F., and J. Wang (2008): "Asset Pricing and the Credit Market," .

Martinez-Miera, D., and R. Repullo (2009): "Does Competition Reduce the Risk of Bank Failure?," *mimeo, CEMFI*.

Philippon, T. (2008): "The Evolution of the US Financial Industry from 1860 to 2007: Theory and Evidence," mimeo, NYU.

Rajan, R. (2006): "Has Financial Development Made the World Riskier?," *European Financial Management*, 12, 499–533.

Shleifer, A., and R. Vishny (2010): "Unstable Banking," *Journal of Financial Economics*, 97, 306–318.

Wang, J. (1996): "The Term Structure of Interest Rates in a Pure Exchane Economy with Heterogeneous Investors," *Journal of Financial Economics*, 41, 75–110.

Weinbaum, D. (2009): "Investor Heterogeneity, Asset pricing and Volatility Dynamics," *Journal of Economic Dynamics and Control*, 33, 1379–1397.

Xiong, W. (2001): "Convergence Trading with Wealth Effects: An Amplification Mechanism in Financial Markest," *Journal of Financial Economics*, pp. 247–292.