THE LIMITS OF MARKET DISCIPLINE: PROPRIETARY TRADING AND AGGREGATE RISK†

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Abstract. This paper studies the role of external claim-holders heterogeneity on entrepreneurial risk-taking. Market discipline is the mechanism by which the adjustment of cost of capital to the level of risk feeds into managerial incentives and deters excessive risk-taking. We show that risk-taking by entrepreneurs and demand for risky securities by risk-neutral investors (e.g. fund managers or proprietary traders) are mutually reinforcing. Larger risk-neutral fund managers (relative to risk-averse investors) not only undermine market discipline and lead to more risk-taking, but they also benefit more from the upside of risk and become relatively larger if the project pays out, which leads to even more risk-taking in the next period. The model explains documented features of the business cycle: bubble-like asset prices (procyclical run-up in prices and procyclical underpricing of risk), shorter cycles and countercyclical leverage of the non-financial sector.

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> *"We have also seen the emergence of a whole range of intermediaries, whose size and appetite for risk may expand over the cycle. Not only can these intermediaries accentuate real fluctuations, they can also leave themselves exposed to certain small probability risks that their own collective behaviour makes more likely. As a result, under some conditions, economies may be more exposed to financial-sector-induced turmoil than in the past."* (Rajan, 2006).

In 2006, the issuance of non-agency mortgage-backed securities reached almost $920 , a fourfold increase from 2001 in which it was $220 (see Figure 1).[1] These securities turned out ex-post to be very risky and underpriced, and arguably led to an increase in systemic risk and caused the financial crisis of 2007-2009. Figure 2 shows the share of assets by broker and dealer from the Flow of Funds data from 1985 to 2010. While the true proprietary trading firms – the hedge funds – are merged in the Flow of Funds data with households statistics, the broker and dealer statistics provide an indication of the overall behavior of proprietary trading behavior (e.g. Lehman Brothers generated 58% of revenues from proprietary trading in 2006, Gandel, 2010). [2] Should we see the issuance of such risky securities as a failure of governance, that is, a lack of monitoring, a failure to circulate information or a lack of coordination? What role, if any, did proprietary traders play in fostering risk-taking? What is the role of market discipline in preventing such run-ups in risk-taking? What is the impact of monetary or prudential policy on risk-taking?

In this paper we study the role of external claim-holders heterogeneity on entrepreneurial risk-taking incentives. Market discipline is the mechasnism by which the adjustment of the

[1]Other events: in 2000, technology stocks; in 1997, emerging market stocks.

[2]The decrease in the share of assets in 2008 is associated to huge the huge trading losses at investment banks such Deutsche Bank, UBS, etc. The increase in the share of assets by brokers/dealers during "bubble" episodes such as the 2005-2007 run-up (e.g. mortgage bonds in late 1980s before the Savings and Loan crisis, 1994 Mexican bubble and crisis). For an overview of the behavior of hedge funds in 2000, see Brunnermeier and Nagel, 2004

Figure 1.

Global CDO. *Souces*: Sifma.

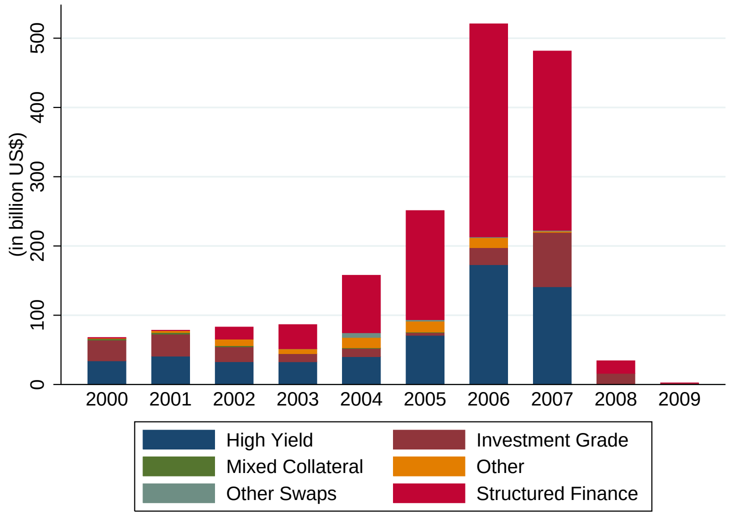


Figure 2.

Share of broker/dealer assets in total US assets. The main episodes of variations are: in the mid 1980s, the deregulation of the Savings and Loans sector allowed the trading of mortgage bonds which benefited broker/dealers until the Savings and Loans crisis; the Mexican crisis in 1994 handed Goldman Sachs heavy losses; in the end of the 1990s, broker/dealers were less exposed to the internet bubble than the general public and their share of assets shrunk until the crash in 2000; in the mid 2000s, the balance-sheet of broker/dealers grew into real estate and credit derivatives until the financial crisis in 2008. *Sources*: Flow of Funds (Federal Reserve).

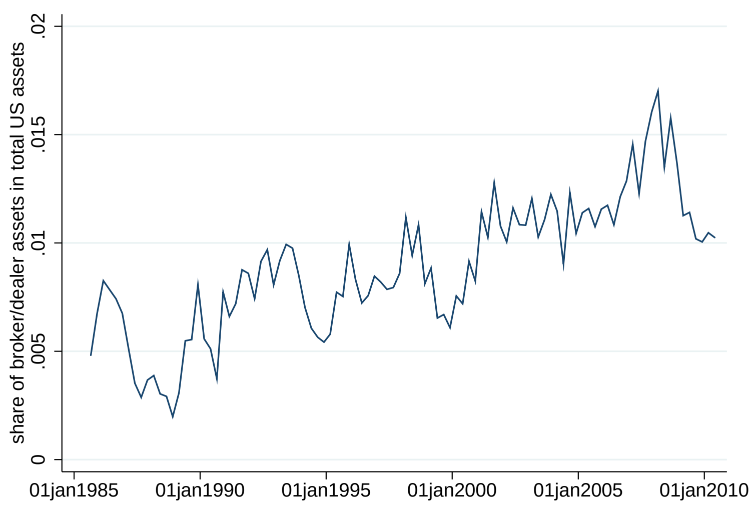


Figure 3.

The allocation of capital by heterogeneous investors.

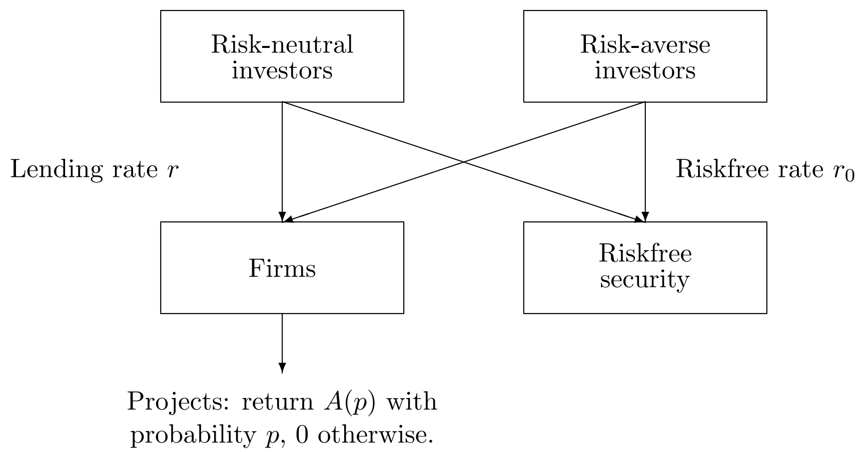
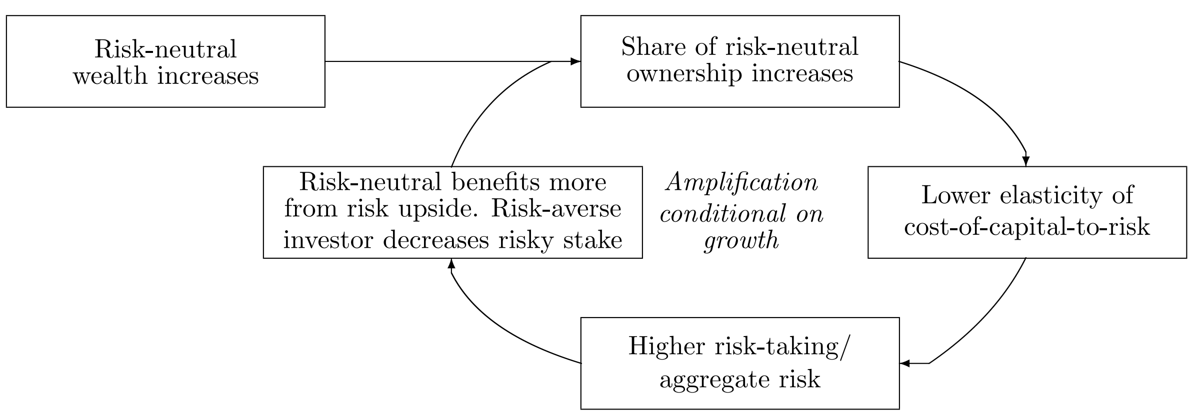


Figure 4.

Risk-taking amplification mechanism. This figure illustrates the amplification of risk. Conditional on the project paying out, the share of risk-neutral ownership of the firm increases: larger risk-neutral fund managers undermine market discipline and lead to more risk-taking, and if the project pays out, their wealth grows more than that of risk-averse investors, leading to more risk-taking in the next period.



cost of capital to risk feeds into managerial incentives and deters risk-taking (Flannery, 2001; Hellwig, 2006). First, we show how the risk-sensitivity of the capital (and therefore the efficiency of market discipline) is positively correlated with aggregate risk-aversion. Second, we show that risk-taking by entrepreneurs and demand for risky securities by risk-neutral investors (e.g. fund managers or proprietary traders) can become mutually reinforcing. Larger risk-neutral investors (relative to risk-averse investors) not only undermine market discipline and lead to more risk-taking, but they also benefit more from the upside (than risk-averse investors) because risky securities represent a larger share of their portfolios. The model explains documented features of the business cycle: bubble-like asset prices (procyclical run-up in prices and procyclical underpricing of risk) and countercyclical leverage of the non-financial sector.[3] Third, we study the effect of policy on risk-taking. In particular, we show that a tightening of monetary policy can induce (rather than deter) risk-taking.

The setup is as follows. There are an infinitely-lived risk-averse investor and a risk-neutral fund manager (with a binding capital-ratio constraint) who provide funds to a penniless entrepreneur who has access to a set of projects with different risk characteristics. The model endogenizes the risk-taking decisions of entrepreneurs as in Blum (2002); Allen and Gale (2004); Boyd and De Nicolo (2005); Martinez-Miera and Repullo (2009). All securities are correctly risk-adjusted (no mispricing, no behavioral types).[4] The main contribution of this paper is to highlight a risk amplification mechanism in this context (Figure 4). If the

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[3]In this paper we focus on long-only investors. Abreu and Brunnermeier (2003) study boom/bust dynamics with exogenous price divergence from fundamentals. In this paper, the "divergence" from fundatementals is endogenous and the consequence of demand for assets from risk-averse investors and risk-neutral fund managers in a production economy with an equilibrium supply of risky security.

[4]In Xiong (2001), arbitrageurs's wealth matters for the risk premium because arbitrageurs (ie. convergence traders) can absorb some of the fluctuations created by noise traders by trading against them (and occasionally amplifying the fluctuations when the wealth shocks are large enough). In this model, all agents has long positions, there are no noise traders and there is no mispricing.

relative wealth of risk-neutral investors is small, their ownership share of the firm is small too, and the elasticity of the cost of capital to risk is high, inducing the entrepreneur to choose a lower risk projects. Since this project represents a higher share of the portfolio of the risk-neutral investor (than the risk-averse investor), the risk-neutral investor benefits more from the upside. If the project pays off, the ownership share by risk-neutral investors increase and the elasticity of the cost of capital decreases inducing the entrepreneur to choose an even riskier project than before and the risk-averse investor rebalances his portfolio into the riskfree asset. As long as the project generates positive returns, the stake of the risk-neutral investors increases and aggregate risk increases.[5]

I. Related literature

I.1. **Corporate governance with heterogeneous claim-holders.** There is a large literature on corporate governance studying how the ownership structure affects the investment decisions of a firm (Hellwig, 2006). The question is how to give incentives to shareholders and investors to exert control and monitor, for instance through the production of information (Faure-Grimaud and Gromb, 2004; Holmstrom and Tirole, 1993).[6] The higher the stake of external investors into the firm, the easier it is to provide such incentives, but this large stake can generate liquidity costs (Bolton and Von Thadden, 1998). Blum (2002) presents of model of bank risk-taking with homogeneous external investors. He analyzes the ability to commit to a level of risk at the financing stage affects aggregate risk. Allen and Gale (2000); Challe and Ragot (2010) study the role of limited liability in inducing endogenous bubbles.[7]

[5]To prevent one type to dominate the economy (and a degenerate asymptotic cross-sectional distribution of wealth), we set the parameters so that the relative wealth of risk-neutral arbitrageurs is a random-walk.

[6]Boot and Schmeits (2000) analyze risk-taking in a conglomerate model with an imperfect monitoring technology.

[7]Fostel and Geanakoplos (2008); Kiyotaki and Moore (1997) describe collateral-driven cycles (see also He and Krishnamurthy (2008) and Brunnermeier and Pedersen (2009) for the role of varying margins)

In this paper, we abstract from informational asymmetries commitment issues and limited liability and analyzes how the distribution of wealth across heterogeneous investors affect risk-taking managerial incentives. If the largest potential shareholder is very risk-averse, the entrepreneur will have an incentive to choose a safe project to raise as much capital as possible from him.

I.2. **Asset pricing with heterogeneous investors.** The asset pricing properties of models in which agents have different risk-aversion parameters carries over to this model (Dumas, 1989; Wang, 1996; Longstaff and Wang, 2008; Weinbaum, 2009). In particular, the portfolio of risk-tolerant agent is biased towards the risky security so that adverse shocks to risky securities affect it more and reduce its contribution to aggregate risk averse. Such logic explains why there is a negative correlation between risky return and the risk premium. One contribution of this paper is to endogenize the supply of securities. In particular, when the risk premium is low, this is precisely when entrepreneurs want to issue risky securities, which links the issuance of risky securities to the aggregate risk-aversion and risk premium.

I.3. **Aggregate risk.** This argument in Acemoglu and Zilibotti (1997) is that growth and wealth accumulation decrease the volatility of the economy. In this paper, we focus on the 2007-2009 financial turmoil that centered on developed economies by showing how the accumulation of wealth in certain parts of the financial sector might be destabilizing. A large literature has looked at the consequences of technological waves on the real economy and the financial sector (Jovanovic and Rousseau, 2003). The uncertainty associated to the new technology or the financial innovation generates an amplification of the business cycle through learning with booms and busts (Barbarino and Jovanovic, 2007; Hobijn and Jovanovic, 2001; Biais, Rochet, and Wolley, 2009). Another view is that booms and busts are not caused primarily by exogenous technology shocks, but by the demand for risky securities from risk-tolerant intermediaries. The emergence of new entities such as private equity and hedge funds as very large players on financial markets might have increased not only the risk-bearing capacity of the economy but also the issuance of risky securities.[8]

I.4. **The impact of policy.** The key theme of policy literature is whether prudential or monetary policy can supplement market discipline. In this environment, we study the role of interest-rate policy and of prudential regulation. Monetary policy in this model operates through a credit channel (Champonnois, 2009). A tightening of monetary policy (with an increase of the riskfree rate) induces the risk-averse investor to rebalance its portfolio towards the riskfree security and therefore undermines the market discipline by increasing the importance of the risk-tolerant fund manager for the decisions of the entrepreneur.[9]

If a goal of policy is to smooth fluctuations (Bordo and Jeanne, 2002), then it needs to induce a stable share of risk-tolerant ownership in the firm. For example, a money-supply neutral policy (with a countercyclical interest-rate policy) lowers the riskfree rate as the size of the fund manager increases relative to the risk-averse investor) and incentivizes the risk-averse investor not to rebalance its portfolio as the size of the risk-neutral fund manager increases. Equivalently, prudential regulation with procyclical capital requirement could prevent the fund manager from taking over the ownership of the firm. Alternatively, capital ratio that would increase with the fund size would limit the decrease in market discipline.

[8]See also Philippon (2008) for theory and evidence on the equilibrium between the demand and supply of financial services.

[9]This is related to Landier, Sraer, and Thesmar (2010) who show that a monetary tightening in 2004 led the mortgage lender Century Corporation to offer riskier loans. See also Adrian and Shin (2010) who describe a risk-shifting channel of monetary policy.

II. Model

There are two infinitely-lived agents, a risk-averse investor with log-utility and a risk-neutral fund manager. The risk-averse investor maximizes

$$U = \max E_0 \sum \beta^t \log c_t.$$

With probability δ, the risk-neutral investor is forced to consume $1-\theta$ of the funds wealth in a given period. Under this constraint he maximizes

$$V = \max E_0 \sum \beta^t c_t^*.$$

The two investors consume a single good and have access to two investment opportunities. They can invest in a riskfree security return r_0 and capital requirements force the fund manager to put at least a share ω is the riskfree security. The investor and the fund manager can invest in a risky project project that pays r with probability p or zero otherwise. The project is chosen and run by a risk-neutral entrepreneur, who lives for one-period.[10] More specifically, he chooses among constant-return projects in which the productivity $a(p)$ of a given project is itself a function of the probability p. Projects with a higher probability of generating revenues have a lower productivity (ie. $a(p)$ is a decreasing function).

The timing is as follows in a given period. The entrepreneur chooses a project with a probability p and productivity $a(p)$ and proposes a rate r to external investors. Given the return r and risk p of the project, the risk-averse investor and the fund manager make their portfolio decisions. The project generates revenues or not and external claimants are repaid. When the risk-averse investor and the risk-neutral fund manager makes their

[10]The entrepreneur can be interpreted as a mortgage originator as in Landier, Sraer, and Thesmar (2010). More generally, he represents the "sell-side" in the financial sector while the investors represent the "buy side".

portfolio decisions, they take the returns as given, which gives the entrepreneur some market power in making the investment decision.[11]

An important consequence of the timing is that external claimants cannot condition their financing on the risk-profile of the project. But ex-post, they decide how much financing to provide if they think the project is too risky which provides some market discipline.

II.1. **Supply of capital.** The consumption and portfolio decisions are considerably simplified by the assumption on risk-preference (log utility and risk-neutral utility).

Lemma 1 (consumption decisions)**.** *The risk-averse investor consumes a constant fraction* $(1-\beta)$ *of its wealth at the beginning of the period. The risk-neutral fund manager consumes the exogenous fraction* $1-\theta$ *when it is forced to (with probability* δ*) and otherwise saves all his wealth.*

If $\langle p_t, r_t, y_t, x_t \rangle$ are the probability of growth, the return of the risky project, the savings of the risk-averse investor and the capital invested in the project by the risk-neutral fund manager, we have

Lemma 2 (portfolio decisions)**.** *If* $p_t r_t \geq r_0$*, the the risk-averse investors puts* $K(r_t, p_t) = p_t y_t - \frac{r_0 y_t (1-p_t)}{r_t - r_0}$ *into the risky project and* $y_t - K(r_t, p_t)$ *into the riskfree security. The risk-neutral fund manager invests all his savings in the risky project.*

Note that $K(r, p)$ is increasing concave in r and increasing in p. The dynamics are

$$y_{t+1} = \begin{cases} \beta p_t r_t y_t & \text{w.p } p_t \\ \beta \frac{y_t(1-p_t) r_0 r_t}{r_t - r_0} & \text{w.p } 1-p_t \end{cases} \quad ; \quad x_{t+1} = \begin{cases} x_t(\omega r_0 + (1-\omega) r_t) & \text{w.p } p_t(1-\delta) \\ \theta x_t(\omega r_0 + (1-\omega) r_t) & \text{w.p } p_t \delta \\ x_t \omega r_0 & \text{w.p } (1-p_t)(1-\delta) \\ \theta x_t \omega r_0 & \text{w.p } (1-p_t)(1-\delta) \end{cases}$$

[11]Since the entrepreneur has market power, the equilibrium allocation is different from the planner's allocation (Dumas, 1989; Wang, 1996).

If we introduce the relative wealth of the fund manager $z_t = \frac{x_t}{y_t}$, we have

$$\frac{z_{t+1}}{z_t} = \begin{cases} \left(\frac{\omega r_0 + (1-\omega) r_t}{\beta p r_t}\right) & \text{w.p } p_t(1-\delta) \\ \theta\left(\frac{\omega r_0 + (1-\omega) r_t}{\beta p r_t}\right) & \text{w.p } p_t\delta \\ \left(\frac{\omega(r_t - r_0)}{(1-p) r_t}\right) & \text{w.p } (1-p_t)(1-\delta) \\ \theta\left(\frac{\omega(r_t - r_0)}{(1-p) r_t}\right) & \text{w.p } (1-p_t)(1-\delta) \end{cases} \tag{2.1}$$

Assumption 1. *We assume that the (effective) discount rates of the risk-averse and risk-neutral investors are equal*

$$\beta = \theta\delta + (1-\delta) \tag{2.2}$$

Since the risk-neutral investor is forced to consume $1-\theta$ of its wealth with probability δ, then $\theta\delta + (1-\delta)$ is the appropriate discount for its wealth.

Lemma 3. *Under Assumption 1, the state variable z_t is a random walk: $E_t(z_{t+1}) = z_t$.*

Using Equation (2.1), it is easy to show that $E(z_{t+1}) = \frac{z_t[\theta\delta + (1-\delta)]}{\beta} = 1$. The important consequence of Assumption 1 and Lemma 3 is that asymptotically, neither the risk-averse investor nor the risk-neutral investor dominate the economy with their wealth.

In the case of growth, the state variable z will grow if the risk-neutral investor is more exposed to the risky security than the risk-averse agent. Since the portfolio choice of the risk-neutral investor is constrained by the capital requirement ω, this happens if

$$1 - \omega \geq \frac{pr - r_0}{r - r_0} \tag{2.3}$$

We later choose ω small enough so that this condition is generally satisfied.

II.2. **Demand for capital.** In this section only, we omit the subscript t. The profit of the entrepreneur is $p[a(p) - r][x + K(r,p)]$ and the first-order conditions of in the return r and the probability of growth p are

$$\frac{a-r}{r} = \frac{1}{\frac{\partial \log(x+K(r,p))}{\partial \log r}}$$
$$-\frac{pa'(p)}{a-r} = 1+\frac{\partial \log[x+K(r,p)]}{\partial \log p}$$

The first equation is the usual relation between markups and the elasticity of capital supply in monopolistic models: the markup of the entrepreneur is smaller when the supply of capital is more elastic in the return r. The second equation relates the elasticity of productivity and to market discipline and the elasticity of capital supply relative to risk: the productivity elasticity (relative to risk) is convex and risk is higher if there is less market discipline (that is, if the supply of capital is more elastic relative to risk). The entrepreneur increases risk until the decrease in the supply of capital has a higher cost than the benefit of an increase in productivity. Under the condition $\frac{a-r_0}{r_0} \geq \frac{y(1-p)}{x+py}$, the condition first equation implies $\frac{a-r}{a-r_0} = \frac{x+k}{x+py} = 1-\sqrt{\frac{r_0 y(1-p)}{(a-r_0)(x+py)}}$.

The equilibrium depends on one unique state variable $z = \frac{x}{y}$ defined as the ratio of the wealths of risk-neutral fund manager and risk-averse investor. The following Lemma characterizes the equilibrium aggregate risk.

Lemma 4. *With $\theta = -\frac{pa'}{a-r_0}$, the equilibrium probability of growth p is decreasing in the fund manager's relative wealth and in the riskfree rate r_0*

$$\theta = 1+\frac{p}{z+p}+\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\left(\frac{p}{1-p}-1\right) \tag{2.4}$$

Lemma 5. *The share of the portfolio of the risk-averse agent in the risky security is decreasing with z and r_0.*

Introduce $p_{\max}$ defined as $\theta(p_{\max}) = 2$ and $\theta(p_{\min}) = 1$. $p_{\max}$ is the probability of growth if the risk-neutral fund managers are infinitely small ($z = 0$) and the riskfree rate r_0 is zero. $p_{\min}$ is the probability of growth if the risk-neutral fund managers are infinitely large ($z = +\infty$). We make the following assumption on the capital requirement ω:

Assumption 2. *The capital requirement is such that the fund manager is always more exposed to the risky asset than the risk-averse investor (for any z and any r_0)*

$$1 - \omega \geq p_{\max}$$

Assumption 2 imposes that the maximum portfolio share in the risky security for the risk-averse agent (for $z = 0$) is always below that of the risk-neutral fund manager. As discussed before, a consequence of this assumption is that the state variable z increases with good returns from the risky technology and decreases with bad returns and this leads to procyclical risk.

Corollary 1 (Leverage)**.** *The leverage $\frac{a}{a-r}$ and risk-adjusted return $\frac{r}{a}$ is increasing in the growth probability p and the riskfree return r_0 and decreasing in the relative wealth z.*

II.3. **Welfare.**

Corollary 2 (Market discipline)**.** *The elasticity of the supply of capital $\frac{\partial \log[x+K(r,p)]}{\partial \log p}$ and the elasticity of the cost to risk $-\frac{\partial \log(r-r_0)}{\partial \log p}$ are positive, decreasing in the fund manager's relative wealth z and increasing in the probability of growth p.*

The per-period utility of the risk-averse agent is $u = p \log(pry) + (1-p) \log \left[\frac{(1-p)r_0 ry}{r-r_0}\right]$ and the utility of the risk-neutral fund manager is $u^* = x \left(pr + \frac{\omega}{1-\omega} r_0\right)$.

Corollary 3 (Welfare)**.** *The utility of the risk-averse agent is increasing in the probability of growth p. The utility of the risk-neutral fund manager is increasing (resp. decreasing) in the probability of growth p for low p (resp. for high p).*

If we define the objective of a (myopic) planner as the weighted sum of the per-period utility of the risk-neutral and risk-averse investors and the entrepreneur, we have

$$w = \lambda_{\text{risk-averse}} u + \lambda_{\text{risk-neutral}} u^* + \lambda_{\text{entrepreneur}} p(a-r)(x+k)$$

Corollary 4. *The planner is more likely too assess that there is too much risk (in the sense that* $\frac{\partial w}{\partial p} < 0$*) if the welfare weight of the risk-averse agent* $\lambda_{\text{risk-averse}}$ *is high (relative to* $\lambda_{\text{risk-neutral}}$*) and if the risk is high (so that* $\frac{\partial u^*}{\partial p} < 0$*).*

First, since the entrepreneur chooses the level of risk, any marginal risk change generates only a second-order loss and so its utility does not matter for the considerations of the planner. Second, the risk-averse investor always considers that there is too much risk. If he is important enough (as measured by $\lambda_{\text{risk-averse}}$), then the planner will have an incentive to decrease risk. An intervention to decrease risk is then to lower risk is then to decrease the riskfree rate r_0 or to increase the capital requirement of the fund ω.

II.4. **Special case: productivity linear in risk.**

Assumption 3. *We assume that productivity is linear in the probability of growth* p*:* $p \in [1/2, 1]$*,* $a = 1 - hp$*.*[12]

Lemma 6. *The equilibrium risk* $P(z)$ *is decreasing in* z *(and* r_0*) and is determined by*

$$\frac{hp}{1-hp-r_0} = 1 + \frac{p}{z+p} + (2p-1)\sqrt{\frac{r_0}{(1-hp-r_0)(1-p)(z+p)}} \qquad (2.5)$$

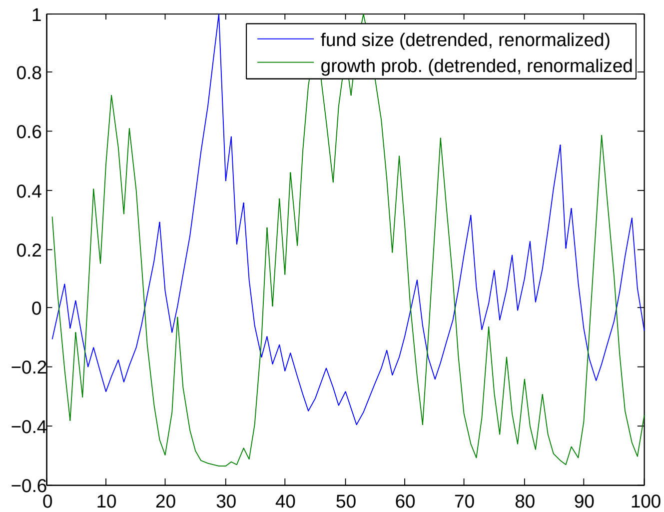
Moreover, $p_{\min} = \frac{1-r_0}{2h}$ *and* $p_{\max} = \frac{2}{3h}$*.*

Assumption 2 is now equivalent to $1 - \omega \geq \frac{2}{3h}$. Figure 5 shows the negative correlation between the relative size of risk-neutral fund managers z and the probability of growth p.

[12]This is without loss of generality. If $a = A(1-hp)$, we can renormalize a and r_0 by A.

FIGURE 5.

Fund size and growth probability. This figures shows the relative fund size z and the probability of growth p (both series are detrended and renormalized).



III. POLICY

As in Champonnois (2008), we study the role of interest-rate policy in affecting the allocation of capital. More precisely, interest-rate policy operates through a credit channel of monetary policy in which higher interest rates lead to a rebalancing of the portfolio of agents.

Corollary 5. *An increase of riskfree rate r_0 by the monetary authority increases aggregate risk in the economy.*

In this model, a higher riskfree rate leads the risk-averse investor to have a lower share of the ownership of the project which, in turn, undermine market discipline and induces the entrepreneur to choose riskier projects.

IV. CONCLUSION

Extensions

- risk-averse hedge fund manager (but still more risk-tolerant than the investors)

- external financing for hedge funds
- several-asset model (similar to Boot and Schmeits 2000)

V. Appendix: Proofs

Proof of Lemma 2. The value function of an infinitely-lived agent with log utility is $V(y) = a + b\log(y)$. The first-order condition ∂k is then

$$\frac{p}{r_0k_0 + rk} = \frac{1}{ry} \quad \Leftrightarrow \quad k = py - \frac{r_0(y-k)}{r}$$

and using $k_0 = y - k$, we find the supply of capital.

Proof of Lemma 3. The expected value of next period state variable is

$$\begin{aligned}
\frac{\beta E(z_{t+1})}{z_t} &= P(z_t)(1-\delta)\frac{(\omega r_0 + (1-\omega)R(z))}{P(z_t)R(z_t)} + P(z_t)\delta\frac{\theta(\omega r_0 + (1-\omega)R(z))}{P(z_t)R(z_t)} + ... \\
&\quad ... + (1-P(z_t))(1-\delta)\frac{\omega r_0}{(1-P(z_t))\frac{R(z_t)r_0}{R(z_t)-r_0}} + (1-P(z_t))(1-\delta)\frac{\theta\omega r_0}{(1-P(z_t))\frac{R(z_t)r_0}{R(z_t)-r_0}} \\
&= (1-\delta)\frac{(\omega r_0 + (1-\omega)R(z))}{R(z_t)} + \delta\frac{\theta(\omega r_0 + (1-\omega)R(z))}{R(z_t)} + (1-\delta)\frac{\omega(R(z_t)-r_0)}{R(z_t)} + \delta\frac{\theta\omega(R(z_t)-r_0)}{R(z_t)} \\
&= (1-\delta) + \delta\theta
\end{aligned}$$

Proof of Lemma 4. The first-order conditions are

$$\begin{aligned}
\partial r: &\quad \frac{(a-r_0)r_0y(1-p)}{(r-r_0)^2} = x + py \\
\partial p: &\quad (a-r)(x+k) + py\left(\frac{r}{r-r_0}\right)(a-r) = -pa'(x+k)
\end{aligned}$$

where $k = K(r,p) = py - \frac{(1-p)r_0y}{r-r_0}$. The condition ∂p can be written as

$$\begin{aligned}
\theta &= 1 - \sqrt{\frac{r_0y(1-p)}{(a-r_0)(x+py)}} + \frac{py}{x+py}\left(1 + \frac{r_0}{(a-r_0)\sqrt{\frac{r_0y(1-p)}{(a-r_0)(x+py)}}}\right) \\
&= 1 + \frac{py}{x+py} + \sqrt{\frac{r_0y(1-p)}{(a-r_0)(x+py)}}\left(\frac{p}{1-p} - 1\right)
\end{aligned}$$

Two remarks

- When $r_0 = 0$, then $\pi = p(x+py)a$. A necessary condition for the risk choice p to be decreasing in x/y is that the elasticity $-\frac{pa'(p)}{a(p)}$ is increasing.

– The first-order condition is $\frac{-pa'(p)}{a} = 1 + \frac{py}{py+x}$. The right-hand side is increasing in p, decreasing in x. The solution $p(x)$ is decreasing in x if the left-hand side crosses the right-hand side from below.

- The condition $pr \geq r_0$ is satisfied if $\theta \geq \frac{a}{a-r_0} \Leftrightarrow -\frac{pa'}{a} > 1$. This last condition imposes that the expected productivity pa is always decreasing. Rewriting with $z = \frac{x}{y}$, we get

$$(\theta - 1)(z + p) - \sqrt{z+p}(2p-1)\sqrt{\frac{r_0}{(1-p)(a-r_0)}} - p = 0$$

There exists a positive z solution to this equation if and only if

$$2 + (2p-1)\sqrt{\frac{r_0}{p(1-p)(a-r_0)}} \geq \theta \tag{5.1}$$

If this condition is satisfied,

$$\begin{aligned}
\sqrt{z+p} &= \frac{(2p-1)\sqrt{\frac{r_0}{(1-p)(a-r_0)}} + \sqrt{(2p-1)^2\frac{r_0 y}{(1-p)(a-r_0)} + 4p(\theta-1)}}{2(\theta-1)} \\
\sqrt{\frac{r_0 y(1-p)}{(a-r_0)(x+py)}} &= \sqrt{\frac{r_0(1-p)}{(a-r_0)}}\left(\frac{\sqrt{(2p-1)^2\frac{r_0}{(1-p)(a-r_0)} + 4p(\theta-1)} - (2p-1)\sqrt{\frac{r_0}{(1-p)(a-r_0)}}}{2p}\right) \\
\frac{pr}{r_0} - 1 &= \left(\frac{\sqrt{(2p-1)^2 + \frac{4(1-p)p(\theta-1)(a-r_0)}{r_0}} - 1}{2}\right)
\end{aligned}$$

Proof of Lemma 5. The weight is $\frac{pr-r_0}{r-r_0}$ so that

$$\begin{aligned}
\frac{\partial\left(\frac{pr-r_0}{r-r_0}\right)}{\partial \log p} &= \frac{pr}{r-r_0} + \frac{r_0(1-p)}{r-r_0}\frac{\partial \log(r-r_0)}{\partial \log p} \\
&= \frac{(1-p)r_0}{r-r_0}\left[\frac{p}{1-p}\left(1 + \frac{(a-r_0)\sqrt{\frac{r_0(1-p)}{(a-r_0)(p+z)}}}{r_0}\right) - \frac{1}{2(1-p)} - \frac{p}{p+z} - \frac{1}{2}\sqrt{\frac{r_0(1-p)}{(a-r_0)(p+z)}}\left(\frac{p}{1-p} - 1\right)\right] \\
&= \frac{(1-p)r_0}{r-r_0}\left[\frac{1}{2}\left(\frac{p}{1-p} - 1\right)\left(1 - \sqrt{\frac{r_0(1-p)}{(a-r_0)(p+z)}}\right) + \frac{p}{p+z}\left(\frac{1}{\frac{1}{\sqrt{\frac{r_0(1-p)}{(a-r_0)(p+z)}}}} - 1\right)\right] \geq 0
\end{aligned}$$

Since $\frac{pr-r_0}{r-r_0} = p - (p+z)\sqrt{\frac{r_0(1-p)}{(a-r_0)(p+z)}}$, the share of the risk-averse agent in the risky asset is decreasing in r_0. It is also easy to show that $\frac{\partial\left(\frac{pr-r_0}{r-r_0}\right)}{\partial z} < 0$ and since p is decreasing in z and r_0, the share of the risk-averse agent in the risky security $\frac{pr-r_0}{r-r_0}$ is decreasing in the relative wealth if the risk-neutral fund manager z and r_0.

Proof of Corollary 1.

Proof. Using $r = r_0\left(1-\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\right) + a\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}$, we have

$$\begin{aligned}
\frac{\partial \log(r/a)}{\partial \log p} &= \left(\frac{a-r_0}{ar}\right)\left(\theta r + a\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\frac{\partial \log(r-r_0)}{\partial \log p}\right) \\
&= \left(\frac{a-r_0}{ar}\right)\left[\theta r_0\left(1-\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\right) + a\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\frac{\partial \log\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}}{\partial \log p}\right] \\
&= \left(\frac{a-r_0}{ar}\right)\left(1-\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\right)\left[\theta r_0 - a\frac{1}{2}\left(\frac{p}{1-p}-1\right)\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\right] \\
&= \left(\frac{r_0(a-r_0)}{ar}\right)\left(1-\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\right)\left[1+\frac{p}{p+z}-\frac{1}{2}\left(\frac{p}{1-p}-1\right)\left(\frac{1-p}{p+z}-\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\right)\right] \\
&= \left(\frac{r_0(a-r_0)}{ar}\right)\left(1-\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\right)\left[1+\frac{1}{2(p+z)}+\frac{\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}}{2}\left(\frac{p}{1-p}-1\right)\right] > 0
\end{aligned}$$

∎

Proof of Corollary 2. We have

$$\begin{aligned}
\frac{\partial \log\left(\sqrt{\frac{r_0 y(1-p)}{(a-r_0)(x+py)}}\right)}{\partial \log p} &= \frac{1}{2}\left(-\frac{pa'(p)}{a-r_0}-\frac{p}{1-p}-\frac{p}{p+z}\right) \\
&= \frac{1}{2}\left[1+\frac{p}{p+z}+\sqrt{\frac{r_0 y(1-p)}{(a-r_0)(x+py)}}\left(\frac{p}{1-p}-1\right)-\frac{p}{1-p}-\frac{p}{p+z}\right] \\
&= -\frac{1}{2}\left(1-\sqrt{\frac{r_0 y(1-p)}{(a-r_0)(x+py)}}\right)\left(\frac{p}{1-p}-1\right) < 0
\end{aligned}$$

Since $\frac{x+k}{x+py} = 1-\sqrt{\frac{r_0 y(1-p)}{(a-r_0)(x+py)}}$, the elasticity of the supply of capital is positive and decreasing in z

$$\frac{\partial \log(x+k)}{\partial \log p} = \frac{p}{p+z}+\frac{1}{2}\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\left(\frac{p}{1-p}-1\right) > 0$$

and the elasticity of the cost of capital to risk is positive and decreasing in z

$$-\frac{\partial \log(r-r_0)}{\partial \log p} = \frac{1}{2(1-p)}+\frac{p}{z+p}+\frac{1}{2}\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\left(\frac{p}{1-p}-1\right) > 0$$

Moreover

$$\begin{aligned}
\frac{\partial^2 \log(x+k)}{\partial \log p^2} &= \frac{pz}{(p+z)^2} + \frac{p^2}{2(1-p)^2}\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}} - ... \\
&\quad ... - \frac{1}{4}\left(\frac{p}{1-p}-1\right)^2 \sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\left(1-\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\right) \\
&= \frac{pz}{(p+z)^2} + \sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\underbrace{\left[\frac{p^2}{2(1-p)^2} - \frac{1}{4}\left(\frac{p}{1-p}-1\right)^2\right]}_{>0} + \frac{1}{4}\left(\frac{p}{1-p}-1\right)^2 \frac{r_0(1-p)}{(a-r_0)(z+p)}
\end{aligned}$$

and

$$\begin{aligned}
-\frac{\partial^2 \log(r-r_0)}{\partial \log p^2} &= \frac{p}{2(1-p)^2} + \frac{pz}{(z+p)^2} + \frac{p}{2(1-p)^2}\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}} \\
&\quad -\frac{1}{4}\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\left(1-\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\right)\left(\frac{p}{1-p}-1\right)^2 > 0
\end{aligned}$$

since the term in $\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}$ is $\frac{2p-(2p-1)^2}{4(1-p)^2} > 0$ whenever $p > \frac{1}{2}$.

Proof of Corollary 3. The utility of the risk-averse agent is $u = p\log(pry) + (1-p)\log\left[\frac{(1-p)r_0 ry}{r-r_0}\right]$. Moreover we have

$$\frac{\partial \log(r-r_0)}{\partial \log p} = -\frac{1}{2}\left[\frac{2p}{p+z} + \frac{1}{1-p} + \sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\left(\frac{p}{1-p}-1\right)\right] \leq 0$$

This implies

$$\begin{aligned}
\frac{\partial u}{\partial p} &= \log\left[\frac{p(r-r_0)}{(1-p)r_0}\right] + \frac{\partial r}{\partial p}\left(\frac{1}{r} - \frac{1-p}{r-r_0}\right) \\
&= \log\left[1 + \frac{pr-r_0}{(1-p)r_0}\right] + \left(\frac{pr-r_0}{pr}\right)\frac{\partial \log(r-r_0)}{\partial \log p} \\
&\geq \left(\frac{pr-r_0}{pr}\right)\left\{\frac{pr}{(1-p)r_0} - \frac{1}{2}\left[\frac{2p}{p+z} + \frac{1}{1-p} + \sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\left(\frac{p}{1-p}-1\right)\right]\right\} \\
&= \left(\frac{pr-r_0}{pr}\right)\left\{\frac{p}{1-p} + \frac{p(a-r_0)}{(1-p)r_0}\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}} - \frac{1}{2}\left[\frac{2p}{p+z} + \frac{1}{1-p} + \sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\left(\frac{p}{1-p}-1\right)\right]\right\} \\
&= \left(\frac{pr-r_0}{pr}\right)\left[\frac{1}{2}\left(\frac{p}{1-p}-1\right)\left(1-\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\right) + \frac{p}{p+z}\left(\frac{1}{\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}}-1\right)\right] \geq 0
\end{aligned}$$

We have

$$\begin{aligned}
\frac{\partial \log(pr)}{\partial \log p} &= 1 + \left(\frac{r-r_0}{r}\right)\frac{\partial \log(r-r_0)}{\partial \log p} \\
&= \left(\frac{r-r_0}{2r}\right)\left[2 + \frac{2(p+z)}{1-p}\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}} - \frac{2p}{p+z} - \frac{1}{1-p} - \sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}\left(\frac{p}{p-1} - 1\right)\right] \\
&= \frac{2z+1}{1-p}\left(\frac{r-r_0}{2r}\right)\left[\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}} - \frac{p-\frac{z}{2z+1}}{p+z}\right]
\end{aligned}$$

The term in bracket is decreasing in p (since in particular $\sqrt{\frac{r_0(1-p)}{(a-r_0)(z+p)}}$ is decreasing in p), so that for high risk (low p), a lower risk increases the expected return, while for low risk (high p), an increase in risk leads to higher expected return.

Note: when $z = 0$, then $\frac{\partial \log(pr)}{\partial \log p} < 0$ and when $z = +\infty$, $\frac{\partial \log(pr)}{\partial \log p} > 0$.

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