Time-Varying Correlations Between Credit Risks and Determinant Factors

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Abstract

This study provides a perspective on the correlation between credit risks and their determinants over time, as it has been an important issue to identify variables in determining credit risks. If the impact of variables on credit risks varies over time, different points in time have different coefficients to reflect different credit conditions. Without complex assumption of parametric form in advance, we construct a time-varying coefficient model to characterize the coefficient and significance dynamics of determinant factors with a large amount of data. Both single factor and multi-factor models provide strong evidence to support that coefficient estimates and their corresponding significance are markedly changed after credit crises. The estimated results also reveal a remarkable time-varying correlation between stock and credit markets. This study further evaluates the credit spreads in the out-of-sample period with the default intensity pricing model to demonstrate that pricing errors can be efficiently reduced by considering the time-varying correlations between credit spreads and their determinants. There are two further findings of interest. First, the industry-wide credit risk diminished during difficult credit conditions. Second, under the consideration of firm-specific and industry-wide risk factors, the macroeconomic variable is still extremely relevant in explaining credit spreads.

Keyword: time-varying coefficient, credit determinant, intensity model, credit default swap, credit crisis.

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1. Introduction

The determinants for credit risks have been widely studied (e.g., Collin-Dufresne *et al.*, 2001; Elton *et al.*, 2001; Eom *et al.*, 2004; Longstaff *et al.*, 2005; Longstaff & Rajan, 2008; Bhansali *et al.*, 2008; Tang & Yan, 2010). Many variables have been proved to be strongly correlated with credit conditions, including some firm-specific financial factors and common systematic components. Ericsson *et al.* (2009) further showed that time series parameter estimates of credit risk determinants have a clear trend (parameters tend to continue rising in the sample period) and economic significance varies over time. However, in traditional time-series regression models, coefficients are assumed to be constant for a long period. If the coefficients between credit risks and their explanatory variables are different in time, constant coefficients may not adequately model the changing credit market. This study investigates the changes of time-varying coefficients and corresponding significance of potential credit determinants to identify their estimation performance over time for a fresh look to better facilitate the management of credit challenges.

As the current credit crunch was triggered by the subprime mortgage crisis leading to clustered defaults of financial institutions, investment banks made poor decisions and rating agencies also failed to recognize the worst possible scenario. After struggling with the most volatile and unnerving period on record of the global economy, it is important to verify whether the difficult macro environment and serious credit conditions have changed the correlations between credit risks and their determinant factors. In order to understand how credit risks are dynamically influenced by covariates, a time-varying coefficient model is provided to characterize coefficient dynamics and their corresponding significance. The estimated results reveal that coefficients change over time, especially during serious credit conditions. The corresponding significance levels of all factors are also different in time. As the correlations between credit risks and their determinants are significantly different when struggling with great credit challenges, these time-varying correlations could be a crucial issue in determining credit risk.

Four objects are investigated. First, we construct a time-varying coefficient regression model to identify the dynamic relationships between credit spreads and their potential determinant factors. In order to avoid the necessity of strong parametric assumptions, the dynamic relationships are estimated in a time pointwise manner using a sufficiently large amount of data without assuming any parametric forms. Additionally, rather than using corporate bond yield spreads, the credit default swap (CDS) spreads are applied to recognize credit conditions directly to eliminate any disturbance from interest rates. Therefore, we precisely investigate the time-varying influences of credit determinants on cross-section CDSs, the most widely traded credit derivatives. Second, as prior researchers have identified many credit determinants with traditional linear regression models assuming constant coefficients, the constructed time-varying coefficient regression model is applied to these potential credit determinants one by one to identify the specific impact of each factor over time. The shape of all coefficient dynamics reveals that the relationship between credit spreads and their determinants apparently change over time. Third, according to the *p*-value and adjusted *R*-square dynamics of each factor during the entire sample period, substantial explanatory credit determinants are selected to construct multi-factor models. Thus, the model fitting performance of time-varying coefficient regression model before and after the credit crisis can be further identified. Fourth, to figure out how much the credit spread estimation performance can be improved with time-varying coefficients, we construct a CDS pricing model based on default intensity model to compare the out-of-sample pricing results of time-varying coefficients to those of traditional constant coefficients.

Duffie and Gˆarleanu (2001) identified three types of default events in their framework: firm-specific defaults, industry-wide defaults, and economy-wide defaults. We would like to measure the impact on credit risks by these three types of determinant factors. Prior research also shows that credit spreads driven by firm-specific factors are as important as systematic factors in determining credit spread changes.[1] Recent research by Longstaff and Rajan (2008), Bhansali *et al.* (2008), and Tang and Yan (2010) further concludes that the firm-specific risk of individual company accounts for the major portion in the variation of credit spreads. Therefore, we investigate firm-specific determinants, followed by industry and macroeconomic determinants. The estimated results reveal that the coefficients of all firm-specific factors are time-varying and become much more volatile during the credit crunch. Only the leverage ratio, distance to default, and CAPM-beta variables are substantially significant during the whole sample period, while others only provide

[1] See e.g., Collin-Dufresne *et al.* (2001), Elton *et al*. (2001), Eom *et al.* (2004), Longstaff *et al.* (2005).

marked significance during specific time intervals. In particular, sector indicators are not statistically significant but provide high explanatory power. Considering the explanatory power and significance level over time simultaneously, we ascertain several substantial credit determinants to construct multi-factor models. To identify whether the macroeconomic variable can further interpret the default risks that have been explained by firm-specific and sector factors, the variances of estimation errors in the multi-factor model are regressed on gross domestic product (GDP) growth rate. The regression results reveal that GDP growth plays an important role different from firm-specific and industry-wide factors in determining credit conditions.

2. Varying coefficient modeling and credit default swap (CDS) pricing

Linear time-varying coefficient regression models (e.g., Faraway, 1997; Yang *et al.*, 2007) are applied to investigate potential determinant factors on credit risks and their dynamic regression relationships over time. While the coefficients are assumed to be constant in traditional linear regression models, time-varying coefficient models allow the coefficients to be time-varying for modeling the time-change regression relationships. The models are especially useful to investigate relationships between variables of multiple time series when their correlations are prone to change over time due to special events such as the subprime mortgage crisis. The change of time-varying coefficients and their corresponding statistical significance give more insight into the dynamic regression relationships, which are quite different from traditional constant coefficient modeling and analysis. The varying-coefficient modeling approach clearly indicates consideration of the dynamic relationships between credit risks and the potential determinant factors is very important in managing credit risks and especially helpful in enhancing credit derivative pricing performance.

2.1 Time-Varying Coefficient Regression Model

Let $cds_i(t)$ be the CDS spreads of the entity *i* measured at time *t*, for $i = 1,...,n$ and $t = 1,...,T$, and the covariate vector $x_i(t) = (x_{i0}(t),...,x_{im}(t))'$ denotes the intercept term $x_{i0}(t) = 1$ and the *m* potential determinant factors $x_{ij}(t)$ for $j = 1,...,m$ and $i = 1,...,n$. The linear time-varying coefficient regression model of CDS spreads can be written as

$$cds_i(t) = x_i(t)'\beta(t) + \varepsilon_i(t), \qquad (1)$$

where $\beta(t)=(\beta_0(t),\beta_1(t),...,\beta_m(t))'$ in which $\beta_0(t)$ is the intercept term and $\beta_1(t),...,\beta_m(t)$ denotes the coefficient functions associated with their covariates. $\varepsilon_i(t)$ is an error term of an *i.i.d.* random process. The errors are assumed to be correlated within each entity, but are independent between different entities. The unknown coefficient functions can be estimated in a time pointwise manner by the least squares method such that $\beta(t)$ is chosen by minimizing the sum of L_2 norms $\sum_{i=1}^{n}\left\| cds_i(t)-x_i(t)'\beta(t)\right\|^2$, leading to the solution

$$\hat{\beta}(t)=\left(X(t)'X(t)\right)^{-1}X(t)'CDS(t),$$

where $CDS(t)=(cds_1(t),...,cds_n(t))'$ is an *n*-vector of credit spreads for the *n* entities, $X(t)=(x_1(t),...,x_n(t))'$ is an $n\times(m+1)$ matrix formed from *n* entities, *m* covariates, and the intercept. The fitted credit spreads and the residuals, respectively, are $c\hat{d}s_i(t)=x_i(t)'\hat{\beta}(t)$, and $e_i(t)=cds_i(t)-c\hat{d}s_i(t)$.

Plotting the estimated time-varying coefficients $\hat{\beta}(t)$ along with time *t* provides an easy visual examination of the dynamic regression relationships.

For statistical inference, Faraway (1997) provided bootstrap-based testing methods and Shen and Faraway (2004) derived an approximated *F* test for testing two nested models under the Gaussian process assumption. Here, in order to easily oversee the significance of each single factor considered in the model, we plot the pointwise *p*-values along with time *t* to demonstrate the changes in significance levels, and summarize the *p*-value statistics by mean *p*-value to make an overall measurement of significance for each factor. The mean *p*-value (mPV_j) for the *j*th factor is calculated by

$$mPV_j=T^{-1}\sum_{t=1}^{T}pv_j(t), \tag{2}$$

where $pv_j(t)$ is the *p*-value of the *j*th covariate at time *t*.

To simplify the correlation between the fitted and the observed CDS, adjusted *R*-square statistics can be summarized by the following mean adjusted *R*-square statistic (mR_j^2)

$$mR_j^2 = T^{-1}\sum_{t=1}^{T} adjR_j^2(t)\,,$$

where $adjR_j^2(t)$ is the adjusted *R*-square at time *t*.

2.2 Time-Varying Coefficients and CDS Pricing

After identifying the time-varying coefficients and corresponding significance levels of credit determinants, the CDS pricing model can help to distinguish whether such time-varying correlation examination is helpful in improving the evaluation performance of credit derivatives. Considering the difficulties of calibrating the specific dynamic model to individual credit entities through the structural model, this study applies the default intensity model to construct a pricing model and enriches the model with economic underpinning, with structural and economic factors. Consequently, the credit determinants are linked to the evaluation of credit derivatives.

2.2.1 Pricing Model of CDS

CDS is the swap contract for which the default protection buyer makes a series of premium payments to exchange the default payment with the protection seller. With no-arbitrage constraints, the CDS spread is the breakeven spread that makes the present value of premium payments and default payment equal. The present value that protection seller will receive from regular payments at time t_1 to t_T is denoted as the premium leg:

$$PL = G\sum_{c=1}^{T}(t_c - t_{c-1})E(t_c)D(t_c),$$

where *G* is the CDS spread, t_c denotes payment date, $E(t_c)$ represents the expected principal at payment time t_c, and $D(t_c)$ is the discount factor. Assuming defaults occur at payment dates, investors should make the default payment at default and the present value of this default payment is represented as the default leg[2]:

$$DL = \sum_{c=1}^{T}[E(t_{c-1}) - E(t_c)]D(t_c).$$

To prevent leaving arbitrage opportunities, the CDS spread is the breakeven spread that makes the default leg equal to the premium leg:

[2] The default is assumed to only occur on payment date, but can be easily generalized as in Hull and White (2008).

$$G = \frac{\sum_{c=1}^{T} [E(t_{c-1}) - E(t_c)] D(t_c)}{\sum_{c=1}^{T} (t_c - t_{c-1}) E(t_c) D(t_c)}. \quad (3)$$

For the sake of simplicity, the principal is set at 1, thus the expected principal at payment time is

$$E(t_c) = 1 \cdot E[S(t_c)|\Omega_t],$$

where $E[S(t_c)|\Omega_t]$ is the expected cumulative survival probability at time t_c, $t_c > t$, conditional on Ω_t, where Ω_t denotes the information filtration at time *t*.

2.2.2 Modeling Default Intensity

Based on the default intensity model, we define the default intensity as an affine function of explanatory factors as in Duffie and Lando (2001) and Wu and Zhang (2008).[3] In order to easily model different risks represented by different factors, the factors are sorted into three risk components- firm-specific, industry-wide, and economy-wide parts. We construct the extended form of the default intensity model as

$$\lambda_{i,t} = \gamma_{0,i} + \gamma'_{1,i} X_{i,t} + \gamma'_{2,i} Y_{k(i),t} + \gamma'_{3,i} Z_t + \varepsilon_{i,t}. \quad (4)$$

where *X*, *Y*, and *Z* are independent basic affine processes to denote firm-specific, industry-wide, and economy-wide explanatory factors, respectively. $\gamma_{o,i}$ is the intercept, γ' denotes the transposition of coefficient γ. $X_{i,t}$ is an vector to represent firm-specific risk factors for entity *i* at time *t*. $Y_{k(i),t}$ denotes industry-wide risk factors and is common to all entities *i* in the same sector *k*, while Z_t is the economy-wide factor and thus common to all entities. $\varepsilon_{i,t}$ is identified as disturbances of the i entity that are not measured by explanatory factors and are independent of each explanatory factor. The Equation (4) can be simplified as

$$\lambda_{i,t} = \gamma_{0,i} + \gamma'_i F_{i,t} + \varepsilon_{i,t}, \quad (5)$$

where $F_{i,t}$ denotes the explanatory factors for entity *i* at time *t* with dimension $m \times 1$, $F_{i,t} \in \mathbb{R}^m$.

The dynamics of explanatory factors in physical measure are represented as

$$dF_{i,t} = -\varphi_i F_{i,t} dt + dB^P_{i,t},$$

where φ_i is an $m \times m$ transition matrix, and restricted to a diagonal matrix yielding independent

[3] Lando (1998) applied the default intensity modeling framework in zero recovery defaultable bond pricing.

explanatory factors. $B_{i,t}^{P}$ denotes a vector of standard Brownian motion at time t under physical measurement *P*. By Euler approximation, the discrete-time version of the factor dynamics can be shown as

$$F_{i,t} = \varphi_i F_{i,t-\Delta t} + \varepsilon_{i,t},$$

where ϕ_i is an $m \times m$ autoregressive coefficient matrix, Δt represents the time interval, and $\varepsilon_{i,t} \sim N(0,\Sigma)$ denotes the $m \times 1$ normal innovation vector with zero mean and diagonal covariance matrix Σ.

According to the default intensity model, the expected cumulative survival probabilities at time τ, $\tau \geq t$, are denoted as

$$S_{i,t}(\tau) = E^{\mathrm{Q}}\left[S_i(\tau)\middle|\Omega_t\right] = E^{\mathrm{Q}}\left[exp\left(-\int_t^{t+\tau}\lambda_{i,u}du\right)\middle|\Omega_t\right].$$

From Equation (5) and technical conditions described in Duffie, Pan, and Singleton (2000), we obtain

$$\begin{aligned} S_{i,t}(\tau) &= E^{\mathrm{Q}}\left[exp\left(-\int_t^{t+\tau}(\gamma_{i,0}+\gamma_i'\ F_{i,u}+\varepsilon_{i,u})du\right)\middle|\Omega_t\right] \\ &= E^{\mathrm{Q}}\left[exp\left(-\int_t^{t+\tau}(\gamma_{i,0}+\gamma_i'\ F_{i,u})du\right)\middle|\Omega_t\right]\cdot E^{\mathrm{Q}}\left[exp\left(-\int_t^{t+\tau}\varepsilon_{i,u}du\right)\middle|\Omega_t\right] \\ &= exp\left[\gamma_{0,i}(\tau)+\gamma_i'(\tau)F_{i,t}^{\mathrm{Q}}\right]. \end{aligned}$$

As the physical survival probabilities of explanatory factors are not relevant for the pricing of financial derivatives, we specify the market price of risk as an affine model of explanatory factors as

$$\eta_{i,t} = \gamma_{i,0}^{\eta} + \gamma_i^{\eta\prime} F_{i,t},$$

where $\eta_{i,t}$ denotes the market price of default risk of entity *i* at time *t*, $\gamma_{0,i}^{\eta}$, is the intercept, and γ_i^{η} is an $m \times m$ transition matrix. Therefore, the dynamics of explanatory factors under the risk neutral measure Q can be derived as follows

$$dF_{i,t} = -\left[\gamma_{0,i}^{\eta} + (\gamma_i^{\eta} + \varphi_i)F_{i,t}\right]dt + dB_{i,t}^{\mathrm{Q}},$$

with boundary conditions $\gamma_{0,i}(0) = 0$, and $\gamma_i(0) = 0$ at time zero, the coefficients $\gamma_{0,i}$ and γ_i in Equation (5) can be solved through numerical calculations with Riccati ordinary differential equations.

3. Data

To investigate the determinants of credit risks and their time-varying correlations, the CDS spread is examined with its greatest trading activity in the credit market. Our sample period is from the first quarter of 2005 to the third quarter of 2009, to include the time period spanning the subprime mortgage crisis. All data is from Compustat and Datastream, except some stock prices (including opening, closing, daily highs and lows of stock prices which are included to calculate volatilities) are checked and collected from the online Google Finance database.

3.1 Credit Spreads

Since CDS is taken for default protection to get default payments when a credit event occurs, the credit spread of CDS essentially reveals the credit risk of its underlying entity. All daily closing quotes of investments grade CDSs on U.S. companies available in the Datastream database are included, except those with private underlying entities or unavailable corresponding firm-specific financial covariates. There are 130,037 observations and 109 entities in the whole sample period.

3.2 The Determinants of Credit Spreads

We contained various determinants proposed by prior research as covariates to examine how well these factors explain the credit spread changes over time[4]. The covariates investigated in this paper are listed in Table 1 and described as follows.

(1). Leverage Ratio

The structural approach proposed by Black and Scholes (1973) and Merton (1974) suggested that risk premiums are determined by the debt-to-firm value ratio (or firm leverage ratio) and the volatility of the firm's operations. Many articles based on this structural framework also treated these two variables as the most important portions in valuing credit risks.[5] Therefore, we include both of them as the first two covariates to understanding how their corresponding coefficients vary over time. Each firm's leverage ratio is defined as

$$Leverage\,ratio_i(t) = \frac{Book\ Value\ of\ \ Debt_i(t)}{Market\,Value\ of\ \ Equity_i(t) + Book\ Value\ of\ \ Debt_i(t)}.$$

[4] Owing to the low explanatory power, some reports for several covariates are omitted, such as stock price, firm's operating cash flow, and the volatility of cash flow.

[5] See e.g., Leland and Toft (1996), Duffie and Lando (2001), Collin-Dufresne *et al.* (2001), Francois and Morellec (2004), and Ericsson *et al.* (2009).

Table 1. Variable Description and Predicted Signs for the Coefficients in Time-Varying Coefficient Regression Model

Variable	Frequency	Description	Predicted Sign
Leverage Ratio	Quarterly	The book value of debt divided by the firm value.	+
Volatility	Daily	The square root of the drift-independent and minimum-variance unbiased variance estimator from additional information provided by opening, high, low, and closing stock prices.	+
Distance to Default	Quarterly	Richly constructed by volatility-adjusted measure of leverage.	-
CAPM-beta	Monthly	The measurement of sensitivity of a company's stock price to the Standard & Poor's 500 Index Price for Companies (S&P 500).	+
Market Value	Daily	The products of stock prices and shares (including trading and non-trading issues) (billions).	-
Relative Firm Size	Daily	Divide the market value of each individual firm by the S&P 500 Index Price and then take the natural logarithm value.	-
Sector Indicator	-	Eight sectors are included and seven sector indicators are identified.	+

(2). Volatility

The other important determinant in the structural approach is firm volatility, which can be extracted from implied volatility of stock options. However, many observations did not have corresponding public traded options in the sample period. This study provides another suitable substitute for volatility to preserve information contained from different firms.[6] Therefore, stock volatility for each firm was calculated with the drift-independent volatility estimation approach proposed by Yang and Zhang (2000). This approach is independent of drift motion and opening jumps of stock prices to estimate volatility more accurately. By applying this minimum-variance unbiased variance estimator using additional information provided by opening, high, low, and closing stock prices, volatility differences of entities are preserved and updated weekly to measure the volatility changes more accurately. Calculation details are provided in Appendix A.

(3). Distance to Default

The distance to default variable is richly constructed with a volatility-adjusted measurement of leverage. Following many prior research observations of this combination factor, Duffie *et al.* (2007) provided evidence that the distance to default significantly affects the default rate with substantially greater influence than other

[6] Collin-Dufresne *et al.* (2001) took VIX index as the proxy for the firm-specific volatilities of all firms. This index is a weighted average of implied volatilities for a range of options on the S&P 500 index. In Ericsson *et al.* (2009), the volatility was computed using exponentially weighted moving average model on daily returns from each company.

covariates (Crosbie and Bohn, 2002; Vassalou and Xing, 2004). In line with Duffie *et al.* (2007), for a given firm i, the distance to default at time t is defined as

$$DD_i(t) = \frac{\ln(AV_i(t)/L_i(t)) + \left(\mu_{A,i} - \frac{1}{2}\sigma_{A,i}^2\right)T}{\sigma_{A,i}^2\sqrt{T}},$$

where $DD_i(t)$ is the distance to default of firm i, $i = 1,...,n$, at time t, $t = 1,...,T$, $AV_i(t)$ is the market value of firm assets, $\mu_{A,i}$ and $\sigma_{A,i}^2$ denote the mean rate of asset growth and asset volatility of firm i, respectively, and $L_i(t)$ is the liability defined as

$$L_i(t) = SD_i(t) + \tfrac{1}{2} LD_i(t),$$

where $SD_i(t)$ and $LD_i(t)$ are the book value of short-term and long-term-debts, respectively, of firm i at time t. Firm asset value $AV_i(t)$, growth $\mu_{A,i}$, and volatility $\sigma_{A,i}^2$ are calculated by call-option pricing model proposed by Merton (1974).[7]

(4). The CAPM-beta

Many researchers have provided evidence for a strong interaction between credit and stock markets (Fama & French, 1993; Whitelaw, 1994; Jagannathan & Wang, 1996; Cremers, 2002; Chen *et al.*, 2009; Zhang *et al.*, 2009; Bhamra *et al.*, 2010). Demchuk and Gibson (2006), Vassalou and Xing (2004), and Duffie *et al.* (2007) also proved that stock related variables have a strong impact on credit spreads. Considering individual stocks and corresponding credit derivatives are related to the same firm-specific information, the CAPM-beta and the following two covariates, firm size and firm market value, are incorporated to investigate the interaction between stock and credit markets. Thus, we can justify whether the stock and credit markets are intrinsically correlated over time.

CAPM is a well known asset pricing theory which measures the sensitivity of a stock to market movements. The theory proposed this sensitivity with *beta*, hereafter referred to as CAPM-beta. A stock with *beta* greater

[7] Seeing the market value of equity as the option premium on firm asset value to strike at liability value $L_i(t)$ before time T, the asset value thus can be derived from the market value of equity. To iteratively calculate the asset value and volatility at each time point, we assume the initial value of asset value $AV_i(1)$ is the sum of the equity value plus liability. The discount rate is the federal funds rate. The equity value is defined as the product of daily closing stock prices and outstanding shares obtained from Compustat.

than 1.0 tends to amplify market movements, and thus is at a higher risk in the stock market. From CAPM, the higher risk premium is then obtained to compensate for higher risk bearing. Correspondingly, the credit spread represents the credit risk compensation in the credit market. In order to understand how these two different markets interact, we take CAPM-beta as a superior measurement to distinguish the linkage between stock and credit markets.

(5). Market Value

By adding the market value variable, we would like to identify how market and credit risks correlate with each other in time.[8] Jarrow and Turnbull (2000) proposed that market and credit risks are intrinsically correlated. They suggested that if the unexpected default probability changes, refer to credit risk, the market value of the firm is then affected, refer to market risk. Conversely, when the market value of a firm unexpectedly changes, it also affects the default probability. In other words, the decrease in equity value leads to ownership transformation from stockholders to bondholders, raising the default probability, and thus expands the credit spread. Therefore, the market value has been regarded as an important component in many credit determinants, such as the leverage and distance to default factors. In this study, firm market value is treated as an additional relevant covariate to investigate the correlation between these two market risks. The product of stock price and outstanding shares is defined as the proxy for market value.

(6). Relative Firm Size

This covariate measures the comparative performance of firms in the stock market. If the market risk comes from the losses caused by the changes in stock prices, the relative firm size factor is provided to consider comparative market risk. Suppose that, during great appreciation, a firm only sustains its market value, the firm performs worse than others. On the contrary, during a recession, a firm which can maintain the same market value already performs very well. In line with Demchuk and Gibson (2006), Collin-Dufresne *et al.* (2001), and Duffie *et al.* (2007), we take the stock index as a proxy for business climate and divide the firm value by stock index to derive the relative firm size variable. The comparative performance in market value is

[8] Market risk can be defined as the gains and losses on asset value caused by the changes in market prices (such as stock prices).

taken into consideration to improve the market value variable to further examine the relationship between market and credit risks. Since the CDS samples examined in this paper are traded in the United States, the market value of each individual firm is divided by the S&P 500 index and then a natural logarithm is taken to form the relative firm size variable.

(7). Sector Indicator

Sector indicators measure common risks which are consistent in the same industry for different entities. Whereas many researchers treated industry-wide risks as an important component in credit risks, we include sector indicators to clarify industry-wide risks (Duffie & Gˆarleanu, 2001; Duffie *et.al.*, 2007; Longstaff & Rajan, 2008; Bhansali *et al.*, 2008). Seven sector indicators are identified in this paper as eight sectors are contained in observations. These eight sectors are basic materials, communications, consumers, energy, financial, industrial, technology, and utilities sectors.

4. Estimation Results

4.1 Single Factor Analysis

All factors are estimated individually to measure specific estimation performance through the time-varying coefficient regression model presented in Equation (1). The results show that the coefficients and their corresponding statistical significance of all variables change over time and become much more volatile after the credit crisis. Relative to the extremely volatile coefficient dynamics after the mid-2007 subprime mortgage crisis, coefficient dynamics before credit crisis were smooth. Moreover, the dynamics of corresponding significance also demonstrate that the first four factors become much more relevant to credit risks during worse credit conditions with their markedly raised significance levels. The estimated results of each factor are reported as follows.

(1). Leverage ratio

The estimated results of the leverage ratio variable are plotted in Figure 1. As predicted in the structural model, the leverage ratio positively affects credit risks during the whole sample period. The coefficients are time-varying. The coefficient dynamics raised after July 2007, markedly increased in Oct. 2008, and peaked in Apr. 2009 after the decreasing at the end of 2008. On average, the coefficients after the crisis reached 200, which is over ten times the level before the crisis.

In general, higher leverage accompanies higher risk, and leads to higher default possibility especially during a recession. The incredible increases in coefficients show that the correlation between leverage ratio and credit risk is not fixed at a certain level. The bottom panel in Figure 2 further reveals that after mid-2007 the coefficient dynamics show an extreme increase and the significance levels represented by *p*-value also jump from around 5% to less than 0.01%. The marked increase in significance level reveals the tightened correlation between leverage ratio and credit risks.

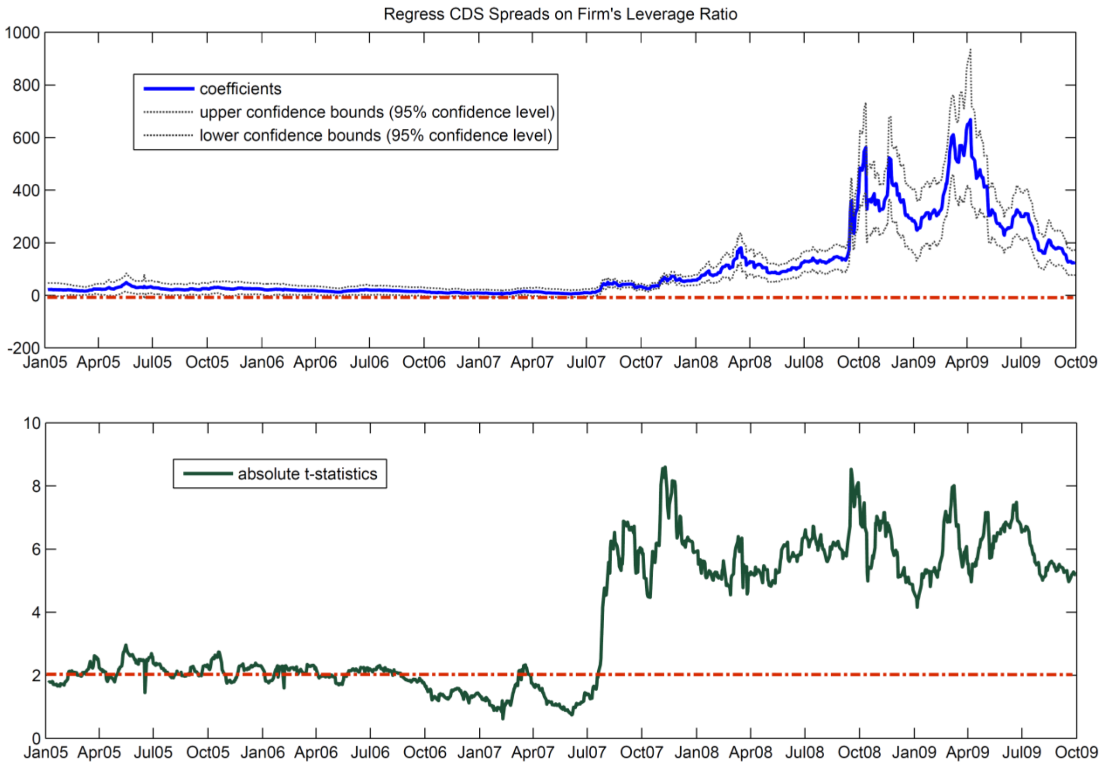

Figure 1. Leverage Ratio Factor
This figure graphs the time-varying estimated results of the leverage ratio factor. The coefficients and absolute *t*-statistics are reported in the top and bottom panels, respectively.

(2). Volatility

During recession, it appears to be much riskier to provide risk protection and thus worse market conditions yield higher market price of risk. Since the coefficient of volatility could be seen as the market price of risk, different market conditions should have different corresponding coefficients.[9] Figure 2 shows that the coefficients are time-varying and markedly increase during economic recession.

In order to compensate for more risk exposure, higher volatility demands a higher risk premium. From Equation (1), the volatility is multiplied by its coefficient to derive the estimated credit spread. If higher volatility accompanies higher risk compensation, the coefficient should be positive. Furthermore, if the risk compensation changes with different market conditions, the coefficient should not be constant. Although before 2008, the volatility variable was insignificant in determining credit risks; after 2008, the volatility became extremely significant with large and positive coefficients. The estimated results reveal that the coefficient of volatility is time-varying and the market price of risk changes with different market conditions.

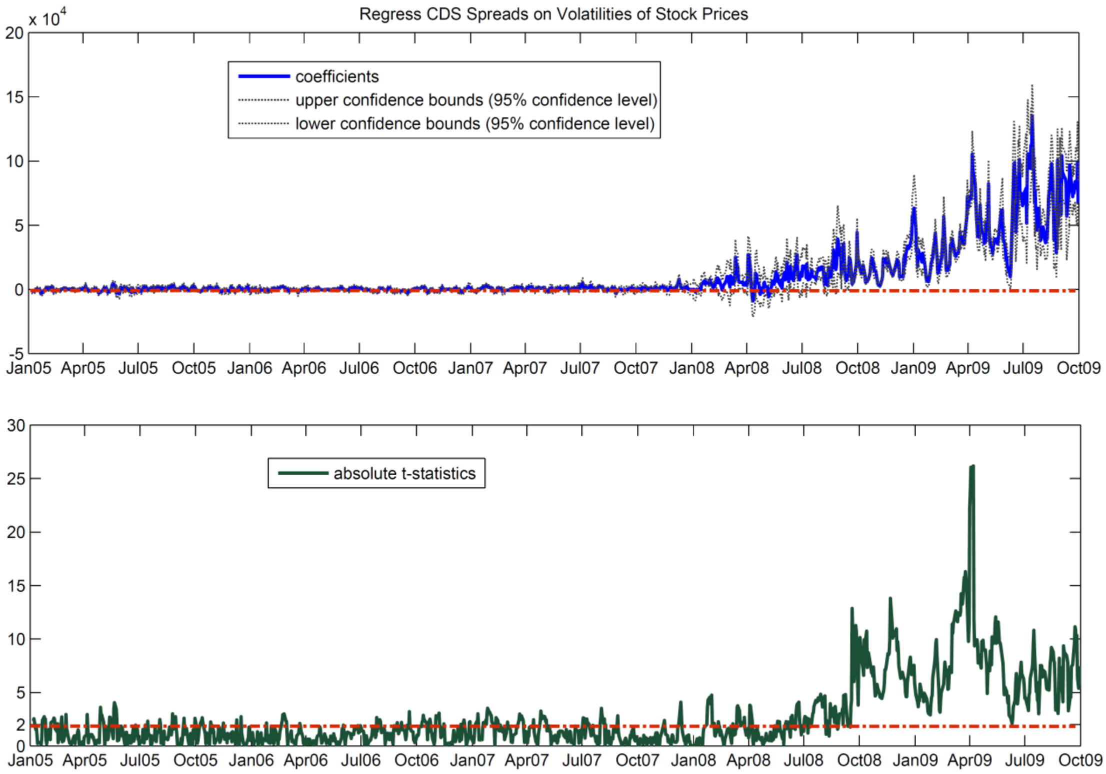

Figure 2. Volatility Factor
This figure graphs the time-varying estimated results of the volatility factor. The coefficients and absolute *t*-statistics are reported in the top and bottom panels, respectively.

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[9] The market price of risk is defined as the risk premium divided by standard deviation, $\frac{risk\ premium}{standard\ deviation}$. From Equation (1), the estimated credit spread is $\hat{cds}_i(t) = x_i(t)'\hat{\beta}(t)$. The coefficient is obtained by $\hat{\beta}(t) = \left(x_i(t)'x_i(t)\right)^{-1} x_i(t)' cds_i(t)$. Since the coefficient of volatility is the measurement of the credit spreads compensated for each unit of volatility, by definition $\hat{\beta}_{Vol}(t) = \frac{credit\ spreads(t)}{volatility(t)}$, the coefficient of volatility can be seen as the market price of risk.

(3). Distance to default

The distance to default variable is negatively significant over time as predicted. As shown in Figure 3, it is noteworthy that the coefficients increased with high absolute *t*-statistic levels after July 2007. Then, the coefficients markedly dropped after Sep. 2008 as credit conditions became more serious with the occurrence of many credit events (e.g. Fannie Mae, Freddie Mac, Lehman Brothers, Merrill Lynch, and American International Group (AIG)). After early 2009, coefficients steadily increased. The coefficients and significance levels markedly changed after the credit crisis. As the actual credit environment improved after 2009, the coefficient dynamic in late 2009 became smooth.

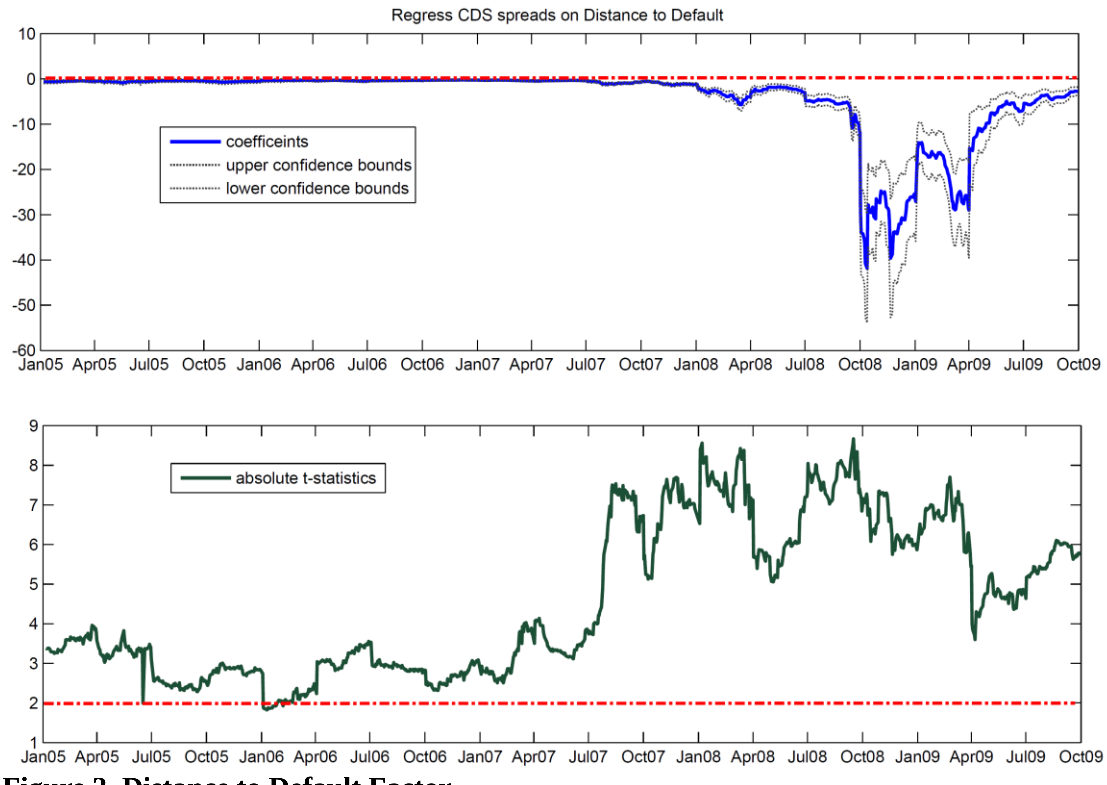

Figure 3. Distance to Default Factor
This figure graphs the time-varying estimated results of the distance to default factor. The coefficients and absolute *t*-statistics are reported in the top and bottom panels, respectively.

(4). CAPM-beta

As predicted, the time-varying regression of credit spreads on CAPM-beta yields positive coefficients. Since the CAPM-beta measures the sensitivity of an individual stock to market movements, the positive coefficients indicate that a stock with greater sensitivity in the stock market accompanies higher credit risks in the credit market. The results support that market risk and credit risk are strongly dependent over time. A riskier stock has a higher expected return in the stock market and is also accompanied by greater spread compensation in the credit market.

The top panel in Figure 4 shows that the coefficient dynamic of CAPM-beta became much more volatile after July 2007 and the extreme value was reached in Apr. 2009. The coefficients can be seen as multipliers, and thus the credit spreads can be estimated by multiplying the CAPM-betas and multipliers at each time point. The marked increase in multipliers after the credit crisis indicates that a higher risk stock demands much more compensation in the credit market during serious credit conditions. Therefore, the risk premium in the credit market is significantly amplified during credit panic. The estimated results support the notion that the stock and credit markets are time-varying correlated especially after a crisis. According to the bottom panel of Figure 4, because the significance levels markedly improved after mid-2008, the correlation between these two markets was much closer after the credit crunch.

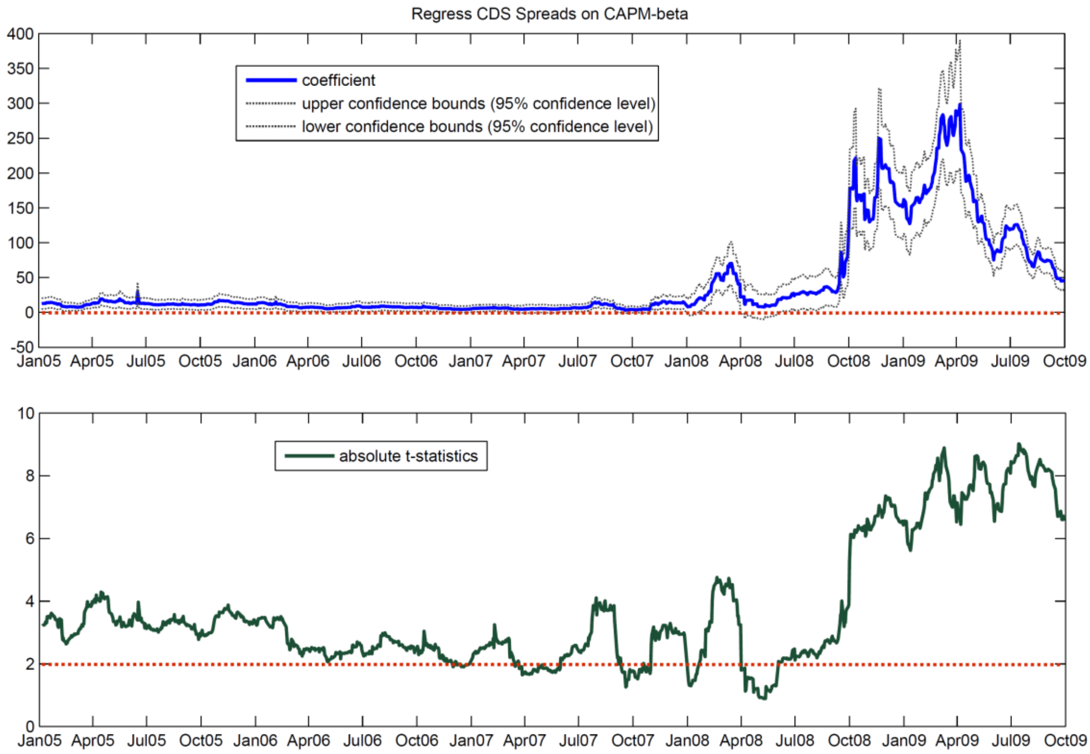

Figure 4. CAPM-beta Factor

This figure graphs the time-varying estimated results of the CAPM-beta factor. The coefficients and absolute *t*-statistics are reported in the top and bottom panels, respectively.

(5). Market Value

Figure 5 displays how market and credit risks are correlated in time. As predicted, most of the time the market value and credit spread are negatively correlated except during Sep. 2008. After Lehman Brothers triggered one of the biggest corporate defaults on record on Sep. 15, 2008, Merrill Lynch were sold to the Bank of America, American International Group (AIG) was being bailed out by the Treasury, and Washington Mutual was seized by the Federal Deposit Insurance Corporation (FDIC) in the same month. During Sep. 2008, the subprime mortgage-induced financial crisis caused the collapse of the fourth-largest U.S. investment bank (Lehman Brothers) and led to the bankruptcies of many financial giants. So many serious credit events in the same month contributed to great erosion of the equity market and turned the correlation between credit spread and market value from negative to positive during Sep. 2008. Apart from some exceptions, the bottom panel in Figure 5 displays that after the credit crisis unfolded, the coefficients were no longer significant. In sum, this variable provided good explanatory power before the credit crisis, but afterward, in spite of some exceptions during Oct. 2007, late 2008, and early 2009, this variable became insignificant.

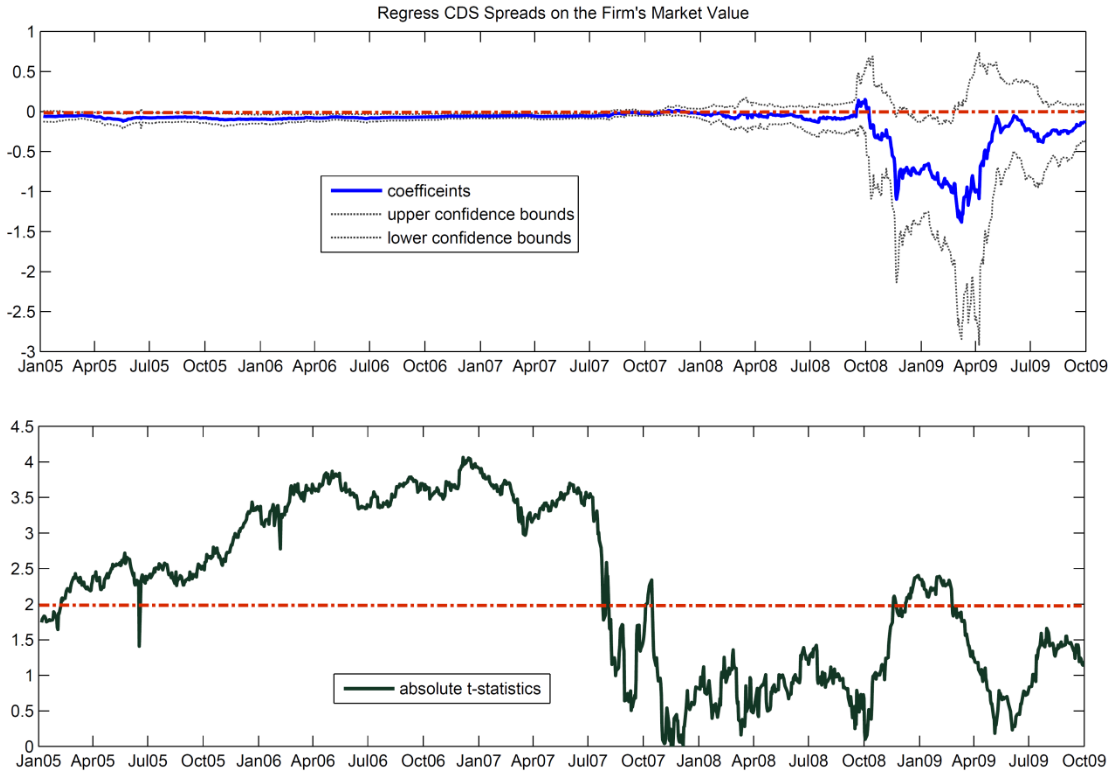

Figure 5. Firm Market Value Factor

This figure graphs the time-varying estimated results of firm market value factor. The coefficients and absolute *t*-statistics are reported in the top and bottom panels, respectively.

(6). Relative firm size

This variable combines the firm market value with stock index to provide richer information and to further investigate the correlation between market and credit risks. Figure 6 shows that, by considering the stock index in the market value variable, the new composite factor has negative correlation with credit spread during the whole sample period; this correlation became more volatile after 2008. Compared with the firm market value factor displayed in Figure 5, the corresponding significance levels of this composite variable are greatly improved as shown in the bottom panel of Figure 6. The corresponding significance levels are always below 5% except in Sep. 2008, when many major credit events came from financial giants. During this period of exception, the negative coefficients correspondingly jumped up and led the credit spread to be less sensitive to the relative firm size. Since the negative coefficients of the relative firm size variable are only insignificant during the worst scenario in financial market in Sep. 2008, this composite factor supports that credit and market risks are usually negatively correlated. The higher relative market value causes a lower default probability, and vice versa.

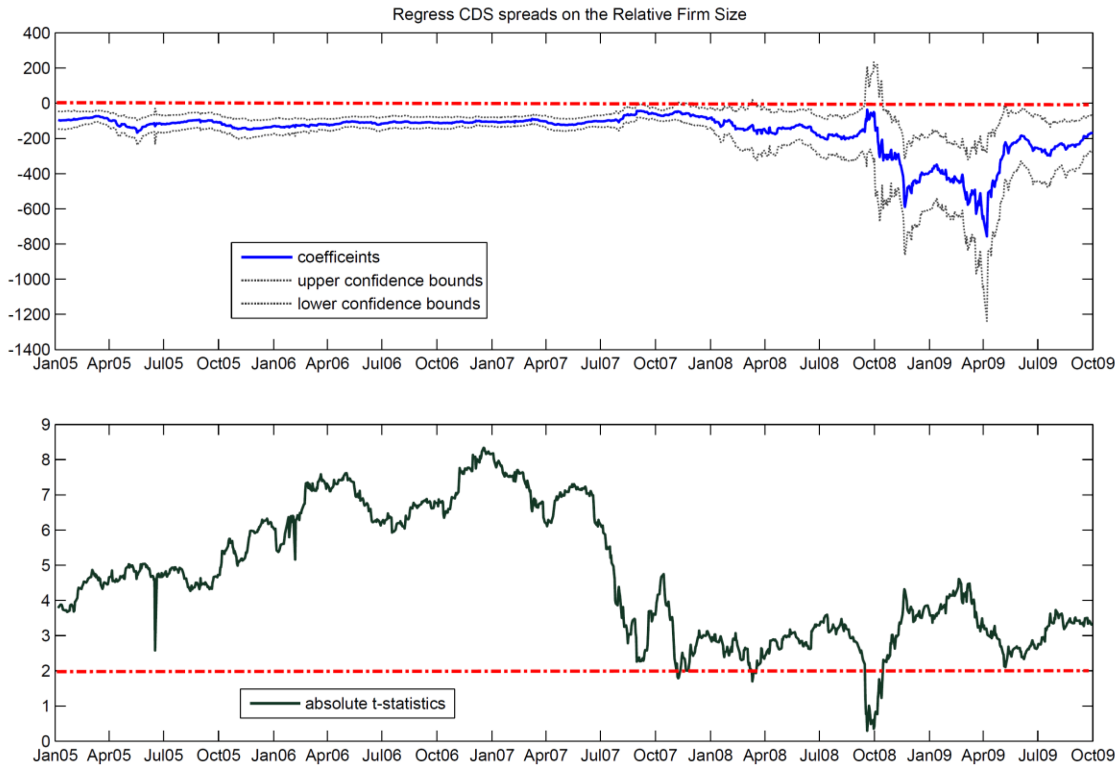

Figure 6. Relative Firm Size Factor
This figure graphs the time-varying estimated results of the market size factor. The coefficients and absolute *t*-statistics are reported in the top and bottom panels, respectively.

(7). Sector indicators

The coefficients of sector indicators are also time-varying. As shown in Figure 7, the coefficient dynamics of all sectors change in time especially after mid-2007. The coefficient dynamic shape of the fifth sector, the financial sector, is particularly different from others. The results coincide with the economic reality that the subprime mortgage crisis unfolded from the U.S. financial market, and then losses spread out throughout the entire economy. Therefore, after mid-2007, the time series of coefficients of the fifth sector markedly raised and the explanatory power was also improved with higher absolute *t*-statistics, while the estimates of other sectors were relatively smooth with clearly different shapes. Although the time series of absolute *t*-statistics in sector indicators are often insignificant, the adjusted *R*-square of sector indicators reach 0.1307 on average, only lower than the distance to default and relative firm size factors. This strong explanatory power indicates that the sector indicators really can provide some helpful information in determining credit spreads.

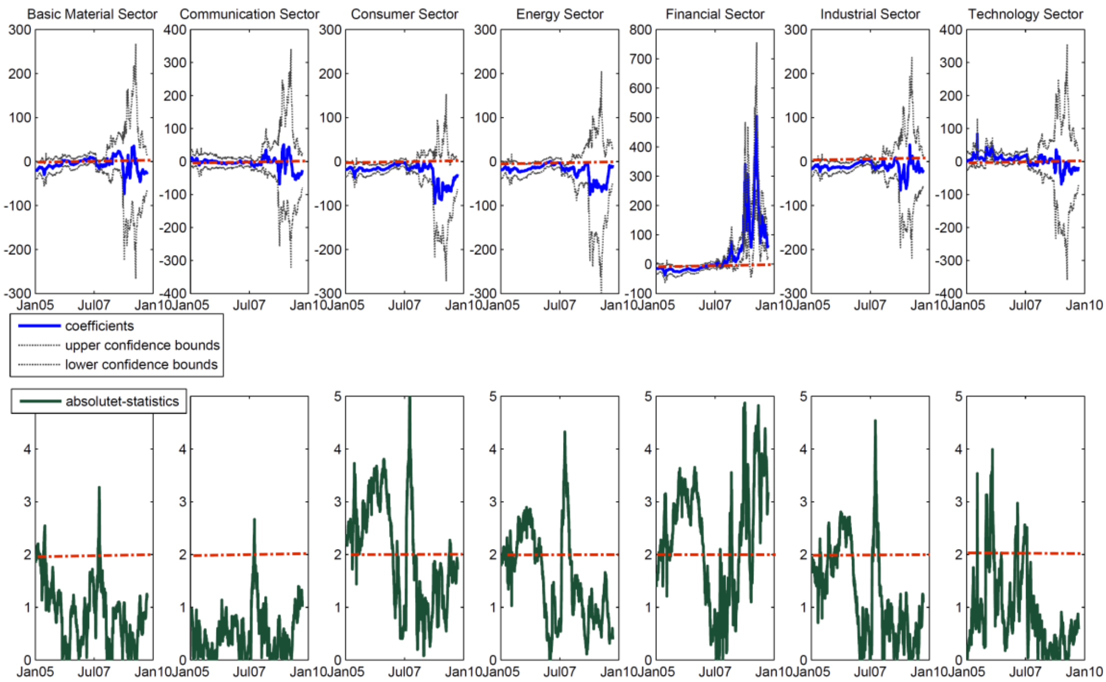

Figure 7. Sector Indicators
Since the sample includes eight sectors, seven indicators are defined. This figure graphs the time-varying estimated results of the sector indicators. The coefficients and absolute *t*-statistics are reported in the top and bottom panels, respectively. From left to right panels, the seven sectors are basic materials, communications, consumers, energy, financial, industrial, technology, and utilities sectors.

In order to summarize the time-varying estimated results, all statistics are averaged in time with equal weight in Table 2. The sample period is separated into two parts. The first part is from Jan. 1, 2005 through June 30, 2007 to denote the time period before the credit crisis. The second part is from July 1, 2007 through Sep. 30, 2009 to represent the time period after the credit crisis unfolded. According to Panel C in Table 2, on average, all variables are significant in determining credit spreads. However, compared with Panel A and B, we find that the mean coefficient and mean *t*-statistics of all variables after the crisis are quite different from before the crisis. All mean coefficients significantly increase after the credit crunch; the significant levels of some variables are greatly enhanced after the crisis (such as leverage ratio, volatility, distance to default, and CAPM-beta variables). On average, after the crisis the market value variable became irrelevant with low absolute mean *t*-statistics (1.1409). Therefore, the estimated results reveal that credit determinants are time-dependent and their coefficients are time-varying especially during serious credit conditions.

Table 2. Summary of the Average Estimated Results
The CDS spreads are regressed on factors one by one. Since the time-varying regression results of all factors show that the coefficients are markedly changed after the subprime crisis unfolded, we separate the estimated results into three panels. The mean value of total sample period results are reported in Panel C; Panel A summarizes the mean estimated results from Jan. 1, 2005 to June 30, 2007; and Panel B summarizes from July 1, 2007 to Sep. 30, 2009. mR^2 is the mean adjusted *R*-square statistic and mPV is the mean *p*-value.

Factor	Panel A: Jan. 1, 2005-June 30, 2007				Panel B: July 1, 2007-Sep. 30, 2009				Panel C: Total Sample Period			
	mean coefficient	mean *t*-statistics	mR^2	mPV	mean coefficient	mean *t*-statistics	mR^2	mPV	mean coefficient	mean *t*-statistics	mR^2	mPV
Leverage Ratio	18.3144	1.8852	0.0247	0.0916	200.1991	5.8150	0.2343	0.0032	104.8243	3.7543	0.1244	0.0496
Volatility	95.2331	0.1285	0.0095	0.3644	22577.0711	4.3015	0.1660	0.1642	10788.2754	2.1133	0.0839	0.2692
Distance to Default	-0.4262	-2.9501	0.0680	0.0090	-8.5551	-6.3911	0.2692	0.0000	-4.2926	-4.5868	0.1637	0.0047
CAPM-beta	9.3421	2.8091	0.0621	0.0159	79.0657	4.7188	0.1777	0.0299	42.5048	3.7174	0.1171	0.0225
Market Value	-0.0725	-3.1180	0.0767	0.0090	-0.2442	-1.1409	0.0075	0.3290	-0.1542	-2.1777	0.0438	0.1612
Relative Firm Size	-112.9193	-6.0993	0.2511	0.0000	-218.2342	-3.0967	0.0773	0.0217	-163.0103	-4.6711	0.1684	0.0103
Sector Indicators			0.1217				0.1407				0.1307	
Indicator 1	-5.5187	-0.5486	-	0.3818	-7.0161	-0.3290	-	0.5073	-6.2309	-0.4441	-	0.4415
Indicator 2	-1.2964	-0.1977	-	0.7103	-7.1741	-0.3588	-	0.5736	-4.0920	-0.2743	-	0.6453
Indicator 3	-14.9440	-2.5205	-	0.0750	-33.6425	-1.5118	-	0.2479	-23.8376	-2.0408	-	0.1572
Indicator 4	-13.3906	-1.7845	-	0.1721	-30.7237	-1.2841	-	0.3224	-21.6347	-1.5465	-	0.2436
Indicator 5	-16.5157	-2.3688	-	0.0883	92.2511	2.0748	-	0.1682	35.2171	-0.2553	-	0.1263
Indicator 6	-9.6843	-1.4729	-	0.2066	-12.9043	-0.7708	-	0.5196	-11.2158	-1.1389	-	0.3555
Indicator 7	12.2880	1.2723	-	0.3201	-7.9687	-0.1780	-	0.6562	2.6533	0.5825	-	0.4800

4.2 Multi-factor Models

Table 3 is an overview of the statistical significance of each variable over time. The distance to default factor is the most significant variable followed by relative firm size, CAPM-beta, and leverage ratio factors. The other factors are relatively less significant as their *mPVs* are higher than 0.1. By selecting the factors with significant level of 10%, we construct the first multivariate estimation model, named Model 1. Then, considering the high adjusted *R*-square provided by sector indicators, these indicators are added in Model 1 to form Model 2.

Table 3. Report of Mean *P*-Value

The *p*-value of each factor is averaged over time and presented in the second column denoted as *mPV*. Since there are seven sector indicators, we summarize the mean *p*-value of all sector indicators at first and then divide the summation by seven to obtain the *mPV* for sector indicators.

Factor	Mean *p*-value (*mPV*)
Leverage Ratio	0.0496
Volatility	0.2692
Distance to Default	0.0047
CAPM-beta	0.0225
Market Value	0.1612
Relative Firm Size	0.0103
Sector Indicators	0.3499

(1). Model 1

The leverage ratio factor is omitted for its collinearity with distance to default factor.[10] Therefore, the first multivariate estimation model includes the most significant factors over time to yield the following function:

$$cds_i(t) = \beta_0(t) + CAPMbeta_i(t)'\beta_1(t) + DD_i(t)'\beta_2(t) + RFS_i(t)'\beta_3(t) + \varepsilon_i(t), \quad (6)$$

where $CAPMbeta_i(t)$, $DD_i(t)$, and $RFS_i(t)$ are *n*-vectors to denote the factor for the *i*th CDS at time t of CAPM-beta, distance to default, and relative firm size, respectively. The estimated results, displayed in Figure 8, reveal that the coefficient dynamics change over time and become much more volatile under worse credit conditions.

[10] Since the distance to default variable is constructed as a rich volatility-adjusted measurement of leverage, the similarity in definition between these two variables implies the collinearity possibility. In addition to Model 1, we constructed another multivariate estimation model (not reported) with four covariates, including the distance to default, CAPM-beta, leverage ratio, and relative firm size factors, and recognize a remarkable collinearity.

Table 4 is an overview of the estimated results from which the averages of statistics are calculated with equal weights. The mean *t*-statistics in all sample periods shown in Panel B for Model 1 indicate that, on average, all factors contained in Model 1 are significant at 5% significance level. Moreover, the mR^2 in the last row in Panel B for Model 1 shows that the model provides better explanatory power of 0.3422 after the crisis relative to 0.3327 before the crisis and 0.3372 in the total sample periods. The coefficient and significance dynamics of all variables also display marked changes after the credit crisis.

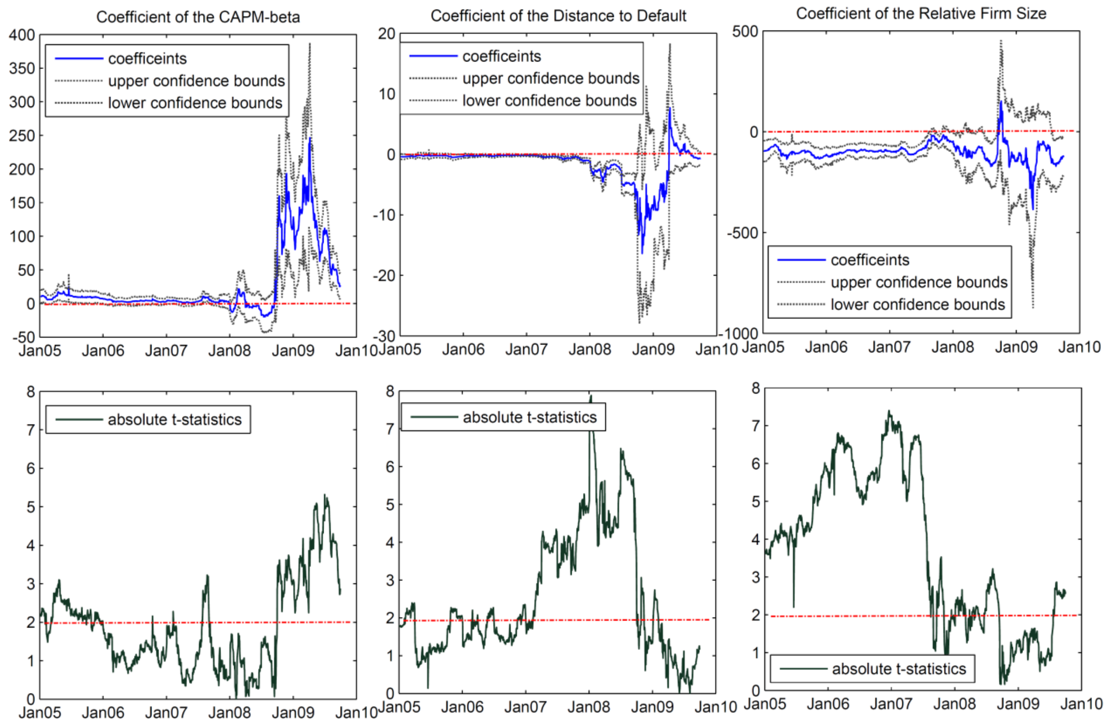

Figure 8. Multi-Factor Model 1
This figure graphs the time-varying estimated results of multi-factor model 1. From left to right columns report the estimated results of covariate CAPM-beta, distance to default, and relative firm size. Their coefficients and absolute *t*-statistics are reported in the top and bottom panels, respectively.

(2). Model 2

Although the mean *p*-value of sector indicators are not small enough to be suitable determinants in valuing credit spreads, Table 2 shows that the stand-alone mR^2 of these indicators is 0.1307. This value is only lower than the relative firm size variable, 0.1684, distance to default variable, 0.1637, and followed by the leverage ratio variable, 0.1244. Thus, aside from the covariates already considered in Model 1, Model 2 contains the sector indicators as additional relevant credit determinants to form the following time-varying coefficient model:

$$cds_i(t) = \beta_0(t) + CAPMbeta_i(t)'\beta_1(t) + DD_i(t)'\beta_2(t) + RFS_i(t)'\beta_3(t) + \sum_{k=1}^{7} I_{k(i)}\beta_{3+k}(t) + \varepsilon_i(t), \quad (7)$$

where $I_{k(i)}$ is the sector indicator and equals 1 while the entity *i* belongs to sector *k*, and $I_{k(i)} = 0$, otherwise.

The estimated results of sector indicators are displayed in Figure 9, but the figures of CAPM-beta, distance to default, and relative firm size are omitted as they are quite similar to the results in Model 1. From Figure 9, the coefficients and their corresponding significance of sector indicators in Model 2 also change over time. Similar to the results of specific sector indicators shown in Figure 7, the financial sector indicator in Model 2 plots an entirely different coefficient shape from others. The report in all sample periods shown in Panel B in Table 4 shows an improvement in mR^2 from 0.3372 in Model 1 to 0.3744 in Model 2, and only the CAPM-beta factor in Model 2 fails to reach the 5% significance level. Thus, on average, the explanatory power of CAPM-beta is restricted after other firm specific determinants and sector indicators have already been considered. Interestingly, after July 2007, the difference in mR^2 between Model 1 and Model 2 shows that sector indicators only improve the average explanatory power by 0.026. Although, on average, the sector indicators improve the explanatory power from 0.3327 to 0.38 before July 2007, this improvement of sector indicators is restricted after the credit crisis. The industry-wide component seems to become less important in determining credit risk after the credit crisis.

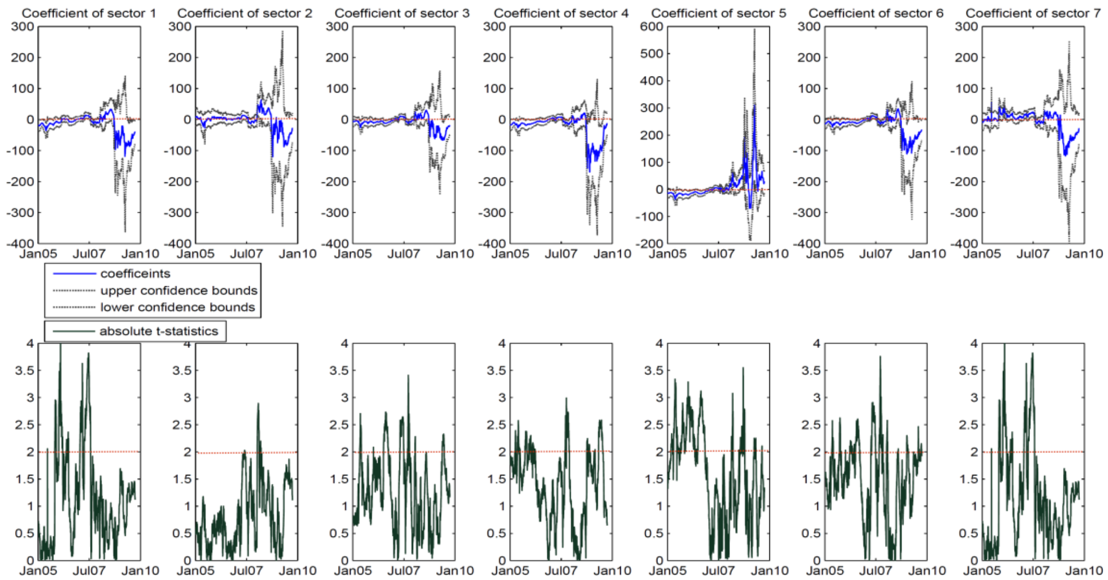

Figure 9. Sector Indicators in Multi-Factor Model 2
This figure graphs the time-varying estimated results of the sector indicators. The coefficients and absolute *t*-statistics are reported in the top and bottom panels, respectively. From left to right panels, the seven sectors are basic materials, communications, consumers, energy, financial, industrial, technology, and utilities sectors.

Table 4. Summary of the Average Estimated Results of Multi-Factor Models

Estimated results are averaged in corresponding report time period in this table. Panel A displays the estimated results of each single factor. The coefficients of CAPM-beta, distance to default, and relative firm size factors are listed in column 1, 2, and 3, respectively. Panel B reports the estimated results of multi-factor estimation models. Model 1 and 2 are reported in column 1 and 2, respectively. The multiple factors contained in Model 1 are the factors with the *mPV* lower then 0.1, and they are CAPM-beta, distance to default, and relative firm size factors. Sector indicators are added in Model 2 due to their high mR^2 reported in Table 2. The *t*-statistics are reported in parenthesis.

Panel A: Single Factor

	1. CAPM-beta			2. Distance to Default			3. Relative Firm Size		
	Before July 2007	After July 2007	All sample	Before July 2007	After July 2007	All sample	Before July 2007	After July 2007	All sample
Constant	21.2602 (6.7350)	18.5142 (3.3341)	19.9541 (5.1174)	39.2176 (10.2822)	177.9305 (14.3574)	105.1937 (12.2205)	237.2029 (6.9250)	630.7085 (3.8122)	424.3660 (5.4445)
Coefficient	9.3421 (2.8091)	79.0657 (4.7188)	42.5048 (3.7174)	-0.4262 (-2.9501)	-8.5551 (-6.3911)	-4.2926 (-4.5868)	-112.9193 (-6.0993)	-218.2342 (-3.0967)	-163.0103 (-4.6711)
mR^2	0.0621	0.1777	0.1171	0.0680	0.2692	0.1637	0.2511	0.0773	0.1684

Panel B: Multiple Factors

		1. Model 1			2. Model 2		
		Before July 2007	After July 2007	All sample	Before July 2007	After July 2007	All sample
Constant		227.6035 (6.9662)	266.0731 (2.5744)	245.9008 (4.8773)	222.2279 (6.6199)	312.6270 (2.7227)	265.2245 (4.7663)
Coefficient	CAPM-beta	8.8311 (2.9967)	49.8863 (2.1632)	28.3582 (2.6003)	6.0865 (1.5954)	49.6310 (1.8229)	26.7976 (1.7036)
	Distance to Default	-0.1824 (-1.4926)	-4.6049 (-3.9848)	-2.2859 (-2.6780)	-0.2587 (-1.9101)	-3.0071 (-2.9945)	-1.5659 (-2.4259)
	Relative Firm Size	-109.2432 (-6.0816)	-78.8900 (-1.6346)	-94.8063 (-3.9665)	-101.2517 (-5.4665)	-98.0274 (-1.8008)	-99.7181 (-3.7230)
	Sector Indicator	No	No	No	Yes	Yes	Yes
	mR^2	0.3327	0.3422	0.3372	0.3800	0.3682	0.3744

In order to display the changes of explanatory power over time, the dynamics of adjusted *R*-squares are plotted in Figure 10. The adjusted *R*-square clearly changes in time. Moreover, the time series of the adjusted *R*-square in Model 1 and 2 are much closer after July 2007, which are indicated in the gray regions. During some periods in these areas, the adjusted *R*-square in Model 2 is even lower than Model 1. Thus, adding sector indicators during these periods can not provide more valuable information about credit risks. A possible interpretation is that since all sectors were affected during the economic recession, the industry-wide risk component was dominated by the systematic component and became a poor indicator in valuing credit risks. However, it is noteworthy that owing to the worst scenario in the financial market during Sep.-Oct. 2008, as many financial institutions contagiously defaulted, the extremely different coefficient dynamic appeared in the financial sector (as shown in the fifth column in Figure 9) leading to a performance improvement in Model 2.

In short, in line with prior researchers, we agree that sector indicators provide useful information to enhance the explanatory power in valuing credit spreads different from firm-specific determinants (Duffie & Gˆarleanu, 2001; Duffie *et al.*, 2007; Longstaff & Rajan, 2008; Bhansali *et al.*, 2008). Additionally, we further suggest that although industry-wide risk can be treated as another type of credit risk, it is diminished during credit crises.

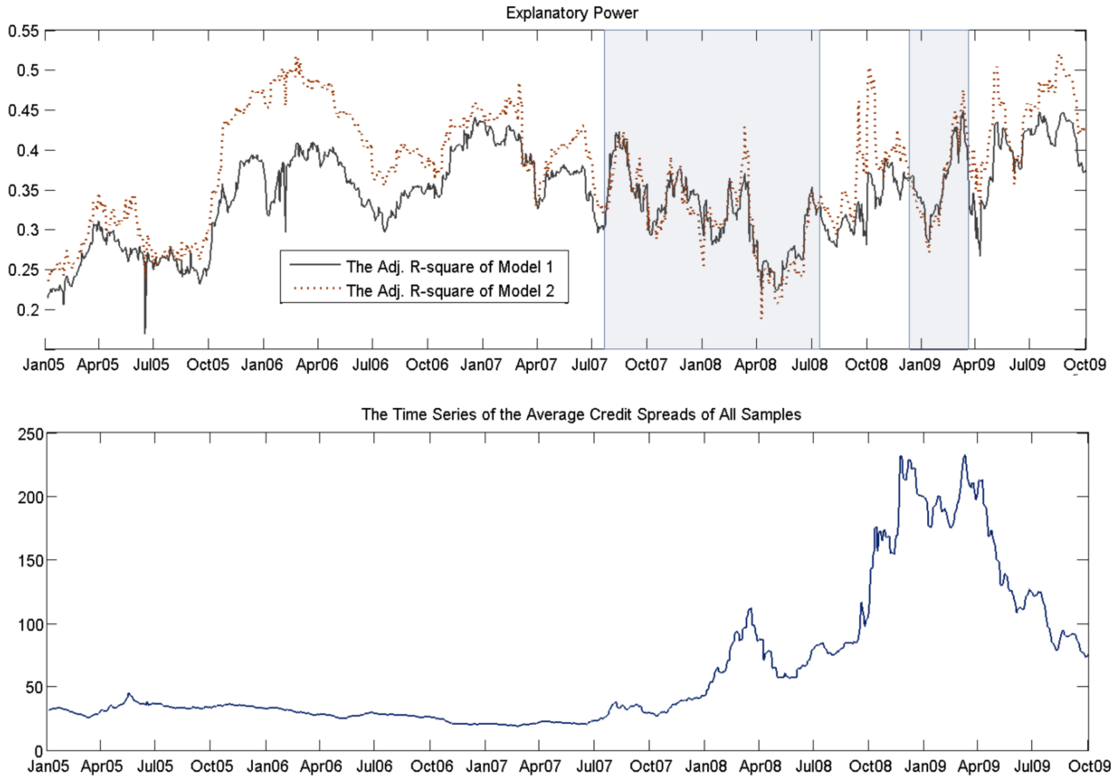

Figure 10. Explanatory Power
This top panel graphs the explanatory power of Model 1 and 2 while the bottom panel shows the average market quotes of all samples over time. The gray areas are where the adjusted *R*-square value of Model 2 is close to (or lower than) the value in Model 1.

4.3 Macroeconomic Conditions and Credit Market

Many prior researchers demonstrated that macroeconomic indicators are important in determining default risks, we would like to further explore whether the macroeconomic default covariate plays an important role in explaining credit risks after firm-specific and sector factors have already been considered (Das *et al.*, 2007; Lo, 1986; Lennox, 1999; McDonald & Van de Gucht, 1999; Collin-Dufresne *et al.*, 2001; Altman *et al.*, 2005; Duffie *et al.*, 2007; Longstaff & Rajan, 2008). Following the definition in the previous section, the residuals of Model 2 are the credit spreads not explained by the firm-specific and sector variables. While general GDP growth rate provides economy-wide information, we take the quarterly released GDP growth rate of the U.S. as the macroeconomic proxy to examine its explanatory power for the residuals of Model 2 through the following regression model for each entity $i = 1,...,n$,

$$\log\left[e_i(t)^2\right] = \alpha_i^R + \beta_i^R GDP(t) + \varepsilon_i^R(t),$$

where α_i^R denotes the intercept term, β_i^R is the coefficient of GDP, and $\varepsilon_i^R(t)$ represents the error term for $t = 1,...,T$, assuming that the mean of the error term $\varepsilon_i^R(t)$ is zero. The regression model of squared residuals $e_i(t)^2$ is associated with the behavior of the variance function of $\varepsilon_i(t)$ in Model 2, conditioning on the firm-specific and sector factors. In addition, the natural logarithmic transformation is applied to enhance the constant error variance assumption of the model and ensure positive definiteness on the time-varying variance function. Thus, $\log\left[e_i(t)^2\right]$ is a measurement of log error variance in Model 2.

As displayed in Figure 11, almost all samples (102 out of 109) support that GDP growth significantly affects the error variance in Model 2 with negative correlation. The mean value of coefficients (*t*-value) β^R is -1.8772 (-16.4255) versus 8.0161 (39.5152) for intercepts (*t*-value) α^R, and 0.0256 for *p*-value. Therefore, the error variance of Model 2 can be explained by GDP growth over time, after considering firm-specific and sector effects. The higher conditional variance accompanies the lower GDP growth rate, and vice versa. Thus, the conditional time-varying variance function of Model 2 is countercyclical.

Furthermore, the average adjusted *R*-square reaches 0.2024. The dramatic significance and high adjusted *R*-square levels support that GDP growth really contributes to credit determination in terms of variation over time even after various useful covariates have already been considered. Consequently, the macroeconomic conditions provide valuable information in addition to firm-specific and sector factors in determining credit risks. The results reveal that GDP growth rate can be a critical factor to bridge the gap between credit market and macroeconomic conditions.

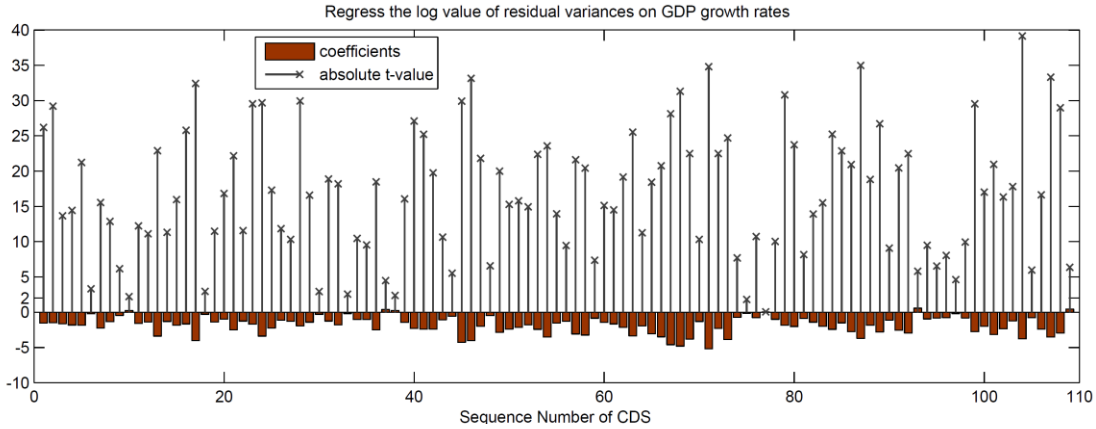

Figure 11. The coefficient and *t*-value of regressing residual variance on GDP growth rate
The coefficient β_i^R for each entity *i* is represented by a dark red bar, and its corresponding absolute *t*-value is the vertical line marked with $\times$.

5. Dynamic Calibration of CDS

5.1 Dynamic Calibration with Time-Varying Coefficients

We now calibrate the CDS spreads with the pricing model constructed in section 2 to justify the valuation performance with time-varying coefficients. As discussed in section 4, three important firm-specific factors are included for their significant correlation with credit spreads. In order to depict the industry-wide and economy-wide default risks, the sector index and GDP growth rate are also included. As shown in Equation (4), the default intensity is defined as the following affine function of explanatory factors to contain economic underpinning,

$$\lambda_{i,t} = \gamma_{0,i} + \gamma'_{1,i} X_{i,t} + \gamma'_{2,i} Y_{k(i),t} + \gamma'_{3,i} Z_t + \varepsilon_{i,t}.$$

where $\gamma_{0,i}$ is the intercept for entity i, $\gamma'_{u,i}$ denotes the transposition of coefficient $\gamma_{u,i}$, $u = 1,...,3$. $X_{i,t}$ is an 3×1 vector to denote CAPM-beta, distance to default, and relative firm size factors for entity i at time t. $Y_{k(i),t}$ is the sector index which is common to all entities in the same sector *k* to represent the industry-wide risk for entity *i* at time *t*, while Z_t denotes GDP growth rate to represent the macroeconomic factor and thus is common to all entities at each time point *t*. $\varepsilon_{i,t}$ is identified as disturbance to denote the estimation error that is not measured by explanatory factors and is independent to each explanatory factor.

Because the estimated results in section 4 show that the coefficients of all factors are time-varying and dramatically change after the subprime crisis unfolded, we estimate the out-of-sample CDS spreads with daily updated parameters. The in-sample period is from Jan. 3, 2005 through Aug. 31, 2009, and the out-of-sample valuation is from Sep. 1, 2009 to Sep. 30, 2009. The parameters at time t are calibrated by observations within one year and then applied to estimate the CDS spreads at time $t+1$. The default intensity at time $t+1$ is modified as follows

$$\lambda_{i,t+1} = \gamma_{0i,t} + \gamma'_{1i,t} X_{i,t+1} + \gamma'_{2i,t} Y_{k(i),t+1} + \gamma'_{3i,t} Z_{t+1} + \varepsilon_{i,t+1}.$$

Thus, all parameters are calibrated with observations within one year before the pricing date, updated daily, and then applied to evaluate the CDS spreads of the next out-of-sample date. Therefore, parameters during the out-of-sample valuation period are continually updated with new information released.

5.2 Dynamic Calibration Results and Comparison

The dynamics of continually updated parameters also show the time-varying characteristics of parameters. To further identify whether the valuation performance can be improved by updating parameters, we compare the valuation results with time-varying parameters to those with traditional constant parameters. The constant parameters are calibrated from Jan. 3, 2005 through Aug. 31, 2009 to contain all information provided during the whole in-sample period. These parameters are applied to value the CDS spreads during the out-of-sample period without updates. The root mean squared error (RMSE) is calculated to quantify the difference between the out-of-sample valuation results and the real market quotes of CDSs. For each entity, $RMSE_i$ is the square root of mean squared error (MSE) for entity *i* and defined as

$$RMSE_i = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(G^i_{Model,t} - G^i_t\right)^2},$$

where the $G^i_{Model,t}$ is the valuation result of CDS i at time t derived from the pricing model, G^i_t is the market quote of CDS i at time t, and T is the total valuation time period.

For an overview of the valuation performance of all CDSs, we summarize $RMSE_i$ of the 109 CDSs and obtain

$$RMSE = \frac{1}{N}\sum_{i=1}^{N} RMSE_i \cdot$$

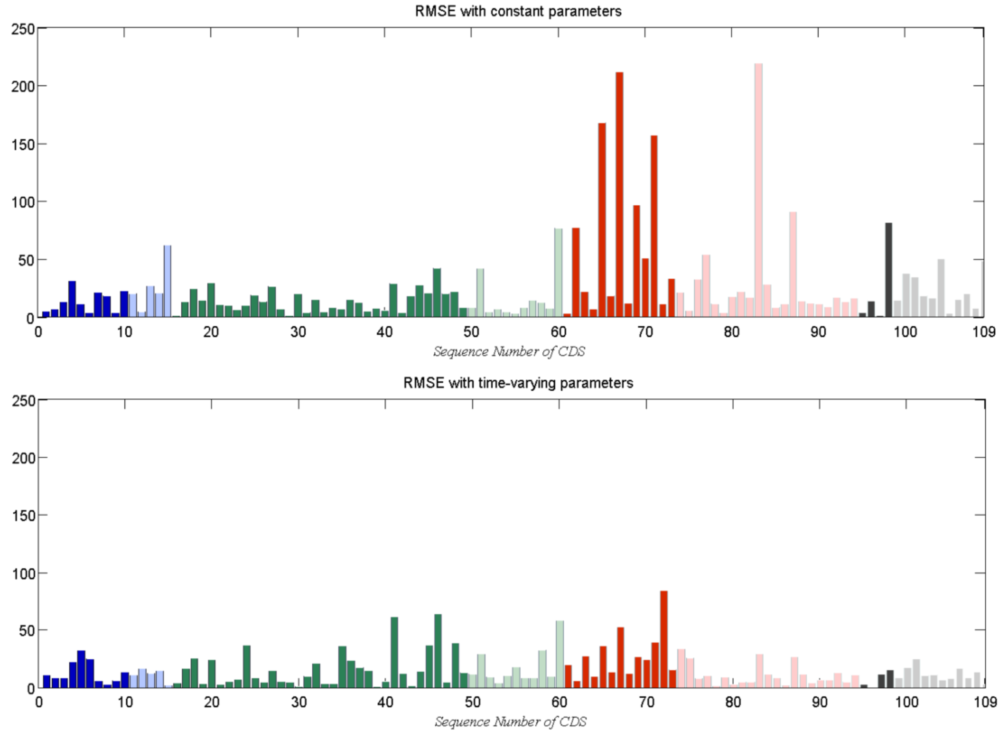

Figure 12. The $RMSE_i$ of all CDSs with constant and time-varying parameters

The top panel displays the $RMSE_i$ of out-of-sample valuation results of CDSs with constant parameters. The parameters are calculated from Jan.03, 2005 through Aug. 31, 2009 to estimate the credit spreads in out-of-sample period from Sep. 01, 2009 to Sep. 30, 2009. The bottom panel shows the valuation results of $RMSE_i$ with time-varying parameters in the same out-of-sample period examined in top panel. CDSs in different sectors are different colors. From left to right, the sectors are basic materials, communications, consumers, energy, financial, industrial, technology, and utilities sectors in order.

The RMSE of time-varying parameters is 16.2067 which dramatically outperforms the result of constant parameters, 26.0251. A thirty-eight percent estimation error is decreased by updating parameters. Since the parameters vary over time especially during a crisis, calibrating credit spreads with traditional constant parameters fail to depict such volatile credit conditions, corresponding to the estimated results in section 4. The $RMSE_i$ of constant and time-varying parameters are displayed in Figure 12. This figure shows that the estimation performance is greatly improved with time-varying parameters. The $RMSE_i$ with constant parameters are markedly high in some CDSs and most are located in the financial sector (the most damaged sector during the subprime mortgage crisis). Thus, failing to renew the relationship between credit spreads and covariates would more likely result in extremely poor estimation of volatile entities during a credit crunch. Consequently, those large $RMSE_i$ estimated with constant parameters are markedly diminished with time-varying parameters as shown in the bottom panel of Figure 12.

6. Conclusions

We argue in this paper that the time-varying correlations between credit spreads and their determinants play a crucial role in valuing credit spreads. As investors are more risk averse under worse credit conditions, the credit spreads may react differently to risk factors during different market conditions. Therefore, the time-varying coefficient regression model is applied to investigate the relationship changes between credit spreads and their determinants from observations without assumption in parametric form. The evidence emerging from single factor estimated results strongly support correlations are time-varying especially after a credit crunch.

This study further constructs two multi-factor models to investigate time-varying fitting results with these substantially significant credit determinants. The estimated results display time-varying coefficients in all factors and reveal different explanatory power over time. From these time-varying multi-factor models, we have two more findings of interest. First, although generally industry-wide risk is important in explaining credit risks, its explanatory power remarkably decreases during worse credit conditions. A possible interpretation is that industry-wide risk is dominated by economy-wide risk during credit crises. As the subprime mortgage crisis dispersed to affect all sectors and led to serious recession in the whole economy, the industry-wide credit crash expanded to be a systematic problem.[11] Thus, after the credit crisis, credit risks between different sectors are no longer so different. Second, the residual variances in the second multi-factor model (named Model 2 which includes sector indicators) are significantly explained by GDP growth. The estimation biases widen during recessions and narrow during expansions. Thus, even after including

[11] Bhansali *et al.* (2008) also concluded that after crisis the systematic credit risk has become a much larger fraction of total credit risk. However, they suggested that after crisis the industry-wide risk levels have remained relatively constant and display small increases after mid-2007.

firm-specific and industry-wide factors, macroeconomic variables still play important roles in explaining credit risks.[12]

Furthermore, in order to clarify whether the time-varying consideration is important in managing default risks, we evaluate the out-of-sample theoretical CDS spreads of time-varying coefficients to compare with the results of traditional constant coefficients. Intuitively, the valuation performance is obviously enhanced by considering time-varying coefficients. From RMSE, a thirty-eight percent estimation error is improved and the extremely high estimation errors represented in the financial sector (the most damaged sector during the subprime mortgage crisis) with traditional constant coefficients are also markedly cut down. Since the changes in credit conditions lead to a different market price for risk, investors ask different risk premiums for compensation. Accordingly, the compensation multipliers (coefficients) related to risk factors should be different over time to correspond with different credit conditions. Therefore, this study provides a new perspective on the importance of considering the correlation changes between credit spreads and their determinant factors to manage default risks more efficiently. As the correlations are demonstrated to be much more volatile after credit crises, such time-varying effects should be thoroughly considered to manage credit challenges.

[12] Considering estimated residuals in multi-factor are not always positive (sometimes credit spreads are underestimated or overestimated), we regressed the natural logarithm of residual variance on GDP growth to ascertain whether the variance of the estimation biases derived from firm-specific and sector indicators can be further explained by GDP growth. Our estimated results support that GDP growth significantly affects the variance of credit residuals. These results differ from Das *et al.* (2007), as they regressed residual defaults on GDP growth and concluded that GDP growth is not statistically significant in explaining the residual correlation of defaults.

Appendix

Following Yang and Zhang (2000), the minimum-variance unbiased variance estimator can be derived by

$$V = V_{open} + KV_{close} + (1-K)V_{RSY}, \tag{A.1}$$

where V_{open} and V_{close} are the variances for the opening and closing prices of stocks, defined as

$$V_{open} = \frac{1}{T-1}\sum_{t=1}^{T}\left(open_t - \overline{open}\right)^2, \tag{A.2}$$

$$V_{close} = \frac{1}{T-1}\sum_{t=1}^{T}\left(close_t - \overline{close}\right)^2, \tag{A.3}$$

where T denotes the time period, and

$$\overline{open} = \frac{1}{T}\sum_{t=1}^{T} open_t,$$

$$\overline{close} = \frac{1}{T}\sum_{t=1}^{T} close_t.$$

We assume $T = 5$ for examination and thus the variance V is updated weekly. The $open_t$ and $close_t$ in Equation (A.2) and (A.3) are the normalized opening and closing prices, respectively, defined as

$$open_t = \ln O_t - \ln C_{t-1},$$

$$close_t = \ln C_t - \ln O_t,$$

where O_t and C_t are the opening and closing prices at time t.

The third term on the right-hand side of Equation (A.1) is V_{RSY} which was proposed by Rogers *et al.* (1994) with the following definition.

$$V_{RSY} = \frac{1}{T}\sum_{t=1}^{T}\left[u_t\left(u_t - close_t\right) + d_t\left(d_t - close_t\right)\right], \tag{A.4}$$

where u_t and d_t are the normalized high and low prices at time t, respectively, and are defined as

$$u_t = \ln H_t - \ln O_t,$$

$$d_t = \ln L_t - \ln O_t.$$

The constant scaled multiple K in Equation (A.1) is chosen to minimize the unbiased variance estimator V. As proposed in Yang and Zhang (2000), the solution of K is

$$K = \frac{0.34}{1.34 + \frac{T+1}{T-1}}.$$

Furthermore, in order to update the variance V weekly, we assume $T = 5$ for examination.

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