The Problem of Estimating the Volatility of Zero Coupon Bond Interest Rate

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Abstract

We analyse the impact of the methodology used to estimate the zero coupon bond yield term structure on the resulting volatility of spot rates with different maturities. The aim of this research is to estimate the current volatility of spot rates in opposition to implicit forward interest rate volatilities which can be observed directly in the markets through interest rate derivative contracts. To have accurate estimates of the volatility of *spot* interest rates is an important issue both from a practical and a theoretical point of view. We focus our attention on three aspects: (1) the model used to estimate the yield curve (Nelson and Siegel, Svensson & Vasicek and Fong), (2) the hypothesis assumed about the variance of the error term of the estimation procedure of spot rates from the Treasury bond markets and (3) the actual data set employed in the estimation. In fact we proceed to estimate term structure of interest rate volatilities using a large transaction data set of U.S. Treasury securities provided by GovPX. We also consider two additional interest rates data sets: estimations of the Federal Reserve Board using the Svensson method from off-the-run bonds, estimations of the US Department of the Treasury using spline functions from on-the-run securities. We find strong evidence that the resulting zero coupon bond yield volatility estimates as well as the correlation coefficients among spot and forward rates depend significantly on these three elements.

Keywords: Volatility Term Structure; Term Structure of Interest Rates; EGARCH;

JEL Classification: E43; F31; G12; G13; G15

1.- Introduction

The main objective of this paper has to do with the importance of knowing what is behind the data we so often use in finance for many purposes, such as calibrations of models, valuations, risk measurement, or design of hedging strategies. Particularly we focus our attention to what we will call the "term structure of interest rates volatility" that is the relationship between **zero coupon bond yield volatilities** and their term to maturity. We should emphasize in the precise variables we are going to deal with. First we specifically concentrate on "zero coupon bond yields (or spot rates), not forward rates neither yields to maturity. And second by volatility we refer expressly to current volatility not implicit volatility.

Although some interest rate volatilities can be considered as an observable variable, for instance, through the quotations of contracts such as caps, caplets, floors, these volatilities are not the variable we are interested on for two reasons. The first one is that they are implicit volatilities instead of current volatilities. The second one is that these volatilities are volatilities of forward interest rates instead of zero coupon bond yields.

Just recall that caps and floors are quoted under the so called Libor Market Model and this model assumes that forward rates with tenor $[T_i, T_{i+1}]$ follows a driftless lognormal process under the forward measure, that is

$$df(t, T_i, T_{i+1}) = f(t, T_i, T_{i+1}) \cdot \sigma(t, T_i. T_{i+1})\, d\omega(t) \qquad [1]$$

where $f(t, T_i. T_{i+1})$ forward rate observable at t with tenor $[T_i ; T_{i+1}]$ $(t \le T_i \le T_{i+1})$, $\omega(t)$ is a standard Wiener process under the forward probability measure and $\sigma(t, T_i, T_{i+1})$ the instantaneous volatility of $f(t, T_i, T_{i+1})$.

It can be easily shown than under the non-arbitrage hypothesis, the price of these contracts which are options on forward rates depend on its implicit volatility that is given by this expression:

$$\sigma_i = \left(\frac{\int_t^{T_i} \sigma^2(u, T_i, T_{i+1})\, du}{T_i - t} \right)^{1/2} \qquad [2]$$

This is a variable that is observable, the variable under which many interest rate derivatives are quoted.

But this variable differs from the one we are interest in two fundamental aspects:

First, if we look at this expression we can see that it is a forward looking measure of the volatility. In fact this volatility is an average of the volatility of the forward rate from t, the present moment up to the maturity of the forward or the option T_i. On the contrary we are interested on the current volatility $\sigma(t, T_i. T_{i+1})$.

But the other difference is that we want to estimate the volatility of zero coupon bond yields not the volatility of a forward rate. And $\sigma(t, T_i. T_{i+1})$ is the volatility of a forward rate $f(t, T_i. T_{i+1})$.

The main problem concerning zero coupon bond yields is that they are not observable variables (except for very short maturities). So we are talking about the problem of estimating the volatility of a non observable variable or, to be more precise the volatility of a set of non-observable variables: zero coupon bond yields with different maturities. The term structure of interest rate volatilities

There are at least three reasons why the term structure of volatilities as we have defined it can be of most interest:

First, we would like to point out that it is of interest from a theoretical point of view. All structural models of the term structure of interest rates assume implicitly not only a particular shape or behaviour of the term structure of interest rates but also a particular shape and a particular dynamics for the volatility term structure. So to know which the actual volatility term structure is can help us to decide whether a model describes more o less accurately the real world.

Second, the volatility term structure is a necessary input for calibrating many interest rate models and particularly the so called "volatility consistent models". Within this category we can find models such as Black, Derman and Toy model one of the most popular tools among practitioners or some extended versions of Hull and White model. Moreover, nowadays some researchers are trying to jointly model the term structure of interest rates and the term structure of volatilities and their relationships (see for instance Cielak and Polava, 2010).

Third, in many topics of Macrofinance or Monetary Policy the term structure of interest rate plays a decisive role. Examples of this can be the problem of testing expectations hypothesis, the estimation of risk premium in bond markets, the ability of volatility to capture the economic uncertainty and its forecasting power with respect to the business cycle, the problem of the volatility transmission along the yield curve.

To illustrate the first point we can use as an example the well known Vacicek model.
It assumes that the very short interest rate follows a normal mean reverting process:

$$dr(t) = \alpha(\bar{r} - r(t))dt + \sigma\, dW(t) \qquad [3]$$

Where $r(t)$ is the very short interest rate, α, $\bar{r}$ and σ are the model parameters and $W(t)$ a standard Wiener process. Using a non arbitrage argument, the current price of a zero coupon bond with maturity at T, $v(t,T)$ is given by:

$$v(t,T) = A(t,T) \cdot e^{-r(t) \cdot B(t,T)} \qquad [4]$$

equations where $A(\ )$ and $B(\ )$ are two functions that depend on term to maturity of the bond $(T\text{-}t)$ and the model parameters alpha and sigma. From the price of zero coupon bonds we can obtain the term structure of interest rates that is the value of all spot rates $r(t,T)$ where $T\text{-}t$ is the interest rate term to maturity.

$$R(t,T) = -\frac{\ln(A(t,T))}{T-t} + \frac{B(t,T)}{T-t} r(t) \qquad [5]$$

The only stochastic element in this equation is *r*(*t*) and it is very easy to see that the volatility of any interest rate and so applying Ito's lemma we obtain that the volatility at *t* of a zero coupon bond yield with maturity at *T* is given by this expression:

$$\sigma_R(t,T) = \frac{\sigma}{\alpha(T-t)}(1 - e^{-\alpha(T-t)}) \qquad [6]$$

This is the expression of the term structure of interest rate volatilities under Vasicek's model. If we analyse this formula we can see many features of interest. First that it depends only on *T-t* the current spot rate term to maturity and model parameters α and σ and so it does not change along time that is the volatilities of spot rates with a given maturity is constant. And also it is immediately that the volatility term structure is a decreasing function of (*T-t*) the term to maturity.

The last issue is a consequence of the dynamics assumed for the short rate *r*(*t*) in Vasicek's model.As it is a mean reverting process, the short rate *r*(*t*) tends to the long term mean rate $\bar{r}$. But long term spot rates can be considered as an average of the present and future short rate. So the volatility of the long rate is smaller that the volatility of the short rate σ and it will be smaller the greater the speed of convergence to the long term mean rate.

A similar argument can be developed for Cox Ingersoll and Ross model. In this case the short rate is assume to follow a mean reverting process but now the volatility of the short rate is made dependent of its level

$$dr(t) = \alpha(\bar{r} - r(t))dt + \sigma\sqrt{r(t)}d\omega(t)_t \qquad [7]$$

Using the same argument than before we can obtain the formula for the volatility term structure:

$$\sigma_R(t,T) = \frac{\sigma\sqrt{r(t)}}{T-t} B^*(t,T) \qquad [8]$$

where B^*( ) is a function that depends on the model parameters. Now we can see that the volatility of a zero coupon bond with a given maturity depend on *r*(*t*) and so it changes as time passes and *r*(*t*) changes. But in any case what we can see that volatility term structure can only change in a very particular way. If r(t) increases all the volatility term structure moves upwards and vice versa. If *r*(*t*) decreases then the whole volatility term structure goes down. That is changes, in the shape of the volatility term structure are not allowed under the Cox, Ingersoll and Ross world.

This limited variety of shapes and dynamics of the volatility term structure can be solved in part using multifactor models of the term structure. But even within these multifactor term structure models there is also an inherent structure and a precise dynamics for the volatilities of spot interest rates with different maturities. For instance in affine term structure models such as Longstaff and Shcwartz (1992) or Fong and Vasicek (1992) the level and shape of the volatility term structure is perfectly determined by the values of the state variables, the short rate and its volatility.

There is also another family of term structure models called "term structure consistent models" characterised for matching perfectly the observed term structure of interest rates. In these sort of models the observed term structure of interest rates is introduced as an input and the model is solved we get back the actual zero coupon bond yields.

The first model of this family was Ho and Lee model and its continuous time limit version is given by this stochastic differential equation.

$$dr(t) = \theta(t)dt + \sigma d\omega(t) \qquad [9]$$

The drift *θ(t)* depends on the present term structure of interest rates. And in this case the volatility term structure is given by this equation:

$$\sigma_R(0,T) = \sigma \qquad [10]$$

That means that is constant and the same for all maturities. That is the volatility of the very short rate is the same than the volatility of 30 year interest rate.

Another model of this family is Hull and White. This model is also a mean reverting model and also we introduce the term structure of interest rates in the draft.

As in Ho and Lee model, if we obtain the equation of the value of zero coupon bonds we would recover the term structure of interest rates that we have introduced in the model.

In this case the volatility term structure that we obtain is the following:

$$\sigma_R(t,T) = \frac{\sigma}{\alpha(T-t)}(1 - e^{-\alpha(T-t)}) \qquad [11]$$

which is exactly the same than in Vasicek model. In fact, Hull and White is sometimes referred as extended Vasicek's model.

So we can see that there is a family of models that can match perfectly the observed term structure of interest rates but they are very restrictive with respect to the possible shapes and dynamics of the volatility term structure.

Finally, there is another family of models that matches perfectly not only the term structure of interest rates but also the volatility term structure of interest rates. They need as an input both the term structure of interest rates and the volatility term structure, that is, the volatility of spot rates with different maturities. The most popular modes is Black, Derman and Toy whose continuous time limit version is

$$d\ln r(t) = \left[\theta(t) + \frac{\sigma'(t)}{\sigma(t)}\ln r(t)\right]dt + \sigma(t)d\omega(t) \qquad [12]$$

where θ(t) and σ(t) are chosen in such a way that the models fit exactly the current term structure of interest rates and the current term structure of spot rates volatilities. This is of course the continuous time limit version of the model but in practice this model is implemented using a double binomial tree and it needs as inputs the term structure of interest rates and the volatility of spot rates with maturity at each tree knot.

But even in this case, the future behaviour of the volatility tem structure is completely determined by the inputs that we have introduced in the model and particularly by the term structure of volatilities we have used as input of the model.

In summary, all structural models assume implicitly a particular shape and a particular dynamics for the term structure of spot interest rate volatilities. This is a good enough reason to analyse which is the actual shape and the actual behaviour of the term structure of interest rate volatilities.

But as we said at very beginning there are more reasons why the problem of the estimation of the volatility term structure is of interest.

Many financial problems such as product valuations, risk measurements, hedging strategies, depend on spot interest rate volatilities and correlations. As a very simple example let's think of the calculation of the VaR of a bond portfolio. Value at Risk depends, above all, on the volatility of spot interest rates and their correlations.

In practice these estimates of interest rate volatilities are obtained following a two step process:

First step: As zero coupon bonds are not observable we download from database providers zero coupon yield curves or we can proceed to estimate them directly from market data. From the swap market, spot rates can be recovered using bootstrapping techniques and from the Treasury bond market, spot rates are estimated approximating a particular functional form to a theoretical discount function that values as accurately as possible bond prices.

Second step: We proceed to estimate the volatility from the time series of zero coupon rates with different maturities that has been estimated in the first step.

But we must be aware of the fact that when proceeding in this way we are estimating the volatility of the sum of two elements: the zero coupon bond yield itself and a "small" error term.

So, the question we would like to answer is to what extent the valuations, risk measurements, hedging strategies, etc. that depends so crucially on interest rate volatilities are "contaminated" by the particular set of estimated spot rate data employed.

Or put this question in another way: does it matter the method we (o someone else) have used to estimate the term structure of interest rates for the resulting estimates of the volatility term structure?

This is in fact the main question we would like to analyse empirically in this article.

But there are more reasons why an accurate estimation of the volatility term structure can be an important issue. In other fields such as macrofinance or monetary policy the variance and the relationship among spot rates with different maturities can play an essential role.

As an example of this point we can think, for instance, in the problem of testing of the Pure Expectations Hypothesis according to which forward rates are unbiased estimates of future spot rates

As far as forward rates depend of pairs of spot rates, the results of most of the test of this hypothesis[1] will depend, in the end, on the variances and covariances among sets of spot rates with different maturities.

Then, do the outcomes of these experiments depend on the actual set of spot rates eventually chosen for testing this hypothesis? Moreover, do the correlations among forward rates depend on the method used to estimate spot rates?

In this paper, we analyse to what extend both the error structure imposed in fitting the zero coupon yield curve (YC) and the composition of the data set affect on the resulting volatility term structure (VTS). We check whether these assumptions in the underlying YC determine the VTS even more than the own method used for fitting the VTS.

We analyze three different data sets according to the securities to be included. First, we examine the Treasury YC estimates of the Federal Reserve Board posted on its website[2] and commented by Gürkaynak *et al.* (2007). They use a weighted version of the SV method from prices of all the outstanding off-the-run bonds. Among other securities, they exclude in their estimation all Treasury bills and the on-the-run and the "first-off-the-run" issues of bond and notes. Second, we analyze the YC reported by the U.S Department of the Treasury.[3] We consider this data set even when they use a different YC method, a quasi-cubic hermite spline function. The inputs are the yields for the on-the-run securities. They include four maturities of most recently auctioned bills (4-, 13-, 26-, and 52-week), six maturities of just-issued bonds and notes (2-, 3-, 5-, 7-, 10- and 30-year), plus the composite rate in the 20-year maturity range. Finally, we run our own estimations using both the unweighted and weighted versions of SV from prices of all the bills, bonds, and notes in the data set provided by GovPX. This daily data set includes the prices of all actual transactions and quotes provided by GovPx from January 1994 to December 2006.

We call "unweighted" the fitting of each yield curve fitting method assuming homoscedasticity in the bond price errors. The term structure is estimated by ordinary least square (OLS). In the weighted estimation process, we assume a structure of weighted pricing errors and apply a generalized least square (GLS) method.[4] This overweighting of the short term bonds in the price errors minimization program has become widely used by practitioners and central banks. In the homoscedastic case, the short term end of the curve can be erratic. An error in short term bond prices induces an error in the estimation of short interest rates greater than the error in long term interest

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[1] These test can be more less sophisticated from a simple linear regression to Vector Autorregresive Models or cointegration techniques.
[2] http://www.federalreserve.gov/econresdata/researchdata.htm
[3] http://www.ustreas.gov/offices/domestic-finance/debt-management/interest-rate/yield.shtml
[4] Weights are the reciprocal of the modified Macaulay's duration of each security.

rates produced by the same error in long term bond prices. In the heteroskedastic case, the variance of the residuals is constrained to be small for short maturity securities.

Once alternative estimates of YC are obtained, we proceed to estimate interest rate volatilities. We consider both model-free volatilities computed using a 30-day rolling window estimator and model-implied volatilities from conditional volatility models (GARCH models). A standard GARCH(1,1) model is traditionally assumed to estimate volatility of daily interest rates (see, e.g., Longstaff and Schwartz, 1992). However, since Hamilton (1996), the EGARCH model has been widely used for analyzing the volatility of daily short term and very short term interest rates. Andersen and Benzoni (2007) have documented that an EGARCH representation for the conditional yield volatility provides a convenient and successful parsimonious model for the conditional heteroskedasticity in these series. According to the Schwarz and Akaike Information Criterion (SIC and AIC respectively), we choose the EGARCH (1, 1) model to estimate interest rate volatility.

We show statistically significant differences between estimates of the term structure of interest rate volatilities depending on the sample selection and the heteroskedasticity structure of errors used to estimate the YC. These differences are observed mainly in the short-term, but also in the long-term volatility. Differences between alternative volatility estimation methods from the same zero coupon series are almost negligible. This inspection could have significant consequences for a lot of issues related to risk management in fixed income markets.

2.- The impact of the methodology for estimating spot rates on volatilities and correlations

The hypothesis that we are going to test in this research is that the method used to estimate the zero coupon bond yields may have important consequences in the resulting volatility of the time series of interest rates. There are at least three reasons for that. First, the greater or smaller degree of flexibility of the model used to describe the tem structure of interest rates. Second, the structure of the variance of the error term imposed in the fitting process. And third,the actual set of assets included in the sample.

2.1. The flexibility of the model

The estimation of the spot rates consist of finding a functional form that approximates the theoretical discount function, $D(t)$, and to replicate at a given instant as accurately as possible a set of bond prices:

$$P_k = \sum_{T=T_1^k}^{T_n^k} C_T^k \cdot D(T,\bar{b}) + \varepsilon_k \qquad k=1,2,\dots,m \qquad [13]$$

where P_k is the price of bond k, C_T^k are the cash flows (coupon and principal payments) generated by bond k, $D(T,\bar{b})$ the discount function that we want to approximate and that depends on a vector of parameters $\bar{b}$ and ε_k is an error term.

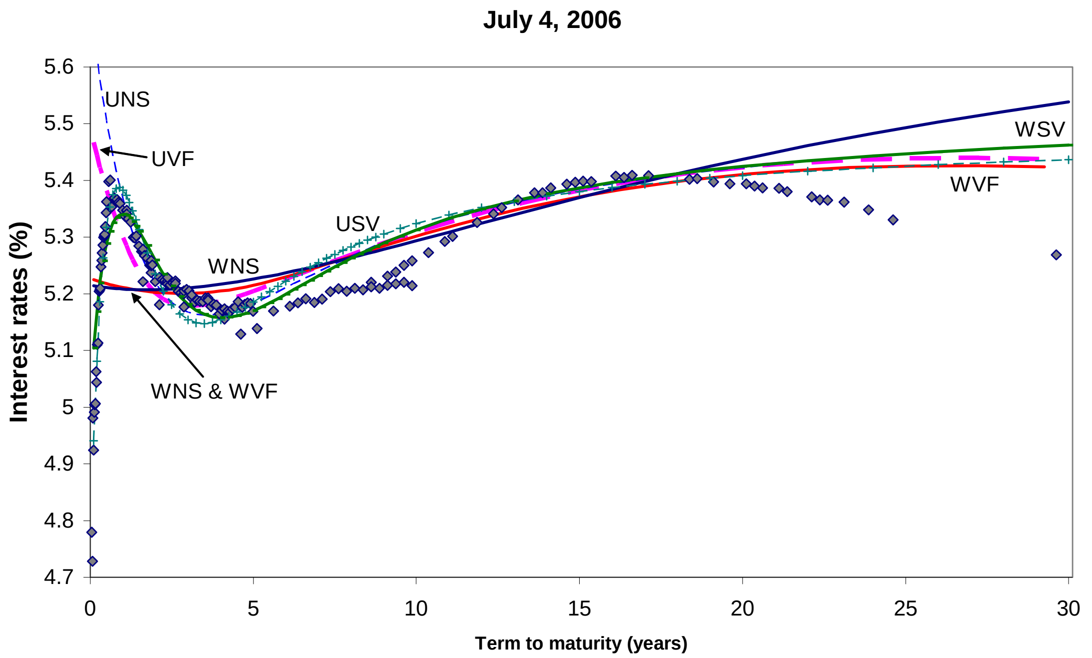
$D(T, \bar{b})$ can be approximated using mainly two sets alternative functional forms:

- non-parametric models such as McCulloch or Vasicek and Fong (1982)
- parametric models such as Nelson and Siegel (1987) or Svensson (1994)

However, these models can present different degrees of flexibility to describe the term structure of interest rates. For instance some models may have a greater ability to describe the hump so often observed in the yield curve or the behaviour of long term interest rates meanwhile others can be more rigid in the adjustment of the actual yield curve; in this case, some models may produce more or less volatile interest rate estimates in some tranches of the yield curve. If a model is too rigid to adapt to the actual shape of the yield curve it may produce in some regions of the yield curve estimates of the spot rates that fluctuate less than real spot rates do. However, a more flexible model may capture more adequately the real behaviour of interest rates.

Let's see an example in the following picture that represents the yields to maturity of bonds and bills in the US Treasury market the 4^{th} of July of 2006. The vertical axis represents interest rates and the horizontal axis the term to maturity of the bonds traded in that market. The dots correspond to the yields to maturity of these securities. Zero coupon bonds, although a different variable must follow a similar pattern. We can see here the hump that can be observed quite often in the US market. It is no easy to find a function that can capture this feature of the term structure. For instance we can have a look at the solid lines that represent estimations of the term structure using alternative models. We can see that two of these curves (red and blue lines) just cross the data in the short end of the yield curve and so they do not describe correctly the behaviour of actual interest rates. However we can see that the third model (green line) has captured pretty well this hump. If as time passes and the hump disappear moving towards a flat term structure the first two models will not change their estimates of these interest rates meanwhile the third one will register this movement. So the third model will produce higher volatile estimates of the yield curve for these maturities.

Figure 1.- Alternative estimations of the term structure of interest using models with different degrees of flexibility



2.2.- The impact of the assumption about the error term variance

The second element in the process of estimation of zero coupon yields that can impact on the resulting volatility of interest rates is the structure of the variance of the error term ε_k of model [13].

The first estimates of the term structure of interest rates usually assumed homoskedasticity and so the model could be estimated using OLS.

$$VAR[\varepsilon_k^2] = \sigma^2 \qquad [14]$$

But this is not neutral assumption. In fact a small error in a short term bond price produces an important error in its yield to maturity. On the contrary, a big error on the price of a long term bond affects very slightly its yield to maturity. We should not forget that in this model the dependent variable are bond prices. And so, if we assume homoskedasticity we give the same importance to errors in the price of all bonds and that means that we are penalizing very heavily errors in the yields of long term bonds. So, to assume homoskedasticity implies forcing the adjustment in the long end of the yield curve but at the cost of relaxing the adjustment the curve for short maturities.

To correct this problems some authors suggested to penalize the valuation errors of the short term bonds and particularly it is usually suggested to correct the variance of the error term making it proportional to the bond duration, that is:

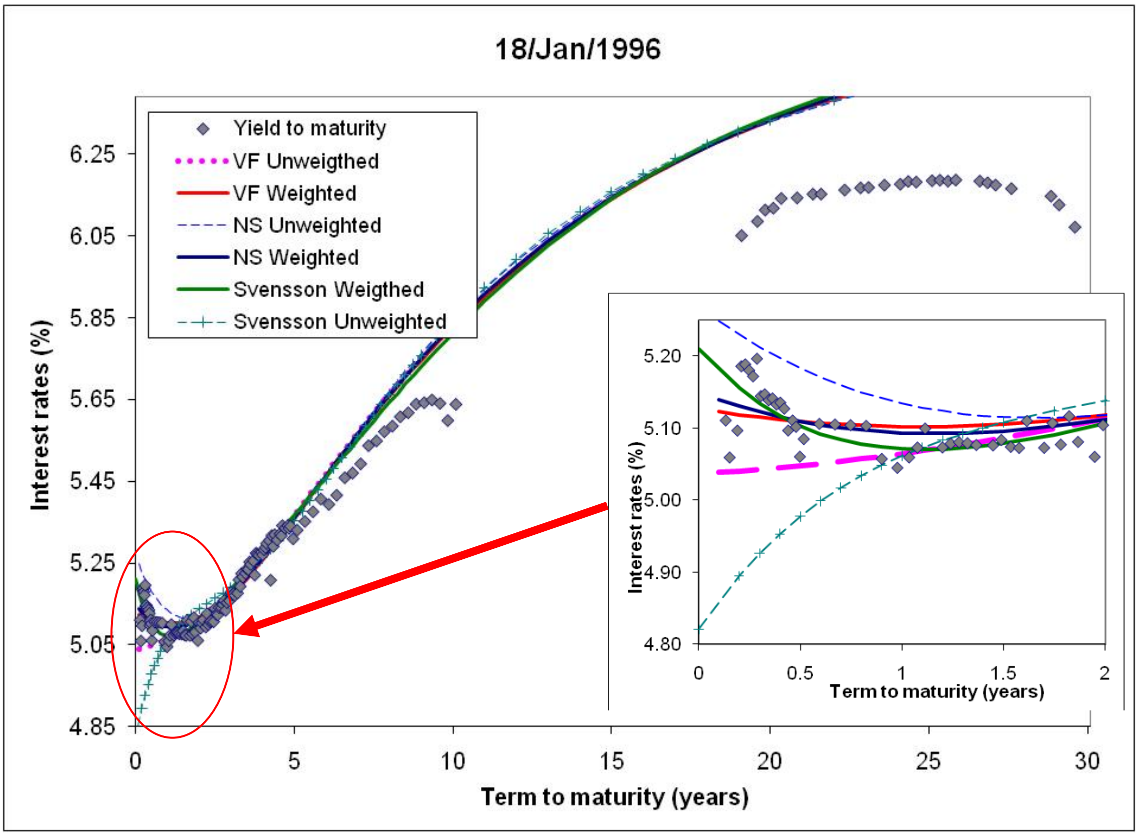
$$VAR[\varepsilon_k^2] = \left(\frac{\partial P_k}{\partial Y_k}\right)^2 \cdot \sigma^2 = \left(\frac{D_k \cdot P_k}{1+Y_k}\right)^2 \cdot \sigma^2 \qquad [15]$$

where D_k is the k-bond duration, y_k its yield to maturity and P_k its price.Then the model is adjusted using GLS.

When doing that in fact what we are doing is to force the adjustment of the short term interest rates. But this is not free: it implies relaxing the adjustment of long term interest rates.

What we claim is that this correction of the variance of the error term affects not only to the accuracy of the estimates but also to the volatility of these estimates. Moreover, it may cause an important impact on the relative fluctuation of the estimated long and short term spot rates and so it may affect significantly the shape of the volatility term structure.

Figure 2.- The importance of the variance of the error term



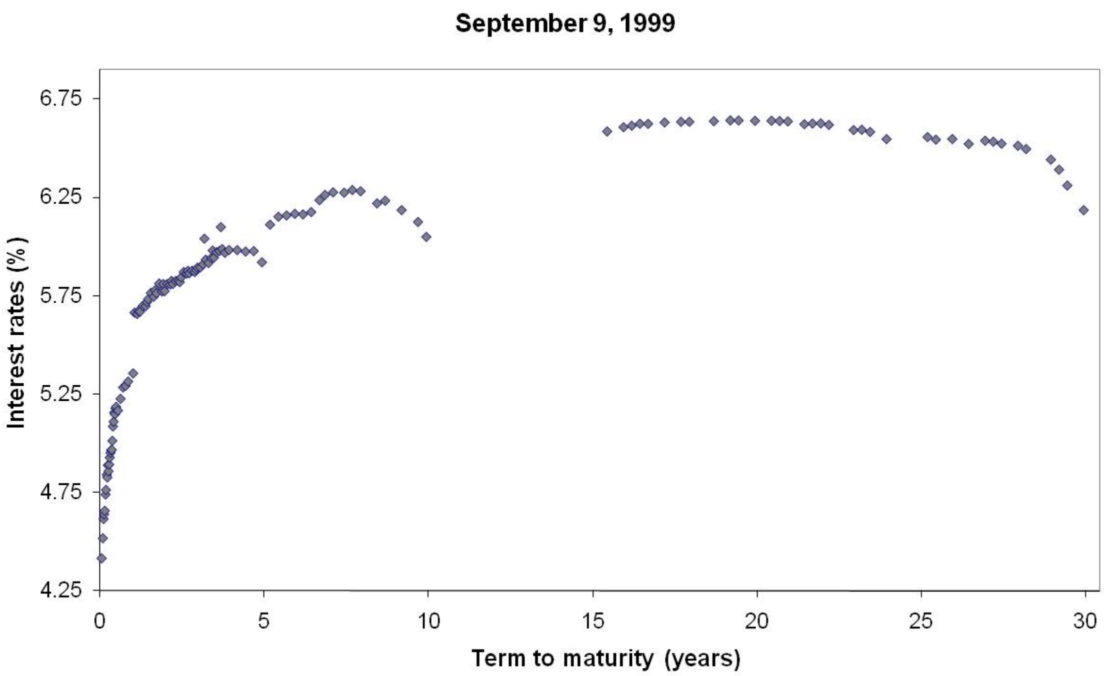
Let's have a look at this figure 2 corresponding to alternative term structure estimations the 18th of January 1996. The solid lines represent three different estimates of the yield curve using the heteroskedatic structure. We can see that these three curves describe quite well the data for short maturities. On the contrary, the dotted lines represent estimates of the term structure using the same yield curve models but applying the unweighted or homoskedatic scheme for the variance of the error terms. We can see that the adjustment is pretty bad for short maturities because when making this assumption the model does not care the adjustment in this side of the yield curve. It only pays attention to what happen in the other side. So we wonder if the assumption about the structure of the variance of the error term affects not only the quality of the adjustment but also the volatility of the estimated interest rates when we proceed to estimate the yield curve day after day.

2.3. The importance of the actual data set

And the third problem mentioned at the beginning of this section has to do with the actual set of data included in the estimation of the yield curve.

In the Treasury market are traded securities with important differences, apart from the term to maturity. For instance we can come across bonds with very different degrees of liquidity. And we all know that liquidity may have an important impact on prices. Let's see Figure 3.

Figure 3.- The impact of liquidity on yields to maturity



Usually when a bond is just issued it concentrates the most of the trading volume as most investors and fund managers are trying to allocate or distribute this new asset in their portfolios or within their clients. But as this bond becomes seasoned and above all when new references are issued the trading volume decreases dramatically and so does its liquidity. And this seems to have an important impact on bond prices.

For instance, have a look at the 10-year bond yield to maturity in Figure 3. This dot here represents the yield to maturity of the last issued bond which is the most actively traded within this segment of the market. The dot at its left represents the 10-year bond that was issued before the last one. Its trading volume is smaller. And so on. These dots represent 10 year bonds that are becoming seasoned and we can see that the yield to maturity is considerably bigger.

So when adjusting the yield curve we have to decide if we include all bonds, only the most liquid ones or just the opposite. But depending on this decision we are estimating, in fact, different things: the spot rates corresponding to average market liquidity level, the spot rates of the most liquid references or the spot rates of seasoned bonds. The level of these interest rates should be different, but probably the volatility is different too.

In addition, some estimates of the term structure eliminate very short bonds because it can make it difficult to describe with a simple function the behavior of the yield curve in this end of the curve.

Some authors use actual transaction prices and some other quoted price (mid bid-ask, bid or the ask prices . . .)

We wonder if all these elements can have a significant impact on the resulting volatility of the estimated zero coupon bond yields and this is what we have tried to answer.

First we have analyzed the first two issues: the effect of the model and the structure of the error term and then the problem of using different sets of data in the estimation of the zero coupon bonds.

3.- Empirical analysis

3.1. Models

To test the first two issues (the impact of the model used and the assumption about the variance of the error term) we have proceed to estimate the term structure of interest rates using three alternative models extensively used in the literature. For each model we will proceed to estimate zero coupon bond yields using the two assumptions about the variance of the error term, $\varepsilon_{k,}$ of model [13] described in section 2.2. These three models are Vasicek and Fong (1982), Nelson and Siegel (1987) and Svensson (1994). So we obtained eventually six different daily estimations of the term structure of interest rates.

Vasicek and Fong is one of the pioneering non-parametric methods used to estimate the yield curve. In view of the shortcomings of the polynomial splines proposed by McCulloch in 1971 for estimating the discount function, Vasicek and Fong presented an exponential splines fitting that obtains a desirable asymptotically flat forward rate curve. This method is not as popular as Nelson and Siegel or Svensson probably because its implementation is more complicated. In fact, it can not be immediately applied. To implement the model a basis of piecewise third-degree exponential polynomials and a method to locate the “knot points” must be assumed. However, there are several reasons why Vasicek and Fong is of interest:

1) VF exhibits a high degree of flexibility as most non-parametric methods to fit a wide variety of shapes of the term structure of spot rates. In fact is as flexible as you want, depending of the number of knots of piece-wise functions.
2) It presents sufficient robustness to produce stable forward rate curves particularly in the long end of the yield curve

The Vasicek and Fong model can be summarized as follows:

They propose the use of a transform of the argument of the function D(T, $\bar{b}$) in order to make the estimation technique easier::

$$T = -\frac{1}{\alpha}\log(1-x) \qquad 0 \leq x \leq 1 \qquad [16]$$

$$D(T, \bar{b}, \alpha) = D\left(-\frac{1}{\alpha}\log(1-x)\right) \equiv G(x) \qquad [17]$$

The advantage of the transformation is that if we assume that D(T, $\bar{b}$) is an exponential like function then G(x) would be a power like function, that is G(x) is an approximate power function that preserves the linearity of the model (with respect to the parameters except α) and can be well fitted by polynomial splines.

Nelson and Siegel method is a simple parametric model of the TSIR. This parsimonious approach imposes a functional form on the forward rates.

Nelson and Siegel assume explicitly the following function form for the instantaneous forward rates:

$$f_T = \beta_0 + \beta_1 \exp\left(-\frac{T}{\tau}\right) + \beta_2 \frac{T}{\tau}\exp\left(-\frac{T}{\tau}\right) \qquad [18]$$

where T is the term to maturity and (β_0 , β_1 , β_2 , τ) is the vector of four parameters to be estimated.

A number of authors have interpreted the first three parameters, β_0, β_1 and β_2, as the specific factors that drive the yield curve: level, slope, and curvature. In concrete, β_0 is related to the long-run level of interest rates. Literature pays less attention to the last parameter τ. It is usually fixed at a prespecified value. Diebold and Li (2006) justify this simplification in terms of simplicity, convenience and "numerical trustworthiness by enabling us to replace hundreds of potentially challenging numerical optimizations with trivial least-squares regressions". Anyway, we estimate the four parameters as in the original proposal of Nelson and Siegel. Our experience shows that the value of this parameter is much more volatile than the values of the other three parameters, but it plays a relevant role. This parameter determines the inflection point and decides the position of the possible hump in the yield curve.

Nelson and Siegel method has a number of advantages over the spline methods. It is simple, it results in more stable yield curves, it requires fewer data points, and it does not require finding an appropriate location of knot points which joint up a series of splines. However, Vasicek and Fong model allows for a much higher degree of flexibility than parametric models. Specifically, the individual curve segments can move almost independently of each other (subject to the continuity and differentiability constraints), so that separate regions of the curve are less affected by movements in nearby areas. Therefore Vasicek and Fong approach incorporates a wider variety of yield curve shapes than Nelson and Siegel method. Thus Nelson and Siegel model is unable to accommodate humped shapes that are sometimes observed in the bond market.

Gürkaynak *et al.* (2006) emphasize that convexity[5] makes it difficult for fitting the entire term structure, especially those securities with maturities of twenty years or more. They maintain that convexity tends to pull down the yields on longer-term securities, giving the yield curve a concave shape at longer maturities. We corroborate that Nelson and Siegel method tends to have the forward rates asymptote too quickly to be able to capture the convexity effects at longer maturities.

In summary, there exists a trade-off between the complexity cost of the spline methods and the richness of the eligible forms for the TSIR that these models provide.

Svensson can be considered as an extension of Nelson and Siegel model and it assumes that the instantaneous forward rates are governed by six parameters:

$$f_T = \beta_0 + \beta_1 \exp\left(-\frac{T}{\tau_1}\right) + \beta_2 \frac{t}{\tau_1} \exp\left(-\frac{T}{\tau_1}\right) + \beta_3 \frac{T}{\tau_2} \exp\left(-\frac{T}{\tau_2}\right) \qquad [19]$$

where T is the term to maturity and (β_0 , β_1 , β_2 , τ_1 , β_3 , τ_2) the set of parameters to be estimated .

In this sense, Svensson is a more flexible approach which adds two new parameters to Nelson and Siegel that allow a second "hump" in the forward rate curve and also provides a better fitting of the convex shape of the yield curve in the long end.

3.2.- Data

We obtain intraday U.S. Treasury security quotes and trades for all issues between January 1994 and December 2006 (2,864 trading days) from the GovPX database.[6] GovPX consolidates and posts real-time quotes and trades data from six of the seven major interdealer brokers (with the notable exception of Cantor Fitzgerald). Taken together, these brokers account for about two-thirds of the voice interdealer broker market. In turn, the interdealer market is approximately one half of the total market (see Fleming, 2003). Hence, while the estimated bills coverage exceeds 90% in every year of the Fleming's GovPX sample (Jan 97 – Mar 00), the availability of thirty-year bond data is limited because of the prominence of Cantor Fitzgerald at the long-maturity segment of the market. According to Mizrach and Neely (2006), voice-brokered trading volume began to decline after 1999 as electronic trading platforms (e.g., eSpeed, BrokerTec) became available. In fact, GovPX does not provide aggregate volume and transaction information from May 2001.[7] Therefore, we assume an imperceptible impact

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[5] Convexity is understood as that obtained from the second-order approximation of the change in the log price of the bond.

[6] GovPX Inc. was set up under the guidance of the Public Securities Association as a joint venture among voice brokers in 1991 to increase public access to U.S. Treasury security prices.

[7] After ICAP's purchase of GovPX in January 2005, ICAP PLC was the only broker reporting through GovPX.

of the decline in GovPX market coverage on our estimates since we consider the midpoint prices and yields between bid and ask at 5 pm.

The GovPX data set contains snapshots of the market situation at 1 pm, 2 pm, 3 pm, 4 pm, and 5 pm. Each snapshot includes detailed individual security information such as CUSIP, coupon, maturity date, and product type (indicator of whether the security is trading when issued, on the run, or active off the run). The transaction data include the last trade time, size, and side (buy or sell), price (or yield in the case of bills), and aggregate volume (volume in millions traded from 6 pm previous day to 5 pm. The quote data include best bid and ask prices (or discount rate Actual/360 in the case of bills), and the mid price and mid yield (Actual/365).

Our initial sample relies on the information at 5 pm, i.e. last transaction taking place during "regular trading hours" (from 7:30 am to 5:00 pm Eastern Time, ET) if available, or quote data otherwise. We complement the GovPX data with official data on the dates of the last issue and of the first interest payment, and the standard interest payment of each Treasury security.[8] This information is publicly available on the U.S. Treasury Website.

To obtain a good adjustment in the short end of the yield curve, we consider all the Treasury bills. In this term to maturity segment, bills are very much more actively traded than old off-the-run notes and bonds.[9] Thus, we include only the Treasury notes and bonds that have at least one year of life remaining. Since the number of outstanding bills with terms to maturity between 6-month and 1-year declines considerably during year 2000 and the 1-year Treasury bill is no longer auctioned beginning March 2001, we also consider Treasury notes and bonds with remaining maturities between 6- and 12-month from 2001.

We also apply other data filters designed to enhance data quality. First, we do not include transactions associated with "when-issued" and cash management, or trades and quotes related to callable bonds and TIPS (Treasury Inflation-Protected Securities). Second, when two or more different securities have the same maturity, we only consider trades and quotes of the youngest one, i.e. the security with the last auction date. Finally, we exclude yields that differ greatly from yields at nearby maturities.[10] In certain dates, we apply an *ad hoc* filter. We observe occasionally that deleting a single data point in the set of prices used to fit the yield curve can produce a notable shift in parameters and also in fitted yields improving notably the fitting. This phenomenon is also commented by Anderson and Sleath (1999).

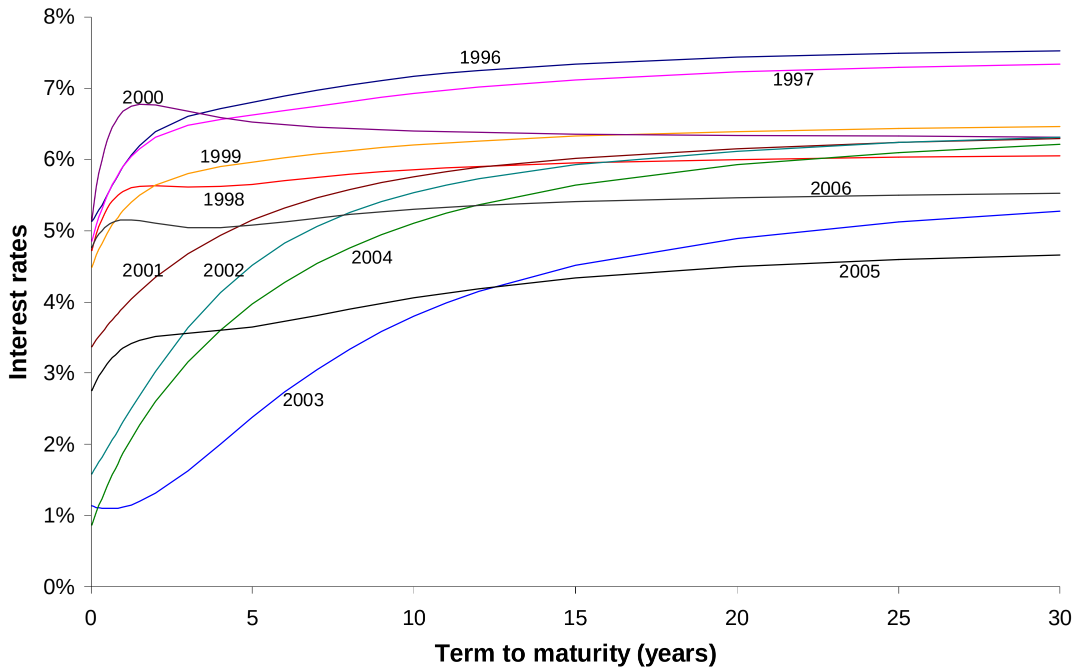
Controlling for market conventions, we recalculate the price of each security in a homogeneous fashion to avoid effects of different market conventions depending on maturities and assets. Every price is valued at the trading date in an actual/actual day-count basis. In the case of Treasury bills, firstly we obtain the price at the settlement date from the last trade price if available or from the mid price between bid-ask

[8] "Standard interest payment" field gives indirectly information to identify callable bonds and TIPS (Treasury Inflation-Protected Securities).
[9] Also, Fleming (2003) emphasizes that GovPX bill coverage is larger than bond and note coverage.
[10] These cases include interdealer brokers' posting errors like those mentioned by Fleming (2003).

otherwise.[11] In both cases, the GovPX reported price is a discount rate using the actual/360 basis. Secondly, we compute the yield-to-maturity as a compound interest rate using the actual/actual. Thirdly, we calculate "our" price at the trading date using the yield-to-maturity obtained in the previous step.[12] In the case of Treasury notes and bonds, the price is directly reported in the data as the last trade price or the mid price. From this price we apply the mentioned second and third steps to obtain "our" homogeneous price.[13]

As a summary of the results, in figure 4 we picture the estimated term structures of interest rates corresponding to the first working day of June during the eleven years of our sample. These estimates were obtained using the Nelson and Siegel model and assuming heteroskedasticity. As mentioned at the beginning of section 3 we got six different estimates of this term structure using the same set of data.

Figure 4.- Term structure of interest the first working day of June during the sample period. (01.96-12.06) .



The results have been summarised in tables 1 and 2. Table 1 show the sum of squared residuals using the six methods. As expected, the homoskedatic estimates produce lower squared residual than the corresponding heteroskedastic estimates. We can also see that the non parametric model (Vasicek and Fong) seems to produce, on average, smaller residuals than the parametric models and Svensson (an extension of Nelson and Siegel) also show a better adjustment than the latter model.

[11] We do not consider the reported mid yield. This is a simple interest with actual/365 basis, except for more than 6-month remaining maturity bills which are valued using the bond equivalent yield.

[12] Note that the settlement date is in most cases a working day after the trading date.

[13] We control for the special amount of the first interest payment in just-issued securities.

3.3 Volatility estimates

Once we the set of successive term structures of spot rates was obtained, we extracted 27 different spot rates with maturities ranging from one week up to 15 years

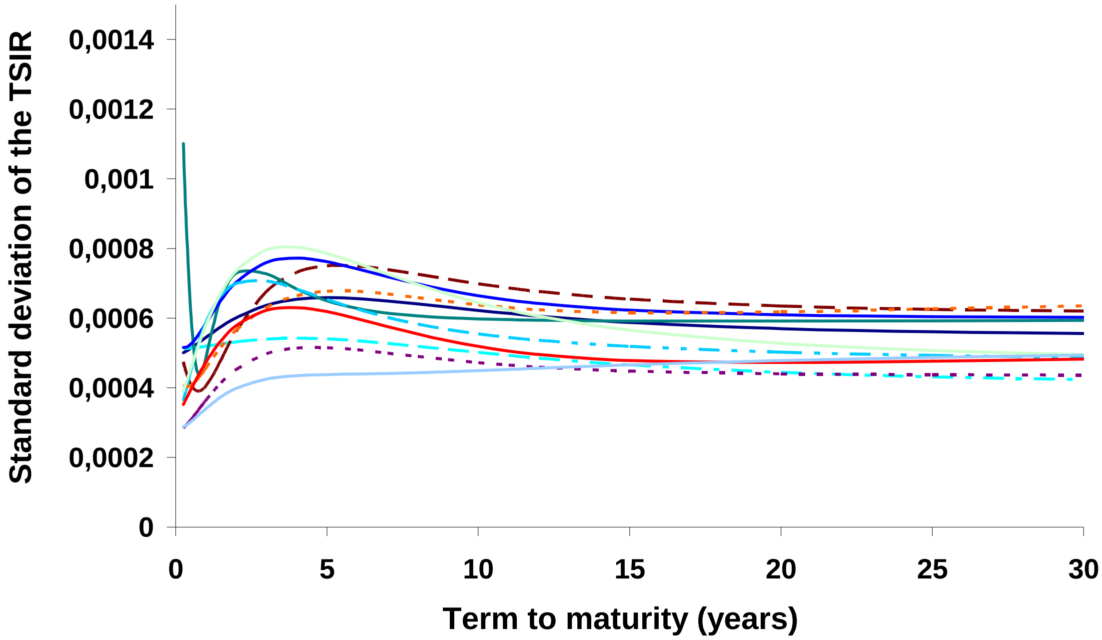
From these data sets we use two alternative methods to estimate the volatility.

– A simple standard deviation using a 30-day rolling window

– We considered different specifications of the well known family of the conditional volatility models. Finally we choose the EGARCH(1,1) model proposed by Nelson in 1991 which allows for asymmetric impacts of the innovations

The results obtained are illustrated in figure 5 where we depict the term structure of volatilities corresponding to estimations of spot interest rates using Nelson al Siegel with homoskedatic errors and using the simple standard deviation for estimating the interest rate volatilities.

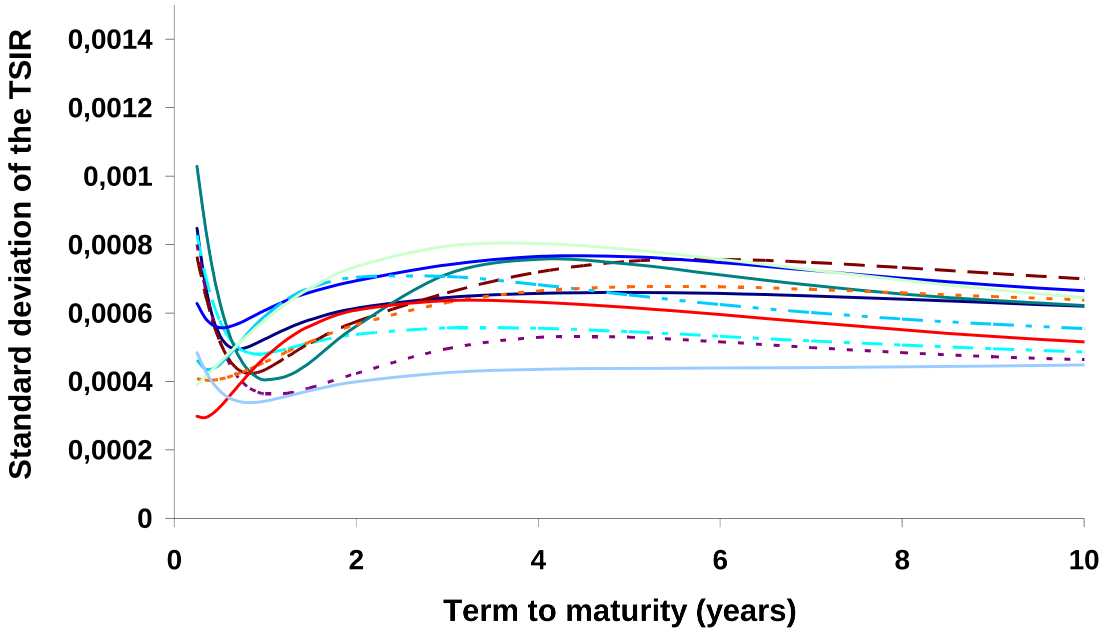
Figure 5.- Volatility term structure estimates corresponding to the first working day of June during the sample period. (01.96-12.06)



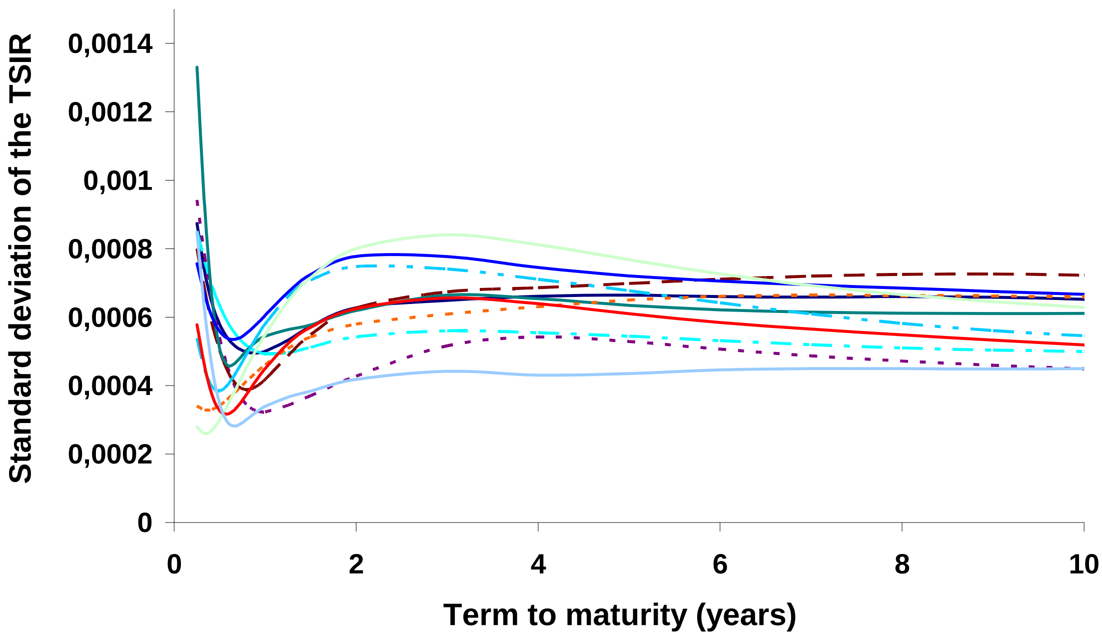
The first point that we can point out when we observe figure 5 is the great diversity of shapes that the volatility term structure has taken during the sample period. We can see increasing curves, humped curves, double humped curves. . . . It is evident that single factor models of the term structure can hardly capture this variety of profiles and shapes of the volatility.

The second result has to do with the impact of the different assumptions about the variance of the error term.

Figure 6.- The impact of the assumption about the variance of the error term
Panel A.- Svensson model with homoskedastic error. VTS corresponding to the first working day of June during the sample period



Panel B.- Svensson model with heteroskedastic error. VTS corresponding to the first working day of June during the sample period



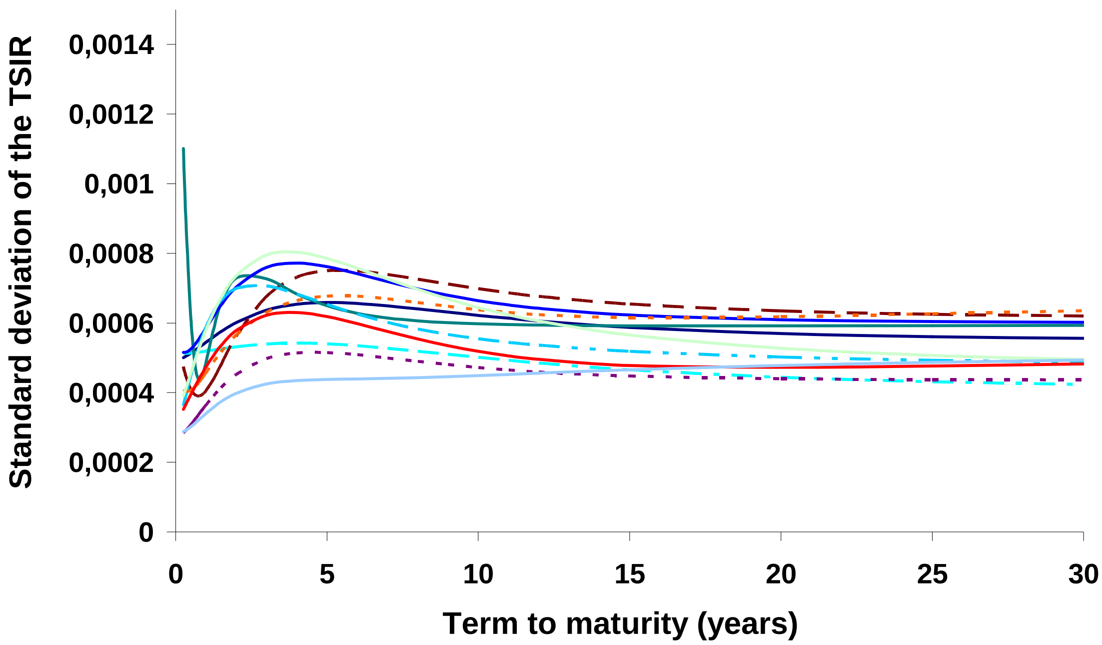
The first one is that although this volatilities term structure have been chosen randomly but we can see that the shape of the term structure changes significantly depending of the weighting scheme we had chosen to estimate the term structure of interest rates. Panel A represents the homoskedstic assumption and panel B the heterokedasitc one and in both cases we used Svensson model

The second thing we can observe is that humps shifted towards the left and also for very short maturities, the weighted scheme produced estimates of spot rates with a higher volatility. All this is due to the fact that using the heteroskedastic assumption forces the adjustment in the short end of the curve. On the contrary, the volatility estimates of very long term rates are higher when the weighted scheme is introduced.

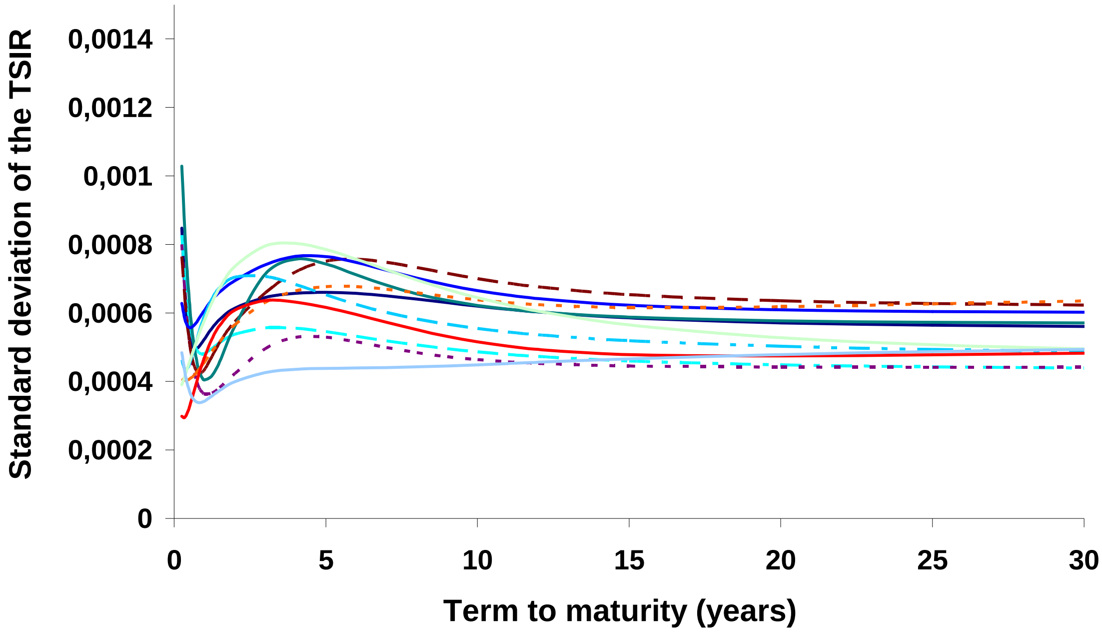
However the impact of using these alternative assumptions about the variance of the error term may depend on the flexibility of the model. For instance in the case of Nelson and Siegel, penalising errors in short term bond prices produces very unstable adjustment of the whole yield curve due to the difficulty of capturing with a very simple functional form the steep slopes of the yield curve observed during some sample periods. (See figure 7 panel A and figure 8 panel A)

Figures 7 and 8 compare the volatility estimates using the three models.

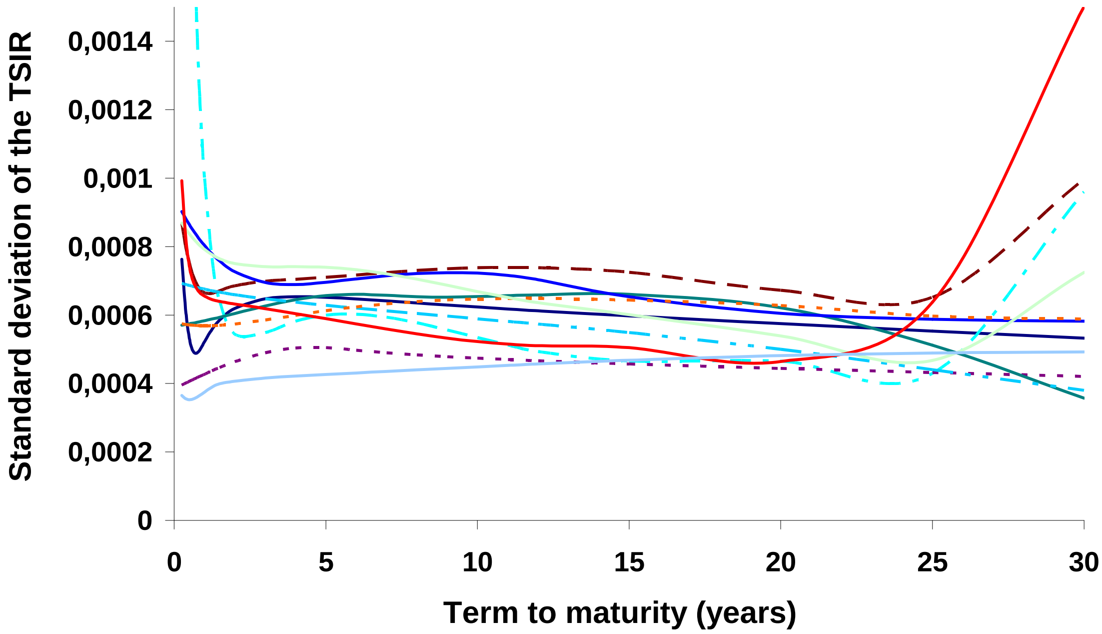
Figure 7.- Volatility estimates and term structure model. Homoskedatic assumption
Panel A.- Nelson & Siegel



Panel B.- Svensson model.



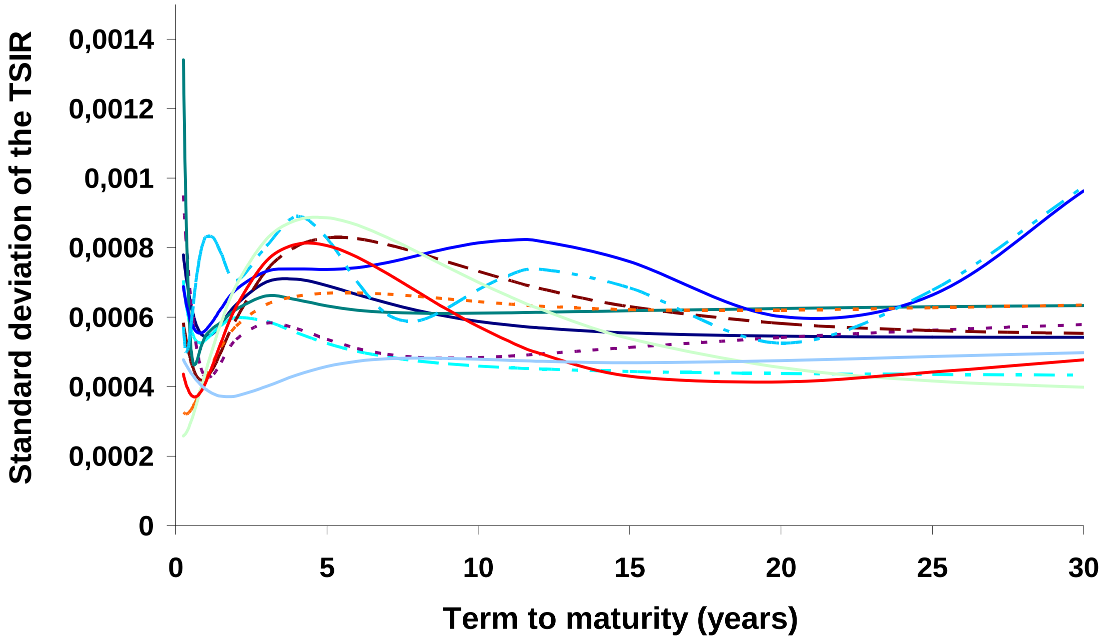
Panel C.-Vasicek model.



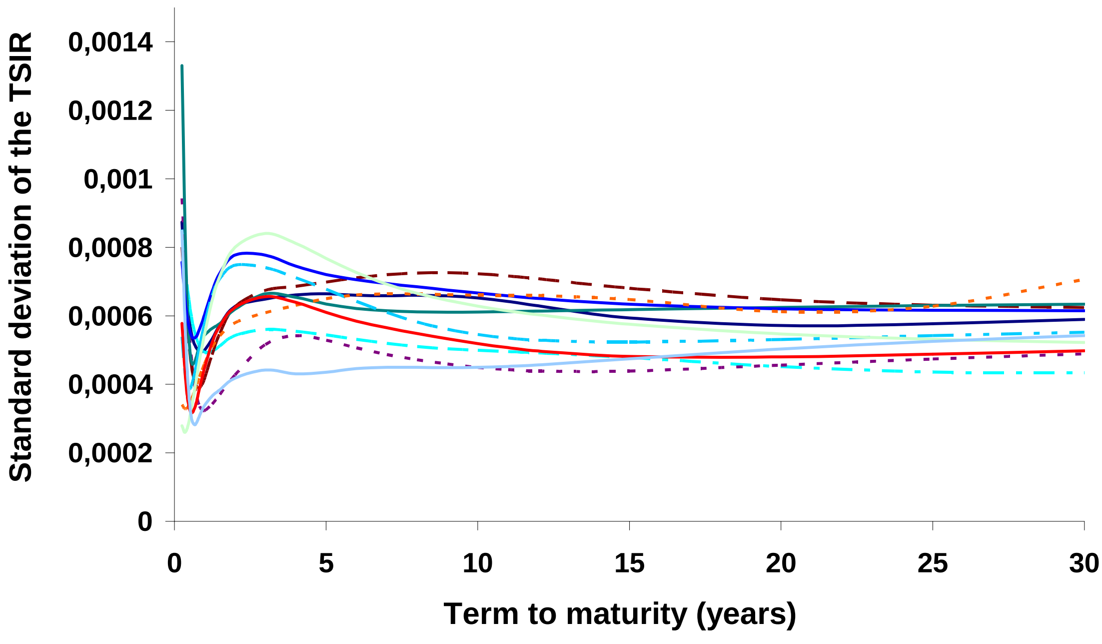
The first thing we can observe is that the non-parametric model (Vasicek and Fong) produces quite different estimates of the term structure of volatilities both in shape and level, an outcome particularly acute in the long end. These pictures show clearly the importance of the model employed to estimate the yield curve in the resulting interest rate volatilities. These differences increases when we assume heteroskedasticity .

Figure 8.- Volatility estimates and term structure model. Heteroskedastic assumption

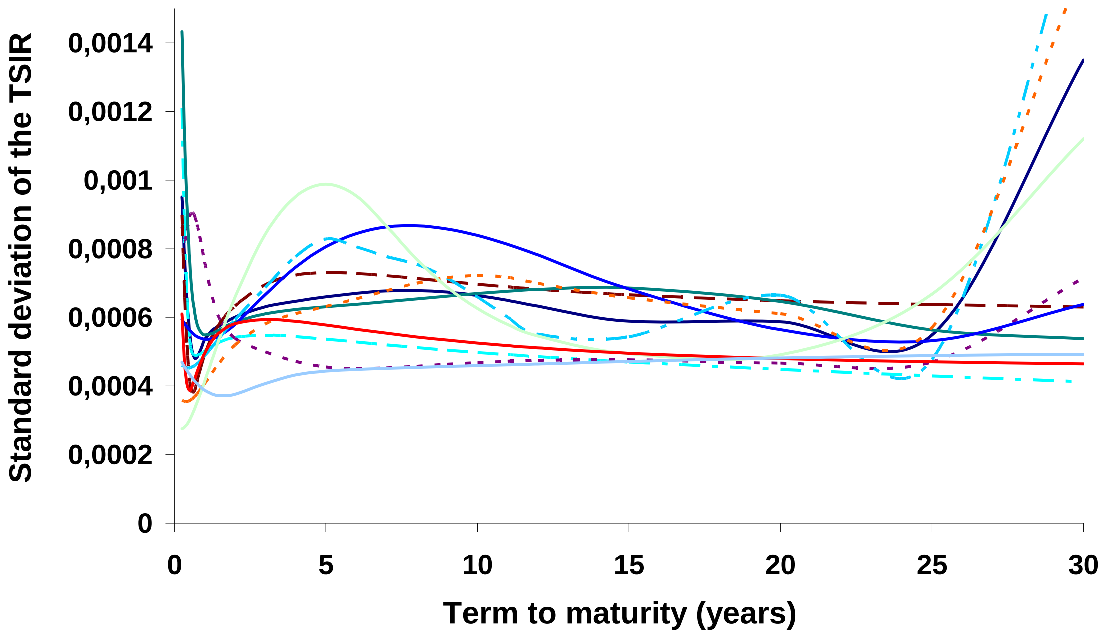
Panel A. Nelson and Siegel



Panel B. Svensson



Panel C. Vasicek and Fong



Finally we should highlight that these results are robust with respect to the particular specification we had used to model volatility, the historical volatility or the exponential GARCH. The comparison between models seems to produce the same results.

3.4.- Tests of differences in volatility estimates

To test if these differences are significant from a statistical point of view we applied a sign test. This test allows checking if two alternative models produce significant differences in the resulting spot rate volatilites.

This test assumes as null hypothesis that given two alternative models to estimate zero coupon bond yields, the probability that one of them produces a higher volatility estimate than the other for a given day is 50 %, that is the null hypotheses assumes that the method used to estimate the spot rates do not produce significant differences in the resulting zero coupon bond yield volatilities.

As we used a 30 day window to estimate the volatilities we selected one out of thirty estimates from our 2717 daily volatility estimates in order to avoid autocorrelation problems. Eventually, we had 91 independent volatility estimates for each maturity.

Under the null hypothesis the number of times that one method produces a higher volatility estimate than the other (x) is distributed according to a binomial random variable with parameters N=91 and p=0,5. As N is big enough we eventually assumed that X can be approximated by a normal distribution with mean $N \cdot p$ and variance $N \cdot p \cdot (1-p)$, that is $X \sim N(55{,}5;\ 27{,}75)$. The results are summarised in Table 4.

These tables show which method (compared by pairs) produces lower volatility estimates. Panel A and panel B indicates these result for both alternative volatility specifications: historical volatility and EGARCH model respectively.

Looking at figures 6 to 8 and Table 4 there are various issues that should be highlighted. The first one is that alternative models produce a significantly different volatility term structure, differences that seem to affect all maturities. Particularly, the most rigid model (Nelson and Siegel) seems to provide the less volatile zero coupon rates above all for the shortest and longest maturities.

Second, the assumption about the variance of the error term has a strong effect on the ensuing spot rate volatilities. On the whole, Nelson and Siegel provide more stable estimates for very short maturities in both weighting schemes (homoskedastic and heroskedastic) meanwhile Vasicek and Fong produces less volatile spot rates for the longest maturities. For medium term rates the results also depend on the model used to estimate the yield curve although Vasicek and Fong is the method that presents lower volatilities between 2 and 7 year term.

Another interesting point is that the results do not depend on the specification employed to estimate the volatility (historical or EGARCH). Moreover if in the second case (EGARCH) the sign test had been applied using a bigger set of data (for instance one out of fifteen instead of one out of thirty something justifiable when using a conditional volatility model) the results would be even more similar.

4.- The importance of the data set.

In this section we proceed to check if the assets eventually selected in the estimation of the term structure of interest rates have a significant impact on the resulting volatility estimates of spot rates.

For analyzing this issue we proceed to compare four different estimates of the term structure of interest rates corresponding to the US Treasury market:

(1) Our own estimates using Svensson model with two different specifications for the variance of the error term: (1.a) homoskedastic and (1.b) heteroskedastic error variance scheme described in section 2.2,
(2) Yield curve estimates elaborated by the Federal Reserve Board posted on its website (they also used Svensson model) and
(3) Yield curves reported by U.S. Department of Treasury.

Apart from differences in the model, the main differences of these four estimates are the following:

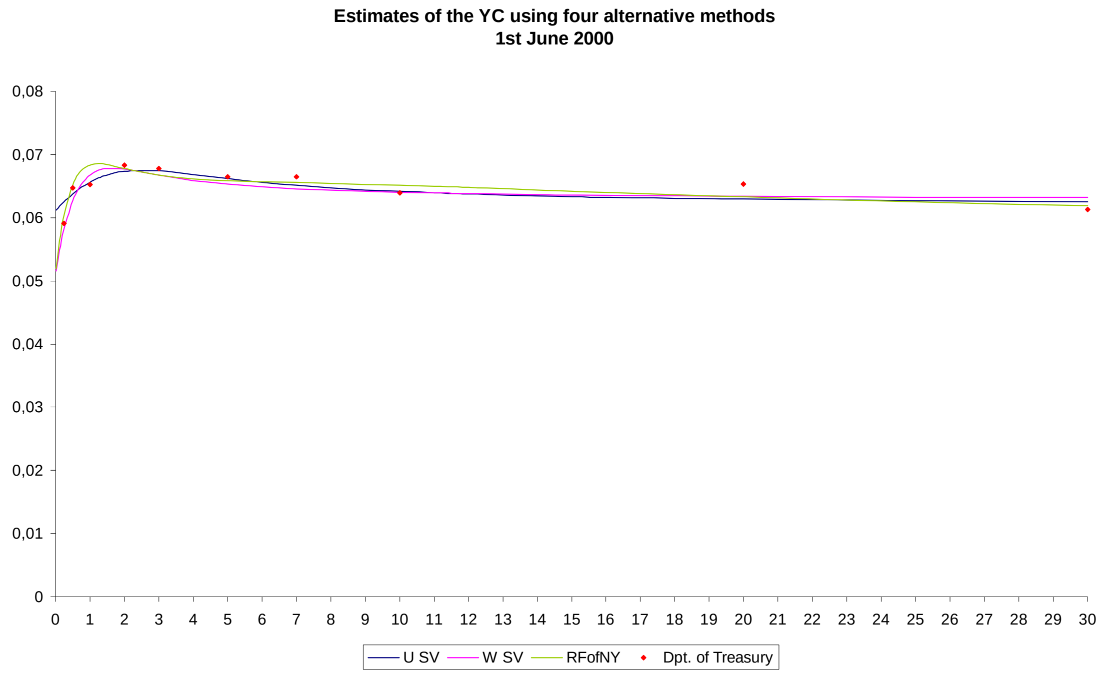
- (1.a) and (1.b) uses GovPX data consisting of actual trades (mid bid-ask quotes from 2001) at 5 p.m. of nearly all bonds and bills traded in the market

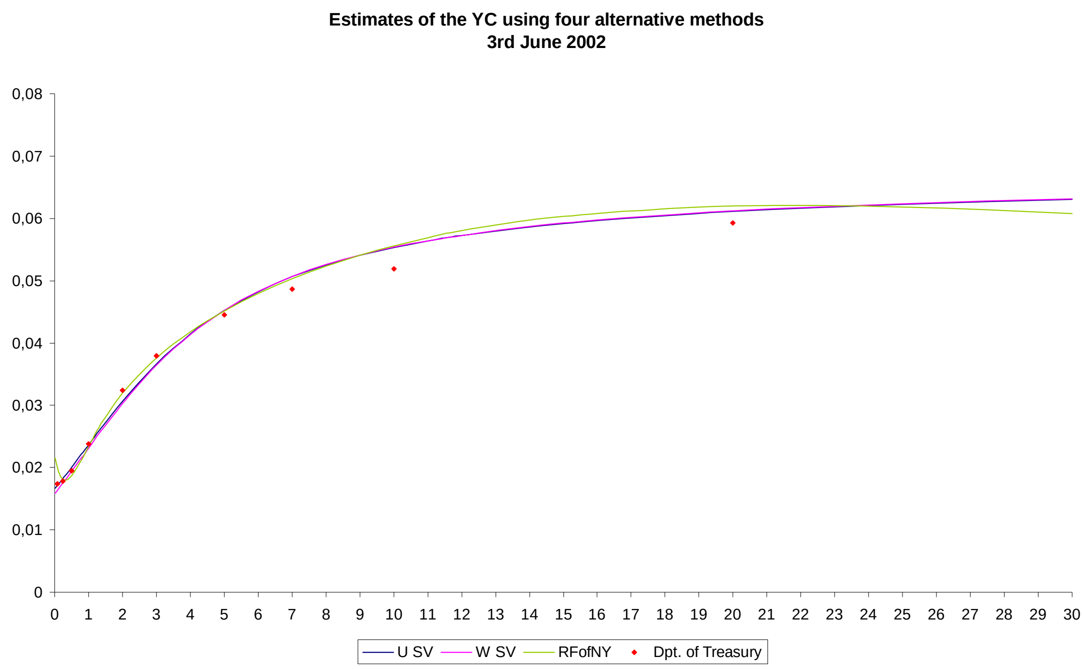
- (2) They use quotes of Treasury securities excluding all securities with less than three months to maturity, all Treasury bills , all twenty-year bonds , the on-the-run and the first-on-the-run bonds and "other issues that we judgmentally exclude on an *ad hoc* basis"

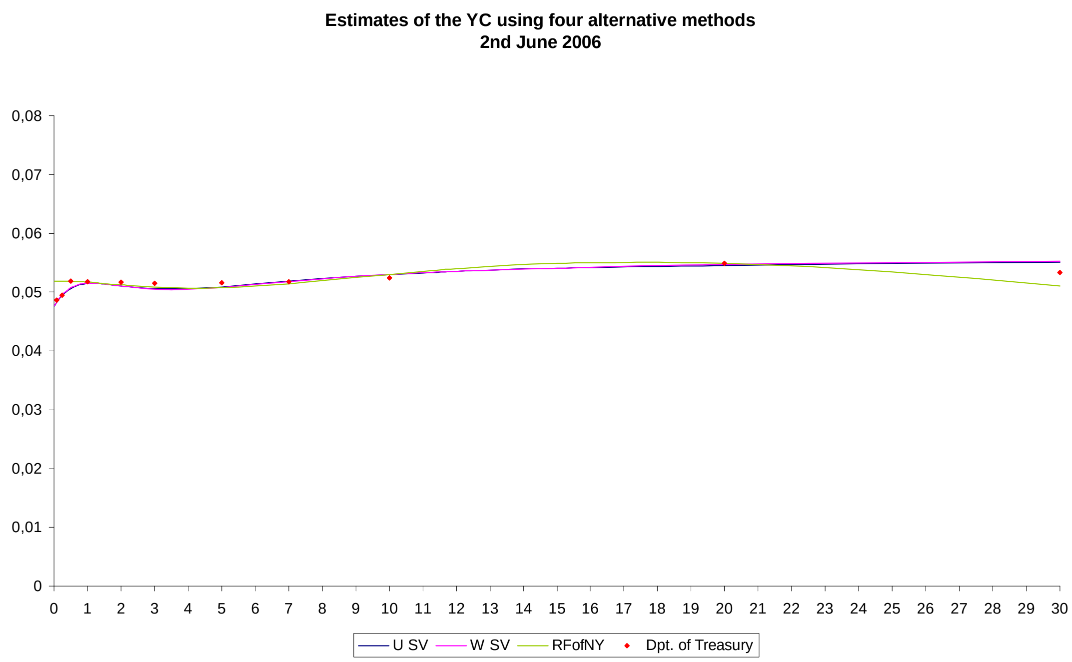
- (3) They use bid-side yields from on-the run securities (these are the assets that the Federal Reserve removed from its sample). They use four maturities of most recently auction bills (4-13-26-and 52 week) six maturities of just issued bonds and notes (2-, 3-. 5-, 7- 10 and 30- year) plus the composite rate in the 20 year maturity range.

In figure 9 we can see some of these estimates corresponding again to the first working day of some of the years of the sample:

Figure 9.- Term structure estimates using different data sets from the Treasury market







These pictures illustrates some of the differences that affect both the level and the shape of the term structure. We can outline at least three points:

- The differences in level correspond mainly to the estimations of the Department of Treasury
- We can appreciate important differences in the shapes of the other three estimations although they all used Svensson's model. The Federal Reserve produces estimates with a concave shape in the long end of the yield curve: this may have a very important impact on the forward rates with longer maturities
- The unweighted scheme tends to produce a different adjustment in the short end of the yield curve.

As in section 3, we proceed to estimate the term structure of volatilities using two alternative specification: the sample standard deviation and the EGARCH (1,1) . To test if there are significant differences we apply again the sign test described in the former section and the outcomes are summarised in Table 6

On the whole, we can state that differences are quite important. The Federal Reserve estimates produce a significantly lower spot rate volatility than other estimates including those of the Department of Treasury (except for very short maturities). At the same time, the Department of Treasury estimates seem to produce less volatile spot rates than our estimates although these results are no so clear when the GARCH model is used to estimate volatilities. In fact, in this second case, Nelson and Siegel with homoskedastic assumption produce lower volatilities in the range of maturities between one and two years.

5.- Impact on the correlation of forward rates

Forward rates play a key role in many financial issues. For instance in the implementation of interest rates models as Heath, Jarrow and Morton or in many product valuations where correlations among forward rates are crucial (for instance in swaptions) or for testing The Expectations Hypothesis . . .

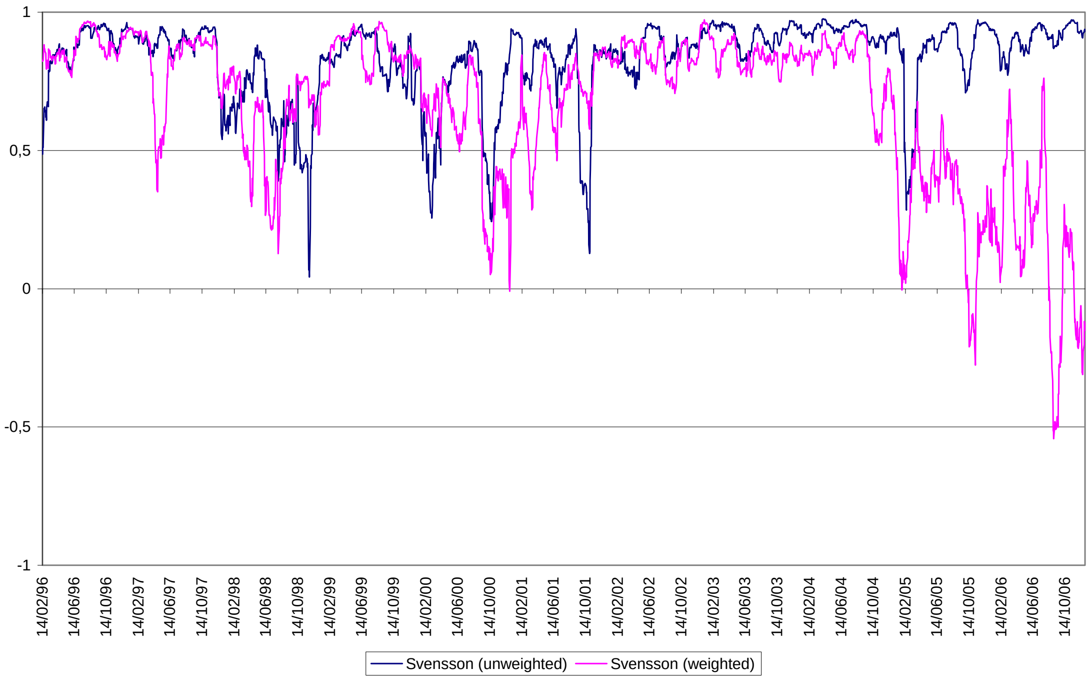
But at the same time forward rates can be very sensitive to the method employed in estimating the yield curve and so the correlations among forward rates . . . We have to highlight the fact that the forward rates are very sensitive to the shape of the yield curve particularly in the very long end.

To test if the way used to estimate the term structure of interest rates have a significant impact on correlations among forward rates we have proceed to estimate them.

First we have estimated the correlations using the six sets of zero coupon bond estimates from GovPex data base considered in section 3: Nelson and Siegel, Vasicek and Fong and Svensson models using both the homoskedastic and the heteroskedastic assumptions.

We can observe that this correlation coefficient estimates differ significantly from one model to another. Analyzing the results tahere some patterns that can be pointed out. The first one is that unweighted schemes produce lower correlation coefficients. This is illustrated in figure 10 where we have represented the evolution of the correlation coefficient between two year and ten year forward rates (with six month tenor) corresponding to Svensson model using the homoskedastic (or unweighted) and the heteroskedastic (or weighted) assumption about the error term.

Figure 10.- The impact of the assumption about the error term. Correlations between 2 year forward rate and five year forward rate using Svensson's model and two alternative assumptions about the variance of the error term



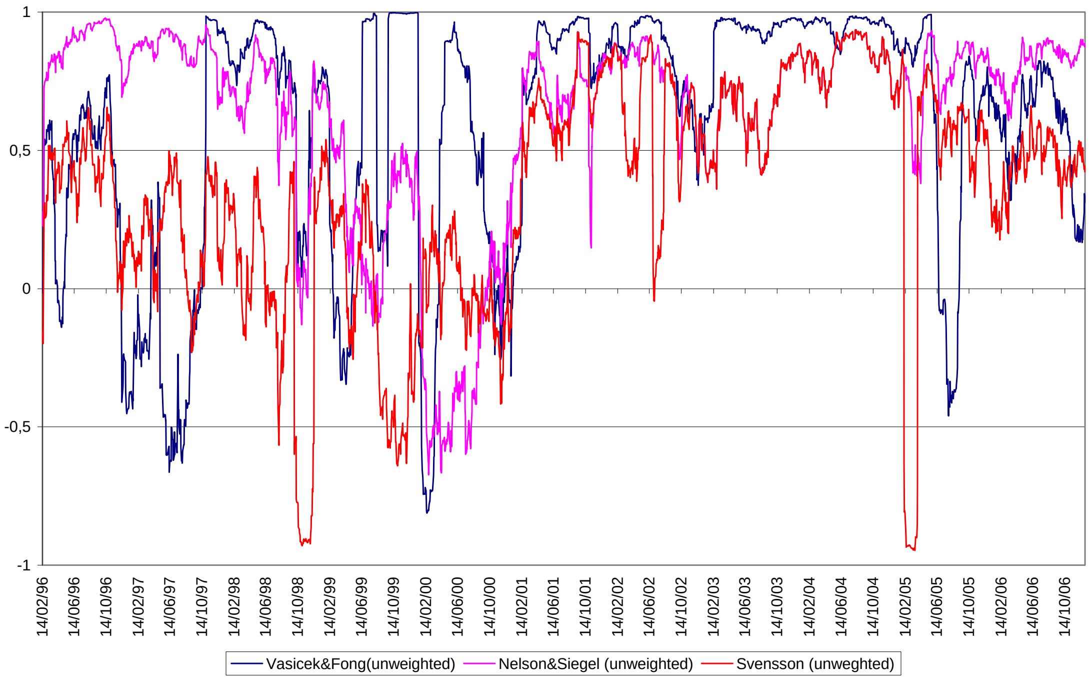
This result that can be generalized to all models and maturities and it is a consequence of the fact that using the weighted scheme forces the adjustment of the yield curve in such a way that affects the entire curve, making the resulting changes in the yield curve less "linear".

And the second result that can be highlighted is that Svensson model produces the lowest correlation coefficients compared with the other two alternative models.

This is illustrated in figure 11. We can also see that these differences were higher during the first halve of the sample period where the spread between short and medium term rates was very high.

In table 6 we present the outcomes of applying the sigh test to corroborate if these differences are significant from a statistical point of view. These results confirm that weighted models produce lower correlations and also that Svensson models is the one that for most maturities

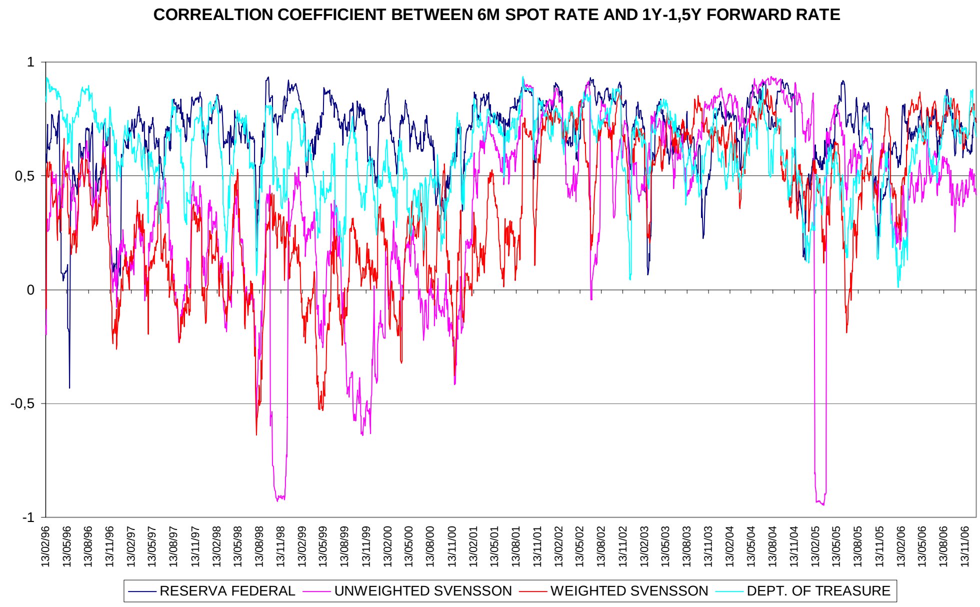
Figure 11.- The impact of the model on correlation coefficients. Correlations between two year and five year forward rates using Vasicek and Fong, Nelson and Siegel and Svensson (unweighted)



So in the next pictures we have illustrated the evolution of the correlation coefficient between different pairs of forward rates using the four alternative estimates of the yield curve of section 4.

The first one represents the correlations using again a 30 days window between six month spot rates and the one year forward rates with a six month tenor

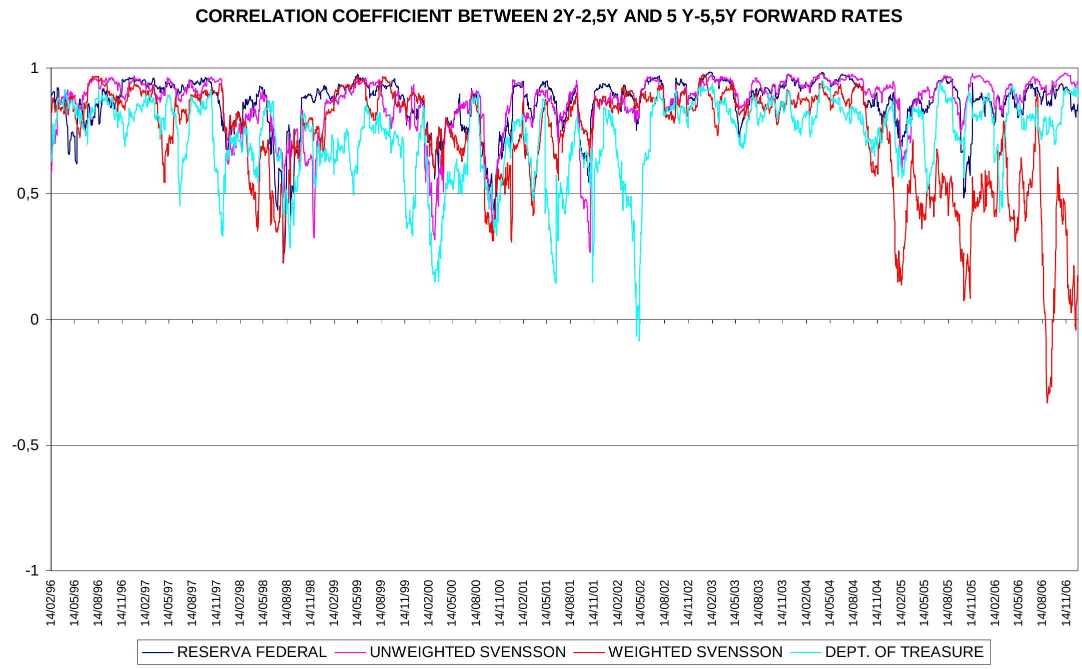
Figure 10.- Correlations between 6m month spot rate and one year forward rate with six month tenor



We can see that differences are huge during the first half of the sample period.The Federal Reserve estimates produced the most stable and highest estimates. Some of these differences may come from the fact that Federal Reserve dropped from the sample those assets with a maturity lower than three moths. That means that they did not care too much about what happened in the very short end of the yield curve. However, if we have a look at the yield curve during the first years of the sample the term structure had a very steep slope in the shortest maturities. And forward rates a very sensitive to the slope of the yield curve. So the Federal Reserve estimation has eliminated or al least softened some of the sharp changes and movements that forward rates experienced during this part of the sample period.

On the contrary the correlations among mid maturity forward rates the differences are not so severe. The only one with a different behavior corresponds to our estimates using unweighted scheme.

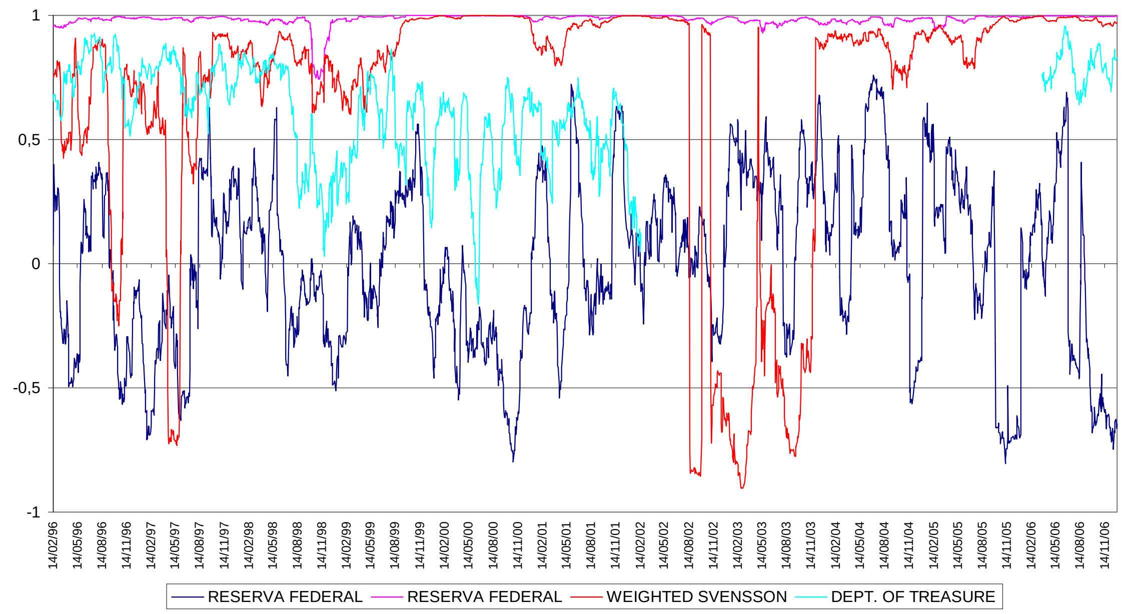
Figure 11.- Correlations between two year spot rate and five year forward rates with six month tenor



But the most astonishing results were obtained when estimating correlations between medium and long term forward rates. It must be taken into account that forward rates in the very long end of the yield curve are very sensitive to the slope of the zero coupon yield curve. As we can see in figure 12, the differences are dramatic.

Figure 12.- Correlations between ten year and thirty year forward rates with six month tenor

CORRELATION COEFFICIENT BETWEEN 10Y-10,5Y AND 29,5 Y-30Y FORWARD RATES



As we can see the estimates, of the Federal Reserve indicates that 10 year forward rate and 30 year forward rate are nearly linearly independent meanwhile our estimates would suggest just the opposite, that they are practically linearly dependent particularly if we use Svensson with the homosketastic scheme. The method applied by the Department of the Treasury produces forward rates with a behaviour in the mid point between our estimates and those of the Federal Reserve.All these differences are corroborated using the sign test described above whose results are shown in table 7.

5.- Conclusions

Although alternative methodologies proposed to estimate zero coupon bond yields from Treasury market can accurately replicate bond prices very important differences may appear in the volatilities and correlations among the resulting series of spot and forward rates with different maturities particularly in the short and long ends of the range of maturities

Differences may arise from three sources:

(1) The particular model used to estimate the yield curve and its relative flexibility to replicate the humps and s-shapes usually observed in the bond markets yield curves

(2) The hypothesis assumed about the variance of the error term during the estimation process of spot rates from Treasury bond market prices

(3) The actual data set employed in the estimation

These elements affect the resulting estimates of the spot rate volatility but also uit has a dramatic impact on the correlations among forward rates with different maturities.

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Table 1.- Summary statistics of TSIR estimations

This table reports the yearly averages of the sum of squared residuals from the daily estimation of the term structure of interest rates obtained by applying unweighted and weighted versions of Vasicek and Fong (1982), Nelson and Siegel (1987), and Svensson (1994). Additional columns show the average number of observations per day, the number of trading days in the year and the average maturity of the longest bond included in the daily estimation

	Sum of Squared Residuals (SSR) per day						Observ	Days	Maturity
Year	UVF	WVF	UNS	WNS	USV	WSV	per day	per year	longest bond
1996	12.3	12.4	15.9	28.7	14.2	14.8	151.6	259	29.8
1997	8.4	8.7	11.7	32.2	9.8	10.2	150.1	261	29.8
1998	38.2	40.8	37.6	58.4	35.5	38.6	146.7	261	29.7
1999	32.6	59.2	71.8	79.2	70.0	78.1	134.4	261	29.8
2000	17.6	20.8	36.3	40.0	33.6	38.1	120.0	260	29.8
2001	13.3	18.5	44.0	42.6	43.5	45.9	114.6	258	29.5
2002	14.6	20.8	62.3	60.9	62.3	64.6	113.3	261	28.6
2003	7.6	10.5	41.9	44.3	41.9	38.3	115.2	261	27.6
2004	5.8	14.2	27.0	30.6	27.0	27.4	123.8	262	26.6
2005	10.7	24.8	16.9	48.7	15.2	16.3	129.6	260	25.6
2006	14.0	14.3	13.7	20.8	13.1	13.8	157.0	260	28.6
Avg	15.9	22.3	34.5	44.2	33.3	35.1	132.4	260	28.7

Table 2.- Summary of estimated main parameters

This table reports the yearly averages and standard deviations (in parenthesis) of the estimated value of the main parameters of the unweighted and weighted versions of Vasicek and Fong (1982), Nelson and Siegel (1987) and Svensson (1994)

Panel A: Vasicek and Fong (1982), Nelson and Siegel (1987)

	UVF		WVF		UNS				WNS			
	α	Knot	α	Knot	β_0	β_1	β_2	τ	β_0	β_1	β_2	τ
1996	5.76	2.07	5.67	1.18	7.21	-1.74	-0.93	2.73	7.06	-2.08	-0.34	2.09
	(0.17)	(1.37)	(1.59)	(2.56)	(0.19)	(0.20)	(1.19)	(0.27)	(0.29)	(0.51)	(1.01)	(2.18)
1997	2.25	24.09	4.46	4.95	6.33	-1.18	-1.25	1.58	6.31	-1.74	-0.11	2.09
	(1.75)	(37.04)	(1.60)	(10.86)	(0.27)	(0.29)	(0.86)	(0.29)	(0.21)	(0.23)	(0.36)	(1.58)
1998	3.22	5.88	3.26	3.86	6.06	-0.55	1.20	0.92	6.13	-0.51	0.79	1.32
	(1.69)	(11.62)	(1.56)	(7.07)	(0.27)	(1.10)	(2.57)	(0.31)	(0.25)	(0.66)	(1.85)	(0.98)
1999	5.18	24.26	5.59	19.75	6.14	-2.54	-3.59	1.36	6.10	-2.70	-2.05	2.18
	(2.28)	(13.01)	(1.97)	(14.47)	(0.20)	(1.30)	(0.85)	(0.21)	(0.39)	(1.18)	(3.00)	(1.52)
2000	10.63	31.63	8.00	20.63	6.31	-4.62	-4.80	1.36	6.27	-4.77	-2.26	2.32
	(1.43)	(2.67)	(4.39)	(11.50)	(0.16)	(0.36)	(1.74)	(0.26)	(0.31)	(0.35)	(4.23)	(1.26)
2001	4.38	30.06	5.49	23.84	6.23	-5.03	-5.89	1.72	6.32	-5.35	-5.10	1.92
	(0.65)	(2.58)	(3.73)	(8.98)	(0.27)	(0.39)	(0.76)	(0.12)	(0.26)	(0.35)	(0.72)	(0.17)
2002	4.52	27.80	4.71	20.61	6.14	-4.51	-3.73	2.00	6.23	-4.97	-1.45	3.07
	(1.72)	(1.02)	(2.28)	(9.91)	(0.22)	(0.79)	(0.86)	(0.15)	(0.19)	(0.62)	(1.92)	(0.93)
2003	3.08	17.69	3.97	2.94	5.01	-1.24	-1.77	2.11	4.81	-1.73	0.01	2.25
	(1.41)	(28.63)	(0.89)	(13.05)	(0.24)	(0.71)	(0.43)	(0.10)	(0.29)	(0.67)	(0.13)	(1.41)
2004	4.36	1.90	3.91	8.91	5.11	0.14	-1.69	1.80	5.28	-0.48	-1.00	4.58
	(0.22)	(0.43)	(1.20)	(22.56)	(0.24)	(0.24)	(0.46)	(0.15)	(0.28)	(0.33)	(0.82)	(2.71)
2005	4.90	15.81	4.91	10.03	6.14	-2.11	-2.27	1.89	6.12	-2.48	-1.11	2.46
	(2.46)	(19.22)	(2.52)	(14.18)	(0.67)	(1.85)	(2.28)	(0.61)	(0.65)	(1.77)	(2.39)	(1.73)
2006	5.76	2.07	5.67	1.18	7.21	-1.74	-0.93	2.73	7.06	-2.08	-0.34	2.09
	(0.14)	(5.57)	(0.50)	(1.11)	(0.25)	(0.16)	(0.89)	(0.26)	(0.18)	(0.22)	(0.79)	(0.89)
1996-	5.39	4.15	5.03	0.64	6.93	-1.29	-0.55	2.88	6.71	-1.68	0.00	1.52
2006	(0.38)	(6.10)	(0.93)	(0.14)	(0.37)	(0.35)	(0.33)	(0.24)	(0.35)	(0.42)	(0.00)	(0.75)

Panel B: Svensson (1994)

	USV						WSV					
	β_0	β_1	β_2	τ_1	β_3	τ_2	β_0	β_1	β_2	τ_1	β_3	τ_2
1996	7.21	-0.27	-0.08	0.73	0.12	1.48	7.27	-0.21	-0.04	1.06	0.09	1.96
	(0.26)	(0.07)	(0.06)	(0.18)	(0.04)	(0.09)	(0.25)	(0.07)	(0.05)	(0.35)	(0.04)	(1.21)
1997	6.94	-0.21	-0.07	0.69	0.10	1.51	6.98	-0.18	-0.03	0.97	0.08	2.09
	(0.38)	(0.03)	(0.01)	(0.03)	(0.01)	(0.05)	(0.38)	(0.07)	(0.04)	(0.29)	(0.03)	(2.12)
1998	6.00	-0.54	0.14	0.96	0.26	1.47	6.08	-0.25	0.01	1.13	0.12	1.76
	(0.19)	(0.51)	(0.28)	(0.37)	(0.25)	(0.16)	(0.16)	(0.15)	(0.09)	(0.35)	(0.07)	(0.24)
1999	6.34	-1.08	0.45	0.83	0.53	0.95	6.49	-0.72	0.31	1.51	0.35	1.58
	(0.27)	(0.55)	(0.33)	(0.30)	(0.27)	(0.24)	(0.24)	(0.01)	(0.01)	(0.33)	(0.01)	(0.34)
2000	6.03	-0.19	0.11	0.74	0.09	0.91	6.10	-0.75	0.37	0.85	0.37	0.88
	(0.26)	(0.45)	(0.18)	(0.13)	(0.22)	(0.18)	(0.28)	(0.18)	(0.11)	(0.27)	(0.09)	(0.27)
2001	6.13	-1.02	0.41	0.95	0.49	1.00	6.20	-0.71	0.30	1.48	0.34	1.49
	(0.20)	(0.13)	(0.04)	(0.13)	(0.06)	(0.13)	(0.21)	(0.01)	(0.01)	(0.27)	(0.01)	(0.27)
2002	6.31	-0.08	-0.03	1.34	0.02	1.31	6.36	-0.71	0.28	1.45	0.33	1.45
	(0.16)	(0.14)	(0.07)	(0.27)	(0.07)	(0.35)	(0.26)	(0.01)	(0.03)	(0.55)	(0.00)	(0.51)
2003	6.23	-0.10	-0.03	1.72	0.03	1.72	6.11	-0.69	0.30	2.46	0.32	2.36
	(0.27)	(0.01)	(0.01)	(0.12)	(0.01)	(0.12)	(0.39)	(0.04)	(0.05)	(0.93)	(0.02)	(0.77)
2004	6.14	-0.08	-0.02	2.00	0.02	2.00	6.21	-0.69	0.25	1.70	0.32	1.80
	(0.22)	(0.02)	(0.01)	(0.15)	(0.01)	(0.15)	(0.25)	(0.06)	(0.06)	(0.27)	(0.03)	(0.19)
2005	5.01	-0.84	0.36	1.30	0.41	1.35	5.03	-0.57	0.21	1.18	0.28	1.35
	(0.24)	(0.29)	(0.14)	(0.30)	(0.15)	(0.28)	(0.26)	(0.18)	(0.11)	(0.28)	(0.09)	(0.19)
2006	5.10	-0.90	0.41	1.13	0.45	1.17	5.09	-0.19	0.03	0.79	0.09	1.17
	(0.24)	(0.01)	(0.01)	(0.11)	(0.00)	(0.12)	(0.25)	(0.09)	(0.06)	(0.20)	(0.05)	(0.22)
1996-	6.13	-0.48	0.15	1.13	0.23	1.35	6.18	-0.52	0.18	1.33	0.25	1.63
2006	(0.67)	(0.48)	(0.26)	(0.46)	(0.24)	(0.37)	(0.69)	(0.26)	(0.16)	(0.62)	(0.12)	(0.91)

Table 3.- Summary statistics of VTS estimations

Panel A: Historical Volatility

	UNS		USV		UVF		WNS		WSV		WVF	
	Mean	**SD**	**Mean**	**SD**	**Mean**	**SD**	**Mean**	**SD**	**Mean**	**SD**	**Mean**	**SD**
1w	11.12	10.29	18.33	18.84	16.82	12.72	12.62	9.81	14.05	8.62	13.83	9.13
1m	10.52	9.15	15.53	14.7	15.87	11.34	11.11	8.03	12.25	7.74	12.39	8.11
2m	9.9	7.97	12.95	11.21	14.8	9.86	9.78	6.97	10.49	7.02	10.87	7.16
3m	9.44	7.09	11.18	9.13	13.93	8.73	9	6.51	9.33	6.59	9.75	6.59
4m	9.11	6.47	10.07	7.86	13.24	7.91	8.63	6.26	8.66	6.34	9.05	6.3
5m	8.91	6.09	9.44	7.03	12.72	7.37	8.53	6.11	8.37	6.19	8.71	6.15
6m	8.81	5.88	9.12	6.5	12.35	7.04	8.6	6	8.33	6.1	8.65	6.07
7m	8.81	5.79	9	6.15	12.11	6.82	8.78	5.92	8.46	6.04	8.77	6
8m	8.9	5.76	9.01	5.93	11.96	6.68	9.01	5.87	8.68	5.98	8.96	5.94
9m	9.08	5.74	9.11	5.79	11.89	6.57	9.27	5.82	8.96	5.93	9.2	5.89
10m	9.31	5.71	9.27	5.69	11.87	6.48	9.55	5.79	9.25	5.88	9.44	5.85
11m	9.58	5.69	9.48	5.63	11.87	6.41	9.83	5.76	9.54	5.84	9.68	5.81
1y	9.85	5.68	9.7	5.6	11.9	6.34	10.1	5.75	9.83	5.8	9.91	5.78
1y3m	10.64	5.76	10.39	5.64	12.03	6.18	10.85	5.72	10.6	5.74	10.56	5.72
1y6m	11.33	5.88	11.03	5.76	12.18	6.06	11.49	5.73	11.25	5.74	11.11	5.7
2y	12.35	6.09	12.11	6.02	12.45	5.93	12.46	5.77	12.23	5.84	11.97	5.73
3y	13.39	6.29	13.4	6.31	12.77	5.85	13.52	5.94	13.24	6.12	13.01	5.96
4y	13.65	6.31	13.79	6.33	12.91	5.83	13.86	6.07	13.56	6.26	13.53	6.21
5y	13.55	6.19	13.7	6.2	12.94	5.79	13.83	6.09	13.54	6.22	13.72	6.33
6y	13.31	5.97	13.41	5.99	12.9	5.73	13.61	6.01	13.36	6.06	13.68	6.28
7y	13.01	5.72	13.06	5.74	12.81	5.65	13.33	5.87	13.1	5.84	13.5	6.11
8y	12.71	5.47	12.72	5.48	12.71	5.55	13.03	5.69	12.82	5.59	13.26	5.86
9y	12.43	5.25	12.42	5.25	12.59	5.44	12.74	5.5	12.55	5.37	13	5.61
10y	12.18	5.07	12.15	5.06	12.47	5.32	12.47	5.33	12.29	5.17	12.74	5.38
11y	11.96	4.91	11.93	4.9	12.35	5.2	12.21	5.16	12.07	5.01	12.48	5.19
12y	11.77	4.78	11.74	4.78	12.23	5.09	11.98	5.01	11.87	4.88	12.25	5.04
15y	11.35	4.53	11.33	4.53	11.89	4.79	11.41	4.62	11.45	4.58	11.66	4.71
20y	10.97	4.35	10.98	4.37	11.2	4.45	10.85	4.33	11.09	4.33	10.99	4.34
25y	10.79	4.3	10.82	4.34	10.4	4.14	10.72	4.3	11	4.28	10.76	4.28
30y	10.68	4.29	10.73	4.34	11.66	5.73	10.88	4.41	11.09	4.37	12.91	6.04

w stands for week, **m** for month and **y** for year
10.000 times smaller scale

Panel B: Conditional Volatility

	GUNS		GUSV		GUVF		GWNS		GWSV		GWVF	
	Mean	**SD**	**Mean**	**SD**	**Mean**	**SD**	**Mean**	**SD**	**Mean**	**SD**	**Mean**	**SD**
1w	10.94	10.21	15.78	16.6	15.76	11.9	10.27	8.85	12.99	8.34	11.11	8.27
1m	10.35	9.09	13.46	13.11	14.97	10.73	9.18	7.65	11.52	7.59	10.08	7.58
2m	9.75	7.93	11.39	10.18	14.08	9.45	8.79	6.9	9.96	6.95	9.05	6.95
3m	9.28	7.06	9.96	8.41	13.32	8.47	8.24	6.51	8.76	6.58	8.37	6.56
4m	8.96	6.46	9.11	7.35	12.75	7.76	8.02	6.29	8.33	6.38	8.05	6.34
5m	8.77	6.09	8.72	6.7	12.34	7.3	7.86	6.15	7.75	6.24	8	6.21
6m	8.67	5.88	8.59	6.3	12	7	8.08	6.04	7.95	6.13	8.15	6.11
7m	8.68	5.79	8.62	6.05	11.78	6.81	8.33	5.96	8.22	6.05	8.38	6.03
8m	8.78	5.76	8.75	5.89	11.68	6.68	8.62	5.9	8.52	5.99	8.65	5.97
9m	8.97	5.74	8.92	5.77	11.64	6.58	8.94	5.85	8.82	5.93	8.93	5.91
10m	9.22	5.71	9.12	5.69	11.64	6.5	9.25	5.81	9.14	5.88	9.19	5.87
11m	9.49	5.69	9.35	5.63	11.67	6.42	9.56	5.78	9.45	5.84	9.45	5.83
1y	9.78	5.68	9.6	5.6	11.72	6.35	9.86	5.76	9.74	5.8	9.7	5.8
1y3m	10.58	5.76	10.32	5.64	11.89	6.18	10.68	5.73	10.54	5.74	10.39	5.74
1y6m	11.28	5.88	10.98	5.76	12.07	6.06	11.36	5.73	11.19	5.74	10.96	5.71
2y	12.31	6.09	12.04	6.02	12.36	5.93	12.37	5.77	12.16	5.84	11.84	5.73
3y	13.35	6.28	13.35	6.3	12.71	5.84	13.43	5.94	13.18	6.11	12.92	5.96
4y	13.61	6.3	13.75	6.32	12.85	5.82	13.77	6.07	13.51	6.25	13.46	6.2
5y	13.51	6.18	13.66	6.18	12.88	5.78	13.75	6.09	13.49	6.21	13.65	6.31
6y	13.26	5.96	13.37	5.97	12.83	5.72	13.55	6	13.3	6.05	13.59	6.26
7y	12.97	5.7	13.02	5.72	12.76	5.63	13.28	5.85	13.05	5.82	13.4	6.09
8y	12.66	5.45	12.67	5.47	12.65	5.53	12.96	5.67	12.81	5.58	13.15	5.84
9y	12.38	5.24	12.36	5.24	12.53	5.42	12.65	5.49	12.49	5.35	12.87	5.59
10y	12.12	5.05	12.1	5.05	12.41	5.3	12.37	5.31	12.24	5.16	12.59	5.36
11y	11.9	4.9	11.87	4.89	12.28	5.19	12.1	5.15	12	5	12.33	5.17
12y	11.7	4.77	11.67	4.77	12.17	5.07	11.86	5	11.81	4.86	12.09	5.02
15y	11.29	4.52	11.27	4.52	11.8	4.78	11.29	4.62	11.37	4.57	11.53	4.7
20y	10.91	4.34	10.92	4.36	11.12	4.44	10.74	4.33	11.01	4.33	10.89	4.33
25y	10.73	4.29	10.76	4.33	10.29	4.14	10.54	4.32	10.87	4.28	10.62	4.28
30y	10.63	4.29	10.67	4.33	11.26	5.44	10.61	4.41	10.88	4.37	12.08	5.7

w stands for week, **m** for month and **y** for year
10.000 times smaller scale

Table 4.- Test of equal volatilities

	Panel A. Historical volatility								
Maturity	UNS-UVF	UNS-USV	UVF-USV	WNS-WVF	WNS-WSV	WVF-WSV	UNS-WNS	UVF-WVF	USV-WSV
1w	UNS^{a}	UNS^{a}	-	-	WNS^{a}	WVF^{a}	UNS^{a}	-	USV^{a}
1m	UNS^{a}	UNS^{a}	-	WNS^{b}	WNS^{a}	WVF^{b}	UNS^{a}	-	USV^{a}
3m	UNS^{a}	UNS^{a}	USV^{b}	WNS^{b}	WNS^{a}	-	UNS^{a}	WVF^{b}	USV^{a}
6m	UNS^{a}	UNS^{a}	USV^{b}	-	WSV^{a}	WSV^{a}	UNS^{a}	WVF^{b}	WSV^{b}
1y	UNS^{a}	-	USV^{a}	WVF^{b}	WSV^{a}	WSV^{a}	UNS^{b}	WVF^{a}	WSV^{a}
1.5y	UNS^{a}	-	USV^{a}	-	-	-	UNS^{b}	WVF^{a}	-
2y	UNS^{a}	-	USV^{a}	WVF^{b}	-	WVF^{c}	UNS^{c}	WVF^{a}	USV^{a}
3y	UVF^{a}	-	UVF^{a}	WVF^{a}	WSV^{b}	WVF^{a}	UNS^{a}	UVF^{a}	USV^{a}
5y	UVF^{a}	UNS^{a}	UVF^{a}	WVF^{c}	WSV^{a}	-	UNS^{b}	UVF^{a}	-
7y	UVF^{a}	-	UVF^{a}	-	WSV^{a}	WSV^{c}	UNS^{b}	WVF^{a}	-
10y	UNS^{c}	-	USV^{a}	WNS^{b}	WSV^{c}	WSV^{a}	UNS^{a}	WVF^{a}	USV^{c}
15y	UNS^{a}	USV^{a}	USV^{a}	WNS^{a}	-	WSV^{a}	UNS^{a}	WVF^{a}	USV^{a}
20y	UNS^{a}	UNS^{b}	USV^{a}	-	WNS^{b}	-	UNS^{c}	WVF^{a}	USV^{a}
25y	UNS^{a}	UNS^{a}	UVF^{a}	WVF^{b}	-	WVF^{a}	UNS^{a}	UVF^{a}	USV^{a}
30y	UNS^{a}	UNS^{a}	UVF^{b}	WNS^{a}	-	WSV^{b}	UNS^{a}	UVF^{b}	USV^{a}
	Panel B. Conditional Volatility (E-GARCH)								
Maturity	UNS-UVF	UNS-USV	UVF-USV	WNS-WVF	WNS-WSV	WVF-WSV	UNS-WNS	UVF-WVF	USV-WSV
1w	UNS^{a}	UNS^{b}	USV^{a}	WNS^{a}	WNS^{a}	WVF^{a}	-	WVF^{a}	USV^{a}
1m	UNS^{a}	-	USV^{a}	WNS^{a}	WNS^{a}	WVF^{a}	-	WVF^{a}	USV^{a}
3m	UNS^{a}	-	USV^{a}	WVF^{a}	WNS^{a}	WVF^{a}	UNS^{b}	WVF^{a}	USV^{a}
6m	UNS^{a}	USV^{a}	USV^{a}	-	-	-	UNS^{b}	WVF^{a}	WSV^{a}
1y	UNS^{a}	USV^{a}	USV^{a}	-	WNS^{b}	WVF^{b}	UNS^{b}	WVF^{a}	-
1.5y	UNS^{a}	USV^{a}	USV^{a}	WVF^{a}	-	WVF^{a}	-	WVF^{a}	USV^{a}
2y	-	USV^{a}	USV^{a}	WVF^{a}	WNS^{a}	WVF^{a}	-	WVF^{a}	USV^{a}
3y	UVF^{a}	USV^{a}	UVF^{a}	WVF^{a}	-	WVF^{a}	-	WVF^{c}	USV^{a}
5y	UVF^{a}	UNS^{a}	UVF^{a}	-	-	-	-	UVF^{a}	WSV^{a}
7y	UVF^{a}	UNS^{a}	UVF^{a}	-	-	-	-	UVF^{b}	WSV^{b}
10y	UNS^{a}	USV^{a}	USV^{a}	WNS^{b}	WNS^{b}	-	-	WVF^{b}	-
15y	UNS^{a}	USV^{a}	USV^{a}	WNS^{a}	WNS^{a}	-	UNS^{a}	WVF^{a}	-
20y	UVF^{a}	UNS^{a}	UVF^{a}	WNS^{b}	WNS^{a}	-	UNS^{a}	-	-
25y	UVF^{a}	UNS^{a}	UVF^{a}	WVF^{a}	WNS^{a}	WVF^{a}	UNS^{a}	-	-
30y	UVF^{a}	UNS^{a}	UVF^{a}	-	-	-	UNS^{a}	-	WSV^{a}

Note: $^{a}p<0.01$; $^{b}p<0.05$; $^{c}p<0.1$

Table 5.- Differences in volatility estimates and the sample data

			Panel A. Historical volatility			
Maturity	USV-WSV	USV-DoT	USV-FR	WSV-DoT	WSV-FR	DoT-FR
1 month	USV^{a}	USV^{b}	FR^{a}	DoT^{c}	FR^{a}	-
3 months	USV^{a}	DoT^{a}	FR^{a}	DoT^{a}	FR^{a}	-
6 months	WSV^{b}	DoT^{a}	FR^{a}	DoT^{a}	FR^{a}	-
1 year	WSV^{b}	DoT^{a}	FR^{a}	DoT^{a}	FR^{a}	-
1.5 years	-	n.a.	-	n.a	FR^{a}	n.a.
2 years	USV^{a}	DoT^{a}	FR^{a}	DoT^{a}	FR^{a}	FR^{a}
5 years	-	DoT^{a}	FR^{a}	DoT^{a}	FR^{a}	FR^{a}
7 years	-	DoT^{a}	FR^{a}	DoT^{a}	FR^{a}	FR^{a}
10 years	USV^{c}	-	FR^{a}	DoT^{a}	FR^{a}	-
15 years	USV^{a}	n.a.	-	n.a	FR^{a}	n.a.
20 years	USV^{a}	DoT^{a}	FR^{a}	DoT^{a}	FR^{a}	FR^{a}
25 years	USV^{a}	n.a.	FR^{a}	n.a	FR^{a}	n.a.
30 years	USV^{a}	DoT^{a}	FR^{a}	DoT^{a}	FR^{a}	-
			Panel B. Conditional volatility (E-GARCH)			
Maturity	USV-WSV	USV-DoT	USV-RF	WSV-DoT	WSV-FR	DoT-FR
1 month	USV^{a}	-	-	DoT^{a}	FR^{a}	-
3 months	USV^{a}	DoT$^{(b)}$	FR^{b}	DoT^{a}	FR^{a}	-
6 months	WSV^{a}	-	FR^{b}	WSV^{a}	-	FR^{a}
1 year	-	-	-	-	WSV^{c}	-
1.5 years	USV^{a}	n.a.	USV^{c}	n.a.	-	n.a.
2 years	USV^{a}	USV^{a}	USV^{a}	-	-	FR^{a}
5 years	WSV^{a}	DoT$^{(a)}$	FR^{a}	-	FR^{c}	FR^{b}
7 years	WSV^{a}	USV^{b}	FR^{c}	-	-	FR^{a}
10 years	-	USV^{b}	-	-	-	FR^{c}
15 years	-	n.a.	-	n.a.	-	n.a.
20 years	-	USV^{a}	FR^{a}	-	FR^{a}	FR^{a}
25 years	-	n.a.	FR^{a}	n.a.	FR^{a}	n.a.
30 years	WSV^{a}	-	FR^{a}	WSV^{b}	FR^{a}	FR^{b}

Note: $^{a}p<0.01$; $^{b}p<0.05$; $^{c}p<0.1$

Table 6.- Test of equal correlations

Maturity	UNS-UVF	UNS-USV	UVF-USV	WNS-WVF	WNS-WSV	WVF-WSV	UNS-WNS	UVF-WVF	USV-WSV
$R_{0;0.5}$-$F_{1;1.5}$	-	USV^{a}	USV^{a}	WNS^{a}	-	WSV^{b}	WNS^{a}	WVF^{a}	WSV^{a}
$R_{0;0.5}$-$F_{10;10.5}$	-	USV^{a}	USV^{a}	WVF^{a}	WSV^{a}	WSV^{a}	WNS^{a}	WVF^{b}	WSV^{a}
$R_{0;0.5}$-$F_{29.5\,;\,30}$	UVF^{a}	USV^{a}	UVF^{a}	WVF^{a}	-	WVF^{a}	WNS^{a}	WVF^{c}	USV^{b}
$F_{1;1.5}$-$F_{10;10.5}$	UNS^{a}	USV^{a}	USV^{a}	WNS^{a}	WNS^{a}	WSV^{c}	WNS^{a}	WVF^{a}	USV^{a}
$R_{2;2.5}$-$F_{5;5.5}$	UNS^{a}	USV^{a}	USV^{a}	WNS^{a}	WSV^{a}	WSV^{a}	WNS^{a}	WVF^{a}	WSV^{b}
$R_{2;2.5}$-$F_{10;10.5}$	UNS^{a}	USV^{a}	USV^{a}	WNS^{a}	WNS^{a}	WSV^{a}	WNS^{a}	WVF^{a}	WSV^{a}
$R_{10;10.5}$-$F_{29.5;30}$	UVF^{a}	UNS^{a}	UVF^{a}	-	-	WSV^{a}	WNS^{b}	WVF^{b}	WSV^{a}

Note: $^{a}p<0.01$; $^{b}p<0.05$; $^{c}p<0.1$
Table shows which model produces a smaller correlation between daily changes of pairs of forward rates

Table 7.- Test of equal correlations. Differences among sample data.

Maturity	USV-WSV	USV-DoT	USV-FR	WSV-DoT	WSV-FR	DoT-FR
$R_{0;0.5}$-$F_{1;1.5}$	WSV^{a}	USV^{a}	-	WSV^{a}	WSV^{a}	DoT^{a}
$R_{0;0.5}$-$F_{10;10.5}$	WSV^{a}	USV^{a}	FR^{c}	WSV^{a}	WSV^{a}	FR^{a}
$R_{0;0.5}$-$F_{29.5\,;\,30}$	USV^{b}	DoT^{a}	FR^{a}	DoT^{a}	FR^{a}	-
$F_{1;1.5}$-$F_{10;10.5}$	USV^{a}	USV^{a}	-	DoT^{a}	FR^{a}	FR^{a}
$R_{2;2.5}$-$F_{5;5.5}$	WSV^{b}	DoT^{a}	FR^{a}	WSV^{a}	WSV^{a}	DoT^{a}
$R_{2;2.5}$-$F_{10;10.5}$	WSV^{a}	USV^{a}	FR^{a}	-	-	FR^{a}
$R_{10;10.5}$-$F_{29.5;30}$	WSV^{a}	DoT^{a}	FR^{a}	DoT^{a}	FR^{a}	FR^{a}

Note: $^{a}p<0.01$; $^{b}p<0.05$; $^{c}p<0.1$
Table shows which model produces a smaller correlation between daily changes of pairs of forward rates