The Effects of Illiquidity and Lock-Ups on Portfolio Weights

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ABSTRACT

Using several recently proposed portfolio policies, we study the effects of a partial lock-up, or transaction costs, on portfolio performance and the weight of the illiquid asset in both in- and out-of-the-sample tests. We use REITs as a proxy for a low liquidity asset. Our first approach follows that of Brandt and Santa-Clara (2006) and de Roon, Guo and ter Horst (2009). In an unconditional setting, we find that the weight for our illiquid asset is in general lower that in prior studies, and is reduced to values close to empirically observed weights for real estate, once a lock-up is introduced, producing a potential answer to an earlier puzzle. Our results also indicate that the Brandt and Santa-Clara (2006) methodology and the Kan and Zhou (2007) shrinkage strategy, suggested as more stable than many traditional asset allocation approaches, still result in extreme and unstable weights and reduced out-of-the-sample performance. The Brandt, Santa-Clara and Valkanov (2009) model with rebalancing costs, which we apply to an asset allocation problem, substantially reduces the noise from conditional signals, and improves performance.

Keywords: Asset Allocation; Illiquidity; Lock-Up; Multi-period Portfolio Policy; REITs

JEL Classification: G11

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1. Introduction

Many assets are relatively illiquid at least in the short run. The recent financial crisis has efficiently demonstrated how e.g. hedge funds and real estate funds can be forced to introduce lock-ups, or extend them, during severe market conditions. Venture capital is another example of a relatively illiquid asset. Institutional portfolios typically hold many types of such illiquid or temporarily locked-up assets. Whereas there are some theoretical studies of the effect of illiquidity on portfolio choice, see e.g. Longstaff (2001), Schwartz and Tebaldi (2006), and Vath, Mnif and Pham (2007), there is a rather limited amount of studies of how the ex-ante knowledge of a potential illiquidity problem or a lock-up affects the weights of the illiquid asset and the other assets in the portfolio. Typically the studies of the effects of illiquidity on portfolio choice rely on adjusting the moments (return, and/or variance) of the return distribution for the illiquid asset[1], such as in the Bond, Hwang and Richards (2006) analysis of U.K. commercial real estate in a multi-asset portfolio.[2]

We study empirically the effect of a relatively illiquid asset on optimal *portfolio weights* and *Sharpe ratios* in a multi-asset portfolio using two rather novel, alternative methods. These are the multi-period optimization method by Brandt and Santa-Clara (2006), complemented by a lock-up for the illiquid asset in line with de Roon, Guo and ter Horst (2009) for hedge funds, and secondly, by introducing rebalancing costs in a multi-period framework, following

[1] See e.g. Cao and Teiletche (2007) for ways to deal with estimation problems for illiquid assets.
[2] Exceptions are studies such as by Ghysels and Pereira (2008), who directly model the relationship between illiquidity and the conditional distribution of returns for a sample of NYSE stocks, and González and Rubio (2007), who build in a reference for liquidity in the utility function, as well as impose constraints on illiquidity in the mean-variance portfolio optimization problem using Spanish stocks. In the market microstructure literature, illiquidity is also analyzed through its effects on bid-ask spreads, as well as returns, as in Amihud and Mendelson (1986), who show that expected stock returns are an increasing function of illiquidity costs.

Brandt, Santa-Clara and Valkanov (2009). Based on the results of e.g. Glascock, Lu and So (2000), and Oikarinen, Hoesli and Serrano (2009), who present evidence for the linkages between securitized and direct real estate[3], we use a U.S. monthly series for Real Estate Investment Trusts (REITs) as a proxy for the relatively illiquid asset class of real estate.[4] We start by investigating the effect of a complete rebalancing lock-up (for a number of periods) in the illiquid asset in a portfolio also including stocks, bonds, and money market investments, both in an unconditional as well as conditional setting, in and out of the sample, and with or without short sales constraints for the illiquid asset. Next, we consider optimal trading strategies under proportional transaction costs. As shown by our results in the paper, these can effectively be modeled as partial lock-ups as it clearly becomes sub-optimal to rebalance an asset in the presence of high transaction costs.

We contribute to the prior literature on portfolio optimization, especially with low liquidity assets, in several ways. First, since REIT data are available from the early 1970s, we produce empirical results of asset allocation with low liquidity assets using one of the longest possible time series available for a relatively illiquid asset class. Second, we produce the first tests of the multi-period portfolio policies of Brandt and Santa Clara (2006) in an out-of-sample context including empirically implemented short sales constraints, hence producing evidence with respect to performance stability issues. Third, we contribute to the literature

[3] Bond and Hwang (2003) also show that direct and securitized real estate have a similar volatility process, while Pagliari, Scherer and Monopoli (2005) report that the return and volatility of REITs and direct real estate are undistinguishable from a statistical perspective once leverage and property mix are accounted for.

[4] The relative liquidity of REITs is much lower than that of stocks, motivating our use of them as a proxy for a relatively illiquid asset. Brounen, Eichholtz and Ling (2009), for instance, calculate a liquidity ratio defined as the inverse of Amihud's (2002) illiquidity measure, and report dramatically lower values for REITs as compared to stocks. By using securitized real estate data, we also avoid many of the measurement problems associated with direct real estate. The reliability of the estimated correlation and of volatility patterns for direct real estate have been questioned due to problems with data quality (appraisal smoothing and time aggregation creating artificial autocorrelation). Therefore direct real estate data typically requires rather ad hoc de-smoothing to ensure comparability with stock and bond return dynamics. Direct real estate data are also available only at the quarterly frequency, thus posing practical problems in accurate portfolio parameter estimation due to lack of data for long enough time periods.

on the optimal weight for real estate in a multi-asset portfolio. Prior studies have typically been unable to solve the contradiction (pointed out e.g. by Chun and Shilling, 1998, and Geltner *et al.,* 2007) between the high theoretical weights[5] for real estate in portfolio optimization studies, and the low empirical weight observed in institutional asset portfolios.[6] We report evidence on the extent to which the ex-ante knowledge of a lock-up in real estate reduces the weight for that asset, as well as creates a hedging demand in the other assets, and show that the optimal weights may come close to the empirically observed weights for real estate in institutional portfolios.[7]

Our setup also provides an extended out-of-sample study of an estimation error corrected portfolio policy recently proposed in the literature. In general, it has been shown that portfolio policies shrinking weights toward equal weights (1/N) or the short-sales-constrained minimum variance portfolio (despite the fact that the investor may have low risk aversion) is beneficial because estimation errors, especially in historical means but also in co-variances, tend to be quite serious in practice (DeMiguel, Garlazzi and Uppal, 2009). Much of the previous literature has focused on historical frontier portfolios, which are known to be quite

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[5] Typically, optimization studies have reported weights between 15% to 30% for real estate in a multi-asset portfolio, see e.g. Ennis and Burik (1991), Ziobrowski and Ziobrowski (1997), Hoesli, Lekander and Witkiewicz (2004) and Fisher, Geltner and Pollakowski (2007).

[6] See e.g. Bond, Hwang and Richards (2006), who despite efforts to adjust for illiquidity, report suggested weights of 20% for real estate, while the average empirical weight in U.K. pension fund portfolios in 2003 was 6%. Partially successful efforts to explain the discrepancy also include Chun, Ciochetti and Shilling (2000) and Craft (2001), who argue that the weight which should be allocated to real estate is much more in line with the actual institutional weight when an asset-liability framework is used rather than an asset only framework, and Kallberg, Liu and Greig (1996), who consider the effects of real estate market imperfections, such as indivisible assets and no short sales.

[7] If portfolio rebalancing for the real estate part of the portfolio is not possible, its relative weight (and the weight difference between the actual and the desired weight) in a portfolio with at least some stocks and bonds as well, is likely to increase during a bear market (given that real estate falls by less than stocks). If furthermore, as reported by Hung, Onayev and Tu (2008), REITs in a time-varying setting add value to the portfolio only in up markets (due to their lower correlation with other assets then), being partially locked-up in real estate during periods when it is less useful can hurt the portfolio. Rational investors aware of a potential lock-up should anticipate it, and initially invest less in such an asset, which may explain the empirically observed lower weight for real estate as compared to theoretical weights estimated under a perfect rebalancing assumption.

unstable out-of-sample. Tu and Zhou (2008) and DeMiguel, Garlazzi and Uppal (2009) further develop estimation error corrected shrinkage and find that it is possible to improve upon out-of-sample portfolio performance by combining minimum variance and equally weighted portfolios. We study the merits of implementing such shrinkage estimators with the Brandt and Santa-Clara (2006) model. Moreover, our study contributes to the literature on asset allocation by applying the Brandt, Santa-Clara and Valkanov (2009) stock selection model with rebalancing costs into an asset allocation framework, including illiquid assets. Finally, we also contribute by testing multi-asset strategies for a data set including at least part of the recent financial crisis.

In our in-sample tests for the time period from 1982 to 2008, we find that the weight for the illiquid asset (REITs) is in general lower that in prior studies of real estate weights, and is reduced to values below 10% once a lock-up for REITs is introduced. These values come close to empirically observed values for real estate. The in-sample results are rather similar in our unconditional and conditional tests. An analysis of the certainty equivalents for the unlocked versus locked strategies indicate an annualized lock-up cost of around 1.5% to 2% for the illiquid asset.

The strategies we study include strategies based on the Brandt and Santa-Clara (2006) methodology, and the Kan and Zhou (2007) shrinkage strategy. In Brandt and Santa-Clara (2006), the authors report that their strategy results in more stable (in-sample) weights than the traditional approach to asset allocation. The Kan and Zhou (2007) shrinkage strategy in turn is suggested to improve performance under parameter uncertainty. When applied to asset classes in an out-of-the-sample framework, both strategies produce quite extreme and unstable weighs, and substantially reduced performance in terms of Sharpe ratios. The weight for REITs in these analyses is driven by its relatively more positive performance during the beginning of our time period (i.e., the estimation period for the conditional analysis), and the imposition of a short-sales constraint only for the illiquid asset magnifies the effect.

In the second part of our empirical study, we report results from an asset allocation model with rebalancing costs for the low liquidity asset. This asset allocation version of the Brandt, Santa-Clara and Valkanov (2009) model proves successful in reducing the weight changes suggested by noisy conditioning instruments, and produces superior Sharpe ratios.

The structure of this paper is as follows. First, in section 2, the methodologies used in our analyses are presented. In section 3, we present the data. Results from different portfolio strategies are reported in section 4, and final conclusions are given in section 5.

2. Methods to Capture the Illiquidity of an Asset

2.1. General

In this paper, we will use two different approaches to analyze optimal portfolio strategies (both static and dynamic, implemented both in in-sample as well as out-of-the sample frameworks). These are the methodology of Brandt and Santa-Clara (2006) complemented with a lock-up in line with de Roon, Guo and ter Horst (2009) (in their case, applied to hedge funds)[8], and the parametric portfolio policies of Brandt, Santa-Clara and Valkanov, hereafter BSV (2009), where rebalancing costs can easily be taken into account. We will describe these approaches in the following sub-sections.

2.2. Multi-Period Asset Allocation with a Lock-Up

2.2.1. The Unconditional Case

If the first and second moments of asset returns exhibit predictability, a dynamic trading strategy is called for. If, due to illiquidity, one asset class is temporarily "locked-up" (i.e. the amount invested in it cannot be changed, at least not downwards), a dynamic trading strategy may also be called for, since the lock-up can generate a systematic hedging demand, affecting the demand for other asset classes during the subsequent time periods. However, computing optimal dynamic trading strategies has proven to be problematic, since closed-form solutions are seldom available. Different numerical solution methods such as solving partial differential equations, Monte-Carlo simulations, and discretizing the state-space have been used in the literature, while practitioners still mainly rely on the static Markowitz approach.

Brandt and Santa-Clara (2006) develop a novel approach to dynamic portfolio selection which is easy to implement, and allows the use of most of the refinements developed for the Markowitz model, such as portfolio constraints, shrinkage estimation, and the combination of prior beliefs with the information contained in historical return data (i.e. the estimation of

[8] De Roon, Guo and Ter Horst (2009) study the effect of a lock-up for hedge funds in a portfolio. Using U.S. data from December 1989 to December 2007 for stocks, bonds, and hedge funds in both an unconditional framework, and in a conditional framework with the market dividend-price ratio as the state variable, they find that a three-month lock-up for hedge funds costs the investor 4.2% per annum. Investors compensate the lock-up by making adjustments to their equity and bond holdings.

dynamic trading strategies). Their method solves the portfolio problem in one step as the optimal choice (which maximizes the investor's utility) is determined among simple multi-period trading strategies. In a single-period setting with i.i.d. returns, their solution leads to the well-known Markowitz solution. With some return predictability, their approach is related to that of Ferson and Siegel (2001), who model conditional means and covariances as known functions of the state variables, and then derive optimal portfolio weights by maximizing a mean-variance utility function. The resulting portfolio weights can then be shown to be functions of the state variables. Brandt and Santa-Clara (2006) instead model the portfolio weights directly as functions of the state variables, and find the coefficients that maximize the investor's utility.

The key idea in the methodology of Brandt and Santa-Clara (2006) is to consider all the paths through which, in a multi-period setting, an initial unit of money can "travel" through the investment period. Assume two risky asset classes A and B, and two time periods. Following the notation of Brandt and Santa-Clara (2006), denote by R_t^f and R_{t+1}^f the invested amount (one) plus the risk-free rate at the time points t (at the beginning of the first period, ranging from t to t+1) and t+1 (at the beginning of the second period, ranging from t+1 to t+2). Let r_{t+1}^A and r_{t+1}^B stand for the end-of-period excess returns when investing in the asset classes A and B over the first time period (from t to t+1). Then a two-period excess return $r^p_{t\rightarrow t+2}$ for a portfolio investing in the risk-free rate, and the asset class A (with some beginning-of-period weights $w^A{}_t$ and $w^A{}_{t+1}$ for investments in the risky asset A) is

$$
\begin{aligned}
r^p_{t\rightarrow t+2} &= (R_t^f + w^A{}_t\, r_{t+1}^A)\,(R_{t+1}^f + w^A{}_{t+1}\, r_{t+2}^A) - R_t^f R_{t+1}^f \\
&= w^A{}_t\,(r_{t+1}^A R_{t+1}^f) + w^A{}_{t+1}\,(R_t^f r_{t+2}^A) + (w^A{}_t\, r_{t+1}^A)(w^A{}_{t+1}\, r_{t+2}^A) \\
&\approx w^A{}_t\,(r_{t+1}^A R_{t+1}^f) + w^A{}_{t+1}\,(R_t^f r_{t+2}^A) \qquad (1)
\end{aligned}
$$

i.e. in the last step above, the cross-product of the two excess returns and the two weights is assumed to be approximately equal to zero due to the excess returns being expected to be small over short time horizons. Brandt and Santa-Clara (2006) argue that the magnitude of the cross-product (the compounding term) is typically of the order of $1/100^{th}$ of a percent per year. They also study the impact of ignoring the compounding terms in a model for monthly excess stock and bond returns, with rebalancing frequencies from monthly to annual, and investment horizons ranging from 1 to 20 years. They conclude that, consistent with their intuition, the compounding terms are relatively unimportant for short horizons.[9]

A generalization of equation (1) for multiple risky assets is straightforward. For two risky assets, the two-period portfolio return will be

$$
\begin{aligned}
r^p_{t\to t+2} &\approx w^A_t\,(r_{t+1}^A\,R_{t+1}^f) + w^B_t\,(r_{t+1}^B\,R_{t+1}^f) + w^A_{t+1}\,(R_t^f\,r_{t+2}^A) + w^B_{t+1}\,(R_t^f\,r_{t+2}^B) \\
&\approx w'_t\,(R_{t+1}^f\,r_{t+1}') + w'_{t+1}(R_t^f\,r_{t+2}')
\end{aligned}
\qquad (2)
$$

where w'_{t+1} and w'_{t+1} are weight vectors and r_{t+1}' and r_{t+2}' are return vectors. The portfolio problem boils down to solving for the risky asset weights which maximize the investor's utility function, i.e. to e.g. solve a two-period quadratic utility optimization problem of the following form for an investor:

$$
\max E_t\,[r^p_{t\to t+2} - \gamma / 2\,(r^p_{t\to t+2})^2] \qquad (3)
$$

[9] However, for horizons beyond five years, the quality of their approximation deteriorates substantially.

where γ is the coefficient of relative risk aversion. Given a time series from t to T, the optimal weight matrix w´´ for a two-period dynamic strategy is given by:

$$w'' = 1/\gamma \left[\sum_{t=1}^{T-2} r^{p}_{t\rightarrow t+2}{}' \, r^{p}_{t\rightarrow t+2} \right]^{-1} \left[\sum_{t=1}^{T-2} r^{p}_{t\rightarrow t+2}{}' \right] \tag{4}$$

where the first set of elements of w´´ represents the fraction of wealth invested in the risky assets in the first period, and the second set of elements represents the fraction of wealth invested in the risky assets in the second period.

Next, following de Roon et al (2009), assume that a portfolio includes one liquid asset, and one illiquid asset, i.e. assume that asset A is liquid whereas whatever amount is invested in asset B in the first period, remains fixed for the next period (a two-period lock-up). In that case, the two-period portfolio excess return takes the following form:

$$\begin{aligned} r^{p}_{t\rightarrow t+2} &= (R_t^{f} + w^{A}_{t}\, r_{t+1}^{A})\,(R_{t+1}^{f} + w^{A}_{t+1}\, r_{t+2}^{A}) - R_t^{f} R_{t+1}^{f} + w^{B}_{t}\, r^{B}_{t\rightarrow t+2} \\ &\approx w^{A}_{t}\,(r_{t+1}^{A} R_{t+1}^{f}) + w^{A}_{t+1}\,(R_t^{f} r_{t+2}^{A}) + w^{B}_{t}\, r^{B}_{t\rightarrow t+2} \end{aligned} \tag{5}$$

where $r^{B}_{t\rightarrow t+2}$ is the two-period return for the illiquid (locked-up) asset B.

3.2.2. The Conditional Case

Conditional portfolio policies can be implemented in a straight-forward fashion by allowing portfolio weights to be determined (typically linearly) by observable state variables z_t. For the liquid asset A and the illiquid asset B, this implies that:

$$w^A_t = \beta_{A1}' z_t, \quad w^A_{t+1} = \beta_{A2}' z_{t+1} \quad \text{and} \quad w^B_t = \beta_B z_t . \qquad (6).$$

Although z_t could be an S-dimensional vector of state variables at time t, in this paper, we are using one state variable at a time due to data limitations. The two-period return for the conditional strategy with one liquid and one illiquid asset will then be:

$$r^p_{t\to t+2} \approx \beta_{A1}' z_t \,(r_{t+1}^A R_{t+1}^f) + \beta_{A2}' z_{t+1} (R_t^f r_{t+2}^A) + \beta_B z_t \, r^B_{t\to t+2} , \qquad (7)$$

where the βs can be viewed as the unconditional weights in a portfolio problem with scaled returns (returns scaled by the state variable). The investment problem is then to find the set of parameters β that maximize a multi-period quadratic utility as in equation (3). The unconditional weights that maximize the conditional expected utility at all dates should also maximize the unconditional expected utility.

The unconditional and conditional methods above can be generalized from the two-period asset allocation problem to a L-period problem with lock-up constraints for some risky assets. While a straightforward optimization can give negative unconditional weights, non-negativity constraints can easily be implemented in the unconditional case. Also, in the conditional case, negative weights can be ruled out by empirically restricting the size of the parameters β to values which, together with in-sample values of the state variable, do not result in negative amounts being invested in the underlying assets. Also, shrinkage estimation e.g. can be implemented. For more details concerning the methods, see Brandt and Santa-Clara (2006) and de Roon, Guo and ter Horst (2009).

2.4. Asset Allocation with Rebalancing Costs

Another way to capture asset illiquidity is to include costly rebalancing into an asset allocation model. It has theoretically been shown that transaction costs have two main effects on portfolio policies: 1) portfolio rebalancing consumes portfolio return (related to proportional transaction costs per trade and the amount/frequency of rebalancing) and 2) the optimal amount of rebalancing itself is affected. Magill and Constantinides (1976) and others show that the total utility of a portfolio strategy net of transaction costs is maximized when the investor only rebalances partially towards upcoming target weights in a "trading range".

Our rebalancing strategy will build on the approach by Brandt, Santa-Clara and Valkanov (2009). There, the authors propose a new method for optimization of portfolios with a large number of assets. They model the portfolio weight of each asset as a function of the asset's characteristics cross-sectionally. The method is related to that of Brandt and Santa-Clara (2006), where the optimal weights are modeled as functions of the state variables over time. However, here the weight invested in each asset is modeled as the same function (with common coefficients) of asset-specific variables, i.e. the portfolio problem can be formulated as a statistical estimation problem. The optimal portfolio weights are obtained by maximizing the conditional expected utility of an investor with CRRA (constant relative risk aversion) preferences. In practice, this can be obtained e.g. through a method of moments estimator, i.e. the advantage of their method is that, given the low dimensionality of the parameter vector, it is computationally easy to optimize the portfolio with nonlinear optimization methods. The method also easily allows for e.g. shrinkage estimation or short sales constraints.

While the methods of BSV (2009) are developed for a multiple asset case, they can also be used, in a conditional setting, for a smaller set of asset classes. We use this method since it allows for the inclusion of periodic transaction (rebalancing) costs. With transaction costs, the return of a portfolio net of such costs is:

$$r^{p}_{t+1} = \sum_{i=1}^{N} w_i \, r_{i,t+1} - c_{i,t} \, [w_{i,t} - w_{i,t-1}] \tag{8}$$

where $c_{i,t}$ is the periodic rebalancing cost, modeled as being proportional to the weight change for the asset i in the portfolio, between periods t and t+1. Following BSV, we define a no trade region of a certain distance (parameter *k*) such that only large enough weight updates trigger a trade and incur transaction costs. The *k*=0 case coincides with the no transaction cost case where full update from last period's realized weights (taking into account technical update of weights due to realized interim return) is always optimal, whereas for sufficiently high *k* no update will take place. Thus, higher values of *k* effectively reduces portfolio turnover. We set this parameter to value *k*=0.2 and consider separately the sensitivity of our results to this choice. The portfolio optimization is carried out on portfolio returns net of optimized transaction costs. We implement these tests both in-sample and out-of-sample by utilizing past transaction cost optimized weights over 10 years estimation window keeping the last weights for one month ahead, in a rolling fashion.

2.5. Application Details

In the first part of this paper, we will estimate optimal portfolio weights both in the unconditional as well as the conditional case, using real estate as the asset with a lock-up, and stocks and bonds as the two other asset classes (in excess of the risk-free rate). As state variables, we will in line with de Roon, Guo and ter Horst (2009) only use one state variable at a time out of the following set: the dividend yield, the default spread, and the term premium.[10] Our basic return interval is the monthly one, and the multi-period asset allocation problem is that of 3 periods, i.e. 3 months.[11] In strategies with a lock-up, real estate is assumed to have a 3-period lock-up, while investments in stocks and bonds can freely be rebalanced in the beginning of each month. Both unconstrained as well as short sales constrained strategies (short sales constraints only for the real estate asset)[12] will be estimated for an investor having quadratic utility as in Brandt and Santa-Clara (2006). More specifically, we estimate the following portfolio strategies (in an unconditional and conditional framework, with or without short sales constraints):

1. The Brandt and Santa-Clara (2006) model with γ =5 (base case), with initial weights scaled to sum to 1 (i.e. the tangency portfolio). We use excess returns in estimations, so during sub-periods other than the initial one in a multi-period strategy, the weights do not need to sum up to one, meaning that the remaining portion is borrowed/lent in the risk-free rate.

2. An optimally combined Kan and Zhou (2007) portfolio strategy (the intuitive interpretation of which is a shrinkage towards 1/N), using the global minimum variance portfolio and a γ =5.

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[10] De Roon, Guo and Ter Horst (2009) also test the short-term interest rate as a state variable.

[11] The three-month interval seems to be a reasonable proxy for the actual lock-up often present in real estate funds. The choice of the length of the horizon for the asset allocation problem is also related to data availability. In our case, having about 30 years of data at the monthly frequency, a 3-month horizon leaves us with a reasonable number of observations: an overall number of N units of 3-month intervals, and for the ten-year estimation periods, a number of 120 observations per period.

[12] We impose a short sales constraint only on real estate, since stock market and government bond weights are easily altered in practice through trading in corresponding futures contracts, which enable even large positive and negative investment weights with minimal trading costs.

The second part of the paper utilizes monthly one-step ahead returns only, but imposes transaction costs on real estate. The modeling of transaction costs and portfolio weight optimal rebalancing follows BSV (2009).

We will not only estimate in-sample optimal weights, as in Brandt and Santa-Clara (2006) and de Roon, Guo and ter Horst (2009), but also extend the analysis to rolling out-of-sample strategy outcomes.[13] Our out-of-sample strategy is largely in line with BSV (2009), i.e. we estimate optimal weights using data for an estimation period (in our case, the first ten years, i.e. 1972.01–1981.12), and then apply the weights to the first out-of-sample quarter. To facilitate comparison, the in-sample strategies are also estimated using data only from 1982 forwards. In both the unconditional and conditional strategies, we study non-overlapping quarters, i.e. the conditional strategy moves forward in steps of 3 months. When moving forward in the conditional strategies, contrary to BSV (2009), we do not enlarge the estimation period, but keep it constant (equal to the last 10 years) by dropping the oldest quarter.

The set of resulting monthly out-of-sample outcomes will be analyzed using Sharpe ratios, the certainty equivalent, and, naturally, we will focus on the resulting weights for the real estate assets as compared to a setting without a lock-up for it. The significance between the Sharpe ratios for a strategy without a lock-up, and its locked-up pair, will be compared and significance tested in line with the serial correlation preserving bootstrapping methods by Ledoit and Wolf (2008).

[13] Only BSV (2009) report results from a method like the one above (but also including rebalancing costs) for out-of-sample strategies. They also extend their analysis to portfolios with weight constraints (long-only).

The second part of the paper utilizes monthly one-step ahead returns only, but imposes transaction costs on real estate. The modeling of transaction costs and portfolio weight optimal rebalancing follows BSV (2009). Since our purpose is to study the impact of the relatively higher illiquidity of real estate as compared to the other assets, it is above all the relative difference in transaction costs which will be of importance. We use an assumption of a rebalancing cost of 0.5% for real estate, and 0% for the other assets.

3. Data

Our data set consist of stock, bond, and REIT data for the U.S. from January 1972 to December 2008. For stocks, we use the value-weighted CRSP index. For bonds, we use the Fama Bond Portfolio (Treasuries), with maturities greater than 10 years, also obtainable from CRSP. Returns for real estate are computed from the FTSE NAREIT U.S. Real Estate Index (All REITs). The 1-month Treasury bill is used as a proxy for the risk-free rate. As instruments in the conditional analyses, we use the term spread (10 year Federal government bond yield, downloaded from www.federalreserve.gov, in excess of the 3 month T-Bill rate), the dividend yield (measured as the 1-month return difference between the returns on the CRSP value-weighted index in its total return and price index forms), and the default spread (the difference between Moody's yield on seasoned corporate all-industries AAA- and BAA-rated bonds, also from www.federalreserve.gov).

Descriptive statistics for our data are reported in Table 1, and a correlation matrix of the state variables and asset returns is reported in Table 2. Table 1 shows that during our time period, REITs have offered higher returns and risks as compared to stocks during two sub-periods: from 1972 to 1981, and from 2002 to 2008. During 1992 to 2001, in turn, the assets have offered returns more in line with what is typically expected, with stock returns being highest both in terms of risk and return (a return of roughly 1% per month, and a monthly volatility of 1.5%), REITs offering a return of 0.9% and a volatility of 1.1%, and bonds a return of 0.7% and a volatility of 0.7%. Table 2 in turn shows that the correlations between stocks, bonds, and REITs have been rather low, with the bond-REIT correlation being the highest (0.5984), but also that the alternative instruments used in our conditional analyses have rather low simultaneous correlations with the assets.

4. Results from Different Portfolio Strategies

We start by analyzing results from multi-period portfolio strategies with and without a lock-up for real estate in section 4.1. These strategies follow the approaches in Brandt and Santa-Clara (2006), and de Roon, Guo and ter Horst (2009) for hedge funds, with the addition that we focus on out-of-sample results, and include short-selling restrictions for real estate. In section 4.2., we instead model the illiquidity of real estate with rebalancing costs in line with BSV (2009).

4.1. Multi-Period Portfolio Strategies with a Lock-Up for Real Estate

First, as a basis for comparison, we estimate unconditional, unconstrained multi-period (3-month) in-sample strategies for the assets classes of stocks, bond, and REITs, using data for the time period of 1982-2008. We leave the time period of 1972-1981 outside at this point, so that we later can compare these strategy outcomes to out-of-sample strategy outcomes for the same period, when using the first 10 years as an estimation period. The results are reported in Panel A of Table 3 for strategies without and with a 3-month lock-up for REITs.

Table 3, Panel A shows that when short sales are allowed for, the change in Sharpe ratios[14] is significant (falling from 0.5855 to 0.4153) when a 3-month lock-up is introduced for REITs. The average REIT weight is above 16.7% in the strategy for the $\gamma = 5$ investor (Column 1) in the base case, and falls to 8.3% when a lock-up is introduced for REITs (Column 2). The Kan and Zhou (2007) shrinkage strategy produces lower Sharpe ratios, and the change in them is not significant when introducing a lock-up for REITs. Here the REIT weights are even smaller, falling from 13% to a negative one of -8.5% in the strategy with a lock-up.

When a short-sales constraint is introduced for REITs (Panel B of Table 3, only for the $\gamma = 5$ strategy), the change in Sharpe ratios is not significant, but the fall in weights is even larger for REITs, from 23.6% to 8.3%. The results for the locked-up case are rather similar with and without short sales constraints, indicating that the short sales constraint is typically not binding for the illiquid asset weights during the first of the three months in the multi-period strategies.

Interestingly, whereas our results when there is no lock-up for the illiquid asset (i.e., real estate) are very similar to those from earlier studies (i.e. a real estate weight of between 15% to 25%), our results from locked-up strategies, especially with short sales constraints, resemble empirically observed weights, i.e. weights which typically are below 10%.

Next, we study conditional in-sample strategies using the dividend yield as the conditioning instrument.[15] Table 4 reports results for our two strategies without and with a lock-up for our

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[14] The change in Sharpe ratios is tested using the serial correlation preserving bootstrapping method by Ledoit and Wolf (2008), with B=1,000 bootstrap re-samples, and expected block size b=5.

[15] The results, not reported here, are rather similar also using the other instruments.

illiquid asset, without and with a short-sales constraint for the illiquid asset in Panels A and B, respectively. Again, the reduction in Sharpe ratios brought by the lock-up is significant for the first strategy, but not the second one. The weights for the illiquid asset falls even more dramatically, to negative values in both cases in Panel A, and to 8.3%, i.e. as in the unconditional case, when a short sales constraint is imposed for the illiquid asset.

In Brandt and Santa-Clara (2006), their method is demonstrated using in-sample conditional multi-period strategies with stocks and bonds (and the T-bill) as assets. Their method results in more stable weights than the traditional approach to asset allocation, where conditional expected returns are obtained from an in-sample regression of returns on state variables, and the Markowitz method is applied to these conditional expected returns together with an unconditional covariance matrix. We want to test the stability of the Brandt and Santa-Clara (2006) approach on our problem in a full out-of-sample setting. We therefore estimate portfolio weights using a ten-year rolling estimation period, and apply the weights for the subsequent 3-month horizons. We do this for all the strategy alternatives in Tables 3 and 4.

Table 5 reports the out-of-sample performance of unconditional portfolio policies with and without a lock-up for the illiquid asset, and without and with short sales constraint for it in Panels A and B, respectively. Sharpe ratios for the $\gamma = 5$ strategy are now much lower than their in-sample values in Table 3 were, indicating a much poorer out-of-sample performance. The Kan-Zhou strategy is less affected by the switch to an out-of-sample framework, as might be expected for a shrinkage strategy similar to the Bayes-Stein correction in the Markowitz framework. Both strategies also result in substantial fluctuation in sub-period portfolio weights, as well as extreme weights, as evidenced by the high weight turnover statistics in Table 5.

The introduction of a lock-up for the illiquid asset actually increases the average weight for it in the strategy with the on average higher weight turnover, i.e. the $\gamma = 5$ strategy, but substantially reduces the Sharpe ratio, so that the strategy now looses to the Kan-Zhou strategy in the lock-up case. A closer inspection of the time-varying portfolio weights in these strategies (not reported here) reveal that the high average weight for real estate is largely due to its good performance and hence superior weight in the 1970's (a period used here, but not in the in-sample tests, since an estimation period is required for the out-of-the-sample weights). Especially in the less risk averse and more parameter sensitive $\gamma = 5$ strategies, this effect is magnified. The results for the Kan-Zhou strategy are very similar to those in Table 3 both in terms of Sharpe ratios as well as average weights and weight falls for the illiquid asset.

Table 6 finally reports the out-of-sample performance of our conditional strategies using, as before, the dividend yield as the state variable. The pattern is the same as for the unconditional ones, but now with even a higher weight turnover. Here the introduction of a lock-up leads to a significant reduction in Sharpe ratios for strategy one, and increases in the weight of the illiquid asset when short sales are possible, but a reduction when they are restricted for. The weight for the illiquid asset is again, as in Table 5, much higher than in the in-sample cases.

Analyzing the changes in the certainty equivalents (CEVs) give some indication of the costs of illiquidity for the strategies. In the in-sample cases, the differences between the CEV for the unconstrained vs. the locked-up cases in Tables 3 and 4 are around 2%-1.5% for the first strategy, and around 2%-1% for the Kan-Zhou one. In the out-of-the-sample tests, these differences increase to values around 3% to 3.5% for our first strategy, indicating a higher cost for the lock-up in out-of-sample tests.

4.2. Strategies with Varying Transaction Costs for the Illiquid Asset

In section 4.1, we studied the effects of introducing a lock-up for the illiquid asset, and the corresponding reduction in Sharpe ratios and CEVs due to obstacles for efficient portfolio rebalancing. We found evidence of significant reductions in Sharpe ratios especially for the γ=5 strategy, and a radical drop in the portfolio weights for the illiquid asset in the in-sample strategies. Our out-of-the-sample strategies in turn produced extreme volatility in portfolio weights, and a worse out-of-sample behavior.

A lock-up of the illiquid asset can be seen as an infinite transaction cost for it. A perfect lock-up may also indirectly enhance portfolio volatility by creating an (in our previous analysis unbounded) hedging demand in the liquid assets. Also the mechanical use of conditional information may lead to excess volatility. Boundaries given by transaction costs may enhance behavior by permitting weight changes in the illiquid asset when motivated by strong enough conditioning information, but limiting the extent of weight changes. A transaction costs approach is also a more empirically valid approach to most portfolio rebalancing problems given illiquid assets. Next, we thus study the effects of varying transaction costs in a setting with conditioning information.

We apply the method of BSV (2009), which allows for predictability over time as well as in the cross-section, to our three asset classes. We estimate both strategies with only cross-sectional predictability, as well as conditional strategies with time series predictability as well. As the conditioning instrument, we use dividend yield, and as the cross-sectional instrument, the past 12-month return for the asset classes in question, proxying for a potential momentum factor.

As described earlier in equation (8), the BSV (2009) methodology estimates the optimal weights as deviations from a benchmark portfolio, given rebalancing costs. We take as the starting point two alternative sets of weights from in-sample results in section 4.1. These are: 60%-20%-20% for bonds, stocks, and REITs, respectively (set 1), and 70%-20%-10% (set 2). These weights are close to the average weights in Tables 3 and 4 for our risky asset classes. As monthly transaction costs, we use different alternatives in the 0% to 2% region for the illiquid asset only, since we aim at studying how differences in rebalancing costs influence the development of optimal portfolio weighs.

Table 7 reports the results of our analyses using a cross-sectional instrument only (measured based on the past 12 months, data not used in the execution of the strategy), while Table 8 reports the results from using both the cross-sectional and the time-series instrument (which in itself only requires lagged data for one period).

The results indicate that while higher transaction costs hurt the strategies in terms of somewhat lower average returns, they actually improve Sharpe ratios due to relatively higher benefits from a lower portfolio volatility. It is also notable that these ratios are higher than the ones in part 4.1 of this paper. The highest Sharpe ratio (a ratio of 0.8027) in Table 7 is produced by the portfolio with a lower REIT weight (10%) in the benchmark, leading to an average actual weight (average over time) of 13.4% for the illiquid asset.

The conditional strategies further improve performance, and again strategies with higher costs of adjusting the weights dominate. The highest Sharpe ratio in table 8 is produced by a strategy in Panel B, i.e. a strategy with a lower weight for the illiquid asset in the benchmark (10% in the benchmark), and the highest rebalancing costs (5%), leading to a smaller standard deviation for the portfolio return. This strategy has a Sharpe ratio of 0.8110 and an average weight of 13.9% for REITs in the actual strategy). However, the differences between the strategies are generally small and insignificant.

In this section, we have studied the effect of actual transaction costs. Our results indicate that conditional strategies such as the ones in this paper perform better when portfolio rebalancing is costly, since they put boundaries for extreme weight changes.

5. Summary and Conclusions

Assets with a low liquidity may harm portfolios by preventing optimal weight changes. They may also cause a hedging demand in the other assets. Portfolio optimization models which take such effects into consideration by other means than by imposing liquidity costs on the moments of the return distribution have only recently been proposed. We contribute to the empirical study of such models by not only investigating the in-sample performance, but also by conducting the (first) out-of-the sample tests of the performance of the multi-period optimization method by Brandt and Santa-Clara (2006), complemented by a lock-up for the illiquid asset in line with de Roon, Guo and ter Horst (2009), and by empirically implemented short sales constraints for the illiquid asset. We also apply the Brandt, Santa-Clara and Valkanov (2009) model with rebalancing costs into an asset allocation framework, and report evidence in support of its use. Our setup furthermore provides an extended out-of-sample study of an estimation error corrected portfolio policy recently proposed in the literature, the Kan and Zhou (2007) shrinkage estimator.

As a proxy for an illiquid asset, we use REITs data, which is available from the early 1970s, giving us one of the longest possible time series available for a relatively illiquid asset class. Through this, we also contribute to the literature on the optimal weight for real estate in a multi-asset portfolio. Prior studies have typically been unable to solve the contradiction between the high theoretical weight for real estate in portfolio optimization studies, and the low empirical weight observed in institutional asset portfolios.

In our in-sample tests for the time period from 1982 to 2088, we find that the weight for the illiquid asset (REITs) is in general lower that in prior studies of real estate weights, and is reduced to values below 10% once a lock-up for REITs is introduced. These values come close to observed values for real estate in institutional portfolios.

When testing the recently proposed portfolio strategies, which should lead to more stable weights than in traditional asset allocation approaches, in an out-of-the-sample framework, we still find extreme weights accompanied by a significant instability, and a resulting reduced out-of-the-sample performance. However, the Brandt, Santa-Clara and Valkanov (2009) model with rebalancing costs, which we here introduce to an asset allocation framework, substantially reduces the noise from conditional signals and produces the highest Sharpe ratios in this study. These results indicate that while higher transaction costs hurt the strategies in terms of somewhat lower average returns, they actually improve Sharpe ratios from conditional strategies due to relatively higher benefits from a lower portfolio volatility.

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Table 1. Descriptive Statistics

This table reports descriptive statistics for our raw data. Panel A reports, for the whole time period of January 1972 to December 2008, the means, medians, standard deviations, and skewness and kurtosis values for monthly arithmetic returns for STOCKS (the CRSP value-weighted index), BONDS (the CRSP Fama Bond Portfolio with maturities greater than 10 years), and REITs (the FTSE NAREIT US All-REIT index), together with T-BILL, the 3-month T-Bill rate. We also report corresponding values for our instruments: the default spread DEFSPR (the difference between Moody's yield on seasoned corporate all-industries AAA- and BAA-rated bonds), the dividend yield DIVYIELD (the 1-month return difference between the logarithmic returns on the CRSP value-weighted index in its total return and price index forms), the term spread TERMSPR (the difference between the 10 year Federal government bond yield and the 3-month T-Bill rate), and 3M_T_BILL, (the 3 month T-Bill yield). Skewness and kurtosis values significant at the 5% level (2-sided tests) are denoted by boldface. In Panels B to E, we report means and standard deviations for four sub-periods: 1972 to 1981 (120 obs), 1982 to 1991 (120 obs.), 1992 to 2001 (120 obs.), and 2002 to 2008 (82 obs.).

	STOCKS	BONDS	REITs	T-BILL	DEFSPR	DIVYIELD	TERMSPR	3M_T-BILL
Panel A. 1972-2008								
Mean	0.0086	0.0076	0.0081	0.0048	1.0853	0.0284	1.6117	5.8394
Median	0.0125	0.0073	0.0102	0.0044	0.9450	0.0272	1.7300	5.3450
Stdev	0.0458	0.0284	0.0486	0.0024	0.4376	0.0112	1.3238	2.9643
Skewness	-0.6095	0.5402	-0.6775	0.8548	1.4185	0.2294	-0.5031	0.8581
Kurtosis	2.4676	2.6048	8.1571	1.1828	2.1149	-1.0268	-0.2684	1.1655
Panel B. 1972-1981								
Mean	0.0070	0.0030	0.0077	0.0063	1.2043	0.0386	0.9103	7.7099
Median	0.0052	0.0007	0.0093	0.0054	1.0200	0.0400	1.3450	6.8700
Stdev	0.0486	0.0298	0.0608	0.0027	0.4517	0.0074	1.5128	3.2525
Panel C. 1982-1991								
Mean	0.0137	0.0127	0.0086	0.0062	1.3451	0.0356	2.1348	7.6193
Median	0.0156	0.0116	0.0061	0.0061	1.2150	0.0350	2.3050	7.6350
Stdev	0.0479	0.0317	0.0332	0.0016	0.4298	0.0063	0.9739	1.8312
Panel D. 1992-2001								
Mean	0.0104	0.0069	0.0093	0.0037	0.7201	0.0184	1.6398	4.5051
Median	0.0149	0.0068	0.0108	0.0039	0.6900	0.0180	1.4100	4.9050
Stdev	0.0421	0.0216	0.0342	0.0009	0.1131	0.0055	1.1397	0.9861
Panel E. 2002-2008								
Mean	0.0008	0.0081	0.0064	0.0021	1.0660	0.0176	1.8262	2.5310
Median	0.0107	0.0091	0.0199	0.0016	0.9500	0.0173	2.2050	1.7750
Stdev	0.0433	0.0291	0.0636	0.0013	0.3676	0.0029	1.3019	1.4963

Table 2. Correlation Matrix of Asset Returns and Instruments

This table reports correlation coefficient between our assets (total returns) and instruments using data for the whole time period from January 1972 to December 2008. The assets are: returns for STOCKS (the CRSP value-weighted index), BONDS (the CRSP Fama Bond Portfolio with maturities greater than 10 years), and REITs (the FTSE NAREIT US All-REIT index), together with T-BILL, the 3-month T-Bill rate. The instruments are: the default spread DEFSPR (the difference between Moody's yield on seasoned corporate all-industries AAA- and BAA-rated bonds), the dividend yield DIVYIELD (the 1-month return difference between the logarithmic returns on the CRSP value-weighted index in its total return and price index forms), the term spread TERMSPR (the difference between the 10 year Federal government bond yield and the 3-month T-Bill rate), and 3M_T_BILL (the 3 month T-Bill yield).

	STOCKS	BONDS	REITs	T-BILL	DEFSPR	DIVYIELD	TERMSPR	3M_T-BILL
STOCKS	1							
BONDS	0.1718	1						
REITs	0.1795	0.5984	1					
T-BILL	0.0445	0.0075	-0.0134	1				
DEFSPR	0.1088	0.0709	0.1077	0.3758	1			
DIVYIELD	0.0737	0.1011	0.0653	0.6864	0.5545	1		
TERMSPR	0.1216	0.0431	0.0921	-0.4968	0.1559	-0.0340	1	
3M T-BILL	0.0398	0.0319	0.0043	0.9826	0.4122	0.7160	-0.4834	1

Table 3. The Performance of Unconditional In-Sample Portfolio Strategies

This table reports statistics concerning the performance and characteristics of estimated multi-period (3-month) unconditional and in-sample portfolio strategies for data for the time period of January 1982 to December 2008. Our assets are stocks (the CRSP value-weighted index), bonds (the CRSP Fama Bond Portfolio with maturities greater than 10 years, and the FTSE NAREIT US All-REIT index together with the 3-month T-Bill rate. In Columns 1 and 2, we report results from two dynamic strategies with no lock-up for any asset, while in Columns 3 and 4, results from these strategies with a 3-month lock up for real estate are reported. The two strategies are: the optimal portfolio strategy for a γ =5 investor with initial (period 1) weights scaled to sum to 1, and an optimal Kan and Zhou (2007) portfolio strategy (the intuitive interpretation of which is a shrinkage towards 1/N), using the global minimum variance portfolio and a γ =5. In Panel A, no short sales restrictions are imposed, while in Panel B, a short sales constraint is imposed for REITs (only the first strategy is possible to compute by our method of empirically restricting the REIT weight to nonnegative values). The statistics reported are the mean excess return and the standard deviation, the Sharpe ratio, and the certainty equivalent (CEV), followed by average relative weights and weight turnover measures for the risky assets (stocks, bonds, and real estate). The weight turnover statistics are calculated as the mean absolute value of weight changes implied by the strategy between specific sub-quarter periods. Significance testing of Sharpe ratios between strategies without a lock-up, and their locked-up versions, uses serial correlation preserving bootstrapping methods by Ledoit and Wolf (2008) with B=1,000 bootstrap re-samples, and expected block size b=5. Sharpe ratios significantly different from each other in pairwise strategy comparisons (Columns 1 vs. 3, and Columns 2 vs. 4 in Panel A; Columns 1 vs. 2 in Panel B) at the 10% level are denoted boldface.

Panel A: no constraints for short sales	No lock-up		3-m. lock-up for REITs	
	Column (1): γ=5, initial weights sum up to 1	Column (2): Kan-Zhou (2007) strategy	Column (3): γ=5, initial weights sum up to 1	Column (4): Kan-Zhou (2007) strategy
Mean excess return	0.0065	0.0039	0.0042	0.0037
St. deviation	0.0384	0.0359	0.0354	0.0364
Sharpe ratio	**0.5855**	0.3794	**0.4153**	0.3477
Certainty equivalent	0.0844	0.0591	0.0639	0.0545
Average bond weight	0.6262	0.6976	0.7203	0.8027
(weigh turnover)	(0.3208)	(0.2870)	(0.5817)	(0.3053)
Average stock weight	0.2070	0.1725	0.1971	0.2822
(weigh turnover)	(0.5836)	(0.2547)	(0.1899)	(0.2451)
Average REITs weight	0.1668	0.1299	0.0826	-0.085
(weigh turnover)	(0.6042)	(0.1467)	(0.0266)	(0.0331)

Panel B: short-sales constraint for REITs	No lock-up	3-m. lock-up for REITs
	γ=5, initial weights sum up to 1	γ=5, initial weights sum up to 1
Mean excess return	0.0060	0.0042
St. deviation	0.0387	0.0354
Sharpe ratio	0.5379	0.4153
Certainty equivalent	0.0780	0.0639
Average bond weight	0.6043	0.7202
(weigh turnover)	(0.3569)	(0.5815)
Average stock weight	0.1600	0.1973
(weigh turnover)	(0.4844)	(0.1897)
Average REITs weight	0.2357	0.0825
(weigh turnover)	(0.4639)	(0.0265)

Table 4. The Performance of Conditional In-Sample Portfolio Strategies

This table reports statistics concerning the performance and characteristics of estimated multi-period (3-month) conditional in-sample portfolio strategies for data for the time period of January 1982 to December 2008. Our assets are stocks (the CRSP value-weighted index), bonds (the CRSP Fama Bond Portfolio with maturities greater than 10 years, and the FTSE NAREIT US All-REIT index together with the 3-month T-Bill rate. The dividend yield has been used as the state (conditioning) variable. In Column 1, we report results from an optimal portfolio strategy for a γ =5 investor with initial (period 1) weights scaled to sum to 1, with no lock-up for any asset, while in Column 2, results from a strategy with a 3-month lock up for real estate are reported. In Panel A, no short sales restrictions are imposed, while in Panel B, a short sales constraint is imposed for REITs (only the first strategy is possible to compute by our method of empirically restricting the REIT weight to nonnegative values). The statistics reported are the mean excess return and the standard deviation, the Sharpe ratio, and the certainty equivalent (CEV), followed by average relative weights and weight turnover measures for the risky assets (stocks, bonds, and real estate). The weight turnover statistics are calculated as the mean absolute value of weight changes implied by the strategy between specific sub-quarter periods. Significance testing of Sharpe ratios between strategies without a lock-up, and their locked-up versions, uses serial correlation preserving bootstrapping methods by Ledoit and Wolf (2008) with B=1,000 bootstrap re-samples, and expected block size b=5. Sharpe ratios significantly different from each other in pairwise strategy comparisons (Columns 1 vs. 3, and Columns 2 vs. 4 in Panel A; Columns 1 vs. 2 in Panel B) at the 10% level are denoted in boldface.

Panel A: no constraints for short sales	No lock-up		3-m. lock-up for REITs	
	Column (1): γ=5, initial weights sum up to 1	Column (2): Kan-Zhou (2007) strategy	Column (3): γ=5, initial weights sum up to 1	Column (4): Kan-Zhou (2007) strategy
Mean excess return	0.0065	0.0095	0.0043	0.0071
St. deviation	0.0380	0.0268	0.0350	0.0218
Sharpe ratio	**0.5937**	1.2206	**0.4237**	1.1359
Certainty equivalent	0.0856	0.1418	0.0652	0.1216
Average bond weight	0.6241	0.7078	0.7175	0.7788
(weigh turnover)	(0.3048)	(0.5220)	(0.5678)	(0.4339)
Average stock weight	0.2098	0.301	0.1996	0.2234
(weigh turnover)	(0.5712)	(1.5925)	(0.1840)	(0.3454)
Average REITs weight	0.1661	-0.0088	0.0829	-0.0022
(weigh turnover)	(0.5975)	(2.2129)	(0.0264)	(0.1591)

Panel B: short-sales constraint for REITs	No lock-up	3-m. lock-up for REITs
	γ=5, initial weights sum up to 1	γ=5, initial weights sum up to 1
Mean excess return	0.0060	0.0043
St. deviation	0.0384	0.0350
Sharpe ratio	0.5422	0.4237
Certainty equivalent	0.0787	0.0652
Average bond weight	0.6026	0.7176
(weigh turnover)	(0.3485)	(0.5678)
Average stock weight	0.1600	0.1995
(weigh turnover)	(0.4753)	(0.1841)
Average REITs weight	0.2373	0.0829
(weigh turnover)	(0.4609)	(0.0264)

Table 5. The Performance of Unconditional Out-of-Sample Portfolio Strategies

This table reports statistics concerning the performance and characteristics of estimated multi-period (3-month) unconditional out-of-sample portfolio strategies, using data for January 1972 to December 1981 as the estimation period, the next three months as the first test period, and then quarterly rolling forward (non-overlapping, and keeping the estimation period equally long, i.e. dropping the oldest quarter) until December 2008. Our assets are stocks (the CRSP value-weighted index), bonds (the CRSP Fama Bond Portfolio with maturities greater than 10 years, and the FTSE NAREIT US All-REIT index together with the 3-month T-Bill rate. In Columns 1 and 2, we report results from two dynamic strategies with no lock-up for any asset, while in Columns 3 and 4, results from these strategies with a 3-month lock up for real estate are reported. The two strategies are: the optimal portfolio strategy for a γ =5 investor with initial (period 1) weights scaled to sum to 1, and an optimal Kan-Zhou (2007) portfolio strategy (the intuitive interpretation of which is a shrinkage towards 1/N), using the global minimum variance portfolio and a γ =5. In Panel A, no short sales restrictions are imposed, while in Panel B, a short sales constraint is imposed for REITs (only the first strategy is possible to compute by our method of empirically restricting the REIT weight to nonnegative values). The statistics reported are the mean excess return and the standard deviation, the Sharpe ratio, and the certainty equivalent (CEV), followed by average relative weights and weight turnover measures for the risky assets (stocks, bonds, and real estate). The weight turnover statistics are calculated as the mean absolute value of weight changes implied by the strategy between specific sub-quarter periods. Significance testing of Sharpe ratios between strategies without a lock-up, and their locked-up versions, uses serial correlation preserving bootstrapping methods by Ledoit and Wolf (2008) with B=1,000 bootstrap re-samples, and expected block size b=5. Sharpe ratios significantly different from each other in pairwise strategy comparisons (Columns 1 vs. 3, and Columns 2 vs. 4 in Panel A; Columns 1 vs. 2 in Panel B) at the 10% level are denoted boldface.

Panel A: no constraints for short sales	No lock-up		3-m. lock-up for REITs	
	Column (1): γ=5, initial weights sum up to 1	Column (2): Kan-Zhou (2007) strategy	Column (3): γ=5, initial weights sum up to 1	Column (4): Kan-Zhou (2007) strategy
Mean excess return	0.0060	0.0037	0.0038	0.0037
St. deviation	0.0458	0.0392	0.0494	0.0410
Sharpe ratio	0.4559	0.3308	0.2648	0.3142
Certainty equivalent	0.0595	0.0488	0.0222	0.0442
Average bond weight	0.3665	0.5974	0.4037	0.6788
(weigh turnover)	(1.5729)	(2.5252)	(1.8044)	(1.3409)
Average stock weight	0.4275	0.3301	0.3394	0.3064
(weigh turnover)	(5.2339)	(3.6958)	(1.4361)	(1.2290)
Average REITs weight	0.2060	0.0725	0.2569	0.0148
(weigh turnover)	(5.6137)	(3.2083)	(1.2492)	(1.2418)

Panel B: short-sales constraint for REITs	No lock-up	3-m. lock-up for REITs
	γ=5, initial weights sum up to 1	γ=5, initial weights sum up to 1
Mean excess return	0.0050	0.0033
St. deviation	0.0460	0.0488
Sharpe ratio	0.3778	0.2323
Certainty equivalent	0.0470	0.0181
Average bond weight	0.2938	0.3615
(weigh turnover)	(1.0081)	(1.1239)
Average stock weight	0.2594	0.2614
(weigh turnover)	(1.0547)	(1.2804)
Average REITs weight	0.4469	0.3771
(weigh turnover)	(0.8375)	(0.5661)

Table 6. The Performance of Conditional Out-of-Sample Portfolio Strategies

This table reports statistics concerning the performance and characteristics of estimated multi-period (3-month) unconditional out-of-sample portfolio strategies, using data for January 1972 to December 1981 as the estimation period, the next three months as the first test period, and then quarterly rolling forward (non-overlapping, and keeping the estimation period equally long, i.e. dropping the oldest quarter) until December 2008. Our assets are stocks (the CRSP value-weighted index), bonds (the CRSP Fama Bond Portfolio with maturities greater than 10 years, and the FTSE NAREIT US All-REIT index together with the 3-month T-Bill rate. The dividend yield has been used as the state (conditioning) variable. In Column 1, we report results from an optimal portfolio strategy for a γ =5 investor with initial (period 1) weights scaled to sum to 1, with no lock-up for any asset, while in Column 2, results from a strategy with a 3-month lock up for real estate are reported. In Panel A, no short sales restrictions are imposed, while in Panel B, a short sales constraint is imposed for REITs (only the first strategy is possible to compute by our method of empirically restricting the REIT weight to nonnegative values). The statistics reported are the mean excess return and the standard deviation, the Sharpe ratio, and the certainty equivalent (CEV), followed by average relative weights and weight turnover measures for the risky assets (stocks, bonds, and real estate). The weight turnover statistics are calculated as the mean absolute value of weight changes implied by the strategy between specific sub-quarter periods. Significance testing of Sharpe ratios between strategies without a lock-up, and their locked-up versions, uses serial correlation preserving bootstrapping methods by Ledoit and Wolf (2008) with B=1,000 bootstrap re-samples, and expected block size b=5. Sharpe ratios significantly different from each other in pairwise strategy comparisons (Columns 1 vs. 3, and Columns 2 vs. 4 in Panel A; Columns 1 vs. 2 in Panel B) at the 10% level are denoted boldface.

Panel A: no constraints for short sales	No lock-up		3-m. lock-up for REITs	
	Column (1): γ=5, initial weights sum up to 1	Column (2): Kan-Zhou (2007) strategy	Column (3): γ=5, initial weights sum up to 1	Column (4): Kan-Zhou (2007) strategy
Mean excess return	0.0060	0.0088	0.0037	0.0142
St. deviation	0.0458	0.2937	0.0492	0.2351
Sharpe ratio	**0.4550**	0.1034	**0.2606**	0.2088
Certainty equivalent	0.0594	-2.4264	0.0221	-1.4331
Average bond weight	0.3665	0.0812	0.3988	0.4844
(weigh turnover)	(1.7964)	(4.9620)	(1.9436)	(6.1966)
Average stock weight	0.427	0.1545	0.3415	0.038
(weigh turnover)	(8.6447)	(5.8914)	(1.4451)	(4.3549)
Average REITs weight	0.2065	0.7642	0.2597	0.4776
(weigh turnover)	(8.6524)	(5.7879)	(1.1617)	(3.1537)

Panel B: short-sales constraint for REITs	No lock-up	3-m. lock-up for REITs
	γ=5, initial weights sum up to 1	γ=5, initial weights sum up to 1
Mean excess return	0.0050	0.0032
St. deviation	0.0460	0.0486
Sharpe ratio	0.3781	0.2305
Certainty equivalent	0.0472	0.0183
Average bond weight	0.2936	0.3573
(weigh turnover)	(1.1068)	(1.0015)
Average stock weight	0.2622	0.2657
(weigh turnover)	(1.1576)	(1.0561)
Average REITs weight	0.4442	0.3770
(weigh turnover)	(0.8338)	(0.4672)

Table 7. Portfolio Strategies with Transaction Costs and a Cross-sectional Instrument

This table reports statistics concerning the performance of strategies in line with Brandt; Santa-Clara and Valkanov (2009), using alternative assumptions for the rebalancing cost of the illiquid asset (REITs) and an adjustment parameter k of 0.2. The data is for January 1972 to December 1981 (of which the 12 first months are used to estimate values for the cross-sectional instrument for the first execution of the strategy, and the remaining months are used to execute the strategy month by month, with a rolling estimation period). Our assets are stocks (the CRSP value-weighted index), bonds (the CRSP Fama Bond Portfolio with maturities greater than 10 years, and the FTSE NAREIT US All-REIT index together with the 3-month T-Bill rate. The cross-sectional instrument is the past 12-month return for the assets in question. In Panel A, the benchmark is 60% bonds (B), 20% stocks (S), and 20% REITs (R), whereas in Panel B it is 70% bonds, 20% stocks, and 10% REITs. The statistics reported are the monthly mean excess return and the standard deviation, and the annualized Sharpe ratio, followed by average relative weights and weight turnover measures for the risky assets (stocks, bonds, and real estate). The weight turnover statistic measures total relative transaction costs. It is calculated as the sum of the proportional transaction costs times the optimal absolute weight updates.

Panel A: benchmark 1;	Transaction costs			
B: 60%, S: 20%, R: 20%	0%	0.5%	1%	2%
Mean excess return	0.0062	0.0060	0.0059	0.0057
St. deviation	0.0284	0.0273	0.0264	0.0252
Sharpe ratio	0.7562	0.7613	0.7742	0.7835
Average bond weight	0.5451	0.5557	0.5709	0.5825
(weigh turnover)	(0.0145)	(0.0139)	(0.0129)	(0.0023)
Average stock weight	0.2023	0.1952	0.1972	0.1881
(weigh turnover)	(0.0147)	(0.0149)	(0.0173)	(0.0033)
Average REITs weight	0.2526	0.2491	0.2319	0.2293
(weigh turnover)	(0.0125	(0.0122)	(0.0132)	(0.0023)
Panel B: benchmark 2;	Transaction costs			
B: 70%, S: 20%, R: 10%	0%	0.5%	1%	5%
Mean excess return	0.0062	0.0060	0.0059	0.0057
St. deviation	0.0285	0.0273	0.0263	0.0246
Sharpe ratio	0.7536	0.7613	0.7771	0.8027
Average bond weight	0.6501	0.6477	0.6780	0.6789
(weigh turnover)	(0.0147)	(0.0133)	(0.0072)	(0.0020)
Average stock weight	0.2112	0.1955	0.1859	0.1870
(weigh turnover)	(0.0136)	(0.0168)	(0.0074)	(0.0028)
Average REITs weight	0.1387	0.1568	0.1360	0.1342
(weigh turnover)	(0.0115)	(0.0127)	(0.0065)	(0.0021)

Table 8. Conditional Portfolio Strategies with Transaction Costs and a Cross-sectional Instrument

This table reports statistics concerning the performance of strategies in line with Brandt, Santa-Clara and Valkanov (2009), using alternative assumptions for the rebalancing cost of the illiquid asset (REITs). The data is for January 1972 to December 1981 (of which the 12 first months are used to estimate values for the cross-sectional instrument for the first execution of the strategy, and the remaining months are used to execute the strategy month by month, with a rolling estimation period). Our assets are stocks (the CRSP value-weighted index), bonds (the CRSP Fama Bond Portfolio with maturities greater than 10 years, and the FTSE NAREIT US All-REIT index together with the 3-month T-Bill rate. The cross-sectional instrument is the past 12-month return for the assets in question, and the time-series instrument is the dividend yield. In Panel A, the benchmark is 60% bonds (B), 20% stocks (S), and 20% REITs (R), whereas in Panel B it is 70% bonds, 20% stocks, and 10% REITs. The statistics reported are the monthly mean excess return and the standard deviation, and the annualized Sharpe ratio, followed by average relative weights and weight turnover measures for the risky assets (stocks, bonds, and real estate). The weight turnover statistic measures total relative transaction costs. It is calculated as the sum of the proportional transaction costs times the optimal absolute weight updates.

Panel A: benchmark 1; B: 60%, S: 20%, R: 20%	Transaction costs			
	0%	0.5%	1%	5%
Mean excess return	0.0067	0.0064	0.0061	0.0059
St. deviation	0.0304	0.0288	0.0276	0.0256
Sharpe ratio	0.7635	0.7698	0.7656	0.7984
Average bond weight	0.5435	0.5309	0.5703	0.5809
(weigh turnover)	(0.0197)	(0.0122)	(0.0097)	(0.0025)
Average stock weight	0.1934	0.2071	0.2083	0.1764
(weigh turnover)	(0.023)	(0.0117)	(0.0108)	(0.0022)
Average REITs weight	0.2631	0.2620	0.2214	0.2427
(weigh turnover)	(0.019)	(0.0099)	(0.0091)	(0.0021)
Panel B: benchmark 2; B: 70%, S: 20%, R: 10%	Transaction costs			
	0%	0.5%	1%	5%
Mean excess return	0.0067	0.0065	0.0062	0.0059
St. deviation	0.0310	0.0292	0.0278	0.0252
Sharpe ratio	0.7487	0.7711	0.7726	0.8110
Average bond weight	0.6452	0.6484	0.6730	0.6770
(weigh turnover)	(0.0214)	(0.0132)	(0.0118)	(0.0036)
Average stock weight	0.2034	0.2056	0.2006	0.1840
(weigh turnover)	(0.0283)	(0.0124)	(0.0157)	(0.0043)
Average REITs weight	0.1514	0.1460	0.1264	0.1389
(weigh turnover)	(0.0199)	(0.0100)	(0.0117)	(0.0033)