

Does Modeling Framework Matter?

A Comparative Study of Structural and Reduced-Form Models

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January 4, 2011

Abstract

This study provides a rigorous empirical comparison of structural and reduced-form credit risk frameworks. As major difference we focus on the discriminative modeling of the default time. In contrast to the previous literature, we calibrate both approaches to the same data set, apply comparable estimation techniques, and assess the out-of-sample prediction quality on the same time series of CDS prices. As our empirical implementations of both approaches rely on the same market information we are able to judge whether empirically the model structure itself makes an important difference. Interestingly, our study shows that the models' prediction power are quite close on average indicating that for pricing purposes the modeling type does not greatly matter compared to the input data used. Still, the reduced-form approach outperforms the structural for investment-grade names and longer maturities. In contrast the structural approach performs better for shorter maturities and sub-investment grade names.

JEL Classification: G13

Keywords: Credit risk, structural models, reduced-form models, default intensity, stationary leverage, credit default swaps.

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We thank Michael Brennan, Wolfgang Bühler, David Lando, Markus Konz, Natalie Packham, participants of the *International conference on price, liquidity, and credit risks* 2008 Konstanz, the *Campus for Finance* 2009 Vallendar and seminar participants at Aarhus School of Business for helpful comments and suggestions. Financial support by the Deutsche Forschungsgemeinschaft (DFG) through the Graduate School IME *Information Management and Market Engineering* is gratefully acknowledged. The views expressed herein are our own and do not necessarily reflect those of the Bundesbank.

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1. Introduction

The empirical literature on defaultable claim valuation is a fast spreading research field. Structural and reduced-form models have been tested for different markets including corporate bonds and credit default swaps. The empirical performance of structural models is rather poor up to now. While early studies conclude that models consistently underpredict spreads (Jones, Mason, and Rosenfeld (1984), Ogden (1987), Lyden and Saraniti (2000)), both under- and overprediction with large pricing errors are found in later empirical approaches (Eom, Helwege, and Huang (2004)). Tests of the reduced-form models appear to be more successful (Duffee (1999), Driessen (2005), and Bakshi, Madan, and Zhang (2006)). Empirical studies of credit derivatives typically focus on reduced-form models (Houweling and Vorst (2005), Longstaff, Mithal, and Neis (2005), Chen, Cheng, Fabozzi, and Liu (2008)).¹ This is not very surprising, since these models are perceived as being flexible enough to be calibrated to arbitrary market data. Thus, the reduced-form approach seems to be ideally suited for the purpose of credit spread modeling and derivative pricing and one might be tempted to abandon the structural in favor of the reduced-form approach.

Despite the challenge in practical implementation structural models clearly stand out due to the economic insights concerning the risk structure of interest rates and its relation to fundamentals. It is encouraging that some recent empirical studies present more favorable findings on structural models. Leland (2004) shows that the models fit

¹Few recent studies make use of credit default swap prices with structural models, i.e., Chen, Fabozzi, Pan, and Sverdløve (2006), Ericsson, Reneby, and Wang (2008), and Huang and Zhou (2008).

actual default frequencies reasonably well. Schaefer and Strebulaev (2008) document quite accurate predictions of the sensitivities of debt returns to equity even for the simplest structural model (Merton (1974)).

This paper contributes to this ongoing debate by empirically comparing structural and reduced-form models of credit risk. To our knowledge, this is the first study providing a rigorous empirical test of both model classes on the same dataset. The previous literature typically relies on equity and balance sheet data when calibrating structural models while using bond data for reduced-form models. It is therefore almost impossible to assess the impact of the model type on the valuation results. In contrast, we calibrate both approaches to bond *and* equity data. To assess the quality of the models, we focus on the models' ability to explain credit default swap (CDS) prices. CDS prices lead the price discovery process, are less constrained by liquidity effects, and are a cleaner credit risk indicator than bond spreads (Blanco, Brennan, and Marsh (2005), Ericsson, Reneby, and Wang (2008)). They are therefore well suited to assess the models' ability to capture credit risk.² By using the same input data, applying comparable estimation techniques, and assessing the out-of-sample prediction quality on the same time series of CDS prices we are able to judge whether empirically the model structure itself makes an important difference between structural and reduced-form approaches.

As major difference between the approaches we focus on the discriminative modeling of the default time. Default is predictable in the structural case, but it becomes a purely random event in reduced-form models. The two approaches imply a significantly different behavior of spreads. This is most obvious for short-term spreads. They are predicted to decline to zero as the maturity goes to zero in the first case, while they remain positive also for very short maturities for the second case. This difference is not only important

²However, Nashikkar, Subrahmanyam, and Mahanti (2010) show that the CDS spreads might not fully capture the credit risk due to frictions in the arbitrage between the CDS and the bond market.

on its own but might well lead to differences in the ability to explain longer-term premia and the behavior of premia across rating classes. However, *ex ante*, differences between approaches are less clear-cut.

From each approach we choose one representative. Leverage and risk-free interest rates have found to be significant in explaining credit spreads (Ericsson, Jacobs, and Oviedo (2009), Collin-Dufresne, Goldstein, and Martin (2001)). They are modeled as state variables within both our structural and our reduced-form approach. A stationary leverage process triggers default when reaching zero in our structural model, while the same leverage process enters the default intensity of our reduced-form model.

Our study shows that both models perform quite similarly on average in out-of-sample tests. Neither approach consistently outclasses the other one. The similar average prediction power reached through a comparable empirical test design indicates that many of the differences documented in the literature so far were due to other reasons such as different input data, calibration methods, and sampling design. Our empirical study shows that once a comparable test design is applied for both frameworks, the reduced-form approach outperforms the structural approach for investment-grade names and longer maturities. In contrast the structural approach performs better for shorter maturities and sub-investment grade names.

The following sections are organized as follows: The next section introduces the valuation framework and describes the representatives chosen on the structural and the reduced-form side. In Section 3 the empirical methodology, datasets, and estimation results are given. The out-of-sample CDS predictions with both frameworks can be found in Section 4. Finally, conclusions that summarize and comment on the findings end the study.

2. Valuation Framework

A valuation model for defaultable claims consists of three components, the model for the default time, the model for the magnitude of default, and the interest rate model that characterizes the dynamics of the risk-free term structure. The fundamental difference between the structural and the reduced-form approach lies in how the models specify the timing risk of default. While structural models assume that default occurs when an exogenously modeled asset value hits some lower boundary, the reduced-form models use an exogenous intensity process to specify the default time. Default is predictable in the first case, but it becomes a purely random event in the second case.³ In fact, Jarrow and Protter (2004) argue that the crucial difference between the models comes from the information assumed known by the modeler.

In order to focus on this discriminative modeling of the default time we specify identical models for the other two components. In particular, we rely on the recovery-of-treasury assumption⁴ for both settings and use identical models for the interest rate risk. The state variables and their dynamics are chosen to be the same in both settings; namely, the short-term interest rate r and the leverage ratio l .

2.1. Common Components

More precisely, for the dynamics of the short rate, we assume a Vasicek (1977) process:

$$dr_t = \kappa_r(\theta_r - r_t)dt + \sigma_r dW_1^Q. \quad (1)$$

³See Uhrig-Homburg (2002) for a more detailed discussion of the structural differences. Duffie (2005) provides a comprehensive mathematical background for both type of models.

⁴I.e., the firm recovers a fraction of an otherwise identical default free security. Note that in the seminal work of Merton (1974) the magnitude of default is determined endogenously from the relation between the firm value and the promised payments to the debtor. In contrast, more recent structural models and most empirical implementations relax this elegant though restrictive relation stemming from the option analogy. In this sense recent structural models including the one we are focussing on feature some kind of hybrid character.

Here r_t is the risk-free interest rate, κ_r is the mean-reversion rate, θ_r is the long-run mean, σ_r is the volatility of the short rate, and W_1^Q is a Brownian motion under the risk-neutral measure. For the second state variable, we rely on the ratio of debt value K_t to asset value V_t and assume leverage ratios to be stochastic but stationary. Define log-leverage

$$l_t = \ln \frac{K_t}{V_t} \quad (2)$$

and let l_t follow a mean-reverting process of the form⁵

$$dl_t = \kappa_l(\theta_l(\cdot) - l_t)dt - \sigma_v dW_2^Q \quad (3)$$

where κ_l is the mean-reversion rate of the leverage to its long-run mean $\theta_l(\cdot)$, σ_v is a volatility parameter, and W_2^Q is a Brownian motion under the risk-neutral measure such that $dW_1^Q dW_2^Q = \rho dt$. To establish (3) assume that V_t follows a geometric Brownian motion $dV_t/V_t = (r_t - \delta)dt + \sigma_v dW_2^Q$ with payout rate δ and asset volatility σ_v . Moreover, let $\ln K_t$ follow some stationary process $\ln K_t = \kappa_l(\ln V_t - \nu - \ln K_t)dt$. Here, ν is a buffer parameter for the distance of log-asset value to log-debt value. The idea behind this mean-reversion process is that when $\ln K_t$ is less than $(\ln V_t - \nu)$ the firm acts to increase $\ln K_t$, and vice versa. Therefore, the firms adjust outstanding debt levels in response to changes in asset value such that mean-reverting leverage ratios result. From Itô's Lemma (3) results with

$$\theta_l(r_t) = \frac{\delta + \frac{\sigma_v^2}{2} - r}{\kappa_l} - \nu = -\frac{r}{\kappa_l} - \bar{\nu}. \quad (4)$$

2.2. Default-Timing in Reduced-Form and Structural Settings

To specify the default time within the *reduced-form setting* we rely on Lando's (1998) doubly stochastic valuation framework and define the default intensity to be

$$\lambda_t = a + c \cdot l_t, \quad (5)$$

⁵Collin-Dufresne and Goldstein (2001) first proposed this dynamics in the context of structural models.

with constants a and c . Obviously, here the critical choice is the selection of the state variables driving credit risk. There have been many empirical studies with reduced-form models that either estimated a stochastic process for the unobserved intensity (Duffee (1999), Driessen (2005)), or made use of a credit risk factor as part of the adjusted discount rate (Bakshi, Madan, and Zhang (2006)). Our setup accommodates the second approach, where the leverage process is defined as the credit risk factor that drives the intensity.

Within the *structural setting* the firm defaults at the first passage time τ of the log-leverage ratio l reaching zero:

$$\tau = \inf \{t : l_t \geq 0\} \tag{6}$$

This idea is in line with Black and Cox (1976) and Longstaff and Schwartz (1995) where default happens the first time when the firm value reaches an exogenously⁶ specified boundary. In fact, the resulting model is a specific version⁷ of Collin-Dufresne and Goldstein's (2001) structural model, extending the basic idea of Merton (1974) to (i) stochastic interest rates, (ii) first-passage time, and (iii) stationary leverage ratios.

2.3. Pricing Corporate Debt

Let $v^{theo}(r_t, l_t, T)$ be the time- t theoretical price of a risky discount bond that matures at T . The recovery-of-treasury assumption with recovery rate φ leads to

$$\begin{aligned} v^{theo}(r_t, l_t, T) &= E_t^Q \left(e^{-\int_t^T r_s ds} \cdot \mathbf{1}_{\{\tau > T\}} + e^{-\int_t^\tau r_s ds} \cdot \varphi \cdot b(r_\tau, T) \cdot \mathbf{1}_{\{\tau \leq T\}} \right) \\ &= E_t^Q \left(e^{-\int_t^T r_s ds} (\varphi + (1 - \varphi)\mathbf{1}_{\{\tau > T\}}) \right). \end{aligned} \tag{7}$$

⁶For models with endogenously derived default boundaries, see the empirical study of Anderson and Sundaresan (2000).

⁷In Collin-Dufresne and Goldstein's (2001) most general version the drift of the log-default threshold can be taken as a decreasing function of the spot interest rate to reflect that debt issuances drop during high interest rate periods.

Within the *reduced-form setting*, (7) further simplifies to

$$\begin{aligned} v^{theo}(r_t, l_t, T) &= \varphi \cdot b(r_t, T) + (1 - \varphi) \cdot E_t^Q \left(e^{-\int_t^T r_s ds} \mathbf{1}_{\{\tau > T\}} \right) \\ &= \varphi \cdot b(r_t, T) + (1 - \varphi) \cdot E_t^Q \left(e^{-\int_t^T r_s + \lambda_s ds} \right) \end{aligned} \quad (8)$$

where $b(r_t, T)$ is the price of a riskless bond. Let v^0 be the defaultable bond price with zero recovery as in the last part of Equation (8) in expectation brackets. Due to the affine structure⁸ the analytical solution of the expectation is of the form

$$v^0(r_t, l_t, T) = E_t^Q \left(e^{-\int_t^T R_s ds} \right) = e^{A(t, T) - B(t, T)r_t - C(t, T)l_t} \quad (9)$$

and solves the following PDE:

$$\frac{\partial v^0}{\partial t} + \kappa_r(\theta_r - r) \frac{\partial v^0}{\partial r} + \kappa_l(\theta_l(r) - l) \frac{\partial v^0}{\partial l} + \frac{1}{2} \sigma_r^2 \frac{\partial^2 v^0}{\partial r^2} + \frac{1}{2} \sigma_v^2 \frac{\partial^2 v^0}{\partial l^2} - \rho \sigma_v \sigma_r \frac{\partial^2 v^0}{\partial r \partial l} = (a + r + cl)v^0 \quad (10)$$

with boundary conditions $A(T, T) = 0$, $B(T, T) = 0$, and $C(T, T) = 0$. By taking partial derivatives of v^0 in (9) and replacing into the PDE, the closed-form solutions for $A(t, T)$, $B(t, T)$, and $C(t, T)$ can be reached. Solutions can be found in Appendix A.

Within the *structural setting* we switch from the expectation under the risk-neutral measure E^Q to the expectation under the T -forward measure E^{F_T} to further simplify (7):

$$\begin{aligned} v^{theo}(r_t, l_t, T) &= b(r_t, T) \cdot E^{F_T} \left(1 - (1 - \varphi) \cdot \mathbf{1}_{\{\tau \leq T\}} \right) \\ &= b(r_t, T) \cdot \left(1 - (1 - \varphi) \cdot Q^{F_T}(r_t, l_t, T) \right) \end{aligned} \quad (11)$$

It remains to determine $Q^{F_T}(r_t, l_t, T)$ which is the time- t probability of default occurring before maturity T under the T -forward measure⁹. In general, it is a complex task to derive expressions for the first-passage density. But fortunately, in our case we can follow the ideas of Longstaff and Schwartz (1995), Collin-Dufresne and Goldstein (2001), and

⁸See Duffie and Singleton (1999).

⁹See Geman, El-Karoui, and Rochet (1995).

Eom, Helwege, and Huang (2004) to reach the straightforward implementation given in Appendix B.

Equations (8) and (11) are used to determine coupon bond prices with the portfolio of zeros approach:

$$v_{coup}^{theo}(r_t, l_t, T, C) = \sum_{j=1}^N C \cdot v^{theo}(r_t, l_t, t_j) + v^{theo}(r_t, l_t, T) \quad (12)$$

where C is the coupon fraction with N payments on dates t_j .

3. Empirical Methodology and Estimation Results

The structural and reduced-form models are calibrated to equity and balance sheet data, corporate bond prices, and risk-free interest rates. For both of the models, the interest rate process parameters $(\kappa_r, \theta_r, \sigma_r)$ as well as the initial short rate r_0 , the leverage process parameters (κ_l, θ_l) as well as the initial leverage ratio l_0 , and the correlation between the interest rate process and the asset value process (ρ) enter similarly. Each of the models uses their theoretical bond prices $v_{coup}^{theo}(r_0, l_0, T, C)$ to estimate their unique asset volatility (σ_v) figure. In the reduced-form setup, the two additional constant parameters a and c are also estimated. After calibration of the models to this market information, CDS prices are predicted out-of-sample, without making use of any CDS information used prior. The following section introduces the datasets used in the analysis.

3.1. Data

3.1.1. CDS Data

Time series of CDS prices were retrieved from CreditTrade and Markit for the period between January 2003 and December 2005. We use mid-month observations of 1, 3, 5, 7, and 10-year senior CDSs. The mid-month value is typically on the 15th of each month. In

case that the 15th is a non-working day, the next working day is selected. For each day considered, the indicative bid and ask quotes are averaged to reach a CDS premium, in the end attaining 36 mid-month observations per issuer for the three year period. The credit quality of the issuers varies between Aa and Ba rated by Moody's. The lowest CDS midpoint of 2.4 bps is within the series of the Aa-rated WAL-MART, whereas the highest midpoint is as much as 578 bps for the Baa-rated SPRINT.

3.1.2. Corporate Bond Data

For each firm in the CDS dataset, corresponding deliverable bonds¹⁰ were retrieved from REUTERS. A typical bond in the dataset is senior unsecured and has annual, semiannual or quarterly coupon payments. The final dataset has been constructed after the removal of the bonds with the following properties:

- callable, puttable, or convertible bonds
- perpetual bonds
- index-linked bonds
- floating rate notes
- foreign currency bonds (bonds should be in the same denomination as the CDS)
- any rank else than senior unsecured bonds
- financial companies' bonds

Bonds with non-standard properties are excluded due to the necessity of including intricate techniques in bond price calculations. Financial companies are excluded due to having

¹⁰Gündüz, Lüdecke, and Uhrig-Homburg (2007) note that the smaller the set of deliverable bonds, the lower the delivery option, which reflects the extra premium for the buyer of CDS for the privilege of being able to deliver any bond in case of default. Recently, Jankowitsch, Pullirsch, and Veza (2008) have analyzed the delivery option embedded in CDS prices.

significantly different capital structures. The time span of the bonds match the CDS dataset, with monthly data from January 2003 to December 2005.

3.1.3. Balance Sheet and Stock Market Data

Leverage values are constructed by dividing the book value total liabilities to the sum of market value of equity and total liabilities. Quarterly total liabilities figures were retrieved from REUTERS balance sheet pages, while market value of equity (MVE) is the product of number of outstanding shares times the closing stock price on a given day. MVE figures can be retrieved daily, whereas total liabilities figures are available only quarterly. In order to avoid loss of data, a method similar to Eom, Helwege, and Huang (2004) has been used. For the mid-month dates where bond and CDS data are available, the leverage ratios are computed by making use of the latest available quarterly liabilities figure from balance sheets. For a consecutive three month period after the quarterly announcement, the leverage ratio is constructed from a constant liabilities figure and an MVE figure unique for the day.

In Table 1, the descriptive statistics of the 30 firms in the sample can be found with their leverage ratios, total liabilities, and market capitalizations. For 24 of these firms, CDSs are denominated in USD (US sample); the remaining 6 firms have EUR-denominated CDS contracts (European sample). A description of the complete list of the 86 bonds used in the study with details such as the issue date, maturity date, and coupon amount is placed in Appendix C.

3.1.4. Interest Rate Data

The 3-, 6-monthly, and 1, 2, 3, 5, 7, 10-yearly yields are retrieved for the interest rate calibration process. For the US sample, Constant Maturity Treasuries from the Federal Reserve Board have been used. A detailed explanation of how these series are constructed

Table 1: Descriptive Statistics for the Leverage Ratios

Firm	Avg. Lev. Ratio(%)	Max. Lev. Ratio(%)	Min. Lev. Ratio(%)	Avg. Total Liab. (Mil.USD)	Avg. Market Capital. (Mil.USD)
AKZO NOBEL	53.88	66.41	45.66	9,743	8,421
CARNIVAL	27.62	35.21	21.63	10,218	26,855
CATERPILLAR	65.15	76.01	44.10	27,640	16,202
CITIZENS	61.01	71.84	53.12	6,005	3,840
CVS	38.47	49.95	21.98	5,562	10,014
DEERE	61.00	69.78	54.80	22,847	14,758
DELL	13.86	20.28	11.41	13,861	86,054
E. KODAK	58.41	65.70	50.83	11,235	8,035
ENEL	53.34	60.32	42.91	42,592	37,508
FEDERATED	66.88	78.55	48.22	9,434	4,728
HP	36.87	42.87	31.25	36,703	63,534
HILTON	47.08	60.37	37.85	5,960	6,880
IBM	4.00	4.62	3.34	5,960	144,175
INT. PAPER	57.47	61.81	52.67	24,479	18,173
KPN	52.35	58.16	48.17	17,526	15,918
LOCKHEED	49.47	54.74	45.12	19,375	19,891
MARRIOTT	50.06	59.59	42.43	4,606	4,659
MOTOROLA	35.05	51.87	22.77	18,515	36,940
NORDSTROM	50.65	71.36	21.87	2,865	3,477
NORFOLK	58.40	65.44	47.88	14,873	10,941
NORTHROP	63.10	85.62	49.84	18,390	11,151
PHILIPS	38.62	49.31	33.21	16,617	26,712
SPRINT	55.89	75.33	27.55	30,596	27,820
STORA ENSO	56.83	63.05	51.96	9,368	7,094
TARGET	33.47	42.77	26.14	19,491	39,581
TELECOM IT.	70.16	78.80	63.15	61,438	26,312
TIME WARNER	48.87	57.97	44.39	57,505	60,349
VERIZON	57.24	59.97	53.50	106,088	79,446
WAL-MART	22.53	29.32	17.5	64,975	224,869
WALT DISNEY	36.40	44.89	31.58	27,000	47,812

can be found on the web page of the Federal Reserve Board. For the European sample, the daily estimates of the Svensson (1994) model are used. Deutsche Bundesbank has estimated these parameters from government bonds and they can be found in the Bundesbank homepage. The time span used to calibrate the model is 1998 to 2005 with daily frequency.

3.2. Estimation of the Parameters

3.2.1. Interest Rate Process Parameters

In a first step, we estimate the parameters of the interest-rate process from government yields by means of Kalman filtering. This allows making use of cross-sectional and time series information at the same time.¹¹ The method results in time series of the short rate r_t , plus the Vasicek process parameters κ_r , θ_r , σ_r , and the market price of interest rate risk parameter η . In Table 2, the estimated values for the risk-neutral parameters can be found for US and European interest rates.

Table 2: Kalman Filter Estimates of the Interest Rate Process

Parameter	US	Euro
κ_r	0.247	0.171
θ_r	0.061	0.064
σ_r	0.012	0.021
η	-0.205	-0.162

The risk-neutral (under Q) and physical (under P) processes of the short rate are:
 $dr = \kappa_r(\theta_r - r)dt + \sigma_r dW^Q$ and $dr = \kappa_r(\tilde{\theta}_r - r)dt + \sigma_r dW^P$ where $\theta_r = \tilde{\theta}_r - \frac{\sigma_r \eta}{\kappa_r}$

The mean reversion rates are in accordance with the values previously found in the literature (Babbs and Nowman (1999), Duan and Simonato (1999)). The same is true for the volatility parameter. The risk-neutral long-run mean is relatively high at 6.1 per cent

¹¹See the studies of Duan and Simonato (1999), Geyer and Pichler (1999), and Babbs and Nowman (1999) for examples.

for US and 6.4 per cent for Euro. This converts to a physical mean of 5.1 (US) and 4.4 (Euro) per cent. The overall fit to actual yields lies between a mean absolute error of 28 bps and 121 bps for different maturities. These figures are higher than those of Duffee (1999) who has made use of Kalman filter for a two-factor square root process. However, note that these errors affect both models in a similar way and thus are not expected to cause a significant distortion in relative terms.

3.2.2. Leverage Process Parameters and the Correlation Coefficient

Next, we determine the parameters describing the second state variable common to both approaches, the log-leverage. For its firm-specific parameters $(\kappa_l, \theta_l)^{12}$ the approach of Eom, Helwege, and Huang (2004) is followed (pp. 540-541). The starting point is the dynamics

$$d\ln(V_t/K_t) = [\mu_v + \kappa_l \bar{\nu} - \kappa_l (\ln(V_t/K_t))]dt + \sigma_v dW^P \quad (13)$$

of the (negative) log-leverage under the physical measure P , with a constant expected asset return μ_v and the other constant parameters defined earlier. The constant $\mu_v + \kappa_l \bar{\nu} \equiv \alpha_l$ and κ_l are estimated via a regression of the change in the log-leverage ratio against log-leverage ratio lagged one period. With an estimate of the mean return μ_v , also an estimate for $\bar{\nu}$ is obtained easily. In the implementation, the mean return is estimated from the mean return of the asset value for the period 2001-2005 in monthly intervals, and monthly market leverage ratios are regressed on one month lagged ratios for the same period. Moreover, the correlation between asset returns and the interest rate process is estimated from correlation between daily equity returns and changes in the 3-monthly interest rates, for the same time interval.

The parameter estimates common in both models can be found in Table 3. First, the

¹²In order to simplify notation, the firm-specific index i was not indicated, which otherwise should have been written as κ_l^i, θ_l^i where $i = 1, \dots, 30$.

mean-reversion rate of the leverage κ_l has a value around 3-20 per cent, although very low figures as well as higher figures are also estimated from regressions. These values fall in a consistent range with prior studies: To Fama and French (2002) who reach a value around 7-10 per cent in their regression analysis and to Shyam-Sunder and Myers (1999) who have a sample weighted towards large and financially conservative firms and reach a value around 40 per cent. The correlation coefficient between the stock returns and change in interest rates has a positive value except few firms, mostly lying in a range between 0 and 15 per cent. This is comparable to the figure of Eom, Helwege, and Huang (2004) who report that they have relatively low correlation values all below 15 per cent. They also note that the correlation variable has not been found to effect the spreads significantly.

3.2.3. Asset Volatility and Reduced-Form Model Specific Parameters

In the final step, we determine the model-specific parameters, the asset volatility σ_v and intensity parameters a and c for each model from the prices of corporate bonds. By minimizing the sum of squared errors over each observation day and each bond price, one can reach the bond-implied asset volatility for the structural model on a firm basis:

$$\min_{\sigma_v} \sum_{i=1}^{ObsDays} \sum_{j=1}^{Bonds} (v_{i,j}^{theo}(r_t, l_t) - v_{i,j}^{obs})^2. \quad (14)$$

On the reduced-form side, we simultaneously estimate the adjusted short-rate parameters a and c and the volatility parameter:

$$\min_{\sigma_v, a, c} \sum_{i=1}^{ObsDays} \sum_{j=1}^{Bonds} (v_{i,j}^{theo}(r_t, l_t) - v_{i,j}^{obs})^2. \quad (15)$$

Note that the number of free parameters used to calibrate the models to bond prices differs across approaches. In the structural model asset volatility is the only free parameter, whereas in the intensity case there are three parameters. The results will be

Table 3: Parameters Common to Both Models

Firm	κ_l	$\bar{\nu}$	ρ
AKZO NOBEL	0.070	0.716	-0.043
CARNIVAL	0.231	1.282	0.091
CATERPILLAR	-0.004	1.201	0.037
CITIZENS	0.031	0.727	0.003
CVS	0.102	0.847	0.061
DEERE	0.136	0.438	0.078
DELL	0.009	0.082	0.070
E. KODAK	0.105	0.555	0.045
ENEL	0.152	0.682	0.023
FEDERATED	0.061	0.216	0.133
HP	0.279	0.972	0.090
HILTON	0.105	0.567	0.176
IBM	0.052	2.795	0.049
INT. PAPER	0.197	0.595	0.119
KPN	0.037	0.828	0.005
LOCKHEED	0.118	0.732	0.088
MARRIOTT	0.159	0.693	0.108
MOTOROLA	0.030	1.024	0.048
NORDSTROM	0.002	-2.398	0.116
NORFOLK	0.030	0.432	0.094
NORTHROP	0.056	0.012	0.076
PHILIPS	0.154	1.002	-0.032
SPRINT	0.060	0.314	0.041
STORA ENSO	0.198	0.580	0.009
TARGET	0.083	1.022	0.154
T.COM ITALIA	0.127	0.323	-0.003
TIME WARNER	0.191	0.728	0.070
VERIZON	0.054	0.644	0.025
WAL-MART	0.027	1.053	0.120
WALT DISNEY	0.078	1.011	0.151

analyzed taking into account the number of free parameters.¹³

In the empirical implementation, we have mostly semiannual coupon payments for the bonds in the dataset. Since future coupon payments are of low priority and are rarely recovered in default (see Helwege and Turner (1999)), we set the recovery rate on coupons to 0, letting only the principal payment receive compensation at default. The recovery rate on principal is fixed at 0.5, following the results produced by Altman and Kishore (1996) and recent practice.

Tables D.1 and D.2 in Appendix D provide the estimation results. For the structural model, estimated asset volatilities are mostly between 15-40 per cent except a few outliers. A comparison to option-implied volatilities computed from at-the-money call options with a maturity of June 2007 reveals that the majority of option-implied volatilities is also in this range, indicating that the bond-implied figures are economically reasonable as well. Nevertheless, there are significant outliers such as the values for IBM and DELL. Within the reduced-form model, the a and c figures convert mostly into reasonable values for the default intensities. The next section will present a more detailed analysis of how these figures transfer into default probabilities. Apart from a few outliers, long-run leverage ratios are between 0 and 1 in both models. For most companies, their value falls close to the range of the original input leverage parameters.

To assess the estimation results, the in-sample fit to bond prices can be found in Table 4. In the table, the mean error (ME), the mean absolute error (MAE), and the mean absolute percentage errors (MAPE) are computed. The results indicate that there is a good fit to bond prices with rather low error figures. A better fit is observed with the intensity model, also indicated by the significance test. However, note that the intensity

¹³Although not documented, an alternative version of the intensity model has also been tested in our runs. This model estimated the a and c parameters common to all firms, instead of individual estimation. The out-of-sample prediction results were inferior to both the firm-specific intensity setup and the structural model.

model has three free parameters in estimation whereas the structural model has only one. Also after considering the free parameters, the Akaike Information Criterion values in the lower panel show that the intensity model has a better (lower) value, and thus a better fit. Further analysis in Section 4 will show whether the better in-sample fit to bond prices carries over to an out-of-sample fit to CDS prices.

3.3. Implied Default Probabilities from Bond Prices

Before predicting CDS prices, it might be insightful to compute the default probabilities indicated by the parameter estimates. With the structural model, we compute the 5-year forward risk-neutral probability of default $Q^{Fr}(r_0, l_0, T)$ mentioned in Equation (11) and compare it with the actual default probabilities for the same rating class reported by Moody's (corresponding to a period of 1970-2003). The default probability PD in the intensity setting is:

$$PD = 1 - E^Q \left(e^{-\int_0^T \lambda_s ds} \right) \quad (16)$$

Afterwards, the risk-neutral probability is converted into the forward probability. Model-implied 5-year default probabilities are the average values of the full observation period (36 mid-month observations).

For both models, the default probability figures are close to each other. The intensity model has generated on average around 1% higher default probabilities. Still, in more than one third of the cases the structural model's 5-year implied default probabilities are higher. Actually, the model-implied default probabilities draw a clear picture. Although not strictly monotonous, the higher the actual probability of default, the higher is the model-implied probability. The implied default probabilities averaged across companies with respect to rating classes in Table 5 confirm this observation. The applicable rating class is taken as the rating at the beginning of the observation period (January 2003). Detailed results for each firm are given in Table D.3 in Appendix D. Note also that the

Table 4: Structural and Intensity Models - In-Sample Fit to Bond Prices

Firm	Bonds		Structural			Intensity		
	No. of Bonds	No. of Prices	ME (pts)	MAE (pts)	MAPE (%)	ME (pts)	MAE (pts)	MAPE (%)
AKZO NOBEL	3	103	0.70	2.21	2.10%	0.01	1.65	1.57%
CARNIVAL	2	55	0.02	1.77	1.68%	-0.36	1.80	1.72%
CATERPILLAR	2	72	1.39	2.56	2.30%	0.30	1.54	1.39%
CITIZENS	3	86	1.89	7.94	7.23%	0.06	3.75	3.41%
CVS	9	171	1.13	1.77	1.76%	0.44	1.04	1.03%
DEERE	1	36	0.35	3.48	3.00%	-0.24	3.04	2.63%
DELL	1	36	0.05	1.13	1.04%	-0.02	1.01	0.92%
E. KODAK	2	63	0.00	2.07	2.00%	-0.31	1.79	1.72%
ENEL	1	36	0.27	1.37	1.31%	0.00	1.22	1.18%
FEDERATED	4	144	1.12	3.96	3.47%	-0.06	1.94	1.69%
HP	2	66	0.26	1.02	0.99%	0.00	0.82	0.79%
HILTON	5	180	-0.37	4.22	3.98%	0.01	2.00	1.88%
IBM	10	331	0.23	1.19	1.17%	-0.01	1.03	1.01%
INT. PAPER	1	36	-0.05	2.51	2.28%	-0.21	2.32	2.10%
KPN	1	36	1.66	2.89	2.71%	0.49	2.15	2.01%
LOCKHEED	1	36	0.13	1.64	1.37%	-0.11	1.52	1.27%
MARRIOTT	4	115	0.38	1.62	1.48%	0.00	1.26	1.14%
MOTOROLA	2	72	0.47	2.57	2.23%	-0.08	2.52	2.21%
NORDSTROM	1	36	2.45	3.38	3.21%	-0.02	1.13	1.08%
NORFOLK	3	108	0.54	2.00	1.76%	-0.06	1.46	1.28%
NORTHROP	3	89	2.66	3.50	3.13%	0.20	1.22	1.10%
PHILIPS	2	72	0.35	2.06	1.87%	-0.02	2.10	1.91%
SPRINT	4	144	1.76	3.96	3.60%	0.09	2.80	2.62%
STORA ENSO	1	36	0.09	1.57	1.47%	-0.25	1.63	1.53%
TARGET	3	104	0.64	2.46	2.44%	0.12	1.48	1.46%
T.COM ITALIA	2	71	0.99	2.50	2.33%	0.21	1.74	1.61%
TIME WARNER	4	144	0.00	2.26	2.12%	-0.31	2.18	2.04%
VERIZON	5	180	0.08	2.08	1.85%	-0.08	1.95	1.73%
WAL-MART	1	36	0.05	1.75	1.55%	-0.04	1.25	1.10%
WALT DISNEY	3	108	-0.07	2.21	2.06%	-0.09	1.93	1.79%
Average			0.60	2.54	2.35%	0.01	1.74	1.61%
AIC			196.96			142.47		
Significance Test	Mean Difference		t-statistic			p-value		
Str. - Int.	0.80		35.72			0.000		

“Mean Error (ME)” is the difference between the model and the observed bond price.

“Mean Absolute Error (MAE)” is the absolute value of the difference between the model and the observed bond price.

“Mean Absolute Error (MAPE)” is the percentage value of the division of MAE by the observed bond price.

“AIC” is the Akaike Information Criterion calculated from $2k + n \ln(RSS/n)$ where k is the number of free parameters for the model, n is the number of observations, and RSS is the residual sum of squares.

“Significance Test, Structural - Intensity” is the significance test between the difference of the structural model mean absolute errors and the intensity model mean absolute errors per firm per day, after correcting for autocorrelation and heteroscedasticity with Beck and Katz (1995) approach.

risk-neutral probabilities are always higher than real world probabilities. Although this general relation is in line with theory and other empirical findings, the high values for the risk-neutral probabilities are conspicuous, in particular for the better-rated firms. The standard explanation for this is that besides risk premia compensating for default risk, bond prices also contain other components such as liquidity premia which then lead to higher model-implied default probabilities for both the structural and the reduced-form approach. Another explanation is that standard asset pricing models fail to capture a strong covariation between the pricing kernel on the one hand and the default time and loss rate on the other hand.¹⁴ Moreover, for the above comparison we simply used historical average default frequencies, which can be seen at most as first approximation for current conditional default probabilities.¹⁵

Table 5: Model-Implied and Actual Probabilities of Default, Breakdown to Ratings

Rating (Moody's)	Structural Model-implied 5 year PD	Intensity Model-implied 5 year PD	Actual PD in Rating Class
Aa	8.18%	7.29%	0.24%
A	9.52%	9.49%	0.54%
Baa	13.27%	15.27%	2.16%
Ba	24.56%	22.29%	11.17%

4. Prediction of Credit Default Swap Prices

The final aim with both types of models is to predict the prices of CDSs out-of-sample.

After assuming zero recovery on coupon payments in parallel to the estimation phase, the

¹⁴Chen, Collin-Dufresne, and Goldstein (2009) show that a countercyclical nature of defaults, e.g. through a countercyclical default boundary, generates a better matching of historical and model-implied results. See also Hackbarth, Miao, and Morellec (2006) and Bhamra, Kühn, and Strebulaev (2010) for further macroeconomic equilibrium settings.

¹⁵For a comprehensive comparison of CDS-implied and actual default probabilities, see Berndt, Douglas, Duffie, Ferguson, and Schranz (2005).

fair price of a credit default swap (CDS) with the recovery-of-treasury assumption is:

$$CDS(T^*) = \frac{E_t^Q \left(e^{-\int_t^\tau r_s ds} (1 - \varphi \cdot e^{-\int_\tau^{T^*} r_s ds}) \cdot \mathbf{1}_{\{\tau \leq T^*\}} \right)}{E_t^Q \left(\sum_{i=1}^n e^{-\int_t^{t_i} r_s ds} \cdot \mathbf{1}_{\{\tau > t_i\}} \right)} \quad (17)$$

The denominator is the cumulation of n discount factors which are at time points t_i . The numerator gives the recovered amount in case of default prior to the CDS's maturity (T^*). The recovery leg (the numerator) has to be equal to the premium leg (the denominator) under no-arbitrage assumptions, which will yield the theoretically fair premium $CDS(T^*)$. A simulation algorithm with 2000 runs has been used in order to reach the fair premium. Details of the simulation algorithm can be found in Appendix E.

4.1. CDS Prediction Results

As it is the most common practice in the industry to agree on a 5-year CDS contract, we first concentrate on premia of CDSs with a maturity of 5 years and evaluate the out-of-sample prediction using deviations from observed premia. In Table 6, the mean errors (ME), the mean absolute errors (MAE), and the mean absolute percentage errors (MAPE) for the structural and the intensity model can be found.

The results indicate that both models have mostly underpredicted CDS premia with an average of 21 to 26 bps. For the structural model, underprediction comes not as a surprise but is well documented in the previous empirical literature on structural prediction of spreads. Underprediction is slightly lower for the majority of firms in the reduced-form case, but remarkably, pricing errors show a comparable pattern. The absolute errors for the structural and reduced-form models of 30 and 26 bps respectively are also rather similar, although at least statistically, the difference is significant. The structural model has a higher percentage error (49 per cent) than the intensity model (37 per cent). Still,

Table 6: Structural and Intensity Models - Out-of-Sample Fit to 5-Year CDS Prices

Firm	Structural			Intensity			Test Stat.
	ME (bps)	MAE (bps)	MAPE (%)	ME (bps)	MAE (bps)	MAPE (%)	Diebold- Mariano
AKZO NOBEL	-17.60	17.60	47.31%	-0.70	9.13	24.44%	4.49*
CARNIVAL	-23.99	24.31	43.06%	-4.69	22.95	52.19%	0.43
CATERPILLAR	-11.39	14.15	55.58%	-1.05	6.61	23.09%	3.48*
CITIZENS	-126.19	130.91	58.47%	-115.87	115.87	51.95%	1.83
CVS	-22.89	23.27	64.74%	-20.24	20.24	53.25%	1.50
DEERE	-2.69	7.52	26.80%	-5.95	7.77	23.75%	-0.13
DELL	17.94	19.33	113.91%	10.39	12.95	75.15%	2.46*
E. KODAK	-87.67	87.67	55.14%	-86.67	86.67	50.14%	0.29
ENEL	-14.31	14.49	53.05%	-10.26	10.99	36.77%	4.64*
FEDERATED	-11.49	15.37	32.37%	-12.41	16.93	29.25%	-0.37
HP	-4.52	8.16	22.59%	-3.98	8.65	22.67%	-0.50
HILTON	-55.98	59.91	31.51%	-84.41	84.58	50.02%	-9.54*
IBM	-3.93	6.95	25.63%	-3.49	7.36	25.58%	-0.61
INT. PAPER	-29.36	29.71	42.84%	-32.68	32.98	46.89%	-1.78
KPN	-42.67	42.67	80.92%	-6.09	17.16	27.62%	11.66*
LOCKHEED	-16.07	16.23	39.74%	-16.80	16.80	38.68%	-0.32
MARRIOTT	-13.54	14.66	25.13%	4.81	12.83	27.99%	0.60
MOTOROLA	-42.18	42.18	52.82%	-40.81	40.81	43.53%	0.47
NORDSTROM	-29.00	29.00	72.98%	-13.50	13.85	27.55%	3.65*
NORFOLK	-4.84	10.33	30.91%	-4.35	7.30	19.36%	2.27*
NORTHROP	-31.64	32.28	80.37%	-13.20	13.25	28.95%	5.32*
PHILIPS	-21.36	21.36	41.57%	-16.20	16.31	27.71%	5.69*
SPRINT	-75.96	75.96	71.38%	-70.31	70.31	49.05%	0.81
STORA ENSO	-17.75	20.06	42.67%	8.62	10.63	25.78%	4.46*
TARGET	8.94	10.01	47.38%	-3.55	7.90	29.08%	1.10
T.COM ITALIA	-42.09	42.09	60.43%	-38.06	38.06	50.66%	1.95
TIME WARNER	-39.66	40.97	37.73%	-41.97	42.77	40.90%	-1.24
VERIZON	-14.14	16.81	32.24%	-13.55	16.88	33.00%	-0.07
WAL-MART	-10.64	10.64	59.34%	3.16	5.38	35.54%	4.76*
WALT DISNEY	0.48	13.81	29.94%	0.92	17.51	39.13%	-3.07*
Average	-26.21	29.95	49.29%	-21.10	26.38	36.99%	
AIC	239.95			234.27			
Significance Test	Mean Difference		t-statistic	p-value			
Str. - Int.	3.57		11.63	0.0000			

“Mean Error (ME)” is the difference between the model and the observed CDS price.

“Mean Absolute Error (MAE)” is the absolute value of the difference between the model and the observed CDS price.

“Mean Absolute Error (MAPE)” is the percentage value of the division of MAE by the observed CDS price.

“Diebold-Mariano Test Statistic” is the significance test from Diebold and Mariano (1995) between the structural model absolute errors and the intensity model absolute errors, aiming to test whether the expected value of the difference of time series is significantly different than zero with 95% confidence (indicated by “**”).

“AIC” is the Akaike Information Criterion calculated from $2k + n \ln(RSS/n)$ where k is the number of free parameters for the model, n is the number of observations, and RSS is the residual sum of squares.

“Significance Test, Structural - Intensity” is the significance test between the difference of the structural model mean absolute errors and the intensity model mean absolute errors per firm per day, after correcting for autocorrelation and heteroscedasticity with Beck and Katz (1995) approach.

for 10 out of 30 firms, the structural model leads to lower mean absolute errors. The indifference between the two models is also seen from the 17 insignificant Diebold-Mariano test statistics in the right column. Although, this statistic indicates that the intensity has significantly lower absolute errors for other 11 firms, it should not be forgotten that the intensity model had three free parameters for fitting to bond prices while the structural model had only one. After checking the Akaike Information Criterion values, it can be observed that the figures of the two models are quite close, with the intensity model having a slightly better (lower) value.

On an aggregate level, the comparison shows that the structural and reduced-form models perform quite similarly once a comparable empirical test design is applied for both frameworks. However, one also has to recognize that pricing errors are at considerable levels for both approaches. To put these results into perspective, one should note that our implementation keeps fixed the model specification and the parameter values over time. Thus, we refrain from inconsistently re-calibrating the models to market data on every observation day in a rolling sample estimation. Our pricing errors can be compared to prior research results in at least two ways. First, the testing of the Collin-Dufresne and Goldstein (CDG) model has few examples in the literature. Among them, the studies of Eom, Helwege, and Huang (2004) (EHH) and Huang and Zhou (2008), which compare the CDG model with four other structural models, are most noteworthy. EHH make use of bond data only. They find that the CDG model suffers from an accuracy problem, where predicted bond spreads are either too small or incredibly large. As a result, they reach a percentage error of 269.78 per cent and an absolute percentage error of spread prediction of 319.31 per cent. Interestingly, in their recent follow-up study Huang and Zhou (2008) find much more support for the CDG model, this time using CDS spreads rather than bond prices. Their overall mean absolute percentage pricing error of 47 per cent is quite close to our result of 49 per cent. Secondly, the results can be compared with recent

studies that predict CDS prices using other types of structural and intensity models. For instance, Ericsson, Reneby, and Wang (2008) report mean errors in the range of 10 to 52 bps with the models of Leland (1994), Leland and Toft (1996), and Fan and Sundaresan (2000), whereas Arora, Bohn, and Zhu (2005) have reached 27 to 102 bps with the Merton (1974) model and -80 to 2 bps with the Vasicek/Kealhofer model.¹⁶ These results are well comparable with the mean error of -26 bps and mean absolute error of 30 bps for our structural model. The error figures signify that the CDS price prediction ability of our structural model is competitive with respect to other models used in the literature. On the other hand, Bakshi, Madan, and Zhang's (2006) observable credit risk factor approach in an intensity model has yielded out-of-sample absolute bond yield prediction errors in a range of 26 - 49 bps when log-leverage is selected as the factor. Our results extend Bakshi, Madan, and Zhang's (2006) results to CDS price prediction.

Previous literature documents that structural spreads are too low particularly for low-risk firms. To understand the dependence on credit quality for our setting, the errors are further analyzed by classifying to ratings. Again the applicable rating is taken as of January 2003. Table 7 shows that indeed structural models always underpredict Aa-rated CDSs with percentage errors clearly above 50%. In terms of absolute and percentage errors, the reduced-form approach does a better job and even overpredicts Aa spreads on average. For both models, the mean absolute errors naturally increase as the rating worsens. The models have difficulties especially in reaching the high CDS premia for low rated classes, where almost always underprediction is observed within both approaches. Interestingly, here it is the structural model that performs better. The significance tests indicate that the intensity model outperforms the structural model for investment grade rated CDSs, but the structural model performs better for sub-investment grades. This finding can be traced back to differences in convexity across rating classes. Our structural

¹⁶See Crosbie and Bohn (2003) and Vasicek (1984).

model produces credit spreads that are on average well below market spreads but show a curvature that is not too different from the behavior of market spreads. In contrast, the reduced-form model starts off at high levels for low-risk firms but increases more moderately with a lower convexity across rating classes.

Table 7: Structural and Intensity Models - Out-of-Sample Fit, Breakdown to Ratings

Ratings	Structural			Intensity			t-test
	ME (bps)	MAE (bps)	MAPE (%)	ME (bps)	MAE (bps)	MAPE (%)	Mean Diff. (bps)
5-Year							
Aa	-10.64	10.64	59.34%	3.16	5.38	35.54%	5.25**
A	-9.16	15.33	47.82%	-6.11	12.31	35.56%	3.02**
Baa	-38.10	40.24	50.87%	-29.90	34.61	37.34%	5.64**
Ba	-55.98	59.91	31.51%	-84.41	84.58	50.02%	-24.67**

“Mean Error (ME)” is the difference between the model and the observed CDS price.

“Mean Absolute Error (MAE)” is the absolute value of the difference between the model and the observed CDS price.

“Mean Absolute Error (MAPE)” is the percentage value of the division of MAE by the observed CDS price.

“t-test Structural-Intensity” is the significance test between the difference of the structural model mean absolute errors and the intensity model mean absolute errors per firm per day, after correcting for autocorrelation and heteroscedasticity with Beck and Katz (1995) approach.

“***” represents significance at 1% level. “**” represents significance at 5% level.

4.2. Robustness Check

We perform a range of robustness checks on our empirical design. First, our analysis so far is restricted to 5-year CDS premia only. Second, estimation and out-of sample prediction are based on the same time period.

4.2.1. Term Structure Results

It might well be that structural models can compete with reduced-form models as long as we fix some specific maturity. However, when it comes to explain the whole term structure of CDS spreads, we could guess that structural models do poorly: In contrast to reduced-form models they predict that credit spreads decline to zero as the maturity goes to zero.

Interestingly, Table 8 shows that on average the structural model underpredicts premia for all maturities while the reduced-form model overpredicts one-year premia and underpredicts premia with a longer term to maturity. The mean absolute errors indicate that for maturities below five years the structural model even outperforms the reduced-form model. In particular for the shortest maturity considered, the reduced-form model reveals large pricing errors. Here, for all but three firms, percentage errors are clearly larger than for the structural model. At the long end of the term structure the reduced-form model does better. On average absolute pricing errors are lower, and also on a firm basis, the reduced-form model outperforms the structural model in 20 (5-year CDSs), 21 (7-year CDSs), and 23 (10-year CDSs) out of 30 cases.

Table 8: Structural and Intensity Models - Out-of-Sample Fit to CDS Term Structure

Maturity	Structural			Intensity			t-test
	ME (bps)	MAE (bps)	MAPE (%)	ME (bps)	MAE (bps)	MAPE (%)	Mean Diff. (bps)
1-Year	-21.60	22.94	77.15%	7.54	25.25	184.90%	-2.31**
3-Years	-14.44	22.11	52.18%	-7.55	22.80	57.84%	-0.69**
5-Years	-26.21	29.95	49.29%	-21.10	26.38	36.99%	3.57**
7-Years	-35.61	38.27	54.27%	-30.39	32.05	38.49%	6.22**
10-Years	-50.57	51.74	65.40%	-41.28	41.63	47.24%	10.11**

“Mean Error (ME)” is the difference between the model and the observed CDS price.

“Mean Absolute Error (MAE)” is the absolute value of the difference between the model and the observed CDS price.

“Mean Absolute Error (MAPE)” is the percentage value of the division of MAE by the observed CDS price. “t-test Structural-Intensity” is the significance test between the difference of the structural model mean absolute errors and the intensity model mean absolute errors per firm per day, after correcting for autocorrelation and heteroscedasticity with Beck and Katz (1995) approach. “***” represents significance at 1% level. “**” represents significance at 5% level.

The shape of the average term structure of credit spreads differs between approaches. Similar to average market spreads structural spreads are upward sloping for short- and medium terms. However, the average term structure is much more flat in the reduced-form approach. Although one could expect from theoretical considerations that for short maturities, reduced-form models do a better job we in fact find the opposite result. Huge deviations from short-term market spreads in both directions render the reduced-form model completely out of scope. At this point, the dependence on ratings deserves further

investigations.

The breakdown to rating classes in Table 9 reveals that apart from few exceptions both models underpredict premia and underprediction almost always increases as the credit rating worsens and maturity increases. Exceptions occur only with the reduced-form model for short maturities and good rating classes. We find overprediction for the best three rating classes (Aa, A, Baa) with one-year CDSs, the best two rating classes (Aa, A) with three-year CDSs, and the best rating class (Aa) with five-year CDSs. For both models mean absolute errors are particularly high for short maturities and in percentage terms the models' inability to explain short term premia for good rating classes is shown clearly. This inability is even stronger for the reduced-form model. For longer terms to maturity it is observed from the significance tests that the intensity model outperforms the structural model for investment grade names, while the structural model performs better in pricing sub-investment grade names.

This more detailed comparison delivers several novel findings: The structural approach underpredicts premia on average in all maturity/rating buckets; in contrast, the reduced-form approach comes up with higher premia when the general spread level is low (good credit quality, short maturity). Unfortunately, within these buckets reduced-form model premia are far off from being reasonable. Along both dimensions, maturity and rating, spread curves are only gently inclined for the reduced-form model while structural spreads reveal an upward-sloping term structure of credit spreads for short- and medium-terms and a stronger convexity across ratings. This convex course is much closer to the behavior of market spreads and explains why the structural model performs better for sub-investment grade names.

Table 9: Structural and Intensity Models - Out-of-Sample Fit, Breakdown to Ratings

Ratings	Structural			Intensity			t-test
	ME (bps)	MAE (bps)	MAPE (%)	ME (bps)	MAE (bps)	MAPE (%)	Mean Diff. (bps)
1-Year							
Aa	-7.49	7.49	94.14%	12.61	13.48	252.96%	-5.99*
A	-13.50	14.13	82.25%	11.33	17.17	210.72%	-3.04
Baa	-25.98	28.03	73.57%	7.28	29.86	165.84%	-1.83
Ba	-65.55	65.77	54.22%	-41.89	62.06	84.09%	3.71
3-Year							
Aa	-2.34	8.05	71.52%	9.20	9.86	105.50%	-1.81
A	-4.48	12.29	53.48%	1.91	12.82	62.61%	-0.53
Baa	-21.37	28.57	49.80%	-12.08	28.10	51.48%	0.46
Ba	-38.48	51.19	40.68%	-68.82	73.23	46.70%	-22.04*
5-Year							
Aa	-10.64	10.64	59.34%	3.16	5.38	35.54%	5.25**
A	-9.16	15.33	47.82%	-6.11	12.31	35.56%	3.02**
Baa	-38.10	40.24	50.87%	-29.90	34.61	37.34%	5.64**
Ba	-55.98	59.91	31.51%	-84.41	84.58	50.02%	-24.67**
7-Year							
Aa	-8.94	8.99	41.29%	-1.68	4.39	20.18%	4.59**
A	-16.09	20.65	50.80%	-13.22	15.28	34.66%	5.53**
Baa	-50.42	51.32	58.11%	-41.14	42.42	41.09%	8.88**
Ba	-67.06	70.16	36.45%	-96.54	96.54	57.96%	-26.38**
10-Year							
Aa	-18.27	18.27	67.93%	-7.50	7.66	26.65%	10.62**
A	-27.98	30.44	60.56%	-22.40	22.78	44.43%	7.74**
Baa	-67.89	67.89	70.16%	-53.29	53.61	49.63%	14.24**
Ba	-84.72	84.72	43.05%	-112.67	112.67	65.66%	-27.94**

“Mean Error (ME)” is the difference between the model and the observed CDS price.

“Mean Absolute Error (MAE)” is the absolute value of the difference between the model and the observed CDS price.

“Mean Absolute Error (MAPE)” is the percentage value of the division of MAE by the observed CDS price.

“t-test” is the significance test between the difference of the structural model mean absolute errors and the intensity model mean absolute errors per firm per day, after correcting for autocorrelation and heteroscedasticity with Beck and Katz (1995) approach. “***” represents significance at 1% level. “**” represents significance at 5% level.

4.2.2. Time Out-Of-Sample Analysis

In a final step, we look at whether significant differences in approaches are revealed in a time out-of-sample analysis. In order to check this, the estimation results from the full observation period were used to compute the theoretical CDS prices of mid-month January 2006. By doing this, it is ensured that the out-of-sample analysis does not include the time horizon of estimation.

Table 10 shows the mean errors, mean absolute errors, and mean absolute percentage errors for this point in time. It is observed that the prediction power deteriorates - an expected outcome with a time out-of-sample analysis. Nonetheless, changes in the overall pattern of pricing errors are small. The structural model still outperforms the reduced-form model for short maturities (one-year). For three-year CDS contracts differences in prediction errors are insignificant. Again, at the long-end of the term structure, reduced-form models perform better. Thus once again, the results show that the two models do not consistently outperform one another: for shorter horizons the structural model is better, whereas for longer maturities the intensity model outperforms.

Table 10: Structural and Intensity Models - Time Out-of-Sample Fit (January 2006)

Maturity	Structural			Intensity			t-test
	ME (bps)	MAE (bps)	MAPE (%)	ME (bps)	MAE (bps)	MAPE (%)	Mean Diff. (bps)
1-Year	-7.09	9.10	82.51%	24.77	24.77	343.60%	-15.67**
3-Year	-10.49	20.19	76.54%	2.65	19.31	85.48%	0.87
5-Year	-29.05	35.66	72.39%	-18.37	26.30	44.15%	9.36**
7-Year	-41.97	46.71	70.50%	-34.40	35.21	43.00%	11.50**
10-Year	-60.10	63.27	79.23%	-48.14	48.14	54.04%	15.13**

“Mean Error (ME)” is the difference between the model and the observed CDS price.

“Mean Absolute Error (MAE)” is the absolute value of the difference between the model and the observed CDS price.

“Mean Absolute Error (MAPE)” is the percentage value of the division of MAE by the observed CDS price.

“t-test” is the significance test between the difference of the structural model mean absolute errors and the intensity model mean absolute errors per firm per day, after correcting for autocorrelation and heteroscedasticity with Beck and Katz (1995) approach. “***” represents significance at 1% level. “**” represents significance at 5% level.

5. Conclusions

This study has provided a comparison of the two major credit risk frameworks, the structural and the reduced-form approach. On the one hand, we assessed a structural models' ability to explain CDS prices by using a stationary leverage model calibrated to bond, stock, and balance sheet information. On the other hand, we examined a comparable reduced-form model with the leverage process as the state variable calibrated to the same data. The results show that the models' overall out-of-sample prediction performance is quite close on average in out-of-sample tests. Both models mostly underpredict spreads (with the exception of short-term CDSs in good rating classes within the reduced-form model) and underprediction typically increases as credit-rating worsens and maturity increases. As a consequence, we can not conclude that the reduced-form approach is superior to the structural approach for pricing CDS. Rather, the study shows that for pricing purposes, the discriminative modeling of the default time, i.e. the modeling type, does not greatly matter on an aggregate level compared to the input data used. Still, the reduced-form approach outperforms the structural for investment-grade names and longer maturities. In contrast the structural approach performs better for shorter maturities and sub-investment grade names.

In the light of the information based perspective by Jarrow and Protter (2004) this result does not come at a surprise. The authors argue that the crucial difference between the approaches comes from the information set available by the modeler: structural models rely on the complete knowledge of very detailed information typically held by firm's insiders and reduced form models rely on less detailed information as it is typically observed by the market. Given that our empirical implementations of both approaches rely on exactly the same market information, a similar performance is to be expected.

Blanco, Brennan, and Marsh (2005) and Hull, Predescu, and White (2004) have

shown that the no-arbitrage equality between CDS premia and bond spreads may not perfectly hold. This may be partly due to liquidity premia in bond prices. Recent studies such as e.g. Longstaff, Mithal, and Neis (2005) have investigated the bond and CDS price differences including a liquidity premium in bond prices. In our analysis liquidity differences are not explicitly taken into account. Rather, we ignore the presence of non-default components in both bonds and CDS spreads. It remains for future research to check whether extensions with liquidity yield a better performance of the models on an absolute level. Also empirical tests of model extensions to a macroeconomic equilibrium setting such as Hackbarth, Miao, and Morellec (2006), Bhamra, Kühn, and Strebulaev (2010), and Chen, Collin-Dufresne, and Goldstein (2009) could be a fruitful direction. It is a task for further research to maintain better accuracy of predictions, to compare model performance within stress scenarios such as the recent financial crises, and to find the best performing structural and reduced-form models.

References

- Altman, E. I., and V. M. Kishore, 1996, Almost Everything You Wanted to Know About Recoveries on Defaulted Bonds, *Financial Analysts Journal* 52, 57–64.
- Anderson, R., and S. Sundaresan, 2000, A Comparative Study of Structural Models of Corporate Bond Yields: An Exploratory Investigation, *Journal of Banking & Finance* 24, 255–269.
- Arora, N., J. R. Bohn, and F. Zhu, 2005, Reduced Form vs. Structural Models of Credit Risk: A Case Study of Three Models, *Journal of Investment Management* 3, 43–67.
- Babbs, S. H., and K. B. Nowman, 1999, Kalman Filtering of Generalized Vasicek Term Structure Models, *Journal of Financial and Quantitative Analysis* 34, 115–130.
- Bakshi, G., D. Madan, and F. X. Zhang, 2006, Investigating the Role of Systematic and Firm-Specific Factors in Default Risk: Lessons from Empirically Evaluating Credit Risk Models, *Journal of Business* 79, 1955–1987.
- Beck, N., and J.N. Katz, 1995, What to Do (and Not to Do) with Time-Series Cross-Section Data, *American Political Sciences Review* 89, 634–647.
- Berndt, A., R. Douglas, D. Duffie, M. Ferguson, and D. Schranz, 2005, Measuring Default Risk Premia from Default Swap Rates and EDFs, Working Paper, Bank for International Settlements, No. 173.
- Bhamra, H. S., L. Kühn, and I. A. Strebulaev, 2010, The Aggregate Dynamics of Capital Structure and Macroeconomic Risk, *Review of Financial Studies* 23, 4187–4241.
- Black, F., and J. C. Cox, 1976, Valuing Corporate Securities: Some Effects on Bond Indenture Provisions, *Journal of Finance* 31, 351–368.
- Blanco, R., S. Brennan, and I. W. Marsh, 2005, An Empirical Analysis of the Dynamic Relationship Between Investment-Grade Bonds and Credit Default Swaps, *Journal of Finance* 60, 2255–2281.
- Chen, L., P. Collin-Dufresne, and R. S. Goldstein, 2009, On the Relation Between the Credit Spread Puzzle and the Equity Premium Puzzle, *Review of Financial Studies* 22, 3367–3409.

- Chen, R. R., X. Cheng, F. J. Fabozzi, and B. Liu, 2008, An Explicit, Multi-Factor Credit Default Swap Pricing Model with Correlated Factors, *Journal of Financial and Quantitative Analysis* 43, 123–160.
- Chen, R. R., F. J. Fabozzi, G. G. Pan, and R. Sverdlove, 2006, Sources of Credit Risk: Evidence from Credit Default Swaps, *Journal of Fixed Income* 16, 7–21.
- Collin-Dufresne, P., and R. S. Goldstein, 2001, Do Credit Spreads Reflect Stationary Leverage Ratios?, *Journal of Finance* 56, 1929–1957.
- Collin-Dufresne, P., R. S. Goldstein, and J. S. Martin, 2001, The Determinants of Credit Spread Changes, *Journal of Finance* 56, 2177–2208.
- Crosbie, P., and J. Bohn, 2003, Modeling Default Risk, White Paper, Moody’s KMV.
- Diebold, F. X., and R. S. Mariano, 1995, Comparing Predictive Accuracy, *Journal of Business & Economic Statistics* 13, 253–263.
- Driessen, J., 2005, Is Default Event Risk Priced in Corporate Bonds?, *Review of Financial Studies* 18, 165–195.
- Duan, J. C., and J. G. Simonato, 1999, Estimating and Testing Exponential-Affine Term Structure Models by Kalman Filter, *Review of Quantitative Finance and Accounting* 13, 111–135.
- Duffee, G. R., 1999, Estimating the Price of Default Risk, *Review of Financial Studies* 12, 197–226.
- Duffie, D., 2005, Credit Risk Modeling with Affine Processes, *Journal of Banking & Finance* 29, 2751–2802.
- Duffie, D., and K. J. Singleton, 1999, Modeling Term Structures of Defaultable Bonds, *Review of Financial Studies* 12, 687–720.
- Eom, Y. H., J. Helwege, and J. Huang, 2004, Structural Models of Corporate Bond Pricing: An Empirical Analysis, *Review of Financial Studies* 17, 499–544.
- Ericsson, J., K. Jacobs, and R. Oviedo, 2009, The Determinants of Credit Default Swap Premia, *Journal of Financial and Quantitative Analysis* 44, 109–132.
- Ericsson, J., J. Reneby, and H. Wang, 2008, Can Structural Models Price Default Risk? Evidence from Bond and Credit Derivative Markets, Working Paper, McGill University.

- Fama, E. F., and K. R. French, 2002, Testing Trade-Off and Pecking Order Predictions about Dividends and Debt, *Review of Financial Studies* 15, 1–33.
- Fan, H., and S. Sundaresan, 2000, Debt Valuation, Renegotiations and Optimal Dividend Policy, *Review of Financial Studies* 13, 1057–1099.
- Geman, H., N. El-Karoui, and J. Rochet, 1995, Change of Numeraire, Changes of Probability Measures and Pricing of Options, *Journal of Applied Probability* 32, 443–458.
- Geyer, A. L., and S. Pichler, 1999, A State Space Approach to Estimate and Test Multifactor Cox-Ingersoll-Ross Models of the Term Structure, *Journal of Financial Research* 22, 107–130.
- Gündüz, Y., T. Lüdecke, and M. Uhrig-Homburg, 2007, Trading Credit Default Swaps via Interdealer Brokers, *Journal of Financial Services Research* 32, 141–159.
- Hackbarth, D., J. Miao, and E. Morellec, 2006, Capital Structure, Credit Risk, and Macroeconomic Conditions, *Journal of Financial Economics* 82, 519–550.
- Helwege, J., and C. M. Turner, 1999, The Slope of the Credit Yield Curve for Speculative-Grade Issuers, *Journal of Finance* 54, 1869–1884.
- Houweling, P., and T. Vorst, 2005, Pricing Default Swaps: Empirical Evidence, *Journal of International Money and Finance* 24, 1200–1225.
- Huang, J., and H. Zhou, 2008, Specification Analysis of Structural Credit Risk Models, Working Paper, AFA 2009 San Francisco Meetings.
- Hull, J.C., M. Predescu, and A. White, 2004, The Relation Between Credit Default Swap Spreads, Bond Yields, and Credit Rating Announcements, *Journal of Banking & Finance* 28, 2789–2811.
- Jankowitsch, R., R. Pullirsch, and T. Veza, 2008, The Delivery Option in Credit Default Swaps, *Journal of Banking & Finance* 32, 1269–1285.
- Jarrow, R. A., and P. Protter, 2004, Structural versus Reduced Form Models: A New Information Based Perspective, *Journal of Investment Management* 2, 1–10.
- Jones, E. P., S. P. Mason, and E. Rosenfeld, 1984, Contingent Claims Analysis of Corporate Capital Structures: An Empirical Investigation, *Journal of Finance* 39, 611–625.
- Lando, D., 1998, Cox Processes and Credit Risky Securities, *Review of Derivatives Research* 2.

- Leland, H. E., 1994, Corporate Debt Value, Bond Covenants, and Optimal Capital Structure, *Journal of Finance* 49, 1213–1252.
- Leland, H. E., 2004, Predictions of Default Probabilities in Structural Models of Debt, *Journal of Investment Management* 2, 5–21.
- Leland, H. E., and K. B. Toft, 1996, Optimal Capital Structure, Endogenous Bankruptcy and the Term Structure of Credit Spreads, *Journal of Finance* 51, 987–1019.
- Longstaff, F. A., S. Mithal, and E. Neis, 2005, Corporate Yield Spreads: Default Risk or Liquidity? New Evidence from the Default Swap Market, *Journal of Finance* 60, 2213–2253.
- Longstaff, F. A., and E. S. Schwartz, 1995, A Simple Approach to Valuing Risky Fixed and Floating Rate Debt, *Journal of Finance* 50, 789–819.
- Lyden, S., and D. Saraniti, 2000, An Empirical Examination of the Classical Theory of Corporate Security Valuation, Working Paper, Barclays Global Investors.
- Merton, R. C., 1974, On the Pricing of Corporate Debt: The Risk Structure of Interest Rates, *Journal of Finance* 29, 449–470.
- Nashikkar, A., M. G. Subrahmanyam, and S. Mahanti, 2010, Liquidity and Arbitrage in the Market for Credit Risk, Working Paper, Forthcoming in *Journal of Financial and Quantitative Analysis*.
- Ogden, J. P., 1987, Determinants of the Ratings and Yields on Corporate Bonds: Tests of the Contingent Claims Model, *Journal of Financial Research* 10, 329–339.
- Schaefer, S. M., and I. A. Strebulaev, 2008, Structural Models of Credit Risk are Useful: Evidence from Hedge Ratios on Corporate Bonds, *Journal of Financial Economics* 90, 1–19.
- Schönbucher, P. J., 2003, *Credit Derivatives Pricing Models: Models, Pricing and Implementation*. (John Wiley & Sons West Sussex, England).
- Shyam-Sunder, L., and S. C. Myers, 1999, Testing Static Trade-Off Against Pecking Order Models of Capital Structure, *Journal of Financial Economics* 51, 219–244.
- Svensson, L. E., 1994, Estimating and Interpreting Forward Interest Rates: Sweden 1992–1994, Working Paper, IMF, 94/114.

Uhrig-Homburg, M., 2002, Valuation of Defaultable Claims - A Survey, *Schmalenbach Business Review* 54, 24–57.

Vasicek, O., 1977, An Equilibrium Characterization of the Term Structure, *Journal of Financial Economics* 5, 177–188.

Vasicek, O., 1984, Credit Valuation, White Paper, Moody's KMV.

Appendix

A. Stochastic Intensity Model Solution

The values for $A(t, T)$, $B(t, T)$, and $C(t, T)$ can be derived as:

$$C(t, T) = \frac{c}{\kappa_l} (1 - e^{-\kappa_l(T-t)}) \quad (\text{A.1})$$

$$B(t, T) = \left[e^{-\kappa_r(T-t)} \left(\frac{c}{\kappa_l \kappa_r} + \frac{c}{\kappa_l (\kappa_l - \kappa_r)} - \frac{1}{\kappa_r} \right) \right] + \left[\frac{1}{\kappa_r} - \frac{c}{\kappa_l \kappa_r} - \frac{c e^{-\kappa_l(T-t)}}{\kappa_l (\kappa_l - \kappa_r)} \right] \quad (\text{A.2})$$

$$A(t, T) = -a(T-t) - \Xi - \Upsilon + \Gamma + \Lambda - \Pi \quad (\text{A.3})$$

with

$$\tilde{\theta}_l = \frac{\delta + \frac{\sigma_v^2}{2}}{\kappa_l} - \nu \quad (\text{A.4})$$

$$W = \frac{c}{\kappa_r \kappa_l} + \frac{c}{\kappa_l (\kappa_l - \kappa_r)} - \frac{1}{\kappa_r} \quad (\text{A.5})$$

$$Z = \frac{1}{\kappa_r} - \frac{c}{\kappa_l \kappa_r} \quad (\text{A.6})$$

$$\Xi = \tilde{\theta}_l c \left[(T-t) - \frac{1 - e^{-\kappa_l(T-t)}}{\kappa_l} \right] \quad (\text{A.7})$$

$$\Upsilon = \kappa_r \theta_r \left[W \frac{1 - e^{-\kappa_r(T-t)}}{\kappa_r} + Z(T-t) - \frac{c}{\kappa_l^2 (\kappa_l - \kappa_r)} (1 - e^{-\kappa_l(T-t)}) \right] \quad (\text{A.8})$$

$$\Gamma = \frac{\sigma_v^2 c^2}{2 \kappa_l^2} \left[(T-t) - \frac{2(1 - e^{-\kappa_l(T-t)})}{\kappa_l} + \frac{1 - e^{-2\kappa_l(T-t)}}{2\kappa_l} \right] \quad (\text{A.9})$$

$$\begin{aligned} \Lambda = & \frac{\sigma_r^2}{2} \left[\frac{W^2}{2\kappa_r} (1 - e^{-2\kappa_r(T-t)}) + \frac{2WZ}{\kappa_r} (1 - e^{-\kappa_r(T-t)}) + Z^2(T-t) \right. \\ & - \frac{2Wc(1 - e^{-(\kappa_l + \kappa_r)(T-t)})}{\kappa_l (\kappa_l - \kappa_r) (\kappa_l + \kappa_r)} - \frac{2Zc(1 - e^{-\kappa_l(T-t)})}{\kappa_l^2 (\kappa_l - \kappa_r)} \\ & \left. + \left(\frac{c}{\kappa_l (\kappa_l - \kappa_r)} \right)^2 \frac{1 - e^{-2\kappa_l(T-t)}}{2\kappa_l} \right] \quad (\text{A.10}) \end{aligned}$$

$$\begin{aligned}
\Pi = & \rho\sigma_v\sigma_r \left[\frac{Wc(1 - e^{-\kappa_r(T-t)})}{\kappa_l\kappa_r} + \frac{Zc(T-t)}{\kappa_l} - \frac{c^2(1 - e^{-\kappa_l(T-t)})}{\kappa_l^3(\kappa_l - \kappa_r)} \right. \\
& - \frac{Wc(1 - e^{-(\kappa_l+\kappa_r)(T-t)})}{\kappa_l(\kappa_l + \kappa_r)} - \frac{Zc(1 - e^{-\kappa_l(T-t)})}{\kappa_l^2} \\
& \left. + \frac{c^2(1 - e^{-2\kappa_l(T-t)})}{2\kappa_l^3(\kappa_l - \kappa_r)} \right] \tag{A.11}
\end{aligned}$$

B. Structural Model Solution

Utilizing the framework provided by Longstaff and Schwartz (1995) and Collin-Dufresne and Goldstein (2001), Eom, Helwege, and Huang (2004) arrive at the below formulation (pp. 537-539):

$$Q^{Fr}(r_0, l_0, T) = \sum_{i=1}^n q(t_{i-1/2}; t_0) \quad (\text{B.1})$$

In deriving this formula, t_0 is set equal to 0 and the time is discretized into n intervals as $t_i = iT/n$,
for $i = 1, 2, \dots, n$,

$$q(t_{i-1/2}; t_0) = \frac{N(a(t_i; t_0)) - \sum_{j=1}^{i-1} q(t_{j-1/2}; t_0)N(b(t_i; t_{j-1/2}))}{N(b(t_i; t_{i-1/2}))} \quad (\text{B.2})$$

The sum on the right hand-side of the equation becomes zero when $i = 1$. N is the cdf of Normal distribution. Values for a and b are required to compute $Q^{Fr}(r_0, l_0, T)$ in the structural model. They are given as:

$$a(t_i; t_0) = -\frac{M(t_i, T|X_0, r_0)}{\sqrt{S(t_i|X_{t_j})}} \quad (\text{B.3})$$

$$b(t_i; t_j) = -\frac{M(t_i, T|X_{t_j})}{\sqrt{S(t_i|X_{t_j})}} \quad (\text{B.4})$$

$X = V/K$ is the inverse of the leverage ratio, where M and S are

$$M(t, T|X_0, r_0) = E_0^{Fr}[\ln X_t] \quad (\text{B.5})$$

$$S(t|X_0, r_0) = \text{var}_0^{Fr}[\ln X_t] \quad (\text{B.6})$$

$$M(t, T|X_u) = M(t, T|X_0, r_0) - M(u, T|X_0, r_0) \frac{\text{cov}_0^{Fr}[\ln X_t, \ln X_u]}{S(u|X_0, r_0)}, \quad u \in (t_0, t) \quad (\text{B.7})$$

$$S(t|X_u) = S(t|X_0, r_0) - \frac{(\text{cov}_0^{Fr}[\ln X_t, \ln X_u])^2}{S(u|X_0, r_0)}, \quad u \in (t_0, t) \quad (\text{B.8})$$

What remains is to have closed form solutions for $E_0^{FT}[\ln X_t]$ and $cov_0^{FT}[\ln X_t, \ln X_u]$ which are computed in Eom/Helwege/Huang (pp. 538-539).

$$\begin{aligned}
E_0^{FT}[\ln X_t] &= e^{-\kappa_l t} \left[\ln X_0 + \bar{\nu}(e^{\kappa_l t} - 1) \right. \\
&+ \left(\frac{1}{\kappa_l - \kappa_r} (e^{(\kappa_l - \kappa_r)t} - 1) (r_0 - \theta_r + \frac{\sigma_r^2}{\kappa_r^2} - \frac{\sigma_r^2}{2\kappa_r^2} e^{-\kappa_r T}) \right. \\
&+ \left. \frac{1}{\kappa_l + \kappa_r} \frac{\sigma_r^2}{2\kappa_r^2} e^{-\kappa_r T} (e^{(\kappa_l + \kappa_r)t} - 1) + \frac{1}{\kappa_l} (\theta_r - \frac{\sigma_r^2}{\kappa_r^2}) (e^{\kappa_l t} - 1) \right) \\
&\left. - \frac{\rho\sigma_v\sigma_r}{\kappa_r} \left(\frac{e^{\kappa_l t} - 1}{\kappa_l} - e^{\kappa_r T} \frac{e^{(\kappa_l + \kappa_r)t} - 1}{\kappa_l + \kappa_r} \right) \right] \tag{B.9}
\end{aligned}$$

$$\begin{aligned}
cov_0^{FT}[\ln X_t, \ln X_u] &= e^{-\kappa_l(t+u)} \left[\frac{\sigma_v^2}{2\kappa_l} (e^{2\kappa_l u} - 1) \right. \\
&+ \frac{\rho\sigma_v\sigma_r}{\kappa_l + \kappa_r} \left(\frac{e^{2\kappa_l u} - 1}{2\kappa_l} - \frac{e^{(\kappa_l - \kappa_r)u} - 1}{\kappa_l - \kappa_r} \right) \\
&+ \frac{\rho\sigma_v\sigma_r}{\kappa_l + \kappa_r} \left(\frac{1 - e^{(\kappa_l - \kappa_r)t}}{\kappa_l - \kappa_r} + \frac{e^{2\kappa_l u} - 1}{2\kappa_l} \right. \\
&+ \left. e^{(\kappa_l + \kappa_r)u} \frac{e^{(\kappa_l - \kappa_r)t} - e^{(\kappa_l - \kappa_r)u}}{\kappa_l - \kappa_r} \right) \\
&+ \frac{\sigma_r^2}{2\kappa_r} \left(- \frac{(e^{(\kappa_l - \kappa_r)t} - 1)(e^{(\kappa_l - \kappa_r)u} - 1)}{(\kappa_l - \kappa_r)^2} \right. \\
&- \frac{\kappa_r}{\kappa_l^2 - \kappa_r^2} \frac{e^{2\kappa_l u} - 1}{\kappa_l} + (e^{(\kappa_l + \kappa_r)u} - 1) \frac{e^{(\kappa_l - \kappa_r)t} - e^{(\kappa_l - \kappa_r)u}}{\kappa_l^2 - \kappa_r^2} \\
&\left. + \frac{1}{\kappa_l^2 - \kappa_r^2} (1 - 2e^{(\kappa_l - \kappa_r)u} + e^{2\kappa_l u}) \right) \Big]
\end{aligned}$$

where

$$\bar{\nu} = (\nu - (\delta + \sigma_v^2/2)/\kappa_l) \tag{B.10}$$

From these equations one can obtain $Q^{FT}(r_0, l_0, T)$ required for pricing the bond.

C. List of Bonds Used in Analysis

Table C.1: List of Bonds Used in Analysis

Firm	Bond ID	Issue Date	Maturity Date	Coupon	No. Coupons
AKZO NOBEL	1	17.11.1998	17.11.2008	5.375	1
AKZO NOBEL	2	13.06.2003	14.06.2011	4.250	1
AKZO NOBEL	3	07.05.2002	07.05.2009	5.625	1
CARNIVAL	1	31.01.2001	01.06.2007	7.300	2
CARNIVAL	2	11.05.2004	15.11.2007	3.750	2
CATERPILLAR	1	01.05.2001	01.05.2011	6.550	2
CATERPILLAR	2	29.04.1991	15.04.2006	9.000	2
CITIZENS	1	23.05.2001	15.05.2011	9.250	2
CITIZENS	2	11.03.2002	15.08.2008	7.625	2
CITIZENS	3	12.11.2004	15.01.2013	6.250	2
CVS	1	09.07.2001	15.03.2006	5.625	2
CVS	2	04.11.2002	01.11.2007	3.875	2
CVS	3	25.03.2003	01.11.2007	3.875	2
CVS	4	14.09.2004	15.09.2009	4.000	2
CVS	5	14.09.2004	15.09.2014	4.875	2
CVS	6	22.11.2004	15.09.2009	4.000	2
CVS	7	22.11.2004	15.09.2014	4.875	2
CVS	8	14.09.2004	15.09.2009	4.000	2
CVS	9	14.09.2004	15.09.2014	4.875	2
DEERE	1	17.04.2002	25.04.2014	6.950	2
DELL	1	27.04.1998	15.04.2008	6.550	2
E. KODAK	1	26.06.2001	15.06.2006	6.375	2
E. KODAK	2	10.10.2003	15.11.2013	7.250	2
ENEL	1	13.10.1998	13.10.2008	4.500	1
FEDERATED	1	14.07.1997	15.07.2017	7.450	2
FEDERATED	2	14.06.1999	01.04.2009	6.300	2
FEDERATED	3	06.06.2000	01.06.2010	8.500	2
FEDERATED	4	27.03.2001	01.04.2011	6.625	2
HP	1	16.12.2002	17.12.2007	4.250	2
HP	2	26.06.2002	01.07.2007	5.500	2
HILTON	4	11.05.2001	15.05.2008	7.625	2
HILTON	1	15.04.1997	15.04.2007	7.950	2
HILTON	2	22.12.1997	15.12.2009	7.200	2
HILTON	3	22.12.1997	15.12.2017	7.500	2
HILTON	5	22.11.2002	01.12.2012	7.625	2
IBM	1	01.10.1998	01.10.2008	5.400	2
IBM	2	03.12.1998	01.12.2008	5.400	2
IBM	3	15.01.1999	15.01.2009	5.500	2
IBM	4	22.01.1999	22.01.2009	5.390	2
IBM	5	26.01.1999	26.01.2009	5.400	2
IBM	6	01.08.2002	15.08.2007	4.200	2
IBM	7	27.12.2002	15.12.2006	3.000	2
IBM	8	30.01.2003	15.01.2009	3.500	2
IBM	9	06.02.2003	15.02.2013	4.200	2
IBM	10	01.02.2005	01.02.2008	3.800	2

Firm	Bond ID	Issue Date	Maturity Date	Coupon	No. Coupons
INT. PAPER	1	27.08.2001	01.09.2011	6.750	2
KPN	1	12.04.2001	12.04.2006	7.250	1
LOCKHEED	1	23.11.1999	01.12.2009	8.200	2
MARRIOTT	1	16.01.2001	15.06.2008	7.000	2
MARRIOTT	2	11.01.2002	15.01.2008	7.000	2
MARRIOTT	3	20.09.1999	15.09.2009	7.875	2
MARRIOTT	4	14.06.2005	15.06.2012	4.625	2
MOTOROLA	1	14.01.2002	01.11.2011	8.000	2
MOTOROLA	2	13.11.2000	15.11.2010	7.625	2
NORDSTROM	1	20.01.1999	15.01.2009	5.625	2
NORFOLK	1	26.04.1999	15.04.2009	6.200	2
NORFOLK	2	23.05.2000	15.05.2010	8.625	2
NORFOLK	3	06.02.2001	15.02.2011	6.750	2
NORTHROP	1	01.03.1996	01.03.2006	7.000	2
NORTHROP	2	16.08.2004	16.11.2006	4.079	4
NORTHROP	3	14.04.2000	15.10.2009	8.000	2
PHILIPS	1	16.05.2001	16.05.2008	5.750	1
PHILIPS	2	16.05.2001	16.05.2011	6.125	1
SPRINT	1	25.01.2001	30.01.2006	7.125	2
SPRINT	2	06.05.1999	01.05.2009	6.375	2
SPRINT	3	25.01.2001	30.01.2011	7.625	2
SPRINT	4	21.06.2002	15.03.2012	8.375	2
STORA ENSO	1	29.06.2000	29.06.2007	6.375	1
TARGET	1	26.03.2001	01.04.2007	5.500	2
TARGET	2	10.10.2001	01.10.2008	5.400	2
TARGET	3	02.05.2003	15.05.2018	4.875	2
T.COM ITALIA	1	01.02.2002	01.02.2007	5.625	1
T.COM ITALIA	2	01.02.2002	01.02.2012	6.250	1
TIME WARNER	1	19.04.2001	15.04.2006	6.125	2
TIME WARNER	2	08.04.2002	01.05.2007	6.150	2
TIME WARNER	3	19.04.2001	15.04.2011	6.750	2
TIME WARNER	4	08.04.2002	01.05.2012	6.875	2
VERIZON	1	28.03.2000	15.03.2007	7.600	2
VERIZON	2	21.06.2002	15.06.2007	6.125	2
VERIZON	3	07.09.2001	01.12.2010	7.250	2
VERIZON	4	21.06.2002	15.06.2012	6.875	2
VERIZON	5	26.08.2002	01.09.2012	7.375	2
WAL-MART	1	10.08.1999	10.08.2009	6.875	2
WALT DISNEY	1	28.06.1999	28.06.2010	6.800	2
WALT DISNEY	2	20.06.2002	20.06.2014	6.200	2
WALT DISNEY	3	27.10.1993	27.10.2008	5.800	2

D. Estimation Results

Table D.1: Structural Model Estimation Figures

Firm	σ_v		θ_l	$\theta_{K/V}$
	Estimated	Option-implied		
AKZO NOBEL	0.202	0.246	-1.14	0.43
CARNIVAL	0.559	0.541	-1.41	0.48
CATERPILLAR	0.115	0.509	7.05	>1
CITIZENS	0.213	0.226	-1.70	0.36
CVS	0.263	0.223	-1.14	0.45
DEERE	0.177	0.370	-0.66	0.58
DELL	0.594	0.312	-3.38	>1
E. KODAK	0.255	0.828	-0.84	0.59
ENEL	0.211	0.256	-0.88	0.48
FEDERATED	0.141	0.237	-0.71	0.58
HP	0.392	0.240	-1.08	0.45
HILTON	0.345	0.292	-0.85	0.75
IBM	0.913	0.173	-3.37	>1
INT. PAPER	0.240	0.192	-0.75	0.55
KPN	0.255	0.089	-1.64	0.47
LOCKHEED	0.250	0.235	-0.99	0.49
MARRIOTT	0.279	0.456	-0.88	0.53
MOTOROLA	0.350	0.237	-2.02	>1
NORDSTROM	0.147	0.320	-13.04	<0.01
NORFOLK	0.176	0.299	-1.42	0.40
NORTHROP	0.088	0.229	-0.54	0.62
PHILIPS	0.342	0.377	-1.20	0.44
SPRINT	0.180	0.543	-0.81	0.58
STORA ENSO	0.263	0.323	-0.73	0.57
TARGET	0.365	0.230	-1.38	0.48
T.COM ITALIA	0.131	0.210	-0.56	0.61
TIME WARNER	0.303	0.262	-0.89	0.53
VERIZON	0.204	0.132	-1.20	0.44
WAL-MART	0.429	0.181	-1.13	0.41
WALT DISNEY	0.384	0.208	-1.39	0.64

The last two columns indicate the long-run mean of the log-leverage process and the leverage itself by applying Itô's lemma to the log-leverage process in Equation (3). In calculating the long-run mean of the log-leverage from $\theta_l = -\bar{\nu} - r/\kappa_l$, the short rate r is assumed at a constant 3 per cent.

Table D.2: Intensity Model Parameter Estimates

Firm	a	c	σ_v	$\theta_{K/V}$
AKZO NOBEL	0.057	0.049	0.569	>1
CARNIVAL	0.037	0.001	0.200	0.27
CATERPILLAR	0.030	0.025	0.629	<0.01
CITIZENS	0.045	0.001	0.501	>1
CVS	0.014	0.004	0.200	0.39
DEERE	0.034	0.033	0.014	0.52
DELL	0.020	0.001	0.828	>1
E. KODAK	0.113	0.112	0.002	0.43
ENEL	0.045	0.045	0.009	0.41
FEDERATED	0.043	0.043	0.211	0.71
HP	0.024	0.003	0.335	0.35
HILTON	0.165	0.158	0.001	0.43
IBM	0.022	0.002	0.112	0.03
INT. PAPER	0.024	0.001	0.987	>1
KPN	0.040	0.001	0.164	0.28
LOCKHEED	0.039	0.026	0.026	0.37
MARRIOTT	0.117	0.111	0.477	0.85
MOTOROLA	0.095	0.058	0.003	0.13
NORDSTROM	0.034	0.015	0.042	<0.01
NORFOLK	0.056	0.057	0.007	0.24
NORTHROP	0.036	0.026	0.010	0.58
PHILIPS	0.080	0.056	0.001	0.30
SPRINT	0.081	0.073	0.002	0.44
STORA ENSO	0.104	0.100	0.002	0.48
TARGET	0.055	0.035	0.001	>1
T.COM ITALIA	0.072	0.115	0.001	0.57
TIME WARNER	0.166	0.175	0.002	0.41
VERIZON	0.021	0.001	0.614	>1
WAL-MART	0.015	0.001	0.232	0.31
WALT DISNEY	0.106	0.071	0.356	0.56

The last column indicates the long-run mean of the log-leverage process and the leverage itself by applying Itô's lemma to the log-leverage process in Equation (3). In calculating the long-run mean of the log-leverage from $\theta_l = -\bar{\nu} - r/\kappa_l$, the short rate r is assumed at a constant 3 per cent.

Table D.3: Model-Implied and Actual Probabilities of Default

Firm	Structural Model-implied 5 year PD	Intensity Model-implied 5 year PD	Rating (Moody's)	Actual PD in Rating Class
AKZO NOBEL	8.49%	10.76%	A2-A3	0.54%
CARNIVAL	17.04%	16.45%	A2-A3	0.54%
CATERPILLAR	6.12%	8.83%	A2	0.54%
CITIZENS	19.37%	20.11%	Baa2-Ba3	2.16%-11.17%
CVS	4.99%	5.18%	A2-A3	0.54%
DEERE	9.46%	8.51%	A3	0.54%
DELL	11.43%	8.93%	A2-A3	0.54%
E. KODAK	22.37%	22.79%	Baa1-B1	2.16%-31.99%
ENEL	6.40%	7.48%	Aa3-A1	0.24%-0.54%
FEDERATED	12.21%	12.17%	Baa1	2.16%
HP	10.35%	9.99%	A3	0.54%
HILTON	24.79%	22.44%	Baa3-Ba1	2.16%-11.17%
IBM	8.17%	7.43%	A1	0.54%
INT. PAPER	13.04%	11.19%	Baa2	2.16%
KPN	16.76%	18.01%	Baa1-Baa3	2.16%
LOCKHEED	9.08%	9.66%	Baa2	2.16%
MARRIOTT	13.44%	15.33%	Baa2	2.16%
MOTOROLA	13.23%	14.97%	Baa2-Baa3	2.16%
NORDSTROM	5.54%	11.13%	Baa1	2.16%
NORFOLK	9.63%	11.73%	Baa1	2.16%
NORTHROP	4.32%	11.74%	Baa2-Baa3	2.16%
PHILIPS	10.46%	12.06%	A3	0.54%
SPRINT	13.42%	17.85%	Baa2-Baa3	2.16%
STORA ENSO	19.76%	20.83%	Baa1	2.16%
TARGET	10.07%	8.11%	A2	0.54%
T.COM ITALIA	10.85%	14.59%	Baa1-Baa2	2.16%
TIME WARNER	14.80%	17.81%	Baa1	2.16%
VERIZON	11.22%	10.12%	A2	0.54%
WAL-MART	8.18%	7.29%	Aa2	0.24%
WALT DISNEY	14.53%	14.45%	Baa1	2.16%
Significance Test Str. - Int.	Mean Difference -0.95%		t-statistic -13.01	p-value 0.000

“Significance Test, Structural - Intensity” is the significance test between the model-implied default probabilities of the structural and reduced-form models per firm, after correcting for autocorrelation and heteroscedasticity with Beck and Katz (1995) approach.

E. Simulation Algorithm

For the structural model, paths of the short rate and the leverage ratio are simulated where default occurred at the first time when the log-leverage is larger than zero (leverage is greater than or equal to 1). For a typical 5-year horizon of the maturity of the CDS, the simulation algorithm generates paths and at each time point the log-leverage is checked for whether it has a value higher than zero:

- (i) At first step, the short rate is simulated using an Euler discretization of the Vasicek process: Start with $r_t=r_0$, and generate r_{t+1} through

$$r_{t+1} = r_t + \kappa_r(\theta_r - r_t)\Delta t + \sigma_r\sqrt{\Delta t}\epsilon_t^1 \quad (\text{E.1})$$

where $\epsilon_t^1 \sim N(0, 1)$.

- (ii) Substitute the simulated r_{t+1} into

$$\theta_l(r_{t+1}) = -\bar{\nu} - \frac{r_{t+1}}{\kappa_l} \quad (\text{E.2})$$

- (iii) Generate l_{t+1} through Euler discretization of the leverage process:

$$l_{t+1} = l_t + \kappa_l(\theta_l - l_t)\Delta t - \sigma_v\sqrt{\Delta t}(\rho\epsilon_t^1 + \sqrt{1-\rho^2}\epsilon_t^2) \quad (\text{E.3})$$

Here, note that the Brownian motions of the two processes are correlated with a factor of ρ and $\epsilon_t^2 \sim N(0, 1)$.

- a. If $l_{t+1} < 0$ (log leverage having a negative sign) then no default occurs. The CDS premiums up to this time point are cumulated, when a quarter is complete (typical quarterly payments is assumed). This accumulation constitutes the ‘‘Premium Leg’’ of a CDS.

$$PremLeg_i = PremLeg_{i-1} + \left(e^{-\sum_0^{t_i} r_t \Delta t} \right) \quad (\text{E.4})$$

Here, t_i is the i^{th} premium date. Simulation continues with step (iv).

- b. If $l_{t+1} \geq 0$, default happens. Simulation is terminated and the recovery leg is computed to constitute the numerator of the fair price of a CDS. $\tau = t + 1$ and

$$RecLeg = \left(e^{-\sum_0^{\tau} r_t \Delta t} (1 - \varphi \cdot b(r_\tau, T - \tau)) \right) \quad (\text{E.5})$$

In addition, the accrued premium since the last premium payment is calculated and added to the premium leg. In this implementation, the recovered bond maturity (T) is taken to be the longest dated bond's maturity. According to the intuition, with no recovery on coupons, the longest available bond should be delivered in case the “cheapest-to-deliver” option is available.

(iv) Go back to step (i) to generate r_{t+2} .

For simulating the fair price of a CDS in the reduced-form case, Euler discretizations for the short rate and leverage process as in Equations (E.1) and (E.3) have been used. Following Schönbucher (2003), a uniform random variate U is generated as the trigger level. Let γ be the default countdown process, which is initiated by letting $\gamma(0) = 1$. Different from the structural model described above, step (iii) is replaced by:

(iii) Generate l_{t+1} through Euler discretization of the leverage process:

$$l_{t+1} = l_t + \kappa_l(\theta_l - l_t)\Delta t - \sigma_v\sqrt{\Delta t}(\rho\epsilon_t^1 + \sqrt{1 - \rho^2}\epsilon_t^2) \quad (\text{E.6})$$

Compute the associated default intensity as:

$$\lambda(t+1) = a + cl_{t+1} \quad (\text{E.7})$$

Then at each time step, the default countdown process is decreased by,

$$\gamma(t+1) = \gamma(t)e^{-\lambda(t+1)\Delta t} \quad (\text{E.8})$$

a. If $U < \gamma(t+1)$ then no default occurs. Similar to the structural side, the CDS premiums up to this quarter are cumulated, when a quarter is complete. This is the premium leg of the CDS.

$$PremLeg_i = PremLeg_{i-1} + \left(e^{-\sum_0^{t_i} r_{t_i}\Delta t} \right) \quad (\text{E.9})$$

b. If $U \geq \gamma(t+1)$, default happens and the recovery leg is computed.

$$RecLeg = \left(e^{-\sum_0^{\tau} r_t\Delta t} (1 - \varphi \cdot b(r_\tau, T - \tau)) \right) \quad (\text{E.10})$$

Accrued premiums are taken into account since the last premium payment date, as well.