

Investment Options with Debt Financing and Differential Beliefs

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This version Sept. 2010

JEL classification: G31, G13

Keywords: Capital Structure; Financing Constraints; Differential Beliefs; Endogenous Default; Real Options

Acknowledgements:

We are thankful for discussions and comments from participants at the Annual International Conferences on Real Options, the Annual International Conference on Macroeconomic Analysis and International Finance, Crete (2010) and the International Risk Management Conference, Venice (2009). Both authors are grateful for financial support from the HERMES European Center of Excellence on Computational Finance and Economics of the University of Cyprus.

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Abstract

A contingent claims model with differential beliefs between debt and equity holders about the asset's growth rate and volatility is used to study the impact of differential information between debt and equity holders on firm value, optimal capital structure, the timing of investment and other variables such as the credit spreads. The model explains the existence of debt constraints when debt holders have unfavorable beliefs about the volatility or the growth prospects of the firm. Within this framework, equity holders' volatility choice prior and after the exercise of the investment option is also analyzed which is affected by the stickyness of debt holders beliefs.

I. Introduction

Most studies analyzing capital structure and investment decisions assume homogenous beliefs about the growth and volatility of assets between the different claim holders. Many studies in experimental economics and behavioral finance, however, show that the firm's managers may exhibit overconfidence and optimism (see discussion in Heaton, 2002). Itzhak Ben-David et al. (2007) show empirical evidence based on survey data on the existence of overconfidence managers. They show that overconfident managers invest more and use more debt. Differences in beliefs may exist because of differential information held by different claim holders (e.g., insiders versus outsiders). Grenadier and Wang (2005) use a real options model to analyze information asymmetries between the owner and manager showing that the manager has an incentive to delay investment compared to the optimal case with no agency problems. Hackbarth (2008) (see also Hackbarth, 2009) studies managerial traits and their impact on capital structure decisions. He studies managerial optimism regarding the growth rate and volatility and its effect on the agency costs caused by asset substitution. This study analyzes the impact of debt differential beliefs between equity holders (owner-managers) and debt holders on firm value, the timing of investment, the optimal default decision and other important variables like the credit spreads. A contingent claim approach is utilized that also incorporates equity holders' choice of volatility before and after investment option exercise.

Our work is also related to the literature concerning the existence of financing constraints. Asymmetric information is considered to be one of the reasons for the

existence of debt constraints and credit rationing (see for example, Fazzari et al. 1988 and Stiglitz and Weiss, 1981). Rauh (2006) and Hubbard et al. (1995) show empirical evidence that firms face financing constraints that are attributed to possible debt and equity market frictions. Holod and Peek (2007) focus on the banking sector showing that banks with less asymmetry between internal and external investors face less financing constraints. Whited and Wu (2006) and Gomes et al. (2006) document empirically the significance of financing constraints and show that they represent a risk factor of firm returns. Our paper shows that unfavorable beliefs by debt holders make external finance costly and thus may reduce the usage of debt effectively acting as an endogenous debt constraint. We build on the Mauer and Sarkar (2005) framework which allows the simultaneous study of the impact of differential beliefs on the investment, capital structure and default decisions. Adding asymmetries in information in capital structure models is also motivated in the conclusions of Leland (1998)¹. Our results show that when the debt holders' estimate of the firm's volatility is higher or when the growth rate of assets is lower than the equity holders' estimate, optimal leverage and firm values are reduced. These unfavourable beliefs of debt holders act as an endogenous constraint on the use of debt and create adjustments in the firm's investment policy and capital structure. With differential beliefs and equity holders optimism about either the growth rate or volatility, equity holders delay the exercise of the investment option in order to obtain a higher option value of waiting since debt is overpriced. Our results are consistent with recent empirical findings of Ascioglu et. (2008) who show that firms facing higher information asymmetry will invest less and rely more on internal capital to fund investment. Our

¹ Part of Leland (1998) focuses on asset substitution, which is not investigated in our work.

results also show that equity holders in this case will also delay default. Credit spreads increase because of debt holders unfavourable beliefs that implies a higher cost of borrowing, however, for very high deviations of debt holders and equity holders beliefs credit spreads may decrease because of the low leverage used.

Finally, a model where equity holders can optimally choose assets' volatility before and after the exercise of the investment option is proposed. Under common beliefs between equity and debt holders, firm value is optimized at higher pre-investment volatility and lower post-investment volatility. Higher volatility prior to investment improves the firm's option on unlevered assets, while lower post-investment volatility increases the debt amount raised and the tax benefits of debt (net of bankruptcy costs). With differential beliefs and debt holders maintaining sticky beliefs based on the pre-investment volatility levels, the equity holders' incentive of high pre-investment volatility is reduced. Moreover, in the case of sticky debt holders beliefs, equity holders have an incentive to increase post-investment volatility, since the debt holders' stickyness on prior beliefs allows them to obtain debt financing at low cost.

II. The model

The model without differential beliefs

Following Leland (1994), firm's unlevered assets follow a Geometric Brownian Motion

$$\frac{dV}{V} = \mu dt + \sigma dZ \quad (1)$$

where μ denotes the capital gains of this asset, σ denotes its volatility and dZ is an increment of a standard Weiner process. Similar to Leland (1994), it is assumed that V is unaffected by the firm's capital structure: any coupon payments on debt are financed by new equity, leaving the value of unlevered assets unaffected. Nonetheless, a dividend-like opportunity cost of waiting to invest δ may exist, that captures competitive erosion on the value of assets (e.g., Childs and Triantis 1999, Trigeorgis 1996 ch.9, and Trigeorgis 1991). A low δ affects the (risk-neutral) drift $r - \delta$, used in the valuation, showing that a low δ effectively increases the growth rate of the value of unlevered assets (see also McDonald and Siegel, 1984).

The firm holds an investment option $F(V)$ to pay capital cost I and acquire a potentially levered position $V^L(V) = E(V) + D(V)$ where $E(V)$ and $D(V)$ denote the stochastic values of equity and debt respectively. Under the perpetual investment horizon assumption, the analytic framework of Leland (1994) and Mauer and Sarkar (2005) for the value of the firm is used. Firm value $F(V)$ coincides with the value of current equity holders. Its value derives from the option to optimally select the time (t_I) of investment, taking into consideration that it can be partially financed with debt $D(V)$. Equity holders will, thus, pay the investment costs, receive $D(V)$ (in cash) from debtholders and get a levered equity position $E(V)$ (that also includes the option to default). The money that the firm actually needs to pay (the equity financing, not to be confused with equity value)

equals $I - D(V)$. Therefore, the current equity holders have the option on $\max(E(V) - (I - D(V)), 0)$ which is equivalent to $\max(E(V) + D(V) - I, 0)$.

Equity value conditional on investment and default at V_B equals (see also Leland, 1994, and Mauer and Sarkar, 2005):

$$E(V_I) = V_I - \frac{R}{r} + \tau \frac{R}{r} + \left[\left[\frac{R}{r} \right] - V_B - \left[\tau \frac{R}{r} \right] \right] \left[\frac{V_I}{V_B} \right]^\beta$$

$$\beta = \frac{1}{2} - \frac{(r - \delta)}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{r - \delta}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}} < 0 \quad (2)$$

The parameters τ, R, r denote the tax rate, the coupon and the risk free rate respectively. Equity holders will obtain the value of unlevered assets V_I minus a perpetual stream of coupon payments (second term) plus the tax benefits (third term) plus the option to default, saving the interest payments on debt by giving up the value of assets at default and the tax benefits from that point forward (last term).

Similar to Leland (1994) and Mauer and Sarkar (2005), equation (3) below shows the value of debt $D(V)$ when debt holders have full information about default risk and other parameters. Debt holders will account for foregone interest at default thus accounting for default risk in the determination of $D(V)$. They will also take into consideration bankruptcy costs proportional to V (defined by parameter b).

$$D(V_I) = \frac{R}{r} - \left[\frac{R}{r} \right] \left[\frac{V_I}{V_B} \right]^\beta + (1 - b)V_B \left[\frac{V_I}{V_B} \right]^\beta \quad (3)$$

At the investment trigger, equity holders would want to maximize their position, that is $E(V_I) + D(V_I) - I$. Combining equation (2) with equation (3) gives firm value (equity holders position) at the investment trigger:

$$F(V_I) = (V_I - I) + \frac{\tau R}{r} \left(1 - \left(\frac{V_I}{V_B} \right)^\beta \right) - b V_B \left(\frac{V_I}{V_B} \right)^\beta \quad (4)$$

Firm value at the investment trigger equals the value of unlevered assets plus the expected value of tax benefits until default minus the expected value of bankruptcy costs.

As in Leland (1994), the optimal default trigger is:

$$V_B = \frac{-\beta}{(1-\beta)} (1-\tau) \frac{R}{r} \quad (5)$$

Note that since $\beta < 0$, V_B is positive. The equity holders' option to invest is given by:

$$F(V) = [E(V_I) + D(V_I) - I] \left(\frac{V}{V_I} \right)^a \quad \text{where} \quad (6a)$$

$$a = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{r-\delta}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}} > 1$$

Equivalently, equation (8a) can be re-written as:

$$F(V) = (V_I - I) \left(\frac{V}{V_I} \right)^a + \frac{\tau R}{r} \left(1 - \left(\frac{V_I}{V_B} \right)^\beta \right) \left(\frac{V}{V_I} \right)^a - b V_B \left(\frac{V_I}{V_B} \right)^\beta \left(\frac{V}{V_I} \right)^a \quad (6b)$$

$$= E[V - I] + E[TB] - E[BC]$$

where $E[.]$ in the last line stands for “expected present value”. The last line effectively shows that the value of the firm can be written as the expected value of the unlevered assets (option on unlevered assets) plus the expected value of tax benefits minus the

expected value of bankruptcy costs (as in Mauer and Sarkar, 2005). The net benefits of debt are defined as the difference between the expected tax benefits and the expected bankruptcy costs, i.e., $NB = E(TB) - E(BC)$.

V_I is selected to maximize the current equity holders' option value given by equation (8a) (or equivalently 8b). The first order condition is calculated by applying

$\frac{\partial F}{\partial V} \Big|_{V=V_I} = \frac{\partial V^L}{\partial V} \Big|_{V=V_I}$ and is given in equation (9) below:

$$1 + \beta \left((1 - \tau) \frac{R}{r} - V_B \right) \left(\frac{V_I}{V_B} \right)^\beta \left(\frac{1}{V_I} \right) + \beta \left((1 - b) V_B - \frac{R}{r} \right) \left(\frac{V_I}{V_B} \right)^\beta \left(\frac{1}{V_I} \right) - \alpha \left(\frac{1}{V_I} \right) (E(V_I) + D(V_I) - I) = 0 \quad (7)$$

Mauer and Sarkar (2005) use this framework to study agency issues between equity holders and debt holders. The condition above for the investment trigger is equivalent to their "first-best" condition of firm value maximization.

At the time of investment the equity holders will select the optimal level of the coupon payment that determines optimal capital structure. The coupon payment should be selected simultaneously with the investment trigger, since both the coupon and the investment trigger affect the firm's debt capacity and the risk of default.

Differential information between equity and debt holders

In this section it is assumed that each party may have a different belief about either the growth rate or the volatility of assets, however, each party truthfully communicates its beliefs to the other party. We describe the model of differential

information with respect to volatility. A similar analysis applies for the growth rate and numerical results are presented for both cases.

Equity holders will use their own estimate of volatility to optimize the bankruptcy trigger point. The default trigger determined using their estimate of volatility (which is shown using the symbol e) affects V_B through the auxiliary parameter $\beta(e)$ and equals:

$$V_B(e) = \frac{-\beta(e) R(1-\tau)}{(1-\beta(e)) r} \quad (8)$$

$$\beta(e) = \frac{1}{2} - \frac{(r-\delta)}{\sigma_e^2} - \sqrt{\left(\frac{1}{2} - \frac{r-\delta}{\sigma_e^2}\right)^2 + \frac{2r}{\sigma_e^2}} < 0$$

where σ_e^2 is the estimate of volatility perceived by equity holders. Equity value is then given by:

$$E(V_I) = V_I - \frac{R}{r} + \tau \frac{R}{r} + \left[\left[\frac{R}{r} \right] - V_B(e) - \left[\tau \frac{R}{r} \right] \right] \left[\frac{V_I}{V_B(e)} \right]^{\beta(e)} \quad (9)$$

For a given investment trigger, debt holders will decide on the amount of debt to be given based on their estimate of volatility. Debt holders will determine the amount of debt by:

$$D(V_I) = \frac{R}{r} + ((1-b)V_B(e) - \frac{R}{r}) \left(\frac{V_I}{V_B(e)} \right)^{\beta(d)} \quad (10)$$

Note that debt holders use their own perception of volatility that affects parameter $\beta(d)$ and in turn their perceived probability of default and the expected present value of debt. However, debt holders use the default trigger as determined by equity holders since the default decision is controlled by equity holders.

Equity holders, working backwards, will take into consideration the debt holders' valuation when they decide about the optimal timing of investment, which is found by maximizing firm value:

$$F(V) = [E(V_I) + D(V_I) - I] \left(\frac{V}{V_I} \right)^a \quad (11)$$

where

$$a = \frac{1}{2} - \frac{(r - \delta)}{\sigma_e^2} + \sqrt{\left(\frac{1}{2} - \frac{r - \delta}{\sigma_e^2} \right)^2 + \frac{2r}{\sigma_e^2}} > 1$$

Note that $D(V_I)$ is the value of debt as perceived by debt holders. The optimal investment trigger is then found by solving the following first order condition:

$$\begin{aligned} & 1 + \beta(e) \left((1 - \tau) \frac{R}{r} - V_B(e) \right) \left(\frac{V_I}{V_B(e)} \right)^{\beta(e)} \left(\frac{1}{V_I} \right) + \left(\beta(d) ((1 - b) V_B(e) - \frac{R}{r}) \left(\frac{V_I}{V_B(e)} \right)^{\beta(d)} \left(\frac{1}{V_I} \right) \right. \\ & \left. - \alpha \left(\frac{1}{V_I} \right) \left(\left(V_I - (1 - \tau) \frac{R}{r} + (1 - \tau) \frac{R}{r} - V_B(e) \right) \left(\frac{V_I}{V_B(e)} \right)^{\beta(e)} + \frac{R}{r} + \left((1 - b) V_B(e) - \frac{R}{r} \right) \left(\frac{V_I}{V_B(e)} \right)^{\beta(d)} - I \right) = 0 \end{aligned} \quad (12)$$

Equation (12) includes debt holders' differential beliefs about the volatility, since the debt value incorporates the debt holders' estimate. A similar analysis can be applied for

differential perceptions about the dividend yield (affecting the perceived growth of unlevered assets). By substituting equations (9) and (10) into equation (11), firm value may also be written as the value of unlevered assets plus the tax benefits of debt minus the bankruptcy costs.

The expected present value of the unlevered firm (net of investment costs) $E(V - I)$ equals

$$\left[V_I + \left(V_B(e) \left(\frac{V_I}{V_B(e)} \right)^{\beta(d)} - V_B(e) \left(\frac{V_I}{V_B(e)} \right)^{\beta(e)} + \frac{R}{r} \left(\frac{V_I}{V_B(e)} \right)^{\beta(e)} - \frac{R}{r} \left(\frac{V_I}{V_B(e)} \right)^{\beta(d)} \right] - I \left(\frac{V}{V_I} \right)^{\alpha} \quad (13)$$

The expected present value of tax benefits $E(TB)$ equals

$$\left[\frac{\tau R}{r} - \frac{\tau R}{r} \left(\frac{V_I}{V_B(e)} \right)^{\beta(e)} \right] \left(\frac{V}{V_I} \right)^{\alpha} \quad (14)$$

The expected present value of bankruptcy costs $E(BC)$ equals

$$\left[b V_B(e) \left(\frac{V_I}{V_B(e)} \right)^{\beta(d)} \right] \left(\frac{V}{V_I} \right)^{\alpha} \quad (15)$$

With common beliefs, $\beta(e) = \beta(d)$ and the second term in the value of unlevered assets becomes zero. In this case equation (6b) for firm value is obtained (the non-differential information case). As can be seen by equation (13), debt holders differential beliefs may significantly change the net value of unlevered assets obtained by equity holders and the remaining debt payments after default. In all equations, a common estimate about the

level of the default trigger is being used which is the one determined by equity holders. Equation (14) shows that expected tax benefits are only influenced by the equity holders' beliefs, while equation (15) shows that the level of bankruptcy costs is also affected by the debt holders' beliefs (parameter $\beta(d)$). This result is intuitive since debt holders place emphasis on the value obtained at default but have no interest on the tax benefits of debt which is solely obtained by equity holders.

III. Numerical results and discussion

Differential information between equity holders and debt holders

Table I(a) presents numerical results with varying degree of differential information, in terms of volatility between the two stake holders. The base case parameter values of Leland (1994) are used with an additional assumption of a positive opportunity cost δ of 6%. For the symmetric case we use a volatility $\sigma = 0.25$ for both the equity and debt holders. Other parameters values are as follows: value of unlevered assets $V = 100$, risk-free rate $r = 0.06$, investment cost $I = 100$, bankruptcy costs $b = 0.5$ and tax rate $\tau = 0.35$. The upper panel of the first table shows the results when the debt holders believe that actual volatility is lower than that perceived by the equity holders. In this case, equity holders will invest earlier than in the symmetric case, because they can use higher leverage. Equity holders also default at higher default trigger, compared to the symmetric case. Note that, in this case, firm and debt value increases substantially, since equity holders can acquire inexpensive debt. In the more interesting case where debt

holders believe that volatility is higher, equity holders will delay investment and also default at a later point. This enhances the value of equity and reduces debt and firm value. This effectively acts as a binding constraint on debt, since one observes that debt levels and optimal leverage ratios are lower than those in the symmetric case. Credit spreads exhibit an inverse U-shape. They are lower than in the symmetric case when debt holders perceive lower volatility than equity holders. This is, in general, reversed when their perception is higher, except for very high (unfavorable) asymmetry levels, where credit spreads become lower than those in the symmetric case, because of the extremely low debt levels used.

[Insert table I(a) here]

Table I(b) shows the results for the case of differential information, in terms of growth rate estimates. A higher level of perceived δ implies a lower perceived level of growth. Our results are similar to the case of differential information about volatility, including the behavior of the credit spreads. Significantly, when debt holders perceive a lower growth rate of the assets, equity holders increase the optimal investment trigger, and reduce the optimal default trigger debt levels and leverage ratios. Effectively, lower perceived growth rates by debt holders act as a constraint on the level of debt used.

[Insert table I(b) here]

Our analysis adds to the literature analyzing the underinvestment problem (see, for example, Moyen, 2002 and Mauer and Ott, 2000). In this literature the equity holders delay investment (and thus underinvest) when there is an existing debt and new investments are financed solely with new equity. Equity holders underinvest, since the new investment creates shared benefits with existing debt holders (while equity holders alone bear the extra risk). Leland (1998) and Mauer and Sarkar (2005) discuss overinvestment incentives by equity holders. In Leland (1998), there is overinvestment due to asset substitution, i.e., equity holders invest in riskier project ex-post to agreed debt levels. Similarly, in Mauer and Sarkar (2005), equity holders maximize the value of equity, instead of the total (levered) firm value. Our model provides an alternative explanation based on differential beliefs about the volatility of assets or growth that may justify over or under investment. In the more interesting case analyzed, debt holders have beliefs of higher volatility or lower growth of assets that cause equity holders to underinvest (delay investment) as a way to mitigate the problem of unfavourably priced debt. This result is consistent with recent empirical findings of Ascioğlu et. (2008) that shows that firms facing higher information asymmetry will invest less and rely more on internal capital to fund investment.

Equity holders choice of volatility with debt holders sticky beliefs

In this section our model allows equity holders to choose the optimal level of volatility before and after the investment option is exercised. In contrast to Leland (1994), who studies asset substitution and agency issues, here it is assumed that each volatility level is

truthfully communicated between equity and debt holders prior to the investment, so that debt holders choose the appropriate coupon levels². Within this framework, we investigate two cases: In the first case, volatility choices under common beliefs between equity holders and debt holders is investigated. In the second case, debt holders have differential beliefs about the volatility that mainly affects their estimate of default probability. In this latter case the degree of differentiation and stickiness to early volatility levels proves to be important.

Panel A of table II presents numerical results for the case where equity holders can choose (or affect) the volatility level before investment, ranging from a low volatility ($\sigma_{before} = 0.1$) to a high volatile project ($\sigma_{before} = 0.4$)³. At the time of investment, the equity holders can choose to switch to a different risk profile, irrespective of the initial choice, also ranging between a low volatility ($\sigma_{after} = 0.1$) and a high volatility ($\sigma_{after} = 0.4$) alternative. This setting may reflect situations of start-up firms, or new projects where firms may engage in risky R&D and learning activities to explore alternative potentials (like in Childs and Triantis, 1999). In our setting, the firm may revert to a more “normal” volatility level after these initial highly uncertain R&D investments take place.

[Insert table II here]

² This may be the case for bank loans, where banks require equity holders to verify the type of assets they have invested in.

³ Despite having performed the analysis with more dense volatility choices, we only present the results in increments of 0.1 to preserve space.

Our results show that, for a given volatility prior to investment, firm value is maximized by selecting a low post-investment volatility of assets. A low post-investment volatility increases debt value, leverage ratios and expected tax benefits (net of bankruptcy costs). At lower post-investment volatility, the firm invests earlier and defaults at a higher default trigger. Nevertheless, the higher default trigger does not reflect a higher bankruptcy risk, as seen by the lower credit spreads and the lower bankruptcy costs. High post-investment volatility increases equity value, like in Leland (1994). However, since the option holder receives $E(V_I) - (I - D(V_I))$ and since the debt value decreases substantially, it is not optimal to select a high post-investment volatility. The firm would rather commit to low post-investment volatility levels, so as to increase debt levels and the tax benefits of debt. These results are consistent with Leland (1998) (see discussion in p.1234) regarding the benefits of hedging, i.e., reducing volatility. Leland (1998) shows that the benefits of hedging can be higher leverage, greater tax benefits and lower bankruptcy costs. Similar to our results, Leland (1998) shows that, despite higher leverage, credit spreads are lower at lower volatility levels.

High volatility prior to investment increases firm (option) value consistently with the real options literature. A high volatility choice prior to investment increases the option value on unlevered assets, but may decrease the expected tax benefits of debt (net of bankruptcy costs). In summary, with common beliefs, one observes that firm value is maximized at the highest pre-investment volatility ($\sigma_{before} = 0.4$) and the lower post-investment volatility ($\sigma_{after} = 0.1$).

In panel B, the results where debt holders have different beliefs from equity holders about volatility that affects their perception of default probability are

demonstrated⁴. It is now assumed that debt holders have sticky beliefs about the volatility, by maintaining the belief about volatility levels that exists prior to investment. As can be seen in the results, under this scenario it is no longer the case that equity holders will always select the lower post-investment volatility. This is because it is beneficial for equity holders to switch to higher volatility levels at the investment trigger, since they can raise more debt at relatively low cost (low credit spreads). There is a subtle difference between this result and the “asset substitution” result, e.g., Leland (1998). In our case, the low cost debt is caused by the debt holders’ (suboptimal) stickiness on their original pre-investment volatility regarding the probability to default. Additionally, this analysis demonstrates that, with debt holders having sticky beliefs, it may now be less attractive for equity holders to choose a high pre-investment volatility and this will depend on the degree of lenders’ stickiness.

IV. Summary

In this paper we consider differential beliefs between debt and equity holders, with respect to the volatility or the growth rate of asset value. It is shown that when the debt holders’ perceived estimate of the volatility of assets is higher or when their perceived estimate of the growth rate of assets is lower than that of equity holders, then, equity holders will optimally reduce leverage and delay investment and default. With unfavorable beliefs by debt holders credit spreads increase except for very high deviations in beliefs where credit spreads decrease because of the low leverage used.

⁴ We maintain the assumption that the default trigger is chosen by equity holders and that this is known and used by debt holders in deciding the level of debt financing.

Finally, a model of equity holders' choice of volatility before and after the investment option is developed. Under common beliefs between equity and debt holders, equity holders will optimally choose to engage in risky projects before investment, so as to increase option value. After investment, equity holders will optimally choose a low volatility in order to increase the debt raised and the tax benefits of debt. In the case where debt holders have sticky beliefs about the volatility, based on the pre-investment levels, equity holders may be less motivated to choose high volatility strategies before the exercise of the investment option, because this will pre-commit to a costly debt issue. At the point of investment, however, they may choose to increase risk as much as possible, given the debt holders' beliefs are fixed to pre-investment levels.

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Table I (a): Differential information between debt and equity holders with respect to volatility

	Firm value	Inv. Trigger (V_I)	Bankruptcy Trigger (V_B)	Optimal Capital Structure at Investment Trigger V_I				
				Equity	Debt	Leverage	Coupon	Credit Spread
Base ($\sigma(e)=\sigma(d)=0.25$)	35.42	171.57	57.92	74.82	127.94	0.63	10.84	0.0247
$\sigma(d)=0.1$	71.81	119.89	72.72	18.39	184.34	0.91	13.61	0.0138
$\sigma(d)=0.15$	52.88	140.02	71.54	33.21	169.53	0.84	13.39	0.0190
$\sigma(d)=0.2$	42.03	157.33	66.68	51.85	150.92	0.74	12.48	0.0227
$\sigma(d)=0.3$	31.34	182.54	46.16	101.39	101.37	0.50	8.64	0.0252
$\sigma(d)=0.35$	28.87	190.32	33.66	128.48	74.29	0.37	6.30	0.0248
$\sigma(d)=0.4$	27.41	195.38	22.81	152.02	50.75	0.25	4.27	0.0241

Table I (b): Differential information between debt and equity holders with respect to growth

	Firm value	Inv. Trigger (V_I)	Bankruptcy Trigger (V_B)	Optimal Capital Structure at Investment Trigger V_I				
				Equity	Debt	Leverage	Coupon	Credit Spread
Base ($\delta(e)=\delta(d)=0.06$)	35.42	171.57	57.92	74.82	127.94	0.63	10.84	0.0247
$\delta(d)=0$	53.93	138.64	71.75	31.99	170.76	0.84	13.43	0.0186
$\delta(d)=0.02$	46.58	149.32	69.51	42.32	160.43	0.79	13.01	0.0211
$\delta(d)=0.04$	40.31	160.67	65.08	56.47	146.27	0.72	12.18	0.0233
$\delta(d)=0.08$	31.92	180.87	48.35	96.60	106.17	0.52	9.05	0.0252
$\delta(d)=0.10$	29.57	188.00	37.83	119.47	83.30	0.41	7.08	0.0250
$\delta(d)=0.12$	28.07	193.02	28.16	140.37	62.39	0.31	5.27	0.0245

Base case used for all models: value of unlevered assets $V=100$, risk-free rate $r=0.06$, opportunity cost $\delta=0.06$, volatility $\sigma=0.25$, investment cost $I=100$, bankruptcy costs $b=0.5$ and tax rate $\tau=0.35$. Equity, debt, optimal leverage, optimal coupons and the credit spread are calculated at the investment trigger. Sensitivity analysis is with respect to debt holders perceived estimate of volatility $\sigma(d)$ (panel a) or the opportunity cost $\delta(d)$ (panel b). A higher estimate of $\delta(d)$ implies a lower growth rate of the unlevered assets.

Table II: Firm value when equity holders can choose pre- and post- investment volatility

Panel A: Common information about volatility levels												
σ_{before}	σ_{after}	Unlevered		Expected	Expected	Bankruptcy		Optimal Capital Structure at Investment Trigger V_I				
		Firm value	Firm Value	Tax Benefits	Bankr. Costs	Inv. Trigger (V_I)	Trigger (V_B)	Equity	Debt	Leverage	Coupon	Credit Spread
0.10	0.10	28.70	3.28	28.90	3.48	103.81	54.84	33.38	99.96	0.75	6.75	0.0075
	0.20	22.26	7.09	19.17	4.01	110.62	42.17	45.30	88.03	0.66	6.88	0.0182
	0.30	19.37	8.45	15.04	4.12	114.54	34.90	52.30	81.03	0.61	7.47	0.0322
	0.40	17.79	9.07	12.83	4.12	117.01	30.33	56.90	76.44	0.57	8.40	0.0499
0.20	0.10	36.80	18.04	21.33	2.57	137.64	72.72	44.26	132.53	0.75	8.95	0.0075
	0.20	31.78	19.32	15.76	3.29	146.65	55.90	60.06	116.69	0.66	9.12	0.0182
	0.30	29.34	19.81	13.11	3.59	151.84	46.25	69.35	107.40	0.61	9.90	0.0322
	0.40	27.93	20.06	11.60	3.72	155.11	40.19	75.45	101.30	0.57	11.13	0.0499
0.30	0.10	46.67	28.50	20.66	2.49	180.56	95.39	58.06	173.85	0.75	11.74	0.0075
	0.20	41.74	29.24	15.80	3.31	192.40	73.36	78.76	153.13	0.66	11.97	0.0182
	0.30	39.26	29.53	13.40	3.67	199.19	60.69	90.96	140.91	0.61	12.99	0.0322
	0.40	37.82	29.67	11.99	3.85	203.47	52.72	98.98	132.89	0.57	14.60	0.0499
0.40	0.10	56.03	37.42	21.16	2.55	233.53	123.34	75.13	224.81	0.75	15.18	0.0075
	0.20	50.93	37.92	16.45	3.44	248.89	94.87	101.92	198.06	0.66	15.48	0.0182
	0.30	48.34	38.12	14.08	3.85	257.72	78.54	117.68	182.33	0.61	16.81	0.0322
	0.40	46.82	38.22	12.67	4.07	263.24	68.21	128.05	171.93	0.57	18.89	0.0499
Panel B: Differential beliefs between equity and debt holders about volatility at the investment trigger												
0.20	0.10	26.90	16.97	11.70	1.77	157.68	47.86	94.31	82.46	0.47	5.89	0.0114
	0.20	31.78	19.32	15.76	3.29	146.65	55.90	60.06	116.69	0.66	9.12	0.0182
	0.30	44.73	31.87	18.82	5.96	126.42	57.33	34.99	141.74	0.80	12.27	0.0266
	0.40	68.41	55.07	22.27	8.92	105.14	51.17	23.02	153.75	0.87	14.17	0.0322
0.30	0.10	32.94	27.38	6.84	1.27	220.09	38.35	169.02	62.85	0.27	4.72	0.0151
	0.20	34.32	26.48	9.57	1.74	215.06	44.31	141.08	90.82	0.39	7.23	0.0196
	0.30	39.26	29.53	13.40	3.67	199.19	60.69	90.96	140.91	0.61	12.99	0.0322
	0.40	49.73	40.21	15.44	5.93	174.13	65.79	57.63	174.22	0.75	18.22	0.0446

Parameter values are: value of unlevered assets $V=100$, risk-free rate $r=0.06$, opportunity cost $\delta=0.06$, investment cost $I=100$, bankruptcy costs $b=0.5$ and tax rate $\tau=0.35$. Equity, debt, optimal leverage, optimal coupons and the credit spread are calculated at the investment trigger. Sensitivity analysis is with respect to volatility of unlevered assets before and after the investment. In Panel B, at investment trigger, differential beliefs exist because debt holders have sticky beliefs and use σ_{before} , while equity holders use σ_{after} . Firm Value (column 3) equals unlevered firm value (column 4) plus the expected tax benefits (column 5) minus the expected bankruptcy costs (column 6). Calculations are defined in equations 11-15.