Private equity: strategies for improving performance[‡]

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Abstract

Existing research suggests the median private equity manager does not create excess returns over public markets net of fees. In this paper, we first confirm this result using a factor model that allows for leverage, illiquidity and volatility clustering. The model explains 70 to 90 per cent of the variation in returns. Our model also gives rise to two methods of improving on conventional private equity performance via a synthetic exposure. We test the robustness of the synthetic methods using historical and simulated data.

Keywords: buyouts; venture capital; conditional overlay; GARCH.

JEL: G12; G14

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For the purpose of this article, private equity is defined as covering both leveraged buyouts and venture capital firms. Buyout firms apply leverage to acquire companies. Venture capital firms provide financing to start up companies. More details on the characteristics of both types of firms are provided in the appendices to the paper.

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1. INTRODUCTION

The use of private equity by institutional investors was pioneered by US institutional investors in the mid-1970s, and especially by the Yale University endowment under the leadership of David Swensen: Yale University currently allocates 26 per cent of its assets to private equity (Swensen, 2000).[1] Following the success of this model, other institutions followed with 5 per cent of institutional assets now being invested in private equity globally (Baldridge et al., 2010). Yet, few investors have been able to match the returns enjoyed by US endowments. In the private equity industry, institutional investors such as these endowments are referred to as Limited Partners (LPs), and fund managers as General Partners (GPs).[2] Lerner et al. (2007) measure the return differential between US endowments and other Limited Partners and find a difference of 21 per cent per annum over the period from 1991 to 1998 inferring that the ability to access superior managers is key to successful investing in the private equity industry.[3]

Despite the stellar performance enjoyed by some Limited Partners, many academic studies suggest that the median private equity manager has not created excess returns after fees and stale prices are taken into account.[4] Berk and Green's (2004) model of rational financial intermediation predicts that financial intermediaries will provide zero excess returns to their investors while capturing a rent that is commensurate with their abilities, while Bias et al. (2010) suggest that the net returns delivered by

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[1] See http://www.yale.edu/investments/

[2] See appendix A for a full description of industry terms.

[3] For Endowments, alumni networks lead to access to superior General Partners (GPs) and the ability to co-invest (invest alongside the manager, thereby saving on fees). The discussion as to why institutions allocate to private equity if they cannot access these managers falls outside the scope of this paper. Moskowitz and Vissing Jorgensen (2002) suggest the private non-pecuniary benefits must be large.

[4] See Swensen (2000), Moskowitz and Vissing Jorgensen (2002), Kaplan and Schoar (2005), Phalippou and Zollo (2005), Conroy and Harris (2007) and Franzoni et al. (2009).

financial intermediaries could even turn negative. Private equity fees are substantial, with estimates starting from 7 per cent per annum (Phalippou, 2009)[5] rising to 12 per cent based on the Yale Endowment's experience (Swensen, 2000).[6] Phalippou et al. (2005, 2007) suggest an average return to US private equity of minus 330 basis points after all the costs are taken into account.

Further, the use of stale pricing of private equity assets (i.e., relying on lagged market valuations of assets for computing returns) overstates the excess returns created by managers. Private equity fund managers smooth reported quarterly returns, and during periods of sharp falls in public markets tend to lag valuations of the non-traded assets which make up the majority of their illiquid portfolios. This creates an artificial stability in unit prices that feeds into returns (Gompers and Lerner, 1997, and Anson, 2006).

We investigate the tension between the evidence for high excess returns of some private equity managers and the doubts raised once one allows for fees and stale prices. Our paper makes the following contributions to the extant literature. First, we tailor the factor model introduced by Bird et al. (2010), which adjusts for leverage, illiquidity and volatility clustering,to measure the performance of private equity investing and improve our understanding of the fundamental drivers of private equity excess returns. [7] We substantiate the findings of previous research that there are no

[5] The 7 per cent fee estimate by Phalippou (2009) consists of 2 per cent per annum base fee, 2.5 per cent portfolio fees (transaction, advisory and director fees, taken directly out of the fund holdings) and 2.5 per cent performance fees. Performance fees are 20 per cent above an 8 per cent hurdle rate.

[6] In the case of Yale, the 12 per cent fee estimate reflects the performance fees on the 21 per cent return differential for US endowments. In addition, Lerner et al. (2007) estimate 20 per cent of investors use fund of funds, which contributes an additional 1 per cent base and 10 per cent performance fees.

[7] The model is a general model for alternative investments, initially developed for hedge funds. For a more extensive discussion of the adopted methodology see Bird et al. (2010).

excess returns from private equity investments. Our model is able to explain between 70 to 90 per cent of the variation in private equity returns.

Second, based on the high explanatory power of our factor model, we introduce two methods for producing synthetic exposure to private equity that also improve the performance of private equity. These methods can be applied independently or combined to increase the reward to risk ratios for investors in the asset class.

The first method is aimed at the institutional investor (the Limited Partner) and demonstrates how a passive factor based allocation to private equity reduces fees during the initial years, when the investor pays fees on committed capital, but the manager remains underinvested. Investors set aside capital for commitment for periods of 10 to 14 years and management fees are paid on committed, rather than invested capital. On average only 50 per cent of committed capital is invested over the commitment period as the fund manager searches for investment opportunities. Thus, investors tend to over-commit by a factor of up to 2 times.[8] As we shall see, the use of a factor model approach reduces the need for any such over-commitment.

The second method is primarily aimed at the private equity manager (the General Partner). We suggest a mechanical overlay strategy has the ability to improve excess returns and reduce risk during periods of market stress when managers are unable to sell holdings. Because of the illiquid nature of the underlying holdings, private equity managers are unable to quickly adjust portfolio positions to changes in market sentiment (unlike, for example, hedge funds). One option is to trade investments in

[8] A 2 per cent base fee is then effectively charged at 3 to 4 per cent (Ljungqvist and Richardson, 2003 and Phalippou and Gottschalg, 2009).

the secondary market at deep discounts. However, these actions can lead to large market distortions.[9] The method we apply in this paper overcomes illiquidity and is similar to that previously proposed for hedge funds by Healy and Lo (2009).

The remainder of the article is structured as follows. In section 2, we review background literature on style models used when evaluating private equity performance, while in section 3 we develop the linear model used in this study. Our private equity data source is examined in section 4 and we present our empirical findings in Section 5. Section 6 provides two strategies for improving private equity performance while section 7 offers conclusions and suggestions for further research.

2. BACKGROUND

A large body of academic evidence suggests that private equity as an industry, measured by returns provided either by managers or directly through analysis of the underlying holdings, does not outperform public markets once account is taken of fees. Table 1 provides a summary of several studies in private equity outlying the style factor models used and summarising their findings.

<< INSERT TABLE 1 >>

As can be seen from table 1, most authors do not find evidence of excess returns from private equity investing with those that do exhibiting either a time period or a sample selection bias. For example, Peng (2001) sample period ends before the correction from the technology boom, while Ljungqvist and Richardson (2003) rely on data from

[9] Refer http://www.preqin.com/item/secondaries-deluge-leads-to-asset-pricing-turmoil/101/1098.

a single undisclosed limited partner. In addition, the information contained in table 1 shows that the equity risk premium (commonly proxied by the S&P 500) continues to be the dominant factor employed in academic research, but this premium alone may not capture all the risks associated with private equity investing. The emphasis on the equity risk premium is not surprising, given that many institutional investors continue to benchmark their private equity holdings against such a public benchmark whilst targeting a 3 to 5 per cent out-performance net of fees (Evans, 2008).

Phalippou (2009) concludes that the literature provides us with an interesting puzzle: if the average performance of private equity funds is below the benchmark after fees are taken into account, then why does the marginal investor allocate to private equity? He suggests investors underestimate the impact of fees and rely on biased samples to form their judgement. Swensen (2000), perhaps the most experienced private equity investor, notes that *"the large majority of funds fail to add sufficient value and only the upper quartile of managers are worth investing in."*

3. THE MODEL

Anson (2006) suggests private equity managers provide a package of returns consisting of market risk, liquidity risk, active skill and leverage, and it follows that a good model of private equity returns will combine all these elements, whereas table 1 suggests most literature has traditionally focused mainly on the equity market risk. In this section we review the multifactor model proposed by Bird et al. (2010) as an interpretation of the Intertemporal Capital Asset Pricing Model (ICAPM), in which the equity risk premium is supplemented by additional associated state variables. Under ICAPM the state variables are defined as undesirable outcomes against which investors hedge, thereby creating additional risk premia (Merton, 1973). For instance, in private equity for buyout and venture capital firms, illiquidity is one example of such a state variable, as investors that commit capital to private markets are exposed to illiquidity risk. In addition, investors in leveraged buyouts may be exposed to credit risk as buyout firms take on additional financing, while venture capitalists invest in start up firms and are exposed to small firm and growth risk.

3.1 Adjustment for leverage

Within the private equity industry, buyout managers apply leverage.[10] We follow Bertelli (2007) and transform leveraged returns to deleveraged returns as follows.[11]

$$R^{L}_{it} = R^{DL}_{it} + (R^{DL}_{it} - I_{it}) \cdot (\lambda-1) \quad \Leftrightarrow R^{DL}_{it} = (R^{L}_{it} + I_{it} \cdot \lambda - I_{it}) / \lambda \qquad (1)$$

Where R^{L}_{it} is the leveraged return of sector i at time t, R^{DL}_{it} is the deleveraged return of sector i at time t, I_{it} is the interest expense paid by sector i at time t set equal to a 50/50 mix of the high yield and mezzanine debt[12] rates consistent with observations by Anson (2006), and λ represents the gearing level. For buyout funds we define λ as the sector exposure (equal to debt plus equity positions held by the sector) divided by the equity of the sector. We use a leverage ratio of 3.3, based on an average over the period from 1990 to 2009 (See Anson 2006 and Griffin 2010).

[10] Refer to appendix B for the differences between venture capital (VC) and buyout (BO) strategies. The growth in the private equity industry in recent years has come mainly from the buyout segment and industry practitioners at times use the terms 'private equity' and 'buyout' interchangeably.

[11] Leverage is implicit in buyout strategies to make opportunities profitable. An average over the period is used as investments stretch over 5 to 7 years. Private equity fund managers charge 20 per cent performance ('skill') based fees above an 8 per cent hurdle rate, but can in theory achieve these excess returns by using a combination of beta and leverage, rather than skill. For example, a buyout manager may decide to leverage up 2 times by buying S&P futures and picks up 8 per cent over the hurdle (prior to financing cost). Investors can create the same payoff profile without having to pay performance based fees to the managers.

[12] A hybrid of debt and equity financing used to finance expansion of existing companies. Refer appendix A for more detail.

3.2 Adjustment for illiquidity

For private equity, illiquidity adjustment methods based on serial correlation have been employed by Gompers and Lerner (1997), Jones and Rhodes-Kropf (2003) and Anson (2006). In general these methods rely on regressing private equity market returns against the lequity market premium, lagged one or more periods. Our adjustment builds on the work by Getmansky, Lo and Makarov (2004) who explored several sources of serial correlation in alternative investments, and showed that the most likely explanation of serial correlation is illiquidity exposure.[13]

To remove the illiquidity effects we add lags of the dependent variables on the right hand side of the regression model as liquidity factors. We find that to completely remove the serial dependence, we need to add up to three lags which suggests that private equity managers tend to rely on the previous three quarter valuations as an anchoring point for the present valuation. This is consistent with the number of lags in the multiperiod model of Anson (2006), and is symptomatic of the 'stale pricing' methods employed by managers.

3.3 Risk factors

The model that we use to test for excess returns commences with the CAPM. In the case of venture capital investments, we also include the returns on the NASDAQ as NASDAQ listing represent the main exit strategy for venture capitalists and also captures the small cap/growth tilt inherent in venture capital. For buyouts, we do not apply the NASDAQ returns, but instead apply a 50/50 mix of high yield and

[13] The authors examine market inefficiencies, time varying expected returns, leverage and fee structures as possible sources of serial correlation before reaching their conclusions.

mezzanine debt as an additional variable to the equity risk premium (Anson, 2006).[14] This factor captures the reliance of buyouts on the high yield credit markets which are the main source of their leverage financing.[15]

3.4 Conditioning variable

Conditioning variables are first introduced by Ferson and Schadt (1996) and subsequently examined by Bird et al. (2010) who found them to be useful for explaining alternative investments returns. One potentially useful conditioning variable is the CBOE Volatility Index (VIX) which is a commonly used indicator of market turbulence and is readily observable (Healy and Lo, 2009).[16] We use the VIX to set up a dynamic overlay (hedge): whenever the VIX exceeds a certain threshold level we short the market based risk factors in our model in order to hedge out the portfolio betas. In this manner the portfolio can become "market neutral" during periods of high volatility without having to trade the less liquid underlying assets during periods of market dislocation.

3.5 Mean equation

Having identified the relevant risk variables, we use the models as represented by equations (2a) and (2b) to measure whether significant excess returns are created by private equity managers. Equations (2a) and (2b) are CAPM models extended to leverage and illiquidity risk factors. For venture capital, the NASDAQ is added as an

[14] High yield represents the returns on the Barclays Capital US high yield index. Mezzanine debt is represented by the returns on the Venture Economics Mezzanine debt index. In leveraged buyouts, mezzanine capital is used in conjunction with other securities to fund the purchase price of the company being acquired. Typically, mezzanine capital will be used to fill a financing gap between less expensive forms of financing (e.g., senior loans, second lien loan, high yield financings) and equity.
[15] In addition we test combined effects using both NASDAQ and high yield simultaneously for venture capital and buyouts, currency effects using the trade weighted U.S. dollar index (DXY), and international effects using the MSCI Europe, Australasia and Far East (EAFE). W do not find these factors significant.
[16] http://www.cboe.com/micro/vix/historical.aspx

additional factor (2a). For buyouts, a 50/50 mix of mezzanine debt and high yield is added (2b).

$$R^{DL}_{it} - R_{ft} = \alpha_i + \beta_{i,m} (R_{mt} - R_{ft}) + \sum_{j=1}^{n} [\beta_{i,L} (R^{DL}_{ij,t-j} - R_{f,t-j})] + \beta_{i,NQ} (R_{NQ,t} - R_{mt}) + \varepsilon_{it} \quad (2a)$$

$$R^{DL}_{it} - R_{ft} = \alpha_i + \beta_{i,m} (R_{mt} - R_{ft}) + \sum_{j=1}^{n} [\beta_{i,L} (R^{DL}_{ij,t-j} - R_{f,t-j})] + \beta_{i,HY} (R_{HY,t} - R_{ft}) + \varepsilon_{it} \quad (2b)$$

Where R^{DL}_{it} is the deleveraged rate of return of manager i at time t, R_{ft} is the risk free rate at time t and α_i is the intercept term not explained by the multi-factor model. In terms of factor exposure, $(R_{mt} - R_{ft})$ represents the equity risk premium and $(R^{DL}_{i,t-j} - R_{f,t-j})$ the illiquidity premium based on the lagged dependent variables where we include up to three lags ($n=3$). Coefficients $\beta_{i,m}$ and $\beta_{i,L}$ measure the sensitivity of excess returns of manager i to the market and illiquidity premium.

The additional factors are represented by $(R_{NQ,t} - R_{mt})$ for equation (5a) reflecting the differential returns on NASDAQ stocks and larger capitalised companies, and $(R_{HY,t} - R_{ft})$ reflecting the credit premium obtained by investing in a mix of high yield and mezzanine debt for equation (5b).

Equations (3a) and (3b) extend equations (2a) and (2b) to include the Treynor and Mazuy (1966) market timing component $(R_{mt} - R_{ft})^2$. Under Treynor and Mazuy a significant positive coefficient ($\beta_{i,timing}$) suggests positive market timing ability[17].

[17] See Treynor and Mazuy (1966) or Bird et al. (2010) for a more detailed explanation.

$$R^{DL}_{it} - R_{ft} = \alpha_i + \beta_{i,m}\,(R_{mt} - R_{ft}) + \sum_{j=1}^{n} [\beta_{i,L}\,(R^{DL}_{ij,t-j} - R_{f,t-j})] + \beta_{i,NQ}\,(R_{NQ,t} - R_{mt}) + \beta_{i,timing}\,(R_{mt} - R_{ft})^2 + \varepsilon_{it} \quad (3a)$$

$$R^{DL}_{it} - R_{ft} = \alpha_i + \beta_{i,m}\,(R_{mt} - R_{ft}) + \sum_{j=1}^{n} [\beta_{i,L}\,(R^{DL}_{ij,t-j} - R_{f,t-j})] + \beta_{i,HY}\,(R_{HY,t} - R_{ft}) + \beta_{i,timing}\,(R_{mt} - R_{ft})^2 + \varepsilon_{it} \quad (3b)$$

3.6 Variance equation

We find evidence of conditional clustering of variance in the residuals as the squared residuals exhibit long memory effects.[18] As private equity funds use leverage, we use maximum likelihood estimation to apply the GJR-GARCH version by Glosten, Jagannathan and Runkle (1993) which caters for asymmetry in the GARCH process in up and down markets. Negative returns increase volatility more than positive returns due to the use of leverage. Under this model given the information set Ω at $t-1$ then $\varepsilon_{it} \mid \Omega_{t-1} \sim N(0,h_{it})$ and the conditional volatility is

$$h_{it} = \gamma_0 + \gamma_1\varepsilon^2_{i,t-1} + \gamma_2 h_{i,t-1} + \gamma_3\varepsilon^2_{i,t-1}I_{t-1} \quad (4)$$

where $I_{t-1} = 0$ if $\varepsilon_{i,t-1} \geq 0$, and $I_{t-1} = 1$ if $\varepsilon_{i,t-1} < 0$.

In this case, γ_0 represents a constant intercept impacting the long run unconditional volatility, γ_1 a weighting to the previous period's squared shock, γ_2 a weighting to the previous period's predicted volatility and γ_3 sensitivity to negative returns shocks.
In private equity it is important to test the GJR model in order to understand whether managers reduce volatility in returns as markets decline (the 'stale pricing' effect).

[18] For completeness, we also test more basic ARCH(1)..ARCH(4) models but find the squared error terms of the models retain the long memory effects. We are not aware of previous studies on the use of GARCH for private equity. Elyasiani, Getmansky and Mansur (2008) are among the first to investigate other alternative investments such as hedge funds using GARCH(1,1).

4. THE DATA

For our sample, we rely on quarterly data on the return on private equity funds using the Venture Economics indices,[19] which cover over 2,184 private equity funds. We use the aggregate performance of composites from January 1990 to December 2009. As a consequence our findings will reflect an overall average performance across funds across this time period. Venture Economics gathers its data by surveys sent to private equity funds and thus relies on self-reporting. These surveys are voluntarily filled in on a confidential basis and not audited. A pooled return of all funds is then calculated by Venture Economics by treating all funds as a single fund by combining their monthly cash flows. This cash flow series is used to calculate a time-weighted rate of return.[20]

Chen et al. (2002) and Kaplan and Schoar (2005) suggest biases do not occur in the Venture Economics database, despite its voluntary reporting nature. First, Venture Economics' dataset is based on anonymous reporting. Thus there is no incentive to bias performance data upwards as funds cannot be marketed through the database. Second, private equity funds have a long and fixed lifespan (10 to 14 years). There is no incentive for a private equity manager to force early closure and so discontinue reporting returns, as long as fees can be collected. Third, Born et al. (2005) note that

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[19] Venture Economics remains the most frequently used database in terms of academic research, see Kaplan and Schoar (2005), Conroy and Harris (2007) and Phalippou and Gottschalg (2009).

[20] For example, the quarterly return R_t of an index at time t, or $R_t = (Assets\ valuation_t + Distributions_t - Cash\ inflows_t) / (Assets\ valuation_{t-1})$ whereby the *Asset valuations*, *Distributions* and *Cash inflows* are the aggregated numbers reported by industry participants. This method implicitly creates an asset weighted return and most closely matches the method that many investors use in measuring the return on their portfolio. Similar to a market-value weighted index in the equity market, this pooled method is considered the most appropriate method for presenting the aggregate performance of private equity funds. Venture Economics does not provide access to the cash flows themselves, but only the aggregate measures.

successful and unsuccessful projects are intrinsically linked together within the performance of a fund, thus, unsuccessful investments are always included.

Sample selection bias and time period bias may have contributed to conflicting findings on the ability of private equity to generate excess returns. We aim to minimise both biases by maximising the extent and the length of the sample used. Because of the private nature of the industry and its confidentiality, data is extremely limited. Venture Economics provides by far the most extensive data set available for research.[21] Selection bias, as well as the anonymous nature of the database, are an important limitation of our research, and can mean that our results are still biased either upwards or downwards.[22]

5. EMPIRICAL RESULTS

5.1 Descriptive statistics

The descriptive statistics for the data are shown in table 2.

<< INSERT TABLE 2 >>

Based on the excess return column in table 2, the descriptive statistics suggest deleveraged returns of most strategies exceed the risk free rate prior to adjustment for market and illiquidity risk factors. The skewness in returns reflects the large upside optionality (positive skew) arising in venture capital when portfolio companies go

[21] Grabenwarter and Weidig (2005) note that Venture Economics actively tries to complete data retrospectively by asking managers to add data on so far unreported funds and that this possibly biases the sample towards successful funds. But since the database is not used for marketing, but mainly to measure performance against peers, there is also an incentive to skew results downwards.

[22] One other and unique bias to private equity is reported by Phalippou and Zollo (2005) and known as the "living dead" bias. The extent of this bias depends on the valuation of non-exited investments at the end of the sample period. Phalippou and Zollo were given access to data enabling them to calculate this bias by aggressively writing off residual values (rather than maintaining them at the last reported values) on funds over 10 years old, over the period from 1980 to 1996. They argue the bias accounts for up to 2.5 to 3.5 per cent of performance. However, it may be argued that Phalippou and Zollo's accounting methods are too aggressive. The evidence on the size of this bias is inconclusive at this stage and falls outside the scope of this paper.

public at large multiples to their original investment. Significant autocorrelation in venture capital reflects strategies of investing in smaller, illiquid companies. Buyout returns are more normally distributed than venture capital, as buyout funds invest in large and liquid companies. The negative skew in buyout returns comes from incurring large losses when buyouts fail.

5.2 Performance of GARCH models

Estimated excess returns disappear in the pricing models with conditional heteroskedasticity (Table 3), consistent with findings from literature from Table 1. While regression constants are estimated in the range of 0.0034 per cent per quarter (or 1.3 per cent per annum) for venture capital and 0.0011 per cent per quarter (or 0.44 per cent per annum) for buyouts, these returns are not significantly different from zero.

<< INSERT TABLE 3 >>

As can be seen from Table 3, equity (β_m) and illiquidity (β_L) exposure explain most of the returns. The significance of lagged valuations is also evident; $\beta_{L,t-1}$ to $\beta_{L,t-3}$ suggest that managers rely on up to three lags to value current assets. In addition, the NASDAQ (β_{NQ}) and the high yield/mezzanine debt (β_{HY}) factors are significant return drivers for all venture capital strategies and all buyout strategies, respectively.

We find evidence of ARCH (γ_1) and GARCH (γ_2), suggesting volatility persistence and clustering. These effects are more persistent in venture capital than buyout firms. More importantly, the sign on the asymmetry coefficient (γ_3) is evidence that the conditional volatility of these (self-reported) returns is higher during rising markets.

This significant asymmetry coefficient is symptomatic of the practice of venture capital managers to report returns that are based on internal asset valuation. We find that at the peak of the tech bubble in 1999-2000, venture capital manager reported returns that are much higher than those predicted by the factor model. On the other hand, during periods of market declines, the volatility of the errors decreases, pointing to an element of 'stale pricing' whereby managers are more predisposed to revaluing assets upwards than they are to revaluing assets downwards.

The model is able to explain 71 per cent of the variation in returns for all venture capital firms and 86 per cent of the variation in returns for all buyout firms. This means that factor models offer relative transparency of return drivers in an asset class widely viewed as characterized by poor manager disclosure. In addition, private equity has traditionally been viewed as the domain of a handful of privileged institutions who can afford the fees and expertise required to access private market opportunities. However if similar performance can be obtained using a mix of public markets instruments, factor models can offer a low cost alternative to direct investment.

5.3 Stability of factors

<< INSERT FIGURE 1>>

Figure 1 shows seven year rolling averages for the betas in our model, which are calculated by regressing rolling 28 data periods (7 years of 4 quarters) of private equity returns against the identified risk factors.[23] The betas are relatively stable and could be used to replicate private equity returns. The graphs in Figure 1 also show that alphas for venture capital and buyouts have drifted around zero over most of our sample period. Positive excess returns during the 1990s tech bubble have been offset by the negative excess returns over 2000-2008, suggesting that alpha is episodic in nature for the private equity industry.

6 Strategies for improving performance

The high R^2 from our factor model and the stability of the betas has important implications for investors otherwise unable to access superior managers. Below we outline two strategies to replicate the returns generated by investments in both venture capital and buyouts.

6.1 Strategy 1: Applying a factor approach during the commitment period

Comment [FoB1]: Is this correct. I thought that the approach was applied over the entire period but directed to solve the problem that fees are paid on committed capital, only about 50% of which is invested. Also does not the synthetic approach solve the illiquidity problem and should not this also be stressed (which would suggest some rewriting within the section). If I am right then maybe the heading should be" "Applying a factor approach to overcome commitment and illiquidity problems"

The first application of the factor model is aimed at the institutional investor (Limited Partner) and is used to reduce the fees paid by institutional investors during the commitment period.[24]

We rely on the illiquid asset model by Alexander and Takahashi (2001) to demonstrate the performance over a typical 12 year commitment period of directly investing in private equity via active managers versus gaining the exposure via a

[23] The 7 year period is selected to have sufficient data points to regress against, and also to coincide with the maximum period needed to realise private equity investments (refer appendix B).
[24] Note that strategy 1 is not applicable to the General Partner (the private equity fund manager), as the General Partner's existence is predicated on his ability to find suitable direct investments for his clients. The General Partner will not engage in factor exposure that the client can easily create himself. Furthermore, the General Partner does not suffer from the commitment problem: cash will be called from the institutional investors (Limited Partners) only when investment opportunities arise.

passive factor model.[25] Yale has been investing in buyouts since 1973 and venture capital since 1976. Based on Yale's experience, Alexander and Takahashi (2001) developed a model for assessing the impact of changing investment levels during the commitment period (the period for which an institution commits to a private equity program with a manager, in general between 10 to 14 years) which has been fitted in terms of parameters based on Yale's actual experience.[26]

Figure 2 represents the investment pattern of a typical capital commitment based on Yale's default settings for contribution rates,[27] fund life, committed capital and capital growth rates. When we cross-reference Yale's settings with information provided by DeBrito et al. (2006) for the Venture Economics database from 1985 to 2004, we find that they substantiate the patterns provided by Yale. Figure 2 suggests that once capital is committed for the 12 year period, the majority of contributions by the investor are called up by the manager over the first 5 years of the commitment period. During this period the manager is searching for companies in which to invest, and starts distributing dividends and capital gains proceeds from year 3 onwards. Based on Yale's experience, 56 per cent of committed capital is invested over the 12 years and this is represented by the horizontal line in Figure 2. Thus, an over commitment of capital of about 1.8 times is suggested by private equity managers.

<< INSERT FIGURE 2>>

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[25] For this analysis, we assume the fees to obtain factor exposure through e.g. futures or swaps in the derivatives markets are negligible as the number of required transactions is small. In general, transaction costs are low (a few basis points) and negotiable depending on investment size. Fees do not significantly alter the overall conclusions.

[26] Refer Appendix C for a full description of the Yale model.

[27] Contribution refers to the capital called up each year from the investor whenever the manager finds suitable investments. Refer to Appendix A for more detail.

The fact that fees are typically paid on committed, rather than invested, capital inflates the cost of investing in private equity, especially during the first 5 years when the over commitment is greatest. The first strategy we propose enables investors to use a replicating portfolio to gain exposure to private equity without being subject to the costs associated with over-commitment. In the next paragraphs we describe the three steps used in a simulation where we compare the outcome from a direct investment in private equity with obtaining the exposure by producing replicating portfolios based upon the factors in our model.

1. **Simulation of direct investment returns.**

   We 'block' bootstrap by resampling with replacement from the available 20 year history (from 1990-2009) of active manager returns. This creates 1,000 runs of alternative 20 year paths of industry returns.[28] The 'block' refers to the resampling of continuous blocks of time (rolling 4 quarter periods) to capture the 3 period lagged dependencies detected in our factor model.

2. **Simulation of passive factor model returns.**

   Risk factors are resampled based on the same block bootstrap method and then combined to form passive factor model returns at time t ($R_{clone,t}$).

$$R_{clone,t} = \sum_{k=1}^{n} w_k F_{kt} \tag{5}$$

[28] In effect, we create another 1,000 runs x 20 years x 4 rolling quarters = 80,000 data points to overcome the limited historical data set.

The risk factors F_{kt} used to build up the replicating portfolio returns are to Table 3 but since there is no α in equation (5), unlike in equations (2a) and (2b), the weights are determined by regressing without intercept.

Venture capital manager returns

For venture capital, the long position in the liquid synthetic asset consists of a 40 per cent weight to the S&P 500 and 60 per cent to the NASDAQ, to which we add an illiquidity premium, which is described in more detail below.

Buyout manager returns

For buyouts we apply a 10 per cent weight to the S&P 500, 80 per cent to high yield / mezzanine debt and 10 per cent to cash.

The illiquidity premium

In the original model based on equations (2a) and (2b) we measure the illiquidity premium by using serial dependence up to four lags, and find for venture capital that illiquidity is an important driver of returns. In practice, we cannot invest in lagged dependents. To simulate the contribution from illiquidity premium using a market based factor, we use an option based model proposed by Golts and Kritzman (2010), whereby a long position in an illiquid asset equals a long position in a liquid asset and a short position in a liquidity option.

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Golts and Kritzman (2010) suggest committing capital into illiquid assets such as private equity means giving away the option to trade, thereby creating a liquidity premium. The optionality and asymmetric profile of illiquidity has been recognized by many authors, including Merton (1981). The short liquidity option proxy consists of the sale of a quarterly at-the-money put option on the S&P 500. Following Golts and Kritzman (2010), the S&P 500 is used as a reference asset for the liquidity option, suggesting falls in the S&P 500 are a proxy for an increased demand for liquidity. For more details on the liquidity option model and other possible liquidity proxies investigated for private equity we refer the reader to Appendix D.

3. **Simulation of returns to investors**.

For direct investment simulation, we create 12 year commitment periods, where we replace the constant growth rate in the Yale model with the simulated results from step 1 as the capital committed is gradually invested. The simulated returns affect the net asset value and distribution rates and the eventual rate of return experienced by the investor. For example, based on the formulas listed in appendix C, a period of high simulated asset returns, leads to high distribution rates in the Yale allocation model.

For passive factor model exposure we assume 100 per cent investment on a passive basis from day one. The factor model offers a number of immediate advantages. First, there is the absence of active management fees. Second, it is possible to instantly invest in all of the factors.[29]

Table 4 examines the impact of a simulated allocation to private equity based on direct investment and the factor based approach using 1,000 simulated runs with a 12 year commitment period.

<<INSERT TABLE 4>>

Venture Capital

As can be seen from Table 4, for venture capital, the factor model underperforms the direct investment by 0.7 per cent (14.2 – 13.5), although the reward to risk ratio has slightly increased.[30] A 10.7 per cent tracking error is observed between the factor based approach and the direct investment approach over the 12 year period. We suggest this is due to the smoothed returns reported by the active managers. The factor model's worst possible run suggests a loss of minus 16.6 per cent per annum over a 12 year period, compared to minus 1.2 per cent per annum for the direct investment. Arguably, the increased downside risk of the factor model is caused by inclusion of the NASDAQ as a risk factor, and its post tech bubble decline. On the other hand, it can also be argued that the numbers understate the risk associated with direct investment due to the managers being unwilling to adequately adjust down the value of the assets during down markets.

[29] Mezzanine debt is less liquid. For the mezzanine debt component, for the first 2 years we use the liquid high yield credit market as a proxy investment, consistent with industry practice of the time allowed to get set in the asset class.
[30] Risk is defined as the standard deviation of possible outcomes at the end of the 12 year period, as per table 4.

Results in Table 4 also suggest that the best 12 year period for a direct investment consists of a return of 123 per cent per annum.[31] Once again, we argue that this is due to the overstatement of net asset values on the upside during the tech bubble (as we find from our GARCH model). We suggest the 36.1 per cent per annum best 12 year period generated by the factor model represents a more conservative estimate.[32]

Buyouts

The results for buyouts in Table 4 suggest the replicating approach is able to track buyouts with only a 1.9 per cent tracking error. This compares favorably to the tracking error we found for venture capital which we suggest is a consequence of buyouts being more liquid and thus less prone to issues of stale pricing. The various risk measures are marginally higher for the replicated portfolio than for direct investing. When this is combined with a slightly higher return for the replicated portfolio, then it is not surprising that they deliver similar reward-to-risk ratios. Again, it has to be remembered that the smoothing methods used by the managers are likely to understate the risks associated with direct investing.

In conclusion, the results from simulating private equity strategies are promising for both venture capital and buyouts. The potential in terms of fee savings and liquidity gains are substantial. For both venture capital and buyouts, the investor ends up with a comparable risk/reward ratio and an appreciation of the downside risk which may be understated when relying on managers' smoothed returns. The more liquid nature of

[31] This reflects four quarters in 1999 being resampled 12 times in a row as one of the possible outcomes.
[32] Unlike 123 per cent per annum, the ability to achieve 36.1 per cent per annum after fees, for 12 years in a row would be considered remarkable, but not impossible by many investors.

buyout investments means a lower tracking error can be achieved than for venture capital.

6.2 Strategy 2: Applying factor models to implement a conditional hedge

The second application of the factor model is aimed at private equity managers (General Partners) and is used to reduce the downside risk experienced in their investment portfolios.[33]

For this section we assume the position of a private equity manager who has no differential skill and is generating returns comparable to that achieved by the industry. In other words, the General Partner is managing a diversified pool of private equity funds at different stages of life, equating to a 100 percent investment in the industry at all times.[34]

6.2.1 Usefulness of a hedging overlay

Another use of the high explanatory power in the factor model is the use of dynamic overlays to reduce downside risk. Our analysis starts with the question of whether it is actually worthwhile to hedge downside risk, given that managers tend to smooth returns (offer stale pricing) during periods of market downturns and thus understate the downside risk associated with investment via active private equity managers.

<< INSERT TABLE 5 >>

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[33] In theory, Limited Partners could apply the same hedging method we propose in this section in proportion to the capital invested with the General Partner. For the illustrative purpose of this paper, it is assumed the hedging decision is outsourced to the General Partner.

[34] In practice, the hedge can be adjusted depending on the General Partner's underlying portfolio characteristics.

Table 5 shows the performance of venture capital and buyout firms during up and down markets. The Treynor Mazuy coefficients (β_{timing}) suggest that the performance of managers deteriorate during down markets. Although the coefficients are not statistically significant, they are overwhelmingly negative. Figure 3 depicts the relationship in a graph, where the curved line provides a quadratic estimation of a best fit line.

<< INSERT FIGURE 3 >>

From Figure 3, we see that the relationship is concave, rather than convex,[35] suggesting that private equity funds underperform during down markets despite the fact that they seem to lag valuations. The small firms and levered buyouts that play an important role in private equity strategies tend to rely on sub-investment grade debt, and financing becomes more difficult to obtain for these firms during periods of market stress than is the case for large investment grade firms with more established credit facilities. Figure 3 thus suggests to the potential performance improvement of hedging the risk factor exposure during periods of market stress.

6.2.2 Developing a hedging program

A successful hedging program requires that the risk factors in a linear risk model account for a significant fraction of the variability in the manager's returns, as we have found to be the case in our model. As Healy and Lo (2009) note, hedging these factors only during periods when the portfolio is deemed to be at higher risk and forgoing the overlay during other periods may seem like market-timing, but in fact is

[35] Under Treynor and Mazuy (1966) a convex relationship suggests a manager with good market timing ability: as the market declines, the manager's fund value should decline by less. As the market gains, the manager's fund value should increase by more. The outcome of figure 3 suggests the opposite.

closer to volatility-timing, a considerably less daunting challenge. In fact, there is evidence that volatility is both time-varying and persistent, as we detect GARCH effects. Investors do respond dynamically to sharp changes in risk, which is consistent with a conditional implementation of beta overlay strategies. Based on Healy and Lo, a hedge R_{ht} for strategy i can be constructed as follows:

$$R_{ht} = -\sum_{k=1}^{n} \beta_{ik} F_{kt} \tag{6}$$

Such that the sum of the R_{ht} and R_{it} contains no factor exposures.

$$R_{ht} + R_{it} = \alpha_i + \varepsilon_{it} \tag{7}$$

Following equation (7), the investor is expected to hold a combination of the direct investment and the conditional hedging portfolio (the overlay). We apply an overlay method based on a number of assumptions regarding investor behaviour. First, we use the volatility index (VIX) as an important indicator of deteriorating market conditions.[36] Second, we assume the hedge ratio used is determined by the investors' long term view on the asset class. We initially set the long term hedge ratios based on the information contained in Table 3, i.e. for venture capital we use 0.3 sensitivity to the S&P 500 and 0.4 to the NASDAQ. For buyouts we use a sensitivity of 0.1 to the S&P 500 and 0.8 to the high yield index as a base case.[37]

[36] Various indicators exist to detect the presence of a 'risky' environment. Healy and Lo (2009) suggest a moving average system or an absolute VIX number. In our model, we test various hedge levels whenever the quarterly VIX in the preceding period exceeds long term historical stock volatility of 15 per cent to 20 per cent. We test the use of 25 per cent, 30 per cent and 35 per cent as the benchmark.
[37] Our multifactor hedges require investors to be able to short the S&P 500 and NASDAQ futures for venture capital. In addition, for buyouts, only large investors may be able to purchase tailored credit default swaps (CDS) in the OTC market to obtain the short high yield /mezzanine credit spread exposure and it should be noted that CDS may not be always available to match these specifications.

The hedging process is easy to implement, especially given the limited number of parameters involved. We start by measuring the impact of the conditional hedge based on a single path: the historical returns realised under the market conditions existing from 1990 to 2009.

<< INSERT TABLE 6 >>

As can be seen from Table 6, the outcome from adopting the conditional hedge is very pleasing, increasing returns for both venture capital and buyout firms while at the same time reducing the risk (defined by standard deviation). The skew improves, as the distribution becomes more right-tailed in both cases. For venture capital the kurtosis increases, for buyouts it decreases. In both cases we consider this an improvement. From Table 2, we found venture capital to be right skewed (reflecting the possibility of exiting private companies at high earnings multiples in the public capital markets) and buyouts to be left skewed (reflecting the possibility of deal failure). Thus, the change in kurtosis under the conditional overlay increases the likelihood of positive tail events for venture capital, but at the same time decreases the possibility of negative tail events for buyouts. Specific downside risk measures such as semi-deviation (the standard deviation of down quarters only) and worst possible quarter also improve.

<< INSERT FIGURE 4 >>

Figure 4 graphs the effect of the conditional overlay in reducing the concavity evident in Figure 3. As for Figure 3, the curved line provides a quadratic estimation of a best fit line and the positive quadratic relationship suggests applying the conditional hedge improves market timing.

6.2.3 Robustness tests

We conduct robustness tests by varying the VIX trigger for implementing the hedge and the betas to the risk factors in order to test the sensitivity of the hedge for VIX levels of 25, 30 and 35 per cent and changing betas in the conditional hedge to the equity risk premium, the NASDAQ and the high yield premium.

<<INSERT TABLE 7A>>

Table 7A reports the impact of changing the S&P betas under different VIX triggers in the hedging formula. The base case (default) hedge parameters are highlighted in the shaded rectangle. A positive incremental return and risk reduction is noted if we vary the S&P beta over a wide range.[38]

<<INSERT TABLE 7B>>

Table 7B provides comparable information to table 7A, but instead varies the NASDAQ factor for venture capital and the high yield premium for buyouts under different VIX trigger ranges. From the information provided in Tables 7A and 7B, the hedge is robust to a wide range of settings in the NASDAQ and high yield parameters.

[38] The exception to this is the S&P beta to buyouts. Buyouts have a low base case beta of 0.1 to the S&P. Thus, if we move the conditional hedge to have an S&P Beta above 0.1, a net short position is created. For example, if the direct buyout investment has a beta of 0.1, and a beta of -0.5 is applied to the conditional hedge, the net portfolio would have an equity beta of -0.4). As can be seen from the table, the volatility of the combined investment increases once a net short position on the equity markets is taken out, as the original direct investment in buyouts had limited equity exposure.

So far we have discussed the hedge from a historical return perspective. As a final robustness test, we now compare the impact of the hedge under simulated market conditions. We simulate returns provided by the managers and the associated capital market factors required to set up the hedge (the S&P 500, the NASDAQ, the high yield premium and the VIX) for 1,000 runs of 20 years. By simultaneously re-sampling the data across four rolling quarters (rows) as well as across economic variables (columns) we preserve the serial dependence within variables and also the relationships across variables. If the VIX is found to be above the hedge threshold during the current quarter, the hedging overlay is applied for the next quarter. At the end of each 20 year run, the difference in return and volatility between the unhedged and the conditionally hedged asset are determined. These differences are determined for each of 1,000 runs of simulations and our findings are summarised in Table 7C.

<< INSERT TABLE 7C>>

Table 7C suggests that for the majority of runs, an increase in returns and at least some risk reduction is achieved when applying the hedge across a large range of possible capital market conditions. For venture capital, we can be confident at the 90 per cent level of an increase in the reward-to risk ratio with the median outcome being an increase in returns of 1.2 per cent and a reduction in risk of 2.3 per cent. For buyouts, the findings are only slightly less favourable, with a 75 per cent likelihood of an improvement in the reward-to-risk ratio, where the median outcome is an increase in returns of 0.4 per cent with a reduction in risk of 3.8.

7. CONCLUSIONS

We do not find evidence of excess returns in private equity after adjusting for equity risk, illiquidity, leverage and volatility clustering. This is consistent with the arguments put forward by Berk and Green (2004) and Biais et al. (2010) and the fee concerns raised by Phalippou (2009). It also concurs with recent findings by Ang et al. (2009) who, based on an extensive review, conclude that "there is little convincing evidence of superior risk adjusted returns to private equity and venture capital. Arguably some recent alternative vehicles simply repackage certain systematic factors in much more expensive forms."[39] Confirming this argument, we find that 70 to 90 per cent of the variation in private equity returns is explained by the style tilts introduced in our model. Further, we find that managers do engage in stale pricing and are much more reticent to adjust valuations downward than they are to adjust them upwards.

We suggest two strategies that can improve performance of this asset class for investors by reducing fees and creating liquidity. The strategies can be used independently or combined and suggest a review by investors of the role that private equity can play in their asset allocation given that private equity exposure may not require a static commitment for a 10- to 14-year period. Our first proposed strategy serves to reduce fees and increase liquidity during the commitment period. Furthermore, for buyouts, we suggest it raises the possibility of a form of capital structure arbitrage: similar returns to buyout managers can possibly be obtained by investing in a high yield credit. Our second proposed strategy suggests how excess returns, the reward to risk ratio, and liquidity can be improved by applying a

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[39] Refer Ang, Goetzmann and Schaeffer (2009) in 'Evaluation of Active Management for the Norwegian Government Pension Fund'.
http://www.regjeringen.no/upload/FIN/Statens%20pensjonsfond/rapporter/AGS%20Report.pdf

conditional overlay to improve protection during periods of market stress. We test both strategies for robustness using historical data and simulation models.

A number of areas are interesting for further research. First, while the literature suggests that the median private equity manager underperforms public markets, there is also acknowledgement that U.S. Endowments have been very successful private equity investors. A wide dispersion of manager returns and the persistence of top quartile performance by managers has been documented (Aigner et al., 2008). Research into the characteristics of these successful managers has the potential to improve the excess returns accruing to investors.

Second, investors who, unlike the U.S. endowments, find they cannot access top quartile managers can research ways of implementing either fee reductions (e.g. through co-investment)[40] or gaining a synthetic exposure along the lines that we have described in this paper. Although our rolling beta analysis suggests that the betas are fairly constant, this may not continue to be the case as the private equity industry continues to evolve. As more data points become available, greater insight can be gained into the variability of these factors.

Third, biases have not been central to our discussion. Some biases are unique to private equity. The 'living dead' bias reported by Phalippou and Zollo (2005) reduces reported excess returns and investors may in reality experience an underperformance versus public markets. Apart from the lack of available data, some biases such as 'self selection' bias (whether or not funds decide to include themselves within a particular

[40] Typically, co-investors are existing limited partners. Co-investments are made outside of the existing fund and as such co-investors rarely pay management fees or carried interest on individual investments. Private equity firms offer co-investments to their largest and most important investors as an incentive to invest in future partnerships.

database) are notoriously hard to measure. Thus, our paper serves only as a starting point for further research on the fees, transparency and valuation methods currently offered by the industry.

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Appendix A: Private equity industry terms

The following definitions are sourced from *the Thomson Reuters & The Australian Private Equity & Venture Capital Association Limited Yearbook 2009* and VC Experts' *The Glossary of Private Equity and Venture Capital* (www.vcexperts.com/vce/library/encyclopedia/glossary.asp).

Capital Call
When a venture capital firm has decided where it would like to invest, it approaches its investors in order to draw down the money. The money will already have been committed to the fund but this is the actual act of transferring the money.

Committed Capital
Capital committed by investors. Cash to the maximum of these commitments may be requested or drawn down by the private equity managers usually on a deal-by-deal basis. To the extent that capital invested does not equal capital committed, limited partners will have their private equity returns diluted by the much lower cash returns earned on the uninvested portion.

Early Stage Venture Capital
This is a fund investment strategy involving investment in companies for product development and initial marketing, manufacturing and sales activities. Revenues exist, but since this is a capital-intensive stage, profits are minimal if they exist at all.

General Partner (GP)
The partner in a limited partnership responsible for all management decisions of the partnership. The GP has a fiduciary responsibility to act for the benefit of the limited partners (LPs), and is fully liable for its actions.

Latter Stage Venture Capital
A fund investment strategy which provides financing for the growth of a company that has moved beyond the expansion stage to increase its sales volume and generate consistent growth. It is considered the last venture capital stage of financing prior to a liquidity event (i.e. an IPO or acquisition of the company).

Leveraged Buyout (LBO)
A takeover of a company, using a combination of equity and borrowed funds. Generally, the target company's assets act as the collateral for the loans taken out by the acquiring group. The acquiring group then repays the loan from the cash flow of the acquired company. For example, a group of investors may borrow funds, using the assets of the company as collateral, in order to take over a company. Or the management of the company may use this vehicle as a means to regain control of the company by converting a company from public to private. In most LBOs public shareholders receive a premium to the market price of the shares.

Limited Partner (LP)
An investor in a limited partnership who has no voice in the management of the partnership. LPs have limited liability and usually have priority over GPs upon liquidation of the partnership.

Mezzanine Debt
A hybrid of debt and equity financing that is typically used to finance the expansion of existing companies. Mezzanine financing is basically debt capital that gives the lender the rights to convert to an ownership or equity interest in the company if the loan is not paid back in time and in full. It is generally subordinated to debt provided by senior lenders such as banks and venture capital companies.

Seed Stage Venture Capital
An investment strategy involving portfolio companies at its earliest phase of development to promote a business concept before a company is started. Capital invested in companies at this point have not yet fully established commercial operations, and may also involve continued research and product development. Because it is the earliest stage of development, it is considered as the riskiest of the various financing stages.

Appendix B: Characteristics of venture capital and buyout firms

Private equity investments are defined as either venture capital (VC) or buyout (BO) funds. The following table from Anson (2006) summarises the key differences.

	Venture Capital (VC)	**Buyout (BO)**
Company	Start up	Mature
Competitive advantage	New technology	Distribution, marketing, production
Financing	Equity	Debt
Target IRR [1)]	40-50%	20-30%
Shareholder position	Minority	Control of company
Board Seats	1 or 2	All
Valuation	Compare to other companies	Discounted cash flow
Investment Strategy	Finance, innovation	Improve operating efficiency
Time to exit	2-5 years	4-7 years
Exit option	IPO, acquisition	IPO, acquisition or recapitalisation

IRR refers to the *Internal Rate of Return*, or the solution to the following equation:

$$-I_0 + \sum_{t=1}^{T} CF_t \; / \; (1+IRR)^t + NAV_T \; / \; (!+IRR)^T = 0$$

Where I_0 is the initial investment, CF_t is the net distribution at time t, and NAV_T is the estimated net asset value of yet to be liquidated holdings.

Appendix C: Simulation model

C1. Yale allocation model (standard settings)

The following allocation model to private equity is based on the Yale Endowment's experience. The model is able to incorporate and respond to actual experience. A more detailed description may be found in Alexander and Takahashi (2001).

Parameters	**Description**	**Initial Settings (Yale)**
RC	Rate of contribution	25% in year one, 33.3% in year 2, 50% in subsequent years
CC	Capital commitment ($)	$100
L	Life of fund (years)	12
B	Factor describing changes in the rate of distribution over time	2.5
G	Annual Growth rate	13%
Y	Yield (%)	0%
Variables		**Formula**
PIC	Paid in capital	$\sum_{0}^{t-1} C_t$
RD	Rate of distribution	Max[Y,(t/L)^B]
Outputs		**Formula**
C	Capital contributions ($)	$C_t=RC_t(CC-PIC_t)$
D	Distributions ($)	$D_t=RD*[NAV_{t-1}*(1+G)]$
NAV	Net Asset Value	$NAV_t=[NAV_{t-1}*(1+G)] + C_t-D_t$

The model is initially applied to a single path. C2 creates a simulation model (multiple paths).

C2. Combination with bootstrap (1000 runs)

G uses resampled output.

Venture Economics provides 20 years of quarterly observations (1990-2009), thereby offering 80 historical data points. An additional 1000 runs of industry growth during the 12 years of commitment are created by using resampling with replacement.

To arrive at the returns to investors, for each path the percentage invested capital is calculated based on the Yale model. The uninvested capital is assumed to be invested at the risk free rate. Thus, a weighted average return is calculated over the life of the partnership.

Appendix D: Modelling liquidity risk through options

Introduction

Under the Golts and Kritzman (2010) model:

Illiquid asset = long position in liquid asset + short position in liquidity option

Golts and Kritzman suggest the S&P 500 can be a useful reference asset for the liquidity option when changes in liquidity are associated with declines in the S&P 500. The reference asset for the liquidity option can be changed depending on data availability. Golts and Kritsman suggest e.g. trading volume, market depth, volatility, or other risk indices associated with illiquidity.

Put option pricing

In their work, Golts and Kritsman (2010) rely on the use of *cliquet* options. A *cliquet* is a series of at-the-money options, with periodic settlement resetting the strike value at the then current price level, at which time the option locks in the difference between the old and new strike and pays that out as the profit. In our paper we simulate a series of plain vanilla European at the money put options instead of using one cliquet. For the synthetic asset we are creating, quarterly European options are easier to implement in the marketplace (exchange traded and liquid).[41] They also provide a lower (more conservative) estimate in terms of the liquidity option price.[42] The plain vanilla European put option pricing formula is given below (Black and Scholes, 1973).

$$P(S,t) = (1-N(d_2))\, K\, e^{-r(T-t)} - (1-N(d_1))\, S$$

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = \frac{\ln(S/K) + (r - \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$$

Whereby $P(S,t)$ is the price of the European put option, S is the price of the underlying reference asset, K is the strike of the option, r is the annualized risk-free interest rate, $T-t$ the time to expiry of the option, σ the implied volatility of the asset returns obtained from Bloomberg[43] and $N(x)$ the standard normal cumulative distribution function

Additional considerations

Commonly used illiquidity proxies such as bid-ask spread or trading volume are not available for private equity. Prior to settling on the liquidity option model we investigate different possible proxies for the illiquidity premium. We consider Listed Private Equity (based on the LPX 50 index returns) but we find that Listed Private Equity does not generate a premium, but instead underperforms public equity from 1993-2009.
A second considered illiquidity premium is based on data provided by Kenneth French.
http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
In this case we examine the returns of the lowest decile stocks in terms of market capitalisation minus the returns on the highest decile stocks. This differential can be used to proxy the illiquidity premium on the basis that venture capital firms tend to invest in very small, fast growing companies. We find the results unsatisfactory and the factor also overlaps with the Nasdaq factor already included in equations (5a) and (5b).

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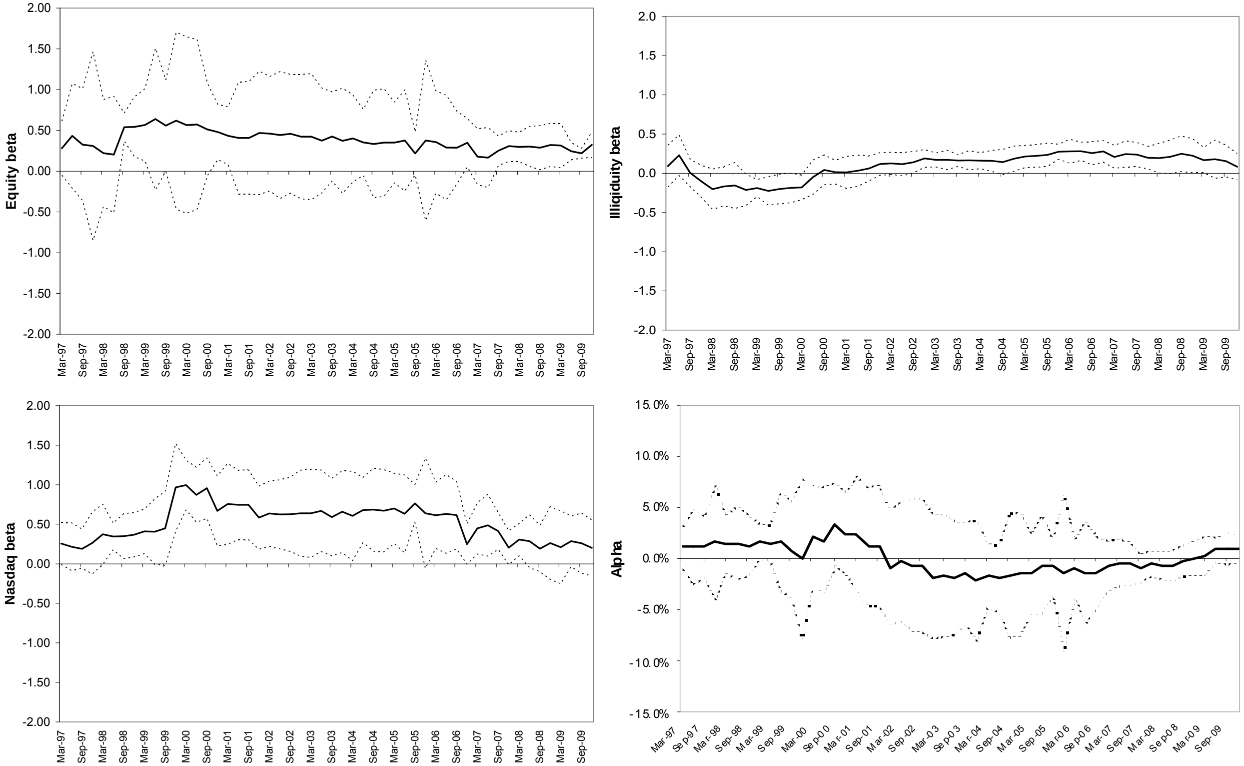
[41] http://www.cboe.com/Products/indexopts/quarterly_spx_spec.aspx
[42] The cliquet is used for investors seeking pricing in advance of a series of events. In general, the pricing of cliquet options is slightly higher than for plain vanilla options, as the reset increases the chance of finishing in the money, refer Resheff and Shparber (2004).
[43] S&P 500 implied volatility data is estimated using Bloomberg from March 2005. Implied volatility prior to that date is based on the differential between implied and historical volatility estimated by Corrado and Miller (2005).

Figure 1: Seven year rolling beta and alpha experience with 95 per cent confidence bands

The solid lines in figure 1 represent the seven year rolling betas in our model, which are calculated by regressing rolling 28 data periods (7 years x 4 quarters) of private equity returns against the identified risk factors. The dotted lines represent the 95 per cent confidence bands.

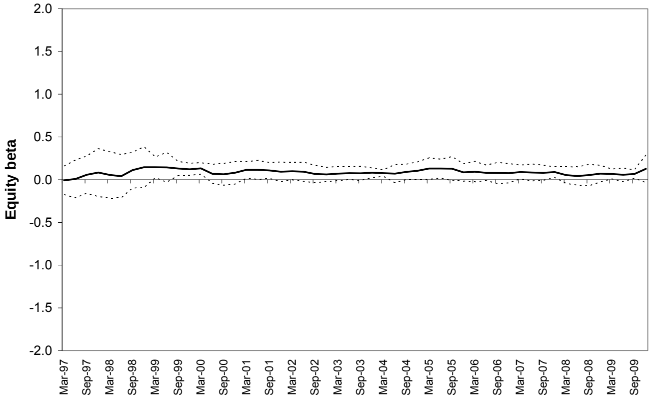
1A. Venture Capital

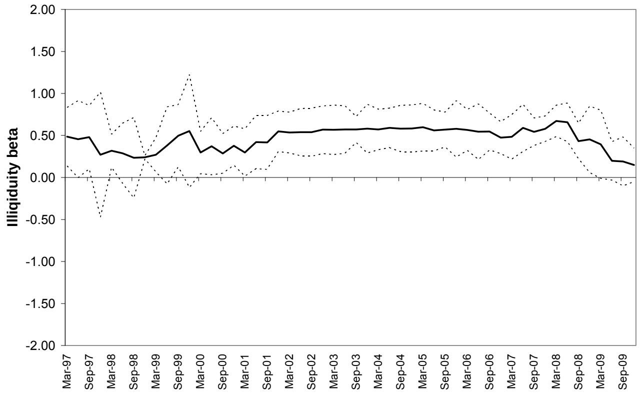
For Venture Capital a period of excess returns is detected in the 1990s coinciding with the technology boom, during which a high exposure to equity and NASDAQ markets is undertaken by Venture Capital managers. More recently, the illiquidity beta increased, suggesting Venture Capital has turned more illiquid, as investors' interest turned towards Buyouts and away from Venture Capital.

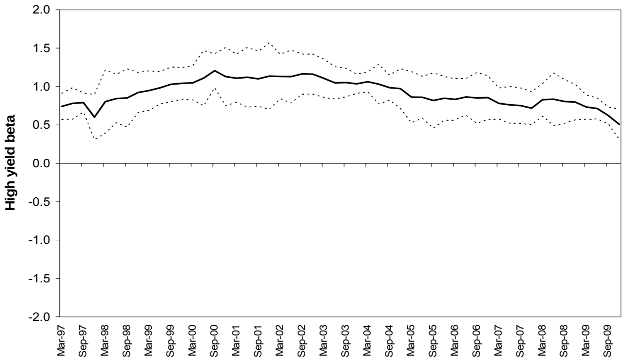


1B. Buyouts

For Buyouts there is limited evidence of equity exposure or excess returns throughout the period. Buyouts continue to have a high exposure to the high yield markets, reflecting the leveraged nature of the strategy.







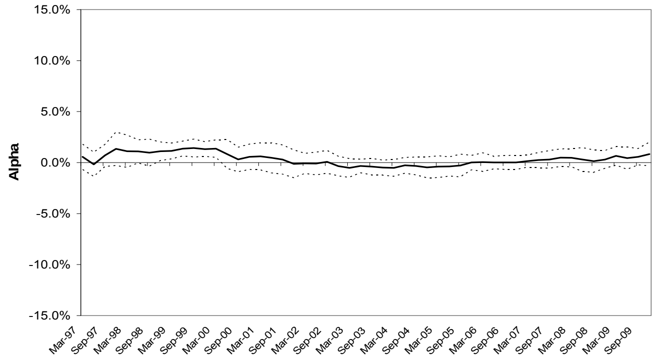


Figure 2 Base case for private equity allocation simulation model

Figure 2 suggests capital investment patterns based on Yale's default settings for contribution rates, fund life, committed capital and capital growth rates. On average, 56 per cent of committed capital is invested over the 12 year period. A peak in invested capital is reached in year 5.

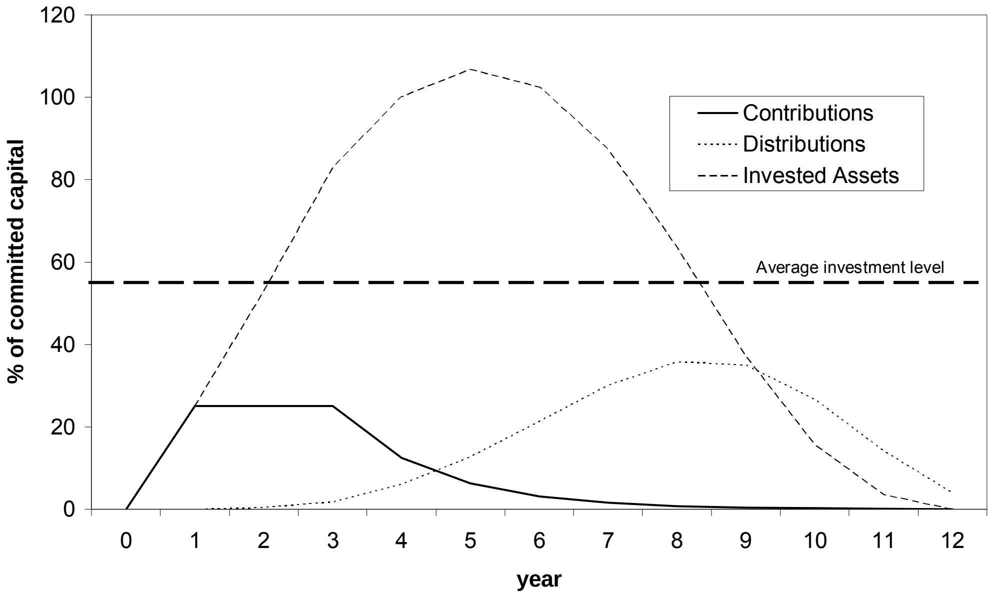
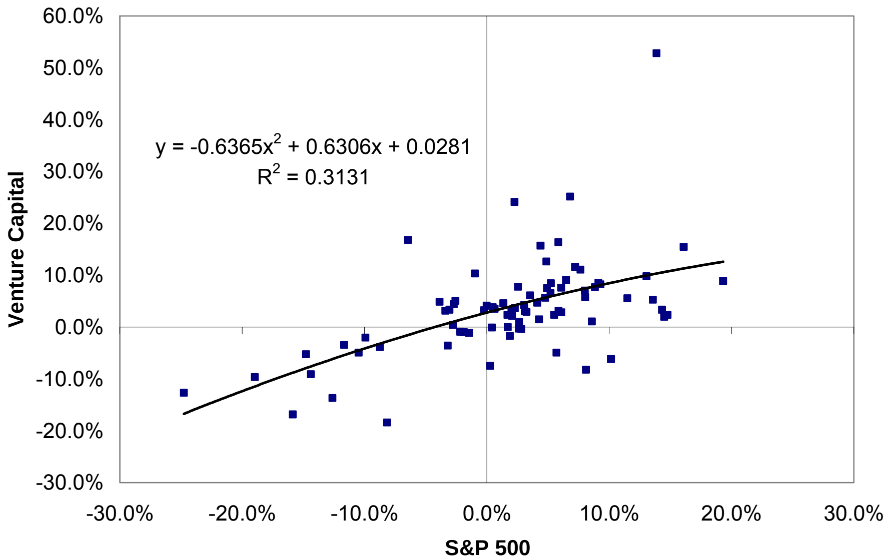


Figure 3 Evidence of market timing skill 1990 – 2009 (prior to conditional overlay)

Figure 3 shows the performance of private equity (venture capital and buyout) firms on the y-axis versus equity up and down markets on the x-axis. A convex relationship between private equity firms and the S&P 500, suggests private equity has defensive abilities. A concave relationship indicates that private equity firm returns deteriorate disproportionally as the equity market declines.



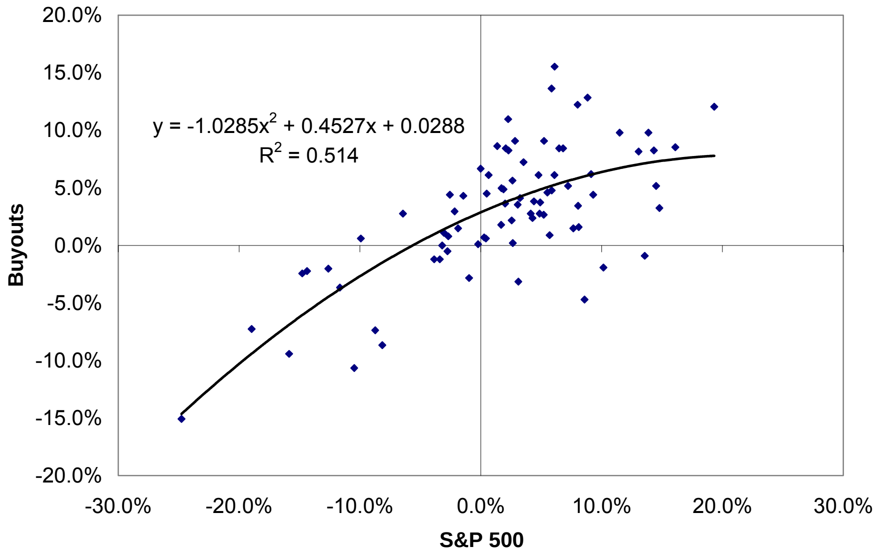
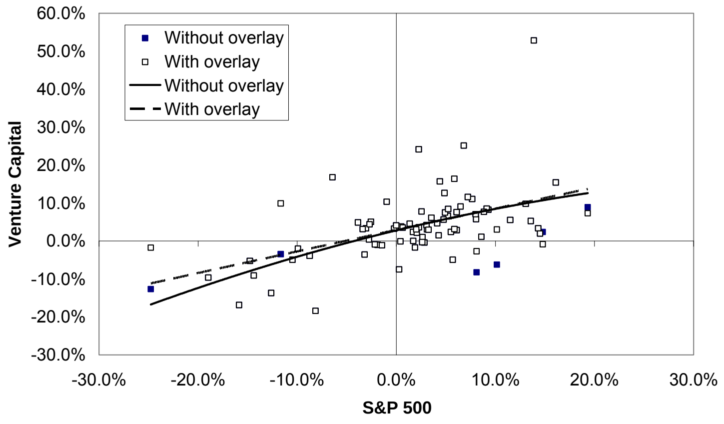


Figure 4 Impact of conditional overlay 1990-2009



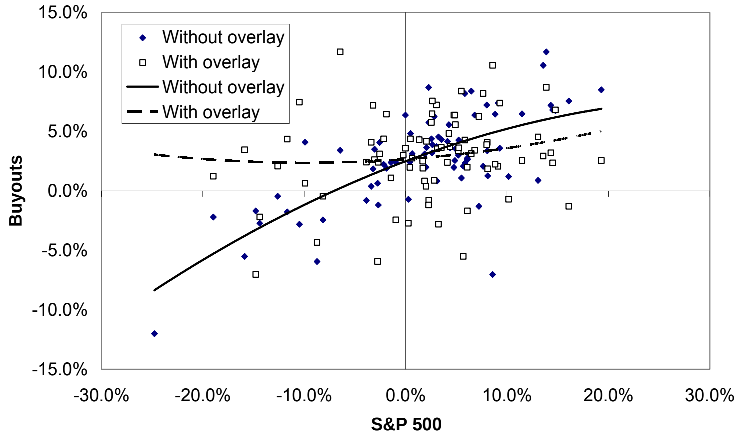


Table 1 An overview of private equity literature on factor (style) analysis

Paper	Factors (style benchmarks)	Results
Gompers and Lerner (1997)	1 factor: S&P 500.	Excess return found, but reduced after stale pricing adjustment.
Swensen (2000)	1 factor: S&P 500.	Buyouts underperform compared to an equally leveraged version of the S&P 500.
Peng (2001)	1 factor: S&P 500 or NASDAQ.	Excess return found, but possibly reflects the sample period which ends with the tech bubble in 1999.
Moskowitz and Vissing-Jorgensen (2002)	1 factor: CRSP index.	No excess return found.
Quigley and Woodward (2003)	1 factor: S&P 500 or NASDAQ.	Excess return found, but based on self-built index, and performance to 1999 (time period bias).
Kaplan and Schoar (2005)	1 factor: S&P 500.	No excess return found.
Jones and Rhodes-Kropf (2003)	1 factor: CRSP value weighted index of NYSE, AMEX and NASDAQ stocks, lagged up to three periods.	Fama and French small cap and value premium. No excess return found.
Ljungqvist and Richardson (2003)	1 factor: S&P 500.	5 per cent excess return found, but from a single undisclosed LP, with 73 investments (sample bias).
Ick (2005)	1 factor: S&P 500, NASDAQ, Russell 2000, Dow Jones Industrial Average, MSCI World.	No excess return found.
Anson (2006)	1 factor, S&P 500, lagged up to 4 periods.	Excess return found, but reduced after stale pricing.
Phalippou et al. (2005, 2007)	1 factor: S&P 500.	Negative 3 per cent excess return after assuming biases.
Conroy and Harris (2007)	1 factor: S&P 500.	No excess return found.
Korteweg and Sorensen (2008)	3 factors: equity premium, Fama and French small cap and value premium, and Monte Carlo correction.	No excess return found.
Franzoni, Nowak and Phalippou (2009)	S&P 500, Fama and French small cap and value premium, liquidity factor corresponding to a long position in high liquidity stocks and a short position in low liquidity stocks (Pástor and Stambaugh, 2003).	No excess return found.

Table 2 Returns to private equity: Descriptive Statistics

Descriptive statistics are calculated using quarterly data from March 1990 to December 2009. The data are net of fees log returns in USD from the Venture Economics (VE) database. The total number of observations is 80 quarters. The total return is annualised by multiplying log quarterly results by 4. Excess returns are the deleveraged return less the 3 month T-bill rate. The s annualised standard deviation is the standard deviation of the log quarterly return scaled by the square root of four. Beta is the estimated slope coefficient from the regression of strategy returns on the S&P 500 index. Liquidity refers to the first order serial correlation.

	Total (%pa)	Excess (%pa)	Standard deviation (% pa)	Best quarter	Worst quarter	Beta	Skew	Kurtosis	Liquidity
Early/Seed VC	13.82%	10.0%	23.4%	58.8%	-22.8%	0.63	1.57	6.51	0.67
Seed Stage VC	6.70%	2.8%	17.3%	29.8%	-22.1%	0.36	0.69	1.81	0.25
Early Stage VC	14.06%	10.2%	23.7%	59.2%	-23.1%	0.64	1.56	6.40	0.67
Balanced VC	14.74%	10.9%	18.3%	53.9%	-17.1%	0.65	1.96	10.57	0.44
Latter Stage VC	14.76%	10.9%	14.9%	37.0%	-18.3%	0.57	0.50	4.80	0.42
All venture	14.41%	10.5%	19.0%	52.8%	-18.4%	0.63	1.60	8.45	0.57
Small buyouts	11.09%	6.9%	10.3%	13.8%	-12.0%	0.27	0.00	0.09	0.37
Medium buyouts	12.62%	8.3%	17.7%	55.7%	-14.5%	0.32	2.51	13.63	0.13
Large buyouts	10.94%	6.7%	12.6%	18.9%	-18.8%	0.30	-0.39	1.44	0.25
Mega buyouts	10.87%	6.6%	12.3%	18.8%	-15.7%	0.33	-0.32	1.13	0.13
All buyouts	11.17%	6.9%	11.3%	15.5%	-15.1%	0.32	-0.56	0.87	0.28

	(5a) $R^{DL}_{it} - R_{ft} = \alpha_i + \beta_{i,m} (R_{mt} - R_{ft}) + \sum (R^{DL}_{i,t-j} - R_{f,t-j}) + \beta_{i,NQ} (R_{NQ,t} - R_{mt}) + \varepsilon_{it}$ $h_{it} = \gamma_0 + \gamma_1\varepsilon^2_{i,t-1} + \gamma_2 h_{i,t-1} + \gamma_3\varepsilon^2_{i,t-1}I_{t-1}$					(5b) $R^{DL}_{it} - R_{ft} = \alpha_i + \beta_{i,m} (R_{mt} - R_{ft}) + \sum (R^{DL}_{i,t-j} - R_{f,t-j}) + \beta_{i,HY} (R_{HY,t} - R_{mt}) + \varepsilon_{it}$ $h_{it} = \gamma_0 + \gamma_1\varepsilon^2_{i,t-1} + \gamma_2 h_{i,t-1} + \gamma_3\varepsilon^2_{i,t-1}I_{t-1}$				
	Early/Seed Venture Capital	Seed Venture Capital	Early Stage Venture Capital	Balanced Venture Capital	Latter Stage Venture Capital	All Venture Capital	Small Buyout	Medium Buyout	Large Buyout	Mega Buyouts	All Buyouts
α	0.0007	0.0001	0.0003	0.0047	0.0038	0.0034	-0.0010	0.0004	-0.0003	-0.0020	0.0011
β_m	0.2630***	0.2356**	0.2443***	0.3937***	0.3105***	0.3300***	0.0071	0.0259	0.0341	0.0248	0.0728***
$\beta_{L,t-1}$	0.4262***	0.4329***	0.4297***	0.2166**	0.2521***	0.3060***	0.0584	0.0628	0.1169**	0.1096**	0.0346
$\beta_{L,t-2}$	0.2145***	0.2648***	0.2640***	0.2855***	0.3080***	0.2305***	0.1802***	0.2146***	0.1516***	0.1709***	0.1816***
$\beta_{L,t-3}$	0.0407	-0.0189	-0.0176	0.0076	0.0119	0.0593	0.2023***	0.1346***	0.1156***	0.0455	0.0434
β_{NQ}	0.4116***	0.4609***	0.4608***	0.4829***	0.5156***	0.3637***					
β_{HY}							0.8411***	0.7688***	0.7409***	0.8886***	0.7825***
γ_0	0.0004**	0.0008	0.0008*	0.0012	0.0001	0.0003***	0.0000	0.0001	0.0000	0.0001	0.0000
$\gamma_{1\ (ARCH)}$	0.5418**	0.5001**	0.5071**	0.2883***	0.1194	0.4554**	0.0467	0.6828	0.2303	-0.0931	0.1088**
$\gamma_{2\ (GARCH)}$	0.5557***	0.3759	0.3763	0.3796	0.8081	0.5273***	0.6494	0.2517	0.8210***	0.1884	0.9030***
γ_3	-0.6962***	-0.5994**	-0.6052**	-0.4337**	-0.0903	-0.5594***	0.0070	-0.1596	-0.01812	0.7389	-0.2781***
$\gamma_1 + \gamma_2 + 0.5\ \gamma_3$	0.7490	0.5763	0.5808	0.4511	0.8825	0.7030	0.6996	0.8547	0.9602	0.0953	0.8727
t-dist errors	14.46	16.02	15.88	18.92	6.57	13.88	15.64	3.89	3.30**	23.79	20.60
Adj R^2	0.7021	0.7128	0.7119	0.6227	0.7594	0.7142	0.8563	0.6851	0.7842	0.8431	0.8638
Log Likelihood	135.45	133.35	132.13	127.42	155.40	151.35	225.20	202.99	214.37	218.70	226.95

* significant at the 10 per cent level ** significant at the 5 per cent level *** significant at the 1 per cent level

Table 3 Results of GARCH model for quarterly deleveraged private equity fund returns (Augmented CAPM) 1990-2009

Table 3 reports estimates of the augmented CAPM based on quarterly data from March 1990 to December 2009 for mean equations (2a) and (2b) and variance equation (4). In the mean equation, the dependent variable ($RDL_{it} - Rf_t$) is the deleveraged excess return of strategy i at time t and is regressed on the market risk premium $(Rm_t - Rf_t)$, the illiquidity premium $(RDL_{i,t}\text{-}1 - R_{f,t-1})$, the NASDAQ premium $(RNQ_{,t} - Rm_t)$ for venture capital or the credit premium $(RHY_{,t} - Rf_t)$ for buyouts. In the variance equation, the conditional variance of strategy i (h_{it}) is regressed on the error of the previous period $(\varepsilon^2_{i,t-1})$ and the variance of the previous period $(h_{i,t-1})$. The mean and variance equations used for each model are shown at the top of the respective tables.

Table 4 Performance of direct investment versus factor model during commitment period

Table 4 presents a comparison of direct investment versus the factor based approach to private equity investing for 1000 simulation runs of 12 year commitment periods. Results are based on cash flow patterns from the Yale allocation model. Simulated returns are generated by resampling (with replacement) over the period 1990 to 2009.

	Venture Capital		Buyouts	
	Direct investment	Factor model	Direct investment	Factor model
Return (%pa)	14.2%	13.5%	8.2%	8.8%
Risk (%)	11.8%	8.7%	2.0%	2.4%
Reward to risk	1.2	1.5	4.0	3.7
Best period (%pa)	123.0%	36.1%	15.1%	15.9%
Worst period (%pa)	-1.2%	-16.6%	2.6%	1.5%
Tracking error (%)		10.7%		1.9%

Returns are presented net of fees in USD and reflect the average annual return of possible return outcomes at the end of the 12 year commitment period. Risk reflects the standard deviation of average return outcomes at the end of the 12 year period. Reward to risk equals return divided by risk. Best period reflects the average annual return under the best possible 12 year outcome. Worst period reflects the average annual return under the worst possible 12 year period. Tracking error is calculated as the standard deviation of the difference in outcome between the direct investment and the factor model over the 12 year period.

Model	(6a) $R^{DL}_{it} - R_{ft} = \alpha_i + \beta_{i,m} (R_{mt} – R_{ft}) + \sum (R^{DL}_{i,t-j} - R_{f,t-j}) + \beta_{i,NQ} (R_{NQ,t} – R_{mt}) + \beta_{i,timing} (R_{mt} - R_{ft})^2 + \varepsilon_{it}$ $h_{it} = \gamma_0 + \gamma_1\varepsilon^2_{i,t-1} + \gamma_2 h_{i,t-1} + \gamma_3\varepsilon^2_{i,t-1} I_{t-1}$						(6b) $R^{DL}_{it} - R_{ft} = \alpha_i + \beta_{i,m} (R_{mt} – R_{ft}) + \sum (R^{DL}_{i,t-j} - R_{f,t-j}) + \beta_{i,HY} (R_{HY,t} – R_{mt}) + \beta_{i,timing} (R_{mt} - R_{ft})^2 + \varepsilon_{it}$ $h_{it} = \gamma_0 + \gamma_1\varepsilon^2_{i,t-1} + \gamma_2 h_{i,t-1} + \gamma_3\varepsilon^2_{i,t-1} I_{t-1}$				
	Early/Seed Venture Capital	Seed Venture Capital	Early Venture Capital	Balanced Venture Capital	Latter Stage Venture Capital^	All Venture Capital	Small Buyout	Medium Buyout	Large Buyout	Mega Buyouts	All Buyouts
α	0.0040	0.0042	0.0040	0.0069	-0.0020	0.0043	-0.0021	0.0015	-0.0004	-0.0038	0.0017
β_m	0.2073**	0.0175	0.2073**	0.3537***	0.2806***	0.3152***	0.0086	0.0291	0.0355	0.0348	0.0751***
$\beta_{L,\ t-1}$	0.4259***	0.2373***	0.4259***	0.2266***	0.3149***	0.2196***	0.0662	0.0454	0.1190**	0.1110***	0.0278
$\beta_{L,\ t-2}$	0.2659***	0.1823*	0.2659***	0.2371***	0.3042***	0.2212***	0.1825***	0.2150***	0.1508***	0.1785***	0.1717***
$\beta_{L,\ t-3}$	-0.0284	0.1883**	-0.0284	-0.0149	0.0947&	0.0717	0.2015***	0.1276**	0.1149***	0.0551	0.0269
β_{NQ}	0.4605***	0.5736***	0.4605***	0.3637***	0.4385***	0.4189***	0.8338***	0.7546***	0.7383***	0.8910***	0.7801***
β_{HY}											
β_{timing}	-0.4716	-0.3700	-0.4716	-0.2360	0.5433	-0.1336	-0.0002	-0.0767	0.0128	0.0655	-0.0646
γ_0	0.0007**	0.0011	0.0007**	0.0012	0.000	0.0001	0.0002	0.0001	0.0000	0.0001	0.0000***
$\gamma_{1\ (ARCH)}$	0.4989**	0.7197	0.4989**	0.3291	0.0858	0.4413	-0.1218	0.7917	0.2495	-0.1307	0.0375
$\gamma_{2\ (GARCH)}$	0.3724	0.4023	0.3724	0.3526	0.7011***	0.7720***	0.1388	0.2563	0.8053***	0.4677	0.9719***
γ_3	-0.5986**	-0.7798	-0.5986**	-0.4914	0.2431	-0.6563	-0.3843	-0.2953	-0.2071	0.5568	-0.2359***
$\gamma_1 + \gamma_2 + 0.5\ \gamma_3$	0.5720	0.7321	0.5720	0.4360	0.9085	0.8851	-0.4363	0.9003	0.9513	0.6154	0.8914
t-dist errors	15.66	662.07	15.66	3.69	7.54	2.82**	491.90	4.11	3.25	23.59	22.89
Adj R^2	0.7030	0.4310	0.7030	0.5853	0.7424	0.7056	0.8542	0.6752	0.7815	0.8321	0.8596
Log Likelihood	134.13	114.73	134.13	137.28	164.00	159.69	226.11	203.10	214.38	218.19	226.91

* significant at the 10 per centlevel ** significant at the 5 per centlevel *** significant at the 1 per centlevel

^ A dummy variable is introduced as a variance regressor for the period from January 1995 to December 2009.to stabilise the GARCH model.

Table 5 Results of GARCH model for quarterly deleveraged private equity fund returns (Augmented Treynor and Mazuy model) 1990-2009

Table 5 presents the Treynor and Mazuy (market timing) model based on mean equations (3a) and (3b) and variance equation (4). The variables are as defined for table 3. In addition, in the mean equation, a market timing factor $(R_{mt} - R_{ft})^2$ is added.

Table 6 Impact of conditional overlay 1990-2009

	without overlay		with overlay	
	VC	BO	VC	BO
Return (%pa)	14.4%	11.2%	16.0%	12.1%
Risk (%pa)	19.0%	7.8%	18.5%	7.2%
Skew	1.6	-0.8	1.7	-0.4
Kurtosis	8.5	2.2	9.3	0.6
Reward to risk	0.8	1.4	0.9	1.7
Downside risk measures				
Semi-deviation (%)	7.8%	3.7%	1.6%	1.7%
Worst quarter (%)	-18.4%	-12.0%	-18.4%	-7.0%

Table 6 represents the impact on Venture Capital (VC) and Buyouts (BO) of applying the conditional overlay on historical quarterly data. Returns are presented net of fees in USD based on annualised quarterly data. Risk is calculated by multiplying quarterly standard deviation by the square root of 4. Reward to risk is defined as the return divided by the risk. Skew represents the skew of the quarterly returns. Kurtosis refers to the excess kurtosis versus a normal distribution. Semi-deviation refers to the standard deviation of down quarters only. Worst quarter represents the worst possible quarter over the period measured.

Table 7 Robustness check for hedge overlay parameters (1990-2009)
7A Sensitivity versus the S&P 500

Venture Capital (VC)				**Buyouts (BO)**			
Incremental Return (%pa) ^		**VIX**		**Incremental Return (%pa) ^**		**VIX**	
S&P 500 beta	**25**	**30**	**35**	**S&P 500 Beta**	**25**	**30**	**35**
0.0	3.1%	2.0%	1.3%	**0.0**	0.2%	1.0%	0.8%
0.1	2.8%	1.9%	1.3%	**0.1**	-0.1%	0.9%	0.8%
0.2	2.6%	1.8%	1.3%	**0.2**	-0.4%	0.8%	0.7%
0.3	2.3%	1.7%	1.3%	**0.3**	-0.7%	0.8%	0.7%
0.4	2.0%	1.6%	1.2%	**0.4**	-1.0%	0.6%	0.7%
0.5	1.7%	1.6%	1.2%	**0.5**	-1.2%	0.6%	0.7%
0.6	1.5%	1.5%	1.2%	**0.6**	-1.5%	0.5%	0.6%
0.7	1.2%	1.4%	1.1%	**0.7**	-1.8%	0.4%	0.6%
0.8	0.9%	1.3%	1.1%	**0.8**	-2.0%	0.3%	0.6%
0.9	0.6%	1.2%	1.1%	**0.9**	-2.3%	0.3%	0.5%
1.0	0.4%	1.2%	1.1%	**1.0**	-2.6%	0.2%	0.5%
Risk reduction (%pa)*		**VIX**		**Risk reduction (%pa)***		**VIX**	
S&P 500 beta	**25**	**30**	**35**	**S&P 500 Beta**	**25**	**30**	**35**
0.0	4.5%	2.2%	1.5%	**0.0**	0.7%	4.7%	4.7%
0.1	6.0%	2.8%	2.0%	**0.1**	3.8%	7.3%	7.1%
0.2	7.0%	3.1%	2.3%	**0.2**	4.4%	8.6%	8.2%
0.3	7.6%	3.2%	2.5%	**0.3**	2.5%	8.6%	8.0%
0.4	7.8%	3.1%	2.4%	**0.4**	-1.8%	6.4%	6.6%
0.5	7.5%	2.8%	2.1%	**0.5**	-8.2%	6.3%	4.0%
0.6	6.8%	2.2%	1.6%	**0.6**	-16.3%	-0.4%	2.0%
0.7	5.6%	1.5%	0.9%	**0.7**	-25.9%	-5.5%	-4.3%
0.8	4.1%	0.5%	0.1%	**0.8**	-36.4%	-11.6%	-10.7%
0.9	2.1%	-0.6%	-1.0%	**0.9**	-48.0%	-18.3%	-15.8%
1.0	-0.2%	-1.9%	-2.2%	**1.0**	-60.4%	-25.6%	-23.6%

^ Incremental return is calculated as the additional percentage return compared to the unhedged portfolio
* Risk reduction is calculated as the percentage reduction in annualised standard deviation compared to the unhedged portfolio
The numbers in the highlighted rectangle represent the base case for the hedge.

7B Sensitivity versus the NASDAQ (for VC) and the High Yield premium (for BO)

Venture Capital (VC)				Buyouts (BO)			
Incremental Return (%pa) ^		**VIX**		**Incremental Return (%pa) ^**		**VIX**	
NASDAQ beta	**25**	**30**	**35**	**High Yield Beta**	**25**	**30**	**35**
0.0	-1.4%	-0.4%	-0.1%	**0.0**	-0.3%	-0.1%	-0.1%
0.1	-0.6%	0.1%	0.2%	**0.1**	-0.3%	0.0%	0.0%
0.2	0.2%	0.6%	0.5%	**0.2**	-0.2%	0.2%	0.2%
0.3	1.0%	1.1%	0.9%	**0.3**	-0.2%	0.3%	0.3%
0.4	1.7%	1.6%	1.2%	**0.4**	-0.2%	0.4%	0.4%
0.5	2.5%	2.1%	1.5%	**0.5**	-0.2%	0.5%	0.5%
0.6	3.3%	2.5%	1.9%	**0.6**	-0.2%	0.6%	0.6%
0.7	4.1%	3.0%	2.2%	**0.7**	-0.2%	0.8%	0.8%
0.8	4.9%	3.5%	2.5%	**0.8**	-0.1%	0.9%	0.9%
0.9	5.6%	4.0%	2.9%	**0.9**	-0.1%	1.0%	1.0%
1.0	6.4%	4.5%	3.2%	**1.0**	-0.1%	1.2%	1.1%
Risk reduction (%pa)*		**VIX**		**Risk reduction (%pa)***		**VIX**	
NASDAQ beta	**25**	**30**	**35**	**High Yield Beta**	**25**	**30**	**35**
0.0	5.3%	0.0%	1.1%	**0.0**	7.1%	4.1%	4.1%
0.1	7.4%	1.1%	1.6%	**0.1**	8.6%	5.1%	5.1%
0.2	7.9%	2.1%	1.6%	**0.2**	9.9%	6.0%	6.0%
0.3	8.4%	2.7%	2.1%	**0.3**	9.9%	6.8%	6.8%
0.4	7.5%	2.7%	2.1%	**0.4**	9.8%	7.2%	7.2%
0.5	5.8%	2.7%	2.1%	**0.5**	9.0%	7.6%	7.6%
0.6	3.7%	2.7%	1.6%	**0.6**	7.8%	7.6%	7.6%
0.7	0.6%	1.6%	1.3%	**0.7**	6.0%	7.6%	7.6%
0.8	-3.7%	1.1%	1.1%	**0.8**	3.8%	7.2%	7.2%
0.9	-7.9%	0.0%	0.0%	**0.9**	1.1%	6.7%	6.7%
1.0	-12.6%	-1.6%	-1.0%	**1.0**	-1.9%	5.9%	5.9%

^ Incremental return is calculated as the additional percentage return compared to the unhedged portfolio
* Risk reduction is calculated as the percentage reduction in annualised standard deviation compared to the unhedged portfolio
The numbers in the highlighted rectangle represent the base case for the hedge.

Table 7C Performance of conditional overlay (1000 runs of 20 years)

	Venture Capital	
Percentile	Incremental Return (%pa)^	Risk Reduction (%pa)*
0.05	-0.2%	-0.2%
0.10	0.0%	0.2%
0.25	0.5%	1.2%
0.50	1.2%	2.3%
0.75	2.0%	4.0%
0.90	3.0%	6.2%
0.95	3.6%	7.9%

	Buyouts	
Percentile	Incremental Return (%pa)^	Risk Reduction (%pa)*
0.05	-0.3%	-1.5%
0.10	-0.2%	-0.7%
0.25	0.0%	0.4%
0.50	0.4%	3.8%
0.75	1.0%	7.8%
0.90	1.5%	10.6%
0.95	1.8%	12.0%

^ Incremental return is calculated as the additional percentage return compared to the unhedged portfolio
* Risk reduction is calculated as the percentage reduction in annualised standard deviation compared to the unhedged portfolio