Estimating insurer´s capital requirements through Markov switching models in the Solvency II framework

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ABSTRACT: Solvency II will transform the system for determining capital requirements for insurers. The new regulatory framework proposes a standard model, but at the same time, it encourages the use of internal models of self-evaluation and risk management. This paper attempts to assess the adequacy of Markov switching models for the design of internal models of insurers' equity risk exposure. We have used monthly data from four of the main European indices for the period between January1990 and January 2010. The comparison of models across different statistical criteria and backtesting shows the superiority of Markov switching models over simpler models, to capture insurers' equity risk. Subsequently, we compared capital requirements resulting from applying these models against the Solvency II proposal. The results showed that the funds needed to take the equity risk are dependent on the specification used. Also, the capital raised by Markov switching specifications exceeds those of the standard model. This means that companies using the standard model or another based on similar assumptions are underestimating the risk actually assumed.

I. Introduction

The new UE solvency regulation, known as Solvency II, involves the revision of standards for evaluating the financial condition of insurance companies. Under the new framework, the determination of capital requirements can be done through a standard model or internal models previously approved by the regulator. On 10 November 2009, the ECOFIN (Council of Ministers of Economy and Finance of the European Union) has adopted the new Directive, and, therefore, the transposition into national standards should be carried out before the end of 2012. The standard model development was carried out through a total of five quantitative impact studies (QIS) conducted by the Committee of European Insurance Supervisors (CEIOPS). The new legislation promotes the use of alternative internal models for managing risk and determining capital requirements.

Equity risk is included within the market risk module and, under the new Solvency II Directive (Commission of the European Communities, 2009), measures the sensitivity of the value of assets, liabilities and financial instruments to changes in the level or volatility of market share prices. This risk is the main component of the investment portfolios of European insurers, accounting for 34.3% in 2007. The model of normal returns implicit in the calculation of the standard formula has been chosen for simplicity. However, the normality assumption may underestimate the tail of the distribution of losses and not capture the variations in volatility possibly variability, making it less suitable to represent longer periods of time (Hardy, 2001). The returns may have other properties such as serial autocorrelation in the mean, volatility not constant over time (heteroscedasticity) and clusters of volatility. Autoregressive models AR (p) and ARMA are adequate to capture the serial dependence that exists among the asset returns. In addition, the variance may vary over time, in which case the ARCH and GARCH models may be appropriate as conditional variance modeled itself on the basis of past values of the variable itself (Engle 1982, Bollerslev, 1986). In subsequent proposals, the models been have adapted to incorporate the so-called leverage effect among which are the exponential GARCH or

EGARCH (Nelson, 1991) and the GJR-GARCH (Glosten et al. 1993). However, the models mentioned above are not able to capture sudden changes in market behavior that happen in crisis situations or to sudden changes in government policy. The transformation of the previous models introducing regime switching may be better suited to measuring the risk of equity. In this sense, some studies have shown the superiority of these models for series of short-term yields (Li and Lin, 2004, Maekawa et al., 2005; Rapach and Strauss, 2005; and Sajjad, Coakley and Nankervis, 2008). However, in the framework of Solvency II, market risk is determined by the VaR at 99.5% within one year. For this reason, some studies have evaluated the adequacy of the models using other periods of analysis (Hardy (2001), Won and Chang (2005) and Hardy et al. (2006)).

This paper analyzes the suitability of Markov regime-switching for building internal models of equity risk. Thus, we analyze their ability to capture the dramatic changes observed in the behavior of markets. Several models have been calibrated to the monthly series of major European indices and then compared using different criteria. Subsequently, Monte Carlo simulation has been carried out to calculate capital requirements under the specifications of Solvency II. This study contributes to existing literature providing a new approach that has not yet been applied to the European market. Furthermore, unlike previous studies, we have included a variety of regime switching models and the selection has been done through the implementation of backtesting and not based solely on statistical criteria. The results could be very useful for the calibration of the standard model and, also, for insurance companies wishing to opt for an internal model.

The paper is structured as follows. In section 2 we review the previous literature. Subsequently, section 3 outlines the models that will be analyzed. Section 4 discusses the series of indices used in the empirical analysis. Section 5 presents the estimation and evaluation of models, and section 6 makes a comparative analysis of the results of applying the proposed models for determining capital requirements. Finally, conclusions are presented.

II. Theoretical Framework

The evaluation of equity risk models must take into account the particularities of this type of exposition in the case of insurance companies. This is very important because the models that best project the behavior of short term returns may not be the most suitable for analyzing long term risk. In this sense, most studies have shown the superiority of different specifications of regime switching models, but using series of short term yields (Li and Lin, 2004, Maekawa et al., 2005; Rapach and Strauss, 2005; and Sajjad, Coakley and Nankervis, 2008). However, a number of authors have focused on long term risk analysis for insurers evaluating several basic RS models using monthly returns and obtained similar results. Thus, Hardy (2001) evaluated the suitability of different models, concluding that the Regime Switching Lognormal Model provides a better fit than the other models tested. Bayliff and Pauling (2003) conducted a comparison of the lognormal model, lognormal autoregressive, GARCH and RSLN, and concluded that the models that have mean reversion and fat tails are the ones that provide the best fit for long-term data. Wong and Chan (2005) employed two new models, the mixture autoregressive (MAR) and the mixture of ARCH (MARCH), for monthly returns of the TSE 300 and S&P 500. The results of this paper show that new models fit worse than the regime switching (RSLN) alternative, although they may be useful for modeling tails. A similar study was performed by Hardy et al. (2006) who compared a large number of models estimated by maximum likelihood for the return of the S&P-500, reaching the conclusion that regimen switching has the best fit in the left tail of the distribution and, therefore, is more suitable for risk assessment. Finally, Boudreault and Panneton (2009) conducted a comparison of different multivariate models for monthly returns in Canadian, American, British and Japanese markets. In general, it appears that GARCH models have a better adjustment than regime switching, but the latter represent the thick tails of the distribution much better.

3. Models Considered for Equity Risk Assessment.

Table 2 lists the specifications of the models that have been evaluated for consideration as alternatives to incorporate into an internal model. In particular, we have considered the lognormal model, because it is implicit in the Solvency II standard framework, GARCH and EGARCH models and their variants of regime switching. Thus we try to evaluate if the addition of regime switching is suitable for modeling the long term risk of equity.

Models description

Regimen switching lognormal model (RSLN)

Regime switching was introduced by Hamilton (1989), who described an autoregressive regime switching process. In Hamilton and Susmel (1994) several regime-switching models are analyzed, varying the number of regimes and the form of the model within regimes. The regime-switching lognormal model was proposed for the purpose of modeling long-term equity returns in Hardy (2001), with further discussion in Hardy (2003). This model uses a Markov chain $i = \{1,2 \ldots .\}$ which represents the evolution of the state of the economy, which can be in two possible situations known as regimes. In each of the regimes, returns follow a normal independent distribution where the parameters are different for each regime i, namely:

$$y_t = \mu_i + \sigma_i Z_t \quad (i = 1,2) \; (1)$$

Where: $Z_t \sim N(0; 1)$; $t = 1,2, \ldots n$; and i represents each of the regimes.

Regimen switching GARCH

This model was described by Gray (1996) who proposes an alternative that combines the main features of the GARCH model within a regime-switching framework. This was subsequently modified by Klaassen (2002), and Marcucci (2005) compares a set of GARCH, EGARCH and GJR-GARCH models within an MS-GARCH framework (Gaussian, Student's t and Generalized Error Distribution for innovations) in terms of their ability to forecast S&P100 volatilities. The model analyzed is a RS-GARCH(1,1) with two regimes and constant in the equation of the mean, i.e.:

$$y_t = \mu_{i,t} + \varepsilon_t \text{ where } \varepsilon_t = \sigma_{i,t} z_t \quad (2)$$

Unlike RSLN model, which assumes a constant volatility, in regimen switching GARCH models volatility varies according to an ARMA process, so that the equation for the variance for each regime is given by:

$$\sigma_{i,t}^2 = \omega_i + \alpha_i(\varepsilon_{t-1})^2 + \beta_i \sigma_{t-1}^2 \quad (i = 1,2) \ (3)$$

Cai (1994) and Hamilton and Susmel (1994) argue that MS-Garch models are intractable because the conditional variance depends on the entire past history of the data. Gray (1996) proposes the possibility of constructing a measure σ_t^2 that is not path dependent:

$$\sigma_t^2 = p_{1,t}(\mu_1^2 + \sigma_{1,t}^2) + (1 - p_{1,t})(\mu_2^2 + \sigma_{2,t}^2) - (p_{1,t}\mu_1 + (1 - p_{1,t})\mu_2)^2 \ (4)$$

Under this approach the conditional variance depends on the regime alone, and not on the entire history of the process

Regimen Switching E-GARCH

The asymmetric behavior of stock volatility has been well described in the literature. Nelson (1991) proposes the EGarch model to capture this asymmetry. Equity returns tend to display more volatility in the case of a negative shock than after a positive shock of equal magnitude. The equation for the variance for each regime is given by:

$$\log(\sigma_{i,t}^2) = \omega_i + \alpha_i \left[\left| \frac{\varepsilon_{t-1}}{\sqrt{\sigma_{p_t,t-1}^2}} \right| - \sqrt{2/\pi} \right] + \beta_i \log(\sigma_{t-1}^2) + \delta_i \frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}} \ , (i = 1,2) \ (5)$$

The logarithmic form for the conditional variance σ_t^2 ensures positive values, avoiding the constraints of non negativity characteristics in the estimation of Garch models. As Henry (2008) says, since δ tipically has a negative sign, a negative innovation generates more volatility than a positive one of equal magnitude.

Models estimation

As Hardy et al. (2006) says, the estimation for this models can be problematic as the likelihood surface has many local maxima, many of which are very close to the global maximum. This means that many parameter sets are more or less equally valid under maximum likelihood estimation, even though the outcome may look very different. Those models assume that the regime switching are exogenous and there are fixed probability for each regime changes, where the regime is the realization of a M-state Markov chain (Hamilton, 2005):

$$\Pr\left(S_{t=j}\middle|S_{t-1}=i,S_{t-2}=k,\ldots..\right)=\Pr\left(S_{t=j}\middle|S_{t-1}=i\right)=p_{ij} \quad (6)$$

The transition matrix P contains transition probabilities p_{ij}, giving the probability that state i will be followed by state j. The estimation of the parameters can be done maximizing by numerical optimization the log-likelihood function:

$$Log\, f(Y1,\ldots\ldots\ldots.,Y_T;\theta)=\sum_1^T \log f(y_t,\Omega_{t-1};\Theta) \quad (7)$$

In the case of two regimes the value of the conditional probability density of the return is:

$$f(y_t,\Omega_{t-1};\Theta)=\sum_{i=1}^{2}\sum_{J=1}^{2}\pi_{i,t-1}\cdot p_{ij}\cdot f(y_t\mid S_t=j,\Omega_{t-1};\Theta) \quad (8)$$

Where the probability that the observed regime at time t evolves according to the following equation:

$$\pi_{i,t}=\frac{\sum_{J=1}^{2}\pi_{i,t-1}\cdot p_{ij}\cdot f(y_t\mid S_t=j,\Omega_{t-1};\Theta)}{f(y_t,\Omega_{t-1};\Theta)} \quad (9)$$

Table 1. Specification of the models used in the analysis

Model	Specification [1]
Normal	$y_t = \mu + \sigma Z_t$
GARCH (1,1)	$y_t = \mu + \varepsilon_t;\;\; \varepsilon_t = \sigma_t z_t;\;\; \sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$
EGARCH (1,1)	$y_t = \mu + \varepsilon_t;\;\; \varepsilon_t = \sigma_t z_t; \log(\sigma_t^2) = \omega + \alpha_1 \left\lvert \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right\rvert + \gamma_1 \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_1 \log(\sigma_{t-1}^2)$
RSLN (k regímenes)	$y_t = \mu_i + \sigma_i Z_t;\;\; (p_t = 1, \ldots, k)$
RS-GARCH (1,1)	$y_t = \mu_i + \varepsilon_t;\;\; \varepsilon_t = \sigma_{i,t} z_t;\;\; \sigma_{i,t}^2 = \omega_i + \alpha_i (\varepsilon_{t-1})^2 + \beta_i \sigma_{t-1}^2;\;\; (i = 1{,}2)$
RS-EGARCH (1,1)	$y_t = \mu_i + \varepsilon_t; \varepsilon_t = \sigma_{i,t} z_t;\; \log(\sigma_{i,t}^2) = \omega_i + \alpha_i \left[\left\lvert \frac{\varepsilon_{t-1}}{\sqrt{\sigma_{p_t,t-1}^2}} \right\rvert - \sqrt{2/\pi} \right] + \beta_i \log(\sigma_{t-1}^2) + \delta_i \frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}}\;, (i = 1{,}2)$

IV.- Empirical Analysis of The Series

The data analyzed to calibrate equity risk comprises monthly observations of four of the main European stock market index returns (FTSE 100, CAC40, DAX and IBEX-35). Because insurance companies must protect the interests of their policyholders in the medium and long term, it is common to use this data frequency [see Hardy (2001), Panneton (2003), Wong and Chan (2005), Hardy et al. (2006) or Boudreault and Panneton (2009)], and even the CEIOPS has calibrated the solvency standard European model on a quarterly basis. Descriptive statistics and the information relative to normality (Jarque Bera Test), are shown in table 2.

Table 2. Descriptive statistics, autocorrelation and normatility test of the main Market Index

	CAC	FTSE	IBEX	DAX
Mean	0,0027	0,0033	0,0068	0,0061
Median	0,0123	0,0075	0,0100	0,0137
Maximum	0,1259	0,1088	0,2342	0,1937
Minimum	-0,1923	-0,1395	-0,2340	-0,2933
Std,Dev,	0,0573	0,0428	0,0666	0,0638
Skewness	-0,5326	-0,5756	-0,2675	-0,8101
Kurtosis	3,4354	3,6043	4,2821	5,4654

[1] $Z_t \sim N(0;1)$: z_t is a standardized noise, i.e., consisting of independent variables with zero mean and variance equal to 1. In this paper we have used a normal white noise, but we can use other asymmetric distributions such as Student t or generalized error distribution (GED).

JBstatistic	13,1325	16,9036	17,3706	83,0442
Probability	0,0014	0,0002	0,0002	0,0000
Observations	238	240	216	229

As can be seen in the different indices (Table 2 and Fig. 1), the mean return is not very important and the skewness is significant and negative, implying a possible leverage effect in the data, and the kurtosis is significant higher than that of a Gaussian distribution, indicating fat-tailed returns. This is consistent with the values found at Jarque-Bera statistic, which rejects the hypothesis of normality in the yields behavior of different indexes.

As shown in Fig.2, yields have clusters of volatility. This feature can be seen in the figure of the square of the logarithmic returns. For this reason it is important to determine which model best fits the behavior of the variance over time. The simple autocorrelation function (ACF) and partial autocorrelation function (PACF), represented in Fig. 3, of the squared returns show a strong dependence structure, which implies the existence of dependence in the variance of monthly returns. This means that the models not considering a constant volatility over time may be more appropriate for assessing market risk.

Fig.1. Histogram of monthly logarithmic returns and time series plot

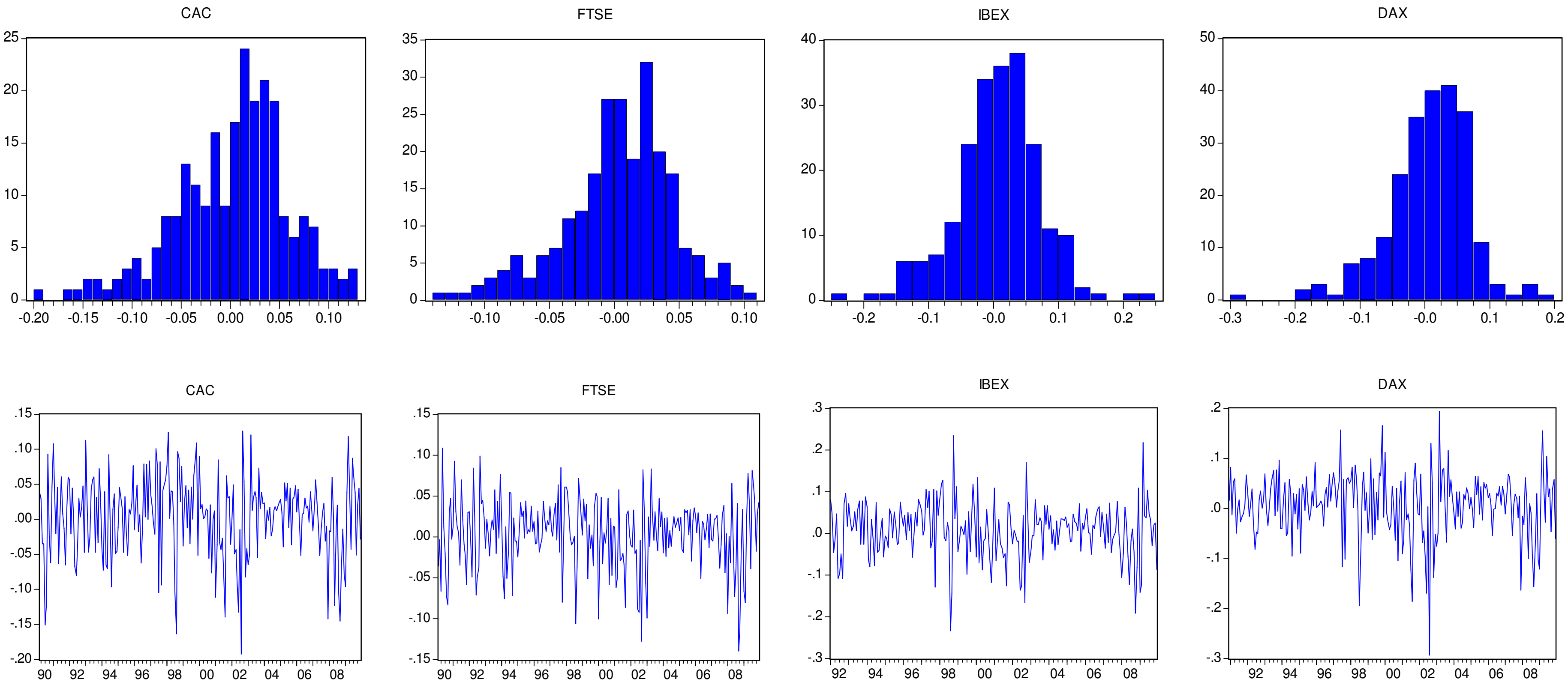
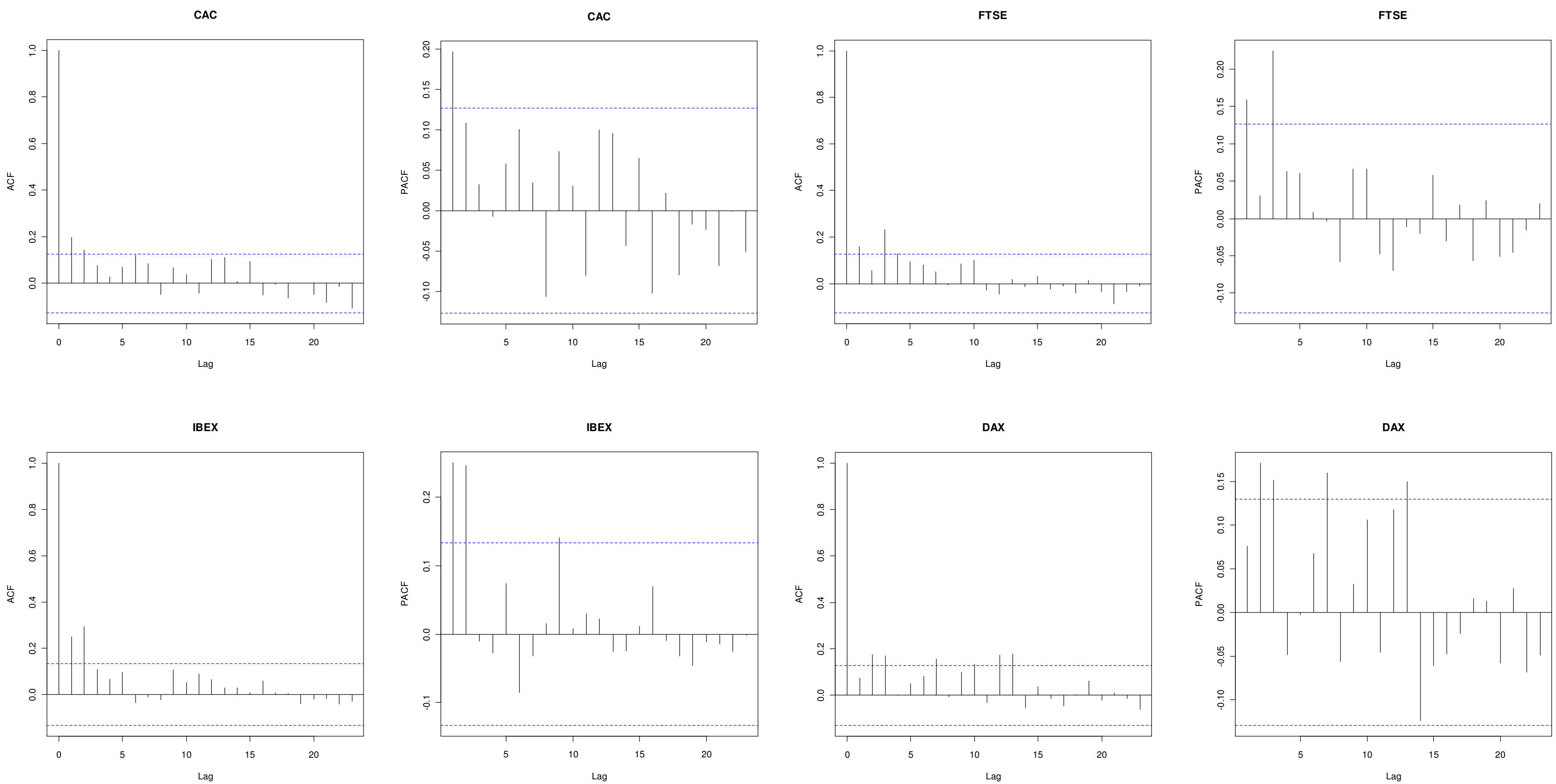


Fig. 2.- Auto correlation function (ACF) and partial auto correlation function (PACF) plots of squared returns.



V.- Models Estimation and Comparison

In this section we present the results of estimating the models from the analyzed series and comparing them, using different statistical criteria. All the models have been estimated using TSM, an Ox package Developed by James Davidson, and E-Views 6. Table 4 shows the parameters resulting from the maximum likelihood estimation. The model of two regimes (RSLN2), as is the case with the results obtained by Hardy (2001.2006) provides a more stable regime with positive expected returns, and a more volatile one with a negative expected return. The high significance of the parameters of the equation for the variance of the GARCH models is consistent with the persistence of volatility in the empirical data. As can be seen, the estimation of regime switching models involves a large number of parameters, and thus the complexity of the estimation. In fact, E-GARCH Regime switching could not be estimated since the algorithm presented convergence problems and a robust solution could not be obtained.

Table 3. Model´s parameters estimated by maximum likelihood

		CAC 40	FTSE 100	IBEX-35	DAX
LOGNORMAL	**Intercept**	0.0026	0.0033	0.0068	0.0060
	S.D.	0.0572	0.0425	0.0664	0.0636
Garch (1,1)	**Intercept**	0.0062	0.0067	0.0106	0.0096
	intercept garch	0.0004	0.0001	0.0001	0.0003
	Alfa	0.1819	0.1989	0.1735	0.1408
	Beta	0.7162	0.7624	0.8103	0.8011
EGARCH(1,1)	**Intercept**	0.0059	0.0068	0.0106	0.0086
	intercept egarch	-1.4569	-1.0561	-0.7445	-0.7298
	assymetry (nu)	-0.1951	-0.1063	-0.0818	-0.0512
	Alfa	0.3117	0.3213	0.2891	0.2554
	Beta	0.7949	0.8762	0.9077	0.9047
RSLN2	**P(1|2) P(2|1)**	0.9316/0.1011	0.9560/0.0220	0.9572/0.0507	0.9777/0.0170
		Regime 1:	**Regime 1:**	**Regime 1:**	**Regime 1:**
	Mean/S.D.	0.0152/0.0394	0.0111/0.0204	-0.0042/0.0831	-0.00388 /0.08393
		Regime 2:	**Regime 2:**	**Regime 2:**	**Regime 2:**
	Mean/S.D.	-0.0156/0.07221	-0.00126/0.05064	0.01898/0.0375	0.0140/0.0388
RSLN3	**P(1|1),P(2|1)**	0.9304/0.0614	0.3649/0.0217	0.8281/0.1657	0.9699/0.0002
	P(2|2),P(3|2)	0.69513/0.30264	0.6303/0.96587	0.9449/0.0546	0.5638/0.4330
		Regime 1:	**Regime 1:**	**Regime 1:**	**Regime 1:**
	Mean/S.D.	0.01004/0.03803	0.0588/0.0035	-0.01261/0.11705	-0.00497/0.08502
		Regime 2:	**Regime 2:**	**Regime 2:**	**Regime 2:**
	Mean/S.D.	-0.0648/0.0205	-0.0033/0.0504	0.0199/0.0378	-0.0033/0.0365
		Regime 3:	**Regime 3:**	**Regime 3:**	**Regime 3:**
	Mean/S.D.	0.0671/0.0081	0.0110/0.0201	-0.0028/0.0661	0.0410/0.0242
RS-GARCH	**P(1|2)/P(2|1)**	0.2609/0.8890	0.1506/0.0632	0.95677/ 0.0439	0.9126/0.2439
		Regime 1:	**Regime 1:**	**Regime 1:**	**Regime 1:**
	Intercept	-0.0061	-0.0876	-0.0073	0.0184
	Intercept garch	0.0498	0.0000	0.0698	0.0306
	Alpha	0.0343	0.0757	0.2112	0.0516
	Beta	0.8141	0.0430	0.274	0.8353
		Regime 2:	**Regime 2:**	**Regime 2:**	**Regime 2:**
	Intercept	0.0181	0.0102	0.0185	-0.0228
	Intercept garch	0.0000	0.0213	0.0342	0.0000
	Alpha	0.5745	0.1539	0.1401	0.0478
	Beta	0.3864	0.6889	0.5065	0.9753

This table shows the parameters of the models that have been considered in the analysis. The first three models do not consider regime switching. The parameters presented are those relating to the mean (intercept) and variance equation, which in the case of the lognormal is a constant while in GARCH and E-GARCH models it has a functional form. The other three models incorporate regime switching and therefore, for each regime the parameters of the mean and variance, as well as transition probabilities are reported.

Comparison of models by statistical criteria

The model selection must be based on the principle of parsimony, which states a preference for simpler models when they provide a similar adjustment to the data. For this purpose statistical criteria that analyze the value and likelihood function controlled by the number of parameters are used. In particular, this section takes into account the AIC criteria (Akaike information criteria) proposed by Akaike (1973), SC (Schwartz criteria) proposed by Schwartz (1978) and HQC (Hannan Queen). For each model analyzed Table 5 shows the values of the logarithm of the likelihood function and AIC, SBC and HQC criteria. As can be seen, in general, Regime switching models outperform the other models. In turn, the lognormal model, in which is based the calculation of capital in Solvency II, shows the worst fit to the empirical series of all indexes. However, the account within this model of two regimens (RSLN2) results in the best overall fit of all models tested, outperforming Garch and E-Garch models in all the indexes considered. For RSLN3 models and RS-GARCH, the overall fit is very good but its complexity penalizes against the simplest version (RSLN2).

Table 4. Comparison of different models through statistical criteria

	CAC 40	FTSE 100	IBEX-35	DAX
	LOGNORMAL			
Log likelihood	1.4421	1.7376	1.2923	1.3351
SBC	1.4191	1.7148	1.2674	1.3114
HQC	1.4278	1.7234	1.2767	1.3203
AIC	1.4337	1.7293	1.2831	1.3264
	GARCH(1,1)			
Log likelihood	1.4808	1.6962	1.3651	1.3968
SBC	1.4348	1.6507	1.3154	1.3493
HQC	1.4523	1.6680	1.3651	1.3968
AIC	1.4640	1.6796	1.3466	1.3793
	E-GARCH			
Log likelihood	1.5017	1.7931	1.3666	1.3422
SBC	1.4442	1.7362	1.3044	1.2829
HQC	1.4659	1.7578	1.3666	1.3422
AIC	1.4806	1.7723	1.3435	1.3204
	RSLN 2			
Log likelihood	1.4943	1.8335	1.3807	1.4378
SBC	1.4254	1.7652	1.3060	1.3666
HQC	1.4515	1.7911	1.3340	1.3934
AIC	1.4691	1.8086	1.3529	1.4116
	RSLN3			
Log likelihood	1.5230	1.8438	1.3944	1.4497
SBC	1.3851	1.7072	1.2451	1.3073
HQC	1.4373	1.7590	1.3010	1.3610
AIC	1.4726	1.7940	1.3389	1.3973
	RS-GARCH			
Log likelihood	1.5030	1.8288	1.3962	1.4489
SBC	1.3876	1.7147	1.2713	1.3298
HQC	1.4313	1.7579	1.3180	1.3747
AIC	1.4608	1.7871	1.3496	1.4050

The criterion of Akaike (AIC) selects the model that takes higher value of the difference between the log-likelihood function under the j-th model and the number of parameters, ie $l_j - k_j$. Schwarz Bayesian Criterion (SBC) would prefer the model with highest value of $l_j - \frac{1}{2}k_j \cdot \ln n$.The Hannan-Quinn Criterion (HQC) is an alternative to (AIC) and (SBC). It is given as $\mathrm{HQC} = n \ln\left(\frac{\mathrm{RSS}}{n}\right) + 2k \ln\ln(n).$ where k is the number of parameters, n is the number of observations and RSS is the fitted residual sum of squares of a minimum that results from linear regression or from non-linear global optimization.

As the aim of this study is the selection of appropriate models for the measurement of equity risk in insurance, it is not sufficient to assess the overall fit. Instead, it is necessary to evaluate the fit to extreme values. In this sense, it could be that the models with higher values provide a good global fit but not to the extreme values. Under these models, extreme values are often considered as outliers, but from a risk management perspective, they are of crucial significance because they determine the maximum loss to which the insurer is exposed.

Therefore, one must evaluate the extent to which residuals exceed the test of normality, especially in the left tail of the distribution. In the case of non normality of residuals, the adjustment provided by the model is not adequate. Residuals, for regime switching models, can be calculated, either by assigning the residuals to each submodel according to its conditional probability or using only the residuals associated to the submodel with a higher probability. The TSM software uses the first option.

The test of normality was done using the Jarque-Bera statistic (Jarque and Bera, 1980, 1987). Table 5 shows how the models do not take into account the existence of regimes that do not exceed the test of normality with 99% of confidence. However, all regime switching models pass the Jarque Bera test. These results are in line with those obtained by Hardy et al. (2006) for the TSE and SP500 indexes.

Table 5. Normality test of residuals.

Model	CAC 40	FTSE 100	IBEX-35	DAX
Lognormal	13.132 (0.001)	17.156 (0.000)	17.370 (0.000)	83.044 (0.000)
Garch	14.432 (0.000)	14.097 (0.000)	12.235 (0.002)	40.373 (0.000)
Egarch	6.371 (0.041)	11.326 (0.003)	24.888(0.000)	38.791 (0.000)
RSLN2	4.0341 (0.133)	3.9079 (0.142)	1.5482 (0.461)	1.4582 (0.482)
RSLN3	3.861 {0.145}	4.0396 (0.133)	6.0575 (0.048)	4.0139 (0.134)
RS-Garch	2.7844 (0.249)	3.5996 (0.165)	0.7938 (0.672)	2.8378 (0.242)

Jarque Bera test uses the skewness (S) and kurtosis (C) of the waste and takes the following expression $Q = \frac{n}{6}\left(S^2 + \frac{(C-3)^2}{4}\right)$. Under the assumption that the residuals are normal Q statistic has a χ 2 distribution with two degrees of freedom.

In addition to analyzing normality, the correct specification of the models requires analyzing whether the residuals and their squares are uncorrelated. A frequently used test is the Ljung and Box Q (1979). Table 7 shows the p-values associated with this statistic for the residuals (R) and squared residuals (R^2). As can be seen in all models except the lognormal, the residuals and their squares are uncorrelated.

Table 6. Q-stat on residuals (RES) and squared residuals (RES2).

Q(j)	NORMAL		GARCH		EGARCH		RSLN2		RSLN3		RSGARCH	
	RES	RES2	RES	RES2	RES	RES2	RES	RES2	RES	RES2	RES	RES2
FTSE												
1	0,284	0,013	0,841	0,528	0,962	0,521	0,807	0,668	0,417	0,915	0,396	0.513
3	0,439	0,000	0,836	0,570	0,814	0,604	0,354	0,239	0,214	0,252	0,469	0.916
6	0,241	0,000	0,497	0,881	0,513	0,918	0,333	0,573	0,161	0,592	0,326	0.824
9	0,230	0,000	0,380	0,959	0,402	0,996	0,370	0,720	0,253	0,585	0,306	0.405
12	0,417	0,002	0,560	0,943	0,544	0,919	0,591	0,795	0,417	0,627	0,511	0.268
1	0,112	0,002	0,083	0,929	0,166	0,934	0,472	0,123	0,638	0.962	0,139	0,419
3	0,261	0,001	0,285	0,991	0,495	0,923	0,867	0,168	0,818	0.896	0,446	0,504
6	0,597	0,002	0,579	0,790	0,792	0,341	0,990	0,102	0,940	0.720	0,755	0,402
9	0,535	0,004	0,443	0,802	0,507	0,390	0,812	0,087	0,824	0.598	0,628	0,363
12	0,269	0,006	0,390	0,881	0,459	0,541	0,527	0,130	0,902	0.713	0,398	0,445
DAX												
1	0,453	0,247	0,327	0,772	0,433	0,805	0,745	0,635	0,454	0,712	0,577	0,178
3	0,644	0,002	0,433	0,988	0,506	0,990	0,936	0,584	0,693	0,228	0,835	0,236
6	0,785	0,007	0,553	0,957	0,616	0,954	0,971	0,657	0,883	0,312	0,924	0,387
9	0,608	0,002	0,532	0,872	0,533	0,900	0,862	0,787	0,634	0,385	0,712	0,257
12	0,604	0,000	0,598	0,866	0,607	0,848	0,921	0,680	0,791	0,266	0,857	0,278
IBEX												
1	0,564	0,000	0,291	0,549	0,372	0,642	0,891	0,056	0,963	0,353	0,778	0,815
3	0,501	0,000	0,530	0,928	0,601	0,969	0,987	0,006	0,997	0,337	0,837	0,657
6	0,755	0,000	0,857	0,845	0,901	0,974	0,986	0,019	0,972	0,330	0,959	0,618
9	0,829	0,000	0,830	0,616	0,947	0,930	0,965	0,043	0,908	0,296	0,900	0,585
12	0,813	0,000	0,792	0,708	0,901	0,638	0,957	0,075	0,907	0,203	0,903	0,479

The null hypothesis of this test for the lag k is that there is autocorrelation for orders over k. The statistic is defined as $\mathbf{Q} = \mathbf{T}(\mathbf{T}+\mathbf{2})\sum_{\mathbf{j=1}}^{\mathbf{k}} \frac{\boldsymbol{\tau}_{\mathbf{j}}^{\mathbf{2}}}{\mathbf{T-j}}$, where $\boldsymbol{\tau}_{\mathbf{j}}$ is the j/th autocorrelation and $\mathbf{T}$ the number of observations. $\mathbf{Q}$ is asymptotically distributed as a $\boldsymbol{\chi}^{\mathbf{2}}$ with degrees of freedom equal to the number of autocorrelations.

Backtesting

The models we have discussed in the previous section are designed to analyze the equity risk assumed by the insurer through the calculation of Value at Risk (VaR). Formally, VaR is the loss level such that there is a probability p that they are equal to or greater than Y^*:

$$VaR_p(Y) = Prob(Y \geq Y^*) = p \quad (10)$$

In parametric models, quantiles are direct functions of the variance, and therefore, GARCH class models and regimen switching, present a dynamic measure of VaR defined as:

$$VaR_{t+1}^{p}(r) = \mu + \sigma_{t+1} F_p^{-1}(z) \; (11)$$

The diagnostic tests used in previous studies [Hardy (2001), Panneton (2003), Wong and Chan (2005), Hardy et al. (2006) and Boudreault and Panneton (2009)] may not be appropriate for choosing the best model. In this sense, Hansen (1996) and McLachlan and Peel (2000) showed that the standard likelihood test cannot be employed for testing the single-regime versus the MS model. Furthermore, Sarma et al. (2003) show that different methodologies can yield different VaR measures for the same portfolio and can sometimes lead to significant errors in risk measurement. Alternatively, in this work, in line with Sajjad et al. (2008), we have chosen to supplement the selection of models with the implementation of a VaR backtesting for different models for each of the series. This analysis consists of evaluating the number of times the index's losses exceed the VaR in the period. In this regard, we built a hit sequence, which takes the value 1 if the loss exceeds the VaR. As Campbell (2005) says, the accuracy of the model depends if the variable sequence meets the unconditional dichotomous coverage and independence properties2. These two properties of the hit sequence confirm that the hit sequence is identically and independently distributed as a Bernoulli random variable with probability α. In our study we compared the performance of both properties through the test of Kupiec (1995), also known as proportion of Fails (POF) and Christoffersen (1998).

$$POF = 2ln\left(\left(\frac{1-\hat{\alpha}}{1-\alpha}\right)^{T-I(\alpha)}\left(\frac{\hat{\alpha}}{\alpha}\right)^{I(\alpha)}\right) \; (12)$$

In this test statistic, if the proportion of VaR violations is exactly equal to α, then the POF test takes the value zero, indicating no evidence of any inadequacy in the underlying VaR measure. As the proportion of VaR violations differs from α, the POF test statistic grows indicating mounting evidence that the proposed VaR measure either systematically understates or

[2] The unconditional coverage property states that the proportion of failures of the model should be similar to the level of significance. Moreover, the property Independence means that the sequence of failures can not show dependence.

overstates the portfolio's underlying level of risk. Christofersen's (1998) Markov test examines whether or not the likelihood of a VaR violation depends on whether or not a VaR violation occurred on the previous day.

$$POF = -2ln\left(\frac{(1-\pi)^{n_{00}+n_{01}}\pi^{n_{01}+n_{11}}}{(1-\pi_0)^{n_{00}}\pi_0{}^{n_{01}}(1-\pi_1)^{n_{10}}\pi_1{}^{n_{11}}}\right) \quad (13)$$

Overall we can see that the models RSLN2 and RS-Garch pass all tests for all indices at different levels of confidence, except in the case of the CAC-40 for a confidence level of 90%. Something similar happens in the case of E-Garch model, with the exception of the series on the IBEX-35, also exceeds the unconditional and Independence test. Similarly, the lognormal model and the Garch, pass the test for low levels of confidence but, consistent with previous statistical analysis, fail over for high levels of confidence in most of the indices. Finally, we would like to point out that in some cases the best fitting model is specific to the data set used, hence the selection process should be carried out ad hoc. Thus we can conclude that only RS-Garch models pass the tests of normality, homoskedasticity, autocorrelation, unconditional coverage and independence. Therefore, despite the greater complexity, are best suited to model the risk of equity through internal models. Noted that RSLN2 model also presents a good balance between simplicity and fit and only failed to pass the homoskedasticity test on the IBEX-35.

Table 7. POF and Christoffersen test for the four indices

CAC-40						
	LOGNORMAL	GARCH	E-GARCH	RSLN	RSLN·3	RS-GARCH
POF TEST (99.5%)	0.0426	0.1635	0.4978	0.1224	0.0000	0.1226
POF TEST (99%)	0.0147	0.0147	0.3362	0.3093	0.0002	0.3093
POF TEST (95%)	0.1529	0.3747	0.5616	0.0598	0.1529	0.0533
POF TEST (90%)	0.8621	0.1960	0.8621	0.0065	0.0135	0.0001
CHRISTOFFERSEN LR TEST (99.5%)	0.7115	0.7820	0.8539	1.0000	0.3319	1.0000
FTSE-100						
	LOGNORMAL	GARCH	E-GARCH	RSLN	RSLN·3	RS-GARCH
POF TEST (99.5%)	0.1668	0.0095	0.0438	0.8505	0.8505	0.8505
POF TEST (99%)	0.0497	0.0497	0.1409	0.8505	0.7893	0.7893
POF TEST (95%)	0.0969	0.0299	0.0299	0.7893	0.3535	0.7640
POF TEST (90%)	1.0000	0.2959	0.2958	0.5424	0.5104	0.8307
CHRISTOFFERSEN LR TEST (99.5%)	0.0207	0.6446	0.7127	0.9271	0.9271	0.5791
IBEX-35						
	LOGNORMAL	GARCH	E-GARCH	RSLN	RSLN·3	RS-GARCH
POF TEST (99.5%)	0.1288	0.1288	0.1288	0.9377	0.1411	0.4278
POF TEST (99%)	0.5875	0.2606	0.5875	0.3753	0.0372	0.9118
POF TEST (95%)	0.2144	0.7126	0.5051	0.0439	0.0008	0.5633
POF TEST (90%)	0.8913	0.4507	0.4507	0.0671	0.0671	0.8913
CHRISTOFFERSEN LR TEST (99.5%)	0.7713	0.7713	0.7713	0.9232	1.0000	0.8467
DAX						
	LOGNORMAL	GARCH	E-GARCH	RSLN	RSLN·3	RS-GARCH
POF TEST (99.5%)	0.0077	0.0374	0.1488	0.8896	0.8896	0.1488
POF TEST (99%)	0.0404	0.0404	0.3043	0.3348	0.3348	0.6526
POF TEST (95%)	0.4543	0.4543	0.8907	0.1472	0.0701	0.4410
POF TEST (90%)	0.8419	0.6409	0.2639	0.0649	0.0093	0.6409
CHRISTOFFERSEN LR TEST (99.5%)	0.6366	0.7061	0.7061	0.9254	0.9254	0.7778

Table 7 shows the p-values from the Likelihood Ratio test of Kupiec (1995) for unconditional coverage (Percentage of Failures) and Christoffersen (1998) for independence, and the number of failures of each model.

VI. Comparison of Capital Requeriments Trough Markov Switching Models Against Solvency II Starndard Model

In this section we compare the capital resulting from the use of previously tested models, compared to the requirements established in the Standard Model. In this way we try to show the differences in the quantification of risk capital for equity through regime switching models that have shown a better fit to the time series of various European indices.

Capital requirements in the standard model (QIS4 and QIS5)

As discussed in the preceding paragraph, the quantification of risk in the Solvency II standard model is made using the analytical VaR, which has been widely selected as a measure in the financial markets, having analytical solution and allow the integration of different risks. The parameters and assumptions used to calculate capital requirements correspond to the most adverse shock that may occur to one year with a confidence level of 99.5%.

Thus, as we advance in the introduction to this paper, the amount of capital for equity risk in QIS4 is calculated assuming a 32% drop for the investments in global developed market indices (OECD / EEA countries) and 45% for the rest of the market. These factors were determined using average yields and net nominal exchange rate risk of the global index MSCI developed markets for the period 1970/2005 (quarterly). After obtaining the individual quantities associated with developed markets and other markets, these are added using a linear correlation coefficient of 0.75. Since our work focuses on the analysis of a single index for a developed market, the results should be compared with the factor of 32%. After the financial crisis, the new quantitative impact study (QIS5) reduces this percentage to 30% but it introduces, according to the new Solvency II Framework Directive (article 106), a symmetric adjustment of -9 percent. The base levels of the two stresses are 39% and 49%, depending if the assets are classified as global or other.

A new element that is included in QIS4 is the possibility of taking into consideration an alternative, which considers a damping effect or "dampener". Under this new proposal would also set the maximum load at 32 percent, but it is possible to obtain reductions in respect of that amount depending on the duration of the liabilities and the cyclical component of the index. The theoretical framework considers the existence of mean reversion in profitability of the indices and therefore does not need to have as much equity when liabilities are not required in the short term, since it is assumed that the market will recover. This buffering effect only applies to liabilities with duration greater than three years. In this case the standard stressor (32%) is an adjustment for damping cyclical component that takes into consideration the duration of liabilities. The capital charge for risk of the overall index is then:

$$Mkt_{dampener,\mathrm{Global}} = VM \cdot \left(\alpha \cdot \left(F(k) + G(k) \cdot c(t)\right) + 0{,}32 \cdot (1-\alpha)\right) \quad (14)$$

Where:

- VM, market value of global index.
- α, Proportion of reserves linked to liabilities with duration greater tan 3 years.
- F(k) y G(k), Coefficients from next table, where k is liabilities duration.

K	*F(k)*	*G(k)*
3-5 años	29 %	0,20
5-10 años	26 %	0,11
10-15 años	23 %	0,08
+ 15 años	22 %	0,07

- C(t) is the cyclical component.

Capital requirements with regimen switching internal models

Here we present the results of estimating capital requirements to a portfolio that contains any of the indices that have been used in this study. For this purpose a simulation of 100,000 scenarios for Latin Hypercube method to the time horizon of one year. Since the adjustment has been made from monthly data and the calculation of capital annually, it is necessary to perform temporary aggregation of simulated yields (Klein, 2002, Chan et al., 2008, Chan et al., 2009). To analyze the problem of temporal aggregation is useful to define the concept of A_T accumulation factor. Be P_t the monthly value unprocessed of time series (eg series CAC40 index) at time t, for t = 0,1, ... n and define the performance logarithmic t-th month as $y_t = \ln \frac{P_t}{P_{t-1}}$. The series of logarithmic returns for the month m can be constructed as:

$$Y_T = \ln \frac{P_{mT}}{P_{m(T-1)}} = \sum_{t=m(t-1)+1}^{mT} y_t \quad (15)$$

For: T = 1, 2, ..., N, where N = [n/m] an integer.

The A_T factor accumulation or growth rate of the market value of the index can be defined as:

$$A_T = \frac{P_{mT}}{P_{m(t-1)}} = \exp(Y_T) \ (16)$$

Therefore, the one year factor (m = 12) based on monthly logarithmic returns can be obtained easily by:

$$A_{12} = \exp(y_t + y_{t+1} + y_{t+2} + \cdots + y_{t+11}) \ (17)$$

And thus the value of the index in month 12, i.e. after one year is equal to:

$$P_{12} = P_0 A_{12}.(18)$$

Table 8 shows the capital requirements that result from applying the different models previously evaluated, calibrated as in the case of the standard model for VaR (99.5%). As can be seen these factors significantly exceed the amount referred to in the standard model. In addition, the normal return model, implicit in the calculation of the Solvency II capital, significantly underestimates the amount of capital compared to other models. For all the indices evaluated, the models of best fit, RSLN 2 and RS-GARCH regimes, capital charges result in substantially higher than the standard model and those estimated by lognormal models, GARCH and E-GARCH. This means that the use of simpler models could lead to an underestimation of the risk actually assumed and the calculation of loads below those required. Furthermore, in the case of IBEX, DAX, CAC, best-fit models suggest requirements around 39 percent, i.e. very close to the stage of stress that has been recently included in QIS5. It is also important to note that the FTSE generally presents a lower risk and, consequently, capital needs that in no case exceed 31 percent which shows that the determination of capital needs are specific to each risk factor.

Table 1. Comparison of capital requirements through alternative models

	NORMAL	GARCH	EGARCH	RSLN2	RSLN3	RSGARCH
			CAC			
VaR (99,5%)	-32,4%	-31,4%	-34,5%	-37,5%	-38,5%	-39,9%
VaR (99%)	-29,6%	-28,3%	-29,9%	-35,0%	-36,5%	-37,2%
			DAX			
VaR (99,5%)	-32,5%	-33,5%	-35,3%	-41,0%	-40,2%	-36,7%
VaR (99%)	-29,8%	-29,9%	-31,4%	-38,8%	-36,5%	-34,0%
			IBEX			
VaR (99,5%)	-33,1%	-35,1%	-36,3%	-40,4%	-40,4%	-36,6%
VaR (99%)	-30,4%	-31,2%	-32,0%	-38,2%	-38,1%	-33,9%
			FTSE			
VaR (99,5%)	-25,2%	-27,5%	-27,7%	-30,6%	-31,0%	-31,2%
VaR (99%)	-23,0%	-24,2%	-24,1%	-28,3%	-28,7%	-28,2%

VII. Conclusions

The new European Union solvency regulation for insurance companies, known as Solvency II, involves the revision of standards for assessing the financial situation in order to improve the measurement and control of risk. After the adoption in November 2009 of the new directive, is being carried out the last quantitative impact study (QIS5) and the standard formula will be the final. Under the new framework, the determination of capital requirements can be achieved through a standard or internal models previously approved by the regulatory authority. However, there have been few studies that attempt to analyze the models that can most adequately measure the equity risk. The lognormal return model for analyzing the underlying equity risk in the calculation of QIS5 has been chosen for simplicity and transparency and provides a reasonable approximation for small time periods. However, the normality assumption can seriously underestimate the tail of the loss distribution (extreme results) and inadequately capture the variability in volatility. Other proposed alternatives in the literature like Garch models are not able to capture sudden behavior changes in the market. The transformation of the previous models introducing regime switching may be better suited to measuring the risk of equity. For this reason the current study has examined the adequacy of Markov switching to design internal models for the insurers' equity risk exposure. We have used monthly data from four of the main european indices for the period between January of 1990 and January 2010. The comparison of the models across different statistical criteria and backtesting show the superiority of Markov switching over simpler models, for capturing insurers' equity risk. For all the indices evaluated, regime switching models (RSLN 2 and RS-GARCH) show the best fit. Furthermore, capital charges result in substantially higher than those estimated by lognormal models, GARCH and E-GARCH. This means that the use of simple models could lead to an underestimation of the actual risk assumed and the calculation of charges below those required. Furthermore, in the case of IBEX, DAX, CAC, regime switching models suggest capital requirements around 39 percent, i.e. very close to the stage that has been recently included in QIS5. This means that the legislation revision in the last quantitative impact study is in line with the capital charges estimated by the proposed models. The results are highly relevant as they imply that European insurance companies that opt for internal models based on the assumption of normality are underestimating the risk in accordance with developments in the European equity markets. There is also evidence that the Garch models in their different forms, do not capture enough adverse market movements and confirm the adequacy of RSLN2 and RS-Garch to build internal models focused in the assessment and measurement of equity risk.

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