A Sign Test of Cumulative Abnormal Returns in Event Studies Based on Generalized Standardized Abnormal Returns

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Abstract. Corrado and Zivney (1992) have presented a sign test, which provides well-specified inferences in event studies. However, the sign test is derived only for a one-day event window. This paper examines a new sign test (SIGN-GSAR-T), which is derived by developing the existing sign test for testing in addition to one-day abnormal returns also cumulative abnormal returns (CARs). The new test statistic is developed by adopting the procedure of generalized standardized abnormal returns (GSARs) presented by Kolari and Pynnönen (2010b). Simulations with real returns show that the statistic SIGN-GSAR-T has competitive empirical power properties and is robust against event-induced volatility. Moreover, if the event-dates are clustered, the test statistic SIGN-GSAR-T outperforms both the examined parametric and nonparametric test statistics.

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1. Introduction

Researchers use event study methods to measure stock price reactions to events and many event studies rely on parametric test statistics. Standardized parametric event study tests presented by Patell (1976) and Boehmer, Musumeci and Poulsen (BMP) (1991) have been more popular than conventional nonstandardized tests in testing abnormal security price performance, because of their better power properties. Harrington and Shrider (2007) have argued that, in short-horizon testing of mean abnormal returns, tests that are robust against cross-sectional variation in the *true* abnormal return should always be used. They have found that the BMP test statistic is a good candidate for a robust parametric test in conventional event studies.[1] Although many event studies rely on parametric test statistics, a disadvantage of parametric statistics is that they embody detailed assumptions about the probability distribution of returns. Nonparametric statistics do not usually require as stringent assumptions about return distributions as parametric tests. [e.g. Cowan (1992)].

The sign tests are nonparametric tests often used in event studies. Also nonparametric procedures like the sign tests can be misspecified, if an incorrect assumption about the data is imposed. For example Brown and Warner (1980) and (1985), and Berry, Gallinger and Henderson (1990) have

[1]Conventional event studies are defined as those focusing only on mean stock price effects. Other types of event studies include (for example) the examination of return variance effects [Beaver (1968) and Patell (1976)], trading volume [Beaver (1968) and Campbell and Wasley (1996)], accounting performance [Barber and Lyon (1997)] and earnings management procedures [Dechow, Sloan, and Sweeney (1995) and Kothari, Leone, and Wasley (2005)].

demonstrated that a sign test assuming an excess return median of zero is misspecified. Corrado and Zivney (1992) have introduced a sign test based on standardized excess returns that does not assume a median of zero, but instead uses a sample excess return median to calculate the sign of an event date excess return. The results of simulation experiments presented in Corrado and Zivney (1992) indicate that their sign test provides reliable and well-specified inferences in event studies. They have also reported that their version of the sign test is better specified than the ordinary t-test and has a power advantage over the ordinary t-test in detecting small levels of abnormal performance.

The parametric tests derived by Patell and BMP can be applied to testing cumulative abnormal returns (CARs) over multiple day windows. Corrado and Zivney (1992) have derived the sign test only for testing one-day abnormal returns (ARs). Kolari and Pynnönen (2010b) have derived a nonparametric rank test of CARs, which is based on generalized standardized abnormal returns (GSARs). They have found that their rank test has superior (empirical) power relative to popular parametric tests both at short and long CAR-window lengths. Their test statistic has also been shown to be robust to abnormal return serial correlation and event-induced volatility. Kolari and Pynnönen (2010b) have also suggested that GSARs derived by them can be used to extend the sign test in Corrado and Zivney (1992) for testing CARs. Hence, in an effort to overcome previous pitfalls in the test statistics, and thereby provide more powerful test methods for common practice in event studies, this paper presents new sign test statistics (SIGN-GSAR-T and SIGN-GSAR-Z) based on GSARs. These statistics can be used equally well for testing simple day ARs and CARs.

Cowan (1992) has also derived a sign test for testing CARs and his test is called generalized sign test. The generalized sign test compares the proportion of positive ARs around an event to the proportion from a period unaffected by the event. In this way the generalized sign test takes account of a possible asymmetric return distribution under the null hypothesis. Cowan (1992) has reported that the generalized sign test is well specified for event windows of one to eleven days. He has also reported that the test is powerful and becomes relatively more powerful as the length of the CAR-window increases.

In empirical simulations, the new sign test statistics presented in this paper are compared with the generalized sign test derived by Cowan (1992), the rank test derived by Kolari and Pynnönen (2010b) as well as the parametric tests derived by Patell and BMP, and the ordinary t-test. The results of the current paper show that especially the test statistic SIGN-GSAR-T has several advantages over previous testing procedures. First, it is robust against a certain degree of cross-correlation caused by event day clustering. For example, according to Kolari and Pynnönen (2010a) it is well known that event studies are prone to cross-sectional correlation among abnormal returns when the event day is the same for sample firms. For this reason the test statistics cannot assume independence of abnormal returns. They have also shown that even when cross-correlation is relatively low, event-date clustering is serious in terms of over-rejecting the null hypothesis of zero average abnormal returns, when it is true. Also in this paper it is reported that when the event-dates are clustered, all test statistics, except the the test statistic SIGN-GSAR-T and the rank test derived by Kolari and Pynnönen (2010b), over-reject the null hypothesis both for short and long

CAR-windows. Second, the test statistic SIGN-GSAR-T seems to be robust to the event-induced volatility. Third, it proves to have also good empirical power properties. Thus, the SIGN-GSAR-T test procedure makes available a nonparametric test for general application to the mainstream of event studies.

The paper is organized as follows. Section 2 introduces the distribution properties of the sign of the GSAR. Section 3 presents the test statistics SIGN-GSAR-T and SIGN-GSAR-Z together with the asymptotic distributions for both of the test statistics. Section 4 describes the simulation design and summarizes the test statistics against which the new sign tests are compared with. The empirical results are presented in Section 5, and Section 6 concludes.

2. The Sign of the GSAR

In forthcoming theoretical derivations, the following explicit assumption is made:

Assumption 1 *Stock returns r_{it} are weak white noise continuous random variables with*

$$\begin{aligned} E[r_{it}] &= \mu_i \text{ for all } t, \\ \mathrm{var}[r_{it}] &= \sigma_i^2 \text{ for all } t, \\ \mathrm{cov}[r_{it}, r_{is}] &= 0 \text{ for all } t \neq s, \end{aligned} \tag{1}$$

where i refers to the i^{th} stock and t and s are time indexes.

Let AR_{it} represent the abnormal return of security i on day t, and let day

$t = 0$ indicate the event day.[2] The days $t = T_0 + 1, T_0 + 2, \ldots, T_1$ represent the estimation period days relative to the event day, and the days $t = T_1 + 1, T_1 + 2, \ldots, T_2$ represent event window days, again relative to the event day. Furthermore L_1 represents the estimation period length and L_2 represents the event period length. Standardized abnormal returns are defined as

$$\mathrm{AR}'_{it} = \mathrm{AR}_{it}/S(\mathrm{AR}_i), \tag{2}$$

where $S(\mathrm{AR}_i)$ is the standard deviation of the regression prediction errors in the abnormal returns computed as in Campbell, Lo and MacKinlay (1997, Sections 4.4.2–4.4.3).

The cumulative abnormal return (CAR) from day τ_1 to τ_2 with $T_1 < \tau_1 \leq \tau_2 \leq T_2$ is defined as

$$\mathrm{CAR}_{i,\tau_1,\tau_2} = \sum_{t=\tau_1}^{\tau_2} \mathrm{AR}_{it}, \tag{3}$$

and the time period from τ_1 to τ_2 is often called a CAR-window or a CAR-period. Then the corresponding standardized cumulative abnormal return (SCAR) is defined as

$$\mathrm{SCAR}_{i,\tau_1,\tau_2} = \frac{\mathrm{CAR}_{i,\tau_1,\tau_2}}{S(\mathrm{CAR}_{i,\tau_1,\tau_2})}, \tag{4}$$

where $S(\mathrm{CAR}_{i,\tau_1,\tau_2})$ is the standard deviation of the CARs adjusted for forecast error [see Campbell, Lo and MacKinlay (1997, Section 4.4.3)]. Under the

[2]Abnormal returns are operationalized in Section 4.

null hypothesis of no event effect both AR'_{it} and $\mathrm{SCAR}_{i,\tau_1,\tau_2}$ are distributed with mean zero and (approximately) unit variance.

In order to account for the possible event-induced volatility Kolari and Pynnönen (2010b) re-standardize the SCARs like BMP (1991) with the cross-sectional standard deviation to get re-standardized SCAR

$$\mathrm{SCAR}^*_{i,\tau_1,\tau_2} = \frac{\mathrm{SCAR}_{i,\tau_1,\tau_2}}{S(\mathrm{SCAR}_{\tau_1,\tau_2})}, \tag{5}$$

where

$$S(\mathrm{SCAR}_{\tau_1,\tau_2}) = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(\mathrm{SCAR}_{i,\tau_1,\tau_2} - \overline{\mathrm{SCAR}}_{\tau_1,\tau_2})^2} \tag{6}$$

is the cross-sectional standard deviation of $\mathrm{SCAR}_{i,\tau_1,\tau_2}$s and

$$\overline{\mathrm{SCAR}}_{\tau_1,\tau_2} = \frac{1}{n}\sum_{i=1}^{n}\mathrm{SCAR}_{i,\tau_1,\tau_2}. \tag{7}$$

Again $\mathrm{SCAR}^*_{i,\tau_1,\tau_2}$ is a zero mean and unit variance random variable. The generalized standardized abnormal returns (GSARs) are defined similar to Kolari and Pynnönen (2010b):

Definition 1 *The generalized standardized abnormal return (GSAR) is defined as*

$$\mathrm{GSAR}_{it} = \begin{cases} \mathrm{SCAR}^*_{i,\tau_1,\tau_2}, & \textit{in CAR-period} \\ \mathrm{AR}'_{it}, & \textit{otherwise,} \end{cases} \tag{8}$$

where $\mathrm{SCAR}^*_{i,\tau_1,\tau_2}$ *is defined in equation (5) and* AR'_{it} *is defined in equation (2).*

Thus the CAR-window is considered as one time point in which the GSAR equals the re-standardized cumulative abnormal return defined in equation (5), and for other time points GSAR equals the usual standardized abnormal returns defined in equation (2).

The time indexing is redefined such that the CAR-window of length $\tau_2-\tau_1+1$ is squeezed into one observation with time index $t=0$. Thus, considering the standardized cumulative abnormal return as one observation, in the testing procedure there are again L_1+1 observations of which the first L_1 are the estimation period (abnormal) returns and the last one is the cumulative return.

Kolari and Pynnönen (2010b) have suggested that the GSARs can be used to extend the sign test in Corrado and Zivney (1992) for testing CARs. This can be achieved by defining the sign of the GSAR like:

Definition 2 *The sign of the generalized standardized abnormal return* $GSAR_{it}$ *is*

$$G_{it} = \mathrm{sign}[\mathrm{GSAR}_{it} - \mathrm{median}(\mathrm{GSAR}_i)], \tag{9}$$

where $\mathrm{sign}(x)$ *is equal to +1, 0, -1 as* x *is* >0, $=0$ *or* <0.

If $T = L_1 + 1$ is even, the corresponding probabilities for the sign of the GSAR for values +1, 0 and -1 are

$$\Pr[G_{it} = 1] = \Pr[G_{it} = -1] = \frac{1}{2} \tag{10}$$

and

$$\Pr[G_{it} = 0] = 0. \tag{11}$$

If $T = L_1+1$ is odd, the corresponding probabilities for the sign of the GSAR for values $+1$, 0 and -1 are

$$\Pr[G_{it} = 1] = \Pr[G_{it} = -1] = \frac{T-1}{2T} \tag{12}$$

and

$$\Pr[G_{it} = 0] = \frac{1}{T}. \tag{13}$$

The expectations, variances and covariances of the sign of GSAR are presented in Appendix A for even and odd T, and summarized in Proposition 1.

Proposition 1 *The expectation for the sign of the GSAR defined in (9) is*

$$E[G_{it}] = 0 \tag{14}$$

for T being even or odd. Furthermore the variance and covariance of the sign of the GSAR are

$$var[G_{it}] = \begin{cases} 1, & \textit{for even } T \\ \frac{T-1}{T}, & \textit{for odd } T \end{cases} \tag{15}$$

and

$$cov[G_{it}, G_{is}] = \begin{cases} -\frac{1}{T-1}, & \textit{for even } T \\ -\frac{1}{T}, & \textit{for odd } T. \end{cases} \tag{16}$$

Furthermore i=1,...,n and t≠s.

3. The Test Statistics SIGN-GSAR-T and SIGN-GSAR-Z

The null hypothesis of no mean event effect, reduces to

$$H_0 : \mu = 0, \tag{17}$$

where μ is the expectation of the (cumulative) abnormal return. Like Kolari and Pynnönen (2010b) suggested, this paper introduces a new sign test statistic (called hereafter SIGN-GSAR-T), which can be used for testing the presented null hypothesis. The test statistic SIGN-GSAR-T is defined as

$$t_{\mathrm{SGT}} = \frac{Z_1\sqrt{T-2}}{\sqrt{T-1-Z_1^2}}, \tag{18}$$

where

$$Z_1 = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} G_{i0}/S(G), \tag{19}$$

with

$$S(G) = \sqrt{\frac{1}{T}\sum_{t\in\mathcal{T}}(\frac{1}{\sqrt{n_t}}\sum_{i=1}^{n_t} G_{it})^2}, \tag{20}$$

in which n_t is the number of nonmissing returns in the cross-section of n-firms on day t and $\mathcal{T} = \{T_0+1, \ldots, T_1, 0\}$. The Z_1 statistic in equation (19) is the sign test derived by Corrado and Zivney (1992) for testing single event-day abnormal returns.

Proofs of the Theorem 1 and Theorem 2 regarding the asymptotic distributions of Z_1 and the test statistic SIGN-GSAR-T defined in equations (19) and (18), respectively, are presented in Appendix B for both cases T being even and odd.

Theorem 1 (Asymptotic distribution of Z_1): *For a fixed T, under the assumption of cross-sectional independence, the density function of the asymptotic distribution of the test statistic Z_1 defined in equation* (19) *when $n \to \infty$, is*

$$f_{Z_1}(z) = \frac{\Gamma\left[(T-1)/2\right]}{\Gamma\left[(T-2)/2\right]\sqrt{(T-1)\pi}}\left(1-\frac{z^2}{T-1}\right)^{\frac{1}{2}(T-2)-1}, \tag{21}$$

for $|z| \leq \sqrt{T-1}$ and zero elsewhere, where $\Gamma(\cdot)$ is the Gamma function.

Thus, Theorem 1 implies that $(Z_1)^2/(T-1)$ is asymptotically Beta distributed with parameters $1/2$ and $(T-2)/2$.

Corrado and Zivney (1992) conjecture that for sufficiently large sample size, the Central Limit Theorem implies that the distribution of Z_1 should converge to normality. By Theorem 1 we can conclude that the asymptotic normality holds only if also T is large enough. This follows from the fact that in equation (21)

$$\left(1 - \frac{z^2}{T-1}\right)^{\frac{1}{2}(T-2)-1} \to e^{-\frac{1}{2}z^2} \tag{22}$$

and the normalizing constant

$$\frac{\Gamma\left[(T-1)/2\right]}{\Gamma\left[(T-2)/2\right]\sqrt{(T-1)\pi}} \to 1/\sqrt{2\pi} \tag{23}$$

as $T \to \infty$, implying the limiting $N(0,1)$-distribution.

Theorem 2 (Asymptotic distribution of the test statistic SIGN-GSAR-T): *Under the assumptions of Theorem* 1*,*

$$t_{\mathrm{SGT}} = Z_1\sqrt{\frac{T-2}{T-1-(Z_1)^2}} \xrightarrow{d} t_{T-2}, \tag{24}$$

as $n \to \infty$*, where* Z_1 *is defined in equation* (19)*,* $\xrightarrow{d}$ *denotes convergence in distribution, and* t_{T-2} *denotes the Student* t*-distribution with* $T-2$ *degrees of freedom.*

Given that the t-distribution approaches the $N(0,1)$-distribution as the degrees of freedom $T-2$ increases, also the null distribution of the test statistic t_{SGT} approach the standard normal distribution as $T \to \infty$.

Remark 1 *Using facts about statistics based on signs (see Appendix A), it is easy to show that*

$$\mathrm{var}[\overline{G_0}] = \begin{cases} \frac{1}{n}, & \text{for even } T \\ \frac{T-1}{nT} \approx \frac{1}{n}, & \text{for odd } T. \end{cases} \tag{25}$$

where $\overline{G_0} = \frac{1}{n}\sum_{i=1}^{n} G_{i0}$. *Thus, under the assumption that* $\mathrm{var}[\overline{G_0}] = \frac{1}{n}$, *a useful test statistic for the null hypothesis (17) is*

$$t_{\mathrm{SGZ}} = \frac{\overline{G_0}}{\sqrt{\mathrm{var}[\overline{G_0}]}} = \overline{G_0}\sqrt{n}, \tag{26}$$

for which the null distribution converges rapidly to the standard normal distribution, $N(0,1)$, *as the number of firms increases. We henceforth refer to this statistic as SIGN-GSAR-Z.*

The simplicity of the test statistic SIGN-GSAR-Z makes it an attractive alternative to the test statistic SIGN-GSAR-T. This is particularly the case when the event days across the sample firms are not clustered. However, in the presence of event day clustering, which causes cross-sectional correlations between the returns, the SIGN-GSAR-T can be expected to be much more robust than the SIGN-GSAR-Z test statistic.

Asymptotic Distributions: Cross-Sectional Dependence (Clustered Event Days)

Cross-sectional dependence due to clustered event days (the same event days across the firms) changes materially the asymptotic properties of the test statistics and in particular those statistics that do not account for the cross-sectional dependence.

As stated in Lehmann (1999, Sec. 2.8), it is still, frequently true that the asymptotic normality holds provided that the average cross-correlation, $\overline{\rho}_n$, tends to zero rapidly enough such that

$$\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1,i\neq j}^{n}\rho_{ij}\to\gamma \tag{27}$$

as $n\to\infty$.

In financial applications this would be the case if there are a finite number of firms in each industry and the return correlations between industries were zero. In fact this is a special case of so called m-independence. Generally, a sequence of random variables X_1, X_2,..., is said to be m-independent, if X_i and X_j are independent if $|i-j|>m$. In cross-sectional analysis this would mean that the variables can be ordered such that when the index difference is larger than m, the variables are independent. [See Kolari and Pynnönen (2010b)].

In such a case, we can show in the same manner as in Kolari and Pynnönen (2010b) that the result in (27) holds. More precisely, assuming that for any fixed t, G_{it} defined in equation (9) are m-independent, $i=1,2,...,n$, $(n>m)$, the correlation matrix of $G_{1t},...,G_{nt}$ is band-diagonal such that all ρ_{ij} with $|i-j|>m$ are zeros. It is straightforward to see that in such a correlation matrix there are $m(2n-m-1)$ nonzero correlations in addition to the n ones on the diagonal. Thus, in the double summation (27) there are $m(2n-m-1)$ non-zero elements, and it can be written such that

$$\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}\rho_{ij}=\frac{m(2n-m-1)}{n}\widetilde{\rho}_n\to\gamma, \tag{28}$$

where $\widetilde{\rho}_n$ is the average of the $m(2n-m-1)$ cross-correlations in the band-diagonal correlations matrix and $\gamma=2m\widetilde{\rho}$ is a finite constant with $\widetilde{\rho}=\lim_{n\to\infty}\widetilde{\rho}_n$ and $2m=\lim_{n\to\infty}m(2n-m-1)/n$.

Thus, under the m-independence the asymptotic distribution of the test statistic SIGN-GSAR-Z is

$$t_{\mathrm{SGZ}} \to N(0, 1+\gamma). \tag{29}$$

This implies that the test statistic SIGN-GSAR-Z is not robust to cross-sectional correlation of the return series. Typically $\gamma > 0$, which means that t_{SGZ} will tend to over-reject the null hypothesis.

The limiting distributions of the test statistic SIGN-GSAR-T turns out to apply also under m-independence. This follows from the fact that if the asymptotic normality holds under the m-independence such that the limiting correlation effect is $1+\gamma$, then using the scaled variables, $G_{it}/\sqrt{1+\gamma}$, in place of the original variables, all the results in Theorem 1 and Theorem 2 follow, because in Z_1 defined in (19) and t_{SGT} defined in (18) are invariant to the scaling of the observations (the zero-one sign of the GSARs). Therefore, the theoretical derivation indicates that when the event-dates are clustered, the test statistic SIGN-GSAR-T behave better than the test statistic SIGN-GSAR-Z .

4. Simulation Design

In this section the simulation design, which is used to examine the empirical behavior of the test statistics SIGN-GSAR-T and SIGN-GSAR-Z, is presented. Like for example Kolari and Pynnönen (2010b) have concluded, the optimality of a test can be judged on the basis of size and power. Within a class of tests of given size (Type I error probability), the one that has the maximum power (minimum Type II error probability) is the best. A testing procedure is robust, if the Type I error rate is not affected by real data issues such as non-normality, event-induced volatility, autocorrelation and cross-correlation of returns. Consequently, the aim of our simulations is to focus on the robustness and power properties of the tests. Non-normality, autocorrelation, and other data issues are captured in the simulation by using actual return data instead of artificially simulated data. Event-induced volatility effects are investigated by introducing volatility change within the event period, and the effect of cross-sectional correlation is examined by setting the same event day in the return series for each firm in the sample.

4.1 *Sample construction*

The well-known simulation approach presented by Brown and Warner (1980), and widely used in several other methodological studies [e.g. Brown and Warner (1985), Corrado (1989), Cowan (1992), Campbell and Wasley (1993), and Cowan and Sergeant (1996)], is also used in this paper. From the data base 1,000 portfolios each of 50 stocks are constructed with replacements. Each time a stock is selected, a hypothetical event date is randomly generated and the event day is denoted as day "0". The results are reported for event day $t = 0$ abnormal return AR(0) and for cumulative abnormal returns CAR$(-1, +1)$, CAR$(-5, +5)$ and CAR$(-10, +10)$. The estimation period is comprised of 239 days prior to the event period, hence days from -249 to -11. The event period is comprised of 21 days, hence days from -10 to +10. Therefore, the estimation period and event period altogether comprises of 260 days. In order for a return series to be included, no missing returns are allowed in the last 30 days from -19 to +10.

In earlier studies [e.g. Charest (1978), Mikkelson (1981), Penman (1982) and Rosenstein and Wyatt (1990)] it has been found that the event period standard deviation is about 1.2 to 1.5 times the estimation period standard deviation. Therefore, the increased volatility is introduced by multiplying the cumulated event period returns by a factor $\sqrt{c}$ with values $c = 1.5$ for an approximate 20 percent increased volatility, $c = 2.0$ for an approximate 40 percent increased volatility and $c = 3.0$ for an approximate 70 percent increased volatility due to the event effect.[3] To add realism the volatility factors c are generated for each stock based on the following uniform distributions $U[1, 2]$, $U[1.5, 2.5]$ and $U[2.5, 3.5]$. This generate on average the variance effects of 1.5, 2.0 and 3.0. Furthermore for the no volatility effect experiment $c = 1.0$ is fixed.

For investigating the power properties a similar method as for example Campbell and Wasley (1993) is used. Hence, for single-day event period [AR(0)] the abnormal performance is artificially introduced by adding the indicated percent (a constant) to the day-0 return of each security. While, in the multiday setting [CAR$(-1, +1)$, CAR$(-5, +5)$ and CAR$(-10, +10)$], abnormal performance is introduced by selecting one day of the CAR-period at random and adding the particular level of abnormal performance to that day's return. By this we aim to mimic the real situations, where there can be the information leakage and delayed adjustment. That is, if the markets are inefficient, information may leak before the event, which shows up as abnormal behavior before the event day. Delays in the event information show up as abnormal return behavior after the event day.

[3] Because $\sqrt{1.5} \approx 1.2$, $\sqrt{2.0} \approx 1.4$ and $\sqrt{3.0} \approx 1.7$.

Also the effect of event-date clustering on the test statistics is studied. The effect of event-date clustering is examined by constructing again from the data base 1,000 portfolios each of 50 stocks, but all stocks in the portfolio have exactly the same event date.

4.2 *Abnormal return model*

The abnormal behavior of security returns can be estimated through the market model

$$r_{it} = \alpha_i + \beta_i r_{mt} + \epsilon_{it}, \tag{30}$$

where again r_{it} is the return of stock i at time t, r_{mt} is the market index return at time t and ϵ_{it} is a white noise random component, which is not correlated with r_{mt}. The resulting ARs are obtained as differences of realized and predicted returns on day t in the event period

$$\mathrm{AR}_{it} = r_{it} - (\hat{\alpha}_i + \hat{\beta}_i r_{mt}), \tag{31}$$

where the parameters are estimated from the estimation period with ordinary least squares. According to Campbell, Lo and MacKinley (1997) the market model represents a potential improvement over the traditional constant-mean-return model, because by removing the portion of the return that is related to variation in the market's return, the variance of the AR is reduced. This can lead to increased ability to detect event effects.

4.3 *Test statistics*

Next the test statistics, which are used in the simulations, are presented. The ordinary t-test (ORDIN) is defined as

$$t_{\mathrm{ORDIN}} = \frac{\overline{\mathrm{CAR}}_{\tau_1,\tau_2}}{S(\overline{\mathrm{CAR}}_{\tau_1,\tau_2})}, \tag{32}$$

where

$$\overline{\mathrm{CAR}}_{\tau_1,\tau_2} = \frac{1}{n}\sum_{i=1}^{n} \mathrm{CAR}_{i,\tau_1,\tau_2}, \tag{33}$$

in which $\mathrm{CAR}_{i,\tau_1,\tau_2}$ is defined in equation (3) and $S(\overline{\mathrm{CAR}}_{\tau_1,\tau_2})$ is the standard error of the average cumulative abnormal return $\overline{\mathrm{CAR}}_{\tau_1,\tau_2}$ adjusted for the prediction error [see again Campbell, Lo and MacKinlay (1997, Section 4.4.3)]. The ordinary t-test statistic is asymptotically $N(0,1)$-distributed under the null hypothesis of no event effect.

Patell (1976) test statistic (PATELL) is

$$t_{\mathrm{PATELL}} = \sqrt{\frac{n(L_1-4)}{L_1-2}}\,\overline{\mathrm{SCAR}}_{\tau_1,\tau_2}, \tag{34}$$

where $\overline{\mathrm{SCAR}}_{\tau_1,\tau_2}$ is the average of the standardized CAR defined in equation (7), and L_1 is again the length of the estimation period. Also the test statistic derived by Patell is asymptotically $N(0,1)$-distributed under the null hypothesis.

The Boehmer, Musumeci and Poulsen (1991) test statistics (BMP) is

$$t_{\text{BMP}} = \frac{\overline{\text{SCAR}}_{\tau_1,\tau_2}\sqrt{n}}{S(\text{SCAR}_{\tau_1,\tau_2})}, \tag{35}$$

where again $S(\text{SCAR}_{\tau_1,\tau_2})$ is the cross-sectional standard deviation of SCARs defined in (6), and $\overline{\text{SCAR}}_{\tau_1,\tau_2}$ is defined in equation (7). Also the test statistic t_{BMP} is asymptotically $N(0,1)$-distributed under the null hypothesis.

We follow Kolari and Pynnönen (2010b) and define the demeaned standardized abnormal ranks of the GSARs as

$$U_{it} = \text{Rank}(\text{GSAR}_{it})/(T+1) - 1/2, \tag{36}$$

where $i = 1, ..., n$ and $t \in \mathcal{T} = \{T_0 + 1, ..., T_1, 0\}$ is the set of time indexes including the estimation period for $t = T_0 + 1, ..., T_1$ and to the CAR for $t = 0$, with $T_0 + 1$ and T_1 the first and last observation on the estimation period, and $T = L_1 + 1 = T_1 - T_0 + 1$ is the total number of observations with L_1 estimation period returns and the one CAR. Then the generalized rank test statistic (GRANK) is defined as

$$t_{\text{GRANK}} = Z_2 \sqrt{\frac{T-2}{T-1-Z_2^2}}, \tag{37}$$

where

$$Z_2 = \frac{\overline{U_0}}{S_{\overline{U}}} \tag{38}$$

with

$$S_{\overline{U}} = \sqrt{\frac{1}{T}\sum_{t\in\mathcal{T}}\frac{n_t}{n}\overline{U}_t^2} \tag{39}$$

and

$$\overline{U}_t = \frac{1}{n_t}\sum_{i=1}^{n_t} U_{it}. \tag{40}$$

Furthermore n_t is the number of valid GSARs available at time point t, $t \in \mathcal{T} = \{T_0+1, ..., T_1, 0\}$, $T = T_1 - T_0 + 1$ is the number of observations, and $\overline{U}_0$ is the mean $\overline{U}_t$ for $t = 0$ (CAR). According to Kolari and Pynnönen (2010b) the asymptotic distribution of the test statistic GRANK is Student t-distribution with $T-2$ degrees of freedom. Again given that the t-distribution approaches the $N(0,1)$-distribution as the degrees of freedom $T-2$ increases, also the null distribution of the test statistic t_{GRANK} approach the standard normal distribution as $T \to \infty$.

The generalized sign test statistic presented by Cowan (1992) is

$$t_{\text{COWAN}} = \frac{w - n\widehat{p}}{\sqrt{n\widehat{p}(1-\widehat{p})}}, \tag{41}$$

where w is the number of stocks in the event window for which the CAR is positive and n is again the number of the stocks. Furthermore

$$\widehat{p} = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{m_i}\sum_{t=T_0+1}^{T_1} S_{it}, \tag{42}$$

where m_i is the number of non-missing returns in the estimation period for security-event i and

$$S_{it} = \begin{cases} 1 & \text{if } \mathrm{AR}_{it} > 0 \\ 0 & \text{otherwise.} \end{cases} \tag{43}$$

According to Cowan (1992) the generalized sign test statistic (SIGN-COWAN) is asymptotically $N(0,1)$-distributed under the null hypothesis.

4.4 *The data*

The data in this simulation design consists of daily closing prices of 1,500 the U.S. traded stocks that make up the S&P 400, S&P 500, and S&P 600 indexes. S&P 400 covers the mid-cap range of stocks, S&P 500 the large-cap range of stocks and S&P 600 the small-cap range of stocks. Five percent of the stocks having the smallest trading volume are excluded. Therefore, 72 stocks from S&P 600, two stocks from S&P 400 and one stock from S&P 500 are excluded. The sample period spans from the beginning of July, 1991 to October 31, 2009. S&P 400 index was launched in June in 1991, which is why the sample period starts in the beginning of July, 1991. Official holidays and observances are excluded from the data. By using actual (rather than artificial) stock returns in repeated simulations, a reliable and realistic view about the comparative real data performance of the test statistics in true applications is attained.

The returns are defined as log-returns

$$r_{it} = \log(P_{it}) - \log(P_{it-1}), \tag{44}$$

where P_{it} is the closing price for stock i at time t.

5. Empirical Results

This section discusses the results from the simulation study. First, the sample statistics of the abnormal returns, the cumulative abnormal returns and the test statistics are presented. Second, the properties of the empirical distributions of the test statistics are presented. Third, the rejection rates are reported. The rejection rates are also reported in the cases where the event-induced volatility is present. Fourth, the power properties of the test statistics are presented. The power properties are also presented in the cases where the event-dates are clustered.

5.1 *Sample statistics*

Table 1 reports sample statistics from $1,000$ simulations for the event day abnormal returns and for the cumulative abnormal returns: CAR$(-1,+1)$, CAR$(-5,+5)$ and CAR$(-10,+10)$. It also reports sample statistics for the test statistics for AR(0), CAR$(-1,+1)$, CAR$(-5,+5)$ and CAR$(-10,+10)$. Under the null hypothesis of no even effect test statistics ORDIN, PATELL, BMP, SIGN-COWAN and SIGN-GSAR-Z should be approximately $N(0,1)$-distributed. Strictly speaking, the asymptotic distributions of GRANK and SIGN-GSAR-T should be t-distributions with $T-2$ degrees of freedom. However, with $T-2$ equal to 238, the normal approximation should be valid and so the null distributions of the test statistics GRANK and SIGN-GSAR-T approach the standard normal distribution. Hence, we can conclude that under the null hypothesis of no event effect all the test statistics should have zero mean and (approximately) unit variance.

[Table 1]

Considering only on the single abnormal returns AR(0) in Panel A of Table 1, it can be noted that means of all the test statistics are statistically close to zero. For example (in absolute value) the largest mean of -0.024 for the PATELL statistic is only 1.113 standard errors away from zero. In longer CAR-windows the means of the test statistics, albeit small, start to deviate significantly away from the theoretical value of zero. Considering on the 3-day CARs, CAR$(-1, +1)$, in Panel B of Table 1, we see that only the means for PATELL and BMP deviate significantly away from zero. While, considering on the 11- and 21-day CARs, CAR$(-5, +5)$ and CAR$(-10, +10)$, in Panels C and D of Table 1, it can be noticed that means for almost all the test statistics deviate significantly away from the theoretical value. Nonetheless, it can be seen that the means of the test statistics PATELL and BMP deviate more rapidly and clearly from the theoretical value of zero than the means of the other test statistics. As well, it can be seen that the mean of the test statistic GRANK seems to deviate more slowly from the theoretical value of zero than the means of the other test statistics. Importantly, all standard deviations of the test statistics are close their theoretical values of unity.

5.2 *Empirical distributions*

Table 2 reports Cramer-von Mises normality tests for ORDIN, PATELL, BMP, SIGN-COWAN and SIGN-GSAR-Z, and Cramer-von Mises tests for GRANK and SIGN-GSAR-T against a t-distribution with 238 ($= T - 2$) degrees of freedom. Departures from normality (t-distribution for GRANK and SIGN-GSAR-T) of the statistics are typically not statistically significant for the AR(0) and CAR$(-1, +1)$, i.e., in the short CAR-windows. Only the normality of the test statistic PATELL is rejected for CAR$(-1, +1)$ and the test statistic SIGN-GSAR-Z for both AR(0) and CAR$(-1, +1)$. In the long CAR-windows (11 and 21 days) the normality is rejected for almost every test statistic. The results indicate that particularly for short CAR-windows a sample size of $n = 50$ series seems to be large enough to warrant the asymptotic t-distribution for SIGN-GSAR-T.

[Table 2]

In Figure 1 empirical quantiles of test statistic SIGN-GSAR-T are displayed from 1,000 simulations against theoretical quantiles of test statistic SIGN-GSAR-T for AR(0), CAR$(-1, +1)$, CAR$(-5, +5)$ and CAR$(-10, +10)$. Only the test statistic SIGN-GSAR-T is considered, because it is derived in this paper and because Cramer-von Mises tests reject the normality of the test statistic SIGN-GSAR-Z for both short and long CAR-windows. In Figure 1 on vertical axis are the Student t quantiles with $T - 2 = 238$ degrees of freedom and on horizontal axis are the test statistics SIGN-GSAR-T. If the statistic follow the theoretical distribution depicted on the vertical axis, the plots should be close to the 45 degree diagonal line. According to Figure 1 the empirical distributions of the test statistics SIGN-GSAR-T and Student t-distributions seem to match quite well, because the plots lie quite well on the straight line.

[Figure 1]

5.3 *Rejection rates*

Columns 2–4 in Table 3 report the lower tail, upper tail and two-tailed rejection rates (Type I errors) at the 5 percent level under the null hypothesis of no event mean effect with no event-induced volatility. Almost all rejection rates are close to the nominal rate of 0.05 for short CAR-windows of AR(0) and CAR$(-1, +1)$. Only PATELL statistic tends to over-reject the null hypothesis for the two-tailed tests and SIGN-GSAR-Z statistics tends to over-reject for left and right tail tests as well as two-tailed tests. For the longer CAR-windows of CAR$(-5, +5)$ and CAR$(-10, +10)$ again all the other test statistics except PATELL, BMP, SIGN-COWAN and SIGN-GSAR-Z reject close to the nominal rate with rejection rates that are well within the approximate 99 percent confidence interval of $[0.032, 0.068]$. For the longer CAR-windows the PATELL tends to over-reject in addition of the two-tailed tests also on the lower tail. The BMP statistic tends to somewhat over-reject the null hypothesis for two-tailed test for CAR$(-10, +10)$ and the SIGN-COWAN statistic tends to over-reject the null hypothesis for CAR$(-10, +10)$ for the upper tail test. The SIGN-GSAR-Z statistic over-rejects the null hypothesis again for left and right tailed tests as well as two-tailed tests. It seems that the tails of the test statistic SIGN-GSAR-Z are fat, which may be the reason why the Cramer-von Mises test rejects the normality of the test statistic SIGN-GSAR-Z in every case.

[Table 3]

Columns 5–13 in Table 3 report the rejection rates under the null hypothesis in the cases where the event-induced variance is present. ORDIN and

PATELL tests over-reject when the variance increases, which is a well-known outcome. At the highest factor of $c = 3.0$ the Type I errors for both ORDIN and PATELL are in the range from 0.2 to 0.3 in two-tailed testing, that is, five to six times the nominal rate. The SIGN-GSAR-Z statistic over-rejects the null hypothesis again for left and right tail tests as well as two-tailed tests. Note that because test statistic SIGN-COWAN takes only account to the sign of the difference between AR and zero, and not for example the sign of the difference between AR and its median, the event-induced volatility does not have an impact on the rejection rates of the test statistics SIGN-COWAN. Hence, the test statistics BMP, GRANK, SIGN-COWAN and SIGN-GSAR-T seem to be the best options in the cases where the event induced volatility is present.

5.4 *Power of the tests*

5.4.1 *Non-clustered event days*

The power results of the test statistics for two-tailed tests are shown in Panels A to D of Table 4 and graphically depicted in Figures 2 to 5. The zero abnormal return line (bold face) in each panel of Table 4 indicates the Type I error rates and replicates the Column 4 in each panel of Table 3. The rest of the lines of Table 4 indicate the rejection rates for the respective ARs shown in the first column.

[Table 4]

[Figures 2–5]

There are four outstanding results. First, at all levels of ARs (positive or negative), ORDIN, which is based on non-standardized returns is materially less powerful than the other test statistics that are based on standardized returns. Second, the test statistic GRANK seems to be one of the most powerful tests for shorter CAR-windows as well as for the longer CAR-windows. Third, both the test statistic SIGN-GSAR-T and SIGN-GSAR-Z seem to have good power properties, but SIGN-GSAR-Z seems to be somewhat more powerful than the test statistic SIGN-GSAR-T in every case. However, it should be noted that test statistic SIGN-GSAR-Z also over-rejects the null hypothesis. Fourth, SIGN-COWAN seems to be more powerful than SIGN-GSAR-T, but less powerful than SIGN-GSAR-Z.

5.4.2 *Clustered event days*

Table 5 reports the Type I error and power results of the tests with clustered event-days. The zero abnormal return line (bold face) in each panel again indicates the Type I error rates at the 5 percent level under the null hypothesis of no event mean effect. Consistent with earlier results [e.g., Kolari and Pynnönen (2010a)], test statistics like ORDIN, PATELL and BMP are prone to material over-rejection of the true null hypothesis of no event effect. The results reported in Table 5 indicate that also the test statistic SIGN-COWAN and SIGN-GSAR-Z are prone to material over-rejection of the true null hypothesis of no event effect. According to Table 5 test statistics GRANK and SIGN-GSAR-T are much more robust to cross-correlation caused by event day clustering. However, a notable distinction of the power results in Table 5 of these statistics compared to those in Table 4 is that the powers tend to be discernibly lower in the clustered case. This is due to the information loss caused by cross-correlation. The problem is discussed in more detail in Kolari and Pynnönen (2010a).

[Table 5]

In summary, the derived test statistic SIGN-GSAR-T as well as the test statistic GRANK statistic are quite robust to clustered event days. In addition the well established asymptotic properties of SIGN-GSAR-T, its robustness against event-induced volatility, and competitive power properties make it a recommended robust testing procedure in event studies.

6. Conclusion

This paper has proposed the nonparametric sign tests SIGN-GSAR-T and SIGN-GSAR-Z based on GSARs. These tests extend the single day sign test statistic presented by Corrado and Zivney (1992) to efficient testing of CARs. Also, the theoretical asymptotic distributions of the statistics have been derived when the estimation period is finite. The proposed testing procedure based on SIGN-GSAR-T, in particular, has advantages of being well specified under the null hypothesis of no event mean effect and being robust to event-induced volatility and cross-correlation (clustered event days) of the returns. Simulation results with actual stock returns also show that the SIGN-GSAR-T test statistic has good empirical power properties. The results of this paper suggest the use of the test statistic SIGN-GSAR-T particularly in the cases where the event days are clustered.

References

Barber, B. M. and Lyon, J. D. (1997): Detecting long-run abnormal returns: The empirical power and specification test statistics, *Journal of Financial Economics*, 81, 175–213.

Beaver, W. H. (1968): The information content of annual announcements, *Journal of Accounting research Supplement*, 6, 67–92.

Berry, M. A., Gallinger, G. W. and Henderson, G. V. (1990): Using daily stock returns in event studies and the choice of parametric versus nonparametric test statistics, *Quarterly Journal of Business and Economics*, 29, 70–85.

Boehmer, E., Musumeci J. and Poulsen, A. B. (1991): Event-study methodology under conditions of event-induced variance, *Journal of Financial Economics*, 30, 253–272.

Brown, S. J. and Warner, J. B. (1980): Measuring security price performance, *Journal of Financial Economics*, 8, 205–258.

Brown, S. J. and Warner, J. B. (1985): Using daily stock returns: The case of event studies, *Journal of Financial Economics*, 14, 3–31.

Campbell, C. J. and Wasley C. E. (1993): Measuring security price performance using daily NASDAQ returns, *Journal of Financial Economics*, 33, 73–92.

Campbell, C. J. and Wasley C. E. (1996): Measuring abnormal trading volume for samples of NYSE/ASE and NASDAQ securities using parametric and non-parametric test statistics, *Review of Quantitative Finance and Accounting*, 6, 309–326.

Campbell, J. Y., Lo, A. W. and MacKinlay A. C. (1997): *The Econometrics of Financial Markets*, Princeton, NJ: Princeton University Press.

Charest, G. (1978): Dividend information, stock returns, and market efficiency, *Journal of Financial Economics*, 6, 265–296.

Corrado, C. J. (1989): A nonparametric test for abnormal security price performance in event studies, *Journal of Financial Economics*, 23, 385–395.

Corrado, C. J. and Zivney, T. L. (1992): The specification and power of the sign test in event study hypothesis tests using daily stock returns, *Journal of Financial and Quantitative Analysis*, 27, 465–478.

Cowan, A. R. (1992): Nonparametric event study tests, *Review of Quantitative Finance and Accounting*, 2, 343–358.

Cowan, A. R. and Sergeant, A. M. A. (1996): Trading frequency and event study test specification, *Journal of Banking & Finance*, 20, 1731–1757.

Dechow, P., Sloan R. and Sweeney, A. (1995): Detecting earnings management, *Accounting Review*, 70, 3–42.

Harrington, S. E. and Shrider, D. G. (2007): All events induce variance: Analyzing abnormal returns when effects vary across firms, *Journal of Financial and Quantitative Analysis* 42, 229–256

Kolari, J. W. and Pynnönen, S. (2010a): Event study testing with cross-sectional correlation of abnormal returns, *Journal of Financial Research*, 23, 3996–4025.

Kolari, J. W. and Pynnönen, S. (2010b): Nonparametric rank tests for event studies, *Working Paper*.
Available at SSRN: http://ssrn.com/abstract=1254022

Kothari, S. P., Leone, A. and Wasley C. (2005): Performance-matched discretionary accruals, *Journal of Accounting and Economics*, 39, 163–197.

Lehmann, E. L. (1999): *Elements of Large-Sample Theory*, New York, Springer-Verlag.

Mikkelson, W. H. (1981): Convertible calls and security returns, *Journal of Financial Economics*, 9, 237–264.

Patell, J. A. (1976): Corporate forecasts of earnings per share and stock price behavior: Empirical test, *Journal of Accounting Research*, 14, 246–276.

Penman, S. H. (1982): Insider trading and the dissemination of firms forecast information, *Journal of Business*, 55, 479–503.

Pynnönen, S. (2010): The joint distribution of an arbitrary linear transformation of internally studentized least squares residuals of a linear regression, *Working Papers of the University of Vaasa, Department of Mathematics and Statistics, 16*,
Available at: http://lipas.uwasa.fi/julkaisu/ewp.html

Rosenstein, S. and Wyatt, J. G. (1990): Outside directors, board independence, and shareholders wealth, *Journal of Financial Economics*, 26, 175–191.

A Appendix: The Properties of the Sign of the GSAR

We derive the theoretical expectation and variance of G_{it} as well as the theoretical covariance between G_{it} and G_{is}, $t \neq s$, $t, s = 1, \ldots, T$, in both of the cases $T = L_1 + 1$ being even and odd.

Using equations from (10) to (13) it is straightforward to see that

$$E[G_{it}] \quad = \quad 0 \tag{A.1}$$

and

$$\mathrm{var}[G_{it}] = \begin{cases} 1, & \text{for even } T \\ \frac{T-1}{T}, & \text{for odd } T. \end{cases} \tag{A.2}$$

Again, if $t \neq s$, it is straightforward to verify the following probabilities

$$\Pr[G_{it}G_{is} = 1] = \begin{cases} \frac{\frac{T}{2}-1}{T-1}, & \text{for even } T \\ \frac{T-3}{2T}, & \text{for odd } T, \end{cases} \tag{A.3}$$

$$\Pr[G_{it}G_{is} = 0] = \begin{cases} 0, & \text{for even } T \\ \frac{2}{T}, & \text{for odd } T \end{cases} \tag{A.4}$$

and

$$\Pr[G_{it}G_{is} = -1] = \begin{cases} \frac{\frac{T}{2}}{T-1}, & \text{for even } T \\ \frac{T-1}{2T}, & \text{for odd } T. \end{cases} \tag{A.5}$$

Furthermore for T being even

$$\text{cov}[G_{it}, G_{is}] = E[G_{it}G_{is}] = -\frac{1}{T-1} \tag{A.6}$$

and for T being odd

$$\text{cov}[G_{it}, G_{is}] = E[G_{it}G_{is}] = -\frac{1}{T}. \tag{A.7}$$

B Appendix: The Asymptotic Distributions of Z_1 and SIGN-GSAR-T

The following Lemmas are utilized in the proofs of Theorem 1 and Theorem 2. Proofs of these Lemmas can be obtained as special cases from Pynnönen (2010).

Lemma 1 *Define*

$$\boldsymbol{x} = \mathbf{Q}\boldsymbol{y}, \tag{B.1}$$

where $\mathbf{Q}$ *is a* $T \times T$ *idempotent matrix of rank* $r \leq T$ *and* $\boldsymbol{y} = (y_1, \ldots, y_T)'$ *is a vector of independent* $N(0, 1)$ *random variables, such that* $\boldsymbol{y} \sim N(\mathbf{0}, \mathbf{I})$, *where* $\mathbf{I}$ *is a* $T \times T$ *identity matrix. Furthermore, let* $\boldsymbol{m}$ *be a* T *component column vector of real numbers such that* $\boldsymbol{m}'\mathbf{Q}\boldsymbol{m} > 0$. *Then*

$$z_m = \frac{\boldsymbol{m}'\boldsymbol{x}/\sqrt{\boldsymbol{m}'\mathbf{Q}\boldsymbol{m}}}{\sqrt{\boldsymbol{x}'\boldsymbol{x}/r}} \tag{B.2}$$

has the distribution with density function

$$f_{z_m}(z) = \frac{\Gamma(r/2)}{\Gamma\left[(r-1)/2\right]\sqrt{r\,\pi}} \left(1 - \frac{z^2}{r}\right)^{\frac{1}{2}(r-1)-1}, \tag{B.3}$$

where $|z| < \sqrt{r}$, *and zero otherwise, and where* $\Gamma(\cdot)$ *is the gamma function.*

Lemma 2 *Under the assumptions of Lemma* 1

$$t_m = z_m \sqrt{\frac{r-1}{r - z_m^2}} \tag{B.4}$$

is distributed as the Student t-distribution with $r - 1$ *degrees of freedom.*

Proof of the Theorem 1: The proof of the theorem is adapted from Kolari and Pynnönen (2010b). In order to derive the asymptotic distribution of the Z_1 defined in equation (19), the G_{it}s defined in (9) are collected to a column vector $\boldsymbol{G}_i = (G_{i,T_0+1}, G_{i,T_0+2}, \ldots, G_{i,T_1}, G_0)'$ of $T = T_1 - T_0 + 1$ components, where the prime denotes transpose and $i = 1, \ldots, n$ with n the number of series. Then by assumption the random vectors $\boldsymbol{G}_i$s are independent and, by Proposition 1, identically distributed random vectors with zero means and identical equicorrelation covariance matrices such that

$$E\left[\boldsymbol{G}_i\right] \;=\; \mathbf{0} \tag{B.5}$$

and

$$\operatorname{cov}[\boldsymbol{G}_i] = \begin{cases} (1-\varrho)\mathbf{I} + \varrho\boldsymbol{\iota}\boldsymbol{\iota}', & \text{for even } T \\ \frac{T-1}{T}[(1-\varrho)\mathbf{I} + \varrho\boldsymbol{\iota}\boldsymbol{\iota}'], & \text{for odd } T. \end{cases} \tag{B.6}$$

Again $i = 1, \ldots, n$, where $\boldsymbol{\iota}$ is a vector of T ones, $\mathbf{I}$ is a $T \times T$ identity matrix, and

$$\varrho = -\frac{1}{T-1}. \tag{B.7}$$

Thus, the covariance matrix in (B.6) becomes

$$\Sigma = \operatorname{cov}[\boldsymbol{G}_i] = \begin{cases} \frac{T}{T-1}\left(\mathbf{I} - \frac{1}{T}\boldsymbol{\iota}\boldsymbol{\iota}'\right), & \text{for even } T \\ \left(\mathbf{I} - \frac{1}{T}\boldsymbol{\iota}\boldsymbol{\iota}'\right), & \text{for odd } T. \end{cases} \tag{B.8}$$

It should be noted that the matrix $\mathbf{I} - T^{-1}\boldsymbol{\iota}\boldsymbol{\iota}'$ is an idempotent matrix of rank $T-1$, which implies that Σ is singular in both of the cases for T being even or odd.

However, because $\boldsymbol{G}_i$s are independent with zero means and finite covariance matrices (B.8), the Central Limit Theorem applies such that

$$\sqrt{n}\,\bar{\boldsymbol{G}} \xrightarrow{d} \left(\frac{T}{T-1}\right)^{\frac{1}{2}} \boldsymbol{x}, \tag{B.9}$$

when T is even and

$$\sqrt{n}\,\bar{\boldsymbol{G}} \xrightarrow{d} \boldsymbol{x}, \tag{B.10}$$

when T is odd, as $n \to \infty$, where

$$\boldsymbol{x} \sim N(\mathbf{0}, \mathbf{Q}) \tag{B.11}$$

with the (idempotent) singular covariance matrix

$$\mathbf{Q} = \mathbf{I} - \frac{1}{T}\boldsymbol{\iota}\boldsymbol{\iota}', \tag{B.12}$$

and in $(B.9)$ and $(B.10)$, $\bar{\boldsymbol{G}} = (\bar{G}_{T_0+1}, \ldots, \bar{G}_{T_1}, \bar{G}_0)'$ with

$$\bar{G}_t = \frac{1}{n}\sum_{i=1}^{n} G_{it}, \tag{B.13}$$

where $t \in \{T_0+1, \ldots, T_1, 0\}$. Note that the sum of $G_{i,t}$ over the time index t is zero for all $i = 1, \ldots, n$, i.e., $\boldsymbol{\iota}'\boldsymbol{G}_i = 0$ for all $i = 1, \ldots, n$, which implies that $\boldsymbol{\iota}'\bar{\boldsymbol{G}} = 0$.

Let $\boldsymbol{\iota}_0$ be a column vector of length $T = T_1 - T_0 + 1$ with one in position in the event day $t = 0$ and zeros elsewhere. In terms of the T-vector $\bar{\boldsymbol{G}}$ and under the assumption that $n_t = n$ for all $t \in \{T_0+1, \ldots, T_1, 0\}$, we can write the Z_1-statistic in equation (19) as

$$Z_1 = \frac{\boldsymbol{\iota}_0'\bar{\boldsymbol{G}}}{\sqrt{\bar{\boldsymbol{G}}'\bar{\boldsymbol{G}}/T}} = \frac{\boldsymbol{\iota}_0'\bar{\boldsymbol{G}}/\sqrt{(T-1)/T}}{\sqrt{\bar{\boldsymbol{G}}'\bar{\boldsymbol{G}}/(T-1)}}. \tag{B.14}$$

Defining in Lemma 1

$$\boldsymbol{m} = \boldsymbol{\iota}_0 \tag{B.15}$$

and

$$\mathbf{Q} = \mathbf{I} - \frac{1}{T}\boldsymbol{\iota}\boldsymbol{\iota}', \tag{B.16}$$

we obtain

$$\boldsymbol{m}'\mathbf{Q}\boldsymbol{m} = \frac{(T-1)}{T}, \tag{B.17}$$

such that the ratio z_m in (B.2) becomes

$$z_m = \frac{\boldsymbol{\iota}_0'\boldsymbol{x}/\sqrt{(T-1)/T}}{\sqrt{\boldsymbol{x}'\boldsymbol{x}/(T-1)}}, \tag{B.18}$$

the distribution of which, after arranging term, has the density function,

$$f_{z_m}(z) = \frac{\Gamma\left[(T-1)/2\right]}{\Gamma\left[(T-2)/2\right]\sqrt{(T-1)\pi}}\left(1-\frac{z^2}{T-1}\right)^{\frac{1}{2}(T-3)} \tag{B.19}$$

for $|z| < \sqrt{T-1}$ and zero elsewhere.

Because of the convergence results in $(B.9)$ and $(B.10)$ and that the function

$$h(\bar{\boldsymbol{G}}) = \frac{\boldsymbol{\iota}_0'\bar{\boldsymbol{G}}/\sqrt{(T-1)/T}}{\sqrt{\bar{\boldsymbol{G}}'\bar{\boldsymbol{G}}/(T-1)}} \tag{B.20}$$

is continuous, the continuous mapping theorem implies $h(\bar{\boldsymbol{G}}) \xrightarrow{d} h(\boldsymbol{x})$. That is,

$$Z_1 = \frac{\boldsymbol{\iota}_0'\bar{\boldsymbol{G}}/\sqrt{(T-1)/T}}{\sqrt{\bar{\boldsymbol{G}}'\bar{\boldsymbol{G}}/(T-1)}} \xrightarrow{d} \frac{\boldsymbol{\iota}_0'\boldsymbol{x}/\sqrt{(T-1)/T}}{\sqrt{\boldsymbol{x}'\boldsymbol{x}/(T-1)}} = z_m, \tag{B.21}$$

which implies that the density function of the limiting distribution of Z_1 for fixed T, as $n \to \infty$, is of the form defined in equation $(B.19)$, completing the proof of Theorem 1.

Proof of the Theorem 2: By the proof of Theorem 1, $Z_1 \xrightarrow{d} z_m$, where z_m is defined in equation $(B.18)$ with $r = T-1$. Again because the function $g(z) = z\sqrt{(T-2)/(T-1-z^2)}$ is continuous, for $|z| < \sqrt{T-1}$, the continuous mapping theorem implies $Z_{SGT} = g(Z_1) \xrightarrow{d} g(z_m)$. That is,

$$Z_{\text{SGT}} \xrightarrow{d} z_m\sqrt{\frac{T-2}{T-1-z_m^2}}, \tag{B.22}$$

where the distribution of the right hand side expression is by Lemma 2 the t-distribution with $T-2$ degrees of freedom, completing the proof of Theorem 2.

Table 1: Sample statistics

The table reports sample statistics from 1,000 simulations for the event day abnormal returns and for the cumulative abnormal returns: CAR$(-1,+1)$, CAR$(-5,+5)$, and CAR$(-10,+10)$. It also reports the sample statistics for the test statistics ORDIN [Eq. (32)], PATELL [Eq. (34)], BMP [Eq. (35)], GRANK [Eq. (37)], SIGN-COWAN [Eq. (41)], SIGN-GSAR-T [Eq. (18)] and SIGN-GSAR-Z [Eq. (26)] for AR(0), CAR$(-1,+1)$, CAR$(-5,+5)$ and CAR$(-10,+10)$. The data is based on 1,000 simulations for portfolios of size $n = 50$ securities with an estimation period of 239 days and event period of 21 days. The event day is denoted as $t = 0$. Cumulative abnormal returns CAR$(-d,+d)$ with d =0, 1, 5 and 10 are computed around the event day. The data consist of securities belonging to S&P 400-, S&P 500- and S&P 600-indexes from July, 1991 to October, 2009. The returns are calculated with the help of the market model presented in equation (30). Superscripts a, b and c correspond to the significance levels 0.10, 0.05 and 0.01.

Test statistics							
Panel A: AR(0)	Mean	Med.	Std.	Skew.	Kurt.	Min.	Max.
AR(0), %	0.004	-0.008	0.413	-0.082	1.018	-1.688	1.641
ORDIN	0.008	-0.019	1.053	-0.079	0.701	-3.878	3.694
PATELL	-0.024	-0.036	1.113	-0.193	1.170	-6.178	3.837
BMP	-0.013	-0.033	1.000	-0.013	0.146	-4.000	3.777
GRANK	0.002	-0.010	0.974	0.056	0.071	-3.518	3.375
SIGN-COWAN	-0.002	-0.042	0.958	0.059	-0.120	-3.475	2.999
SIGN-GSAR-T	-0.016	0.000	0.990	0.041	-0.206	-2.630	2.997
SIGN-GSAR-Z	-0.016	0.000	1.082	0.037	-0.226	-2.828	3.111
Panel B: CAR$(-1,+1)$	Mean	Med.	Std.	Skew.	Kurt.	Min.	Max.
CAR$(-1,+1)$, %	-0.010	-0.029	0.671	-0.019	0.146	-2.288	2.096
ORDIN	-0.018	-0.040	0.988	-0.028	0.306	-3.759	3.329
PATELL	-0.067^{b}	-0.085	1.077	0.133	0.113	-3.380	4.059
BMP	-0.054^{a}	-0.088	1.023	0.159	0.083	-3.208	3.856
GRANK	-0.001	0.011	1.021	0.088	0.189	-3.332	3.963
SIGN-COWAN	0.042	0.108	1.020	0.063	0.127	-3.475	3.832
SIGN-GSAR-T	0.028	0.000	1.036	0.050	0.082	-3.225	3.610
SIGN-GSAR-Z	0.031	0.000	1.128	0.038	0.045	-3.677	3.960

(Continued)

Table 1, continued.

Test statistics							
Panel C: $CAR(-5,+5)$	Mean	Med.	Std.	Skew.	Kurt.	Min.	Max.
CAR(-5,+5), %	-0.076	-0.027	1.269	-0.114	0.346	-5.178	4.455
ORDIN	-0.060^{b}	-0.020	0.959	-0.183	0.433	-3.977	3.441
PATELL	-0.132^{c}	-0.108	1.107	-0.034	0.363	-4.005	3.992
BMP	-0.113^{c}	-0.117	1.036	0.088	0.157	-3.417	3.603
GRANK	0.016	0.062	1.038	0.011	0.113	-3.073	3.334
SIGN-COWAN	0.085^{b}	0.102	0.973	-0.028	-0.081	-2.764	3.334
SIGN-GSAR-T	0.065^{b}	0.000	0.993	0.018	-0.005	-2.804	3.284
SIGN-GSAR-Z	0.071^{b}	0.000	1.084	0.030	0.034	-3.111	3.677
Panel D: $CAR(-10,+10)$	Mean	Med.	Std.	Skew.	Kurt.	Min.	Max.
CAR(-10,+10), %	-0.056	-0.029	1.800	-0.136	0.018	-5.442	4.845
ORDIN	-0.038	-0.015	0.967	-0.225	0.149	-3.332	2.749
PATELL	-0.130^{c}	-0.108	1.105	-0.287	0.852	-5.208	4.129
BMP	-0.100^{c}	-0.117	1.042	0.011	0.033	-3.092	3.759
GRANK	0.071^{b}	0.063	1.053	-0.049	0.282	-3.645	3.942
SIGN-COWAN	0.180^{c}	0.162	1.013	-0.052	0.295	-3.296	3.536
SIGN-GSAR-T	0.148^{c}	0.241	0.997	-0.059	0.370	-3.275	3.423
SIGN-GSAR-Z	0.158^{c}	0.283	1.090	-0.088	0.413	-3.677	3.677

Table 2: Cramer-von Mises tests of the distributions

The table summarizes the results of Cramer-von Mises tests for testing the goodness of fit for different test statistics for AR(0), CAR$(-1,+1)$, CAR$(-5,+5)$ and CAR$(-10,+10)$. Test statistics ORDIN [Eq.(32)], PATELL [Eq.(34)], BMP [Eq.(35)], SIGN-COWAN [Eq.(41)] and SIGN-GSAR-Z [Eq. (26)] are tested against the standard normal distribution and test statistics GRANK [Eq.(37)] and SIGN-GSAR-T [Eq.(18)] are tested against the Student t-distribution with 238 $(= T-2)$ degrees of freedom. Superscripts a and b correspond to the significance levels 0.05 and 0.01. See Table 1 for details of the simulation setup.

	AR(0)	CAR(-1,+1)	CAR(-5,+5)	CAR(-10,+10)
ORDIN	0.054	0.218	0.350	0.196
PATELL	0.164	0.795^b	1.488^b	1.104^b
BMP	0.066	0.625	1.277^b	0.985^b
GRANK	0.074	0.029	0.143	0.541^a
SIGN-COWAN	0.136	0.270	0.916^b	2.994^b
SIGN-GSAR-T	0.361	0.387	0.855^b	2.400^b
SIGN-GSAR-Z	0.918^b	1.006^b	1.288^b	2.871^b

Table 3: Lower tail, upper tail and two-tailed rejection rates

The table reports the lower tail, upper tail and two-tailed rejection rates (Type I errors) at the 5 percent level under the null hypothesis of no event mean effect for AR(0), CAR$(-1,+1)$, CAR$(-5,+5)$ and CAR$(-10,+10)$ and for the test statistics ORDIN [Eq. (32)], PATELL [Eq. (34)], BMP [Eq. (35)], GRANK [Eq. (37)], SIGN-COWAN [Eq. (41)], SIGN-GSAR-T [Eq. (18)] and SIGN-GSAR-Z [Eq. (26)]. The 99 percent confidence interval around the 0.05 rejection rate is $[0.032, 0.068]$. See Table 1 for details of the simulation setup.

	c=1.0			c=1.5			c=2.0			c=3.0		
Test statistic	Left	Right	Two-tail	Left	Right	Two-tail	Left	Right	Two-tail	Left	Right	Two-tail
Panel A: AR(0)												
ORDIN	0.050	0.055	0.058	0.085	0.091	0.113	0.131	0.127	0.162	0.174	0.188	0.261
PATELL	0.064	0.057	0.070	0.109	0.103	0.132	0.136	0.136	0.194	0.183	0.180	0.291
BMP	0.048	0.042	0.045	0.051	0.044	0.041	0.051	0.043	0.045	0.048	0.042	0.044
GRANK	0.044	0.048	0.047	0.041	0.048	0.046	0.042	0.046	0.048	0.043	0.046	0.045
SIGN-COWAN	0.040	0.044	0.037	0.040	0.044	0.037	0.040	0.044	0.037	0.040	0.044	0.037
SIGN-GSAR-T	0.048	0.041	0.042	0.046	0.044	0.041	0.048	0.042	0.042	0.046	0.042	0.042
SIGN-GSAR-Z	0.087	0.071	0.085	0.084	0.073	0.086	0.087	0.074	0.086	0.087	0.070	0.085
Panel B: CAR(-1,+1)												
ORDIN	0.052	0.044	0.047	0.085	0.090	0.108	0.113	0.127	0.165	0.163	0.165	0.250
PATELL	0.067	0.058	0.076	0.112	0.092	0.139	0.149	0.128	0.190	0.203	0.164	0.293
BMP	0.053	0.054	0.054	0.051	0.052	0.051	0.054	0.053	0.054	0.054	0.055	0.054
GRANK	0.049	0.051	0.052	0.052	0.050	0.055	0.051	0.048	0.054	0.048	0.052	0.053
SIGN-COWAN	0.051	0.062	0.055	0.051	0.062	0.055	0.051	0.062	0.055	0.051	0.062	0.055
SIGN-GSAR-T	0.056	0.057	0.058	0.056	0.057	0.059	0.057	0.056	0.057	0.056	0.056	0.058
SIGN-GSAR-Z	0.082	0.091	0.105	0.083	0.092	0.105	0.081	0.090	0.105	0.083	0.091	0.104

(Continued)

Table 3, continued.

	c=1.0			c=1.5			c=2.0			c=3.0		
Test statistic	Left	Right	Two-tail	Left	Right	Two-tail	Left	Right	Two-tail	Left	Right	Two-tail
Panel C: CAR(-5,+5)												
ORDIN	0.052	0.033	0.042	0.092	0.065	0.098	0.127	0.083	0.148	0.173	0.132	0.227
PATELL	0.080	0.054	0.080	0.130	0.093	0.141	0.166	0.112	0.200	0.217	0.149	0.299
BMP	0.066	0.046	0.060	0.063	0.044	0.057	0.072	0.043	0.060	0.067	0.041	0.062
GRANK	0.056	0.054	0.058	0.053	0.050	0.058	0.057	0.051	0.058	0.055	0.053	0.057
SIGN-COWAN	0.042	0.045	0.042	0.042	0.045	0.042	0.042	0.045	0.042	0.042	0.045	0.042
SIGN-GSAR-T	0.041	0.053	0.039	0.041	0.053	0.038	0.040	0.055	0.039	0.040	0.054	0.037
SIGN-GSAR-Z	0.076	0.079	0.091	0.076	0.080	0.090	0.076	0.077	0.092	0.076	0.078	0.090
Panel D: CAR(-10,+10)												
ORDIN	0.053	0.033	0.048	0.090	0.078	0.099	0.120	0.101	0.154	0.174	0.146	0.233
PATELL	0.086	0.050	0.075	0.125	0.089	0.145	0.0159	0.115	0.198	0.213	0.161	0.289
BMP	0.067	0.047	0.069	0.065	0.050	0.066	0.067	0.047	0.065	0.066	0.044	0.069
GRANK	0.053	0.063	0.064	0.053	0.063	0.061	0.051	0.062	0.063	0.053	0.065	0.063
SIGN-COWAN	0.032	0.073	0.060	0.032	0.073	0.060	0.032	0.073	0.060	0.032	0.073	0.060
SIGN-GSAR-T	0.032	0.063	0.054	0.033	0.066	0.065	0.034	0.064	0.054	0.034	0.063	0.054
SIGN-GSAR-Z	0.044	0.092	0.089	0.046	0.094	0.093	0.044	0.092	0.092	0.046	0.092	0.091

Table 4: Non-clustered event days: Powers of the test statistics

The table reports the power results of the test statistics ORDIN [Eq. (32)], PATELL [Eq. (34)], BMP [Eq. (35)], GRANK [Eq. (37)], SIGN-COWAN [Eq. (41)], SIGN-GSAR-T [Eq. (18)] and SIGN-GSAR-Z [Eq. (26)] for two-tailed tests for AR(0), CAR$(-1,+1)$, CAR$(-5,+5)$ and CAR$(-10,+10)$. The zero abnormal return line (bold face) in each panel of this table indicates the Type I error rates. The rest of the lines of this table indicate the rejection rates for the respective ARs shown in the first column. General details of the simulation setup are given in Table 1.

Panel A: AR(0)							
AR	ORDIN	PATELL	BMP	GRANK	SIGN-COWAN	SIGN-GSAR-T	SIGN-GSAR-Z
−3.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000
−2.0	0.996	1.000	0.999	1.000	1.000	0.997	0.999
−1.0	0.720	0.960	0.909	0.958	0.912	0.856	0.910
±0.0	**0.058**	**0.070**	**0.045**	**0.047**	**0.037**	**0.042**	**0.085**
+1.0	0.729	0.942	0.899	0.969	0.928	0.886	0.928
+2.0	0.994	0.999	0.996	0.999	1.000	1.000	1.000
+3.0	1.000	1.000	0.999	1.000	1.000	1.000	1.000
Panel B: CAR(-1,+1)							
AR	ORDIN	PATELL	BMP	GRANK	SIGN-COWAN	SIGN-GSAR-T	SIGN-GSAR-Z
−3.0	0.994	1.000	0.999	1.000	0.999	0.998	1.000
−2.0	0.983	0.965	0.971	0.984	0.957	0.909	0.954
−1.0	0.313	0.595	0.572	0.656	0.528	0.446	0.575
±0.0	**0.047**	**0.076**	**0.054**	**0.052**	**0.055**	**0.058**	**0.105**
+1.0	0.297	0.535	0.514	0.638	0.572	0.479	0.611
+2.0	0.834	0.981	0.958	0.993	0.971	0.928	0.964
+3.0	0.987	1.000	0.999	1.000	0.999	0.995	0.998

(*Continued*)

Table 4 Continued.

Panel C: CAR(-5,+5)							
AR	ORDIN	PATELL	BMP	GRANK	SIGN-COWAN	SIGN-GSAR-T	SIGN-GSAR-Z
−3.0	0.633	0.912	0.892	0.913	0.782	0.716	0.799
−2.0	0.326	0.619	0.614	0.628	0.481	0.438	0.550
−1.0	0.120	0.244	0.239	0.233	0.155	0.146	0.221
±0.0	**0.042**	**0.080**	**0.060**	**0.058**	**0.042**	**0.039**	**0.091**
+1.0	0.085	0.182	0.171	0.234	0.195	0.175	0.254
+2.0	0.312	0.535	0.538	0.659	0.572	0.490	0.614
+3.0	0.616	0.867	0.843	0.920	0.865	0.791	0.856

Panel D: CAR(-10,+10)							
AR	ORDIN	PATELL	BMP	GRANK	SIGN-COWAN	SIGN-GSAR-T	SIGN-GSAR-Z
−3.0	0.355	0.680	0.670	0.655	0.471	0.422	0.551
−2.0	0.184	0.379	0.378	0.368	0.223	0.212	0.299
−1.0	0.088	0.165	0.154	0.141	0.077	0.067	0.139
±0.0	**0.048**	**0.075**	**0.069**	**0.064**	**0.060**	**0.054**	**0.089**
+1.0	0.069	0.126	0.118	0.161	0.150	0.124	0.203
+2.0	0.162	0.316	0.320	0.398	0.349	0.300	0.403
+3.0	0.351	0.609	0.598	0.699	0.613	0.539	0.637

Table 5: Clustered event days: Powers of the test statistics

The table reports the power results of the test statistics ORDIN [Eq. (32)], PATELL [Eq. (34)], BMP [Eq. (35)], GRANK [Eq. (37)], SIGN-COWAN [Eq. (41)], SIGN-GSAR-T [Eq. (18)] and SIGN-GSAR-Z [Eq. (26)] for two-tailed tests for AR(0), CAR$(-1,+1)$, CAR$(-5,+5)$ and CAR$(-10,+10)$ in the case where the event days are clustered. The zero abnormal return line (bold face) in each panel of this table indicates the Type I error rates. The rest of the lines of this table indicate the rejection rates for the respective ARs shown in the first column. General details of the simulation setup are given in Table 1. With the clustered event days in each simulation round first a single event day is randomly selected which is the same for each of the $n = 50$ securities sampled *without* replacement in the next step.

Panel A: AR(0)							
AR	ORDIN	PATELL	BMP	GRANK	SIGN-COWAN	SIGN-GSAR-T	SIGN-GSAR-Z
−3.0	0.995	0.998	0.996	0.990	0.997	0.985	0.995
−2.0	0.968	0.988	0.982	0.953	0.981	0.936	0.980
−1.0	0.692	0.858	0.844	0.716	0.851	0.627	0.852
±0.0	**0.177**	**0.203**	**0.216**	**0.055**	**0.190**	**0.055**	**0.244**
+1.0	0.690	0.838	0.828	0.714	0.839	0.618	0.853
+2.0	0.965	0.989	0.980	0.960	0.985	0.933	0.987
+3.0	0.999	0.999	0.996	0.992	0.998	0.987	0.999
Panel B: CAR(-1,+1)							
AR	ORDIN	PATELL	BMP	GRANK	SIGN-COWAN	SIGN-GSAR-T	SIGN-GSAR-Z
−3.0	0.928	0.976	0.956	0.923	0.954	0.874	0.960
−2.0	0.750	0.895	0.880	0.779	0.858	0.680	0.863
−1.0	0.397	0.563	0.572	0.380	0.529	0.290	0.586
±0.0	**0.202**	**0.244**	**0.258**	**0.080**	**0.215**	**0.059**	**0.254**
+1.0	0.421	0.575	0.582	0.382	0.594	0.323	0.618
+2.0	0.774	0.893	0.880	0.805	0.902	0.745	0.905
+3.0	0.950	0.981	0.967	0.940	0.969	0.909	0.976

(*Continued*)

Table 5 Continued.

Panel C: CAR(-5,+5)							
AR	ORDIN	PATELL	BMP	GRANK	SIGN-COWAN	SIGN-GSAR-T	SIGN-GSAR-Z
−3.0	0.591	0.775	0.779	0.603	0.689	0.456	0.730
−2.0	0.405	0.554	0.577	0.348	0.480	0.240	0.532
−1.0	0.244	0.322	0.355	0.140	0.280	0.104	0.337
±0.0	**0.197**	**0.221**	**0.247**	**0.083**	**0.204**	**0.062**	**0.257**
+1.0	0.278	0.350	0.363	0.182	0.331	0.137	0.392
+2.0	0.450	0.586	0.609	0.425	0.594	0.325	0.623
+3.0	0.654	0.797	0.801	0.662	0.810	0.545	0.806
Panel D: CAR(-10,+10)							
AR	ORDIN	PATELL	BMP	GRANK	SIGN-COWAN	SIGN-GSAR-T	SIGN-GSAR-Z
−3.0	0.442	0.583	0.638	0.387	0.486	0.244	0.587
−2.0	0.327	0.442	0.462	0.227	0.337	0.136	0.381
−1.0	0.247	0.295	0.319	0.128	0.227	0.072	0.280
±0.0	**0.206**	**0.239**	**0.249**	**0.087**	**0.214**	**0.068**	**0.257**
+1.0	0.237	0.287	0.315	0.145	0.300	0.104	0.339
+2.0	0.339	0.451	0.476	0.266	0.447	0.190	0.469
+3.0	0.468	0.614	0.626	0.443	0.619	0.330	0.639

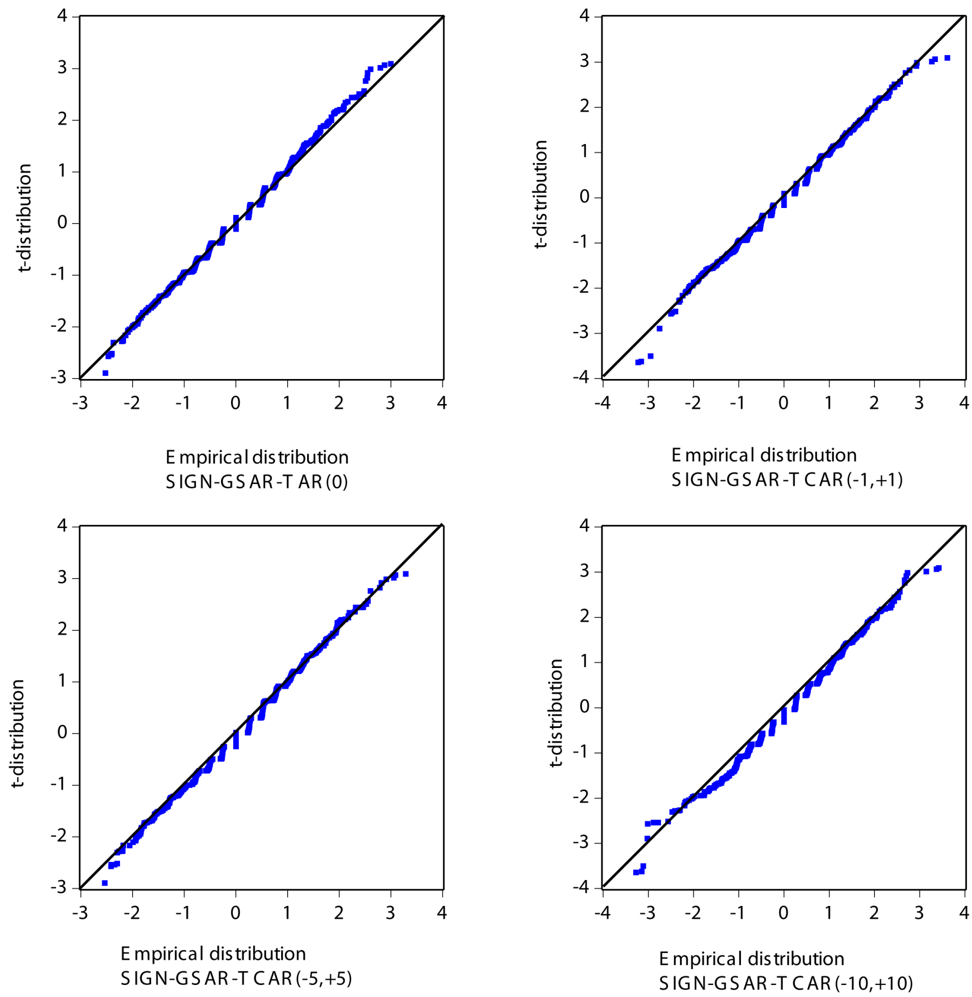

Figure 1: The Q-Q plots of the test statistic SIGN-GSAR-T

The figure illustrates the theoretical quantile-quantiles (Q-Q) for the test statistic SIGN-GSAR-T [Eq. (18)] in cases AR(0), CAR$(-1,+1)$, CAR$(-5,+5)$ and CAR$(-10,+10)$. In vertical axes there are Students t-distributions with 238 $(= T-2)$ degrees of freedom and in horizontal axes there are the test statistics SIGN-GSAR-Ts. The data is based on 1,000 simulations for portfolios of size $n = 50$ securities with non-clustered event days, and with an estimation period of 239 days and event period of 21 days. The event day is denoted as $t = 0$. The data consist of securities belonging to S&P 400-, S&P 500- and S&P 600-indexes from July, 1991 to October, 2009. The returns are calculated with the help of the market model presented in equation (30).

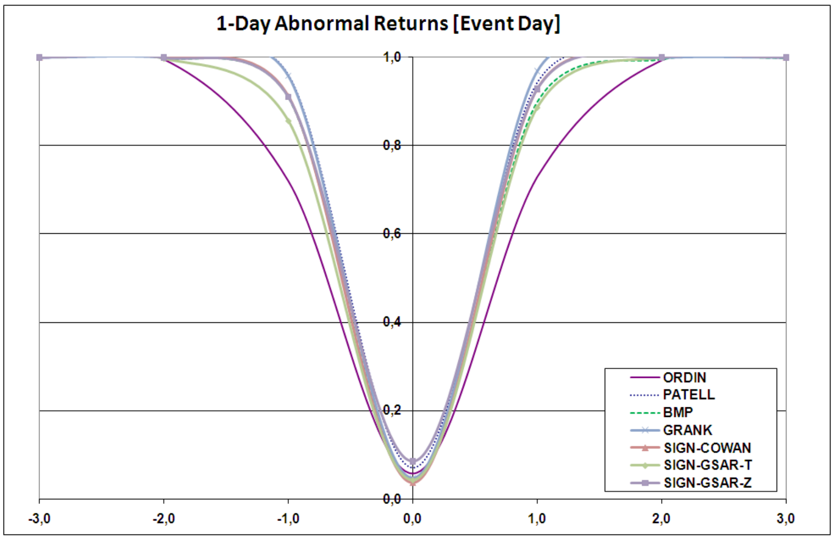

Figure 2: The power results for AR(0)
The figure illustrates the power results of the test statistics for two-tailed tests for testing AR(0) with an AR ranging from −3 percent to +3 percent. General details of the simulation setup are given in the Figure 1.

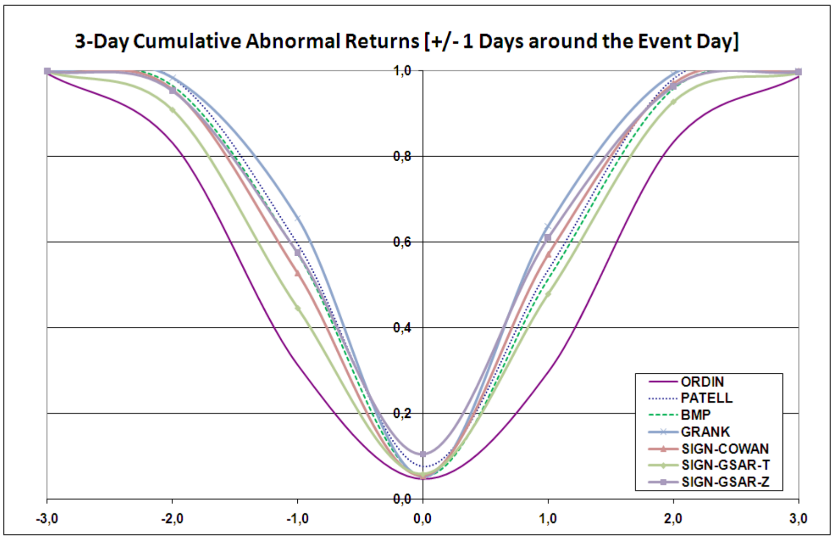

Figure 3: The power results for CAR(−1, +1)
The figure illustrates the power results of the test statistics for two-tailed tests for testing CAR(−1, +1) with an AR ranging from −3 percent to +3 percent. General details of the simulation setup are given in the Figure 1.

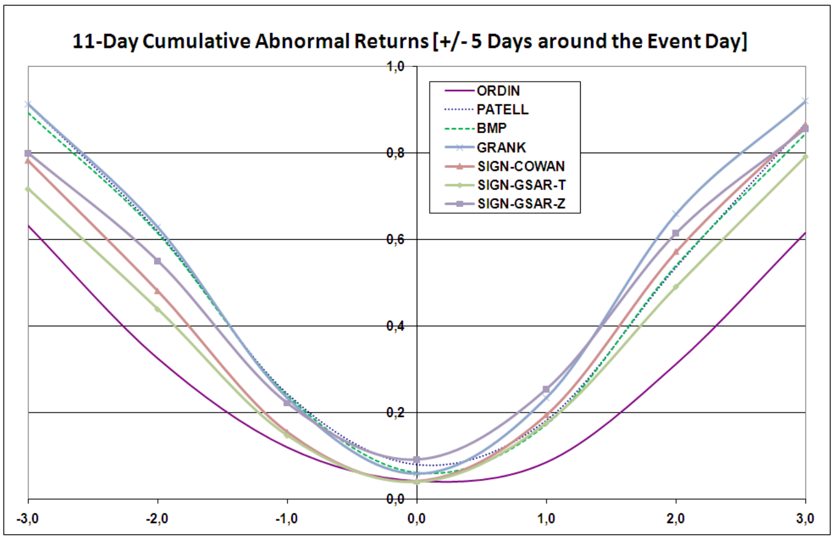

Figure 4: The power results for CAR$(-5, +5)$
The figure illustrates the power results of the test statistics for two-tailed tests for testing CAR$(-5, +5)$ with an AR ranging from -3 percent to $+3$ percent. General details of the simulation setup are given in the Figure 1.

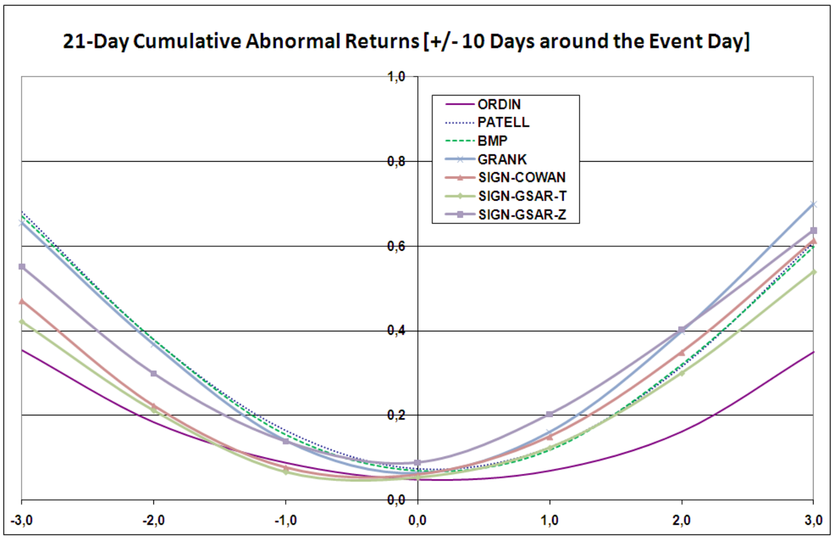

Figure 5: The power results for CAR$(-10, +10)$
The figure illustrates the power results of the test statistics for two-tailed tests for testing CAR$(-10, +10)$ with an AR ranging from -3 percent to $+3$ percent. General details of the simulation setup are given in the Figure 1.