Forecasting the Daily Dynamic Hedge Ratios in Emerging Stock Futures Markets: Evidence from the GARCH models

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Abstract

This paper investigates the forecasting ability of five different versions of GARCH models. The five GARCH models applied are bivariate GARCH, GARCH-ECM, BEKK GARCH, GARCH-X and GARCH-GJR. Forecast errors based on four emerging stock futures portfolio return (based on forecasted hedge ratio) forecasts are employed to evaluate out-of-sample forecasting ability of the five GARCH models. Daily data from December 1999 to December 2009 from Brazil, Hungary, South Africa and South Korea are applied. Forecasts are conducted over two out-of-sample periods, one 2-year period 2008-2009 and one 1-year period 2007. Results show that BEKK model outperforms the other models during the 2-year forecast horizon and the GARCH-X is the best model during the 1-year forecast horizon. The GARCH model performs the worst during both forecast horizons.

JEL Classification: G1, G15

Key Words: Forecasting, Kalman Filter, GARCH, Time-varying beta, Volatility.

1. Introduction

Lately there has been high interest towards the modelling of the optimal hedge ratios (OHR) and alternative hedging strategies applied to the commodity and financial futures (see Choudhry, 2009 for citations). It is now well-known that the principal functions of futures markets are price discovery, hedging, speculation and risk-sharing. Hedgers use these markets as a means to avoid the risk associated with adverse price change in the related cash markets. A hedge is performed by taking simultaneous positions both in cash and future markets--which result in the offset of any loss sustained from an adverse price movement in one market by a favourable price movement in another market. The hedge ratio is simply the number of futures contracts needed to minimize the exposure of a unit worth position in the cash market.[1]

Given the plethora of literature in this field, there are serious gaps in the current research strand in two directions. Firstly, most previous studies confined their attention to more developed and mature financial markets and exchanges. Thus there is a lack of studies involving emerging markets and exchanges. This gap in the literature is more acute in view of the fact that there are informational linkages across global financial markets, which is evidenced in the recent global financial market crisis and the consequent melt down effect of the crisis. Secondly, previous research has evaluated the relative effectiveness of alternative hedging strategies by examining the in-sample and out-of-sample performance of variance reduction of portfolios of returns in the cash and futures markets based on the information of hedge ratios, spot and futures prices of the assets. Surprisingly, however, from a risk management perspective, there has been no attempt to evaluate the forecasting accuracy and

[1] Therefore, an investor holding a long position in the cash market should short h units of futures contracts, where h would be the hedge ratio.

performance of the estimated hedge ratios derived from different econometric models. Indeed, forecasting of optimal hedge ratio/dynamic hedge ratio is important for understanding the role of futures markets in equity trading, program trading, index arbitrage and the development of optimal hedging and trading strategies in fund management. One of the most important properties of forecasting is that it is expansively used in planning or decision-making situations. The forecasting of optimal hedge ratio helps hedger choose appropriate portfolio and allows for portfolio adjustment in dynamic hedging. Generally speaking, given that hedge ratios of various portfolios are predictable, an investor always prefers a portfolio with lower financial capital to reach the maximum of risk reduction. The forecasts of optimal hedge ratio helps investor choose optimal portfolio with suitable futures and reasonable number of futures contract. This paper attempts to fill both of these gaps in the literature.

This article investigates the behaviour of dynamic hedge ratios in four emerging markets using alternative variants of GARCH models and compares the forecasting performance of optimal hedge ratios across those models. More specifically, using daily data from the spot and futures markets of Brazil, Hungary, South Africa and South Korea we estimate the time-varying hedge ratios and compared the forecasting performances of five different GARCH models. The five GARCH under study are the standard GARCH, GARCH-ECM, BEKK GARCH, GARCH-X, and asymmetric GARCH-GJR models. To our knowledge, no previous study empirically investigates the out-of-sample forecasting by different GARCH models of time-varying optimal hedge ratio and then compares the forecasting performance of these models based on the forecasting results.[2] Given the widespread application of the GARCH modelling technique in this area of research, this study provides an illuminating opportunity to investigate this issue as an interesting research strand--filling a gap in the

[2] In previous studies different version of the GARCH models have been used for forecasting of volatility, time-varying beta, etc and then the models compared. See Choudhry and Wu (2008) for citations of some these previous studies.

existing literature. Therefore, this result has important implications for academics, researchers, financial practitioners, and policy-makers.

Choosing of appropriate forecasting methods is an important issue. A large number of different GARCH models have been employed in previous research for forecasting purposes (see Choudhry and Wu, 2008). These different models have different forecasting ability and no superiority is suggested for any particular forecasting model in predicting hedge ratio. The reasonable assumptions are essential for forecasting. For example, the relationship between futures and cash prices for most financial assets is not assured, and if this presupposition is not tenable, forecasts of hedge ratio is not reliable. Time horizon influences hedge ratio forecasts. Different hedging horizon might affect forecasting accuracy for various forecasting methods (Chen et al. 2003). The longer the forecasting horizon, the more data are included and the more accurate the forecasts is. But the market environment might change or unexpected events happen which occurs assumptions less reasonable in a long time horizon (Michael, 1977). Some activities of competitors will affect forecasting accuracy as well. The more competition in the market, the more difficult it is to forecast hedge ratio. In the market that has great competition, competitors can change the course of future events after they make forecast, in order to make themselves more competitive, which makes forecasts invalid (Markridakis, Spyros G., 1989). Forecasting has limitations on its accuracy, but we cannot deny the significance of forecast and neglect its merits with this shadow. In the active derivatives' market, decision-making depends on the quality of the forecasts, and hence forecast of hedge ratio is important and meaningful for hedgers (Roger and Gilbert, 1971).

2. Estimation of Optimal Hedge Ratios and GARCH Models

The notion of OHR emanates within the mean-variance utility framework with the choice of portfolio returns from a simultaneous position both in cash and futures markets that minimize the hedged portfolio variance and maximize the risk-averse investors expected utility (Johnson, 1960, Ederington, 1979). Let F_T and F_{T+1} be the purchase price and settlement price of a future contracts, and C_T and C_{T+1} be the corresponding purchase price and settlement price in the cash market, respectively [see Cecchetti *et al.* (1988), Baillie and Myers (1991), and Kroner and Sultan (1993) for more][3]. Then $p = C_{T+1} - C_T + h\,(F_{T+1} - F_T)$ signifies the random return of simultaneously holding one unit of spot and h units of futures contracts in the portfolio. The hedger's expected return on a simple one-period wealth function is:

$$\mathrm{E}(p) = \mathrm{E}(C_{T+1} - C_T) + h\mathrm{E}(F_{T+1} - F_T) \qquad (1)$$

$$= \mathrm{E}(c) + h\mathrm{E}(f)$$

where the lower case variables refer to change in the asset price. In the cash market, the hedger has a long commitment of one unit of cash positions which is hedged by shorting h units of opposite position in the futures market. The variance of the hedger's portfolio return is:

$$\sigma_p^2 = \sigma_c^2 + b^2\sigma_f^2 + 2h\sigma_{c,f} \qquad (2)$$

where the sources of risk in the portfolio are the volatility of the cash market, the volatility of the futures market, and the covariance between the cash and the futures market. The investor maximizes the following expected utility function, which specifies the trade-off between expected wealth and the volatility of the wealth,

$$\max U(p) = E(p) - r\sigma_p^2 \qquad (3)$$

[3]This section draws extensively from Kroner and Sultan (1993), and Sultan and Hasan (2008).

where r > 0 is the degree of risk aversion. By substituting the definition of volatility from equation (2), the utility maximization function is rewritten as:

$$\max U(p) = E(c) + hE(f) - r[\sigma_c^2 + h^2\sigma_f^2 + 2h\sigma_{c,f}] \tag{4}$$

After taking the first order condition for an extremum with respect to *h* and solving under the condition that the futures returns follow a martingale process, i.e., $E(F_{T+1})=F_T$, yields Johnson's (1960) risk minimizing hedge ratio h^*:

$$h^* = -\frac{\sigma_{c,f}}{\sigma_f^2} = -\frac{\text{cov}(R_c, R_f)}{\text{var}(R_f)} \tag{5}$$

where R_c and R_f denotes return on spot and future indices.[4] It is now well-known in the literature that the conventional hedging model has shortcomings. As the distribution of futures and spot prices are changing through time, h^* which is expressed as the ratio of covariance between futures returns and cash returns and variance of futures returns, moves randomly through time [Checchetti *et al*. (1988), Baillie and Myers (1991), Kroner and Sultan (1993)]. Therefore eq.(5) should be modified as:

$$h_T^* = \frac{\text{cov}(R_{T+1}^c, R_{T+1t}^f \text{I}\Omega_T)}{\text{var}(R_{T+1}^f \text{I}\Omega_T)} \tag{6}$$

In eq. (6), conditional moments are changing as the information set, Ω_T, is updated, consequently, the number of futures contracts held and the optimal hedge ratio will also change through time--hence the *t* subscripts of h_T^*.

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[4] The OHR (equation 5) then is computed as the slope coefficient of the following regression:

$$c_t = \alpha + \beta f_t + \varepsilon_t \tag{7}$$

where c_t and f_t and β (h) are defined as before and ε_t is an error term.[4] A $\beta = 0$ implies unhedged position; $\beta = 1$ signifies a fully hedged position; and $\beta < 1$ implies a partial hedge.

Bivariate GARCH Model

The time-varying hedge ratios are estimated using the following five different variants of GARCH models: standard bivariate GARCH, GARCH-ECM, GARCH-BEKK, GARCH-X, and GARCH-GJR. The following bivariate GARCH(p,q) model is applied to returns from the stock cash and futures markets:

$$y_t = \mu + \varepsilon_t \qquad (8)$$

$$\varepsilon_t / \Omega_{t-1} \sim N(0, H_t) \qquad (9)$$

$$vech(H_t) = C + \sum_{i=1}^{p} A_i vech(\varepsilon_{t-i})^2 + \sum_{j=1}^{q} B_j vech(H_{t-j}) \qquad (10)$$

Where $y_t = (r_t^c, r_t^f)$ is a (2x1) vector containing stock returns from the cash and futures markets. H_t is a (2x2) conditional covariance matrix, C is (3x1) parameter vector of constant, A_i and B_j are (3x3) parameter matrices, and vech is the column stacking operator that stacks the lower triangular portion of a symmetric matrix.

To make estimation amenable, Engle and Kroner (1995) have suggested various restrictions to be imposed on the parameters of A_i and B_j matrices. A parsimonious representation may be achieved by imposing a diagonal restriction on the parameters matrices so that each variance and covariance element depends only on its own past values and prediction errors. The following equations represent a diagonal vech bivariate GARCH(1,1) conditional variance equation(s):

$$H_{11,t} = C_1 + A_{11}(\varepsilon_{1,t-1})^2 + B_{11}(H_{11,t-1}) \qquad (11a)$$

$$H_{12,t} = C_2 + A_{22}(\varepsilon_{1,t-1}, \varepsilon_{2,t-1}) + B_{22}(H_{12,t-1}) \qquad (11b)$$

$$H_{22} = C_3 + A_{33}(\varepsilon_{2,t-1})^2 + B_{33}(H_{22,t-1}) \qquad (11c)$$

In the bivariate GARCH(1,1) model, the diagonal vech parameterization involves nine conditional variance parameters.

Using the bivariate GARCH model, the time-varying hedge ratio can be computed as:

$$h_t^* = \hat{H}_{12,t} / \hat{H}_{22,t} \qquad (12)$$

Where $\hat{H}_{12,t}$ is the estimated conditional covariance between the cash and futures returns, and $\hat{H}_{22,t}$ is the estimated conditional variance of futures returns. Since, the conditional covariance is time-varying, the optimal hedge would be time-varying too.

Bivariate GARCH-ECM Model

When the bivariate GARCH model incorporates the error correction term in the mean equation, it becomes the GARCH-ECM model which is presented as:

$$y_t = \mu + \delta(u_{t-1}) + \varepsilon_t \qquad (13)$$

where u_{t-1} denotes the lagged error-correction term, retrieved from the cointegration regression. Kroner and Sultan (1993), and Sultan and Hasan (2008) noted that the traditional method of estimating the hedge ratio does not incorporate the no-arbitrage condition between the spot and futures markets. For financial markets, the difference between the cash and the futures is the basis, and when the basis becomes large, the markets will correct as arbitrageurs execute trading strategies to exploit this temporary disequilibrium. Econometrically, this is known as the error correction term which is a result of a cointegrating relationship between the cash and futures markets.[5] Therefore, a bivariate GARCH-ECM model will be employed to account for the long-run relationship and basis risk. The time-varying hedge ratio based on the GARCH-ECM model is also expressed as equation 12.

Bivariate GARCH-BEKK Model

[5]See Brenner and Kroner (1995) for further discussion why the basis represents the cointegrating relationship for financial asset prices and their futures contracts.

In the BEKK model as suggested by Engle and Kroner (1995), the conditional covariance matrix is parameterized to:

$$vech(H_t) = C'C + \sum_{k=1}^{k}\sum_{i=1}^{p} A'_{ki}\varepsilon_{t-i}\varepsilon'_{t-1}A_{ki} + \sum_{k=1}^{k}\sum_{j=1}^{q} B'_{kj}H_{t-j}B_{kj} \tag{14}$$

Eqs. (8) and (9) also apply to the BEKK model and are defined as before. In eq(14) A_{ki}, $i = 1,...q$, $k = 1,...k$, and B_{kj} $j = 1,...q$, $k = 1,..k$ are NxN matrices. The GARCH-BEKK model is sufficiently general that it guarantees the conditional covariance matrix, H_t to be positive definite, and renders significant parameter reduction in the estimation. For example, a bivariate BEKK GARCH(1,1) pamerization requires to estimate only 11 parameters in the conditional variance-covariance structure. The time-varying hedge ratio from the BEKK is again represented by equation 12.

Bivariate GARCH-X Model

The GARCH-X model is an extension of the GARCH-ECM model as it incorporates the square of error correction term in the conditional covariance matrix. Lee (1994) contends that, as short-run deviations from the long-run relationship between the cash and futures prices may affect the conditional variance and covariance, then they will also influence the time-varying optimal hedge ratio. In the GARCH-X model, conditional heteroscedasticity may be modelled as a function of lagged squared error correction term--in addition to the ARMA terms in the variance/covariance equations:

$$vech(H_t) = C + \sum_{i=1}^{p} A_i vech(\varepsilon_{t-i})^2 + \sum_{j=1}^{q} B_j vech(H_{t-j}) + \sum_{k=1}^{k} D_k vech(u_{t-1})^2 \tag{15}$$

A significant positive effect may imply that the further the series deviates from each other in the short run, the harder they are to predict. Equation 13 also apply in GARCH-X. The time-varying hedge ratio is again represented by equation 12.

Bivariate GARCH-GJR

Along with the leptokurtic distribution of stock returns data, empirical research has shown a negative correlation between current returns and future volatility (Black, 1976; Christie, 1982). This negative effect of current returns on future variance is sometimes called the leverage effect (Bollerslev et al. 1992).[6] In the linear (symmetric) GARCH model, the conditional variance is only linked to past conditional variances and squared innovations (ε_{t-1}), and hence the sign of return plays no role in affecting volatilities (Bollerslev et al. 1992). Glosten et al. (1993) provide a modification to the GARCH model that allows positive and negative innovations to returns to have different impact on conditional variance.[7] Glosten et al. (1993) suggest that the asymmetry effect can also be captured simply by incorporating a dummy variable in the original GARCH.

$$\sigma_t^2 = \alpha_0 + \alpha u_{t-1}^2 + \gamma u_{t-1}^2 I_{t-1} + \beta \sigma_{t-1}^2 \qquad (16)$$

Where $I_{t-1} = 1$ if $u_{t-1} > 0$; otherwise $I_{t-1} = 0$. Thus, the ARCH coefficient in a GARCH-GJR model switches between $\alpha + \gamma$ and α, depending on whether the lagged error term is positive or negative. Similarly, this version of GARCH model can be applied to two variables to capture the conditional variance and covariance. Equations 8 and 9 also apply here. The time-varying beta based on the GARCH-GJR model is also expressed as equation 12.

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[6] The leverage effect is due to the reduction in the equity value, which would raise the debt-to-equity ratio, hence raising the riskiness of the firm as a result of an increase in future volatility. Glosten et al. (1993) provide an alternative explanation for the negative effect; if most of the fluctuations in stock prices are caused by fluctuations in expected future cash flows, and the riskiness of future cash flows does not change proportionally when investors revise their expectations, the unanticipated changes in stock prices and returns will be negatively related to unanticipated changes in future volatility.

[7] There is more than one GARCH model available that is able to capture the asymmetric effect in volatility. Pagan and Schwert (1990), Engle and Ng (1993), Hentschel (1995) and Fornari and Mele (1996) provide excellent analyses and comparisons of symmetric and asymmetric GARCH models. According to Engle and Ng (1993), the Glosten et al. (1993) model is the best at parsimoniously capturing this asymmetric effect.

The methodology used to obtain the optimal forecast of the conditional variance of a time series from a GARCH model is the same as that used to obtain the optimal forecast of the conditional mean (Harris and Sollis 2003, p. 246)[8]. The basic univariate GARCH(*p*, *q*) is utilised to illustrate the forecast function for the conditional variance of the GARCH process due to its simplicity.

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \qquad (17)$$

Providing that all parameters are known and the sample size is *T*, taking conditional expectation, the forecast function for the optimal *h*-step-ahead forecast of the conditional variance can be written:

$$E(\sigma_{T+h}^2 | \Omega_T) = \alpha_0 + \sum_{i=1}^{q} \alpha_i (u_{T+h-i}^2 | \Omega_T) + \sum_{j=1}^{p} \beta_j (\sigma_{T+h-i}^2 | \Omega_T) \qquad (18)$$

where Ω_T is the relevant information set. For $i \le 0$, $E(u_{T+i}^2 | \Omega_T) = u_{T+i}^2$ and $E(\sigma_{T+i}^2 | \Omega_T) = \sigma_{T+i}^2$; for $i > 0$, $E(u_{T+i}^2 | \Omega_T) = E(\sigma_{T+i}^2 | \Omega_T)$; and for $i > 1$, $E(\sigma_{T+i}^2 | \Omega_T)$ is obtained recursively. Consequently, the one-step-ahead forecast of the conditional variance is given by:

$$E(\sigma_{T+1}^2 | \Omega_T) = \alpha_0 + \alpha_1 u_T^2 + \beta_1 \sigma_T^2 \qquad (19)$$

[8] Harris and Sollis (2003, p. 247) discuss the methodology in detail.

Although many GARCH specifications forecast the conditional variance in a similar way, the forecast function for some extensions of GARCH will be more difficult to derive. For instance, extra forecasts of the dummy variable *I* is necessary in the GARCH-GJR model. However, following the same framework, it is straightforward to generate forecasts of the conditional variance and covariance using bivariate GARCH models, and thus the conditional beta.

3. Data, Data Diagnostics and Forecasting the Time-Varying Hedge Ratio

This paper employs daily data spanning the period from December 1999 to December, 2009 on stock indices and their counterpart futures contracts from Brazil, Hungary, South Africa and South Korea. Forecast of hedge ratio using daily data has tremendous value to money managers who adjust their portfolio as often as daily (Figlewski 1986). The Bovespa index is a total return index weighted by traded volume and is comprised of the most liquid stocks traded in the Sao Paulo Stock Exchange, Brazil. The Sao Paulo Stock Exchange (Bovespa) and the Brazilian Mercantile and Futures Exchanges (BM&F) merged on May 8, 2008, creating BM&FBOVESPA. There are 450 companies traded at Bovespa as on April 30, 2008.[9] The BUX index is the official capitalization-weighted index of the blue-chip shares listed on the Budapest Stock Exchange (BSE).[10] The Johannesburg Stock Exchange (JSE) acquired the South African Futures Exchanges (SAFEX) in 2001 and became the leader in both equities and equity and agricultural derivatives trading in the South African Market.

[9] The BM&FBOVESPA is a Sao Paulo-based stock and futures exchange which is the fourth largest exchange in the America in terms of market capitalization, behind the New York Stock Exchange, NASDAQ, and Toronto Stock Exchange. It is also the tenth largest exchange in the world in terms of market capitalization.

[10] The Budapest Commodity Exchange (BCE) and the Budapest Stock Exchange (BSE) merged on October 2005, which made BSE as one of the main derivatives centres in Central Europe. The BSE played a significant role in the privatisation of many leading Hungarian companies. BSE was one of the first in the world who started to use free-float capitalization weighting instead of the traditional market capitalisation weighting in October 1999.

The FTSE/JSE 40 index consists of the largest 40 South African companies ranked by full market value in the FTSE/JSE All-Share index.[11] The KOSPI 200 index consists of 200 big companies of the stock market division of the Korea Exchange. The KOSPI is calculated as current capitalization (at the time of comparison) divided by the base market capitalization. KOSPI 200 is important because it is listed on futures and option markets and is one of the most actively traded indexes in the world.[12] All futures price indices are continuous series.[13] The data are collected from Datastream. To avoid the sample effect and overlapping issue, two forecast horizons are considered, including one 1-year forecast horizon (2007) and one 2-year forecast horizon (2008 to 2009). All models are estimated for the periods 1999-2006 and 1999-2007, and the estimated parameters are applied for forecasting over the forecast horizons of 2007 and 2008-2009.

Descriptive statistics relating to the distribution of return are presented in Table 1. These statistics are: mean; standard deviation; variance; a measure of skewness, a measure of excess kurtosis (normal value is 3); the Jarque-Bera statistics; and unit root test results of cash and future price indices. Returns are created by taking the difference of the log of the cash and futures indices. The table also presents higher order autocorrelation Q, and ARCH effects in the returns indices series. The values of the skewness statistics indicate that the density function is negatively skewed for both cash and future indices returns for all markets except the cash market of Korea. The values of the excess kurtosis statistic are more than 2 for all countries which suggest that the density function for each country has a fat tail. The values of the Jarque-Bera statistic are high, suggesting the return indices are not normally-

[11] South Africa became the second emerging market to trade index futures when All Share futures were launched on 30 April 1990 in JSE and SAFEX (Smith and Rogers 2006).

[12] On Korea Stock Exchange, the Korea Stock Price Index 200 future was launched in May 1997.

[13] The continuous series is a perpetual series of futures prices. It starts at the nearest contract month, which forms the first values for the continuous series, either until the contract reaches its expiry date or until the first business day of the actual contract month. At this point, the next trading contract month is taken. As indicated by one of the referees, splice bias is introduced when the nearby futures contract are used for estimation changes.

distributed. Judged by the skewness, excess kurtosis and Jarque-Bera statistics, it can be inferred that the returns indices exhibit 'fat-tails' in all markets. The data series have also been checked for stationarity using the Elliott-Rothenberg-Stock Dickey-Fuller generalized least squares (DF-GLS) unit root test.[14] The DF-GLS test results indicate that each of the returns series has no unit root. Tests for autocorrelation in the first moments using the $Q(20)$ statistic indicate that none is present in the any of the indices. Finally, tests for ARCH using Engle's (1982) LM statistic generally support the hypothesis of time-varying variances.

It is important to point out that the lack of a benchmark is an inevitable weak point for studies on time-varying hedge ratio forecasts, since the hedge ratio value is unobservable in the real world. Although the point estimation of hedge ratio generated by the GARCH model is a moderate proxy for the actual hedge ratio value, it is not an appropriate scale to measure a hedge ratio series forecasted with time variation. As a result, evaluation of forecast accuracy based on comparing conditional hedge ratios estimated and forecasted by the same approach cannot provide compelling evidence of the worth of each individual approach. To assess predictive performance, a logical extension is to examine out-of-sample returns. Evaluation of forecast accuracy is thus conducted by forecasting out-of-sample returns of portfolios implied by the computed hedge ratios. The portfolios are constructed as (r^c_t - $\beta_t^* r^f_t$), where r^c_t is the log difference of the cash (spot) prices, r^f_t is the log difference of the futures prices, and β_t^* is the estimated optimal hedge ratio. With the out-of-sample forecasts of time-varying hedge ratio, the out-of-sample forecasts of returns based on the portfolio above can be easily calculated, in which the cash return and futures return are actual returns observed. The relative accuracy of time-varying hedge ratio forecasts can then be assessed by

[14]It is well known that Augmented Dickey-Fuller (ADF) and Phillips-Perron unit root tests have low power in rejecting the null of a unit root and are prone to size distortion. Elliott, Rothenberg and Stock (1996) proposed an alternative DF-GLS test which involves the application of a generalized least squares method to de-trend the data. In the process of performing this test, the autoregressive truncation lag length is determined by the modified Akaike Information Criterion (AIC).

comparing the return forecasts with the actual returns. In this way, the issue of a missing benchmark can be settled.

The methodology of forecasting time-varying hedge ratios will be carried out in several steps. For example for the 2-year forecasting horizon (2008-2009), in the first step, the estimated hedge ratio series will be constructed by GARCH models, from 1999 to 2007. In the second step, the forecasting models will be used to forecast returns over the forecast horizon based on the estimated time-varying hedge ratios and will be compared in terms of forecasting accuracy. In the third and last step, the empirical results of the performance of various models will be produced on the basis of hypothesis tests, looking at whether the estimate is significantly different from the real value, which will provide evidence for comparative analysis of the merits of the different forecasting models.

4. GARCH Results

The GARCH results are reported in tables 2A, 2B, 2C and 2D.[15] The ARCH coefficients (A_{11} and A_{22}) are significant. These parameters indicate the amount of influence past residuals have on current residuals. The GARCH coefficients (B_{11} and B_{22}) represent the influence of past volatility on future volatility. The coefficients are positive and significant in all cases. First, the parameters representing the error correction term (δ_1) in the GARCH-ECM models for each market are negative, large and economically significant.[16] The size of coefficients ranges from -0.2872 (Hungary) to -0.7681 (Korea). The absolute sizes of the parameters suggest that day-to-day deviations do have significant impact on the absolute levels of the cash indices. The result is in sharp contrast to Sultan and Hasan (2008), and Choudhry (2009) whose studies are featured on developed exchanges and commodities'

[15] Many diagnostic tests are not reported or discussed to conserve space. However, they are available upon request.

[16] We find cointegration between cash and futures indices for all four markets. In order to save space these results are not presented but are available on request.

markets. The error correction coefficient in the mean equation of futures return is positive and statistically significant in the cases of Hungary and South Africa.[17] This result may alternatively be interpreted as an increase in short-run deviation lowers the cash returns but increases the future returns. This is a distinguishing feature of the emerging markets as opposed to developed and mature markets where day-to-day deviations do not have much an impact on the absolute levels of the cash and futures returns as such deviations are arbitraged anyway. The error correction coefficients in the mean equation of the cash and futures returns in the GARCH-X model provide very similar results. The error correction coefficients in the conditional variance equations are positive and significant in all tests. This is true for both the cash and futures returns. A significant positive effect may imply that the further the series deviates from each other in the short run, the harder they are to predict. The unconditional variance/covariance terms (C_{11}, C_{12}, C_{21} and C_{22}) are small and significant--suggesting the notion of unconditional risk in these hedging portfolios. Finally, the sign and significance of the covariance parameters indicate positive and significant interaction between the two prices in most cases. For the GARCH models, except the BEKK, the BHHH algorithm is used as the optimisation method to estimate the time-varying beta series. For the BEKK GARCH, the BFGS algorithm is applied. Figure 1 shows the BEKK estimated hedge ratio for all four markets. The graph clearly shows the four hedge ratios are very different from each other. The graphs of other hedge ratios are not provided to save space but are available on request.

5. Measures of Forecast Accuracy

A group of measures derived from the forecast error are designed to evaluate *ex post* forecasts. To evaluate forecasts, different measures of forecast errors (MAE and MSE) are

[17] The result is quite plausible, pointing to the notion that if the error correction term is statistically negative and significant in one equation, then the term would be positive in another equation in a bivariate model.

employed. Mean errors (ME) are employed to assess whether the models over or under-forecast return series.[18] Among them, the most common overall accuracy measure is MSE (Diebold 2004, p. 298):

$$MSE = \frac{1}{n}\sum_{t=1}^{n} e_t^2 \qquad (20)$$

$$MAE = \frac{1}{n}\sum_{t=1}^{n} | e_t | \qquad (21)$$

$$ME = \frac{1}{n}\sum_{t=1}^{n} e \qquad (22)$$

where e is the forecast error defined as the difference between the actual value and the forecasted value. [19]

Diebold and Mariano (1995) develop a test of equal forecast accuracy to test whether two sets of forecast errors, say e_{1t} and e_{2t}, have equal mean value. Using MSE as the measure, the null hypothesis of equal forecast accuracy can be represented as $E[d_t] = 0$, where $d_t = e_{1t}^2 - e_{2t}^2$. Supposing, n, h-step-ahead forecasts have been generated, Diebold and Mariano (1995) suggest the mean of the difference between MSEs $\overline{d} = \frac{1}{n}\sum_{t=1}^{n} d_t$ has an approximate asymptotic variance of

[18] A negative ME indicates model under forecast and a positive ME indicates over forecast.

[19] The lower the forecast error measure, the better the forecasting performance. However, it does not necessarily mean that a lower MSE automatically indicates superior forecasting ability, since the difference between the MSEs may be not significantly different from zero.

$$Var(\bar{d}) \approx \frac{1}{n}\left[\gamma_0 + 2\sum_{k=1}^{h-1}\gamma_k\right] \qquad (23)$$

Where γ_k is the kth autocovariance of d_t, which can be estimated as:

$$\hat{\gamma}_k = \frac{1}{n}\sum_{t=k+1}^{n}(d_t - \bar{d})(d_{t-k} - \bar{d}) \qquad (24)$$

Therefore, the corresponding statistic for testing the equal forecast accuracy hypothesis is $S = \bar{d} / \sqrt{Var(\bar{d})}$, which has an asymptotic standard normal distribution. According to Diebold and Mariano (1995), results of Monte Carlo simulation experiments show that the performance of this statistic is good, even for small samples and when forecast errors are non-normally distributed. However, this test is found to be over-sized for small numbers of forecast observations and forecasts of two-steps ahead or greater.

Harvey *et al.* (1997) further develop the test for equal forecast accuracy by modifying Diebold and Mariano's (1995) approach. Since the estimator used by Diebold and Mariano (1995) is consistent but biased, Harvey *et al.* (1997) improve the finite sample performance of the Diebold and Mariano (1995) test by using an approximately unbiased estimator of the variance of $\bar{d}$. The modified test statistic is given by

$$S^* = \left[\frac{n + 1 - 2h + n^{-1}h(h-1)}{n}\right]^{1/2} S \qquad (25)$$

Through Monte Carlo simulation experiments, this modified statistic is found to perform much better than the original Diebold-Mariano test at all forecast horizons and when the forecast errors are autocorrelated or have non-normal distribution. In this paper, we apply the modified Diebold-Mariano test.

6. Forecast Error Based on Returns Forecasts and Modified Diebold and Mariano Tests Results

As stated earlier, MAE, MSE and ME are the criteria applied to evaluate return forecasting performance. Tables 3A, 3B, 3C and 3D present the MAE, MSE, and ME of return forecast for Brazil, Hungary, South Africa and South Korea, respectively. Each table shows results for both forecast horizons. The results fail to show any one model being superior to the others. Superiority of the models depends upon the market under study and the forecast horizon under consideration. In the case of Brazil, GARCH-ECM seems to perform the best for both periods and the same is true for GARCH-GJR for South Korea. The BEKK outperforms other models for Hungary and South Africa but only for the 1-year (2007) forecast horizon. This mixture of results is not surprising as indicate by Chen et al. (2003) different hedging horizon might affect forecasting accuracy for various forecasting methods. The ME statistics indicates that majority of the models tend to under-predict returns. This is true during both forecast horizons.

Figure 2 shows the return forecast by the different methods and the actual returns over both the forecast horizon for South Africa. All estimates seem to move together with the actual return, but because of the high frequency of the data it is difficult to say which method shows the closest correlation. Figures for other countries are not provided to save space but are available on request.

As stated earlier, Harvey *et al.* (1997) propose a modified version that corrects for the tendency of the Diebold-Mariano statistic to be biased in small samples. Two criteria, including MSE and MAE derived from return forecasts, are employed to implement the modified Diebold-Mariano tests. Each time, the tests are conducted to detect superiority between two forecasting models, and thus there a twenty groups of tests for each market for each forecast horizon.

Each modified Diebold-Mariano test generates two statistics, S_1 and S_2, based on two hypotheses:

1. H_0^1 : there is no statistical difference between the two sets of forecast errors.

   H_1^1 : the first set of forecasting errors is significantly smaller than the second.

2. H_0^2 : there is no statistical difference between the two sets of forecast errors.

   H_1^2 : the second set of forecasting errors is significantly smaller than the first.

It is clear that the sum of the *P* values of the two statistics (S_1 and S_2) is equal to unity. If we define the significance of the modified Diebold-Mariano statistics as at least 10% significance level of *t* distribution, adjusted statistics provide three possible answers for superiority between two rival models:

1. If S_1 is significant, then the first forecasting model outperforms the second.
2. If S_2 is significant, then the second forecasting model outperforms the first.
3. If neither S_1 nor S_2 is significant, then the two models produce equally accurate forecasts.

Tables 4 to 7 presents the numerical and comparison results using the modified Diebold-Mariano tests between the five GARCH models for the four markets during the two different out-of-samples forecast horizons (2008-09 and 2007). The comparison results provide a more clear analysis between two models at a time indicating which model performs better. The Brazilian results (table 4) show that during the 2-year out-of-sample period (2008-09) the BEKK performs the best and the standard GARCH and GARCH-ECM performs the worst. During the shorter period of 1-year GARCH-X does the best and GARCH the worst. During the shorter period using the MSE, the BEKK and the GJR are found to perform equally. For Hungary (table 5) BEKK is the best model and the GARCH-ECM the worst during the longer period. The standard GARCH is the worst and the remaining four models provide similar performance during the shorter out-of-sample period. Both MSE and MAE statistics indicate that GARCH-X and BEKK, GARCH-X and GARCH-GJR, and GARCH-ECM and GARCH-GJR perform equally good. The South African results (table 6) indicate that during the longer period BEKK does the best and GARCH-ECM the worst. During the shorter period, GARCH-X is the worst and GARCH-GJR is the best. Table 7 shows the results from the South Korean tests. The BEKK and the GARCH-ECM provide the best results and the standard GARCH the worst during the longer period. During the shorter period, GARCH-X is the best model and the standard GARCH is the worst. Results also show that the BEKK and the GARCH-GJR perform very similar based on both statistics.

In summary based on the modified Diebold Mariano test across the four markets during the 2-year forecast horizon the BEKK seems to perform best compared to the four models. While the GARCH-X seems to perform the best during the shorter 1-year forecast horizon. Once again different hedging horizon might affect forecasting accuracy for various forecasting methods. The standard GARCH performs the worst during both forecast horizons.

8. Conclusion

This paper empirically estimates the daily time-varying hedge ratio and attempts to forecast the estimated daily hedge ratio of four emerging stock futures markets; Brazil, Hungary, South Africa and South Korea. Knowledge of forecasting ability of optimal hedge ratio/dynamic hedge ratio is important for understanding the role of futures markets in equity trading, program trading, index arbitrage and the development of optimal hedging and trading strategies in fund management. The forecasting of hedge ratio helps hedger choose appropriate portfolio and allows for portfolio adjustment in dynamic hedging.

The paper employs five different GARCH models: standard bivariate GARCH, bivariate GARCH-ECM, bivariate BEKK, bivariate GARCH-X and bivariate GARCH-GJR filter approach to estimate and forecast the beta. The paper thus also provides a comparison between the forecasting ability of the five models. The tests are carried out in two steps. In the first step the estimated hedge ratio series are constructed by GARCH models. In the second step, the forecasting models are used to forecast returns based on the estimated time-varying hedge ratios and are then compared in terms of forecasting accuracy. To avoid the sample effect and overlapping effect, two forecast horizons are considered, including one 1-year forecasts 2007, and one 2-year horizon from 2008 to 2009. Two sets of forecasts are made and the different methods applied are compared. In the third and last step, the empirical results of the performance of the various models are produced on the basis of hypothesis tests, which look at whether the estimate is significantly different from the real value. These will provide evidence for comparative analysis of the merits of the different forecasting models.

Various measures of forecast errors are calculated on the basis of hedge ratio forecasts to assess the relative superiority of alternative models. In order to evaluate the level of forecast errors between portfolio returns based on conditional hedge ratio forecasts and actual returns values, mean absolute errors (MAE), mean squared errors (MSE), and mean errors (ME) are applied. The results from these tests fail to show any one model being superior to the others. Superiority of the models depends upon the country and the forecast horizon under consideration. This mixture of results is not surprising as shown by Chen et al. (2003) different hedging horizon might affect forecasting accuracy for various forecasting methods. The ME statistics indicates that majority of the models tend to under-predict returns. This is true for both forecast horizons.

The last comparison technique used is the modified Diebold-Mariano test. This test is conducted to detect superiority between two forecasting models at a time. Results show during the 2-year forecast horizon the BEKK seems to outperform the other four models. While the GARCH-X seems to perform the best during the shorter 1-year forecast horizon. Once again different hedging horizon might affect forecasting accuracy for various forecasting methods. The standard GARCH performs the worst during both forecast horizons. Thus, if the forecasted hedge ratio is used to provide information of expected return, BEKK is a better choice than other GARCH models for a longer forecast horizon and GARCH-X is a better choice for shorter horizon. The standard GARCH model is the worst during both horizons. The success of GARCH type models in forecasting the time-varying hedge ratio also implies their competence in forecasting conditional second movement, which is crucial in a wide range of decision-making processes involving information of variance and covariance, such as derivative pricing and risk management.

Results presented in this paper advocate further research in this field, applying different markets, time periods and methods. There are potential insights to be gained from examining markets with different institutional features.

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Table 1

Descriptive statistics of stock spot and futures indices return

Statistics	**Brazil**		**Hungary**		**Korea**		**South Africa**	
	Cash Return	Future Return	Cash Return	Future Return	Cash Return	Future Return	Cash Return	Future Return
Mean	.000584	.000573	.000451	.000446	.000232	.000207	.000365	.000358
Variance	.000631	.000768	.000368	.000362	.000257	.000552	.0003136	.0003485
Std. Dev.	.025130	.027715	.019206	.019046	.050726	.023509	.017710	.018670
Skewness	-.265225	-.190646	-.328944	-.293191	0.01479	-0.40041	-.309725	-.81671
Kurtosis	6.92300	5.84232	10.7273	11.6078	16.3872	10.1907	7.38807	12.4973
Jarque-Bera	1703.60	894.03	6538.17	8092.09	19482.5	5690.70	2134.91	10095.46
Stationaryity: t_μ	-8.557[a]	-10.215[a]	-5.675[a]	-4.190[a]	-1.850[c]	-7.624[a]	-5.5801[a]	-6.758[a]
t_τ	-13.599[a]	-15.205[a]	-10.293[a]	-7.988[a]	-3.917[a]	-12.742[a]	-10.127[a]	-11.50[a]
ARCH(1)	51.96	61.43	172.60	144.35	92.42	39.82	672.34	582.08
Q(20)	64.81	41.03	112.08	93.32	274.17	34.07	34.40	28.82

Note: t_μ and t_τ are the Elliot-Rothenberg-Stock Dickey-Fuller generalised least squares (DF-GLS) unit root test statistics with allowance for a constant and trend, respectively. 5% critical values of t_μ and t_τ are -1.948 and -3.190 (see Elliot-Rothenberg-Stock 1996, Table 1). L and Δ signify the level and first difference of a variable respectively.

Table 2A

Parameter Estimates of Conditional Hedging Model of Brazil

Dependent Variable	GARCH	GARCH-ECM	GARCH-X	GARCH-BEKK	GARCH-GJR
μ_1	0.00167 (4.1869)	0.00171 (4.0788)	0.00153 (3.5016)	0.0015[a] (3.5700)	0.00044 (1.1623)
δ_1		-0.5517 (-11.058)	-0.5391 (-9.6045)		
μ_2	0.00178 (4.0558)	0.00179 (3.8751)	0.00131 (2.7487)	0.0015[a] (3.1860)	0.000152 (0.3520)
δ_2		0.0309 (0.0552)	0.0236 (0.3642)		
C_{11}	.000038 (5.4301)	0.000037 (4.7584)	0.00003 (8.3680)	0.0057[a] (14.7939)	0.000028 (6.0484)
A_{11}	0.08968 (8.3020)	0.0901 (7.7493)	0.0611 (6.5836)	0.9284[a] (129.5480)	0.0271 (3.6965)
B_{11}	0.84008 (43.0519)	0.8394 (37.737)	0.8203 (40.657)	0.2882[a] (12.7988)	0.8686 (56.988)
D_{11}			0.3018 (3.7133)		0.1008 (7.5873)
C_{21}	0.000045 (5.5816)	.0000039 (4.8117)	0.000042 (19.999)	0.00192[a] (3.5454)	0.000029 (6.4026)
A_{12}				0.4255[a] (7.9279)	
A_{21}	0.08581 (8.5178)	0.0809 (8.2695)	0.0556 (7.6462)	0.3429[a] (6.5490)	0.01662 (2.5291)
B_{12}				-0.0453 (-1.1904)	
B_{21}	0.83174 (38.2209)	0.8479 (40.859)	0.8191 (60.669)	0.1489[a] (4.1840)	0.8746 (61.703)
D_{21}			0.3382 (3.8261)		0.1059 (8.2422)
C_{22}	0.000060 (5.1883)	0.000048 (4.6111)	0.000052 (20.884)	-0.0000 (-0.0263)	0.000035 (6.5017)
A_{22}	0.10297 (8.3195)	0.0910 (8.4195)	0.0677 (8.6211)	0.8640[a] (55.5352)	0.0140 (1.9442)
B_{22}	0.81131 (91.4276)	0.8396 (37.786)	0.7982 (61.094	0.3844[a] (10.5731)	0.8726 (60.592)
D_{22}			0.5234 (4.474)		0.1266 (8.7021)
ρ	0.885				
L	14261.92	14523.12	14564.08	14060.95	14314.74
φ	0.870	0.907	0.882	0.768	0.900

a, b and c imply significance at 1%, 5% and 10% level respectively; figures in parentheses underneath the coefficients are *t*-statistics. ρ is the within sample correlation coefficient between cash and futures returns. Log-*L* is the log-likelihood and φ signifies the first order serial correlation coefficient in the hedge ratio derived from an AR(1) model.

Table 2B:

Parameter Estimates of Conditional Hedging Model of Hungary

Dependent Variable	GARCH	GARCH-ECM	GARCH-X	GARCH-BEKK	GARCH-GJR
μ_1	0.00070[b]	0.000925	0.00100	0.0006[b]	0.000511
	(2.4880)	(3.1959)	(3.0583)	(2.2056)	(1.8639)
δ_1		-0.2872	-0.3259		
		(-7.1421)	(-6.4825)		
μ_2	0.00077[a]	0.000769	.00080	0.0007[b]	0.000546
	(2.7391)	(2.6250)	(2.3856)	(2.4392)	(2.0474)
δ_2		0.2570	0.1922		
		(6.4762)	(3.8415)		
C_{11}	0.00001[a]	.00000938	.0000165	0.0028[a]	0.0000077
	(10.9715)	(18.881)	(23.639)	(8.7123)	(5.4841)
A_{11}	0.1239[a]	0.1160	0.1098	0.9381[a]	0.0723
	(20.4853)	(41.316)	(10.549)	(102.4943)	(6.6284)
B_{11}	0.8564[a]	0.8623	0.8166	0.3276[a]	0.8815
	(150.1438)	(189.89)	(76.695)	(12.8572)	(83.751)
D_{11}			0.0668		0.06503
			(3.0801)		(4.871)
C_{21}	0.00001[a]	0.00000934	0.0000167	0.0026[a]	0.0000077
	(12.0166)	(131.135)	(185.002)	(7.5923)	(5.7945)
A_{12}				0.30721[a]	
				(11.007)	
A_{21}	0.1228[a]	0.1125	0.1096	0.47638[a]	0.0703
	(20.2455)	(604.43)	(10.893)	(16.892)	(6.7001)
B_{12}				0.05089[a]	
				(4.6618)	
B_{21}	0.8546[a]	0.8644	0.8178	0.07714[a]	0.8809
	(157.3800)	(275.20)	(82.907)	(6.2765)	(83.957)
D_{21}			0.000570		0.0644
			(0.4294)		(4.9555)
C_{22}	0.00001[a]	0.000010	0.0000174	0.0009[a]	0.0000083
	(11.8525)	(24.397)	(28.538)	(5.9948)	(5.8338)
A_{22}	0.1221[a]	0.1103	0.1095	0.9412[a]	0.0663
	(19.3211)	(45.484)	(10.401)	(85.0817)	(6.1583)
B_{22}	0.8549[a]	0.8669	0.8197	0.3174[a]	0.8827
	(147.6708)	(198.39)	(73.739)	(11.1192)	(80.852)
D_{22}			0.0319		0.0667
			(2.2910)		(4.9286)
ρ		0.867			
L	21130.35	16608.88	16664.66	16370.42	16408.86
φ	0.946	0.923	0.929	0.914	0.958

a, b and c imply significance at 1%, 5% and 10% level respectively; figures in parentheses underneath the coefficients are *t*-statistics. ρ is the within sample correlation coefficient between cash and futures returns. Log-L is the log-likelihood and φ signifies the first order serial correlation coefficient in the hedge ratio derived from an AR(1) model.

Table 2C:
Parameter Estimates of Conditional Hedging Model of South Africa

Dependent Variable	GARCH	GARCH-ECM	GARCH-X	GARCH-BEKK	GARCH-GJR
μ_1	.0.00108	0.00117	0.00120	0.0010[a]	0.000695
	(3.8703)	(4.0502)	(4.4157)	(5.5532)	(2.5851)
δ_1		-0.4338	-0.4443		
		(-9.608)	(-9.4336)		
μ_2	0.00113	0.00108	0.00111	0.0011[a]	0.000667
	(3.8665)	(3.5139)	(3.8266)	(5.4526)	(2.2946)
δ_2		0.1104	0.1007		
		(2.2390)	(1.9726)		
C_1	.0000118	0.0000103	0.0000059	0.0033[a]	0.000010
	(13.878)	(7.5822)	(4.2084)	(6.8414)	(5.9980)
A_{11}	0.0973	0,0852	0.0773	0.9296[a]	0.05622
	(12.978)	(12.456)	((9.7907)	(74.8526)	(4.6610)
B_{11}	0.8556	0.8716	0.8629	0.3135[a]	0.8740
	(106.65)	(84.570)	(66.879)	(12.6500)	(68.139)
D_{11}			0.2244		0.05353
			(4.6611)		(3.9538)
C_{21}	0.0000137		0.0000062	0.0043[a]	0.000011
	(40.506)		(4.0007)	(4.4065)	(5.6845)
A_{12}				0.5644[a]	
				(12.648)	
A_{21}	0.0991	0.0668	0.0748	0.1011[b]	0.06125
	(12.804)	(12.455)	(9.1471)	(2.2068)	(4.8399)
B_{12}				1.6750[a]	
				(13.554)	
B_{21}	0.8442	0.8633	.8570	0..03023	0.8637
	(117.32)	(80.866)	(63.329)	(0.22365)	(59.800)
D_{21}			0.2683		0.04998
			(5.6221)		(3.7548)
C_{22}	0.0000183		0.0000084	0.0015[a]	0.000014
	(43.338)		(4.329)4	(5.5141)	(5.2586)
A_{22}	0.1182	0.0999	0.0841	0.8959[a]	0.07294
	(13.274)	(11.703)	(8.8989)	(34.5360)	(4.9030)
B_{22}	0.8234	0.8488	0.8444	0.3718[a]	0.8471
	(101.58)	(65.017)	(57.664)	(10.1902)	(48.007)
D_{22}			0.3297		0.05961
			(6.1532)		(4.0224)
ρ	0.892				
L	16282.23	16528.85	16551.99	16234.62	16295.46
φ	0.901	0.905	0.904	0.685	0.911

a, b and c imply significance at 1%, 5% and 10% level respectively; figures in parentheses underneath the coefficients are *t*-statistics. ρ is the within sample correlation coefficient between cash and futures returns. Log-*L* is the log-likelihood and φ signifies the first order serial correlation coefficient in the hedge ratio derived from an AR(1) model.

Table 2D:

Parameter Estimates of Conditional Hedging Model of South Korea

Dependent Variable	GARCH	GARCH-ECM	GARCH-X	GARCH-BEKK	GARCH-GJR
μ_1	0.00033 (0.3875)	-.000665 (-1.4269)	-0.00098 (-5.1099)	0.0002 (0.1820)	-0.000833 (-0.9215)
δ_1		-0.7681 (-38.837)	-0.5162 (-48.010)		
μ_2	0.00076 (2.1196)	.000574 (1.5774)	0.00077 (28.053)	0.0007^c (1.7670)	.00037 (1.0204)
δ_2		-0.0277 (-2.8819)	0.0241 (24.205)		
C_{11}	0.00029 (9.4802)	.000338 (8.4840)	0.000074 (23.593)	0.0177^a (5.8281)	.00027 (10.619)
A_{11}	0.1080 (9.3424)	0.9951 (7.7938)	0.1510 (4173588.17)	0.8700^a (22.6852)	0.0616 (5.5894)
B_{11}	0.7706 (40.504)	0.2788 (4.9667)	0.1906 (49.758)	0.3431^a (9.1313)	0.7843 (46.611)
D_{11}			1.1876 (171.71)		0.0863 (4.8802)
C_{21}	0.000029 (3.8787)	.000068 (6.3436)	0.000030 (0.000)	0.0017^a (5.1302)	.000020 (4.0582)
A_{12}				0.0152^b (2.2025)	
A_{21}	0.0546 (5.2968)	0.1680 (6.2368)	0.0511 (464.08)	0.3783^a (7.9621)	0.0446 (4.0695)
B_{12}				0.0162^a (3.3122)	
B_{21}	0.8862 (41.955	0.5889 (12.825)	0.4307 (104.97)	0.1783^a (7.3768)	0.8859 (41.126)
D_{21}			0.1018 (206.98)		0.0179 (1.1575)
C_{22}	0.000006 (4.2071)	.0000094 (5.0700)	0.000006 (515.16)	0.0016^a (5.0175)	0.0000066 (5.6595)
A_{22}	0.0705 (8.0460)	0.0772 (7.2589)	0.0561 (4258.92)	0.9626^a (194.853)	0.0356 (4.4173)
B_{22}	0.9184 (97.010)	0.9073 (78.945)	0.9190 (634.87)	0.2536^a (11.1066)	0.9211 (112.85)
D_{22}			0.0040 (39.522)		0.0568 (4.6967)
ρ	0.362				
L	11037.72	11458.61	12029.40	11027.04	11060.60
φ	0943	0.711	0.746	0.832	0.945

a, b and c imply significance at 1%, 5% and 10% level respectively; figures in parentheses underneath the coefficients are *t*-statistics. ρ is the within sample correlation coefficient between cash and futures returns. Log-*L* is the log-likelihood and φ signifies the first order serial correlation coefficient in the hedge ratio derived from an AR(1) model.

Table 3A

Brazil – Forecast Accuracy Results

2008-09

	ME	MAE	RMSE	MSE	Theil U
BEKK	0.0001	0.006	0.0117	0.00014	0.6274
GARCH-ECM	0.000003	0.00007	0.0002	0.00000	0.0138
GARCH	-0.000003	0.00023	0.0005	0.00000	0.0338
GARCH-X	-0.000097	0.00278	0.0041	0.000017	0.2542
GARCH-GJR	-0.000003	0.00010	0.0002	0.000000	0.0161

2007

	ME	MAE	RMSE	MSE	Theil U
BEKK	0.00000	0.0002	0.0004	0.00000	0.051
GARCH-ECM	-0.0000005	0.00005	0.00009	0.00000	0.010
GARCH	-0.00018	0.00210	0.00290	0.000008	0.322
GARCH-X	-0.00124	0.01714	0.02333	0.000545	0.936
GARCH-GJR	-0.00002	0.00017	0.00023	0.000000	0.027

Table 3B

Hungary – Forecast Accuracy Results

2008-09

	ME	MAE	RMSE	MSE	Theil U
BEKK	-0.00042	0.00370	0.0069	0.000048	0.6053
GARCH-ECM	-0.000001	0.00008	0.00016	0.00000	0.0191
GARCH	-0.000007	0.00014	0.00022	0.00000	0.0256
GARCH-X	-0.000003	0.00164	0.00024	0.00000	0.0286
GARCH-GJR	-0.000005	0.00014	0.00026	0.000000	0.0300

2007

	ME	MAE	RMSE	MSE	Theil U
BEKK	-0.0000017	0.00002	0.00005	0.00000	0.008
GARCH-ECM	-0.0000045	0.00006	0.00012	0.00000	0.019
GARCH	0.0000039	0.00004	0.00008	0.00000	0.013
GARCH-X	-0.000011	0.00007	0.00017	0.00000	0.026
GARCH-GJR	-0.000013	0.00006	0.00013	0.000000	0.020

Table 3C

South Africa – Forecast Accuracy Results

2008-09

	ME	MAE	RMSE	MSE	Theil U
BEKK	-0.000007	0.0042	0.0080	0.000063	0.4940
GARCH-ECM	-0.00002	0.0012	0.00016	0.000003	0.0134
GARCH	0.000002	0.00041	0.00073	0.000001	0.0615
GARCH-X	-0.000005	0.00017	0.00031	0.00000	0.0259
GARCH-GJR	-0.00001	0.00040	0.00076	0.000001	0.0621

2007

	ME	MAE	RMSE	MSE	Theil U
BEKK	0.000006	0.00007	0.00013	0.00000	0.019
GARCH-ECM	-0.0000009	0.00014	0.00021	0.00000	0.031
GARCH	0.000004	0.00030	0.00043	0.00000	0.063
GARCH-X	-0.00002	0.00011	0.00019	0.00000	0.027
GARCH-GJR	-0.000016	0.00025	0.00040	0.000000	0.056

Table 3D

South Korea – Forecast Accuracy Results

2008-09

	ME	MAE	RMSE	MSE	Theil U
BEKK	-0.00143	0.01848	0.0404	0.00163	0.738
GARCH-ECM	0.00248	0.00288	0.0046	0.00002	0.108
GARCH	-0.000026	0.00040	0.0006	0.00000	0.016
GARCH-X	-0.00058	0.00274	0.0055	0.000031	0.129
GARCH-GJR	-0.00001	0.00029	0.0005	0.000000	0.012

2007

	ME	MAE	RMSE	MSE	Theil U
BEKK	-0.00004	0.00019	0.00038	0.00000	0.014
GARCH-ECM	0.00009	0.00272	0.00391	0.00002	0.149
GARCH	-0.00007	0.00028	0.00056	0.00000	0.021
GARCH-X	0.00024	0.00307	0.00430	0.00002	0.161
GARCH-GJR	-0.00001	0.00018	0.00036	0.000000	0.013

Table 4

Brazil

Modified Diebold-Mariano Numerical Results

	2008-2009		2007	
Models	MSE	MAE	MSE	MAE
GARCH vs. GARCH-X	0.00012[a]	0.0051[a]	0.00039[b]	0.014[b]
GARCH vs. BEKK	0.00031[b]	0.0082[b]	0.00003[b]	0.0035[b]
GARCH vs. GARCH-ECM	0.00012[a]	0.0051[a]	0.00001[b]	0.0031[b]
GARCH vs. GARCH-GJR	0.00013[b]	0.0054[b]	0.00003[b]	0.0034[b]
GARCH-X vs. BEKK	0.00032[b]	0.0082[b]	0.00039[a]	0.014[a]
GARCH-X vs GARCH-ECM	0.0001[b]	0.0046[b]	0.00039[a]	0.014[a]
GARCH-X vs GARCH-GJR	0.00013[b]	0.0054[b]	0.00039[a]	0.014[a]
BEKK vs GARCH-ECM	0.00032[a]	0.0082[a]	0.00003[a]	0.0035[a]
BEKK vs GARCH-GJR	0.00032[a]	0.0082[a]	0.00003	0.0034[a]
GARCH-ECM vs GARCH-GJR	0.00013[b]	0.0054[b]	0.00003[b]	0.0034[b]

Note:

a imply S_1 is significant indicating the first forecasting model outperforms the second

b imply S_2 is significant indicating the second forecasting model outperforms the first.

Modified Diebold-Mariano Comparison Results

	2008-2009		2007	
Models	MSE	MAE	MSE	MAE
GARCH vs. GARCH-X	>	>	<	<
GARCH vs. BEKK	<	<	<	<
GARCH vs. GARCH-ECM	>	>	<	<
GARCH vs. GARCH-GJR	<	<	<	<
GARCH-X vs. BEKK	<	<	>	>
GARCH-X vs GARCH-ECM	<	<	>	>
GARCH-X vs GARCH-GJR	<	<	>	>
BEKK vs GARCH-ECM	>	>	>	>
BEKK vs GARCH-GJR	>	>	=	>
GARCH-ECM vs GARCH-GJR	<	<	<	<

Table 5

Hungary

Modified Diebold-Mariano Numerical Results

	2008-2009		2007	
Models	MSE	MAE	MSE	MAE
GARCH vs. GARCH-X	0.000007[b]	0.0013[b]	0.000001[b]	0.0006[b]
GARCH vs. BEKK	0.00006[b]	0.0042[b]	0.000001[b]	0.0006[b]
GARCH vs. GARCH-ECM	0.0000006	0.0012[a]	0.000002[b]	0.0007[b]
GARCH vs. GARCH-GJR	0.000007[b]	0.0013[b]	0.000001[b]	0.0007[b]
GARCH-X vs. BEKK	0.00006[b]	0.0042[b]	0.000001	0.0006
GARCH-X vs GARCH-ECM	0.000007[a]	0.0013[a]	0.000002[b]	0.0007
GARCH-X vs GARCH-GJR	0.000007	0.0013[b]	0.000001	0.0007
BEKK vs GARCH-ECM	0.00006[a]	0.0042[a]	0.000002[b]	0.0007[b]
BEKK vs GARCH-GJR	0.00006[a]	0.0042[a]	0.000001[b]	0.0007[b]
GARCH-ECM vs GARCH-GJR	0.000007[b]	0.0013[b]	0.000002	0.0007

See note at the end of table 4

Modified Diebold-Mariano Comparison Results

	2008-2009		2007	
Models	MSE	MAE	MSE	MAE
GARCH vs. GARCH-X	<	<	<	<
GARCH vs. BEKK	<	<	<	<
GARCH vs. GARCH-ECM	=	>	<	<
GARCH vs. GARCH-GJR	<	<	<	<
GARCH-X vs. BEKK	<	<	=	=
GARCH-X vs GARCH-ECM	>	>	<	=
GARCH-X vs GARCH-GJR	=	<	=	=
BEKK vs GARCH-ECM	>	>	<	<
BEKK vs GARCH-GJR	>	>	<	<
GARCH-ECM vs GARCH-GJR	<	<	=	=

Table 6

South Africa

Modified Diebold-Mariano Numerical Results

	2008-2009		2007	
Models	MSE	MAE	MSE	MAE
GARCH vs. GARCH-X	0.00001	0.0022	0.000008[a]	0.002[a]
GARCH vs. BEKK	0.0001[b]	0.0053[b]	0.000008[b]	0.002
GARCH vs. GARCH-ECM	0.00001[a]	0.0022[a]	0.000008[a]	0.002
GARCH vs. GARCH-GJR	0.00002[b]	0.0028[b]	0.000009[b]	0.002[b]
GARCH-X vs. BEKK	0.0001[b]	0.0053[b]	0.000008[b]	0.002
GARCH-X vs GARCH-ECM	0.00001[a]	0.0022[a]	0.000007[b]	0.002[b]
GARCH-X vs GARCH-GJR	0.00002[b]	0.0028[b]	0.000009[b]	0.002[b]
BEKK vs GARCH-ECM	0.0001[a]	0.0053[a]	0.000009[b]	0.002[b]
BEKK vs GARCH-GJR	0.0001[a]	0.0053[a]	0.000009[b]	0.002[b]
GARCH-ECM vs GARCH-GJR	0.00002[b]	0.0028[b]	0.000009[b]	0.002[b]

See note at the end of table 4

Modified Diebold-Mariano Comparison Results

	2008-2009		2007	
Models	MSE	MAE	MSE	MAE
GARCH vs. GARCH-X	=	=	>	>
GARCH vs. BEKK	<	<	<	=
GARCH vs. GARCH-ECM	>	>	>	=
GARCH vs. GARCH-GJR	<	<	<	<
GARCH-X vs. BEKK	<	<	<	=
GARCH-X vs GARCH-ECM	>	>	<	<
GARCH-X vs GARCH-GJR	<	<	<	<
BEKK vs GARCH-ECM	>	>	>	=
BEKK vs GARCH-GJR	>	>	<	<
GARCH-ECM vs GARCH-GJR	<	<	<	<

Table 7

South Korea

Modified Diebold-Mariano Numerical Results

	2008-2009		2007	
Models	MSE	MAE	MSE	MAE
GARCH vs. GARCH-X	0.0005[b]	0.013[b]	0.0002[b]	0.009[b]
GARCH vs. BEKK	0.0019[b]	0.022[b]	0.00003[b]	0.003[b]
GARCH vs. GARCH-ECM	0.0004[b]	0.010[b]	0.00006[b]	0.005[b]
GARCH vs. GARCH-GJR	0.0002[b]	0.007[b]	0.00003[b]	0.003[b]
GARCH-X vs. BEKK	0.0019[b]	0.022[b]	0.0002[a]	0.009[a]
GARCH-X vs GARCH-ECM	0.0004[a]	0.010[a]	0.0002[a]	0.009[a]
GARCH-X vs GARCH-GJR	0.0005[a]	0.013[a]	0.0002[a]	0.009[a]
BEKK vs GARCH-ECM	0.0019[a]	0.022[a]	0.00006[b]	0.005[b]
BEKK vs GARCH-GJR	0.0019[a]	0.022[a]	0.00003	0.003
GARCH-ECM vs GARCH-GJR	0.0004[a]	0.010[a]	0.00006[a]	0.004[a]

See note at the end of table 4

Modified Diebold-Mariano Comparison Results

	2008-2009		2007	
Models	MSE	MAE	MSE	MAE
GARCH vs. GARCH-X	<	<	<	<
GARCH vs. BEKK	<	<	<	<
GARCH vs. GARCH-ECM	<	<	<	<
GARCH vs. GARCH-GJR	<	<	<	<
GARCH-X vs. BEKK	<	<	>	>
GARCH-X vs GARCH-ECM	>	>	>	>
GARCH-X vs GARCH-GJR	>	>	>	>
BEKK vs GARCH-ECM	<	<	<	<
BEKK vs GARCH-GJR	>	>	=	=
GARCH-ECM vs GARCH-GJR	>	>	>	>

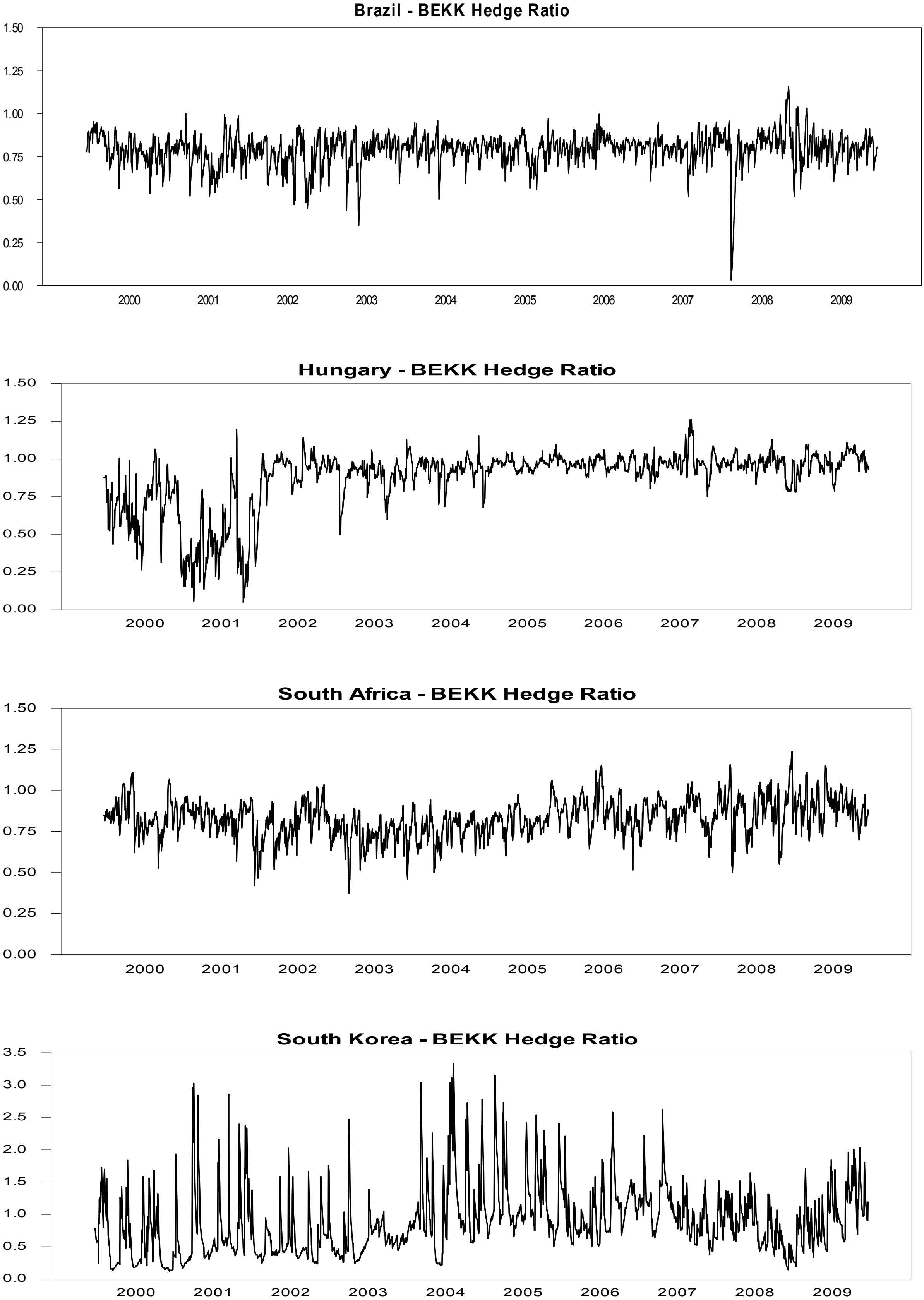
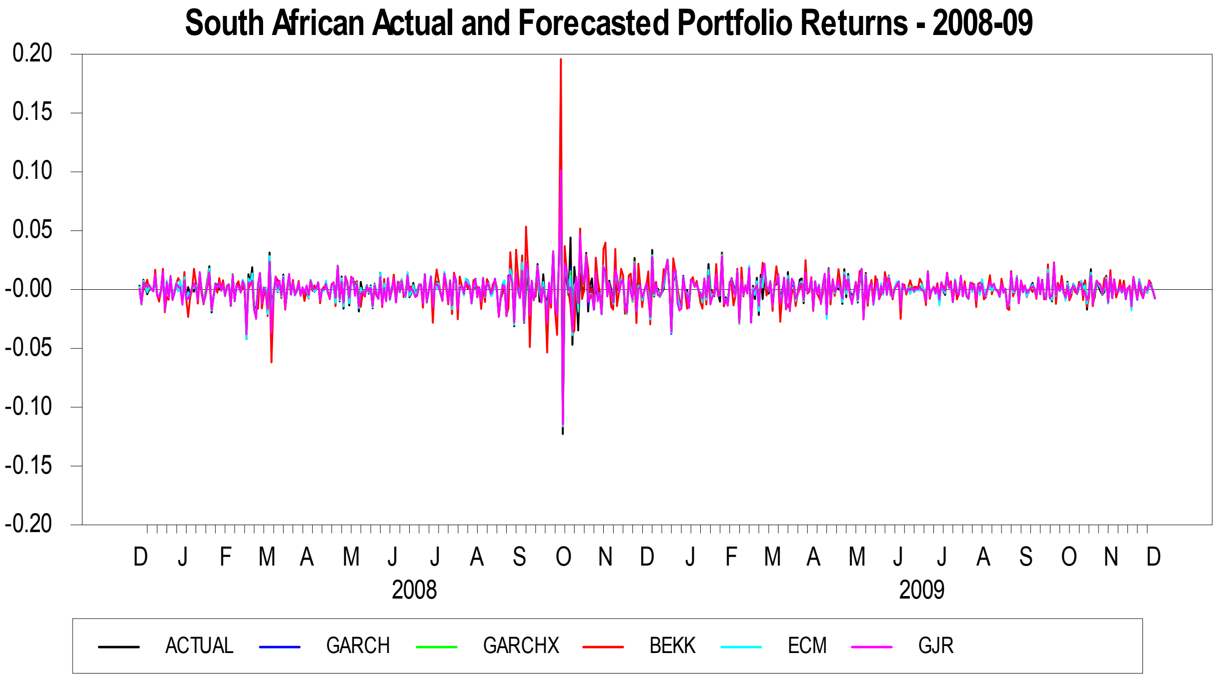


Figure 1 – BEKK Hedge Ratio



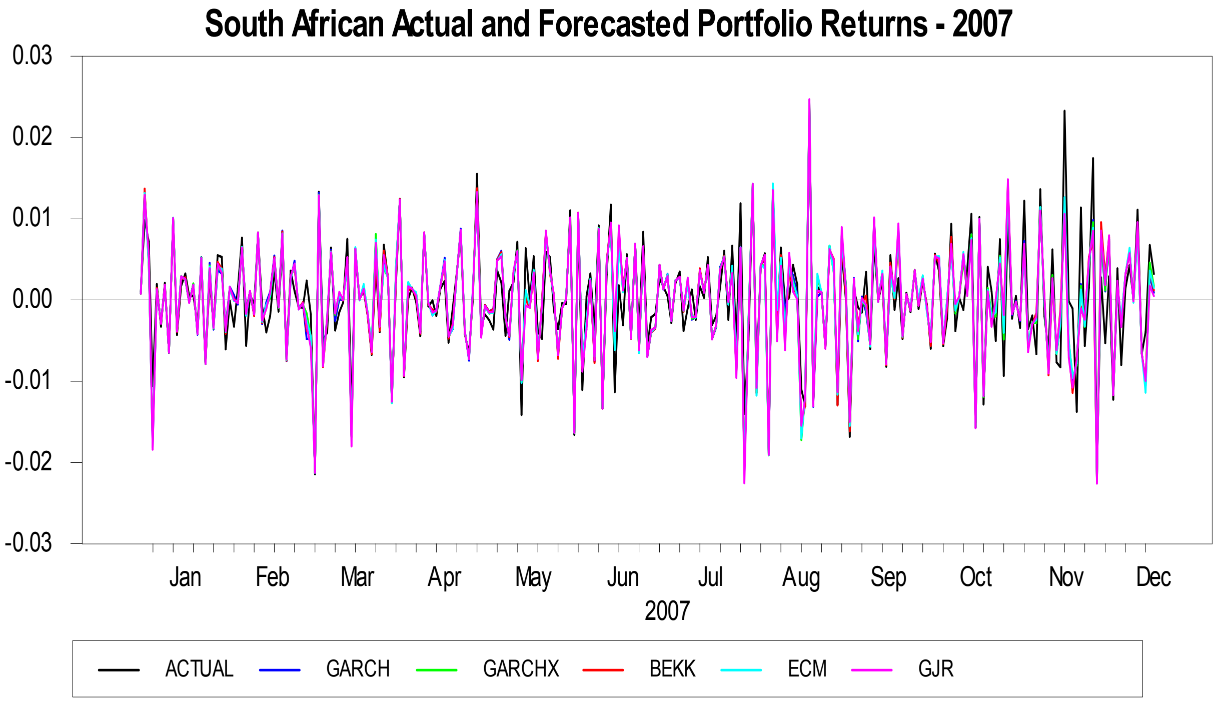


Figure 2 – Out-of-Sample return forecast.