Consumption Moment Risk Factors and Cross-Section of Long-Run Stock Returns

By Andrei Semenov[*]

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Abstract

We introduce the multifactor asset pricing model that includes as risk factors the five Chen *et al.* (1986) macroeconomic variables along with the rates of change in the first four cross-sectional consumption moments. The empirical evidence on the pricing of the economic state variables is sensitive to the experimental design, whereas we find strong evidence that with the limited participation of households in the capital markets the aggregate consumption risk (measured by the rate of change in average consumption) and the background risk in consumption (measured jointly by the rates of change in the higher-order consumption moments) are both significantly priced.

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[*]Department of Economics, York University, 4700 Keele St.Toronto, Ontario M3J 1P3, Canada (e-mail: asemenov@econ.yorku.ca).

1 Introduction

The conventional Sharpe-Lintner Capital Asset Pricing Model (CAPM) (Sharpe (1964), Lintner (1965)) measures the risk of an asset by its beta with the market portfolio. It is observed that in the time-series regressions the stock market indices can account for a large proportion of the intertemporal variability in other stock portfolios. However, this contrasts sharply with the inability of the stock market indices' betas to explain the cross-sectional variation in expected stock returns.

Two alternatives to the CAPM are the Arbitrage Pricing Theory (APT) model and the consumption-based CAPM. The APT suggests that the expected return on a stock can be modeled as a linear function of a number of independent macroeconomic variables or theoretical stock portfolios that are viewed as sources of systematic risk. In the APT model, the single market beta is replaced by a vector of factor-specific beta coefficients that measure sensitivity of the return on a financial asset to changes in systematic risk factors. Based on theoretical arguments, Chen *et al.* (henceforth CRR) (1986), for example, argue that the yield spread between long and short interest rates (the maturity premium), expected inflation and unexpected inflation, industrial production growth, and the yield spread between corporate high- and low-grade bonds (the default premium) can be regarded as factors that systematically influence stock market returns. The evidence on the ability of the implied five-factor model to explain a cross-section of stock portfolio returns is mixed. While CRR (1986) find that the identified sources of risk are significantly rewarded in the stock market, Shanken and Weinstein (henceforth SW) (2006), for example, find the results to be sensitive to alternative techniques for calculating the returns on size portfolios and estimating the factor betas. Using the full-period post-ranking return approach, they find that only the quarterly growth rate of industrial production is significantly priced.

In contrast to the conventional CAPM, the standard representative-agent consumption CAPM (Lucas (1978)) measures the asset's systematic risk by the covariance (beta) of its return with overall aggregate (marginal utility of) consumption. The beta coefficient in this model is known as the consumption beta. An attractive feature of the standard representative-agent consumption CAPM is that it models consumption and savings choices of investors simultaneously in the intertemporal setting, whereas the static asset pricing models (the CAPM and the APT model) ignore consumption decisions. Based on this, one usually argues that this model offers a better measure of systematic risk. Empirical evidence is, however, that, like the CAPM, the standard representative-agent consumption CAPM also fails to explain cross-sectional differences in expected stock returns.

To test whether the set of identified economic state variables have pricing significance in direct competition with a market index and aggregate consumption, CRR (1986) augment the set of risk factors by the inclusion of the market index and the percentage change in real aggregate per capita consumption. These two tests can be regarded as the tests of the APT model against the CAPM and the representative-agent consumption CAPM, respectively. CRR (1986) find that the market indices and average consumption have no independent explanatory influence on pricing.

One deficiency with the approach adopted in CRR (1986) to test the APT model against the consumption CAPM is that this approach implicitly relies on the assumption of market completeness. If the capital markets are complete, then investors can fully insure their future consumption by equalizing, state by state, their optimal consumption growth rates, so that the growth rate of average consumption can be used in place of the consumption growth rate of any particular investor, as in the standard representative-agent consumption CAPM. In the absence of certain contingent-claims markets, the presence of additional independent (of the risk associated with the investment decision) non-tradable (background) risks can make it impossible for investors to completely insure their future consumption and, therefore, equalize their optimal consumption growth rates. The examples of background risks are labor income risk, loss of employment, divorce, disability, etc. These risks cannot be sold on markets and are hence uninsurable. The lack of insurance against background risks represents a type of market incompleteness. The potential of the incomplete consumption insurance hypothesis to help explain the equilibrium behavior of stock and bond returns, both in terms of the level of equilibrium rates and the discrepancy between equity and bond returns, was first suggested by Bewley (1982), Mehra and Prescott (1985), and Mankiw (1986). Constantinides and Duffie (1996), Brav *et al.* (2002), Balduzzi and Yao (2007), and Kocherlakota and Pistaferri (2009) also argue that consumers' heterogeneity induced by market incompleteness can be relevant for asset pricing.

Another shortcoming of the CRR (1986) approach is that, when calculating the real per capita consumption growth rate, CRR (1986) suppose the aggregate consumption in the economy to be an adequate proxy for the consumption of stock market investors. However, it is observed that only a small fraction of individuals in the population participate in the capital markets.[1] Campbell and Mankiw (1990), Mankiw and Zeldes (1991), Basak and Cuoco (1998), Alvarez and Jermann (2000), and Constantinides *et al.* (2002), among others, argue that market frictions, such as transactions costs and limits on borrowing or short sales, can make aggregate consumption in the economy an inadequate proxy for the consumption of stock market investors. Mankiw and Zeldes (1991), for example, document that the consumption growth of stock market investors is more highly correlated with stock returns than the consumption of the agents that do not participate in stock markets. Since the consumption of non-stockholders is irrelevant to the determination of stock prices (but may be a large fraction of aggregate consumption in the economy), it can then be supposed that the changes in per capita consumption of stock holders may have a more significant influence on stock returns compared with the changes in aggregate per capita consumption in the economy.

In this paper we address the both these problems by taking into account incomplete consumption insurance and the limited participation of households in the capital markets. To allow for incomplete consumption insurance, we express the average marginal utility of consumption as a

[1] According to the US Consumer Expenditure Survey, for example, only nearly 19% of households own stocks, bonds, mutual funds, and other such securities. Using data from the Family Expenditure Survey, Attanasio *et al.* (2002) observe that, in keeping with the US, only about 20-25% of UK households own shares directly. Agell and Edin (1990) find that only 18.6% of Swedish households hold common stocks.

Taylor series and show that with incomplete consumption insurance changes (caused by background risk factors) in the normalized centered cross-sectional consumption moments of order two and higher may also (along with the change in average consumption of asset holders, i.e., the first cross-sectional consumption distribution moment) influence (through their impact on the average marginal utility of consumption) the expected excess asset returns. Under the assumption of incomplete consumption insurance, the vector of consumption betas can, therefore, be extended by adding the beta coefficients that measure sensitivity of the asset return to changes in the second and higher moments of the cross-sectional consumption distribution. In this framework, the signs of the risk premia for different moments of the cross-sectional consumption distribution are determined by the agents' preferences and depend on whether a change in the corresponding cross-sectional consumption moment has a positive or negative effect on the average marginal utility of consumption. We show that non-satiation, risk aversion, prudence, temperance, and edginess in the investor's preferences enable us to sign the risk premia for the first four cross-sectional consumption distribution moments. Specifically, we demonstrate that the average consumption and third normalized consumption moment betas should have a positive effect on excess asset returns, while the influence of the second and fourth normalized consumption moment betas is negative.

To investigate whether the defined consumption moment risk factors have pricing significance as against the CRR (1986) economic state variables, we test the multifactor asset pricing model that includes as risk factors the five CRR (1986) macroeconomic variables along with the rate of change in average consumption of asset holders (this factor reflects the influence of the aggregate consumption risk on asset pricing) and the rates of change in the normalized moments of order two to four of the cross-sectional consumption distribution of asset holders (these factors jointly reflect the influence of the background risk in consumption). The multibeta expected return relation associated with this multifactor model nests as special cases the expected return relations associated with (i) the CRR (1986) five-factor model, (ii) the (complete consumption insurance) representative-agent consumption CAPM, and (iii) the incomplete consumption insurance consumption CAPM.

To estimate the model parameters, we use the conventional two-step estimation technique and impose the restrictions implied by the preference theory on the consumption moment factor premia. Using quarterly data, we find that the empirical evidence on the pricing of the CRR (1986) five risk factors is sensitive to the experimental design. By contrast, there is strong evidence that, when the limited participation of households in the capital markets is taken into consideration, the aggregate consumption risk and the background risk are both significantly priced.

The rest of the paper proceeds as follows. In Section 2 we express the average marginal utility of consumption as a function of the normalized cross-sectional consumption distribution moments and argue that changes in the moments of order two and higher (along with the rate of change in per capita consumption of asset holders) are potentially important determinants of excess asset returns. Then, we use the preference theory to sign the risk premia for the consumption moment risk factors and introduce the implied multibeta expected return relation. Section 3 tests empirically the ability of the proposed multibeta model to explain the cross-section of stock market returns. Section 4 concludes.

2 A Multifactor Model of Risk

2.1 Consumption Moment Risk Factors

In the absence of arbitrage opportunity an investor equates the loss in utility associated with buying an additional unit of asset at time t to the resulting increase in discounted expected utility of consumption at time $t+1$:

$$u'\left(C_{k,t}\right) = E_t\left[\delta u'\left(C_{k,t+1}\right) R_{j,t+1}\right]. \tag{1}$$

In this no arbitrage condition δ is the subjective time discount factor, $C_{k,t}$ is the agent k's period t consumption, $R_{j,t+1}$ is the real gross return between time t and $t+1$ on asset (the portfolio of assets) j in which the agent holds a non-zero position, u is the agent's continuously differentiable von Neumann-Morgenstern period utility-of-consumption function,[2] and u' is the first derivative of utility with respect to consumption. E_t is the expectation operator. Expectation in (1) is taken conditional on the set of information available to the agent at time t.

Averaging the no arbitrage condition (1) over all the investors in the economy yields

$$K^{-1}\sum_{k=1}^{K} u'\left(C_{k,t}\right) = E_t\left[\delta K^{-1}\sum_{k=1}^{K} u'\left(C_{k,t+1}\right) R_{j,t+1}\right]. \tag{2}$$

By dividing the left- and right-hand sides of (2) by the time t average marginal utility of consumption, $K^{-1}\sum_{k=1}^{K} u'\left(C_{k,t}\right)$, we obtain

$$E_t\left[SDF_{t+1}R_{j,t+1}\right] = 1, \tag{3}$$

where the stochastic discount factor, or pricing kernel, SDF_{t+1} is the discounted ratio of the average marginal utility of consumption at time $t+1$ to the average marginal utility of consumption at time t:

$$SDF_{t+1} = \delta\frac{K^{-1}\sum_{k=1}^{K} u'\left(C_{k,t+1}\right)}{K^{-1}\sum_{k=1}^{K} u'\left(C_{k,t}\right)}. \tag{4}$$

Suppose that the utility function u is $N+1$ times differentiable and take a Taylor series approximation of the average marginal utility around average consumption $\overline{C}_t = K^{-1}\sum_{k=1}^{K} C_{k,t}$:

$$K^{-1}\sum_{k=1}^{K} u'\left(C_{k,t}\right) \approx u'\left(\overline{C}_t\right) + \sum_{n=2}^{N}\frac{1}{n!}u^{(n+1)}\left(\overline{C}_t\right)\mu_{n,t} = u'\left(\overline{C}_t\right) + \sum_{n=2}^{N}\frac{1}{n!}u^{(n+1)}\left(\overline{C}_t\right)\overline{C}_t^{n}\widetilde{\mu}_{n,t}, \tag{5}$$

where $\mu_{n,t} \equiv K^{-1}\sum_{k=1}^{K}\left(C_{k,t}-\overline{C}_t\right)^n$ is the nth centered moment of the time t cross-sectional consumption distribution and $\widetilde{\mu}_{n,t}$ (henceforth referred to as the nth normalized consumption

[2] As is conventional in the literature, we assume that the utility function has the properties $u' > 0$ (non-satiation) and $u'' < 0$ (risk aversion).

moment) is $\mu_{n,t}$ normalized by $\overline{C}_t^n$, $\widetilde{\mu}_{n,t} \equiv K^{-1} \sum_{k=1}^{K} \left(C_{k,t}/\overline{C}_t - 1 \right)^n$. Here and throughout the paper $u^{(n)}$ denotes the nth derivative of u.

Substituting (5) into the equation for the stochastic discount factor (4) yields

$$SDF_{t+1} = \delta \frac{u' \left(\overline{C}_{t+1} \right) + \sum_{n=2}^{N} \frac{1}{n!} u^{(n+1)} \left(\overline{C}_{t+1} \right) \overline{C}_{t+1}^n \widetilde{\mu}_{n,t+1}}{u' \left(\overline{C}_t \right) + \sum_{n=2}^{N} \frac{1}{n!} u^{(n+1)} \left(\overline{C}_t \right) \overline{C}_t^n \widetilde{\mu}_{n,t}}. \tag{6}$$

Assume that there is a risk-free asset with the real gross return $R_{f,t+1}$. By taking unconditional expectations of the both sides of the no arbitrage condition (3) and applying the covariance decomposition formula, we can explicitly determine the expected excess return on asset j over the risk-free rate, $Z_{j,t+1} \equiv R_{j,t+1} - R_{f,t+1}$, as

$$E \left[Z_{j,t+1} \right] = -cov \left(R_{j,t+1}, SDF_{t+1} \right) R_{f,t+1}. \tag{7}$$

Suppose that the investor can freely trade in the competitive and frictionless capital markets. If the capital markets are complete, then the investors are able to fully insure their future consumption by equalizing, state by state, their intertemporal marginal rates of substitution in consumption and hence, under the assumption of homogenous preferences, their optimal consumption growth rates. Thus, with complete consumption insurance the normalized consumption moments of order two and higher in (6) are all time invariant. As follows from equation (7), in this case the asset's systematic risk is measured by the covariance (beta) of the asset return with the rate of change in average consumption of asset holders.

In the absence of certain contingent-claims markets,[3] the agents are unable to self-insure against background risks and realized consumption growth rates can differ across the investors. If so, the normalized consumption moments of order two and higher are not constant over time and, therefore, as implied by (7), the betas for these additional risk factors (that reflect the influence of the background risk in consumption on asset pricing), along with the beta for the average consumption growth rate (that reflects the influence of the aggregate consumption risk), may also contribute to the expected equity premium.

To examine the signs of the risk premia for the consumption moment risk factors, assume first that there is an (normalized consumption moments of order two and higher preserving) increase in the mean of the cross-sectional consumption distribution. This is the (complete consumption insurance) case when the consumption of each agent in the cross-section increases by the same factor. Because all the investors are assumed to have the same strictly increasing and concave utility function, an increase in the consumption of each investor lowers the time $t+1$ marginal utility of his consumption and hence the average marginal utility of consumption. It follows from (7) that the asset with a greater covariance (beta) with the average consumption growth rate has a higher return when the marginal utility of consumption is low and, inversely, a lower return when the marginal utility is high (i.e., when consumption is most valuable). The inability to insure

[3] Cochrane (1991), Mace (1991), and Nelson (1994), for example, provide empirical evidence of market incompleteness.

against adverse movements in consumption makes the asset riskier to the investor, driving up the risk premium the investor demands to hold the asset.

As implied by equation (7), if $u^{(n+1)} > 0$ ($n = 2, 3, ...$), then an increase in the nth normalized consumption moment (the other consumption moments held fixed) raises the time $t+1$ average marginal utility of consumption. Thus, the asset with a greater covariance (beta) with the growth rate of the nth normalized consumption moment tends to have a higher return when the average marginal utility of consumption is high and, therefore, is, in this sense, less risky, driving down the risk premium required by the investor. If, by contrast, $u^{(n+1)} < 0$ ($n = 2, 3, ...$), then an increase in the nth normalized consumption moment (the other consumption moments held fixed) lowers the time $t+1$ average marginal utility of consumption, thereby implying that the asset with a greater covariance (beta) with the growth rate of the nth normalized consumption moment has a higher return when the average marginal utility is low and, inversely, a lower return when the average marginal utility of consumption is high. This makes the asset riskier to the investor, raising the risk premium demanded by the investor to induce him to hold this asset.

This suggests that the risk premium for the aggregate per capita consumption growth rate should be positive. The signs of the risk premia for the growth rates of the normalized consumption moments of order two and higher depend on the signs of the third and higher derivatives of the agent's utility function. The risk premium for the nth normalized consumption moment growth rate should be positive if $u^{(n+1)} < 0$ and negative if $u^{(n+1)} > 0$.

2.2 Preference Assumptions and the Consumption Moment Risk Premia

In the previous section it was shown that with incomplete consumption insurance a change in the average marginal utility of consumption is no longer a function of a change in average consumption (the first cross-sectional consumption moment) alone (as with complete consumption insurance), but is also a function of changes in higher-order moments of the cross-sectional consumption distribution. The problem with both this and the Arbitrage Pricing Theory (APT) approaches is the unknown number of risk factors. In our framework, this problem translates into deciding how many consumption moments must be taken into account or, in other words, at which point to truncate the Taylor expansion (5).

One way to determine the order at which the expansion should be truncated is to allow data to motivate the point of truncation.[4] This approach consists in repeating the estimation of a model for increasing values of N and truncating the expansion at the point when further increasing in N does not significantly affect the estimation results. As Dittmar (2002) points out, there are at least two difficulties with allowing data to determine the required order of a Taylor expansion. The first one is the possibility of overfitting the data. Another problem is that when a high-order expansion is taken, preference theory no longer guides in determining the signs of risk factor prices. To avoid the latter problem, Dittmar (2002) proposes to let preference arguments determine the point of truncation. He shows that increasing utility, risk aversion, decreasing absolute risk aversion,

[4] See Bansal *et al.* (1993).

and decreasing absolute prudence imply $u^{(4)} < 0$. Since preference assumptions do not guide in determining the signs of the higher-order derivatives, Dittmar (2002) argues that the Taylor expansion terms of order higher than three do not matter for asset pricing and truncates a Taylor expansion after the cubic term.[5] He claims that the advantage coming from signing the Taylor expansion terms outweighs a loss of power due to omitting the terms of order four and higher.

Following Dittmar (2002), we also let preference theory determine the order of the Taylor series approximation. Below, we will show that, in contrast with the sign of the second derivative of the utility function that characterizes the agent's attitudes towards market risk regardless of a specific choice problem, signing the higher-order derivatives of the utility function is based upon the context in which the risk associated with the investment decision arises.

The Taylor series approximation (5) implies that the sign of the contribution of the second normalized consumption moment to the average marginal utility of consumption is determined by the sign of the third derivative of the utility function. Leland (1968), Sandmo (1970), and Drèze and Modigliani (1972), for example, argue that if the agent's absolute risk aversion is decreasing (i.e., $u''' > 0$), then incomplete consumption insurance leads the agents to save more in order to self-insure against the additional variability in their consumption streams caused by background risk factors.[6] Kimball (1990) defines prudence ($u''' > 0$) as a measure of the sensitivity of the optimal choice of a decision variable to risk (of the intensity of the precautionary saving motive in the context of the consumption-saving decision under uncertainty). A precautionary saving motive is positive when $-u'$ is concave ($u''' > 0$) just as an individual is risk averse when u is concave.

The unavailability of insurance against an additional independent background risk with a non-positive expected value raises the aversion of a decision maker to the risk associated with the investment in a risky asset relative to the no background risk case. The preferences with such a property are said to exhibit risk vulnerability.[7] Gollier (2001) shows that, when the additive independent background risk has a non-positive mean, preferences exhibit risk vulnerability if at least one of the following two conditions is satisfied: (i) absolute risk aversion is decreasing and convex and (ii) both absolute risk aversion and absolute prudence are positive and decreasing in wealth. This latter condition is referred to as standard risk aversion, the concept introduced by Kimball (1993).[8]

Non-satiation ($u' > 0$), risk aversion ($u'' < 0$), and prudence ($u''' > 0$) together imply that the absolute risk aversion coefficient, $ARA = -u''/u'$, and the coefficient of absolute prudence,

[5] Brav *et al.* (2002) also limit their analysis to a third-order approximation when using a Taylor series expansion of the equal-weighted average of the household's intertemporal marginal rates of substitution. Cogley (2002) stops at a third-order polynomial when taking a Taylor series expansion of the individual's intertemporal marginal rates of substitution.

[6] Courbage and Rey (2007) stress that a positive third derivative of the utility function is still a necessary and sufficient condition for a positive precautionary saving motive when a non-financial background risk and the financial market risk are independent. They show that the set of sufficient conditions is more complex if the risks are dependent.

[7] See Gollier and Pratt (1996) and Gollier (2001), for example.

[8] Kimball (1993) shows that standard risk aversion implies risk vulnerability.

$AP = -u'''/u''$, are both positive, as required by (ii). Prudence ($u''' > 0$) is also the necessary (but not sufficient) condition for decreasing absolute risk aversion. To complete the set of the conditions necessary for risk vulnerability, we, therefore, need to determine the properties of the utility function required for absolute prudence to be decreasing in wealth (for the set of conditions (ii) to be met) or absolute risk aversion to be convex (for the set of conditions (i) to be met).

Intuitively, the willingness to save is an increasing function of the expected marginal utility of future consumption. Since the marginal utility is decreasing in consumption, at a given level of background risk the absolute level of precautionary savings must also be expected to decline as consumption rises.

Proposition 1. *Absolute prudence is decreasing if and only if $u^{(4)} < -APu'''$. The condition $u^{(4)} < 0$ is necessary for decreasing absolute prudence.*

Proof of Proposition 1. Decreasing absolute prudence implies that

$$AP' = -\frac{u^{(4)}u'' - (u''')^2}{(u'')^2} < 0. \tag{8}$$

In order to prove that the condition $u^{(4)} < 0$ is necessary for decreasing absolute prudence, suppose, in contrast, that $u^{(4)} \geqslant 0$. When $u^{(4)} \geqslant 0$, $u^{(4)}u'' \leqslant 0$ and, therefore, $AP' > 0$. This contradicts the assumption that absolute prudence is decreasing.

Inequality (8) implies that $u^{(4)}u'' - (u''')^2 > 0$ is the necessary and sufficient condition for decreasing absolute prudence. We can rewrite this condition as

$$u^{(4)} < \frac{(u''')^2}{u''} = -APu'''. \tag{9}$$

Since the agent is prudent, the term on the right-hand side of (9) is negative, proving that $u^{(4)} < 0$ is the necessary (but not sufficient) condition for decreasing absolute prudence. ■

The condition $u^{(4)} < 0$ is referred to as temperance.[9] Kimball (1992) defines temperance as a type of behavior when the presence of a background risk leads the agent to reduce his exposure to another independent risk. Menezes and Wang (2005) provide an interpretation of $u^{(4)} < 0$ in the context of choice between pairs of risky prospects. They argue that $u^{(4)} < 0$ can also be interpreted as aversion to relocations of dispersion from the center of a distribution to its tails (aversion to outer risk).

It is easy to check that the condition $u^{(4)} < 0$ is also necessary (but not sufficient) for convex absolute risk aversion. Thus, the assumptions of non-satiation ($u' > 0$), risk aversion ($u'' < 0$), prudence ($u''' > 0$), and temperance ($u^{(4)} < 0$) form the set of necessary (but not sufficient) conditions for the agent's preferences to exhibit risk vulnerability.

It looks natural to assume that for each level of background risk the absolute level of precautionary savings should decline in wealth at a decreasing rate and hence that, like absolute risk aversion, absolute prudence is convex.

[9] See Kimball (1992).

Proposition 2. *Absolute prudence is convex if and only if* $u^{(5)} > -2AP'u''' - APu^{(4)}$. *If preferences exhibit prudence and decreasing absolute prudence, then* $u^{(5)} > 0$ *is the necessary condition for convex absolute prudence.*

Proof of Proposition 2. Absolute prudence is convex if the following condition is satisfied:

$$AP'' = -\frac{A - B}{C} > 0, \tag{10}$$

where $A = (u'')^2 \left(u^{(5)}u'' - u'''u^{(4)}\right)$, $B = 2u''u'''(u^{(4)}u'' - (u''')^2)$, and $C = (u'')^4$.

To prove that $u^{(5)} > 0$ is necessary for convex absolute prudence under prudence and decreasing absolute prudence, assume that $u^{(5)} \leqslant 0$. An agent is prudent ($AP > 0$) if and only if $u''' > 0$. By Proposition 1, we know that the necessary condition for decreasing absolute prudence is that $u^{(4)} < 0$. Then, under prudence and decreasing absolute prudence, $A > 0$. Since $u^{(4)}u'' - (u''')^2 > 0$ is the necessary and sufficient condition for decreasing absolute prudence, prudence and decreasing absolute prudence also imply that $B < 0$. In consequence, $AP'' < 0$, which contradicts the initial assumption that absolute prudence is convex.

It follows from (10) that the necessary and sufficient condition for convex absolute prudence is $A - B < 0$. This condition can be written as

$$u^{(5)} > \frac{2u'''(u^{(4)}u'' - (u''')^2)}{(u'')^2} + \frac{u'''u^{(4)}}{u''} \tag{11}$$

or, equivalently,

$$u^{(5)} > -2AP'u''' - APu^{(4)}. \tag{12}$$

Under prudence and decreasing absolute prudence, the term $-2AP'u''' - APu^{(4)}$ is positive.[10] Thus, $u^{(5)} > 0$ is necessary (but not sufficient) for convex absolute prudence. ■

Lajeri-Chaherli (2004) considers a two-period optimal consumption choice problem with income uncertainty in the second period. She shows that $u^{(5)} > 0$ is a necessary condition for decreasing absolute temperance and labels this condition as edginess.[11] Eeckhoudt *et al.* (2010) interpret edginess "as implying that a decrease in one risk (via second order stochastic dominance) helps to temper the effects of an increase in downside risk of another additive risk".

The restriction of convex absolute prudence enables us to sign the fifth derivative of the utility function and hence to expand the average of investors' marginal utilities of consumption up to the fourth cross-sectional consumption moment, further than it is done in Dittmar (2002).

Therefore, reference to the agent's behavior in the presence of background risk enables us to justify the signs of the third, fourth, and fifth derivatives of u. Combined with the conditions of non-satiation ($u' > 0$) and risk aversion ($u'' < 0$), this makes it possible to sign the risk premia for the changes in average consumption and the second through fourth normalized moments of the cross-sectional consumption distribution. As we argued in Section 2.1, non-satiation ($u' > 0$)

[10] If an agent exhibits prudence, then $AP > 0$ and $u''' > 0$. The condition $u^{(4)} < 0$ is necessary for DAP.

[11] Lajeri-Chaherli (2004) calls the term $-u^{(5)}/u^{(4)}$ absolute edginess.

and risk aversion ($u'' < 0$) imply that the risk premium for the aggregate per capita consumption growth rate should be positive. According to the results in Section 2.1, from prudence ($u''' > 0$) and edginess ($u^{(5)} > 0$) it follows that the risk premia for the second and fourth normalized consumption moments should be negative and, finally, temperance ($u^{(4)} < 0$) implies that the risk premium for the third normalized consumption moment should be positive.

2.3 The Model

As we showed in Section 2.1, the consumption beta theories suggest that assets are priced according to their covariances with the average (over investors) marginal utility of consumption. With complete consumption insurance, the normalized centered moments of order two and higher of the cross-sectional consumption distribution are time-invariant and, therefore, are irrelevant for asset pricing. In this case, the asset's systematic risk is measured by the covariance of the asset return with the growth rate of aggregate per capita consumption. When consumption insurance is incomplete, the covariances (betas) of the normalized moments of order two and higher with the asset returns are not zero and hence the rates of change in these moments (caused by background risk factors) may be viewed as potentially important determinants of asset prices. In Section 2.2 we showed how the preference theory enables us to sign the first five derivatives of the utility function and hence sign the risk premia for the first four cross-sectional consumption moments.

Due to a potentially important role played by the consumption moments of order two to four in explaining the cross-section of asset returns, in this section we examine the model that includes, in addition to the five systematic risk factors discussed in CRR (1986), the rates of change in the first through fourth cross-sectional consumption moments, as suggested by our results in Sections 2.1 and 2.2. More specifically, we consider the following multifactor asset pricing model:

$$\begin{aligned}\mathbf{Z}_t = \boldsymbol{\beta}_0 + \boldsymbol{\beta}_1 MP_t + \boldsymbol{\beta}_2 DEI_t + \boldsymbol{\beta}_3 UI_t + \boldsymbol{\beta}_4 UPR_t + \boldsymbol{\beta}_5 UTS_t + \\ \boldsymbol{\beta}_6 ACG_t + \boldsymbol{\beta}_7 SMG_t + \boldsymbol{\beta}_8 TMG_t + \boldsymbol{\beta}_9 FMG_t + \boldsymbol{\epsilon}_t,\end{aligned} \tag{13}$$

where $\mathbf{Z}_t$ is the $(J \times 1)$ vector of time t real returns on portfolios of risky assets in excess of the return on the risk-free asset with elements $Z_{j,t} \equiv R_{j,t} - R_{f,t}$ $(j = 1, ..., J)$,[12] MP_t is the quarterly growth rate of industrial production led by one quarter, DEI_t is the change in expected inflation, UI_t is unexpected inflation, UPR_t is the excess return of low-grade corporate bonds over high-grade corporate bonds (the default premium), UTS_t is the unanticipated return on long bonds (the term premium), ACG_t is the time t aggregate per capita consumption growth rate, SMG_t, TMG_t, and FMG_t are the time t rates of change in, respectively, the second, third, and fourth normalized consumption moments ($\widetilde{\mu}_{n,t}$, $n = 2, 3, 4$), $\boldsymbol{\epsilon}_t$ is an $(J \times 1)$ vector of idiosyncratic error terms, $\boldsymbol{\beta}_0$ is an $(J \times 1)$ vector of portfolio return intercepts, and $\boldsymbol{\beta}_k$ $(k = 1, ..., 9)$ are $(J \times 1)$ vectors of the portfolios' exposures to the risk factors.

In model (13), the first six risk factors are the same as in the CRR (1986), whereas the last three factors are new. In contrast with the risk factor ACG that measures the aggregate

[12] The use of excess returns makes it easier to impose the bond pricing restrictions that we consider below.

consumption risk that can be hedged using capital market instruments, as we argued above, these new three factors (jointly) capture the influence of background risk factors on excess asset returns. An attractive feature of model (13) is that, in contrast with the APT, which does not provide the identification of the risk factors, the set of the consumption moment risk factors in this model obtain endogenously from the no arbitrage condition (1). This allows to avoid some serious problems arising from an ad hoc specification of a factor structure.[13]

The absence of arbitrage opportunities implies the following exact factor pricing relation:

$$E\left[\mathbf{Z}_t\right] = \gamma_0 \boldsymbol{\iota} + \sum_{i=1}^{9} \gamma_i \boldsymbol{\beta}_i, \tag{14}$$

where $E\left[\mathbf{Z}_t\right]$ is the $(J \times 1)$ vector of expected excess returns, $\boldsymbol{\iota}$ is an $(J \times 1)$ vector of ones, γ_0 is the excess zero-beta rate, and $\boldsymbol{\gamma} \equiv (\gamma_1, ..., \gamma_9)$ is a vector of factor risk premia.

This *unrestricted model* nests a number of the previously considered models as special cases. If none of the consumption moment betas is significantly priced ($\gamma_6 = \gamma_7 = \gamma_8 = \gamma_9 = 0$), then we obtain the multibeta expected return relation associated with the conventional CRR (1986) five-factor model. Alternatively, under the null hypothesis of the consumption-based theories, the betas for the CRR (1986) five economic state variables should all be rejected statistically as having an influence on excess asset returns ($\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = \gamma_5 = 0$). If incomplete consumption insurance plays no role in explaining the cross-section of asset returns, then the risk premia for the normalized consumption moments of order two to four are all zero ($\gamma_7 = \gamma_8 = \gamma_9 = 0$) and, therefore, (14) reduces to the multibeta expected return relation associated with the model with the risk factors being the five economic state variables and the growth rate of aggregate per capita consumption, considered in CRR (1986).

The APT pricing models are agnostic about the preferences of the investors. A virtue of model (14) is that the signs of the consumption moment risk premia are driven by preference assumptions, while the risk factor premia are unrestricted in the multifactor models based on the APT. The results in the previous section suggest that in (14) the risk-return parameters γ_6 and γ_8 should be positive, whereas the parameters γ_7 and γ_9 should be negative.

As emphasized by Stambaugh (1982) and SW (2006), the inclusion of other (than stock portfolios) assets should increase the precision of the estimates of the regression parameters and the power of the tests. As such assets we can use the long-term government bond and low-grade bond portfolios, for example. Employing the long-term government bond and low-grade bond portfolios as assets in (13), we obtain the following two restrictions on the risk-return parameters in pricing relation (14):[14]

$$E\left[UTS_t\right] = \gamma_0 + \gamma_5, \tag{15}$$

$$E\left[UPR_t\right] = \gamma_4. \tag{16}$$

[13] See Campbell, Lo, and MacKinlay (1997). First, choosing factors without regard to economic theory may lead to overfitting the data. The second potential danger is the lack of power of tests which ignore the theoretical restrictions implied by a structural model.

[14] See SW (2006).

Substituting restrictions (15) and (16) into equation (14) and rearranging terms yields the restricted multibeta expected return relation

$$E\left[\mathbf{Z}_t - UPR_t\boldsymbol{\beta}_4 - UTS_t\boldsymbol{\beta}_5\right] = \gamma_0\left(\boldsymbol{\iota}_J - \boldsymbol{\beta}_5\right) + \sum_{i=1}^{3}\gamma_i\boldsymbol{\beta}_i + \sum_{i=6}^{9}\gamma_i\boldsymbol{\beta}_i. \tag{17}$$

Let us call (17) the *restricted model*. The risk-return parameters γ_0, ..., γ_3, γ_6, ..., γ_9 in this model can be estimated by projecting dependent variable $(\mathbf{Z}_t - UPR_t\boldsymbol{\beta}_4 - UTS_t\boldsymbol{\beta}_5)$ on explanatory variables $\boldsymbol{\iota}_J - \boldsymbol{\beta}_5$, $\boldsymbol{\beta}_1$, ..., $\boldsymbol{\beta}_3$, $\boldsymbol{\beta}_6$, ..., $\boldsymbol{\beta}_9$. As implied by restrictions (15) and (16), the estimates of the risk prices γ_4 and γ_5 can be obtained as $\widehat{\gamma}_4 = T^{-1}\sum_{t=1}^{T} UPR_t$ and $\widehat{\gamma}_5 = T^{-1}\sum_{t=1}^{T} UTS_t - \widehat{\gamma}_0$. Since $\widehat{\gamma}_4$ is computed directly from raw data, which are assumed to be measured without error, it is not subject to the errors-in-variables (EIV) complication. The restricted estimator for the UPR risk premium is unbiased and more efficient than its unrestricted counterpart.[15]

3 Data, Estimation, and Testing

In this section we test the ability of the unrestricted (14) and restricted (17) models to explain a cross-section of long-run stock returns.

3.1 The Data

Because the data on individual consumption are available on the quarterly basis only, we take the standard period to be one quarter. The time period extends from 1982:Q1 to 2003:Q4. Throughout the paper, X_t denotes the value of variable X at the end of quarter t. $E_{t-1}\left[X_t\right]$ denotes the expectation of X_t conditional on the information available at the end of quarter $t-1$.

3.1.1 Industrial Production

We define the quarterly growth rate of industrial production as $MP_t \equiv ln\left(IP_t\right) - ln\left(IP_{t-1}\right)$, where IP_t is the Industrial Production Index (INDPRO series) in quarter t from FRED database at Federal Reserve Bank of St.Louis. Variable IP_t is measured as a flow of industrial production during quarter t. Because of this, the MP series may actually measure changes in industrial production with a lag. To make the MP series contemporaneous with other variables, we lead it forward by one quarter.

3.1.2 Inflation

Unexpected inflation is defined as $UI_t \equiv I_t - E_{t-1}\left[I_t\right]$ and the change in expected inflation as $DEI_t \equiv E_t\left[I_{t+1}\right] - E_{t-1}\left[I_t\right]$, where the inflation rate from quarter $t-1$ to t, I_t, is the quarterly first difference in the logarithm of the seasonally adjusted Consumer Price Index (CUSR0000SA0 series) from the US Bureau of Labor Statistics for quarter t, $I_t \equiv ln\left(CPI_t\right) - ln\left(CPI_{t-1}\right)$. The

[15] See SW (2006).

expected inflation variable is defined as $E_{t-1}[I_t] = R_{f,t} - E_{t-1}[RHO_t]$, where $R_{f,t}$ is the three-month Treasury Bill return from Ibbotson and Associates, Inc. and $RHO_t \equiv R_{f,t} - I_t$ is the *ex post* real return on Treasury Bills in quarter t. As in Fama and Gibbons (1984), we model the difference between the *ex post* real rate of interest in quarter t and $t-1$ as $RHO_t - RHO_{t-1} = u_t + \theta u_{t-1}$, so that $E_{t-1}[RHO_t] = (R_{f,t-1} - I_{t-1}) - (\widehat{u}_t + \widehat{\theta}\widehat{u}_{t-1})$.

3.1.3 The Term Premium

As a measure of the unanticipated return on long bonds, we use the term premium variable UTS defined as the excess return on the 20-year US Treasury Constant Maturity Rate over the one-year US Treasury Constant Maturity Rate from FRED database at Federal Reserve Bank of St.Louis.[16]

3.1.4 The Default Premium

Following Liu and Zhang (2008), we define the default premium as the excess return of low-grade corporate bonds over high-grade corporate bonds, $UPR_t \equiv Baa_t - Aaa_t$, where Baa_t and Aaa_t are, respectively, the Moody's Seasoned Baa and Aaa Corporate Bond yields from FRED database at Federal Reserve Bank of St.Louis.

3.1.5 Consumption Moments

The normalized moments of the cross-sectional consumption distribution for each quarter are calculated using quarterly consumption data from the US Consumer Expenditure Survey (CEX), produced by the US Bureau of Labor Statistics (BLS). The CEX data cover the period from 1980:Q1 to 2003:Q4. As suggested by Attanasio and Weber (1995), Brav *et al.* (2002), and Vissing-Jorgensen (2002), we drop all consumption observations for the years 1980 and 1981 because the quality of the CEX consumption data is questionable for this period. Thus, our sample covers the period extending from 1982:Q1 to 2003:Q4. Following Brav *et al.* (2002), in each quarter we drop from the CEX data set households that do not report or report a zero value of consumption of food, consumption of nondurables and services, or total consumption. We also delete from the sample nonurban households, households residing in student housing, households with incomplete income responses, households that do not have a fifth interview, and households whose head is under 19 or over 75 years of age.

As is conventional in the literature, the consumption measure used in this paper is consumption of nondurables and services. For each household we calculate quarterly consumption expenditures for all the disaggregate consumption categories offered by the CEX. Then, we deflate obtained values in 2005 dollars with the CPI's (not seasonally adjusted, urban consumers) for appropriate consumption categories.[17] Aggregating the household's quarterly consumption across these categories is made according to the National Income and Product Account (NIPA) definition of

[16] See Liu and Zhang (2008).

[17] The CPI series are obtained from the BLS through CITIBASE.

consumption of nondurables and services. To mitigate measurement error in individual consumption, we subject the households to a consumption growth filter and use the conventional z-score method to detect outliers. Following common practice for highly skewed data sets, in each quarter we consider the consumption growth rates with z-scores greater than two in absolute value to be due to reporting or coding errors and remove from the sample the households whose real per capita consumption has a z-score greater than two in absolute value for this period.[18]

In the fifth (final) interview, the household is asked to report the end-of-period estimated market value of all stocks, bonds, mutual funds, and other such securities held by the consumer unit on the last day of the previous month as well as the difference in this estimated market value compared with the value of all securities held a year ago last month. Using these two values, we calculate each consumer unit's asset holdings at the beginning of a 12-month recall period in constant 2005 dollars. To take into account the limited participation of households in the capital markets, we consider three sets of households defined as asset holders according to a criterion of asset holdings at the beginning of a 12-month recall period above a certain threshold. First, we consider households that report total assets equal to or exceeding \$1000 (the first set).[19] The second set of asset holders consists of households with the reported amount of asset holdings equal to or exceeding \$5000, and, finally, households that report an estimated market value of all securities held a year ago equal to or greater than \$10000 are grouped in the third set.

Like the Industrial Production Index, individual consumption in the CEX is the flow of consumption during a quarter and is not measured at the end of each quarter as the other variables. To deal with this problem, as in the case with variable MP, subsequent statistical work leads the series of changes in the cross-sectional consumption moments forward by one quarter.

A well documented potential problem with using household-level data is the large measurement error in reported individual consumption.[20] To investigate the effect of measurement error on the changes in the normalized consumption moments, assume that the agent k's consumption in period t is misreported by some stochastic factor $(1 + \epsilon_{k,t})$, so that the observed consumption is $C_{k,t} = C^*_{k,t}\,(1 + \epsilon_{k,t})$, where $C^*_{k,t}$ is the true consumption level. Suppose further that for all k and t, $\epsilon_{k,t}$ has a zero mean value and is independent of $C^*_{k,t}$.

Because $C^*_{k,t}$ and $\epsilon_{k,t}$ are independent,

$$\overline{C}_t = K^{-1} \sum_{k=1}^{K} C_{k,t} \xrightarrow{P} E[C_{k,t}] = E[C^*_{k,t}] \tag{18}$$

[18] The quarterly consumption growth between dates t and $t + 1$ is calculated if consumption is not equal to zero for both of the quarters (missing information is counted as zero consumption).

[19] Since the CEX reports only some limited information about asset holdings, we start with consumer units that report asset holdings equal to or exceeding \$1000, rather than households that report a positive amount of total asset holdings, in order to reduce the likelihood of including households, who do not participate in the capital markets. See Cogley (2002) and Vissing-Jorgensen(2002), for example, for more details.

[20] See Zeldes (1989) and Runkle (1991), for example.

and for any $n \geqslant 2$

$$\widetilde{\mu}_{n,t} \equiv K^{-1} \sum_{k=1}^{K} \left(\frac{C_{k,t}}{\overline{C}_t} - 1 \right)^n \xrightarrow{P} E\left[\left(\frac{C^*_{k,t}\left(1+\epsilon_{k,t}\right)}{\overline{C}^*_t} - 1 \right)^n \right] \neq E\left[\left(\frac{C^*_{k,t}}{\overline{C}^*_t} - 1 \right)^n \right]. \tag{19}$$

As can be seen from the above results, the measurement error of the type assumed here does not affect the cross-sectional mean and, therefore, has no influence on the rate of change in per capita consumption. In contrast with the first cross-sectional moment, the normalized consumption moments of order two and higher can be affected considerably by the multiplicative idiosyncratic measurement error in reported individual consumption. This leads to imprecise estimates of the percentage changes in these moments unless the estimates for two consecutive time periods are biased by the same factor. This assumption, however, is likely to be unrealistic.

As we shall show below, imprecise estimates of the consumption moment risk factors may introduce an EIV complication that can result in contaminated estimates of the risk-return parameters in models (14) and (17). In Section 3.2 we will explain how to reduce this problem.

3.1.6 The Returns Data

We use the quarterly (long-run) returns on four different sets of portfolios: (i) the 25 size and book-to-market portfolios, (ii) the 25 portfolios formed on size and momentum, (iii) the 25 portfolios formed on size and short-term reversal, and (iv) the 25 portfolios formed on size and long-term reversal.[21] The nominal returns (capital gain plus all dividends) on these portfolios are from Kenneth R. French's web page. The nominal quarterly risk-free rate is the 3-month US Treasury Bill secondary market rate on a per annum basis obtained from the Federal Reserve Bank of St.Louis. In order to convert from the annual rate to the quarterly rate, we raise the 3-month Treasury Bill return on a per annum basis to the power of 1/4. The real quarterly returns are calculated as the quarterly nominal returns divided by the 3-month inflation rate based on the deflator defined for consumption of nondurables and services. Excess returns are calculated as the differences between the real equity returns and the real risk-free rate.

3.1.7 Descriptive Statistics

Tables I and II report statistical characteristics of the risk factors in model (13). Table I shows the means, standard deviations, and the correlation coefficients for the factors computed over the sample period from 1982:Q1 to 2003:Q4. The CRR (1986) five macroeconomic factors are frequently significantly correlated among themselves. The strongest correlation is observed between

[21] The 25 size and book-to-market portfolios for July of year t to June of $t+1$ include all NYSE, AMEX, and NASDAQ stocks and are constructed at the end of June of year t as the intersections of five portfolios formed on size and five portfolios formed on the ratio of book equity to market equity. The 25 portfolios formed on size and momentum are the intersections of five portfolios formed on size (market equity) and five portfolios formed on prior (2-12) return. The 25 portfolios formed on size and short-term (long-term) reversal are the intersections of five portfolios formed on size and five portfolios formed on prior (1-1) and (13-60) return, respectively. The latter three groups of portfolios are constructed each month with month t portfolios formed at the end of month $t-1$.

the time series of the percentage change in expected inflation DEI and unexpected inflation UI. A strong positive correlation between these two variables is due to the fact that these two factors both use the expected inflation series $E_{t-1}[I_t]$.

[Table I about here]

[Table II about here]

The rates of change in average consumption and the normalized consumption moments of order two, three, and four are not significantly correlated with the CRR (1986) five systematic risk factors, with the exception of the default premium UPR that is significantly negatively correlated with the normalized consumption moments of order two to four computed for the set of households that report total assets equal to or exceeding \$1000. The correlation becomes insignificant as the definition of asset holders is tightened. Changes in the consumption moments (especially, the rates of change in the second through fourth normalized consumption moments) are mostly strongly positively correlated with each other.

The high collinearity of risk factors usually leads to less precise estimates of the factor betas and hence may be viewed as a potential source of EIV. In Section 3.2 we shall explain how the use of the instrumental variable approach helps mitigate the problem of factor multicollinearity.

Table II reports the autocorrelations and Ljung-Box Q-statistics with five and ten autocorrelations. It is observed that, except for variable UI, the other CRR (1986) macroeconomic risk factors display statistically significant serial correlation and, therefore, are not, in fact, innovations, as required by model (13). Despite this, in order to get comparable results, we prefer to use the original approach to compute these factors, as it is described in CRR (1986) and Liu and Zhang (2008), for example, and do not extract innovations in the series of highly correlated risk factors.

In contrast with the CRR (1986) variables, the consumption moment risk factors are not strongly correlated (and can then be treated as innovations, as required by the model) with the autocorrelation coefficients getting less significant as the threshold of household assets holdings is raised. An exception is the rate of change in average consumption that is found to exhibit long-range dependence for the sets of asset holders with threshold values \$5000 and \$10000.

3.2 Estimation Methodology

To estimate the risk-return parameters in the unrestricted (14) and restricted (17) models, like in many recent studies, we use the following two-step estimation technique. In a first-step time-series regression we project each asset excess return on the macroeconomic variables to estimate the asset betas for these risk factors over the first 10 years of data (40 quarters ending in the last quarter of a calendar year). In the second step, for each quarter of the subsequent year we run a cross-sectional regression (CSR) of the excess asset returns on the first-step estimates of the factor betas to get the vector of the factor risk premia. Then, we move four quarters ahead and repeat these steps. Finally, given time series of the CSR estimates for each risk-return parameter, we test statistical significance of their time-series averages by a t-test based on the usual Fama-MacBeth (1973) standard error of the quarterly time-series means of the parameter estimates.

The problem with this estimation technique is that the true risk factor betas are unobservable and the second-step CSRs use betas estimated from the data. This introduces the EIV problem that potentially could lead to biased estimates of the risk premia. One approach to address the EIV problem is to group stocks into portfolios and then to apply the two-step estimation methodology described above to portfolio returns rather than the returns on individual stocks. This allows to increase the precision of the estimates of the factor betas and hence to mitigate the EIV complication. Following this approach, CRR (1986), for example, group stocks into equally-weighted portfolios according to their total market values at the end of each five-year ranking (prior-beta-estimation) period.

Ball and Kothari (1989) consider the case where systematic risk is measured by market beta and document that betas calculated for the portfolios formed by ranking stocks on their market values at the end of the ranking period (the most immediate past five years) give biased assessments of the portfolio betas beyond the ranking date. They observe that simple annual averages of the estimates of market beta for the ranking periods are biased downward (upward) estimates of the simple annual averages of the postranking-period (the five years following the ranking year) market betas for small (large) size portfolios. Even larger discrepancies between the ranking period and post-ranking period market betas are observed by Ball and Kothari (1989) for the portfolios constructed by ranking stocks according to their returns during the ranking period. It was found that the average portfolios' ranking-period beta estimates substantially understate (overstate) the average postranking betas for low-return (high-return) portfolios. SW (2006) argue that it is quite likely that the reduced spread in betas would lead to biased upward estimates of risk premia.

SW (2006) adapt the portfolio formation approach used by Black *et al.* (1972), Black and Scholes (1974), Chan and Chen (1988), and Fama and French (1992). They form size portfolios based on the market values of stocks at the end of December of each year and then compute the returns on these portfolios in each month of the following year. SW (2006) argue that the use of the returns on such annually updated portfolios enables to avoid the selection biases discussed above. The methodology of generating the portfolio returns that we use in our empirical investigation is similar to the portfolio formation approach recommended by SW (2006). We expect this to help increase the precision of the estimates of the factor betas and hence reduce a potential EIV complication in the second-step CSRs regression.

It is usually assumed in empirical investigations that the risk factor realizations in the conventional CRR (1986) five-factor model are measured without error. The situation is not as neat when the first-step multifactor model includes the rates of change in the second through fourth normalized consumption moments as additional explanatory variables. As we showed in Section 3.1, the measurement error in reported individual consumption may bias the estimates of these factors.

In linear regression models, the presence of measurement errors in the independent variables lead to inconsistent OLS estimators of the regression parameters. As argued by Dagenais and Dagenais (henceforth DD) (1997), in a model with two correlated regressors the OLS estimators of the coefficients associated with the both explanatory variables will generally be inconsistent if there are errors in at least one of the regressors. DD (1994) find that the gaps between the true values of the regression parameters and the probability limits of their OLS estimators increase when the true explanatory variables are more collinear (with similar errors of measurement). As found in DD (1997), when the independent variables are highly correlated the bias of the OLS estimator of the coefficient on the badly measured variable will generally be even larger than it would be in a model with a single regressor. Furthermore, the OLS estimates of the coefficients associated with all other correlated explanatory variables will also be severely contaminated.

The correlation coefficients reported in Table I show that the rates of change in the first four cross-sectional consumption moments are highly correlated among themselves as well as with variable UPR. The results in DD (1997), therefore, suggest that errors in the measurement of the growth rates of the second through fourth normalized consumption moments (documented in Section 3.1) may severely contaminate the first-step OLS estimates of not only the coefficients associated with these variables, but also the coefficients of UPR and the percentage change in average consumption, thereby introducing the EIV complication to the second-step CSRs.

The most common solution to the EIV problem in linear regression models is to resort to instrumental variable techniques. Assume, as in DD (1997), the following multiple linear regression model:

$$\mathbf{Y} = \alpha \boldsymbol{\iota}_T + \mathbf{X}^* \boldsymbol{\beta} + \boldsymbol{\varepsilon}, \tag{20}$$

where $\mathbf{Y}$ is a $(T \times 1)$ vector of observations of the dependent variable, $\mathbf{X}^*$ is a $(T \times K)$ matrix of variables measured without error, $\boldsymbol{\varepsilon}$ is a $(T \times 1)$ vector of error terms ($\boldsymbol{\varepsilon} \sim N(\mathbf{0}, \sigma^2 \mathbf{I}_T)$), and $\mathbf{I}_T$ is the $(T \times T)$ identity matrix. The $(K \times 1)$ vector of factor betas $\boldsymbol{\beta}$, α, and σ^2 are unknown parameters.

Suppose further that the observed $\mathbf{X}$ contains errors of measurement:

$$\mathbf{X} = \mathbf{X}^* + \mathbf{u}, \tag{21}$$

where $\mathbf{u}$ is a $(T \times K)$ matrix of errors in variables, $\mathbf{u} \sim N\left(\mathbf{0}, v^2 \mathbf{I}_T\right)$.

Assume also that $\mathbf{u}$, $\boldsymbol{\varepsilon}$, and $\mathbf{X}^*$ are mutually independent and that

$$var\left[vec\left(\mathbf{u}\right)\right] = \Omega \otimes \mathbf{I}_T, \tag{22}$$

where Ω is a $(K \times K)$ symmetric positive definite matrix, the notation $\otimes$ denotes the Kronecker product of the two matrices, and var designates the variance-covariance matrix. The last assumption implies that the measurement errors $\mathbf{u}$ are independent between observations, but not between variables, as well as that for a given variable the measurement errors are homoskedastic.

DD (1997) suggest new consistent higher moment (based on the third and higher sample moments) estimators of $\theta = \left(\alpha, \boldsymbol{\beta}'\right)'$, which are instrumental variable estimators with the matrix of instrumental variables $\mathbf{H}_1$ defined as

$$\mathbf{H}_1 = (\boldsymbol{\iota}_T, \mathbf{h}_1, ..., \mathbf{h}_7) \tag{23}$$

with $\mathbf{h}_1 = \mathbf{x} \odot \mathbf{x}$, $\mathbf{h}_2 = \mathbf{x} \odot \mathbf{y}$, $\mathbf{h}_3 = \mathbf{y} \odot \mathbf{y}$, $\mathbf{h}_4 = \mathbf{x} \odot \mathbf{x} \odot \mathbf{x} - 3\mathbf{x}\left(E\left[\mathbf{x}'\mathbf{x}/T\right] \odot \mathbf{I}_K\right)$, $\mathbf{h}_5 = \mathbf{x} \odot \mathbf{x} \odot \mathbf{y} - 2\mathbf{x}\left(E\left[\mathbf{x}'\mathbf{y}/T\right] \odot \mathbf{I}_K\right) - \mathbf{y}\left\{\boldsymbol{\iota}_K'\left(E\left[\mathbf{x}'\mathbf{x}/T\right] \odot \mathbf{I}_K\right)\right\}$, $\mathbf{h}_6 = \mathbf{x} \odot \mathbf{y} \odot \mathbf{y} - \mathbf{x}\left(E\left[\mathbf{y}'\mathbf{y}/T\right]\right) - 2\mathbf{y}\left(E\left[\mathbf{y}'\mathbf{x}/T\right]\right)$, and $\mathbf{h}_7 = \mathbf{y} \odot \mathbf{y} \odot \mathbf{y} - 3\mathbf{y}\left(E\left[\mathbf{y}'\mathbf{y}/T\right]\right)$, where the notation $\odot$ denotes the Hadamard (element-by-element) product and $\mathbf{x}$ and $\mathbf{y}$ are, respectively, $\mathbf{X}$ and $\mathbf{Y}$ expressed in mean deviation form.[22]

DD (1997) show that their instrumental variable estimators for linear regression models with errors in the explanatory variables outperform conventional OLS estimators in many respects. They find that in small samples the higher moment estimators have smaller biases than the alternative OLS estimator. Contrary to the conventional OLS estimator, in the presence of errors in the variables the size of the bias of the higher moment estimators is not much affected by the collinearity among the independent variables in small samples.

The result in DD (1997) is that an instrumental variable regression estimator θ^* of $\theta = \left(\alpha, \boldsymbol{\beta}'\right)'$ with the matrix of instrumental variables $\mathbf{H}_2 = (\boldsymbol{\iota}_T, \mathbf{h}_1, \mathbf{h}_4)$ outperforms other higher moment estimators in terms of root mean squared errors. In our empirical investigation we assume that the joint density function of the errors in the explanatory variables is unknown but symmetric and, as suggested by DD (1997), remove $\mathbf{h}_4$ from the definition of $\mathbf{H}_2$ to preserve the consistency of θ^*. We denote the resulting matrix of instrumental variables as $\mathbf{H}_2 = (\boldsymbol{\iota}_T, \mathbf{h}_1)$.

The GMM estimation technique allows to implement easily the instrumental variable approach suggested by DD (1997) in order to avoid the EIV problem in the first-step estimation of the model with the explanatory variables being the five CRR (1986) macroeconomic variables, the aggregate per capita consumption growth rate, and the rates of change in the second through fourth normalized consumption moments. Since the five CRR (1986) macroeconomic variables and the growth rate of average consumption are assumed to be measured properly, they serve as their own instruments in $\mathbf{H}_3 = (\boldsymbol{\iota}_T, \mathbf{h}_1)$, while the percentage changes in the normalized consumption moments of order two to four are instrumented as suggested by DD (1997).

Our analysis shows that the use of the set of instruments $\mathbf{H}_3 = (\boldsymbol{\iota}_T, \mathbf{h}_1)$ allows to overcome the multicollinearity problem underlined in Section 3.1. Table III below reports the summary statistics for the instruments for the consumption moment risk factors, when, following DD (1997), the average consumption growth rate serves as its own instrument and the rates of change in the normalized consumption moments of order two to four are instrumented by the squares of their deviations from the mean. The coefficients of correlation of the instrumental variables among themselves and with the CRR (1986) economic state variables are much lower in absolute value and less significant statistically compared with their counterparts obtained for the original variables.

[Table III about here]

[22] A condition for higher moment estimators to be consistent estimators of $\theta = (\alpha, \boldsymbol{\beta}')'$ is that the joint distribution of the variables in $\mathbf{X}^*$ is not normal.

The coefficients of autocorrelation in the instrumental variables for the rates of change in the second through fourth normalized consumption moments, reported in Table IV, are also less significant compared with those for the factors themselves. The observed autocorrelation coefficients show that the instruments do not deviate significantly from the innovations what makes the use of them as factors in the first-step regression consistent with the theory.

[Table IV about here]

Because all the factor betas are estimated from the data (and, therefore, are assumed to be measured with error), to deal with the EIV problem in the second-step CSRs regression we also use the GMM estimation approach with the matrix of instruments $\mathbf{H}_3 = (\boldsymbol{\iota}_T, \mathbf{h}_1)$, in which all regressors (betas) are instrumented as described in DD (1997).

Following the findings in Section 2.2, in each CSR we estimate the risk premium parameters γ_6, γ_7, γ_8, and γ_9 subject to inequality constraints imposed by the preference theory. Specifically, we restrict the estimates of the risk premia γ_6 and γ_8 to be non-negative and constrain the estimates of γ_7 and γ_9 to be non-positive. Given time series of the CSR estimates for each risk-return parameter, we test statistically whether the time-series average of each risk-return parameter is significantly different from zero at a given significance level. We do this by employing the t-test based on the usual Fama-MacBeth standard error of the quarterly time-series mean of the risk-return parameter estimate. The test statistic is Student t distributed with $(T-1)$ degrees of freedom. Statistical inferences about the significance of the time-series average of each risk-return parameter are made in the usual fashion.[23]

3.3 Estimation Results

Since our sample covers the period beginning in 1982:Q1 and the data for the first 40 quarters are used to estimate the factor betas employed in the first CSR, for each risk-return parameter we have a quarterly series of estimates for the period that starts in 1992:Q1 and ends in 2003:Q4. The time-series averages of risk-return parameters and the test statistics are reported in Table V. The results are displayed separately for the unrestricted (14) and restricted (17) models.

[Table V about here]

[23] An alternative approach to control the second-step EIV problem is to estimate CSRs by means of the usual OLS technique and then to use the t-test based on the EIV-adjusted standard error of the time-series average of the risk-return parameter, as suggested in Shanken (1992) and SW (2006). The problem with using this approach in our case is that the EIV-adjusted standard error of the time-series mean of the parameter estimate obtained from constrained CSRs (as explained above, we restrict the signs of the risk premia γ_6, γ_7, γ_8, and γ_9) frequently is not well defined, which makes this technique of limited use for practical purposes. Due to this drawback of the approach proposed by Shanken (1992) and SW (2006), we decided to use in our empirical analysis the instrumental variable technique, the applicability of which is not restricted to whether the constrained or unconstrained estimation is performed.

3.3.1 Unrestricted Model

The signs of the unrestricted risk premium estimates for DEI, UI, and UPR are sensitive to the choice of the threshold value in the definition of asset holders. Whereas the theory provides no strong guidance in signing the DEI and UI risk premia, it is usually presumed that UPR has a positive price of risk. However, the UPR risk premium is estimated to be significantly negative for the sets of households with the reported amount of asset holdings equal to or exceeding \$1000 and \$10000. The estimated risk premium for UTS (when significantly different from zero) has the wrong sign. Variable MP has a positive risk premium consistent with the results in CRR (1986) and SW (2006). The sign of the unrestricted estimate of the excess zero-beta rate is sensitive to the choice of portfolio and the set of households considered in estimation. In some cases, the excess zero-beta rate is estimated to be significantly negative, what contradicts the theory. There is strong evidence that the aggregate consumption and background risks are both significantly priced. The ACG, SMG, TMG, and FMG price estimates are all statistically significant. This result is robust to the portfolio of stocks and the subset of households defined as asset holders.

3.3.2 Restricted Model

Imposing the bond pricing restrictions (15) and (16) yields risk premium estimates with plausible signs and, as expected, increases the precision of the estimates of the risk-return parameters. In contrast with the unrestricted estimates of the excess zero-beta rate, the corresponding restricted estimates are always positive when significantly different from zero. The restricted estimates of the MP risk premium are frequently negative, but, in most cases, not significantly different from zero. The price of risk estimates for DEI are sensitive to the threshold value in the definition of asset holders. It is significantly positive for the set of households that report total assets equal to or exceeding \$1000 then becoming insignificant for the set of households that report an estimated market value of all securities held a year ago equal to or greater than \$5000 and, finally, getting significantly negative for the set of households with the reported amount of asset holdings equal to or exceeding \$10000. All the (significantly different from zero) restricted price estimates for UI are positive. As expected, the bond return premium UPR is positive and highly significant. The t-test for UTS fails to produce any evidence that this factor is separately rewarded in the stock market. Like their unrestricted counterparts, the risk premium estimates for the rates of change in average consumption and the second through fourth normalized consumption moments are all significantly different from zero.

Exploiting the set of instrumental variables $\mathbf{H}_3 = (\boldsymbol{\iota}_T, \mathbf{h}_1)$ leaves one overidentifying restriction that is used to test the model's validity. We observe that the time-series of Hansen's $J(1)$-statistics and their corresponding p-values exhibit statistically significant positive autocorrelation, which makes it impossible to use Fisher's method to test the joint hypothesis that the $J(1)$-statistics for all the quarters equal zero. Since there is positive dependence among statistical tests for different quarters, the null distribution of Fisher's test statistic is different from a $\chi^2(2T)$ and the evidence for the alternative hypothesis (that the $J(1)$-statistic for at least one quarter is statistically significant) is generally overstated. Because of this, when testing the quality of the restricted model, in Table V we simply report the number of quarters (along with relative frequency (number in brackets) computed over the entire sample period (48 quarters)) for which the null hypothesis that the restricted model is a valid representation of portfolio expected excess returns is not rejected statistically at the 5% and 1% significance levels according to Hansen's $J(1)$-statistic. These results show that, regardless of the portfolio used and the threshold value in the definition of asset holders, in most cases the restricted model is not rejected at the 5% level and is rejected at the 1% level of significance for only a very small number of quarters. This provides strong support for the restricted model.

4 Concluding Remarks

In this paper we examine the multifactor asset pricing model in which the risk factors are the CRR (1986) five economic state variables and the rates of change in the first four cross-sectional moments of the consumption distribution. This model nests the CRR (1986) five-factor model and the consumption-based asset pricing model as special cases. Using a similar (to that in SW (2006)) method of generating portfolio returns we obtain that the null hypotheses of the CRR (1986) five-factor model and the consumption-based asset pricing model are both rejected empirically. The CRR (1986) macroeconomic variables and the consumption moment risk factors are found to be significantly priced.

The conclusion about the five macroeconomic factors studied by CRR (1986) is sensitive to the portfolio used and the threshold value in the definition of asset holders. The empirical evidence from the restricted asset-pricing model is that the macroeconomic factors UI and UPR both have statistically significant influence on the cross-sectional variation in long-run stock returns. There is no evidence that UTS is separately rewarded in the stock market. The evidence for MP and DEI is less consistent across portfolios and sets of asset holders. These results are quite different from the results in CRR (1986) and SW (2006). It, however, should be kept in mind that we use different (from those in CRR (1986) and SW (2006)) data frequency and sample period.

In contrast with the result in CRR (1986), we find strong evidence that the change in aggregate per capita consumption of asset holders (the first moment of the consumption distribution) is a priced risk factor. The discrepancy in the results should be attributed, in our opinion, to not only the differences in the method of generating portfolio returns, data frequency, or sample period, but rather to the facts that (i) under the assumption of limited capital market participation we consider the consumption of asset holders only and not the total consumption in the economy as in CRR (1986) and (ii) in the CSRs for each quarter we constrain the risk premium for the rate of change in average consumption to be non-negative (as suggested by the theory), while the sign of the risk premium for this factor is not constrained in CRR (1986). Imposing the theoretical restriction implied by the preference assumptions increases the power of tests and hence the reliability of the inferences drawn from the observed data.

The preference theory also enables us to sign the risk premia for the rates of change in the second through fourth normalized consumption moments. When investigating the influence on stock returns of these risk factors, we find strong evidence of their significant pricing power. The experiment design plays no role in the conclusion that with the limited participation of households in the capital markets the aggregate consumption risk (measured by the rate of change in aggregate per capita consumption) and the background risk (measured jointly by the rates of change in the normalized consumption moments of order two to four) have an incremental power in explaining the cross-sectional variation in long-run (quarterly) stock returns.

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Table I
Summary Statistics for the Risk Factors

Factor	Mean	SD	Correlation Coefficients							
			MP	DEI	UI	UPR	UTS	ACG	SMG	TMG
			1. CRR (1986) Economic State Variables							
MP	0.0073	0.0119								
DEI	-0.0003	0.0071	$0.214^{\ddagger}$							
UI	0.0012	0.0084	$0.195^{\ddagger}$	$0.754^{\dagger}$						
UPR	0.0104	0.0043	$-0.216^{\ddagger}$	-0.064	0.114					
UTS	0.0170	0.0106	0.128	0.091	$0.308^{\dagger}$	0.083				
			2. Moments of the Cross-Sectional Consumption Distribution							
			Households with Total Assets $\geqslant$ \$1000							
ACG	0.0053	0.0414	0.019	-0.024	-0.112	-0.044	0.014			
SMG	0.0151	0.1423	0.094	0.029	0.033	$-0.224^{\ddagger}$	0.021	$0.288^{\dagger}$		
TMG	0.1725	0.6022	0.105	0.062	0.019	$-0.213^{\ddagger}$	0.141	0.141	$0.859^{\dagger}$	
FMG	0.1600	0.6121	0.129	0.018	-0.026	$-0.205^{\ddagger}$	0.102	$0.258^{\dagger}$	$0.807^{\dagger}$	$0.906^{\dagger}$
			Households with Total Assets $\geqslant$ \$5000							
ACG	0.0054	0.0456	0.006	-0.013	-0.082	-0.013	-0.003			
SMG	0.0270	0.1862	0.102	0.021	0.074	$-0.196^{\ddagger}$	0.056	$0.235^{\dagger}$		
TMG	0.3464	1.1235	0.098	-0.078	0.012	-0.067	0.130	0.110	$0.822^{\dagger}$	
FMG	0.2694	0.8537	0.096	-0.021	0.024	-0.103	0.105	$0.210^{\ddagger}$	$0.882^{\dagger}$	$0.902^{\dagger}$
			Households with Total Assets $\geqslant$ \$10000							
ACG	0.0082	0.0502	-0.037	-0.017	-0.099	0.024	0.011			
SMG	0.0289	0.1835	0.089	0.009	0.003	-0.095	0.036	0.183		
TMG	0.2888	0.8308	0.174	0.083	-0.003	-0.085	0.168	0.034	$0.821^{\dagger}$	
FMG	0.2638	0.9120	0.153	0.030	-0.055	-0.056	0.102	0.158	$0.780^{\dagger}$	$0.873^{\dagger}$

Note.- MP = quarterly growth rate of industrial production, DEI = change in expected inflation, UI = unexpected inflation, UPR = excess return of low-grade corporate bonds over high-grade corporate bonds (the default premium), UTS = unanticipated return on long bonds (the term premium), ACG = rate of change in average consumption, SMG = rate of change in the second normalized consumption moment, TMG = rate of change in the third normalized consumption moment, and FMG = rate of change in the fourth normalized consumption moment. Estimates marked with † and ‡ are statistically different from zero at the 5% and 10% level of significance, respectively.

Table II
Autocorrelation in the Risk Factors

Factor	$\widehat{\rho}_1$	$\widehat{\rho}_2$	$\widehat{\rho}_3$	$\widehat{\rho}_4$	$\widehat{\rho}_5$	$\widehat{Q}_5$	$\widehat{Q}_{10}$
			1. CRR (1986) Economic State Variables				
MP	0.3918†	0.1901	0.1849	0.0564	-0.0549	17.86*	19.60†
DEI	-0.3887†	-0.1187	0.2075‡	-0.1858	0.0427	19.07*	23.71*
UI	0.0832	-0.0014	0.1655	-0.1889	-0.0555	5.82	10.33
UPR	0.9101†	0.8291†	0.7689†	0.7271†	0.7404†	251.51*	444.64*
UTS	0.9208†	0.7835†	0.6231†	0.4447†	0.2643†	165.60*	175.15*
			2. Moments of the Cross-Sectional Consumption Distribution				
			Households with Total Assets $\geqslant$ \$1000				
ACG	0.0712	0.0392	-0.0803	-0.1515	0.1981‡	6.06	15.86
SMG	-0.1176	0.2015‡	-0.1265	0.0248	-0.2423†	10.34	11.69
TMG	0.0770	0.0469	-0.0963	0.1003	-0.2170‡	6.01	7.35
FMG	0.0869	-0.0096	-0.0256	0.0515	-0.1289	2.21	2.48
			Households with Total Assets $\geqslant$ \$5000				
ACG	0.1450	-0.0250	-0.1282	-0.1609	0.1946‡	8.14	20.45†
SMG	-0.0799	0.1023	-0.1711	-0.0285	-0.1843	6.46	9.37
TMG	-0.0787	0.0606	-0.0930	0.0335	-0.0935	2.25	3.42
FMG	-0.0584	-0.0050	-0.0510	0.0241	-0.1217	1.72	4.10
			Households with Total Assets $>$ \$10000				
ACG	0.1104	-0.0290	-0.0372	-0.2368†	0.1924‡	8.63	18.36†
SMG	-0.1148	0.0316	-0.0697	0.0539	-0.1484	3.51	5.23
TMG	0.0784	-0.1102	0.0024	0.0323	-0.1682	3.81	8.43
FMG	0.0327	-0.0974	0.0120	-0.0310	-0.0795	1.43	2.42

Note.- MP = quarterly growth rate of industrial production, DEI = change in expected inflation, UI = unexpected inflation, UPR = excess return of low-grade corporate bonds over high-grade corporate bonds (the default premium), UTS = unanticipated return on long bonds (the term premium), ACG = rate of change in average consumption, SMG = rate of change in the second normalized consumption moment, TMG = rate of change in the third normalized consumption moment, and FMG = rate of change in the fourth normalized consumption moment. ρ_i is the autocorrelation coefficient of order i. Q_m is the Ljung-Box (1978) Q-statistic with m autocorrelations. t-statistics are in parentheses. Estimates of the autocorrelation coefficients marked with † and ‡ are statistically different from zero at the 5% and 10% levels of significance, respectively. Estimates of Q_m statistically significant at the 1% and 5% levels are marked, respectively, with * and †.

Table III
Summary Statistics for the Instrumental Variables

Factor	Mean	SD	Correlation Coefficients							
			MP	DEI	UI	UPR	UTS	ACG	SMG$^+$	TMG$^+$
					Households with Total Assets $\geqslant$ \$1000					
ACG	0.0053	0.0414	0.019	-0.024	-0.112	-0.044	0.014			
SMG$^+$	0.0200	0.0244	0.029	0.019	-0.066	0.058	0.055	0.218‡		
TMG$^+$	0.3584	0.5941	0.052	0.049	-0.037	-0.056	0.091	0.135	0.662†	
FMG$^+$	0.3702	1.5576	0.075	-0.035	-0.082	-0.072	0.059	0.205‡	0.483†	0.844†
					Households with Total Assets $\geqslant$ \$5000					
ACG	0.0054	0.0456	0.006	-0.013	-0.082	-0.013	-0.003			
SMG$^+$	0.0342	0.0604	0.006	-0.020	0.054	0.146	0.071	0.169		
TMG$^+$	1.2472	5.0502	-0.037	-0.196‡	-0.032	0.059	0.031	0.071	0.724†	
FMG$^+$	0.7203	2.2780	-0.021	-0.069	0.025	0.021	0.030	0.171	0.890†	0.730†
					Households with Total Assets $\geqslant$ \$10000					
ACG	0.0082	0.0502	-0.037	-0.017	-0.099	0.024	0.011			
SMG$^+$	0.0333	0.0544	0.215‡	0.153	0.085	0.156	0.128	0.118		
TMG$^+$	0.6881	1.4835	0.162	0.048	-0.037	-0.008	0.076	0.087	0.708†	
FMG$^+$	0.8218	4.2242	0.101	-0.029	-0.086	-0.057	0.068	0.143	0.608†	0.862†

Note.- MP = quarterly growth rate of industrial production, DEI = change in expected inflation, UI = unexpected inflation, UPR = excess return of low-grade corporate bonds over high-grade corporate bonds (the default premium), UTS = unanticipated return on long bonds (the term premium), and ACG = rate of change in average consumption. SMG$^+$, TMG$^+$, and FMG$^+$ are, respectively, variables SMG = rate of change in the second normalized consumption moment, TMG = rate of change in the third normalized consumption moment, and FMG = rate of change in the fourth normalized consumption moment instrumented by $x \odot x$, where x is variable X expressed in mean deviation form. Estimates marked with † and ‡ are statistically different from zero at the 5% and 10% level of significance, respectively.

Table IV
Autocorrelation in the Instrumental Variables

Factor	$\widehat{\rho}_1$	$\widehat{\rho}_2$	$\widehat{\rho}_3$	$\widehat{\rho}_4$	$\widehat{\rho}_5$	$\widehat{Q}_5$	$\widehat{Q}_{10}$
				Households with Total Assets $\geqslant$ \$1000			
ACG	0.0712	0.0392	-0.0803	-0.1515	$0.1981^{\ddagger}$	6.06	15.86
SMG$^+$	0.1322	0.1399	0.0336	-0.0761	0.0559	3.69	4.21
TMG$^+$	0.1332	-0.0237	-0.1021	-0.0419	-0.0903	3.04	5.32
FMG$^+$	0.0041	-0.0080	-0.0356	-0.0402	-0.0372	0.35	0.70
				Households with Total Assets $\geqslant$ \$5000			
ACG	0.1450	-0.0250	-0.1282	-0.1609	$0.1946^{\ddagger}$	8.14	$20.45^{\dagger}$
SMG$^+$	0.0692	0.1263	$0.2473^{\dagger}$	0.1404	0.0302	8.12	10.35
TMG$^+$	-0.0262	0.0296	-0.0049	0.0465	-0.0372	0.41	0.75
FMG$^+$	-0.0449	0.0084	0.0100	0.0271	-0.0678	0.60	1.81
				Households with Total Assets $>$ \$10000			
ACG	0.1104	-0.0290	-0.0372	$-0.2368^{\dagger}$	$0.1924^{\ddagger}$	8.63	$18.36^{\dagger}$
SMG$^+$	-0.0061	0.1010	0.0622	0.0311	0.0344	1.28	2.88
TMG$^+$	-0.0322	-0.0335	-0.0607	0.0006	-0.0408	0.60	6.70
FMG$^+$	-0.0183	-0.0153	-0.0236	-0.0186	-0.0264	0.17	0.44

Note.- MP = quarterly growth rate of industrial production, DEI = change in expected inflation, UI = unexpected inflation, UPR = excess return of low-grade corporate bonds over high-grade corporate bonds (the default premium), UTS = unanticipated return on long bonds (the term premium), and ACG = rate of change in average consumption. SMG$^+$, TMG$^+$, and FMG$^+$ are, respectively, variables SMG = rate of change in the second normalized consumption moment, TMG = rate of change in the third normalized consumption moment, and FMG = rate of change in the fourth normalized consumption moment instrumented by $x \odot x$, where x is variable X expressed in mean deviation form. ρ_i is the autocorrelation coefficient of order i. Q_m is the Ljung-Box (1978) Q-statistic with m autocorrelations. t-statistics are in parentheses. Estimates of the autocorrelation coefficients marked with † and ‡ are statistically different from zero at the 5% and 10% levels of significance, respectively. Estimates of Q_m statistically significant at the 1% and 5% levels are marked, respectively, with * and †.

Table V
Estimates of the Factor Risk Premia
A. Portfolios Formed on Size and Book-to-Market

Constant	MP	DEI	UI	UPR	UTS	ACG	SMG	TMG	FMG
1. Unrestricted Model									
Households with Total Assets $\geqslant$ \$1000									
0.0072	-0.0086	0.0435†	0.0275†	0.0028	0.0074	0.0691†	-0.0442†	0.0710†	-0.3781†
(0.333)	(-1.353)	(4.204)	(3.548)	(1.209)	(1.258)	(3.110)	(-3.378)	(2.670)	(-3.726)
Households with Total Assets $\geqslant$ \$5000									
-0.0540‡	-0.0083	-0.0108‡	-0.0079	0.0001	0.0043	0.0499†	-0.1264†	0.1613†	-1.1583†
(-1.764)	(-1.047)	(-1.703)	(-1.061)	(0.020)	(0.520)	(3.729)	(-4.463)	(2.859)	(-2.922)
Households with Total Assets $>$ \$10000									
-0.0045	0.0053	0.0119	0.0048	0.0000	0.0144‡	0.0249†	-0.0785†	0.3298†	-0.4204†
(-0.174)	(0.688)	(1.478)	(0.716)	(-0.023)	(1.701)	(2.641)	(-3.703)	(2.195)	(-3.143)
2. Restricted Model									
Households with Total Assets $\geqslant$ \$1000									
5% - 37 [0.77], 1% - 46 [0.96]									
0.0132	-0.0339†	0.0394†	0.0220†	0.0080†	0.0065	0.0608†	-0.0382†	0.0528	-0.4618†
(1.286)	(-3.297)	(3.055)	(2.224)	(25.98)	(0.092)	(3.531)	(-3.085)	(1.824)	(-3.794)
Households with Total Assets $\geqslant$ \$5000									
5% - 42 [0.88], 1% - 47 [0.98]									
0.0148†	-0.0213†	-0.0152	0.0032	0.0080†	0.0048	0.0312†	-0.2067†	0.5943†	-1.7270†
(2.060)	(-2.265)	(-1.560)	(0.305)	(25.98)	(0.097)	(3.050)	(-4.163)	(2.880)	(-3.684)
Households with Total Assets $>$ \$10000									
5% - 40 [0.83], 1% - 47 [0.98]									
0.0088	-0.0055	0.0193	0.0287†	0.0080†	0.0108	0.0812†	-0.0779†	0.4903†	-1.2303†
(0.662)	(-0.612)	(1.359)	(2.719)	(25.98)	(0.117)	(3.213)	(-3.163)	(2.224)	(-3.869)

Note.- MP = quarterly growth rate of industrial production, DEI = change in expected inflation, UI = unexpected inflation, UPR = excess return of low-grade corporate bonds over high-grade corporate bonds (the default premium), UTS = unanticipated return on long bonds (the term premium), ACG = rate of change in average consumption, SMG = rate of change in the second normalized consumption moment, TMG = rate of change in the third normalized consumption moment, and FMG = rate of change in the fourth normalized consumption moment. Estimates marked with † and ‡ are statistically different from zero at the 5% and 10% level of significance, respectively. The sample means of the quarterly time series of CSR estimates of the gammas are reported in the main rows. Estimates marked with † and ‡ are statistically different from zero at the 5% and 10% level of significance, respectively. t-statistics are in parentheses. For the restricted model, immediately below the row with the threshold value in the definition of asset holders, it is shown the number of quarters (with relative frequency over the entire estimation period (48 quarters) reported in brackets) for which the model is not rejected statistically according to Hansen's $J(1)$-statistic at the 5% and 1% levels of significance.

Table V (continued)
B. Portfolios Formed on Size and Momentum

Constant	MP	DEI	UI	UPR	UTS	ACG	SMG	TMG	FMG
1. Unrestricted Model									
Households with Total Assets $\geqslant$ \$1000									
0.0767‡	0.0204†	-0.0027	-0.0002	-0.0017	-0.0026	0.0529†	-0.0320†	0.1359†	-0.6461†
(1.755)	(2.480)	(-0.253)	(-0.014)	(-1.107)	(-0.299)	(3.928)	(-2.854)	(2.527)	(-3.664)
Households with Total Assets $\geqslant$ \$5000									
0.0584†	0.0217†	0.0022	0.0206†	-0.0014	0.0032	0.1001†	-0.0483†	0.6191†	-1.6208†
(2.211)	(3.154)	(0.270)	(2.117)	(-0.929)	(0.264)	(3.808)	(-3.332)	(2.395)	(-2.888)
Households with Total Assets > \$10000									
-0.0953†	0.0069	-0.0275†	-0.0245†	-0.0045†	-0.0035	0.1275†	-0.1135†	0.1433‡	-1.6957†
(-2.065)	(0.896)	(-3.355)	(-2.678)	(-2.901)	(-0.386)	(3.835)	(-3.303)	(1.834)	(-2.976)
2. Restricted Model									
Households with Total Assets $\geqslant$ \$1000									
5% - 42 [0.88], 1% - 47 [0.98]									
0.0018	-0.0001	-0.0138	0.0053	0.0080†	0.0179	0.0549†	-0.0633†	0.2009†	-0.8105†
(0.116)	(-0.008)	(-1.615)	(0.609)	(25.98)	(0.169)	(3.670)	(-2.936)	(2.749)	(-4.975)
Households with Total Assets $\geqslant$ \$5000									
5% - 32 [0.67], 1% - 47 [0.98]									
0.0321†	-0.0048	-0.0096	0.0163†	0.0080†	-0.0125	0.1624†	-0.0740†	0.4447†	-2.0473†
(2.879)	(-0.472)	(-1.641)	(2.278)	(25.98)	(-0.161)	(3.651)	(-3.001)	(2.835)	(-4.447)
Households with Total Assets > \$10000									
5% - 37 [0.77], 1% - 48 [1.00]									
0.0223†	-0.0250‡	-0.0157‡	0.0227†	0.0080†	-0.0026	0.1466†	-0.0463†	0.5087†	-2.7214†
(2.541)	(-1.701)	(-1.935)	(2.759)	(25.98)	(-0.044)	(4.526)	(-2.536)	(2.258)	(-5.286)

Table V (continued)
C. Portfolios Formed on Size and Short-Term Reversal

Constant	MP	DEI	UI	UPR	UTS	ACG	SMG	TMG	FMG
				1. Unrestricted Model					
				Households with Total Assets $\geqslant$ \$1000					
0.0206	-0.0028	0.0054	0.0022	-0.0008	0.0167†	0.0322†	-0.0394†	0.2522†	-0.8440†
(0.681)	(-0.431)	(0.828)	(0.410)	(-0.356)	(2.251)	(3.481)	(-3.416)	(2.522)	(-3.737)
				Households with Total Assets $\geqslant$ \$5000					
0.0159	-0.0050	-0.0069	-0.0009	0.0034‡	-0.0120	0.0589†	-0.0695†	0.2992†	-0.6502†
(0.577)	(-1.049)	(-0.578)	(-0.150)	(1.979)	(-0.847)	(2.911)	(-2.715)	(2.858)	(-3.266)
				Households with Total Assets $>$ \$10000					
0.0322	-0.0048	0.0137	0.0005	-0.0010	0.0244‡	0.1708†	-0.1707†	0.1276†	-0.7467†
(0.835)	(-0.636)	(1.172)	(0.078)	(-0.320)	(1.894)	(2.745)	(-2.627)	(2.135)	(-3.467)
				2. Restricted Model					
				Households with Total Assets $\geqslant$ \$1000					
				5% - 36 [0.75], 1% - 44 [0.92]					
0.0045	0.0013	-0.0008	-0.0020	0.0080†	0.0152	0.0397†	-0.0557†	0.0919‡	-0.2133†
(0.479)	(0.152)	(-0.115)	(-0.302)	(25.98)	(0.234)	(3.933)	(-3.434)	(1.829)	(-3.111)
				Households with Total Assets $\geqslant$ \$5000					
				5% - 41 [0.85], 1% - 47 [0.98]					
0.0188‡	-0.0054	-0.0063	0.0033	0.0080†	0.0008	0.0890†	-0.0710†	0.6658†	-0.7732†
(1.979)	(-0.620)	(-0.768)	(0.414)	(25.98)	(0.013)	(3.512)	(-2.693)	(3.046)	(-3.638)
				Households with Total Assets $>$ \$10000					
				5% - 35 [0.73], 1% - 48 [1.00]					
0.0293†	-0.0042	-0.0114‡	0.0086	0.0080†	-0.0096	0.1056†	-0.0290†	0.0850†	-1.1172†
(2.229)	(-0.426)	(-1.788)	(1.179)	(25.98)	(-0.106)	(3.665)	(-2.970)	(2.475)	(-4.312)

Table V (continued)

D. Portfolios Formed on Size and Long-Term Reversal

Constant	MP	DEI	UI	UPR	UTS	ACG	SMG	TMG	FMG
1. Unrestricted Model									
Households with Total Assets $\geqslant$ \$1000									
0.1075†	0.0267‡	0.0184‡	0.0191	-0.0054†	0.0236	0.0935†	-0.0494†	0.6975†	-1.6871†
(3.833)	(1.895)	(1.874)	(1.487)	(-2.597)	(1.538)	(3.027)	(-4.194)	(2.739)	(-3.486)
Households with Total Assets $\geqslant$ \$5000									
0.0232	-0.0021	-0.0018	0.0075	-0.0008	0.0113	0.1618†	-0.0376†	0.4004†	-1.3954†
(1.171)	(-0.186)	(-0.233)	(0.727)	(-0.419)	(0.686)	(3.187)	(-2.305)	(3.964)	(-3.395)
Households with Total Assets $>$ \$10000									
0.0522†	0.0158†	0.0058	0.0098	-0.0049†	0.0131	0.0570†	-0.1316†	0.6065‡	-0.9000†
(2.085)	(2.062)	(1.354)	(1.324)	(-3.216)	(1.189)	(4.405)	(-4.077)	(1.766)	(-3.342)
2. Restricted Model									
Households with Total Assets $\geqslant$ \$1000									
5% - 33 [0.69], 1% - 46 [0.96]									
-0.0010	-0.0190	0.0162‡	0.0151‡	0.0080†	0.0206	0.0502†	-0.0293†	0.3673†	-0.4970†
(-0.094)	(-1.596)	(1.714)	(1.963)	(25.98)	(0.284)	(2.854)	(-3.200)	(2.332)	(-2.821)
Households with Total Assets $\geqslant$ \$5000									
5% - 40 [0.83], 1% - 48 [1.00]									
-0.0100	0.0116	-0.0079	-0.0024	0.0080†	0.0296	0.0545†	-0.0965†	0.2266†	-0.6253†
(-0.483)	(0.825)	(-1.052)	(-0.302)	(25.98)	(0.207)	(3.402)	(-3.453)	(2.492)	(-4.144)
Households with Total Assets $>$ \$10000									
5% - 36 [0.75], 1% - 47 [0.98]									
0.0308†	0.0050	-0.0054	0.0177†	0.0080†	-0.0111	0.0353†	-0.1129†	0.3099‡	-2.0243†
(2.833)	(0.581)	(-0.858)	(2.914)	(25.98)	(-0.148)	(3.233)	(-3.811)	(1.819)	(-4.161)