Is backdating executive stock options always harmful to shareholders?

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Abstract

Current accounting rules relating to the issuance of employee stock options (ESOs) treat ESOs as a corporate expense. Prior to 2005, accounting rules required corporations that issued ESOs to treat at-the-money options differently from in-the-money options for financial reporting purposes. A corporation that granted in-the-money options was required to expense the intrinsic value of the options, while no expense had to be recognized when at-the-money options were granted. Since 2005, corporations have been required to expense all ESOs at their grant-date fair value.

This paper considers the \textit{ex ante} cost to shareholders when the firm awards backdated stock options to its executives, selecting a date when the stock price is below the award date price, thus granting in-the-money rather than at-the-money options. Using a binomial model, we show that the non-transferability of ESOs and the risk aversion of the manager receiving them make it possible for the firm to grant in-the-money ESOs at a lower cost than at-the-money ESOs without reducing the manager’s utility level. This result is always true when the underlying stock’s expected return is equal to the risk-free rate. Only if the underlying stock’s expected return exceeds a certain spread over the risk-free rate and the manager is made no worse off having been awarded in-the-money rather than at-the-money ESOs are shareholders made worse off. For a given level of the risk-free rate, the threshold expected return at which granting in-the-money rather than at-the-money ESOs cannot simultaneously benefit shareholders and managers is positively related to the manager’s level of risk aversion and is negatively related to the manager’s non-option wealth. In the majority of cases, issuing in-the-money ESOs is welfare-enhancing for both managers and shareholders.

\textbf{Keywords:} Employee stock options; backdating; risk aversion; contingent pricing

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1 Introduction

Awarding employee stock options (ESOs) to employees as part of their compensation has been common practice for a number of years. In the late 1990s, academic studies reported finding abnormal patterns in companies’ stock prices surrounding dates when options were granted. Yermack (1997) showed that ESOs granted by compensation committees with flexible meeting schedules were often followed by positive abnormal returns, suggesting that compensation committees arrange for their meetings to take place prior to the announcement of good news. Aboody and Kasznik (2000) and Chauvin and Shenoy (2001), however, proposed that managers manipulated information to achieve favorable stock returns around ESO grant dates. Subsequently, Lie (2005) offered a much simpler explanation for the too-good-to-be-true returns surrounding ESO grants: backdating. That is, when granting ESOs, an earlier date when the stock price was lower would be selected as the grant date rather than the compensation committee’s meeting date. This hypothesis was supported by Narayanan and Seyhun (2005a) and appears positively correlated with CEO influence (Bebchuk et al., 2010).

Following a series of articles in The Wall Street Journal in 2006, regulators began to investigate the extent to which option backdating was a widespread practice among many large corporations, concluding that such a practice was potentially detrimental to shareholders. That is, issuing in-the-money ESOs necessarily make shareholders worse off by diluting shareholder value to a greater extent than at-the-money options. Using language more akin to the relevant accounting rules, the firm bears a greater expense when issuing in-the-money ESOs relative to issuing at-the-money ESOs. Management is, therefore, apparently enriching employees at the expense of shareholders without reporting it to shareholders. Lawsuits were brought against corporations that were allegedly engaging in backdating, based on the fundamental premise that backdating harmed shareholders.

Although the implementation of the Sarbanes-Oxley Act (SOX) in 2002 made backdating less effective, Narayanan and Seyhun (2005b), Collins, Gong and Li (2005) and Heron and Lie (2007) claim that opportunistic behavior is still observed after the passage of SOX. Narayanan and Seyhun (2008) even suggest forward-dating, i.e., the selection of a grant date following the compensation committee meeting date— as a potential ESO-enhancing strategy. Evidence that firms under investigation for options backdating, as well as firms suspected of such practices, have been punished by the markets through lower stock returns seems to confirm that premise (Narayanan, et al., 2007; Bernile and Jarrell, 2009; Carow, et al., 2009). The observed lower stock return could also be explained by the litigation itself.

These lawsuits largely ignored that, regardless of whether the options had been granted at- or in-the-money, corporations are required to report information sufficient to estimate the potential dilution effect on shareholders of all options granted.
From an economic perspective, a key question emerges: Does backdating indeed make shareholders worse off, or are corporations that engage in backdating acting in the best interest of the shareholders and managers while avoiding accounting rules that incorrectly treat ESOs as corporate expenses\(^4\)? For example, prior to 2006, a corporation that granted at-the-money options was not required to expense the cost of the options, while if it granted in-the-money options, it would have to recognize such an expense on its financial statements\(^5\). In response, corporations may have issued in-the-money options, but backdated the award to avoid the differential accounting treatment. Thus, regardless of the economic value of in-the-money versus at-the-money options, corporations may have chosen not to issue and report in-the-money options if doing so may have created an illusion that their earnings were below those of identical firms that issued at-the-money options.

In this paper, we show that issuing in-the-money options can be Pareto-improving, simultaneously benefiting both shareholders and managers. Taking into account the non-transferability of ESOs (which implies that they must be exercised to realize their value) and the risk aversion of the recipients\(^6\), in the majority of cases, issuing in-the-money ESOs makes shareholders better off than if the managers were granted at-the-money options of equivalent utility value.

As Lambert, Larcker, and Verrecchia (1991) show, the value of an ESO from the point of view of a risk-averse manager is not the same as the option cost to the shareholders of the firm granting it. As a result, changing the strike price of an ESO will not have the same impact on its value from the manager’s viewpoint as it does on its value from the shareholders’ viewpoint. If increasing the moneyness of an ESO has a greater impact on the manager’s valuation than on the shareholders’ valuation, one can replace at-the-money options with fewer in-the-money options in a way that increases the manager’s utility (or at least leaves it unchanged) while at the same time enhancing shareholders’ value. If this is the case, then backdating ESOs—that is, awarding in-the-money rather than at-the-money options—does not necessarily harm shareholders. In short, backdating is a way to implement a Pareto-improving solution that accounting rules had made otherwise problematic for the firm.

We use a binomial model to analyze the value of ESOs from both shareholders’ and the man-

\(^4\)See Calomiris (2005) and Hagopian (2006) for a full discussion regarding the problems related to treating ESOs as a corporate expense.

\(^5\)Since 2006, all ESO awards have been treated as an expense, but are tax-deductible at their intrinsic value.

\(^6\)The assumption of risk aversion precludes their valuation in a risk-neutral framework (Kulatilaka and Marcus, 1994; Huddart, 1994; Rubinstein, 1995).
ager’s perspectives. We find that granting in-the-money rather than at-the-money ESOs can simultaneously increase shareholder value and increase the manager’s expected utility. We show that this outcome is always the case when the expected return on the company’s stock is equal to the risk-free rate of interest. However, if the difference between the expected return on the company’s stock and the risk-free rate exceeds a certain threshold, then backdating is costly to shareholders so long as it does not make the manager worse off. This threshold is negatively related to the manager’s tolerance for risk and to his or her non-option wealth, because a more risk-averse or a less wealthy manager will accept a smaller number of in-the-money ESOs in exchange for at-the-money ESOs.

The paper is structured as follows: The next section explains the model. Section 3 compares in-the-money ESOs with at-the-money ESOs, and Section 4 concludes.

2 Model

We represent time by $t = 1, \ldots, T$ and is measured in years. Consider a call option with time to maturity $T$ and strike price $K$ on a stock with a current price of $S_0$, a dividend yield $\delta$ and a return volatility $\sigma$. The risk-free rate is $r$ and the expected annual return on the stock is $\mu$. The option is granted to a manager with non-option wealth $w_0$ and strictly increasing, strictly concave and continuous utility function over wealth $U(\cdot)$. The option is non-transferable—that is, the manager can monetize its value only by exercising it and his or her decision to do so is based on the expected utility of exercising versus keeping the option. Over the life of the option, the manager’s non-option wealth grows at the risk-free rate $r$. The proceeds from exercising the option before maturity also grow at the risk-free rate following the exercise.

We calculate option values using a binomial tree with time steps represented by $\Delta t \leq 1$. A node in the binomial tree is represented by $(i, j)$, where $i$ is the number of up movements to node $(i, j)$ and $j$ representing time, the number of down movements to node $(i, j)$ is then given by $j/\Delta t - i$. $S_{i,j}$ represents the stock price at node $(i, j)$ and, from each node, the stock price may go up by a factor $u = e^{\sigma \sqrt{\Delta t}}$ or down by a factor $d = e^{-\sigma \sqrt{\Delta t}}$, yielding the risk-neutral probability of an up movement $p_{rn} = \frac{e^{(r-\delta)\Delta t} - d}{u - d}$.
2.1 Manager’s decision to exercise an option

If, at node \((i, T)\), the manager has not yet exercised his or her option, he or she exercises if it is in-the-money and his or her utility corresponds to the utility of the sum of the intrinsic value of the option and non-option wealth—that is, \(U(w_0e^{rT} + \max\{(1 - \tau_m)(S_{i,T} - K), 0\})\)–where \(\tau_m\) corresponds to the manager’s income tax rate. If, at node \((i, T - \Delta t)\), the option has not yet been exercised, then it will be exercised if exercising provides the manager with greater utility than keeping the option. To compute expected utility, we must use the subjective probability of an upward movement represented by \(p_s = \frac{e^{(\mu - \delta)\Delta t} - d}{u - d}\). The manager’s decision at node \((i, T - \Delta t)\), given a stock price \(S_{i,T-\Delta t}\) will be to exercise if \(U(w_0e^{rT} + (1 - \tau_m)(S_{i,T-\Delta t} - K)e^{\tau_m\Delta t}) > p_sU(w_0e^{rT} + \max\{(1 - \tau_m)(S_{i+1,T} - K), 0\}) + (1 - p_s)U(w_0e^{rT} + \max\{(1 - \tau_m)(S_{i,T} - K), 0\})\) and not to exercise otherwise\(^7\). Let \(U_{t,T-\Delta t}^*\) denote the manager’s expected time-\(T\) utility given the utility-maximizing decision at node \((i, T - \Delta t)\). At node \((i, T - 2\Delta t)\), the manager exercises if \(U(w_0e^{rT} + (1 - \tau_m)(S_{i,T-2\Delta t} - K)e^{r(2\Delta t)}) > p_sU_{i+1,T-\Delta t}^* + (1 - p_s)U_{i,T-\Delta t}^*\) and keeps the option otherwise. Proceeding in this manner until we reach time 0, the manager’s decision at any node \((i, j), j < T\), is to exercise if \(U(w_0e^{rT} + (1 - \tau_m)(S_{i,j} - K)e^{r(T-j)}) > p_sU_{i+1,j+\Delta t}^* + (1 - p_s)U_{i,j+\Delta t}^*\), and to keep the option, otherwise.

2.2 Option cost to shareholders

The option is granted to the manager by a firm owned by shareholders who bear the cost of the ESO at the time of exercise through a reduction in the their ownership of the firm determined by the difference between the grant price and the price of the stock at time of exercise. The cost of the option to the firm at node \((i, j)\) corresponds to its expected intrinsic value, given the manager’s decision to exercise at subsequent nodes, discounted at the risk-free rate. At node \((i, T - \Delta t)\), the option cost to shareholders is \(C_{i,T-\Delta t}^e = S_{i,T-\Delta t} - K\) if exercised at that node, and \(C_{i,T-\Delta t}^e = e^{-r\Delta t}(p_m \max\{S_{i+1,T} - K, 0\} + (1 - p_m) \max\{S_{i,T} - K, 0\})\) if retained by the manager. Letting \(V_{i,T-\Delta t}^*\) denote the option cost at node \((i, T - \Delta t)\) given the manager’s decision, the option cost at node \((i, T - 2\Delta t)\) is given by \(C_{i,T-2\Delta t}^e = S_{i,T-2\Delta t} - K\) if exercised and \(C_{i,T-2\Delta t}^e = \)

\(^7\)We will assume that the manager exercises only if his or her time-\(T\) utility of doing so is strictly greater than her expected utility of keeping the option.

\(^8\)It may be useful to point out that \(S_{i+1,T} = uS_{i,T-\Delta t}\) and \(S_{i,T} = dS_{i,T-\Delta t}\).
We obtain the option value by processing as just described until time \( j = 0 \) is reached\(^9\)\(^10\).

### 2.3 Value of the option from the manager’s viewpoint

As in Lambert et al. (1991), the value of an option from the manager’s viewpoint is given by its certainty equivalent—that is, the amount of money the manager would accept in exchange of the option. At node \((i, j)\), the certainty equivalent denoted \( m_{i,j} \) is such that:

\[
\text{Node } (i, j): \quad U \left( w_0 e^{rT} + m_{i,j} e^{r(T-j)} \right) = U^*_{i,j},
\]

which gives

\[
m_{i,j} = \left( U^{-1}(U^*_{i,j}) - w_0 e^{rT} \right) e^{-r(T-j)},
\]

where \( U^{-1} \) is the reciprocal of the manager’s utility function.

#### 2.3.1 Example

Figure 1 provides an example of the value of a typical American call option, denoted \( C \), an executive call option from shareholders’ and the manager’s viewpoints, denoted \( C^e \) and \( m \), respectively, with strike price \( K = 40 \) and time to maturity \( T = 3 \) on a stock with \( S_0 = 40, \mu = 5\% \), \( \delta = 2\% \) and \( \sigma = 30\% \). The risk-free rate is \( r = 5\% \), which can be seen as the return on the market portfolio in the absence of systematic risk, time steps are \( \Delta t = 1 \), the manager’s initial non-option wealth is \( w_0 = 100 \) and her utility function is \( U(w) = w^{1-\gamma}/(1 - \gamma) \) with \( \gamma = 2 \). The tax rate is \( \tau_m = 0\% \) in this example (as shown below, the tax rate has no impact on our results). Figure 1 shows that risk aversion and non-transferability induce the manager to exercise at node \((2,2)\), thus reducing the option cost to the firm compared to a typical American call option. In a risk-neutral setting, the value of the option (conditional on the risk neutral probability of an upward movement being

\(^9\)The tax treatment of incentive stock options (ISOs) differs from that of non-qualified options (NQOs). ISOs are taxed at a favorable rate in the hands of manager when exercised whereas NQOs are taxed at the manager’s personal income tax rate when exercised. Until recently, ISOs were not expensed by firms and thus could not generate tax savings when granted. We abstract from these differences in the present paper.

\(^10\)Under the current accounting rules, ESOs are taxed at fair value on the grant date, which would, in the present model, generate tax savings for the firm when the options are granted, i.e. the cost of an option would be \((1 - \tau_c)C^e\) rather than \(C^e\), \( \tau_c \) being the corporate tax rate. All the results obtained herein still hold when corporate taxes are considered in this manner. Under the accounting rules prevailing prior to 2005, at-the-money options were not taxed in the hands of the firm and thus, if in-the-money options were granted as if they were at-the-money (when backdating), then they would not have any tax consequences either, as is the case in the present model.
\[ p_{rn} = \frac{(e^{0.05-0.02} - e^{-0.3})/(e^{3-0.3})}{0.4756} \] is:

\[
C = 58.38 \times 0.4756^3 e^{-3(0.05)} + 3 \times 13.99 \times 0.4756^2 (1 - 0.4756) e^{-3(0.05)} = 9.69.
\]

From shareholders’ viewpoint, the value of the ESO is given by

\[
C_e = 32.88 \times 0.4756^2 e^{-2(0.05)} + 2 \times 13.99 \times 0.4756^2 (1 - 0.4756) e^{-3(0.05)} = 9.59.
\]

As of time 0, the manager’s expected time-\(T\) utility (conditional on the subjective probability of an upward movement being \(p = (e^{0.05-0.02} - e^{-0.3})/(e^{3-0.3}) = 0.4756)\) is given by

\[
U^* = -0.4756^2 \left(32.88e^{0.05} + 100e^{3(0.05)}\right)^{-1} - 2 \times 0.4756^2 (1 - 0.4756) \left(13.99 + 100e^{3(0.05)}\right)^{-1} - (3 \times 0.4756 (1 - 0.4756)^2 + (1 - 0.4756)^3) \left(100e^{3(0.05)}\right)^{-1}
\]

\[
= -0.00794,
\]

yielding a certainty equivalent \(m = e^{-3(0.05)} (0.00794^{-1} - 100e^{3(0.05)}) = 8.39\) for the ESO.

Given the non-transferability of the option, the risk-averse manager exercises the option prior to maturity. Early exercise yields a cost to shareholders that is lower than what it would be for a typical American call option. When valued from the manager’s viewpoint, the option has an even lower value since it is appreciated through a concave utility function.

### 3 Backdating

Backdating options corresponds to selecting a grant date with a lower stock price than on the true grant date. Abstracting from the change in the maturity of the option, such a practice is equivalent to granting in-the-money options rather than at-the-money options. Backdating appears to have been common practice among publicly traded companies in the late 1990s (Lie, 2005; Narayanan and Seyhun, 2005; Bebchuk, et al., 2010), but does it reduce the value of shareholders’ stake?

Consider a manager being granted an at-the-money option with certainty equivalent \(m_0^a\), a time-\(T\) expected utility of \(U \left((w_0 + m_0^a) e^{r \times T}\right)\). If offered to replace the at-the-money option with \(n\) in-the-money options with certainty equivalent \(m_0^i\), the manager will accept if

\[
U \left((w_0 + nm_0^i) e^{r \times T}\right) \geq U \left((w_0 + m_0^a) e^{r \times T}\right),
\]

i.e., if \(n \geq \frac{m_0^a}{m_0^i}\). Let \(C_0^e,a\) and \(C_0^e,i\) denote the cost to shareholders of granting the at-the-money option and the in-the-money option, respectively. For shareholders to agree to grant \(n\) in-the-money
options instead of one at-the-money option, it is necessary that
\[ nC_{e,i}^c \leq C_{e,a}^c. \]  
(2)

To benefit both the manager and shareholders, the quantity \( n \) of in-the-money options to be granted instead of one at-the-money option must be such that
\[ \frac{m_a^0}{m_i^0} \leq n \leq \frac{C_{e,a}^c}{C_{e,i}^c}. \]  
(3)

If condition (3) is not satisfied, then backdating cannot benefit both parties, meaning that if it benefits the manager, then shareholders are made worse off. We prove the following proposition in the appendix.

**Proposition 1** Backdating can always simultaneously benefit shareholders and the manager when the latter is risk-averse and the expected return on the underlying stock is equal to the risk-free rate.

Let \( C_{e,a}^c \) and \( C_{e,i}^c \) denote shareholders’ cost of granting an at-the-money option and an in-the-money option, respectively, and let \( m_a^0 \) and \( m_i^0 \) denote the manager’s certainty equivalent with the at-the-money option and the in-the-money option, respectively. If the manager’s utility function \( U(\cdot) \) is continuous, strictly increasing and strictly concave and if the expected return on the option’s underlying stock is equal to the risk-free rate (\( \mu = r \)), then, when the time step \( \Delta t \) is sufficiently small, the inequality:
\[ \frac{m_a^0}{m_i^0} < \frac{C_{e,a}^c}{C_{e,i}^c} \]
always holds.

Proposition 1 states that awarding in-the-money options rather than at-the-money options can always be done in a manner that benefits both shareholders and the manager when \( \mu = r \), regardless of the strike price of the in-the-money option and regardless of the manager’s tax rate. This result holds when the expected return on the stock is equal to the risk-free rate, the manager attaches the same probability of reaching exercise nodes as shareholders, but the manager’s concave utility function generates a certainty equivalent that is smaller and more sensitive to changes in the strike price than the option cost to shareholders. The larger impact a reduction in the strike price has on the manager’s certainty-equivalent value than on shareholders’ cost makes it then possible to
replace at-the-money options with a sufficiently small number of in-the-money options to reduce shareholders’ cost.

**Proposition 2** The likelihood that a backdating arrangement benefits both shareholders and the manager vanishes when $\mu$ is sufficiently large relative to $r$.

This proposition uses the same notation as in Proposition 1. If $U(\cdot)$ is continuous, strictly increasing and strictly concave and if $\Delta t$ is sufficiently small, then there exists a $\mu^* > r$ such that

\[
\frac{m_0^a}{m_0} \geq \frac{C_{e,a}^{0}}{C_{e}^{0}} \quad \text{for all} \quad \mu \geq \mu^* \\
\frac{m_0^a}{m_0} < \frac{C_{e,a}^{0}}{C_{e}^{0}} \quad \text{for all} \quad \mu < \mu^*.
\]

That is, awarding in-the-money options instead of at-the-money options cannot benefit both shareholders and the manager when the expected return on the underlying stock is sufficiently large relative to the risk-free rate. This result arises because increasing $\mu$ increases the certainty equivalent. Increasing $\mu$ also increases the option cost to shareholders but at a lower rate than the certainty equivalent, the consequence being that the certainty equivalent becomes greater than the option cost when $\mu$ is sufficiently large.

We provide the proof of the following proposition in the appendix: The more risk-neutral the manager’s behavior, which can arise from either a higher initial non-option wealth $w_0$ or a smaller $\gamma$ when $U(x) = x^{1-\gamma}$, the smaller the range of expected stock returns $[r, \mu^*)$ such that backdating executive stock options can benefit both shareholders and the manager. We obtain this result because the lower the manager’s level of risk aversion, the greater the number of in-the-money options required to replace at-the-money options and maintain the manager’s utility.

**Proposition 3** Given the assumptions on the function $U$, then

1. $\mu^*$ is negatively related to $w_0$—that is, the larger the manager’s initial non-option wealth $w_0$, the smaller $\mu^*$;

2. if $U(x) = x^{1-\gamma}$, then $\mu^*$ is positively related to $\gamma$, i.e., the greater the coefficient of relative risk aversion $\gamma$, the greater $\mu^*$,

where $\mu^*$ is as defined in Proposition 2.

Figure 2 shows whether condition (3) is satisfied for different values of the risk aversion coefficient $\gamma$, assuming a utility function $U(x) = x^{1-\gamma}$ for the manager, and different values of the manager’s
initial non-option wealth \( w_0 \). In each case the stock price is \( S_0 = 40 \) and the in-the-money option has an exercise price \( K = 36 \). The option has a ten-year maturity, a three-year vesting period and option values are computed with binomial trees with 1-month time steps. Other parameters are \( r = 5\% \), \( \sigma = 30\% \) and \( \tau_m = 0 \). On these graphs, both the manager and shareholders can benefit from backdating (granting the in-the-money rather than the at-the-money option) when the line, which depicts \( \frac{C^{e,a}_{0}}{C_0} - \frac{m^a_0}{m_0^a} \), is above zero. The figure shows that the greater the expected stock return relative to the risk-free rate, the less likely backdating can be beneficial to both shareholders and the manager. The upper graph of Figure 2 shows that the less risk averse is the manager, the less likely backdating can benefit both parties. The lower graph of Figure 2 shows that a rising initial non-option wealth \( w_0 \) reduces the likelihood that backdating benefit both shareholders and the manager.

4 Conclusion

We show in this paper that the non-transferability of employee stock options (ESOs), combined with the risk aversion of the managers receiving them, may generate situations in which issuing in-the-money ESOs can simultaneously benefit the firm (through lower dilution to shareholders), and the manager (through higher utility). We show that the more risk-averse the manager, the more likely that doing so can benefit both parties. We also show that reducing the manager’s non-option wealth has a similar effect to increasing risk aversion, thus increasing the likelihood that backdating—or issuing in-the-money options—benefits both the manager and shareholders.

A Proof of Proposition 1

Consider two call options on a firm’s stock with the same time to maturity, \( T \). One option is at-the-money, its characteristics being identified with the superscript \( a \), and the other is in-the-money, its characteristics being identified with the superscript \( i \). Let \( N^a = \{ n_1^a, n_2^a, \ldots, n_g^a \} \) denote the exercise frontier of the at-the-money option—that is, the set of exercise nodes that can be reached through a sequence of non-exercise nodes. Let \( \{ S_1^a, S_2^a, \ldots, S_g^a \} \) denote the stock price at each node in \( N^a \), and let \( \{ T_1^a, T_2^a, \ldots, T_g^a \} \) be the time left until the expiration of the option at each node in \( N^a \). Let \( \{ \pi_{1,1}^a, \pi_{2,1}^a, \ldots, \pi_{g,1}^a \} \) represent the subjective probability of reaching each node in \( N^a \) through sequences of non-exercise nodes only, and let \( \{ \pi_{1,2}^a, \pi_{2,2}^a, \ldots, \pi_{g,2}^a \} \) represent the risk-neutral probability of reaching each node in \( N^a \) through sequences of
non-exercise nodes only. Let \( N^i = \{ n^i_1, n^i_2, \ldots, n^i_h \} \), \( \{ S^i_1, S^i_2, \ldots, S^i_h \} \), \( \{ T^i_1, T^i_2, \ldots, T^i_h \} \), \( \{ \pi^{a,i}_1, \pi^{a,i}_2, \ldots, \pi^{a,i}_h \} \) and \( \{ \pi^i_1, \pi^i_2, \ldots, \pi^i_h \} \) represent similar sets for the in-the-money option. Note that \( h \geq g \) that is, the in-the-money option is exercised at least as often as the at-the-money option. The certainty-equivalent value and cost to shareholders of each option are then given by

\[
m^a_0 = e^{-rT}U^{-1}
\left(\sum_{j=1}^{g} \pi^{a,i}_j U \left( (1 - \tau_m)(S^a_j - S_0)e^{rT_j} + w_0e^{rT} \right) + \left( 1 - \sum_{j=1}^{g} \pi^{a,i}_j \right) U \left( w_0e^{rT} \right) \right) - w_0
\]

\[
m^i_0 = e^{-rT}U^{-1}
\left(\sum_{j=1}^{h} \pi^{a,i}_j U \left( (1 - \tau_m)(S^i_j - K)e^{rT_j} + w_0e^{rT} \right) + \left( 1 - \sum_{j=1}^{h} \pi^{a,i}_j \right) U \left( w_0e^{rT} \right) \right) - w_0
\]

\[
C^{a,a}_0 = e^{-rT} \sum_{j=1}^{g} \pi^{a}_j (S^a_j - S_0)e^{rT_j}
\]

\[
C^{a,i}_0 = e^{-rT} \sum_{j=1}^{h} \pi^{a}_j (S^i_j - K)e^{rT_j},
\]

where \( 0 \leq K < S_0 \) and \( U \) is a continuous, strictly increasing and strictly concave function.

If \( \mu = r \), then \( \pi^{a,i}_j = \pi^i_j \), and \( \pi^{a,i}_j = \pi^a_j \) for all \( j \), and granting in-the-money options instead of at-the-money options can benefit both the firm and shareholders if:

\[
\frac{m^a_0}{m^i_0} = \frac{U^{-1} \left( \sum_{j=1}^{g} \pi^{a}_j U \left( (1 - \tau_m)(S^a_j - S_0)e^{rT_j} + w_0e^{rT} \right) + \left( 1 - \sum_{j=1}^{g} \pi^{a}_j \right) U \left( w_0e^{rT} \right) \right) - w_0e^{rT}}{U^{-1} \left( \sum_{j=1}^{h} \pi^{a}_j U \left( (1 - \tau_m)(S^i_j - K)e^{rT_j} + w_0e^{rT} \right) + \left( 1 - \sum_{j=1}^{h} \pi^{a}_j \right) U \left( w_0e^{rT} \right) \right) - w_0e^{rT}} < \frac{\sum_{j=1}^{g} \pi^{a}_j (S^a_j - S_0)e^{rT_j}}{\sum_{j=1}^{h} \pi^{a}_j (S^i_j - K)e^{rT_j}} = \frac{C^{a,a}_0}{C^{a,i}_0}
\]

or, rearranging (4), if

\[
A = \frac{U^{-1} \left( \sum_{j=1}^{g} \pi^{a}_j U \left( (1 - \tau_m)(S^a_j - S_0)e^{rT_j} + w_0e^{rT} \right) + \left( 1 - \sum_{j=1}^{g} \pi^{a}_j \right) U \left( w_0e^{rT} \right) \right) - w_0e^{rT}}{\sum_{j=1}^{g} \pi^{a}_j (S^a_j - S_0)e^{rT_j}}
\]

\[
< \frac{U^{-1} \left( \sum_{j=1}^{h} \pi^{a}_j U \left( (1 - \tau_m)(S^i_j - K)e^{rT_j} + w_0e^{rT} \right) + \left( 1 - \sum_{j=1}^{h} \pi^{a}_j \right) U \left( w_0e^{rT} \right) \right) - w_0e^{rT}}{\sum_{j=1}^{h} \pi^{a}_j (S^i_j - K)e^{rT_j}} = B.
\]

Note that \( A, B < 1 \) due to the strict concavity of \( U \) for any \( \tau_m \in [0, 1] \).

### A.1 The in-the-money option is exercised at time 0

If the in-the-money option is exercised at time 0, then \( h = 1, m^a_0 = (1 - \tau_m)(S_0 - K) \) and \( C^{a,i}_0 = S_0 - K \). The at-the-money option has zero intrinsic value and thus is never exercised at time 0. The strict concavity of \( U \) then gives:

\[
\frac{m^a_0/m^i_0}{C^{a,a}_0/C^{a,i}_0} = \frac{U^{-1} \left( \sum_{j=1}^{g} \pi^{a}_j U \left( (1 - \tau_m)(S^a_j - S_0)e^{rT_j} + w_0e^{rT} \right) + \left( 1 - \sum_{j=1}^{g} \pi^{a}_j \right) U \left( w_0e^{rT} \right) \right) - w_0e^{rT}}{(1 - \tau_m) \sum_{j=1}^{g} \pi^{a}_j (S^a_j - S_0)e^{rT_j}} < 1.
\]
Hence the inequality (4) holds when $\mu = r$ and the in-the-money option is exercised at time 0.

**A.2 The in-the-money option is not exercised at time 0**

If the in-the-money option is not exercised at time 0 and if $\Delta t$ is sufficiently small, then $h$ will be large enough to perform the following operation: Take the values in $\{S^1_t, S^2_t, \ldots, S^h_t\}$ and reduce each of them to construct a set of $h$ stock prices $\{\hat{S}_j\}_{j=1}^h$ such that

1. $\hat{S}_j \leq S^j_t$ for all $j$,
2. $\sum_{j=1}^h \pi^j (\hat{S}_j - K) e^{rT_j} = \sum_{j=1}^g \pi^a_j (S^a_j - S_0) e^{rT_j}$,
3. and because $\hat{S}_j$ is strictly increasing, it must be the case that $\sum_{j=1}^h \pi^j > 0$.

If we replace each $\hat{S}_j$ by its corresponding $S^j_t$, due to $\hat{S}_j \leq S^j_t$ for all $j = 1, \ldots, h$, due to $A < 1$ and due to the properties of $U$. Since $B$ is such that $\sum_{j=1}^h \pi^j U \left( (1 - \tau_m)(S^j_t - K) e^{rT_j} + w_0 e^{rT} \right) > U \left( A \sum_{j=1}^h \pi^j (S^j_t - K) e^{rT_j} + w_0 e^{rT} \right)$, and because $U$ is strictly increasing, it must be the case that $B > A$, meaning that (5) holds in this case as well. This completes the proof of Proposition 1.
B Proof of Proposition 2

Suppose \( r < \mu \). Using the same notation as in the proof of Proposition 1, we have \( \frac{m^0_i}{m^0_0} \geq \frac{C_{i,a}^0}{C_{0}^{i,a}} \) if

\[
A = \frac{U^{-1} \left( \sum_{j=1}^{g} \pi_{j,a}^s U \left( (1 - \tau_m)(S^a_j - S_0)e^{rT} + w_0e^{rT} \right) + (1 - \sum_{j=1}^{g} \pi_{j,a}^s) U \left( w_0e^{rT} \right) \right) - w_0e^{rT}}{\sum_{j=1}^{g} \pi_{j,a}^s (S^a_j - S_0)e^{rT}} \geq B.
\]

(6)

The subjective probability of an upward movement in the stock price is given by \( p_s = \frac{e^{(u-\delta)(\Delta t - \delta)\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}}-e^{-\sigma\sqrt{\Delta t}}} \)
whereas the risk-neutral probability of an up movement is \( p_{rn} = \frac{e^{(r-\delta)\Delta t - \delta\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}}-e^{-\sigma\sqrt{\Delta t}}} \). If \( \mu > r \), then \( p_s > p_{rn} \), meaning that the subjective probabilities of reaching exercise nodes are higher than their risk-neutral counterparts. That is, \( \pi_{j,a}^{s,a} > \pi_{j,a}^a \) and \( \pi_{j,a}^{s,i} > \pi_{j,a}^i \) for all \( j \).

B.1 \( A \) exceeds \( B \) when \( \mu \) is large enough

To show that \( A \) is larger than \( B \) when \( \mu \) is large enough, consider the limiting case where \( \mu \) is such that \( p_s \rightarrow 1 \).

If \( \sigma \) and \( \mu \) are sufficiently high, then the in-the-money option will not be exercised at time 0. If \( \mu \) is such that \( p_s \rightarrow 1 \), then subjective probabilities foresee a stock price that rises every period, which gives \( m^0_0 = (1 - \tau_m)e^{-rT}(u^{T/\Delta t} - K) \). If exercised at time 0, the in-the-money option’s certainty-equivalent value is \( (1 - \tau_m)(S^a_0 - K) \). Therefore, if \( u^{T/\Delta t} - K > e^{rT}(S^a_0 - K) \), the in-the-money option is not exercised at time 0 when \( p_s \rightarrow 1 \). Because \( u = e^{\sigma\sqrt{\Delta t}} \), this case arises when \( \sigma \) is sufficiently high.

B.1.1 The in-the-money option is exercised at time 0

If \( p_s \rightarrow 1 \) and \( \sigma \) is such that the in-the-money option is exercised at time 0, then \( m^0_0 = (1 - \tau_m)(S^a_0 - K) \) and \( C_{0}^{a,i} = S^a_0 - K \). The at-the-money option has zero intrinsic value and thus is never exercised at time 0.

As a consequence:

\[
\frac{m^0_i}{m^0_0} = \frac{U^{-1} \left( \sum_{j=1}^{g} \pi_{j,a}^s U \left( (1 - \tau_m)(S^a_j - S_0)e^{rT} + w_0e^{rT} \right) + (1 - \sum_{j=1}^{g} \pi_{j,a}^s) U \left( w_0e^{rT} \right) \right) - w_0e^{rT}}{\sum_{j=1}^{g} \pi_{j,a}^s (S^a_j - S_0)e^{rT}} \frac{m^0_0}{m^0_i} = \frac{u^{T/\Delta t}S^a_0 - S_0}{\sum_{j=1}^{g} \pi_{j,a}^s (S^a_j - S_0)} > 1.
\]

Therefore, \( A > B \) when \( \mu \) is sufficiently larger than \( r \) and the in-the-money option is exercised at time 0.
B.1.2 The in-the-money option is not exercised at time 0

In this case, we have:

\[
\lim_{\mu \to 1} A = \frac{U^{-1} \left( U \left( (1 - \tau_m)(u^{T/\Delta t}S_0 - S_0) + w_0e^{r_T} \right) \right) - w_0e^{r_T}}{\sum_{j=1}^{g} \pi_j^a(S_j^a - S_0)e^{r_Tj}} = \frac{(1 - \tau_m)(u^{T/\Delta t}S_0 - S_0)}{\sum_{j=1}^{g} \pi_j^a(S_j^a - S_0)},
\]

\[
\lim_{\mu \to 1} B = \frac{U^{-1} \left( U \left( (1 - \tau_m)(u^{T/\Delta t}S_0 - K) + w_0e^{r_T} \right) \right) - w_0e^{r_T}}{\sum_{j=1}^{h} \pi_j^b(S_j^b - K)e^{r_Tj}} = \frac{(1 - \tau_m)(u^{T/\Delta t}S_0 - K)}{\sum_{j=1}^{h} \pi_j^b(S_j^b - K)}
\]

and

\[
\lim_{\mu \to 1} \frac{A}{B} = \frac{(u^{T/\Delta t}S_0 - S_0)/(u^{T/\Delta t}S_0 - K)}{\sum_{j=1}^{g} \pi_j^a(S_j^a - S_0)/\sum_{j=1}^{h} \pi_j^b(S_j^b - K)}.
\]

Because in this case the option is exercised at maturity only, \( h > g \) and thus:

\[
\sum_{j=1}^{h} \pi_j^b(S_j^b - K) = \sum_{j=1}^{g} \pi_j^a(S_j^a - K) + \sum_{j=g+1}^{h} \pi_j^b(S_j^b - K).
\]

Consequently,

\[
\lim_{\mu \to 1} A = \frac{u^{T/\Delta t}S_0 - S_0}{u^{T/\Delta t}S_0 - K} \left( \frac{\sum_{j=1}^{g} \pi_j^a(S_j^a - K) + \sum_{j=g+1}^{h} \pi_j^b(S_j^b - K)}{\sum_{j=1}^{g} \pi_j^a(S_j^a - S_0)}/\sum_{j=1}^{h} \pi_j^b(S_j^b - S_0) / \sum_{j=1}^{h} \pi_j^b(S_j^b - S_0) \right) > 1,
\]

where \( S_j^a = \sum_{j=1}^{g} \pi_j^a S_j^a / \sum_{j=1}^{g} \pi_j^a \) gives the risk-neutral weighted average value of stock prices between \( S_0 \) and \( u^{T/\Delta t}S_0 \). The last inequality holds because \( S_j^a \in (S_0, u^{T/\Delta t}S_0) \) and \( \frac{u^{T/\Delta t}S_0 - S_0}{u^{T/\Delta t}S_0 - S_0} \left( \frac{x-K}{x-S_0} \right) > 1 \) for all \( x \in (S_0, u^{T/\Delta t}S_0) \). As a result, \( A > B \) when \( \mu \) is sufficiently large and the in-the-money option is not exercised at time 0. This result is independent of tax rates.

B.2 There exists a \( \mu^* \) such that \( A(\mu) > B(\mu) \) for all \( \mu \geq \mu^* \)

As in the proof of Proposition 1, if \( \Delta t \) is sufficiently small, we can construct a set \( \{S_j\}_{j=1}^{h} \) such that:

1. \( S_j \leq S_j^b \) for all \( j \),
2. \( \sum_{j=1}^{h} \pi_j^b(S_j^b - K)e^{r_Tj} = \sum_{j=1}^{g} \pi_j^a(S_j^a - S_0)e^{r_Tj} \)
3. and

\[
U^{-1} \left( \sum_{j=1}^{h} \pi_j^b \mu \left( (1 - \tau_m)(S_j^b - K)e^{r_Tj} + w_0e^{r_T} \right) + \left( 1 - \sum_{j=1}^{h} \pi_j^b \right) U \left( w_0e^{r_T} \right) \right) - w_0e^{r_T}
\]

\[
= U^{-1} \left( \sum_{j=1}^{g} \pi_j^a \mu \left( (1 - \tau_m)(S_j^a - S_0)e^{r_Tj} + w_0e^{r_T} \right) + \left( 1 - \sum_{j=1}^{g} \pi_j^a \right) U \left( w_0e^{r_T} \right) \right) - w_0e^{r_T}.
\]
As a result,
\[
\sum_{j=1}^{h} \pi_{j}^{s,i} U \left( (1 - \tau_m) (S_j^i - K) e^{r T_j} + w_0 e^{r T} \right) + \left( 1 - \sum_{j=1}^{h} \pi_{j}^{s,i} \right) U \left( w_0 e^{r T} \right) = U \left( B \sum_{j=1}^{h} \pi_{j}^{s,i} (S_j^i - K) e^{r T_j} + w_0 e^{r T} \right)
\]
and
\[
\sum_{j=1}^{h} \pi_{j}^{s,i} U \left( (1 - \tau_m) (S_j^i - K) e^{r T_j} + w_0 e^{r T} \right) + \left( 1 - \sum_{j=1}^{h} \pi_{j}^{s,i} \right) U \left( w_0 e^{r T} \right) = U \left( A \sum_{j=1}^{h} \pi_{j}^{s,i} (S_j^i - K) e^{r T_j} + w_0 e^{r T} \right).
\]

When replacing each \( \hat{S}_j \) by its corresponding \( S_j^i \) in the last equation, \( A \) being sufficiently large \((A > 1 - \tau_m)\) will give
\[
\sum_{j=1}^{h} \pi_{j}^{s,i} U \left( (1 - \tau_m) (S_j^i - K) e^{r T_j} + w_0 e^{r T} \right) + \left( 1 - \sum_{j=1}^{h} \pi_{j}^{s,i} \right) U \left( w_0 e^{r T} \right) < U \left( A \sum_{j=1}^{h} \pi_{j}^{s,i} (S_j^i - K) e^{r T_j} + w_0 e^{r T} \right)
\]
and hence \( A > B \).

Because a change in \( \mu \) may shift the exercise nodes, the terms \( A \) and \( B \) as defined above do not vary continuously with \( \mu \). Hence a \( \mu^* \) such that \( A(\mu^*) = B(\mu^*) \) may not exist. However, we can show that if \( A(\hat{\mu}) > B(\hat{\mu}) \) for some \( \hat{\mu} \), then \( A(\hat{\mu} + \epsilon) > B(\hat{\mu} + \epsilon) \) for any \( \epsilon > 0 \).

To show this result, first note that \( A(\mu) \) increases with \( \mu \). Therefore, if \( \hat{\mu} \) is such that \( A(\hat{\mu}) \) is sufficiently large for (7) to hold, then (7) holds with \( A(\hat{\mu} + \epsilon) \) for any \( \epsilon > 0 \). Similarly, if \( \hat{\mu} \) is such that \( A(\hat{\mu}) < B(\hat{\mu}) \), then \( A(\hat{\mu} - \epsilon) < B(\hat{\mu} - \epsilon) \) for all \( \epsilon \in (0, \hat{\mu} - r) \). There must then exist a \( \mu^* \) such that \( A(\mu) \geq B(\mu) \) for all \( \mu \geq \mu^* \) and \( A(\mu) < B(\mu) \) for all \( \mu < \mu^* \). This completes the proof of Proposition 2.

C Proof of Proposition 3

1. This proof uses the same notation as in the proofs of propositions 1 and 2 above. As before, we let:
\[
A = \frac{U^{-1} \left( \sum_{j=1}^{g} \pi_{j}^{s,a} U \left( (1 - \tau_m) (S_j^a - S_0) e^{r T_j} + w_0 e^{r T} \right) \right) - w_0 e^{r T}}{\sum_{j=1}^{g} \pi_{j}^{s,a} (S_j^a - S_0) e^{r T_j}}
\]
and
\[
B = \frac{U^{-1} \left( \sum_{j=1}^{h} \pi_{j}^{s,i} U \left( (1 - \tau_m) (S_j^i - K) e^{r T_j} + w_0 e^{r T} \right) \right) - w_0 e^{r T}}{\sum_{j=1}^{h} \pi_{j}^{s,i} (S_j^i - K) e^{r T_j}}.
\]
Suppose that \( \mu > r \), let \( w_0 \) be such that \( A < B \), let \( \tilde{w}_0 > w_0 \) and let \( \tilde{A} \) and \( \tilde{B} \) be the counterparts of \( A \) and \( B \) when the manager’s initial non-option wealth is \( \tilde{w}_0 \). As \( \tilde{w}_0 \to \infty \), the manager’s valuation of an option approaches that of a risk-neutral agent, meaning that:
\[
\lim_{\tilde{w}_0 \to \infty} \tilde{A} = \frac{(1 - \tau_m) \sum_{j=1}^{g} \tilde{\pi}_{j}^{s,a} (\tilde{S}_j^a - S_0) e^{r T_j}}{\sum_{j=1}^{g} \tilde{\pi}_{j}^{s,a} (\tilde{S}_j^a - S_0) e^{r T_j}} > 1 - \tau_m
\]
when \( \mu > r \). Therefore, given \( \mu > r \), it is always possible to increase \( w_0 \) to obtain \( A > 1 - \tau_m \), which then makes equation (7) hold, so that \( A > B \). Therefore, whenever \( \mu \) and \( w_0 \) are such that \( A < B \), there exists a \( \tilde{w}_0 > w_0 \) such that \( \tilde{A} > \tilde{B} \), meaning that the threshold \( \mu^* \) is smaller with \( \tilde{w}_0 \) than with \( w_0 \).

2. If \( U(x) = \frac{x^{1-\gamma}}{1-\gamma} \) and \( \gamma \to 0 \), the manager behaves as if he or she were risk neutral and the proof proceeds as in (1).
References


Figure 1: Value of a call option with strike price $K = 40$ and time to maturity $T = 3$ on a stock with $S_0 = 40$, $\mu = 5\%$, $\delta = 2\%$ and $\sigma = 30\%$. The tax rate is $\tau_m = 0$, the risk-free rate is $r = 5\%$ and a time step is $\Delta t = 1$, the manager’s initial non-option wealth is $w_0 = 100$ and the manager’s utility function is $U(w) = w^{1-\gamma}/(1 - \gamma)$ with $\gamma = 2$. At each node, the tree gives the stock price ($S_{i,j}$), the value of a regular American call option ($C_{i,j}$), the value of an executive stock option from shareholders’ viewpoint ($C_{e,i,j}$) and the value of an executive stock option from the manager’s viewpoint ($m_{i,j}$).
Figure 2: Potential benefit of issuing in-the-money options instead of at-the-money options for different values of the firm's expected stock return. The upper graph assumes $w_0 = 100$ and lets the coefficient of risk aversion vary. The lower graph assumes $\gamma = 2$ and lets $w_0$ vary. Other parameters are as follows: $S_0 = 40$, $r = 5\%$, $\delta = 2\%$, $\sigma = 30\%$, $T = 10$, $\Delta t = 1/12$, $v = 3$, $\tau_m = 0$ and $U(w) = w^{1-\gamma}/(1-\gamma)$. The strike price of the in-the-money option is $K = 36$. 

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