Higher-Moment Model with Errors-in-Variables: An Examination with Cross Section of Expected Returns

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Abstract

This paper proposes measures of systematic skewness and systematic kurtosis as symmetric measures of risk by extending the work of Kraus and Litzenberg (1976). We examine an asset pricing model that incorporates systematic skewness and systematic kurtosis to test the cross section of asset returns within the context of the Fama and MacBeth (1973) two-pass estimation methodology. However, this estimation suffers from the errors-in-variables (EIV) problems that could attenuate the significance of risk premium betas. We propose the Dagenais and Dagenais (1997) higher-moment estimators as a solution for the EIV problems. Our results suggest that although the EIV correction leads to a diminished role of the market beta, the systematic skewness and systematic kurtosis still retain their significance in explaining the cross section of expected returns.

Key words: Systematic skewness; Systematic kurtosis; Fama and MacBeth two-pass estimation; Errors-in-Variables; Higher Moment Estimators.

JEL classification: C13, C31, G12

1. Introduction

Since the validity of the single-factor CAPM has been questioned (Roll (1977), Gibbons, Ross and Shanken (1989), Mills (1995) and Harvey *et al.* (2004)), numerous researchers have shifted their focus to higher-moment models. For example, Samuelson (1970) and Rubinstein (1973) argue that higher moments are relevant to the investor's decision unless the asset returns are normally distributed and the investor's utility functions are quadratic. Kraus and Litzenberger (1976) expand the investor's utility function beyond the second moment in a Taylor series to examine the skewness effect. Campbell and Hentschel (1992) find volatility clustering caused by large stock returns followed by stock returns of similar magnitude but in the opposite direction leads to fat tail distributions. Harvey and Siddque (2000) use the systematic skewness, which is defined as a component of an asset's skewness related to the market portfolio's skewness, to test whether stocks with large negative systematic skewness can earn a higher risk premium.

Given that the empirical stock return distribution is observed to be asymmetric and leptokurtic, this paper seeks to explain how the CAPM model incorporating the systematic skewness and systematic kurtosis, the third and fourth moments of the return distribution, helps us understand the cross-sectional variation in asset returns. We use the Gibbons, Ross and Shanken (GRS) (1989) test of zero pricing errors (i.e. regression intercepts) to examine the validity of the four-moment model. We find that the pricing errors in the traditional CAPM model can be significantly explained by the systematic skewness and systematic kurtosis. Furthermore, we are unable to reject the null hypothesis of zero intercepts when the systematic skewness and systematic kurtosis are included in the CAPM model, suggesting that the higher systematic moments are relevant to the asset pricing.

The interest of this study is to evaluate the relative importance of systematic skewness and systematic kurtosis in explaining the variation of stock returns in cross-section. Usually studies of this type adopt the Fama and MacBeth (1973) two-pass regression procedure: In the first pass, beta estimates are obtained from time-series regression for each underlying asset, and in the second-pass, gammas are estimated cross-sectionally by regressing asset returns on the estimated betas. However, many researches (Litzenberger and Ramaswamy (1979), Gibbons (1982), Shanken (1992), Kim (1995), Jaganathan and Wang (1996) Kan and Zhang (1997)) have raised serious concerns about the errors-in-variables (EIV) problems in the second pass estimation. Criticisms focus on the unobservable market risk factor due to the fact that the market beta is estimated with errors in the second pass of the procedure. Dagenais and Dagenais (1997) argue that such errors in variables will lead to the inconsistency in the Ordinary Least

Squares (OLS) estimators, to larger mean-squared errors, and probably, most importantly to larger than intended sizes of type I errors of student T-tests.

Despite the importance of the EIV problems, there has been little attempt to correct for the problems. Kim (1995) criticises that the explanatory power of the book-to-market equity ratio for average stock returns reported by Fama and French (1992) is exaggerated under the traditional least squares estimation since the EIV problems result in an underestimation of the price of market beta risk and an overestimation of the other cross-sectional regression (CSR) coefficients associated with variables observed without errors (such as the size and the book-to-market-equity). The same criticism applies to more general models such as Fama and French (1993), Cahart (1997) and of course, to the CAPM model incorporating systematic skewness and systematic kurtosis.

The traditional approach to correct the EIV is to identify the appropriate instrumental variables if one can find these instruments correlated with the true variables but unrelated to the measurement errors. However, such variables may often not be readily available (Pal. (1980)). On the other hand, consistent estimators based on the original, unaugmented set of observables are usually available, which motivates many researchers (Reisersol (1950), Madansky (1959), Van Montfort and Bikel and Ritov (1987)) to use information contained in higher order moments of data to construct consistent estimators. Cragg (1997) and Dagenais and Dagenais (1997) argue that if the regressors in the multivariate models exhibit skewness and/or kurtosis in their distributions, the estimators based on moments order higher than two could help alleviate the EIV problems in the models. Since the financial variables are often found to exhibit non-normality, this paper looks at the issue of EIV problems when using the higher-moment model to examine the variation of asset returns in cross section and applies the Dagenais and Dagenais higher-moment estimators (DDHME) to minimize the problems.

When using DDHME to correct the EIV problems in the context of the four-moment model, we find the significance of the market, systematic skewness and systematic kurtosis premia measured by traditional Fama and MacBeth CSR to be overstated. While the systematic skewness and systematic kurtosis premia measured in two-pass CSR are significant in some sub-periods of downturn markets, the results do not hold for these periods when EIV problems are corrected using DDHME. Nevertheless, although the EIV correction leads to a diminished role of the market beta, the systematic skweness and systematic kurtosis still retain their significance in explaining patterns in cross-sectional asset returns with the DDHME method for the examined period of 1992 to 2009.

The paper is organized as follows. First, we propose a four-moment model in which the systematic skewness and systematic kurtosis are estimated as symmetric measures of risk, analogous to the market beta. We then test the validity of this model using the GRS test of zero pricing errors. To further examine the roles of the systematic skewness and systematic kutosis in explaining patterns of cross-sectional stock returns, we use the Fama and MacBeth (1973) two-pass procedure applying for the four-moment model. As the EIV problems arise from the second-pass of the estimation, we next present the estimation of DDHME to correct for the problems. The results and their discussion follow next. Finally, conclusion remarks finish the study.

2. Methodology

2.1 The Four-Moment Model

In the spirit of Kraus and Litzenberger (1976), we define measures of systematic skewness (S_i) and systematic kurtosis (K_p) risk as follows:

$$S_i = \frac{E[\{R_i - E(R_i)\}\{R_m - E(R_m)\}^2]}{E[\{R_m - E(R_m)\}]^3} \tag{1}$$

$$K_i = \frac{E[\{R_i - E(R_i)\}\{R_m - E(R_m)\}^3]}{E[\{R_m - E(R_m)\}]^4} \tag{2}$$

Where R_i and R_m are the returns of asset i and the market index respectively, and $E(R_i)$ and $E(R_m)$ are the expected returns for the asset and the market index respectively.

As the systematic skewness and the systematic kurtosis capture positive and negative deviations of market portfolio returns from its mean return, these measures are considered as symmetric measure of risk, analogous to market beta.

We expand the traditional two-moment CAPM to the four-moment model to examine the variation of asset returns as follows:

$$R_{i,t} - r_{f,t} = \alpha + \beta_1 (R_m - r_f)_t + \beta_2 S_{i,t} + \beta_3 K_{i,t} \tag{3}$$

where $R_{i,t}$ is the return of asset i at time t, $r_{f,t}$ is the risk-free rate of return at time t, $R_{m,t}$ is the return of the market index at time t, $S_{i,t}$ and $K_{i,t}$ are the return premiums of the systematic skewness and systematic kurtosis risk respectively.

One method to understand how the systematic skewness and systematic kurtosis influence asset pricing is to analyse the pricing errors from the four-moment model. We use a multivariate test of

Gibbons, Ross and Shanken (GRS) (1989) to test whether the pricing errors are jointly equal to zero. We apply GRS test to examine (1) whether adding the systematic skewness and systematic kurtosis to the two-moment CAPM model decreases the significance of pricing errors and (2) whether the four-moment model is empirically valid (i.e. the pricing errors tend to zero when both of the higher moment factors are included).

Let T be the number of observations, N be the number of underlying assets and L be the number of regression parameters including the constant term. We compute the GRS-statistic to test whether the intercepts (δ_{0i}) from time series regressions are jointly equal to zero:

H$_0$: $\delta_{0i} = 0 \qquad \forall i = 1,....N$

The GRS-statistic is computed as:

$$\left(\frac{T}{N}\right)\left(\frac{T-N-L}{T-L-1}\right)\left(\bar{R}'_i\,\hat{\Omega}^{-1}\,\bar{R}_i\right)^{-1}\left(\hat{\delta}_0{}'\hat{\Sigma}^{-1}\,\hat{\delta}_0\right) \tag{4}$$

where $\bar{R}_i$ is a vector of sample mean for vector $R_{it} = (R_{1t}, R_{2t},..., R_{Nt})$;

$\hat{\Omega}$ is the sample variance-covariance matrix for R_i;

$\hat{\Sigma}$ is the variance-covariance matrix of the residuals;

$\hat{\delta}_0 = (\delta_{01,}\delta_{02}...\delta_{0N})$ is the vector of the least squares estimators of the pricing errors where δ_{0i} is the intercept of the regression of asset i on L regression parameters.

GRS-statistic has a F-distribution with degrees of freedom N and (T-N-L). The interpretation of the GRS-statistic is equivalent to that of the usual t-statistic on the single intercept term in a univariate regression model. MacKinlay (1985) suggests that the F-test is fairly robust even when the distribution of asset returns exhibits skewness and/or kurtosis. This is important since our data is expected to be skewed and leptokurtic.

2.2 The Errors in the Variables (EIV) Problem in the Cross-Section of Expected Stock Returns

The two-pass procedure of Fama and MacBeth (1973) has been widely used as a standard test for the risk estimation in cross section. Although Shanken (1992) argues that the use of the predictive beta ,

$\hat{\beta}_{t-1}$, in the CSR (1) avoids the problem of spurious cross-sectional relations arising from statistical correlation between returns and the estimated betas and (2) maintains the independence between the explanatory variables, $\hat{\beta}_{t-1}$, and the regression error term, ε_t, in the CSR model, the problem of EIV is critical and may dispute the significance of the explanatory power of the model. We revisit the second-pass cross sectional regression model of estimating the risk-return relations at a specific time t:

$$R_i - r_f = \alpha + \delta_1 \beta_{1,i} + \delta_2 \beta_{2,i} + \delta_3 \beta_{3,i} + \varepsilon_i \qquad i = 1, \cdots, N \tag{5}$$

$\beta_{1,i}$, $\beta_{2,i}$ and $\beta_{3,i}$ are the market beta, the systematic skewness and systematic kurtosis betas of underlying asset i, which are estimated by the first-pass regression procedure of Fama and MacBeth (1973) in equation 3. ε_i is the idiosyncratic error and $R_i - r_f$ is the excess return of underlying asset i and *N* is the total number of underlying assets.

Criticisms of Fama and MacBeth (1973) focus on the unobservable estimated β_t since $\hat{\beta}_{t-1}$ is used as a proxy for the unknown β_t in the second pass of the estimation. Therefore, the independent variable β_t is measured with an error:

$$\hat{\beta}_{t-1} = \beta_t + \zeta_{t-1} \tag{6}$$

where $\hat{\beta}_{t-1}$ is the beta, which is either the market beta, the systematic skewness beta or the systematic kurtosis beta, estimated from the first-pass regression procedure using 30 weeks of data available up to t-1. ζ_{t-1} is the measurement error.

We next present a simple version of the method of moments developed by Dagenais and Dagenais (1997) to minimise the EIV problems.

2.3 Higher Moment Estimators for Multivariate Linear Models with EIV

We analyse the Dagenais and Dagenais method of higher-moment estimators, which considers the case of non-Gaussian distributions of the regression regressors, to correct the EIV problems. This method assumes the normality condition of measurement errors. Let us consider a multivariate model in a general matrix form:

$$Y = \alpha\, i_N + \tilde{X}\beta + u \tag{7}$$

where $\tilde{X}$ is a N×K matrix explanatory variables measured without errors where N is the number of observations and K is the number of regressors in the model. Y is a N×1 vector of observations of the dependent variable. u is a N×1 vector of normal residual errors, independent of the variables contained in $\tilde{X}$. i_N is N×1 unit vector. If the matrix X is observed instead of $\tilde{X}$ where:

$$X = \tilde{X} + V \tag{8}$$

Equation (7), therefore, can be rewritten as:

$$Y = \alpha \, i_N + X\beta + \eta \tag{9}$$

where $\eta = u - V\beta$ and V is a N×K matrix of normally distributed errors in the variables.

We assume V uncorrelated with u but allow for the explanatory variables measured with errors, X , correlated with the error terms. The Dagenais and Dagenais higher moment estimators (α and β) is derived from the following orthogonality conditions:

$$\underset{n \to \infty}{E} \left(Z' \eta / \sqrt{N} \right) = 0 \tag{10}$$

Where $Z = (i_N, z_1, z_2, z_3, z_4)$

$$z_1 = x * x \tag{11}$$

$$z_2 = x * y \tag{12}$$

$$z_3 = y * y \tag{13}$$

$$z_4 = x * x * x - 3X\left[E\left(x' x / N\right) * I_K\right] \tag{14}$$

Where the symbol * denotes the Hadamard element-by-element matrix multiplication operator[1] and variable x and y correspond to X and Y expressed in a mean deviation form.

[1] If $A \in R^{m \times n}$ and $B \in R^{m \times n}$ then $A * B \in R^{m \times n}$ where the elements of $A * B$ are given by $(A * B)_{ij} = A_{ij} B_{ij}$. Note that if x is a matrix and y is a vector, $x * y = (x_1 * y, x_2 * y, \ldots, x_K * y)$ where x_j is the j[th] column of x.

Let $\hat{w}$ is a N×K matrix that represents the difference between the observed X and the estimated $\hat{X}$. $\hat{w}$ is estimated as:

$$\hat{w} = X - \hat{X} = X - Z(Z'Z)^{-1}Z'X \tag{15}$$

To correct for the EIV problems in OLS regressions, we add $\hat{w}$ to the model and re-run the augmented regression by OLS:

$$Y = \alpha \, i_N + X\beta + \hat{w}\varphi + \varepsilon \tag{16}$$

where φ is a N×1 vector of parameters and ε is a vector of the regression errors.

The second-pass CRS model using higher moment estimators to correct the EIV in betas is therefore rewritten as:

$$R_i - r_f = \alpha + \delta_1\beta_{1,i} + \delta_2\beta_{2,i} + \delta_3\beta_{3,i} + \varphi_1\hat{w}_1 + \varphi_2\hat{w}_2 + \varphi_3\hat{w}_3 + \varepsilon_i \qquad i = 1, \cdots, N \tag{17}$$

where $Y = R_i - r_f$ is a N×1 vector of observations of the dependent variable.

$X = \beta$ is a N×3 matrix of explanatory variables

$\hat{w} = X - \hat{X}$ is measured as described in (15).

3. Data

While daily data contains too much noise and monthly data results in limited observations, weekly data is our favored choice. We collect weekly stock returns of more than 2000 companies listed on the ASX stock exchange and traded in Australian dollars from January 1992 to May 2009, yielding approximately 650,000 observations. The advantage of this data is that it contains most of the largest companies that make up more than 95 percent of the Australian market and therefore it is easy to use them as building blocks for portfolio construction. The proxy for the market is the ASX300 index which is the value weighted average of the 300 largest Australian companies based on their market capitalization. A 90-day Bank-Accepted bill rate is used as a proxy for the risk free rate. To accurately reflect the performance of stocks and indices, we use return indices, which assume all dividends and distributions are reinvested to compute weekly returns by taking the first difference of logarithm of the index multiplied by 100.

The portfolio formation is in the spirit of Fama and French (1992) and Harvey and Siddique (2000). We first compute the systematic skewness and kurtosis of each stock using equation (1) and (2) and rank them according to the magnitude of systematic skewness and kurtosis respectively. They are then sorted into five quintile skewness and kurtosis portfolios respectively with approximately 446 stocks in each quintile. According to the ranking, quintile 1 contains the highest systematic skewness (kurtosis) and quintile 5 the lowest. The difference in returns of the highest systematic skewness (kurtosis) portfolios minus the returns of lowest systematic skewness (kurtosis) portfolios captures the return premium that is related to the skewness (kurtosis) risk. Twenty-five portfolios are subsequently formed by the intersection of five systematic skewness and five systematic kurtosis quintiles. The number of stocks in each portfolio varies from 20 to 412. These portfolio returns are then used for empirical tests in the methodology.

4. Results and Discussion

4.1 The existence of skewness and kurtosis in the return data - Summary Statistics

Table 1 presents the summary statistics of 25 weekly Australian stock portfolios for the period January 1992 to May 2009. Consistent with the current literature, the table shows that high skewness or kurtosis portfolios exhibit higher standard deviations of portfolio returns than on average. This suggests the skewness and kurtosis risk do contribute significantly to the volatility of stock returns overall. Controlling for skewness effects, high kurtosis portfolios tend to outperform the low kurtosis ones on average, while it is less apparent that low skewness portfolios outperform the high skewness portfolios if we control for kurtosis effects. The standard unconditional skewness and excess kurtosis are also reported in the table. 14 out of 25 portfolios are negatively skewed while they all exhibit heavy tails. To investigate the normality assumption, we apply Jarque-Bera standard normality test. The Jarque-Bera statistic measures the differences of skewness and kurtosis of the return series with those of the normal distribution. Table 1 shows that the Jarque-Bera statistics of the portfolio returns firmly reject the null hypothesis of the normal distribution. The evidence thus supports our subsequent empirical tests of whether higher moment factors are significant factors in terms of explaining the return variability.

Table 2 presents direct estimates of the CAPM beta, the systematic skewness and the systematic kurtosis of the 25 portfolios. We define the systematic skewness and kurtosis as standardized estimates of the skewness and kurtosis risk respectively, analogous to the CAPM beta. Positive betas in all portfolios indicate that the portfolio performance generally follows the market performance. In economic terms, the market beta is referred to as a financial elasticity of volatility because it measures the sensitivity of asset returns to changes in market returns or market risk. Because the majority of portfolios betas are less than

1, the elasticity of volatility is fairly "inelastic", indicating that the portfolios are less volatile than the market over the examined period. Only 4 out of 25 portfolios have the market betas more than 1 and they are all high-kurtosis portfolios. Panel B and C of table 2 report direct measures of the systematic skewness and systematic kurtosis of the 25 portfolios. In each kurtosis quintile, the estimate of systematic skewness increases almost monotonically from the low to the high skewness portfolios. Similarly, the estimate of the systematic kurtosis nearly monotonically increases from the low to the high kurtosis portfolios in each skewness quintile. We find that portfolios with high market betas result in high direct estimates of the systematic skewness and systematic kurtosis risk. Further, they generally constitute most stocks with high systematic skewness and systematic kurtosis. This strongly suggests that downside and heavy tails risk do contribute significantly to the volatility of asset returns which is evidenced by high market betas on average.

In summary, the normality assumption for the Australian market from 1992 to 2009 is rejected and the impacts of systematic skewness and kurtosis persist strongly in the market. Next we formally test the information in the systematic skewness and systematic kurtosis relative to the four-moment pricing model.

4.2 The validity of the four-moment model - Can skewness and kurtosis explain what other factors do not?

Table 3 provides multivariate tests of intercepts for the two-moment and four-moment models respectively. To check the stability of our results to economic impacts in Australia, we divide the examined period into four sub-periods of 1992-1996, 1997-2001, 2002-2006 and 2007-2009. The periods of 1997-2001 and 2007-2009 are considered as downturn market times as the market experiences the Asian financial crisis, dot-com bubble deflation and global financial crisis while in the remaining periods, the economy are in expansionary phases.

We find that the null hypothesis of zero intercepts in the two-moment model are rejected at 5% in every period examined. This reaffirms and complements the findings of Black, Jensen and Scholes (1972), Gibbons, Ross and Shanken (1989) that the market beta alone insufficiently explains the expected returns. In this case, we further test whether the additional explanatory variables such as the systematic skewness and the systematic kurtosis are able to identify patterns in the asset returns that are not explained by the market premium. We find that the P-value of the GRS-statistic has increased from the two-moment model to the four-moment model in almost every period examined, implying the decreasing of the statistic significance of the pricing errors when the systematic skewness and kurtosis are added to the CAPM model. The test results generally support our argument that if the systematic skewness and kurtosis are relevant in explaining the stock variation, then adding these factors to the two-moment CAPM model will reduce the significance level of the GRS-statistic. Most importantly, we find that the null hypothesis of zero intercepts in the four-moment model are not rejected at 5% in almost every period examined. This empirically supports the validity of our four-moment model and emphasizes the importance of the systematic skewness and systematic kurtosis as explanatory variables in identifying patterns of asset returns that are not explained by the market returns.

4.3 Higher-Moment Estimators for the Four-Moment Model with EIV

As described in the methodology, we use the approach of Fama and MacBeth (1973) two-pass procedure to examine whether the systematic skewness and kurtosis factor loadings analogous to the CAPM market beta contribute significantly to the return premium in cross section. If the variation in the expected returns can be explained by one of the explanatory variables, the average slope of that variable should be insignificantly different from zero. As the EIV problems arise in the second pass, we use DDHME to help alleviate these problems. Table 4 compares the significance of the average estimated values of the market beta, the systematic skewness and systematic kurotsis premia and the t_statistics of zero slope hypotheses using methods of OLS estimators and the higher-moment estimators.

Without EIV problems, we find strong evidence that systematic skewness and kurtosis factors do have the predictive power over average stock returns and there is a significant positive trade-off between the return and the skewness and kurtosis risk for the period 1992-2009. On the other hand, we are unable to reject the hypothesis that on average the market effect is zero and unpredictably different from zero from one period to the next. This result is consistent to the findings of Brooks and Galagedera (2007) who argue that when downside gamma, which is similar to our systematic skewness factor loading but only measured in the downside of the return distribution, is included in the two moment pricing model, the downside gamma appears to be the dominant explanatory variable and the market factor becomes insignificant. Table 4 also presents the Fama-McBeth cross-sectional regressions for 4 sub-periods. It is interesting to observe that the two higher moment factors are heavily priced in the periods of 1997-2001 and of 2007-2009 when the Australian market experiences severe downturns due to the deflation of dot-com bubble in 2000-2001 and the global financial crisis in middle 2007 to 2009. On the other hand, return premia for the systematic skewness and systematic kurtosis effects are not significant when the economy experiences expansionary phases in the periods of 1992-1996 and 2002-2006. In term of a risk-return relationship, the evidence confirms the findings of Fabozzi and Francis (1977), Kim and Zumwalt (1979), Estrada (2002), Post and van Vliet (2006) and Brooks and Galagedera (2007) that the downside risk is considered as an appropriate measure of the security risk. With Australian stocks, we find that the downside risk is captured in both systematic skewness and systematic kurtosis.

When DDHME are used to correct the EIV problems, we found interesting results. First, we confirm the findings of Shanken (1992) and Kim (1995) that when traditional CSR ignores the measurement error in the market beta, the significance of the market premium is overstated. This is indicated by the t_statistic of the null hypothesis of zero δ_i has decreased from the two-pass CSR method to the DDHME method in almost every period examined. While the systematic skewness and systematic kurtosis measured by two-pass CSR are, on average, significant in both 1997-2001 and 2007-2009 sub-periods , their t-statistics reduce substantially to insignificant when their measurement errors are measured by the DDHME. Importantly, the measurement error of the systematic skewness for the 2007-2009 is significant at 1% level and that of the systematic kurtosis is marginally significant at 10% level. This explains why the t_ statistics of the systematic skewness and systematic kurtosis factors drop to insignificant when the measurement errors are taken into account over this period. Overall, although EIV problems may dispute the cross-sectional results in sub-periods, we still find that the systematic skewness and systematic kurtosis are still important in explaining patterns in the asset returns that are not explained by the market beta. In other words, these risk factors should be priced.

5. Conclusion Remarks

We develop the systematic skewness and systematic kurtosis as analogs of the CAPM beta. We test the validity of the CAPM incorporating the systematic skewness and systematic kurtosis in asset pricing. The results reveal the importance of these higher moment factors in identifying patterns of asset returns that are not explained by the market. In cross-sectional analysis using the Fama and MacBeth (1973) two-pass estimation, we find strong evidence that the systematic skewness and systematic kurtosis factors have the predictive power over the average returns. Importantly, when these factors are incorporated in the two-moment pricing model, they appear to be the dominant explanatory variables and the market factor becomes insignificant. As the Fama and MacBeth estimation is criticised for the EIV problems in the second-pass of the estimation, we suggest Dagenais and Dagenais (1997) higher moment estimators as a solution for the EIV problems. Using DDHME to correct the EIV, we find that the significance of the market, the systematic skewness and systematic kurtosis premia measured by the two-pass estimation are overstated. It suggests that the accuracy of linear asset pricing models may diminish in the presence of measurement errors. Nevertheless, cross-sectional tests of the four-moment model with the DDHME as the EIV correction have shown that systematic risk measured by the systematic skewness and systematic kurtosis maintains its significance in explaining the cross-sectional variation in expected returns.

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Table 1. Summary statistics of the 25 portfolios formed by systematic skewness and systematic kurtosis: January 1992- May 2009

The table reports the summary statistics of the 25 portfolios formed from 2234 Australian stocks listed on ASX for the period of January, 1992 to May 2009. Each portfolio is constructed by the intersection of 5 systematic skewness and 5 systematic kurtosis quintiles. Portfolio 1-1 contains the low systematic skewness and kurtosis stocks while portfolio 5-5 contains high systematic skewness and kurtosis stocks. Mean and standard deviation are the first two momens of the return distribution while unconditional skewness and kurtosis are the third and the fourth. Excess kurtosis is equal to the unconditional kurtosis of the portfolio minus 3 which is the unconditional kurtosis of the normal distribution. t_statistics are reported in the parentheses below the coefficients estimates. * and ** denote the statistical significance at the 5 and 1 percent levels.

	Mean	Median	Maximum	Minimum	Std. Dev.	Uncondition	Unconditional	Jarque-Bera	Probability	Number of stocks
Portfolio 1-1	-0.0022	-0.0017	0.1096	-0.2390	0.0225	-1.4410	19.0557	10022.75	0	390
Portfolio 1-2	0.0000	-0.0008	0.0785	-0.0575	0.0149	0.7520	6.2759	489.95	0	201
Portfolio 1-3	-0.0019	-0.0010	0.3964	-1.1519	0.0801	-4.4434	73.9918	135221.80	0	59
Portfolio 1-4	0.0006	-0.0047	0.4273	-0.3344	0.0772	0.6481	9.0099	776.45	0	28
Portfolio 1-5	-0.0063	-0.0054	0.6162	-0.3582	0.0681	1.7256	30.7226	9105.26	0	20
Portfolio 2-1	0.0004	0.0001	0.1610	-0.0895	0.0251	0.4247	6.9663	557.36	0	168
Portfolio 2-2	0.0005	0.0003	0.0875	-0.1291	0.0164	-0.5437	9.6619	1718.12	0	306
Portfolio 2-3	0.0012	0.0018	0.1317	-0.1225	0.0165	-0.5476	15.7817	6205.71	0	211
Portfolio 2-4	-0.0013	-0.0006	0.1427	-0.1472	0.0290	-0.1451	6.5240	223.49	0	39
Portfolio 2-5	-0.0024	-0.0010	0.4518	-0.7327	0.0685	-1.1502	27.1450	16079.44	0	22
Portfolio 3-1	-0.0009	-0.0004	0.6381	-0.2411	0.0475	2.4817	41.6672	56548.78	0	68
Portfolio 3-2	0.0005	0.0008	0.1451	-0.1285	0.0212	-0.0522	8.5311	1154.04	0	190
Portfolio 3-3	0.0004	0.0018	0.3290	-0.1713	0.0314	1.0253	19.5871	10533.33	0	306
Portfolio 3-4	0.0002	0.0012	0.3484	-0.1860	0.0307	1.1774	25.2914	18946.65	0	169
Portfolio 3-5	0.0000	0.0023	0.3449	-0.3074	0.0564	0.2005	11.7306	2663.87	0	55
Portfolio 4-1	-0.0031	-0.0027	0.3368	-0.3431	0.0645	-0.0719	7.2713	657.54	0	42
Portfolio 4-2	-0.0044	-0.0031	0.6419	-0.6131	0.1093	-0.0467	13.3482	3230.64	0	44
Portfolio 4-3	-0.0012	0.0005	0.2125	-0.2171	0.0456	-0.2826	5.9467	339.48	0	151
Portfolio 4-4	0.0001	0.0016	0.0986	-0.2187	0.0294	-1.1985	10.7159	2461.64	0	357
Portfolio 4-5	-0.0003	0.0009	0.2722	-0.2257	0.0512	-0.0361	6.1238	367.75	0	154
Portfolio 5-1	-0.0047	-0.0027	0.3076	-0.4538	0.0684	-0.5941	8.1894	1061.65	0	24
Portfolio 5-2	-0.0010	-0.0009	0.3017	-0.3716	0.0670	0.0974	6.8451	529.28	0	22
Portfolio 5-3	0.0019	-0.0076	0.8820	-0.3600	0.1125	1.4343	10.7384	2199.43	0	46
Portfolio 5-4	-0.0016	0.0017	0.3410	-0.2585	0.0423	-0.3354	12.1648	3184.25	0	153
Portfolio 5-5	-0.0021	0.0006	0.1957	-0.2663	0.0393	-0.9676	10.4294	2222.55	0	412

Table 2. Beta, systematic skewness and systematic kurtosis estimates

The table reports the estimated market beta, systematic skewness and systematic kurtosis of the 25 portfolios formed by the intersection of 5 systematic skewness and 5 systematic kurtosis quintiles. The market beta, systematic skewness and systematic kurtosis are computed as follows:

$$\beta_i = \frac{E[\{R_i - E(R_i)\}\{R_m - E(R_m)\}]}{E[\{R_m - E(R_m)\}]^2}, \quad S_i = \frac{E[\{R_i - E(R_i)\}\{R_m - E(R_m)\}^2]}{E[\{R_m - E(R_m)\}]^3} \quad K_i = \frac{E[\{R_i - E(R_i)\}\{R_m - E(R_m)\}^3]}{E[\{R_m - E(R_m)\}]^4}$$

Panel A: Market Beta

	Low Sys Kurtosis	2	3	4	High Sys. Kurtosis
Low Sys. Skewness	0.311174	0.26816	0.516089	0.711338	1.857418
2	0.243001	0.30416	0.31601	0.974334	1.163463
3	0.555916	0.395354	0.554134	0.651293	0.978625
4	0.567579	0.326248	0.61261	0.741613	1.037039
High Sys.Skewness	0.382436	0.555464	0.80593	0.865672	1.013571

Panel B: Systematic Skewness

	Low Sys Kurtosis	2	3	4	High Sys. Kurtosis
Low Sys. Skewness	0.6693	-0.0438	-0.2763	0.2652	-0.2448
2	0.6354	0.5077	0.4338	0.3883	0.4721
3	1.5265	0.9333	1.289	1.2552	1.1804
4	2.2314	2.1372	2.3703	1.8085	1.9682
High Sys.Skewness	2.8773	3.2358	3.948	2.9127	2.6647

Panel C: Systematic Kurtosis

	Low Sys Kurtosis	2	3	4	High Sys. Kurtosis
Low Sys. Skewness	0.3095	0.1402	0.3916	0.5628	1.2979
2	0.1733	0.2171	0.2876	0.6197	1.2293
3	0.4795	0.3488	0.6112	0.8473	1.2702
4	0.349	0.538	0.7377	0.9571	1.3462
High Sys.Skewness	-0.067	0.7382	0.8243	1.1449	1.4505

Table 3. Multivariate Tests of Intercepts from the Four-Moment Model

The table reports results of the multivariate tests on zero intercepts from the four-moment model. The test-statistic is the Gibbons-Ross-Shanken statistic which follows an F-distribution with degrees of freedom N and T-N-L where T is the total observations, N is the number of portfolios and L is the number of regression parameters in the model including the intercept. The GRS-statistic is computed as $\left(\frac{T}{N}\right)\left(\frac{T-N-L}{T-L-1}\right)\left(\bar{R}'_p\,\hat{\Omega}^{-1}\,\bar{R}_p\right)^{-1}\left(\hat{\delta}_0{}'\hat{\Sigma}^{-1}\hat{\delta}_0\right)$ where $\bar{R}_p$ is a vector of sample mean for vector $R_{pt}=(R_{1t},R_{2t},...,R_{Nt})$; $\hat{\Omega}$ is the sample variance-covariance matrix for R_p ; $\hat{\Sigma}$ is the variance-covariance matrix of the residuals from the time-series regression; $\hat{\delta}_0=(\delta_{01,}\delta_{02}...\delta_{0N})$ is the vector of the least squares estimators for the pricing errors of the time-series regressions where δ_{0i} is the intercept of the regression of portfolio i on L regression parameters. P-values are presented in parentheses. * and ** denote the statistical significance at 5 and 1 percent levels.

Period	Two-moment model GRS-statistic	Four-Moment Model GRS-statistic
1992-1996	0.991885**	0.905154
	(0.0028)	(0.5983)
1997-2001	1.663674**	1.764330
	(0.0022)	(0.3666)
2001-2006	2.009365**	0.808158
	(0.0069)	(0.7310)
2007-2009	1.278295*	1.53517
	(0.0432)	(0.0721)
1992-2009	1.911462**	1.920141**
	(0.0007)	(0.0044)

Table 4. Comparison of Risk Premium Estimates using Traditional CSR vs Dagenais and Dagenais Higher-Moment Estimators Corrected for EIV Problems

The table reports the significance of average risk estimates of the market , the systematic skewness and the systematic kurtosis premia using (1) week-by-week CSR model after estimating the betas in rolling regressions using 30 weeks at a time (T=30) and (2) DDHME for cross-sectional regressions with EIV. The CSR model is $R_{p,i} - r_f = \alpha + \delta_1\beta_{1,i} + \delta_2\beta_{2,i} + \delta_3\beta_{3,i} + \varepsilon_i \quad i = 1,\cdots,N$ .The equation for DDHME cross-sectional regressions with EIV is: $R_{p,i} - r_f = \alpha + \delta_1\beta_{1,i} + \delta_2\beta_{2,i} + \delta_3\beta_{3,i} + \varphi_1\hat{w}_1 + \varphi_2\hat{w}_2 + \varphi_3\hat{w}_3 + \varepsilon_i \quad i = 1,\cdots,N$ where $\beta_{1,i}$ $\beta_{2,i}$ and $\beta_{3,i}$ are obtained from rolling time-series regressions in the first-pass of CRS to estimate risk factor for each week and $\hat{w}_i$ are the difference between the observed β_i and the estimated $\hat{\beta}_i$ .The null hypotheses of $\delta_i = 0$ and $\hat{\varphi}_i = 0$ are tested and $t_k = \dfrac{\bar{\delta}_k - 0}{\sigma_{\hat{\delta}_k}}$ and $v_k = \dfrac{\bar{\varphi}_k - 0}{\sigma_{\hat{\varphi}_k}}$ are reported respectively. * and ** denote the statistical significance at the 5 and 1 percent levels.

Period	Two-pass CSR			Higher-Moment Estimators					
	t_1 (β_{Market})	t_2 $(\beta_{Skewness})$	t_3 $(\beta_{Kurtosis})$	t_1 (β_{Market})	t_2 $(\beta_{Skewness})$	t_3 $(\beta_{Kurtosis})$	v_1 (w_{Market})	v_2 (w_{Market})	v_3 (w_{Market})
1992-1996	-0.77	0.27	-0.86	0.54	-0.27	1.68^	1.24	-0.06	-1.19
1997-2001	-1.01	2.14**	-2.37**	0.85	1.18	1.16	0.49	-0.64	-0.61
2002-2006	0.62	1.03	1.09	-0.65	0.20	0.37	0.44	0.52	0.43
2007-2009	1.97**	-2.03**	-3.77**	-1.07	-0.42	0.10	1.03	-2.00**	1.59
whole period	-1.51	2.80**	-3.41**	-0.66	2.24**	2.23**	0.34	1.16	1.03