Pseudo Maximum Likelihood Estimation of Structural Credit Risk Models with Exogenous Default Barrier*

Santiago Forte **Lidija Lovreta**

December 2009

Abstract

In this paper we propose a novel approach to the estimation of structural credit risk models with exogenous default barrier. The method consists of an iterative algorithm which, on the basis of the log-likelihood function for the time series of equity prices, provides pseudo maximum likelihood (ML) estimates of the default barrier and of the value, volatility, and expected return on the firm's assets. We demonstrate empirically that, contrary to the standard ML approach, the proposed method ensures that the default barrier always falls within reasonable bounds. Moreover, theoretical credit spreads based on pseudo ML estimates offer the lowest credit default swap pricing errors when compared to the options that are usually considered when determining the default barrier: standard ML estimate, endogenous value, KMV's default point, and principal value of debt.

JEL classification: G12, G13

Keywords: Structural credit risk models, default barrier, pseudo maximum likelihood estimation.

* Santiago Forte is at ESADE Business School – Ramon Llull University, Av. Torreblanca 59, E-08172, Sant Cugat del Vallès (Barcelona, Spain); santiago.forte@esade.edu. Lidija Lovreta (corresponding author) is also at ESADE Business School – Ramon Llull University, Av. Torreblanca 59, E-08172, Sant Cugat del Vallès (Barcelona, Spain); lidija.lovreta@esade.edu. This paper has been partially funded by Fundación UCEIF and Banco Santander. Lidija Lovreta also acknowledges financial support from Ministerio de Asuntos Exteriores y de Cooperación, y Agencia Española de Cooperación Internacional (MAEC-AECID).

Introduction

Thirty-five years after Merton's (1974) seminal paper, no consensus yet exists on the ability of structural models to reflect the credit risk of companies.[1] Under the structural setting, debt and equity are treated as contingent claims on the underlying firm's asset value, which, accordingly, becomes the fundamental source of uncertainty driving credit risk. Following this argument, structural models should be able to transform the information on a firm's asset value process provided by equity prices into the information on credit risk provided, in turn, by credit spreads. This ability of structural models to explain observable market levels of credit spreads has been precisely the cornerstone of most empirical tests. Until recently, the broadly accepted conclusion was that in this regard structural models have not been highly successful (Jones et al., 1984; Ogden, 1987; Lyden and Saraniti, 2001; Huang and Huang, 2003; Eom et al., 2004).

The theoretical completeness of structural models has raised a question, however: To what extent could this seemingly poor performance actually be a product of the estimation methods applied? Key determinants of credit spreads – a firm's asset value and volatility, along with the default barrier – represent pure latent variables; thus, any empirical test necessarily represents a simultaneous test of both the structural model at hand and the estimation method itself. Ericsson and Reneby (2005) and Li and Wong (2008) have shown that the empirical performance of structural models is, in fact, largely undermined by traditional approaches to the estimation of the firm asset value and volatility (i.e. proxy and volatility restriction methods). On the contrary, the maximum likelihood (ML) approach – novel in this context and first motivated by Duan (1994, 2000) – provides much greater support for theoretically appealing structural credit risk models.[2]

In a similar vein, increasing attention is being paid to the exact definition of the default barrier. In general, the default triggering firm asset value may either be set exogenously (e.g. Longstaff and Schwartz, 1995) or endogenously obtained inside the model as the optimal decision for equity holders (e.g. Nielsen et al., 1993; Leland, 1994; Leland and Toft, 1996).[3] Although this second alternative seems more appealing, it ignores the potential influence of other factors on the event of default (e.g. debt covenants, liquidity restrictions, insolvency codes). A more recent approach is based on the assumption of an exogenous default barrier; but rather than imposing a somewhat arbitrary value (e.g. the debt's face value, KMV's default point), market data are used to derive this model parameter. Wong and Choi (2009) consider this possibility in the case of the down-and-out call valuation model discussed by Brockman and Turtle (2003). Specifically, they maximize the likelihood function for the time series of equity prices not only as a function of the expected rate of return and volatility of the firm assets, but also as a function of the default barrier. Wong and Choi's paper provides an insightful analysis of some of the drawbacks of using the proxy approach – as in Brockman and Turtle (2003); however, their results also indicate that standard maximization of the likelihood function can generate misleading results. To be precise, at least 25% of their reported barriers are equal to zero, whereas almost 45% are above nominal debt and 25% of those values are above two-and-a-half times the face value of the debt.[4]

Misleading results from likelihood maximization are typically a reflection of an ill-behaved likelihood function – an old problem in statistics that more commonly appears with an increase in the number of unknown parameters – as in the present case. Under these circumstances, however, *ad hoc* procedures can sometimes be defined which, following the spirit of likelihood maximization, are naturally referred to as pseudo maximum likelihood estimation methods (e.g. Gong and Samaniego, 1981). In this paper we propose one such method for the estimation of structural credit risk models with exogenous default barrier. More explicitly, an iterative algorithm is defined, which, based on the log-likelihood function for the time series of equity prices, provides pseudo ML estimates of the default barrier and of the value, volatility, and expected rate of return on the firm's assets. The suggested approach is tested empirically using an international sample of 96 companies, whereas the reference credit risk model corresponds to the modified version of Leland and Toft's (1996) model suggested by Forte (2009). It is shown that – in line with Wong and Choi (2009) – the standard ML approach results in unreal barriers for a substantial proportion of the companies considered. On the contrary, the pseudo ML approach suggested in this paper generates reasonable values that fall in the range of 50.3% to 96.9% of the principal value of debt. In terms of credit default swap (CDS) spread estimation, theoretical credit spreads based on the proposed method provide the lowest pricing errors when compared to other options that are usually considered when specifying the default barrier: standard ML estimate, endogenous value, KMV's default point, and principal value of debt.[5]

It is worth noting that recent studies suggest using CDS data in addition to equity data for the estimation of structural credit risk models. Predescu (2005), for example, derives the joint likelihood function for the time series of equity prices and CDS spreads, where the equity pricing equation corresponds to the same down-and-out call valuation model analyzed by Brockman and Turtle (2003) and Wong and Choi (2009). Additional information on CDS premia guarantees a well-behaved likelihood function and, consequently, standard likelihood maximization provides default barrier estimates within reasonable bounds in this case.[6] Following a different approach, Forte (2009) employs an iterative scheme to derive the time series of firm asset values and the corresponding volatility from the time series of equity prices, whereas the default barrier is calibrated from the time series of CDS spreads. Again, use of both equity and CDS data ensure reasonable results for most of the cases.

In this paper we explicitly refrain from using market data other than equity prices. Although the use of CDS spreads for the estimation of structural models undoubtedly represents an appealing approach, it does not allow for the most common situation in which such information is either unavailable or unreliable. As this is exactly the situation in which information regarding credit risk becomes more valuable, this is the one we presume in this study.

The remainder of the paper is structured as follows. Section 1 describes the structural model setting. Section 2 summarizes the standard ML approach. Section 3 presents the proposed alternative: the pseudo ML estimation approach. Other methods that are usually applied in determining the default barrier are briefly discussed in Section 4. Section 5 offers a full description of our data set. Section 6 provides the empirical results, in terms of both parameter estimates and predicted spreads. The main conclusions are drawn in Section 7.

1. The Structural Model Setting

As our reference credit risk model, we consider the modified version of Leland and Toft's (1996) model suggested by Forte (2009). This model has already been shown to generate reasonable predictions on credit spreads as long as the appropriate default barrier is selected; therefore, it seems suitable for testing the performance of the pseudo ML estimation approach that we propose in this paper. Here we merely describe the main features of the model, referring the interested reader to the original paper for details.

The market value of total assets at any time t, V_t, is assumed to evolve according to the continuous diffusion process:

$$dV_t = (\mu - \delta)V_t dt + \sigma V_t dz, \qquad (1)$$

where μ is the expected rate of return on the asset value, δ is the fraction of the asset value paid out to investors, σ is the asset return volatility, and z is a standard Brownian motion. Default occurs whenever V_t reaches a specific critical point V_b, defined as a fraction β of the nominal value of total debt P:

$$V_b = \beta P. \tag{2}$$

The value of an individual bond d_n, with maturity τ_n, principal p_n, and constant coupon flow c_n, is given by:

$$d_n(V_t, \tau_n) = \frac{c_n}{r} + e^{-r\tau_n}\left[p_n - \frac{c_n}{r}\right][1 - F_t(\tau_n)] + \left[(1-\alpha)\beta p_n - \frac{c_n}{r}\right] G_t(\tau_n), \tag{3}$$

for $n = \{1, \dots, N\}$, where r is the risk-free rate, α represents bankruptcy costs, and expressions for $F_t(\tau_n)$ and $G_t(\tau_n)$ are given in Appendix A. The total debt value is then represented by the sum of all outstanding bonds:

$$D(V_t) = \sum_{n=1}^{N} d_n(V_t, \tau_n). \tag{4}$$

Finally, the equity value is expressed as:

$$S_t = g(V_t) = V_t - D(V_t | \alpha = 0), \tag{5}$$

where $D(V_t|\alpha = 0)$ is the market value of total debt when bankruptcy costs equal zero. This expression follows from the reasoning that the presence of bankruptcy costs affects only creditors who, in case of default, receive only a fraction $(1 - \alpha)$ of the firm's asset value.

2. Standard Maximum Likelihood Estimation

In order to overcome the problem of unobservability of the asset value process and the default barrier parameter β, it is possible to apply ML estimation using the transformation of variables technique – an idea originally introduced by Duan (1994, 2000). When applied to this specific problem, the observable data set of equity values $S = \{S_t;\ t = 1, \dots, T\}$, can be treated as a transformed data set of the unobservable underlying firm asset values $V = \{V_t;\ t = 1, \dots, T\}$. The ML procedure is then carried out by deriving the log-likelihood function for the transformed equity values $L_s(S; \theta)$, where the theoretical equity pricing formula of the structural model at hand, $S_t = g(V_t; \theta);\ t = \{1, \dots, T\}$, serves as a strictly monotonic, one-to-one transformation function. Accordingly, θ represents the set of unknown parameters to be estimated, along with the complete vector of unobservable firm asset values V.

Regardless of the specificities of the underlying structural model, a complete closed-form solution for the log-likelihood function of the observable data set S, could be derived using standard results on differentiable transformations. Accounting for the survivorship issue under the first-passage time framework, and in the simplest case of exogenous and constant default barrier, such a log-likelihood function can be expressed as (Duan et al., 2003, 2004):[7]

$$
\begin{aligned}
L_s(S;\theta) &= L(\hat{V};\sigma,\mu,\beta) \\
&= L_v(\hat{V};\sigma,\mu) + \sum_{t=2}^{T} \ln\left[1 - e^{\left(\frac{-2}{\sigma^2\Delta t}\right) ln\left(\frac{\hat{V}_{t-1}}{V_b}\right) ln\left(\frac{\hat{V}_t}{V_b}\right)}\right] \\
&\quad - \ln[P_{nd}(\sigma,\mu,\beta)] - \sum_{t=2}^{T} \ln\left|\frac{\partial g(\hat{V}_t;\sigma,\beta)}{\partial \hat{V}_t}\right|,
\end{aligned}
\tag{6}
$$

where $\hat{V}$ represents the vector of implied firm asset values for a given set $\{\sigma, \beta\}$, and for the invertible equity pricing equation $\hat{V}_t = g^{-1}(S_t; \sigma, \beta)$.

The first term in expression (6) reflects the log-likelihood function for the time series of the log-normally distributed firm asset values:

$$L_v(\hat{V};\sigma_v,\mu) = -\sum_{t=2}^{T}\ln\hat{V}_t - \frac{T-1}{2}ln(2\pi\sigma^2\Delta t)$$
$$-\frac{1}{2\sigma^2\Delta t}\sum_{t=2}^{T}\left[\ln\left(\frac{\hat{V}_t}{\hat{V}_{t-1}}\right)-\left(\mu-\delta-\frac{\sigma^2}{2}\right)\Delta t\right]^2. \tag{7}$$

The next two terms account for the survivorship issue, with $P_{nd}(\theta)$ actually denoting the survival probability during the entire sample period:

$$P_{nd}(\sigma,\mu,\beta) = \boldsymbol{\Phi}\left[\frac{\left(\mu-\delta-\frac{1}{2}\sigma^2\right)(T-1)\Delta t - ln\left(\frac{V_b}{\hat{V}_1}\right)}{\sigma\sqrt{(T-1)\Delta t}}\right]$$
$$- e^{\left(\frac{2}{\sigma^2}\right)\left(\mu-\delta-\frac{\sigma^2}{2}\right)ln\left(\frac{V_b}{\hat{V}_1}\right)}\boldsymbol{\Phi}\left[\frac{\left(\mu-\delta-\frac{1}{2}\sigma^2\right)(T-1)\Delta t + ln\left(\frac{V_b}{\hat{V}_1}\right)}{\sigma\sqrt{(T-1)\Delta t}}\right], \tag{8}$$

where $\boldsymbol{\Phi}(\cdot)$ refers to the standard normal distribution function.

The fourth and final term in expression (6) reflects the Jacobian of the transformation. Appendix B provides the exact analytical expression for the derivative of the transformation, $g(V_t;\sigma,\beta)$,in the particular case of the model proposed by Forte (2009).

Following the conventional principle of likelihood maximization, the standard approach derives the entire set of unobservable parameters by solving the maximization problem:

$$Max_{\{\sigma,\mu,\beta\}} \quad L(\hat{V};\sigma,\mu,\beta). \tag{9}$$

3. Pseudo Maximum Likelihood Estimation

In most empirical applications, the exogenous default barrier is predefined, either at the face value of the debt, or at a given fraction of this value (e.g. KMV's default point). In such cases, the likelihood function is well-behaved in the parameter space $\theta = \{\sigma, \mu\}$, and numerical maximization is always feasible. The complexity of the problem, however, is further augmented when the default barrier itself belongs to the parameter space, $\theta = \{\sigma, \mu, \beta\}$. In this case, the likelihood function sometimes exhibits a nonstandard behavior when applied to real data, and numerical routines may converge to spurious parameter values, particularly for the default barrier.

As an alternative to the standard ML approach described in the previous section, we suggest a pseudo ML estimation method. Note first that the unrestricted maximization problem in the parameter space $\{\sigma, \mu, \beta\}$ described in (9) could be actually thought of as a restricted maximization problem in the space $\{V, \sigma, \mu, \beta\}$, specifically,

$$Max_{\{V,\sigma,\mu,\beta\}} \quad L(V, \mu, \sigma, \beta) \quad s.t. \;\; \{V_t = g^{-1}(S_t; \sigma, \beta);\; t = 1, \dots, T\}, \tag{10}$$

where the restriction $V_t = g^{-1}(S_t; \sigma, \beta)$ states that for any possible set of parameter values $\{\sigma, \beta\}$, the firm asset value at t is derived by inverting the transformation function. Referring specifically to the problem that default barrier estimates could often reach unreasonable values under the standard maximization approach, we propose an estimation of the set of unknown parameters $\theta = \{\sigma, \mu, \beta\}$, along with the whole vector of a firm's asset values V, by means of the following iterative algorithm:

Step 1. Propose an initial value for the default-to-debt ratio β_0, and estimate the time series of the firm's asset values V, the volatility σ, and the expected rate of return μ, by solving the restricted maximization problem:

$$Max_{\{V,\mu,\sigma\}} \quad L(V,\sigma,\mu|\beta_0) \quad s.t. \;\; \{V_t = g^{-1}(S_t;\sigma,\beta_0);\; t = 1,\dots,T\}. \tag{11}$$

In this way a set of ML estimates V_0, σ_0 and μ_0 is derived conditional on the predefined value of β, β_0.

Step 2. Departing from the obtained set of ML estimates in Step 1, solve the unrestricted maximization problem:

$$Max_{\{\beta\}} \quad L(\beta|V_0,\sigma_0,\mu_0); \tag{12}$$

This will generate a pseudo ML estimate of β, β_1, given the predefined values V_0 , σ_0 and μ_0.

Step 3. If $\beta_1 = \beta_0$, convergence is attained. If not, set $\beta_0 = \beta_1$ in Step 1 and repeat until convergence is achieved.

This algorithm provides estimates of parameter values σ, μ, and β, and of the whole vector of the firm's asset values V that do not necessarily maximize the log-likelihood function globally, as in the standard procedure. It does, however, offer several noteworthy properties. (a) Estimates of σ, μ, and the whole vector of the firm's asset values V are estimates obtained at the global maximum point of the log-likelihood function, conditional upon the default barrier level (Step 1). (b) The default barrier is not arbitrarily fixed, but is determined conditional on the other parameter values and on the whole set of the firm's asset values (Step 2). (c) The final solution of the algorithm guarantees that the equity pricing equation is satisfied for all t, providing a consistent overall set of final parameter estimates (Steps 1 and 3). (d) Empirical results confirm that this final outcome is, in fact, unique, independent of the initial value of β, β_0. (e) The proposed procedure offers the major advantage of generating much more meaningful default barrier estimates than the standard ML approach does. This property is further discussed in Section 5.

4. Other Default Barrier Specifications

For completeness, it seems suitable to include in our analysis other approaches that are usually considered in determining the default barrier. In particular, we account for the endogenous value, KMV's default point, and nominal debt value.

- ***Endogenous value:*** Endogenous default models assume the default point to be optimally chosen by equity holders. It is specifically derived by invoking the smooth-pasting condition (e.g. Leland and Toft, 1996):

$$\left.\frac{\partial g(V_t)}{\partial V_t}\right|_{V_t=V_b} = 0. \tag{13}$$

In the case of the model suggested by Forte (2009), the endogenous default-to-debt ratio is given by:

$$\beta_{END} = \frac{\sum_{n=1}^{N}\left\{e^{-r\tau_n}\left[p_n - \frac{c_n}{r}\right]A(\tau_n) + \frac{c_n}{r}B(\tau_n)\right\}}{P + \sum_{n=1}^{N} p_n\, B(\tau_n)}, \tag{14}$$

where exact expressions for $A(\tau_n)$ and $B(\tau_n)$ are given in Appendix C.[8]

- ***KMV's default point:*** In the KMV methodology, the default point is determined as short-term liabilities plus 50% of long-term liabilities. In terms of the default-to-debt ratio,

$$\beta_{KMV} = \frac{STL + 0.5 \times LTL}{P}. \tag{15}$$

- ***Nominal debt value:*** Under the simplest assumption, the default barrier is set at the face value of the debt:

$$\beta_P = 1. \tag{16}$$

It is also worth noting that under these default barrier specifications, estimates of the firm's asset value and volatility must still be defined. In further empirical tests, and for the aim of rational comparisons, the ML approach will be used in these cases. More formally, we will solve the restricted maximization problem

$$Max_{\{V,\sigma,\mu\}} \quad L(V,\mu,\sigma|\beta_j) \quad s.t. \ \{V_t = g^{-1}(S_t;\sigma,\beta_j);\ t = 1,\dots,T\}, \tag{17}$$

for $j = \{END, KMV, P\}$.

5. Data

Our data set corresponds to the final sample of 96 nonfinancial companies (41 European, 32 US, and 23 Japanese) analyzed by Alonso et al., (2008). This data set comprises the entire period 2002-2004, containing:

- Daily data on market capitalization (close of business) obtained from DataStream.
- Daily data on 1- to 10-year locally denominated swap rates, also gathered from DataStream.
- Accounting items referring to short- and long-term liabilities, interest expenses, and cash dividends, collected from WorldScope.
- Daily data on CDS spreads (mid bid-ask quotes) obtained, at the close of business in London, New York and Tokyo, from CreditTrade. These data include only 5-year contracts denominated in local currency (euro, dollar, or yen). Furthermore, each company contains CDS data for at least two consecutive years, with a minimum of 150 observations per year.[9]

Using these data, we define those model inputs that are treated as known or observable, whether we deal with the standard or pseudo ML estimation method. Namely,

a) Equity value: Daily data on equity value, $S_t;\ t = 1,\dots,T$, will correspond to daily data on market capitalization.

b) Debt's principal value P: Given that P is treated as a constant, we use the sum of the average short-term liabilities (STL) and long-term liabilities (LTL).

c) Debt structure: In line with expression (4), we need to define the debt structure – the number of individual bonds (N) and their corresponding characteristics: time to maturity (τ_n), coupon (c_n), and principal (p_n). In order to resemble the true debt structure as much as possible, we adopt Forte's (2009) approach, and assume that at each instant t the company has ten bonds – one with a maturity of one year and principal equal to STL and nine with maturity ranging from two to ten years, each with principal equal to 1/9 of LTL. The coupon of each bond is determined as the fraction of average interest expenses (IE) proportional to the weight of the principal of each individual bond p_n, over the total principal value of debt P.

d) Payout rate: The payout rate δ is determined as the average annualized payment of interest expense (IE) and cash dividends (CD) to the proxy value of the firm, calculated as the sum of market value of equity and book value of total liabilities.

e) Risk-free interest rate: The risk-free rate for each individual bond is determined according to the swap rate for the corresponding maturity.

f) Recovery rate: Once the estimation of the unknown parameter values and firm asset values has been completed, theoretical stock market implied credit spreads (ICS) can be derived as the spread from issuing, at par value, a hypothetical bond with the same maturity as the CDS spread that serves as a benchmark (five years in our case).[10] In principle, this requires that we define a value for the bankruptcy costs α, which enters in expression (3) through the recovery rate, $(1-\alpha)\beta$. In terms of CDS spread valuation, however, the market practice is to consider a fixed recovery rate of 40%. For the aim of simplicity and more robust comparisons, we also adopt this convention.

Main descriptive statistics for the sample considered are depicted in Table 1. The average company in the sample has market capitalization of approximately $26.7 billion. Equity volatility, defined as the unconditional annualized standard deviation of the continuously compounded returns on equity, ranges around 36.9%. The mean leverage, calculated as the book value of total liabilities over the sum of market capitalization and book value of total liabilities, amounts to 52.7%. Yet the leverage of the companies in the sample varies, with indebtedness ranging from 3.8% to 92.1%. Regarding CDS spreads, the mean level for the entire period considered ranges from 11.33 bp to 306.40 bp on an individual firm basis, with the overall cross-sectional mean for all entities being 71.82 bp. The average number of daily observations per company is 630, whereas the majority of the companies in the sample refer to A and BBB rated issuers.

<Table 1 about here>

6. Results

6.1. Parameter Values

Final results on parameter values are shown in Table 2, where we include estimates provided by the standard ML approach (MLE), estimates from the pseudo ML approach (ALG), and estimates resulting from the assumption of an endogenous default barrier (END).[11] Main descriptive statistics in Panel A indicate that standard maximization of the log-likelihood function leads to default barriers which are, on average, higher than the face value of the debt (mean β_{MLE} of 1.093). In addition, the dispersion is significant, with a minimum of 0.025 and a maximum of 6.762. We should reiterate that these types of results are difficult to reconcile with economic intuition. In the first case, the probability of default is almost nil; in the second situation, the firm is not able to continue running operations, even when the firm asset value is worth as much as 6.7 times the face value of the debt. Moreover, predicted credit spreads will be lack of sense in both cases.[12]

These puzzling results are an indication of a likelihood function that is not well-behaved. Figure 1, for example, presents the behavior of the log-likelihood function for BASF AG. It is apparent that a global maximum is not achieved in this case for any reasonable default-to-debt ratio, and standard log-likelihood maximization actually converges to a misleading value of 2.085. If we look at the whole distribution of default-to-debt ratios in Table 2, Panel B, we conclude that a similar situation is, in fact, repeated for a significant number of companies. Notwithstanding, reasonable values are also achieved for many of the cases. Take, for instance, the log-likelihood function of Bouygues SA shown in Figure 2. Even though this log-likelihood function is relatively flat in terms of the default-to-debt ratio, the obtained value from the standard approach (0.783) represents a reasonable estimate. In summary, we can conclude that the standard ML approach neither rules out nor guaranties reasonable results.

On the opposite side, parameter estimates from both the pseudo ML approach and the endogenous default barrier approach represent meaningful values for all of the companies. More precisely, the default-to-debt ratio β_{ALG} ranges from a minimum of 0.503 to a maximum of 0.969, with an average value of 0.801. On the other hand, the minimum, maximum, and mean values of β_{END} are 0.438, 0.945 and 0.752, respectively. If we compare estimates of the firm asset volatility and of the expected rate of return, both methods provide virtually identical results; besides, they are typically higher than those provided by the standard ML approach. Going back to the instance of BASF AG in Figure 1, we observe that, in effect, both methods generate more rational results than does the standard ML approach. Moreover, analysis of the results for Bouygues SA in Figure 2 reveals an interesting feature: pseudo ML estimates in the case of a well-behaved log-likelihood function as the one of this company are similar to those provided by the standard ML approach. We explore this issue further in Table 3, where the main descriptive statistics for the difference between pseudo ML and standard ML parameter estimates are provided. Although the mean absolute difference between default-to-debt ratios for all companies is 0.431 (median of 0.099), the mean difference among the group of companies with the most reasonable β_{MLE} values (higher than 0.3 and lower than 1) is merely 0.044 (0.016).

<Table 2 about here>

<Figure 1 about here>

<Figure 2 about here>

<Table 3 about here>

Previous results allow for several conclusions. (a) The standard ML approach does not represent a good candidate for the preferred method, as it often provides puzzling results. (b) In cases in which the log-likelihood is well-behaved and the standard ML approach generates rational values, the pseudo ML approach leads to similar results. Notwithstanding, the pseudo ML approach provides reasonable values, even when the standard ML approach seems to fail. (c) We could naturally think of the endogenous default-to-debt ratio as a lower bound for the true value. In other words, factors that differ from the interest of equity holders (e.g. debt covenants, liquidity restrictions, bankruptcy codes) – if present – are expected to move the default barrier upwards.[13] We find further support for the pseudo ML approach in view of this argument, as not only is β_{ALG} higher than β_{END} on average, but this seems to be the general rule on a firm-by-firm basis: it holds for 94 out of the 96 companies in our sample. Yet, β_{ALG} values within reasonable bounds are only a minimum requirement. Assessment of the real precision – and utility – of the proposed method, requires an investigation of its ability to generate sensible credit spread estimates as well; this point is addressed in the next sub-section.

6.2. Implied Credit Spreads

Results on model implied credit spreads (ICS) on the basis of the different estimation methods are provided in Table 4, along with the corresponding CDS spreads. We observe that, in fact, replication of CDS spreads is highly influenced by the chosen estimation method. Although the cross-sectional mean level of CDS spreads (71.82 bp) is almost fully matched by ICS_{ALG} estimates (71.34 bp), this does not hold for other options that either underestimate (45.49 bp for ICS_{END} and 41.07 bp for ICS_{KMV}) or considerably overestimate (194.98 bp for ICS_{MLE} and 223.29 bp for ICS_{P}) CDS premia. In addition, ICS_{ALG} have another desirable characteristic: contrary to other methods – particularly the standard ML approach and the KMV approach – ICS_{ALG} completely follow the pattern and the level of CDS spreads over different rating categories.[14]

<Table 4 about here>

More formal, standard measures of price discrepancy between ICS and CDS series are summarized in Table 5. In particular, pricing errors are measured by: average basis – avb; percentage average basis – avb(%); average absolute basis – avab; percentage average absolute basis – avab(%); and root mean squared error – rmse. Results confirm the initial conclusion set forth in Table 4; that is, among all possible estimation methods, the pseudo ML approach provides the best predictions on CDS spreads. Specifically, the ICS_{ALG} – CDS basis is, on average, -0.48 bp, suggesting that the ICS_{ALG} represent, in practice, an unbiased estimator of the CDS spread; furthermore, the mean absolute basis amounts to 43.01 bp. The second-best option corresponds to ICS_{END}, with an average basis of -26.33 bp and an average absolute basis of 46.88 bp. The systematic underestimation of credit spreads, as suggested also by results in Table 4, is, however, consistent with the underestimation of the default barrier discussed in previous sub-section. The third-best option seems to be provided by KMV's default point: average basis of -30.75 bp and average absolute basis of 58.66 bp. As a counterpart, pricing errors in the light of other methods are sizable, with ICS_{MLE} and ICS_P far above CDS spreads (average basis of 123.16 and 151.47 bp, respectively). The overall conclusion is clear support for the pseudo ML estimation method in comparison with other methods, and particularly in comparison with the standard ML approach. Among all possible options, the endogenous default barrier method represents the second-best option.

<Table 5 about here>

As a final illustration of the empirical performance of the pseudo ML procedure, we assess risk-neutral default probabilities for time horizons ranging from 1 to ten 10 years. Risk-neutral default probabilities across the different rating categories, and on a cross-sectional basis, are presented in Figure 3. Following economic intuition, the estimated probabilities of default increase with both the time horizon and the average credit riskiness. By way of example, the estimated risk-neutral default probabilities for the 5-year horizon are: 1% for an AAA-AA rated company, 2.8% for A, 5.2% for BBB, and 9.3% for BB.

<Figure 3 about here>

6.3. Robustness Check

In order to attain robustness, it seems suitable to verify whether the estimation method suggested in this paper is also capable of providing reasonable results for other structural models, and not merely for the one considered in this study. Of particular interest is the down-and-out call (DOC) barrier option model discussed by Brockman and Turtle (2003), Predescu (2005), and Wong and Choi (2009). Analytical expressions for the DOC pricing equation and the first derivative of the element-by-element transformation are given in Appendix D.[15]

Results from both the standard and pseudo ML estimation method are provided in Table 6. In line with Wong and Choi (2009), standard likelihood maximization results in unreal default-to-debt ratios for a substantial number of companies. Namely, this parameter is higher than the one for as many as 39 companies (41% of the sample), with a maximum of 4.410. On the other hand, pseudo ML estimates represent reasonable values that fall in the range of 0.156 to 0.891, with a mean value of 0.616. Although estimated values for the default-to-debt ratio are lower than those provided in Table 2, this should be interpreted as a product of the differences in the underlying structural models. In fact, results in this case are consistent with those in Predescu (2005): in the light of the same DOC model, while using additional data on CDS spreads, she reports a mean default-to-debt ratio of 0.591. All things considered, we conclude that the proposed method leads to more meaningful results than does the standard ML approach, irrespective of the underlying model.

<Table 6 about here>

7. Conclusions

In this paper we present a new approach for the estimation of structural credit risk models with an exogenous default barrier. Specifically, we introduce an iterative algorithm that provides pseudo ML estimates of the default barrier and estimates of the value, volatility, and expected return on the firm's assets. This new approach is tested empirically on the basis of an international sample of 96 companies. Taking as the reference credit risk model, the modified version of Leland and Toft's (1996) model suggested by Forte (2009), it is confirmed that standard maximization of the log-likelihood function often results in unreal barriers. On the contrary, the pseudo ML approach proposed in this paper generates reasonable values that fall in the range of 50.3% to 96.9% of the nominal debt value. In terms of CDS spread estimation, theoretical credit spreads based on the suggested method generate the lowest pricing errors when compared to the other options that are usually considered when specifying the default barrier: standard ML estimate, endogenous value, KMV's default point, and principal value of debt.

Appendix

Appendix A

We provide specific expressions for $F_t(\tau_n)$ and $G_t(\tau_n)$:

$$F_t(\tau_n) = \Phi[h_{1t}(\tau_n)] + \left(\frac{V_t}{V_b}\right)^{-2a} \Phi[h_{2t}(\tau_n)]; \tag{A.1}$$

$$G_t(\tau_n) = \left(\frac{V_t}{V_b}\right)^{-a+z} \Phi[q_{1t}(\tau_n)] + \left(\frac{V_t}{V_b}\right)^{-a-z} \Phi[q_{2t}(\tau_n)]; \tag{A.2}$$

where

$$q_{1t} = \frac{-b_t - z\sigma^2\tau_n}{\sigma\sqrt{\tau_n}}; \qquad q_{2t} = \frac{-b_t + z\sigma^2\tau_n}{\sigma\sqrt{\tau_n}};$$

$$h_{1t} = \frac{-b_t - a\sigma^2\tau_n}{\sigma\sqrt{\tau_n}}; \qquad h_{2t} = \frac{-b_t + a\sigma^2\tau_n}{\sigma\sqrt{\tau_n}};$$

$$a = \frac{r - \delta - \frac{\sigma^2}{2}}{\sigma^2}; \qquad b_t = \ln\left(\frac{V_t}{V_b}\right); \qquad z = \frac{\sqrt{(a\sigma^2)^2 + 2r\sigma^2}}{\sigma^2}.$$

Appendix B

The derivative of the transformation $g(V_t; \sigma, \beta)$ is given by:

$$\frac{\partial g(V_t; \sigma, \beta)}{\partial V_t} = \frac{\partial[V_t - D(V_t | \alpha = 0; \sigma, \beta)]}{\partial V_t} = 1 - \sum_{n=1}^{N} \frac{\partial d_n(V_t | \alpha = 0; \sigma, \beta)}{\partial V_t}, \tag{B.1}$$

where

$$\frac{\partial d_n(V_t | \alpha = 0, \tau_n; \sigma, \beta)}{\partial V_t} = -e^{-r\tau_n}\left(p_n - \frac{c_n}{r}\right)\frac{\partial F_t(\tau_n)}{\partial V_t} + \left(\beta p_n - \frac{c_n}{r}\right)\frac{\partial G_t(\tau_n)}{\partial V_t}, \tag{B.2}$$

and

$$\frac{\partial F_t(\tau_n)}{\partial V_t} = f(h_{1t})\frac{\partial h_{1t}}{\partial V_t} - \left[\frac{2a}{V_b}\left(\frac{V_t}{V_b}\right)^{-2a-1}\right]\Phi(h_{2t}) + \left(\frac{V_t}{V_b}\right)^{-2a} f(h_{2t})\frac{\partial h_{2t}}{\partial V_t};$$

$$\frac{\partial G_t(\tau_n)}{\partial V_t} = \left[\frac{-a+z}{V_b}\left(\frac{V_t}{V_b}\right)^{-a+z-1}\right]\Phi(q_{1t}) + \left(\frac{V_t}{V_b}\right)^{-a+z} f(q_{1t})\frac{\partial q_{1t}}{\partial V_t} +$$

$$+\left[\frac{-a-z}{V_b}\left(\frac{V_t}{V_b}\right)^{-a-z-1}\right]\Phi(q_{2t}) + \left(\frac{V_t}{V_b}\right)^{-a-z} f(q_{2t})\frac{\partial q_{2t}}{\partial V_t};$$

with

$$\frac{\partial h_{1t}}{\partial V_t} = \frac{\partial h_{2t}}{\partial V_t} = \frac{\partial q_{1t}}{\partial V_t} = \frac{\partial q_{2t}}{\partial V_t} = -\frac{1}{V_t \sigma \sqrt{\tau_n}}.$$

Appendix C

In this case we provide exact expressions for $A(\tau_n)$ and $B(\tau_n)$:

$$A(\tau_n) = \frac{2f\left(a\sigma\sqrt{\tau_n}\right)}{\sigma\sqrt{\tau_n}} + 2a\Phi\left(a\sigma\sqrt{\tau_n}\right); \tag{C.1}$$

$$B(\tau_n) = (a-z)N\left(-z\sigma\sqrt{\tau_n}\right) + (a+z)N\left(z\sigma\sqrt{\tau_n}\right) + \frac{2f\left(z\sigma\sqrt{\tau_n}\right)}{\sigma\sqrt{\tau_n}}. \tag{C.2}$$

Appendix D

The equity pricing equation in Brockman and Turtle (2003) is given by:

$$S_t(\tau) = V_t\Phi(a_t) - Pe^{-r\tau}\Phi\left(a_t - \sigma\sqrt{\tau}\right) - V_t\left(\frac{V_b}{V_t}\right)^{2\eta}\Phi(b_t)$$
$$+Pe^{-r\tau}\left(\frac{V_b}{V_t}\right)^{2\eta-2}\Phi\left(b_t - \sigma\sqrt{\tau}\right), \tag{D.1}$$

where

$$a_t = \begin{cases} \dfrac{\ln\left(\frac{V_t}{P}\right) + \left(r + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}, & for\ P \geq V_b \\ \dfrac{\ln\left(\frac{V_t}{V_b}\right) + \left(r + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}, & for\ P < V_b \end{cases}$$

$$b_t = \begin{cases} \dfrac{\ln\left(\frac{V_b^2}{PV_t}\right) + \left(r + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}, & for\ P \geq V_b \\ \dfrac{\ln\left(\frac{V_b}{V_t}\right) + \left(r + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}, & for\ P < V_b \end{cases}$$

and

$$\eta = \frac{r}{\sigma^2} + \frac{1}{2}.$$

For tractability purposes, and in line with other empirical studies, rebate of zero for the original model is assumed. Finally, the derivative of the transformation is as follows:

$$\begin{aligned} \frac{\partial S_t(V_t;\sigma,V_b)}{\partial V_t} &= \Phi(a_t) + V_t f(q_{1t})\frac{\partial a_t}{\partial V_t} - Pe^{-r\tau} f\left(a_t - \sigma\sqrt{\tau}\right)\frac{\partial a_t}{\partial V_t} \\ &+ (2\eta - 1)\left(\frac{V_b}{V_t}\right)^{2\eta} \Phi(b_t) - V_t\left(\frac{V_b}{V_t}\right)^{2\eta} f(b_t)\frac{\partial b_t}{\partial V_t} \\ &- Pe^{-r\tau}(2\eta - 2)\left(\frac{V_b}{V_t}\right)^{2\eta-3}\left(\frac{V_b}{V_t^2}\right)\Phi\left(b_t - \sigma\sqrt{\tau}\right) \\ &+ Pe^{-r\tau}\left(\frac{V_b}{V_t}\right)^{2\eta-2} f\left(b_t - \sigma\sqrt{\tau}\right)\frac{\partial b_t}{\partial V_t}, \end{aligned} \tag{D.2}$$

where

$$\frac{\partial a_t}{\partial V_t} = \frac{1}{V_t\sigma\sqrt{\tau}}; \qquad \frac{\partial b_t}{\partial V_t} = -\frac{1}{V_t\sigma\sqrt{\tau}}.$$

Footnotes

[1] Merton's (1974) model was subsequently extended by Black and Cox (1976), Geske (1977), Longstaff and Schwartz (1995), Leland (1994), Leland and Toft (1996), Collin-Dufresne and Goldstein (2001), Zhou (2001), and others.
[2] Traditional approaches dominate the literature (Jones et al., 1984; Ronn and Verma, 1986; Ogden, 1987; Anderson and Sundaresan, 2000; Lyden and Saraniti, 2001; Brockman and Turtle, 2003; Delianedis and Geske, 2003; Eom et al., 2004). The maximum likelihood approach (Ericsson and Reneby, 2005; Ericsson et al., 2007; Li and Wong; 2008, Wong and Choi, 2009) is not the unique option, however. Other approaches include simulated maximum likelihood (Bruche, 2004, 2007; Duan and Fulop, 2009) and iterative schemes (Vassalou and Xing, 2004; Forte, 2009).
[3] In Merton's (1974) model, default can occur only at maturity of the debt. Following the ideas of Black and Cox (1976), this assumption was subsequently surpassed by allowing a firm to default at any time if the market value of its assets falls below some critical lower threshold value – the default barrier.
[4] Wong and Choi's results are not tested in light of their accuracy for credit spread estimation; it is very intuitive, however, that credit spreads will be equal to zero for zero default barriers. Furthermore, in the case of default boundaries in the order of two-and-a-half times (or more) the face value of the debt, reasonable credit spread estimates could be derived only at the cost of assuming unreasonably large bankruptcy costs.
[5] The use of CDS spreads as a reference is motivated by a second, not minor, problem in previous empirical tests: the traditional use of corporate-government yield spreads as a benchmark clashes with the evidence on non-credit risk factors in bond premia (Collin-Dufresne, et al., 2001). Accordingly, CDS spreads are increasingly seen as a preferred choice, further providing stronger support for structural credit risk models (Ericsson et al., 2007; Ericsson et al., 2009; Forte, 2009).
[6] Predescu also analyzes the case in which only equity data are employed for the estimation. She concludes that "using only equity prices, the estimation cannot pinpoint the optimal value for the default point for most of the firms" (Predescu, 2005; p. 19)
[7] Maximum likelihood estimation in this context is applicable only in the case where the default barrier is defined as a constant, or a certain time-dependent deterministic function.
[8] See also Alonso et al. (2008).
[9] In their final sample, Alonso et al. (2008) include the year 2001 as well. We decided to exclude that year, however, as CDS series satisfying the inclusion criteria were available for only eight companies.
[10] See Forte (2009) for details.
[11] The convergence criterion for the pseudo ML estimation algorithm is set at $1\mathrm{x}10^{-6}$.
[12] Take, for instance, the upper bound, 6.7. In case of default, and assuming bankruptcy costs of around 30% (Leland, 2004), debt holders receive 4.7 times the face value of the debt. Under these circumstances, the credit spread will actually be negative.
[13] A default barrier below the endogenous value would need to be interpreted as the result of equity holders being forced to continue running a company, even when they wish to declare the firm bankrupt. Limited liability of equity holders rules out this situation, however.
[14] In order to insure that overall conclusions are not affected by the specific choice of the recovery rate, we have replicated the analysis by fixing the recovery rate at 51.31%, as in Huang and Huang (2003), and by setting bankruptcy costs α equal to 0.3 as in Leland (2004). Results, not presented here, single out once again the best fit of ICS_{ALG} estimates.
[15] Maturity of the DOC option is chosen to correspond to the average maturity of the firm's total liabilities of 3.38 years as reported in Stohs and Mauer's (1996) empirical study. In addition, this is close to the hypothetical average maturity of the company's debt of 3.67 years, calculated by following the assumption made in this paper that the company's debt at each instant consists of ten bonds: one with a maturity of one year and principal equal to STL, and nine with maturity ranging from two to ten years, each with principal equal to 1/9 of LTL. The corresponding risk-free rate is determined by interpolating between 3- and 4-year swap rates.

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Table 1. *Descriptive statistics*

	MC (mm $)	Equity Volatility	Leverage	CDS (bp)	Bid-Ask Spread (bp)	No. of daily observations per company	Rating (No. of companies)
Mean	26,705.41	0.37	0.53	71.82	13.29	630	AAA-AA (14)
Median	14,431.07	0.36	0.53	53.37	10.40	712	A (41)
SD	37,858.73	0.09	0.20	62.24	10.31	129	BBB (35)
Min	1,082.89	0.16	0.04	11.33	4.72	418	BB (4)
Max	310,471.20	0.69	0.92	306.40	73.62	759	ND (2)

This table reports the main descriptive statistics on a cross-sectional basis. The overall sample includes 96 nonfinancial companies. MC refers to market capitalization in millions of dollars. Equity volatility is defined as the unconditional historical volatility calculated as the annualized standard deviation of the continuously compounded returns on equity. Leverage is defined as the ratio of the book value of total liabilities over the sum of market value of equity and book value of total liabilities. CDS spreads refer to the mid bid-ask quote.

Table 2. *Parameter estimates*

Panel A. Descriptive Statistics

	Mean	Median	SD	Min	Max
		Standard ML Estimation			
β_{MLE}	1.093	0.909	0.823	0.025	6.762
σ_{MLE}	0.145	0.132	0.089	0.039	0.542
μ_{MLE}	0.021	0.026	0.060	-0.163	0.178
		Pseudo ML Estimation			
β_{ALG}	0.801	0.804	0.085	0.503	0.969
σ_{ALG}	0.164	0.154	0.091	0.041	0.542
μ_{ALG}	0.027	0.027	0.067	-0.164	0.239
		Endogenous Default Barrier			
β_{END}	0.752	0.762	0.097	0.438	0.945
σ_{END}	0.164	0.154	0.091	0.039	0.541
μ_{END}	0.027	0.027	0.067	-0.162	0.239

Panel B. Distribution of Default-to-Debt Ratios

Range	β_{MLE}	β_{ALG}	β_{END}
[0.0 - 0.1]	4	0	0
(0.1 - 0.2]	3	0	0
(0.2 - 0.3]	1	0	0
(0.3 - 0.4]	0	0	0
(0.4 - 0.5]	0	0	2
(0.5 - 0.6]	3	3	7
(0.6 - 0.7]	3	7	14
(0.7 - 0.8]	15	36	44
(0.8 - 0.9]	19	39	24
(0.9 - 1.0]	16	11	5
(1.0 - 1.1]	3	0	0
(1.1 - 1.2]	6	0	0
> 1.2	23	0	0

This table reports the main descriptive statistics of the parameter estimates (Panel A), along with the distribution of default-to-debt ratios (Panel B). Results from the standard ML approach, the pseudo ML approach, and the endogenous default barrier approach are considered.

Table 3. *Differences among parameter values*

	No.	Mean	Median	SD
		All Companies		
$\lvert\beta_{ALG} - \beta_{MLE}\rvert$	96	0.431	0.099	0.784
$\lvert\sigma_{ALG} - \sigma_{MLE}\rvert$	96	0.019	0.001	0.040
$\lvert\mu_{ALG} - \mu_{MLE}\rvert$	96	0.006	0.000	0.019
		$0.3 < \beta_{MLE} < 1.0$		
$\lvert\beta_{ALG} - \beta_{MLE}\rvert$	56	0.044	0.016	0.072
$\lvert\sigma_{ALG} - \sigma_{MLE}\rvert$	56	0.001	0.000	0.002
$\lvert\mu_{ALG} - \mu_{MLE}\rvert$	56	0.000	0.000	0.000

This table provides the main descriptive statistics for the difference between pseudo ML and standard ML parameter estimates.

Table 4. *CDS spreads and ICS estimates*

	All	AAA-AA	A	BBB	BB
CDS	71.82	22.00	51.39	107.02	151.95
ICS_{MLE}	194.98	176.23	225.16	176.40	91.77
ICS_{ALG}	71.34	21.18	48.22	107.53	164.73
ICS_{END}	45.49	11.85	33.26	63.43	125.76
ICS_{KMV}	41.07	13.18	43.68	50.39	24.25
ICS_{P}	223.29	79.98	175.91	320.70	345.53

This table reports mean values for cross-sectional CDS spreads, along with model implied credit spread (ICS) based on the different estimation methods.

Table 5. *Measures of Pricing Discrepancy*

	avb	avb (%)	avab	avab (%)	rmse
ICS_{MLE}	123.16	4.02	158.34	4.44	174.66
	(49.95)	*(1.02)*	*(82.37)*	*(1.18)*	*(92.33)*
ICS_{ALG}	-0.48	0.11	43.01	0.76	52.16
	(-8.61)	*(-0.21)*	*(27.03)*	*(0.57)*	*(32.72)*
ICS_{END}	-26.33	-0.23	46.88	0.76	54.94
	(-25.07)	*(-0.55)*	*(32.88)*	*(0.71)*	*(38.04)*
ICS_{KMV}	-30.75	-0.26	58.66	0.89	66.93
	(-29.09)	*(-0.69)*	*(38.41)*	*(0.86)*	*(42.44)*
ICS_{P}	151.47	2.97	156.65	3.03	177.46
	(98.24)	*(2.10)*	*(105.60)*	*(2.10)*	*(124.28)*

This table provides cross-sectional mean (median) values of the standard measures of credit spread differentials between ICS and CDS series: average basis – avb; percentage average basis – avb(%); average absolute basis – avab; percentage average absolute basis – avab(%); and root mean squared error – rmse.

Table 6. *Parameter estimates: BT*

Panel A. Descriptive Statistics

	Mean	Median	SD	Min	Max
		Standard ML Estimation			
β_{MLE}	1.077	0.934	0.615	0.178	4.410
σ_{MLE}	0.151	0.137	0.091	0.042	0.578
μ_{MLE}	0.001	0.002	0.059	-0.177	0.161
		Pseudo ML Estimation			
β_{ALG}	0.616	0.627	0.125	0.156	0.891
σ_{ALG}	0.170	0.160	0.093	0.042	0.578
μ_{ALG}	0.008	0.002	0.068	-0.177	0.243

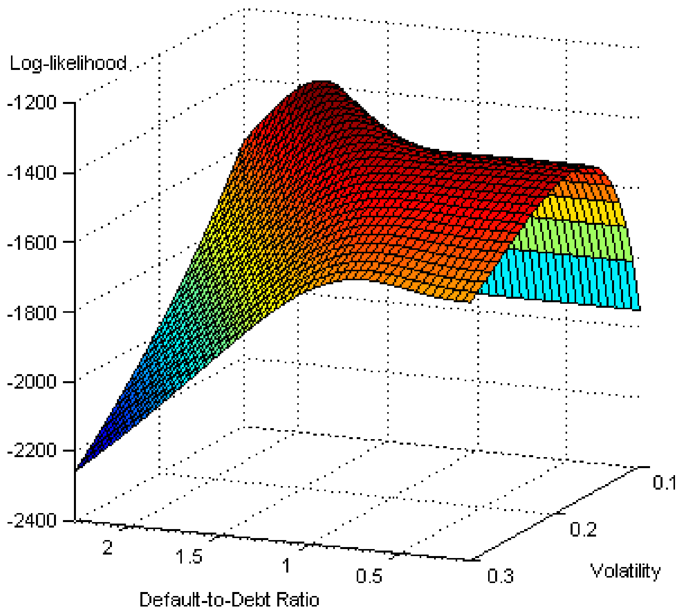
Panel B. Distribution of Default-to-Debt Ratios

Range	β_{MLE}	β_{ALG}
[0.0 - 0.1]	0	0
(0.1 - 0.2]	1	1
(0.2 - 0.3]	0	0
(0.3 - 0.4]	0	7
(0.4 - 0.5]	2	3
(0.5 - 0.6]	5	26
(0.6 - 0.7]	7	37
(0.7 - 0.8]	20	19
(0.8 - 0.9]	9	5
(0.9 - 1.0]	13	0
(1.0 - 1.1]	10	0
(1.1 - 1.2]	9	0
> 1.2	20	0

This table reports, for the case of the Brockman and Turtle's (2003) model, main descriptive statistics of the parameter estimates (Panel A), along with the distribution of default-to-debt ratios (Panel B). Results from the standard ML approach and the pseudo ML approach are considered.

Figure 1. *Behavior of the Log-Likelihood Function for BASF AG*

Company: BASF AG



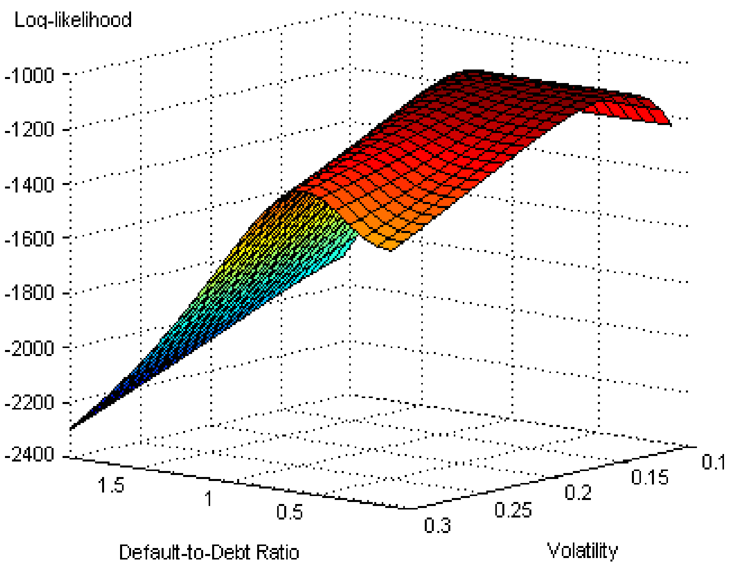
Standard ML Estimation	
β_{MLE}	2.085
σ_{MLE}	0.079
μ_{MLE}	0.037

Pseudo ML Estimation	
β_{ALG}	0.778
σ_{ALG}	0.176
μ_{ALG}	0.047

Endogenous Default Barrier	
β_{END}	0.702
σ_{END}	0.176
μ_{END}	0.047

Figure 2. *Behavior of the Log-Likelihood Function for Bouygues SA*

Company: Bouygues SA

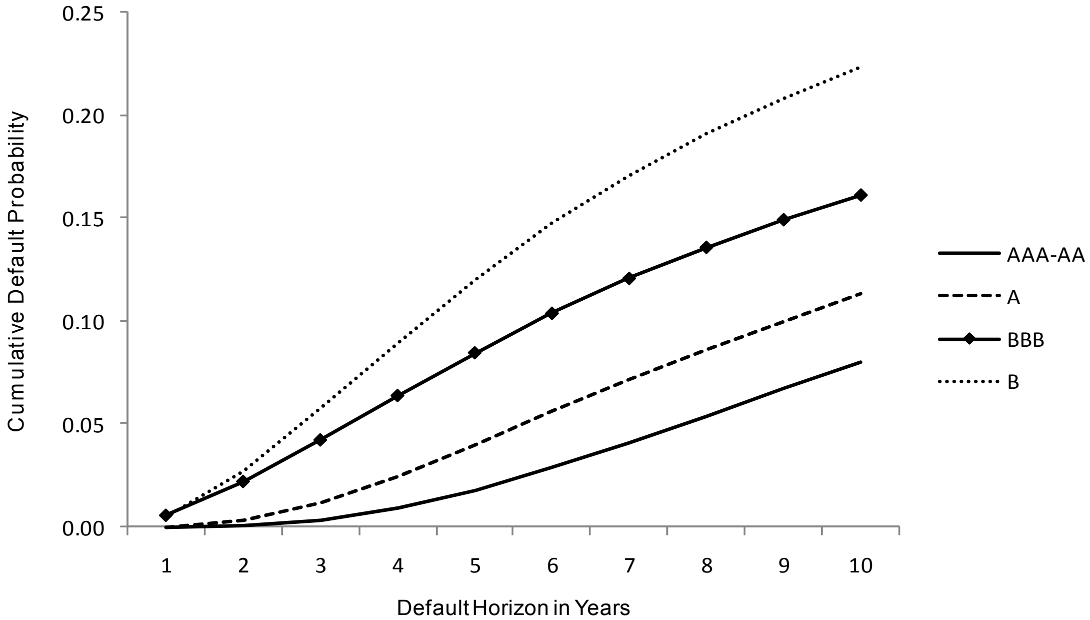


Standard ML Estimation	
β_{MLE}	0.783
σ_{MLE}	0.138
μ_{MLE}	-0.006

Pseudo ML Estimation	
β_{ALG}	0.789
σ_{ALG}	0.138
μ_{ALG}	-0.006

Endogenous Default Barrier	
β_{END}	0.779
σ_{END}	0.138
μ_{END}	-0.006

Figure 3. *Risk-Neutral Default Probabilities*



This figure represents cross-sectional risk-neutral default probabilities by rating category, calculated on the basis of the pseudo ML estimates, and for a default time horizon ranging from 1 to 10 years.