Market Pricing of Credit Linked Notes:

The Case of Retail Structured Products in Germany

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After Lehman's bankruptcy, we observed that the related CLN in Germany, Singapore and Hong Kong became almost worthless. This raises investors by nature the question, if the issue prices of these instruments were fair and adequate to the related risk. There are several studies on the pricing of equity linked notes or rather certificates especially in the European Markets. For Credit Linked Notes there is no comparable study of overpricing. In our opinion the results of such a study will be of special concern because it will shed more light on the reasons for the overpricing. These products are similar to bonds as well as to reverse convertibles and make the retail investors believe, that they are straight bonds. Furthermore, the replicability for the private investor is in opposite to the institutional investor not practicable and the valuation is depending on the individual characteristics scaleable complex. Therefore, we can measure the complexity on an ordered scale, because complexity strictly increases with the number of reference entities and payments days.

In this paper we analyzed the pricing of the 136 outstanding CLN of the four major issuers in the German market. To this end we applied a market based valuation model for CLN, which is based on the reduced model of Jarrow/Turnbull and extended by the single factor Merton model to estimate the joint default probabilities out of assets correlations. The model was calibrated by CDS-spreads and correlations of stock returns for the underlying reference entities.

We found out that the observed CLN are generally overpriced in the primary market: Thereby the result is robust to changes of recovery rates or correlation coefficients. As market maker, issuers of structured financial products participate in almost every transaction and they have the incentive to overprice. The more complex the product and the less transparent the market is, the larger overpricing there tends to be. This consideration is confirmed by the results regarding our major hypothesis of overpricing and the extent of overpricing is positively correlated with the number of underlying reference entities, the coupon rates and the maturity of the contract.

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1 Introduction

1.1 Motivation

The bankruptcy of the fourth-largest US investment bank Lehman Brothers has set off an earthquake in the global financial market. Not only the prices of Lehman shares and bonds went through the floor, but also many other related credit derivatives were strongly affected. One special kind of these derivatives that concerns retail investors is known as Credit Linked Notes (CLN). Compared to structured products faced to institutional investors, CLN usually offer individual investors attractive coupon payment linked to bonds issued by one or several large reference entities, which are generally considered "too big to default". After Lehman's bankruptcy, we observed that the related CLN in Germany, Singapore and Hong Kong became almost worthless. This raises investors by nature the question, if the issue prices of these instruments were fair and adequate to the related risk.

1.2 Literature review and research question

There are several studies on equity linked notes or rather certificates especially on the European Markets. Whereas the first studies of Chen and Kensinger [1990] and Chen and Sears [1990] concentrated on finding significant deviations between quoted prices and theoretical fair values of products in the US market, later studies of e.g. Burth et al. [2001], Brown and Davis [2004] Wilkens et al. [2003] or Gruenbichler and Wohlwend [2005]. ascertained the overpricing in several non-U.S. primary markets for reverse convertibles, discount certificates or endowment warrants. Consequently the following studies were concentrating to illuminate the cause of overpricing. Among others (see e.g. Benet et al [2006] or Entrop et al. [2009]) overpricing seems to be positively related to the replicability and the complexity of the product, which several studies support (See Hernández et al., 2007a and Stoimenov and Wilkens, 2005). According to Wallmeier and Diethelm [2009] the titular high coupon stimulated especially the interests of the retail investors.

For Credit Linked there is no comparable study of overpricing. In our opinion the results of such a study will be of special concern because it will shed more light on the reasons for the overpricing. These products are similar to bonds as well as to reverse convertibles and make the retail investors believe, that they are straight bonds. Furthermore the replicability for the private investor is in opposite to the institutional investor not practicable and the valuation is depending on the individual characteristics scaleable complex. Therefore we can measure the complexity on an ordered scale, because complexity strictly increases with the number of reference entities and payments days.

In order to answer this question, we first identified all major CLN issuers in the German market and gathered the issue prospectuses about their 136 outstanding CLN products. After the data collection, a valuation model was developed to determine only by market prices their theoretical fair values. By analyzing the differences between the calculated fair values and the offered prices, we were able to not only find out the answer to our major question. Several hypotheses we made about the German CLN market could also be verified empirically.

1.3 Structure of this document

In the beginning, we will introduce the underlying theory of credit derivatives with special focus on CLN and state our hypotheses. After that we will explain the details of our valuation framework. The data sources used during the valuation process will then be described in the following section. Having both the model and the data prepared, we will present the results of CLN valuation and test which of the hypotheses we made before the valuation are statistically valid. At last, the knowledge gained by our empirical analysis will be summarized and some suggestions concerning the improvement and extension of our work will also be provided.

2 Products and hypotheses

2.1 Credit Linked Notes for the retail investor

A Credit Linked Note (CLN) is a credit derivative that means a bilateral contract under which the seller sells protection against the credit risk of the reference entity and receives a certain premium from the protection buyer [Fabozzi et al. 2007, p. 67]. Because the payments under the credit derivative are funded using securitization techniques to support all of the potential losses on the underlying, the protection buyer, the issuer of the notes, has not the risk whether the seller or investor will be able to pay in case of the credit event. This enables the participation of a larger group of players including retail investors willing to bear that risk in return for the higher yield.

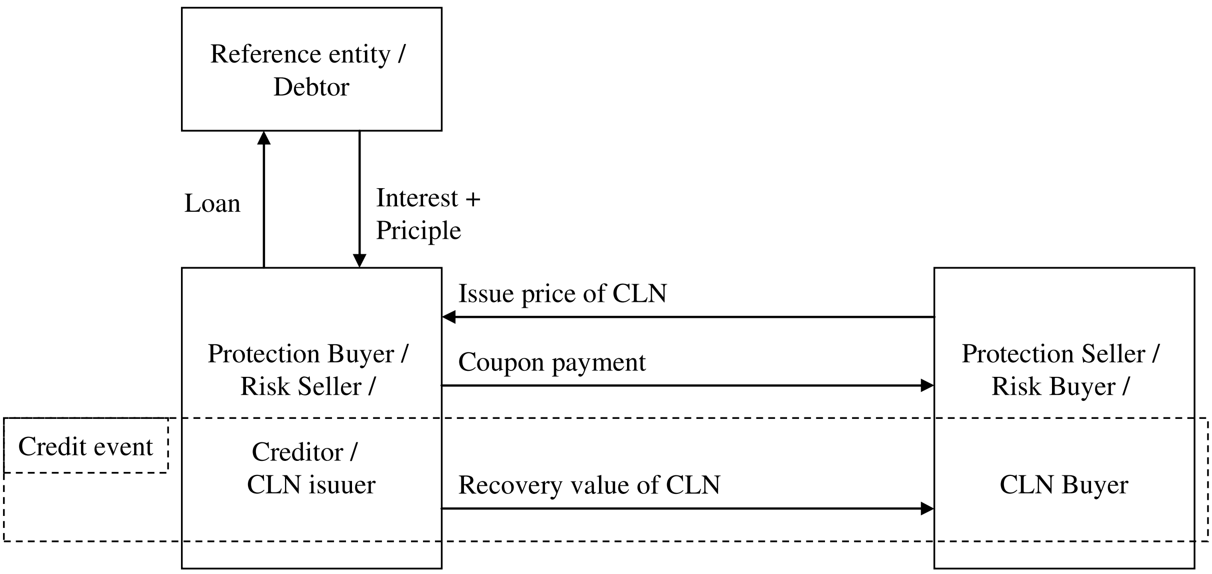


Figure 1: Concept of CLN

The payment structure of CLN is linked to the credit standing of one or several underlying reference assets (see figure 1). In case of a credit event, the coupon payment will be suspended and the investors will receive a recovery rate in form of either cash settlement or physical delivery of the underlying assets. In addition to the credit risk of the reference assets, investors also bear the credit risk of the corresponding CLN issuer.

Depending on the form of the embedded credit derivatives, CLN can be further divided into several categories, each of them has a more specific name accordingly [Telpner 2004, p. 10].

CLN Indication	Embedded Credit Derivatives
Credit Default Note	Credit Default Option
Credit Spread Note	Credit Spread Product
Total Rate of Return Linked Note	Total Rate of Return Swap

Table 1: Categories of CLN

The scope of our work covers Credit Default Notes and Basket Credit Default Notes, which also represent the vast majority of the CLN variants in the retail German market. We will continue to use the generic term CLN since it is more widely known.

2.2 Hypotheses

The fact that the protection buyer of other credit derivative products must bear the risk that the protection seller defaults makes the public offering of most types of credit derivatives in retail market very difficult. Therefore CLN are the only major credit derivative products available for retail investors and they have been marketed worldwide as low risk structured products. In Hong Kong and Singapore for example, CLN have been labeled with the product name of “Mini-Bond”. (See for the market of CLN for Retail Investors Fabozzi et al. [2007], p. 77) However, for individual investors, the construct of CLN is not easy to replicate, because there exists no adequate position in the retail market. Even if martingale probabilities for single entities are available, basket products are difficult to price. A potential information asymmetry in the credit market could motivate CLN issuers to overprice their products at their issuance. Therefore, our major hypothesis is:

- ***Hypothesis 1***

  CLN in the German retail market are generally overpriced.

Besides the major hypothesis of overpricing, we also made several other hypotheses about the possible factors which could influence the significance of overpricing. Since the analyzed CLN contracts are all based on the principle of first to default, we believe the increased risk of those CLN with large number of underlying reference entities were not able to be fully compensated by their insignificantly higher coupon rates compared with single referenced products. Furthermore, calculating fair price of multiple referenced CLN requires expertise and computational facilities which most retail investors do not have. Following the idea in equity linked notes [see e.g. Hernández et all, 2007b] this intransparency might encourage CLN issuers to overprice their products with multiple underlying references even more than those with less entities or a single reference:

- ***Hypothesis 2***

The more underlying reference entities there are, the more significant the overpricing is.

The third hypothesis is about the "first sight effect" of the coupon payments comparable to equity linked bonds [Wallmeier and Diethelm, 2009, p. 12]. There are usually hidden factors behind high coupon rates, which could be e.g. an extremely risky underlying or general high interest level in the market, which the investor does not realize. It could be especially confusing for those CLN with mixed coupon structure, which usually include high rate fixed payments at the beginning, followed by float rate payments. Therefore, height and type of coupon payments could also affect the significance of overpricing:

- ***Hypothesis 3***

  The higher the coupon rate is, the more significant the overpricing is.

CLN with longer maturities incorporates greater risk that the creditworthiness of the underlying reference entities will change. Since longer maturity usually indicates more payment days, the number of payment days will be used as additional control variable during the test. Moreover, during the valuation we have realized that the complexity of calculation strictly increases with the number of payment days. According to our model, it could take days to determine the fair price of a complex CLN product with long maturity which includes many payment days, especially if it has large number of underlying references [See similar Stoimenov and Wilkens, 2005, p. 2980]. Using again the complexity argument, we have similar suspects as for the second hypothesis:

- ***Hypothesis 4***

  The longer the maturity is, the more significant the overpricing is.

We also want to find out if the overpricing has decreased along with the development of the CLN market [See similar Szymanowska et al. 2009, p. 913]:

- ***Hypothesis 5***

  The earlier the date of issue is, the more significant the overpricing is.

Following Szymanowska et al. [2009, p. 917] the market power and marketing strategy of the issuer might also have a significant influence. With the last hypothesis we try to find out if this idea might be correct:

- ***Hypothesis 6***

  The more products an issuer offers, the more significant the overpricing is.

3 Methodology

3.1 Choice of Model

Although there is still no industrial standard valuation model which applies to all credit derivatives at the moment, two basic theoretical approaches are usually applied to model the credit risks [Bielecki and Rutkowski (2002), p. 26]. On one hand, the structured model is developed from the work of Merton [1974]. In compliance with Hui and Lo [2002] the value of a firm is modeled as a continuous diffusion process and a default will happen if the firm value falls below a barrier, which could be interpreted as the liability of the firm. Usually the stock price is used to calibrate the valuation model, since stocks can be seen as options of firm value. On the other hand, the reduced form model [Jarrow/Turnbull, 1997; Duffie/Singleton, 1994] is characterized by the presumptions about the intensity rate, the risk free rate, the correlation and the recovery rate. The default is modeled as a stochastic event depending on the intensity rate of the underlying process, under which macro economic factors and firm specific data can be integrated. The value of related credit derivatives can be modeled as expectations depending on the realization of the involved stochastic process. The calibration of the process is based on the price of defaultable instruments such as bond to build a complete interest and credit spread structure, out of which the probability of default can be derived.

Considering the advantages of the existing credit risk models and the data available, we choose to follow the approach of reduced form model. Our approach can be understood as a multi-borrower Jarrow/Turnbull model, whereby the default probabilities are calculated out of CDS spreads. Then these probabilities are used to calculate the net present value of a CLN contract. In addition, we incorporated the asset correlation of the underlying assets, so that the model is flexible enough to model multiple defaults of the reference entities, which is important for the valuation of CLN with multiple references. Details of our approach will be explained in the following sections.

3.2 Valuation of CLN with single reference entity

3.2.1 CDS spreads and implied probability of default

Since CLN can be interpreted as a bond with embedded CDS, the key is to derive the default probabilities out of the CDS spreads of reference entities. According to the Standard North American Corporate CDS Contract Specification of ISDA [2003], the annual CDS premium is paid quarterly, namely on the 20^{th} of March, June, September and December of each year. Let the date of issue be t_0, for a CDS contract with maturity of n years, the dates of premium payments are $T_n = \{t_{0.25}, t_{0.5}, t_{0.75}, t_1, ..., t_n\}$. On each of these dates, one fourth of the annual CDS spread $0.25 \cdot CDS_n$ will be paid. On the other hand, according to the Standard North American Corporate CDS Converter Specification of ISDA [2009], the recovery rate of a senior CDS is 40% $REC_{CDS} = 0.4$.

Because the data set is limited to yearly CDS contracts, we have to impose assumptions to calculate implied default probabilities on the whole positive scale. To simplify the case, we assume that the amount of recovery rate will also be paid on the next possible CDS premium payment dates in case of default. This means any credit event occurred during a quarter of a year will be settled at the end of the quarter accordingly. Furthermore, for each CDS contract, the probability of default q is assumed to be the same for each quarter of a year. This assumption guarantees that we are able to calculate four probabilities having only one CDS spread.

Based on these two assumptions, we can now discount the expected cash flows which should sum to zero so that no arbitrage is possible. We start with the one year CDS:

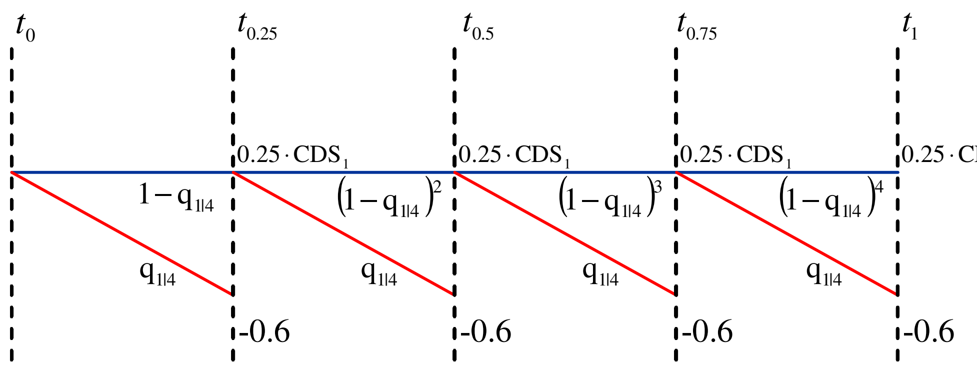


Figure 2: Cash flow of an one year CDS

The CDS spread is set in such a way that the swap is priced fair (See similar Hull and White [2003]):

$$CF_{1,1} \cdot (1+r_{0.25})^{-0.25} + CF_{1,1} \cdot (1-q_{1|4}) \cdot (1+r_{0.5})^{-0.5} + CF_{1,1} \cdot (1-q_{1|4})^2 \cdot (1+r_{0.75})^{-0.75} + CF_{1,1} \cdot (1-q_{1|4})^3 \cdot (1+r_1)^{-1} = 0$$

whereby $CF_{1,1} = -0.6 \cdot q_{1|4} + 0.25 \cdot CDS_1 \cdot (1-q_{1|4})$.[1]

With the spot rate yield curve and CDS spreads prepared before, the quarterly probability of default for one year CDS $q_{1/4}$ can be solved out of this equation. After that, we get the cumulative and probability of default for the end of the first year:

$$q_1 = 1-(1-q_{1|4})^4 \text{ and } q_{1|4} = 1 - \sqrt[4]{1-q_1}.$$

Since CDS contracts with different maturities share the same underlying reference entity, they should also share the same cumulated probability of default during the same period of time. Therefore, the cumulated default probability q_{n-1} for the first year can be adopted while calculating the quarterly probability of default $q_{n-1|4}$ for a CDS with maturity of n years.

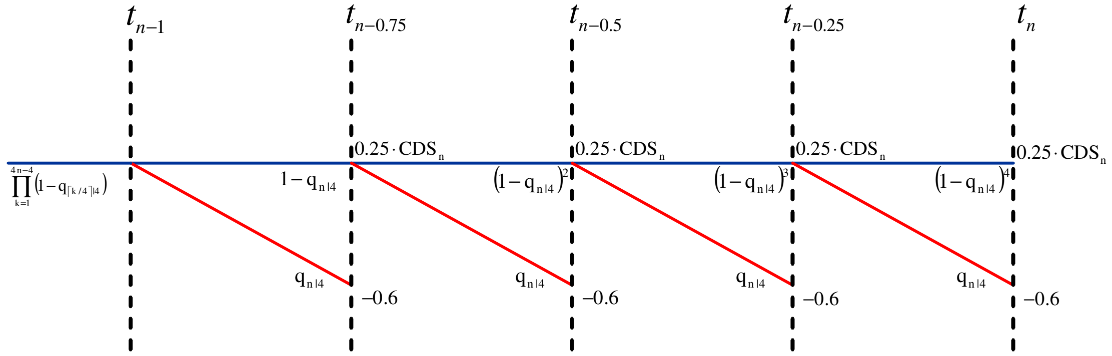


Figure 3: Cash flow of an n year CDS in the n^{th} year

Following this procedure, we can calculate the quarterly probability of default for CDS with maturities of n years, as long as the CDS spreads are available.

$$\sum_{t=1}^{4\cdot n-4} CF_{n,t} \cdot \prod_{k=1}^{t-1} \left(1-q_{\lceil k/4 \rceil|4}\right) \cdot (1+r_{t/4})^{-t/4} + \sum_{t=4\cdot n-3}^{4\cdot n} CF_{n,t} \cdot \prod_{k=1}^{t-1} \left(1-q_{\lceil k/4 \rceil|4}\right) \cdot (1+r_{t/4})^{-t/4} = 0 \ ,$$

whereby $CF_{n,t} = -0.6 \cdot q_{\lceil t/4 \rceil|4} + 0.25 \cdot CDS_n \cdot (1-q_{\lceil t/4 \rceil|4})$ and $\lceil \ \rceil$ represents the ceil function traced back to Gauss. The implied default probabilities $q_{n/4}$ can be solved out of this upper equation. The cumulative probability of default is defined recursively as

$$q_n = \left(1 - (1 - q_{n|4})^4\right) \cdot \left(1 - q_{n-1}\right).$$

In our case, we get the cumulative probability of default q_1, q_2, …, q_n for the end of each year. Furthermore $q_0=0$, since default at the date of issue t_0 is considered to be impossible. The last step is to estimate a continuous curve of cumulated default probability out of the eleven data points we have already calculated. The smoothing method we applied was natural cubic spline interpolation [Press et al., 2007, pp. 120-124]. This results in continuous isotonic function of the cumulative probability of default named $Q(t)=s(q_1,q_2,\dots q_n)$.

3.2.2 Valuation of CLN with fixed coupon rate

Our first goal is to price a CLN with annual or semiannual fixed coupon rate C_f maturing in T_m at a face value of N. The price of issue is V_i. Let the date of issue be T_0 and the following payment dates be $T_1,T_2,\dots,T_m$. Based on the default probabilities curve Q(t), we can estimate

[1] That there is always a unique solution is proven in the appendix.

the cumulated probability of default $Q(T_1\text{-}T_0)$, $Q(T_2\text{-}T_0)$,…,$Q(T_m\text{-}T_0)$ at each of these days accordingly. For a CLN with annual fixed coupon rate C_f, the expected cash flows can be replicated as the following:

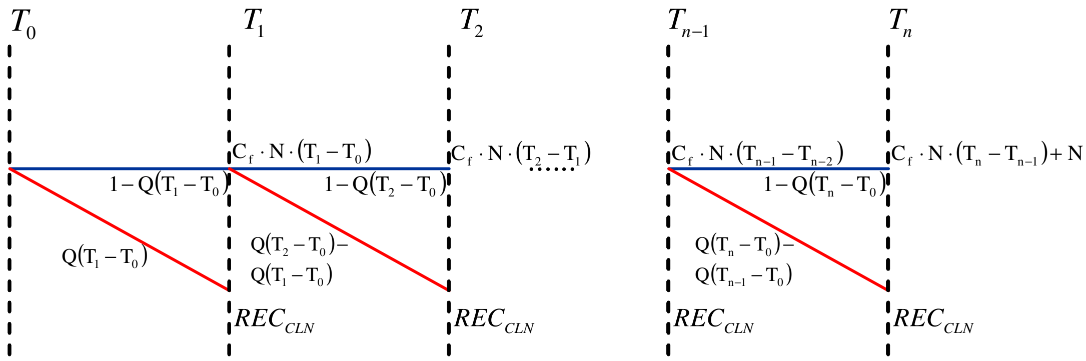


Figure 4: Cash flow of a CLN with fixed coupon payments

One major reason why CLN pricing could be difficult is that this kind of product hardly defaults, because the reference entities are usually giant corporations or sovereigns with excellent creditworthiness. Therefore, it is hard to predict the recovery rate in such a scenario. Along with the financial crisis, we were able to observe the empirical recovery rate for the first time. After Lehman's declaration of bankruptcy on the 15th September 2008, the six affected CLN of DZ Bank were quoted round 10% of their face value. The only Lehman referenced CLN of the Commerzbank was even quoted at 2%[2]. Four weeks later, retail investors found the realized recovery rate for one of these CLN to be 8.8%. Further, in the product specification of the Commerzbank, a cash settlement of 1% nominal value in case of default is defined as an alternative to the delivery of the cheapest bond. Taking all these information into account, we first assumed the recovery rate of CLN to be 8.8% $REC_{CLN} = 0.088$. As second assumption we used the recovery rate given by rating agencies and in the literature $\mathrm{REC}_{\mathrm{CLN}} = 0.35$. In this case the rate is a conservative high value, because the payment is not directly comparable to the recovery rate, since the first one is fixed directly after the credit event. Therefore according to the prospectuses the recovery rate is derived either from bonds quoted or from the average quote of three other banks for a hypothetical claim, which is downwards biased.

If we impose the further assumption, that the issuer is nearly risk free, we can simply discount the expected cash flows back to the date of issue to determine the sum of the present values, which is also the fair price of such an instrument:

[2] Price quoted on the 19th September 2008 from Onvista.

$$V_f = \sum_{t=1}^{m} \left(\left(C_f \cdot N \cdot (T_t - T_{t-1}) + 1^{\mu} \cdot N \right) \cdot \left(1 - Q(T_t - T_0)\right) + REC_{CLN} \cdot N \cdot \left(Q(T_t - T_0) - Q(T_{t-1} - T_0) \right) \right) \cdot (1 + r_{T_t - T_0})^{T_0 - T_t},$$

whereby $\mu = \left\lfloor \frac{t}{m} \right\rfloor$ is the result of the floor function.

To calculate the overpricing we evaluate the difference between the theoretical fair price V_f and the price of issue in reality. $\Delta V = V_i - V_f$. A positive ΔV indicates overpricing and vice versa.

Furthermore, we can assume the price of issue to be fair and estimate the minimal recovery rate which fulfills this assumption.

$$REC_{\min} = \frac{V_i - \sum_{t=1}^{m} \left(C_f \cdot N \cdot (T_t - T_{t-1}) + 1^{\mu} \cdot N \right) \cdot \left(1 - Q(T_t - T_0)\right) \cdot (1 + r_{T_t - T_0})^{T_0 - T_t}}{\sum_{t=1}^{m} N \cdot \left(Q(T_t - T_0) - Q(T_{t-1} - T_0) \right) \cdot (1 + r_{T_t - T_0})^{T_0 - T_t}}.$$

The price difference ΔV and the minimal recovery rate $REC_{\min}$ are two major indicators to determine if a CLN product is overpriced.

3.2.3 CLN with float coupon rate

As mentioned in chapter two, the CLN we include also products with float rate coupon payments. Since they only represent the minority of the entire data set, we did not apply complex interest rate model explicitly for them. Instead, we used analogously to practical approaches the forward rates, which are derived from spot rates on the date of issue, to predict the coupon on each payment day based on the information available at the date of issue. Take a CLN with quarterly coupon payment of three month EURIBOR rate plus b basis points as example, the forward rate for the payment in T_t denoted as r_{T_{t-1},T_t}. Adding the b basis points of bonus, the coupon rate for the period T_{t-1} to T_t is determined as:

$$C_{T_t} = \left(1 + r_{T_{t-1},T_t}\right)^{1/(T_t - T_{t-1})} - 1 + b \cdot (T_t - T_{t-1}).$$

The calculation of fair price and the minimal recovery rate is similar to the case of CLN with fixed coupon rate.

3.3 Valuation of multiple referenced CLN

So far, we are able to valuate all the observed CLN with single reference entity. The next step is to extend our model to handle multiple referenced CLN with o reference entities. Since all the observed CLN follow the principle of "first to default", which means as long as one credit event of any underlying reference entity is identified, the contract will be terminated and paid off immediately. The key issue of pricing a CLN with more than one reference entity is to calculate the probability under which any single or combination of the reference entities will default. Therefore, we need to extend our model to handle joint defaults.

Therefore we choose the single factor model, because it simply assumes that the economic fortune of a creditor depends only on the realization of one underlying latent process, which is interpreted by Merton as the Asset Value Process.[3] In this model, the default probability q_i of the creditor i is the probability under which the latent score variable R_i falls below the threshold value c_i [Hull 2009, pp. 512-516 and pp. 542-547].

$$Q_i(T_k) = \text{Prob}_{T_k}(1_{D_i} = 1) = \text{Prob}_{T_k}(R_i < c_i).$$

The score variable R_i of a portfolio with o creditors can be parameterized using a single systematic factor which represents the general uncertainty of the market. The first step is to calculate the joint default probability of two or more different reference entities regarding a time horizon of T_k, which can be expressed as:

$$Q_{1,2,\ldots o}(T_k) = \text{Prob}_{T_k}(1_{D_1} = 1, 1_{D_2} = 1, \ldots., 1_{D_1} = 1).$$

This means a joint default will occur, if and only if credit events for all creditors occur in the observed period of time. The joint default probability depends on the joint distribution of R_i. Since the vector $R=(R_1,R_2,\ldots,R_o)'$ is o-dimensional normally distributed with the correlation matrix of Corr(R), all the marginal distributions are also standard normal distributed. Therefore, the joint default probability can be calculated by:

$$Q_{1,2,\ldots o}(T_k) = \text{Prob}_{T_k}(R_1 < c_1, R_2 < c_2, \ldots, R_o < c_o) = \Phi_o(c, \text{Corr}(R)),$$

where $c=(c_1,c_2,\ldots,c_o)'$ is the vector of thresholds, Φ_o the distribution function of o-variat normal distribution with correlation of Corr(R). Since c_i depends on the individual default probabilities p_i, the joint default probability can be finally calculated as:

$$Q_{1,2,\ldots o}(T_k) = \Phi_o\left(\left(\Phi_1(Q_1(T_k))^{-1}, \Phi_1(Q_2(T_k))^{-1}, \ldots, \Phi_1(Q_o(T_k))^{-1}\right)^{\text{Transposed}}, \text{Corr}(R)\right).$$

Based on the theory of single factor Merton model, we can calculate the joint default probability of any combination of reference entities within a portfolio given the same time horizon.

$$Q_{\sigma_2(1),\sigma_2(2)}(T_k), Q_{\sigma_3(1),\sigma_3(2),\sigma_3(3)}(T_k), \ldots, Q_{\sigma_{o-1}(1),\sigma_{o-1}(2),\ldots,\sigma_{o-1}(o\text{-}1)}(T_k), Q_{1,2,\ldots o}(T_k),$$

[3] Original work on the application of single factor model for credit risk measurement include the publications of Gordy [2003] and Schoenbucher [2007].

whereby $\sigma_j \in \Re_j$ indicates all possible permutations of the original vector (1,2,…,j). The cardinal number of $\Re_j$ matches the binomial coefficient:

$$\left|\Re_j\right| = \binom{o}{j} = \binom{o}{o-j} = \frac{o!}{j! \cdot (o-j)!}$$

Because the products are first to default notes we are interested in the probability that any reference goes into default:

$$Q_{any}(T_k) = \sum_{i=1}^{o} Q_i(T_k) - \sum_{\sigma_2 \in \Re_2} Q_{\sigma_2(1),\sigma_2(2)}(T_k) + \sum_{\sigma_3 \in \Re_3} Q_{\sigma_3(1),\sigma_3(2),\sigma_3(3)}(T_k) \ldots + (-1)^o \cdot \sum_{\sigma_{o-1} \in \Re_{o-1}} Q_{\sigma_{o-1}(1),\sigma_{o-1}(2),\ldots,\sigma_{o-1}(o\text{-}1)}(T_k) + (-1)^{o+1} \cdot Q_{1,2,\ldots o}$$

For a CLN with o underlying reference entities, we first need the o cumulated default probability curve of each individual entity, which can be derived from their CDS spreads.[4] For each of the payment days $T_1, T_2, \ldots, T_n$, instead of estimating only one single cumulated default probability $Q_{any}(T_k)$. This probability can now be used to calculate the fair value in the same way as we did for the CLN with single reference entity. So far, we have a complete model which is able to valuate all observed CLN products, as long as the data including CDS spreads and assets correlations are available.

4 Data sources and tools

4.1 CLN data

As mentioned before, we identified four major German CLN issuers: the Commerzbank, the DZ-Bank, the Landesbank Baden-Württemberg (LBBW) and the HypoVereinsbank (HVB). According to our model, the common data we need to extract out of each individual product description are:

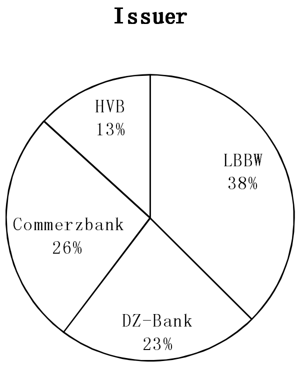
- Date of issue
- Payment dates including the final payment day
- Coupon rate and payment structure
- Underlying reference entities

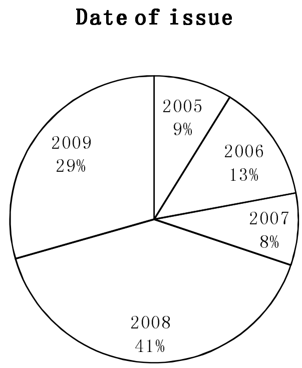
Except for the different product names given by the specific issuer, all these CLN products are constructed similarly. They only differ from each other regarding the following aspects:

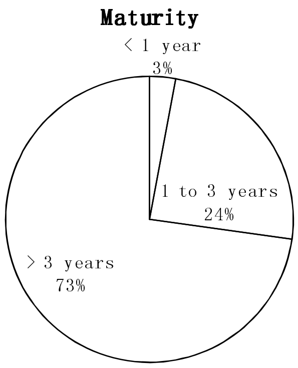
- Number of reference entities: single or multiple
- Type of the reference entities: corporate or national sovereign

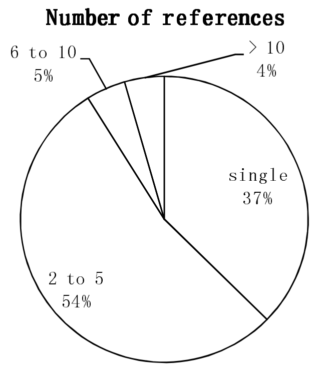
- Coupon type: fixed rate, float rate or mixture of both
- Payment structure: periodic or only at maturity
- Issuing price and final payment: at, under or over par

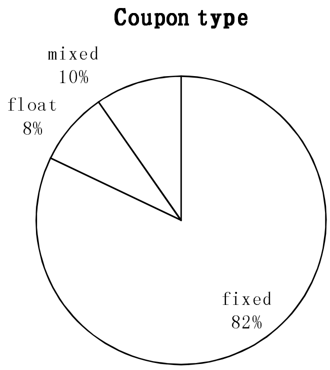
In total, we have observed 136 CLN products issued from December 2004 to September 2009, The major statistics are represented by the following charts:

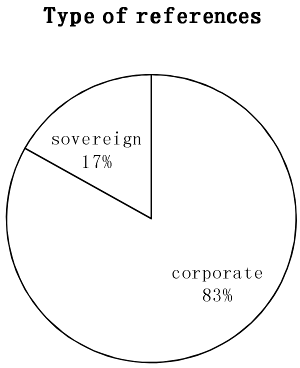












4 When the number of reference entities is bigger than ten, we omitted to calculate the four sums in our empirical study, because their probability mass is nearly zero.

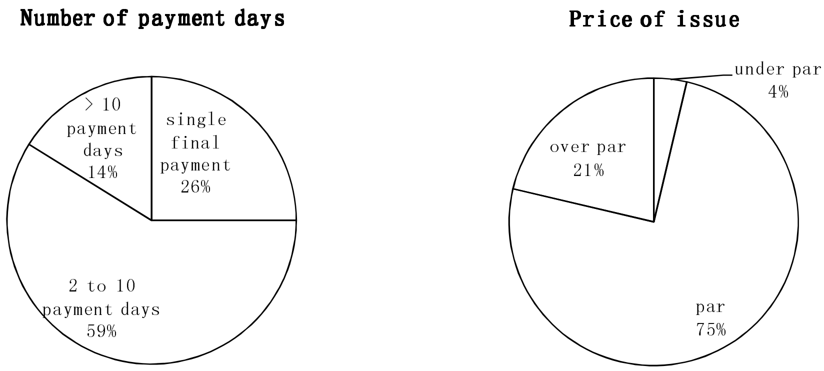


Figure 5: Descriptive statistics of observed CLN

4.2 Other input parameters

Besides the CLN data collected from the individual product description, we also need some other parameters to achieve the valuation according to our model. In order to discount the cash flows to the date of issue, we need the risk free spot rate on daily basis. This yield curve can be estimated using the Nelson-Siegel-Svensson method [Svensson, 1994], which is also used by the German Central Bank. According to the Svensson method, six input parameters are required to calculate the spot rate. For Germany, these parameters are available on a daily basis in form of time series at the Deutsche Bundesbank.

The CDS spreads of the reference entities were retrieved through the Datastream of Thomson Reuters, which use the historical data of CMA as source[5]. Daily quoted closing rates of senior CDS, with maturity from one year to ten years, are available for most of our reference entities needed since 2005.

In order to calculate the joint default probabilities for CLN with multiple underlying reference entities, we need their asset correlation. We used the Yahoo Finance Germany as the data source and took 180 days of stock return before the date of issue for each underlying corporation. As for national sovereign, we took the major stock index in each underlying country and calculated the 180 days return:

[5] www.cmavision.com.

Country	Index
Austira	Austrian Traded Index (ATX)
Belgium	Brussels All Share
Brazil	Brazil BOVESPA
Bulgarian	BSE SOFIX
China	Shanghai SE A Share
Croatia	Croatian Equity MKT (CROEMI)
Colombia	Colombia IGBC Index
Denmark	OMX Copenhagen (OMX20)
Greece	ATHEX All Share
Hungary	Budapest (BUX)
India	India BSE (100) National
Indonesia	Jakarta SE LQ45
Italy	Milan COMIT General
Mexico	Mexico IPC (BOLSA)
Poland	Warsaw General Index
Portugal	Portugal PSI-20
Romania	Romania BET (L)
Russia	Russia RTS Index
South Africa	FTSE/JSE All Share
Spain	Madrid SE General
Sweden	OMX Stockholm 30 (OMXS 30)
Switzerland	Swiss Market (SMI)
Thailand	Bangkok S.E.T. 50
Turkey	ISE National 100
Ukraine	Ukraine KP-DRAGON
UK	FTSE All Share
Venezuela	Venezuela DS Market

Table 2: Overview of selected indices

5 Tests and results

5.1 Overpricing

The major objective of our work is to find out if the outstanding CLN products in the German retail market are overpriced. According to our model, this hypothesis can be verified based on either the difference between the calculated fair price V_f and the price of issue V_i, or the minimal required recovery rate. To enhance comparability, we exhibited the price difference ΔV as the percentage of issuing price. Assuming the recovery rate to be 8.8%, we get the following ordered results:

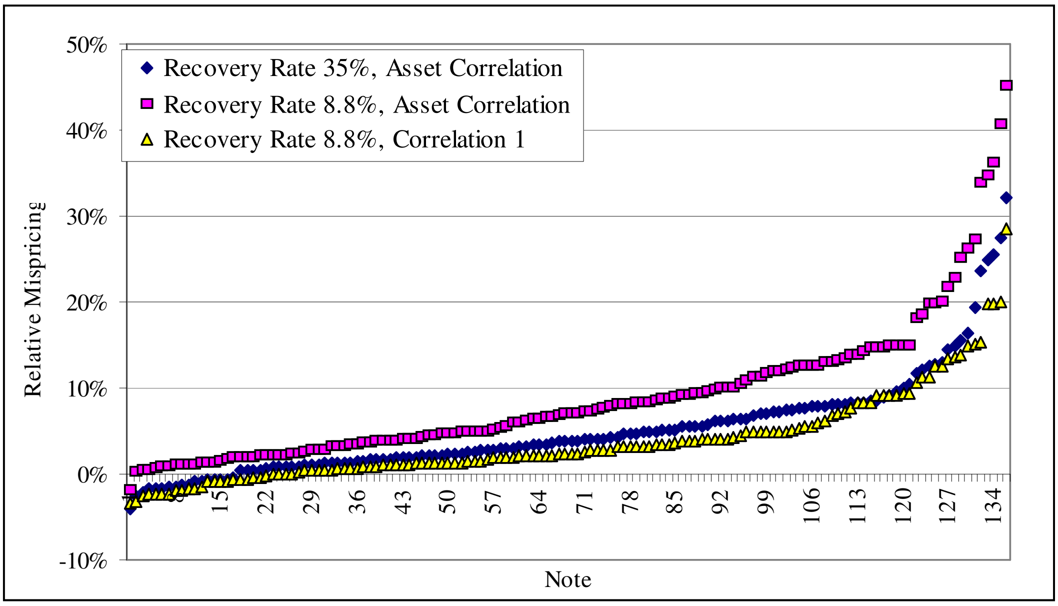


Figure 6: Price difference based on different correlations and recovery rates

As shown in the chart, only one out of the 136 analyzed CLN was issued under the theoretical fair price, all the others were overpriced with maximal difference of up to 45%. In average, the overpricing reached 8.87% with a standard deviation of 0.082. Following the methodology of Burth et al. [2001] a Wilcoxon signed rank test was applied to verify this result. The probability of symmetrical distribution is less than 0.5%. Therefore, the null hypothesis of symmetrically distributed price difference is rejected. In other words, there exists a significant overpricing. Even if we use the 35% recovery rate the result doesn't change. The average overpricing is still 5,12%, according to the Wilcoxon signed rank test the hypothesis of symmetrical distribution can be rejected on a significance level smaller than 0.5%. Among the most mispriced 68 CLN there is only one underpriced. The result appears even more dramatic at the option component and the average overpricing is then 67% and 53% for the two recovery rates.

On the other hand, in order to make the price of issue to be fair, the minimal required recovery rates $REC_{\min}$ are estimated accordingly:

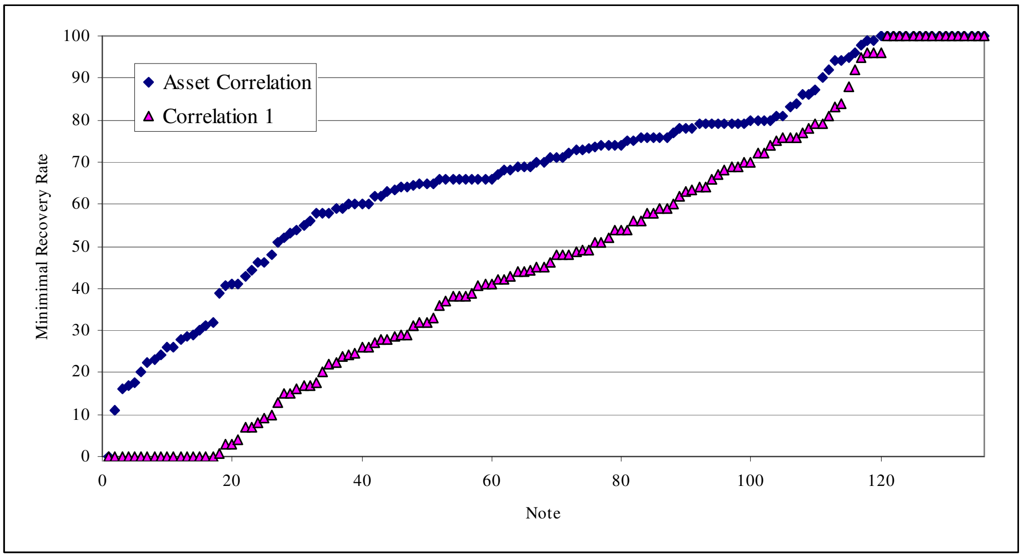


Figure 7: Minimal recovery rate based on different correlations

The average value of required recovery rate is 66%. 17 of the 136 CLN are overpriced even given a recovery rate of 100%. The unrealistic recovery rates prove further that the analyzed CLN are significantly overpriced.

Furthermore, we analyzed the data sample of each individual issuer:

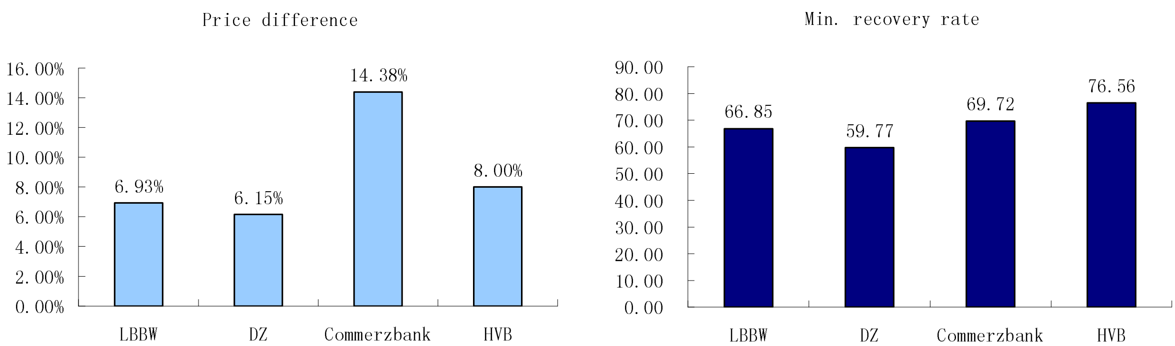


Figure 8: Average price difference and minimal recovery rate grouped by issuer

As is shown in the charts, the hypothesis of overpricing can be verified independently from the CLN issuer.

We also divide our data sample according to their date of issue to find out if the overpricing only exists in a certain period of time:

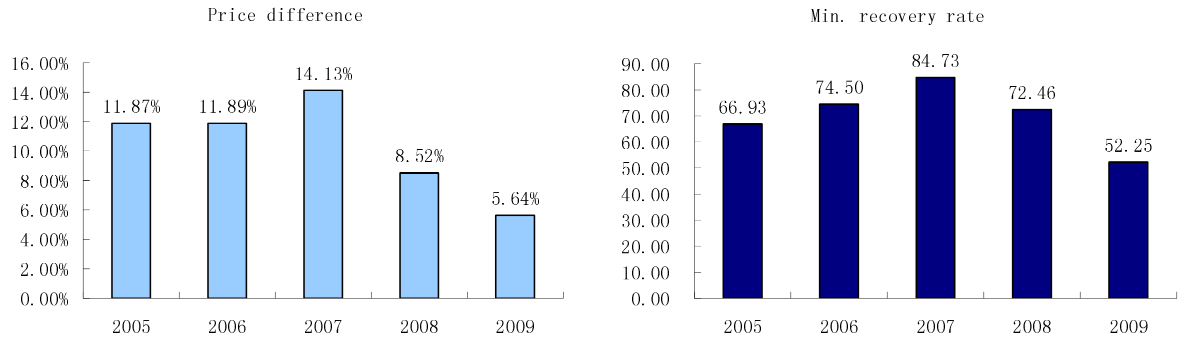


Figure 9: Average price difference and minimal recovery rate grouped by date of issue

The comparison of the average price differences and the average minimal recovery rates of each year indicates that the overpricing exists for the entire observed time horizon from the beginning of 2005 to the middle of 2009 and reached its peak in 2007.

At last, we simulated the two extreme cases of totally independent and totally dependent defaults for CLN with multiple underlying reference entities to test if the results are robust. In the first case naturally all CLN are overpriced. In the latter case we used the maximum of the probabilities of all o entities as the first to default probability.

Since in the case of total dependency these newly calculated combined default probabilities are generally lower than the ones calculated based on Merton model, we will get less significant results of overpricing, which are indicated by smaller positive price differences and lower required minimal recovery rates.

Compared with the results before, we have a smaller average price difference of 3.93 percent, where 121 out of the 136 CLN are overpriced; the average minimal recovery rate is lowered to 47.87 percent and the same 17 products are still overpriced given a recovery rate of 100%. Again, according to the Wilcoxon signed rank test, there exists a significant overpricing even if we push the assumption of defaults correlation to the extreme case.

5.2 Other hypotheses

In order to test the other hypotheses about the possible driving factors which could influence the significance of overpricing, a multiple linear regression analysis is performed which includes the following influence factors as independent variables [see similar e.g. Hernández et al., 2007b, p. 14]:

- Number of reference entities NoR : used to test the Hypothesis 2.
- Coupon rate CR : agreed amount in percentage for CLN with fixed coupon and estimated average value for CLN with variable or mixed coupon, used to test the Hypothesis 3.

- Coupon type CT : either takes the value of 1 for pure fixed coupon or 0 for float rates based coupon, used as additional control variable to test the Hypothesis 3.
- Maturity Mat : duration of contracts expressed in years, used to test the Hypothesis 4.
- Number of payments NoP : number of payment days agreed assuming no default, used to test the Hypothesis 5.
- Date of issue DoI : defined as the distance from the 1st January 2004, used to test the Hypothesis 5.
- Issuer $LBBW$, DZ , HVB : Three larger issuers of the three are defined as independent variables to test the Hypothesis 6.

Two strong correlations were found under these independent variables:

	NoR	***CR***	***DoI***	***Mat***	***NoP***	***CT***	***LBBW***	***DZ***	***Commerzbank***
NoR	1	0.03881	-0.47495	0.18404	0.178906	-0.01228	-0.07692	0.123561	0.325086634
CR	0.03881	1	0.240374	-0.15025	0.160424	-0.41029	-0.09749	-0.22463	0.337980079
DoI	-0.47495	0.240374	1	-0.41845	-0.20782	-0.07002	0.133486	-0.21773	-0.154675602
Mat	0.18404	-0.15025	-0.41845	1	0.811943	-0.31682	-0.33951	0.062097	0.193223232
NoP	0.178906	0.160424	-0.20782	0.811943	1	-0.72108	-0.23509	-0.16282	0.269016238
CT	-0.01228	-0.41029	-0.07002	-0.31682	-0.72108	1	0.034441	0.246969	-0.162794721
LBBW	-0.07692	-0.09749	0.133486	-0.33951	-0.23509	0.034441	1	-0.41211	-0.454858826
DZ	0.123561	-0.22463	-0.21773	0.062097	-0.16282	0.246969	-0.41211	1	-0.319771771
Commerzbank	0.325087	0.33798	-0.15468	0.193223	0.269016	-0.16279	-0.45486	-0.31977	1

Table 3: Correlation matrix of independent variables

These correlations can also be explained intuitively:

- The longer the maturity of a CLN contract is, more probably will it be arranged with more periodical coupon payments in between
- Float rate CLN has more payment days since the coupon are mostly paid every three months, while fixed rate CLN are mostly paid annually

Furthermore, since all the products of HVB are single referenced, which represent the majority of the single referenced CLN, there exist strong correlation between the number of references and the issuer. After regressing each independent variable with all others, we found only the following 8 combinations which fulfill the requirement of not collinear regressor:

1	*NoR*	*CR*	*DoI*	*CT*	*LBBW*	*DZ*	
2	*NoR*	*CR*	*DoI*	*CT*	*LBBW*	*COBA*	
3	*NoR*	*CR*	*DoI*	*CT*	*DZ*	*COBA*	
4	*NoR*	*CR*	*DoI*	*Mat*	*CT*	*DZ*	*COBA*
5	*NoR*	*CR*	*Mat*	*CT*	*DZ*	*COBA*	
6	*NoR*	*CR*	*DoI*	*NoP*	*LBBW*	*COBA*	
7	*NoR*	*CR*	*DoI*	*NoP*	*DZ*	*COBA*	
8	*NoR*	*CR*	*DoI*	*NoP*	*DZ*	*LBBW*	

Table 4: Combinations with uncorrelated regressor

Since both the price difference ΔV and the minimal required recovery rate $REC_{\min}$ can be defined as the dependent variable, we have 16 possible regression equations in total.

5.2.1 Tests based on price difference

Taking the first regression equation as example:

$$\Delta V = c_0 + c_1 * NoR + c_2 * CR + c_3 * DoI + c_4 * CT + c_5 * LBBW + c_6 * DZ + \varepsilon$$

where c_0 to c_6 represents the coefficients to be estimated and ε is the residue. The result can be summarized as the following:

Regression Statistics	
Multiple correlation coeffizient	0.744573
Coefficient of determination	0.554389
Adjusted coefficient of determination	0.533663
Standard error	0.055968
Observations	136

Table 5: Regression statistics of the first regression

ANOVA

	Degree of freedom (df)	*Squared sum (SS)*	*Middle squared sum (MS)*	*Test statistics (F)*	*F critical*
Regression	6	0.502728466	0.083788078	26.74836147	1.56305E-20
Residue	129	0.404086883	0.003132456		
Total	135	0.906815349			

	Coefficient	*Standard error*	*t-Statistics*	*P-Value*	*Under 95%*	*Over 95%*
Intersection	0.077662445	0.034794104	2.232057592	0.027336487	0.008821458	0.146503432
NoR	0.03877702	0.006527177	5.940856492	2.47662E-08	0.025862841	0.051691198
CR	1.665493238	0.447936472	3.718146078	0.000298072	0.779239969	2.551746507
DoI	-0.029948297	0.014344637	-2.087769636	0.038785827	-0.058329511	-0.001567082
CT	-0.070768745	0.014079194	-5.026477014	1.63241E-06	-0.098624775	-0.042912715
LBBW	-0.027509261	0.011350215	-2.423677405	0.016751213	-0.04996594	-0.005052582
DZ	-0.035874048	0.013612335	-2.635407432	0.009433811	-0.062806384	-0.008941711

Table 6: ANOVA of the first regression

Comparing the critical value of F-test and the generated test statistics we can see that the first regression is significant in general, with an adjusted coefficient of determination of 53 percent. Four independent variables have significant impact on the extent of overpricing with a confidence level of 99 percent, namely the number of reference entities, the coupon rate, the coupon type and the issuer DZ Bank. Variables with positive coefficients have positive impact on the extent of overpricing and vice versa.

However, we found out that there exists heteroscedasticity in residues of several independent variables, which makes the results of t-statistics invalid. In case of the first equation, NoR, CR, DoI and NoP are affected. Here is an example of plotting NoR residues:

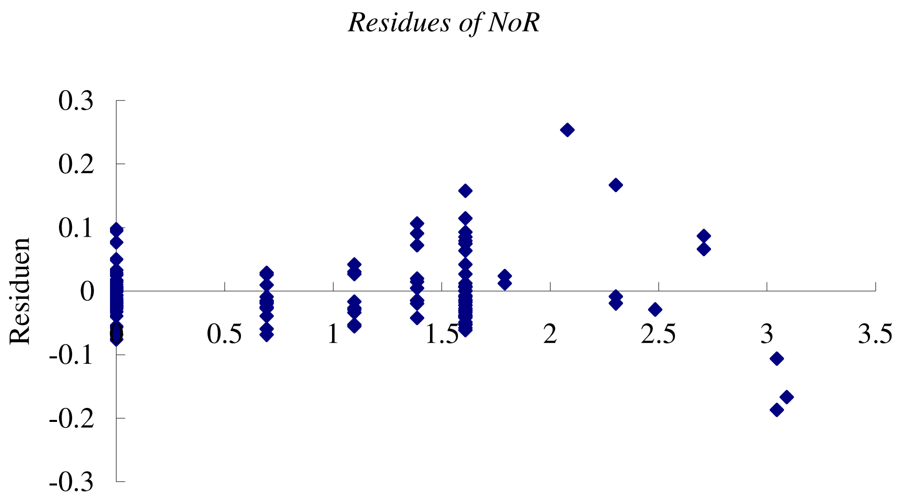


Figure 10: Heteroscedastic residues of NoR

In order to have homoscedastic residues, we applied a weigthed least square estimation on the data of affected independent variables [Griffiths et al., 1993, p. 502]. By applying this transformation to each independent variable with heteroscedastic residues, we get valid t-statistics accordingly. As a result, NoR, CR and CT remain significant.

The same procedure was applied to all other regression equation, the results of all regressions based on the dependent variable ΔV can be consolidated in the following table:

	Significance in regression	*Ajusted R^2*	*NoR*	*CR*	*CT*	*DoI*	*Mat*	*NoP*	*LBBW*	*DZ*	*Coba*
Recovery Rate 8.8% and Asset Correlation	1	53.4%	0.0404 h(p<0.001)	1.6316 h(p<0.001)	-0.0750 h(p<0.001)	-0.0136 h(0.4063)			-0.0289 h(0.0079)	-0.0359 (0.0094)	
	2	51.6%	0.0398 h(p<0.001)	1.6766 h(p<0.001)	-0.0775 (p<0.001)	-0.0266 (0.0713)			-0.0069 (0.5528)		0.0194 (0.1690)
	3	51.9%	0.0385 h(p<0.001)	1.6808 h(p<0.001)	-0.0743 (p<0.001)	-0.0392 h(0.0305)				-0.0148 (0.2646)	0.0179 (0.1836)
	4	53.8%	0.0393 h(p<0.001)	2.0388 (p<0.001)	-0.0673 h(p<0.001)	-0.0169 (0.2740)	0.0096 (0.0144)			-0.0192 (0.1436)	0.0101 (0.4542)
	5	53.7%	0.0412 h(p<0.001)	1.8728 h(p<0.001)	-0.054 (p<0.001)		0.0094 h(0.0067)			-0.0176 (0.1781)	0.0111 (0.4095)
	6	52.7%	0.0374 h(p<0.001)	2.2184 h(p<0.001)		-0.0145 (0.3231)		0.0325 h(p<0.001)	0.0027 (0.8123)		0.0141 (0.3143)
	7	53.4%	0.0379 h(p<0.001)	2.2012 h(p<0.001)		-0.0181 (0.2233)		0.0318 h(p<0.001)		-0.0177 (0.1724)	0.0066 (0.6170)
	8	53.8%	0.0377 h(p<0.001)	2.1727 h(p<0.001)		-0.0183 (0.2117)		0.0305 h(p<0.001)	-0.0140 (0.2380)	-0.0304 h(0.0165)	
Recovery Rate 35% and Asset Correlation	1	52.7%	0.0295 h(p<0.001)	0.8804 h(0.0126)	-0.0622 (p<0.001)	-0.0127 (0.2305)			-0.0222 h(0.0052)	-0.0228 h(0.0113)	
	2	51.2%	0.0299 h(p<0.001)	0.9275 h(0.0111)	-0.0666 (p<0.001)	-0.0107 (0.3205)			-0.0090 (0.2879)		0.0098 (0.3434)
	3	50.9%	0.0287 h(p<0.001)	0.9274 h(0.0113)	-0.0649 (p<0.001)	-0.0134 (0.2183)				-0.0071 (0.4651)	0.0121 (0.2199)
	4	51.3%	0.0288 h(p<0.001)	1.1521 (0.0016)	-0.0604 h(p<0.001)	-0.0075 (0.5142)	0.0041 (0.1548)			-0.0090 (0.3569)	0.0088 (0.3848)
	5	51.5%	0.0300 h(p<0.001)	1.0153 h(0.0059)	-0.0561 (p<0.001)		0.0048 (0.0770)			-0.0083 (0.3928)	0.0092 (0.3928)
	6	47.3%	0.0274 h(p<0.001)	1.5003 h(p<0.001)		-0.0026 (0.8211)		0.0221 h(p<0.001)	-0.0020 (0.8246)		0.0064 (0.5538)
	7	47.8%	0.0278 h(p<0.001)	1.4836 h(p<0.001)		-0.0052 (0.6486)		0.0219 (p<0.001)		-0.0111 (0.2650)	0.0036 (0.7247)
	8	48.5%	0.0271 h(p<0.001)	1.4429 h(p<0.001)		-0.0051 (0.6525)		0.0207 h(p<0.001)	-0.0124 (0.1741)	-0.0195 (0.0707)	

Table 7: Consolidated results of regressions based on price difference dark and light (Column 3: Adjusted R^2 of original regression. Column 4-9 estimated coefficient, in bracket p-value, h indicated new estimation after correction for heteroskedasticity)

Having the price difference as the dependent variable will lead to heteroscedastic residues for almost half of the independent variables. After the weighted least square regression , *NoR* , *CR* , *CT* , and *NoP* were tested to be significant in all corresponding regressions;[6] *Mat* is at 95% and 99% significant in its two regressions; the while the date of issue and the issuers could not be proved to have significant impact on the price difference. Consequently *NoP* seems to be more determined by *CT* than by *Mat*, which can be proven by the fact, that due to the quarterly coupon frequency of the variable notes CT of variable notes is extraordinary high. Therefore CT might be the result of the use of the common bank approach in valuing floaters. Hence a credit risk free floaters are priced correctly, it is unlikely.

5.2.2 Tests based on required minimal recovery rate

In the second step we made the above regressions with the recovery rate as the new dependent variable. In opposite to the earlier regressions only a fistful equations are proved to show

[6] According to a Kolmogorov/Smirnov-test and a Likelihood-ratio-test the hypothesis of non-normal residuals can not be rejected in all expect of two regressions on a 5%-level. Anyhow, after correcting for the heteroskedasticity the effect of non-normality vanishes. Due to the application of DoI in 7 of the 8 regressions as independent variable positive or negative autocorrelation in the residuals can not be verified.

heteroscedasticity and need to be transformed and all residues are normally distributed. Furthermore we get a lower adjusted coefficient of determination compared with the first regression. The results of all regressions based on the dependent variable $REC_{\min}$ can be consolidated in the following table:

	Significance in regression	*Ajusted R²*	*NoR*	*CR*	*CT*	*DoI*	*Mat*	*NoP*	*LBBW*	*DZ*	*Coba*
Recovery Rate 8.8% and Asset Correlation	1	26.7%	6.2123 h(0.0060)	-98.0257 (0.5463)	-27.3149 (p<0.001)	-5.7671 (0.2685)			-1.0189 (0.8044)	-7.4707 (0.1317)	
	2	26.4%	6.6862 h(0.0045)	13.5495 (0.9354)	-29.0858 (p<0.001)	-5.5953 (0.2836)			-0.6171 (0.8808)		-6.5418 (0.1920)
	3	29.2%	7.3843 h(0.0016)	16.4707 (0.9200)	-26.9906 (p<0.001)	-7.7148 (0.1361)				-10.369 (0.0264)	-8.9301 h(0.0278)
	4	36.1%	7.8137 (p<0.001)	-163.9895 (0.2145)	-39.6412 h(p<0.001)	-14.8841 (0.0051)	-5.074 (p<0.001)			-8.0229 (0.0723)	-5.8097 (0.2057)
	5	32.6%	10.4409 (p<0.001)	-243.6709 (0.1405)	-35.0485 (p<0.001)		-3.7503 (0.0035)			-6.5828 (0.1476)	-4.9160 (0.2955)
	6	8.8%	6.5998 h(0.0104)	338.0663 h(0.0438)		-5.1777 (0.3797)		3.3812 (0.1329)	0.7875 (0.8645)		-6.3668 (0.2558)
	7	14.0%	7.4900 h(0.0031)	311.9254 (0.0683)		-8.1734 (0.1600)		2.5090 (0.0056)		-14.2042 (0.0056)	-11.6042 (0.0268)
	8	10.8%	6.5042 h(0.0085)	187.9799 (0.2678)		-6.3208 (0.2813)		1.7019 (0.4620)	-1.7843 (0.7068)	-11.5074 (0.0413)	

Table 8: Consolidated results of regressions based on minimal recovery rate (Column 3: Adjusted R² of original regression. Column 4-9 estimated coefficient, in bracket p-value, h indicated new estimation after correction for heteroskedasticity)

Compared to the results based on price difference, defining the required minimal recovery rate as the dependent variable will lead to significance of the regressors of NoR, CT and Mat. While the significance of NoP and CR could only be verified partially, the impact of issuer and date of issue still can not be robustly proved.

Consolidating the results of all 24 regression equations, we conclude that for the data sample analyzed, the first three hypotheses are valid, while the rest three hypotheses can not be verified statistically.

On the one hand, it is difficult, especially for retail investors, to estimate the default risks with large number of underlying reference entities, since the calculation requires sophisticated model and wide range of data. This asymmetry of information often encourages issuers to overprice their products. That complexity, here measured by the number of reference entities, plays a key role and is supported by our results as well by the size of overpricing, which is in the line with many studies, e. g. Gruenbichler and Wohlwend [2005, p. 372], Hernández et al. [2007b, p. 26], Szymanowska et al. [2009, p. 907] or Stoimenov and Wilkens [2005, p. 2986].

Moreover, a complex coupon structure and long maturity which involves both fixed and float coupon rates might seem to be more attractive for most investors (See for similar results Benet et al. [2006, p. 124] or Hernández et al. [2007a, p. 33]), since they often assess the conditions based on their first sight. Hidden factors behind the high fixed coupon rate usually make these products even more overpriced compared to those with simple coupon structure. This is partly in contradiction to the studies of Burth et al. [2001, p. 13], or Hernández et al. [2007b, p. 26], whose sample include non coupon bearing instruments, but our results coincide with Wallmeier and Diethelm [2009, p. 17].

On the other hand, no significant impact on the extent of overpricing can be proved for the factor date of issue and no significant difference was found among the pricing behaviors of different issuers [see for opposite results Hernández et al. [2007a, p. 32] and Szymanowska et al. [2009, p. 916]) either. In this regard our results are in the line with Wallmeier and Diethelm [2009].

6 Summary and outlook

In this paper we analyzed the pricing of the 136 outstanding CLN of the four major issuers on the German market. To this end we applied a market based valuation model for CLN, which is based on the reduced model of Jarrow/Turnbull and extended by the single factor Merton model to estimate the joint default probabilities out of assets correlations. The model was calibrated by CDS-spreads and correlations of stock returns for the underlying reference entities.

We found out that the observed CLN are generally overpriced in the primary market: Thereby the result is robust to changes of recovery rates or correlation coefficients. Furthermore four of the six hypotheses we made before the valuation were statistically tested to be valid. As for CLN in the German market, we have analyzed the price of issue for the first time and our major finding is widely consistent with those previous results in the literature: As market maker, issuers of structured financial products participate in almost every transaction and they have the incentive to overprice. The more complex the product and the less transparent the market is, the larger overpricing there tends to be. This consideration is confirmed by the results regarding our major hypothesis of overpricing and the extent of overpricing is positively correlated with the number of underlying reference entities, the coupon rates and the maturity of the contract.

There are a lot of possibilities to extend our work. Besides the valuation of CLN on the date of their issue, we can apply the same model to calculate the daily fair prices and compare them to the daily quoted prices. By tracking the daily development of price difference, the hypothesis of product life cycle can also be tested for CLN. Moreover, the change of interest rate can also be modeled more specifically to calculate more accurate fair prices of CLN with float coupon payments. Last but not least, the valuation framework can be used for CLN products issued in other regions to test if the verified hypotheses in the German market are universally valid.

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APPENDIX

Prove for unity and existence of a default probability:

First we look at the one year CDS. Furthermore we assume, that the default probability $q_{0.25}$ is constant trough out one year. This assumption is necessary to shrink the conditions of freedom to one for one given CDS-spread CDS_1. Hence the CDS is fair priced with cds_1, if present value of the expected payments of the riskseller are equal to the payments of the risk buyer under the measure $q_{0.25}$ and discounted by the factor $d_{0.25}$ up to d_1:

$(1- q_{0.25})\cdot c_{0.25}\cdot d_{0.25}+(1- q_{0.25})^2\cdot c_{0.25}\cdot d_{0.5}+(1- q_{0.25})^3\cdot c_{0.25}\cdot d_{0.75}+(1- q_{0.25})^4\cdot c_{0.25}\cdot d_1$

$=0.6\cdot q_{0.25}\cdot d_{0.25}+0.6\cdot q_{0.25}\cdot(1- q_{0.25})\cdot d_{0.5}+0.6\cdot q_{0.25}\cdot(1- q_{0.25})^2\cdot d_{0.75}+0.6\cdot q_{0.25}\cdot(1- q_{0.25})^3\cdot d_1$,

whereby according to ISDA [2009] $c_{0.25} = 0.25\cdot CDS_1$ and 0.6 is the loss given default.

Instead of the default probability $q_{0.25}$ the quarterly probability of survival x is used, which leads to:

$x\cdot c_{0.25}\cdot d_{0.25}+x^2\cdot c_{0.25}\cdot d_{0.5}+x^3\cdot c_{0.25}\cdot d_{0.75}+x^4\cdot c_{0.25}\cdot d_1$

$-0.6\cdot(1-x)\cdot d_{0.25}-0.6\cdot(1-x)\cdot x\cdot d_{0.5}-0.6\cdot(1-x)\cdot x^2\cdot d_{0.75}-0.6\cdot(1-x)\cdot x^3\cdot d_1=0.$

This yields to

$-0.6\cdot d_{0.25} +x\cdot(c_{0.25}\cdot d_{0.25}-0.6\cdot(-d_{0.25}+d_{0.5}+d_{0.75}+d_1))+$

$x^2\cdot(c_{0.25}\cdot d_{0.5}+0.6\cdot d_{0.5}+1.2\cdot d_{0.75}+1.8\cdot d_1)+$

$x^3\cdot(c_{0.25}\cdot d_{0.75}+0.6\cdot d_{0.75}-1.8\cdot d_1)+$

$x^4\cdot(c_{0.25}\cdot d_1+0.6\cdot d_1)=0$

Hence we get a polynomial of fourth degree. Furthermore we can conclude, that for a survival probability of zero the value is strictly negative. Additionally, because of the positivity

$c_{0.25}\cdot d_1+0.6\cdot d_1>0$

of coefficient of x^4 the limits of an infinite survival probability are both positive infinite, leading to a minimum number of one positive survival probability by the intermediate value theorem.

The next step is to prove the unity. By definition a polynomial of fourth degree may have max. three local extrema and therefore two inflection points. Consequently there could be three or one positive solutions to the above equation. After the introduction of a_1 up to a_5 as coefficients, we have:

$-a_1-x\cdot a_2+x^2\cdot a_3-x^3\cdot a_4+x^4\cdot a_5=0.$

Derivation leads to

$-a_2+2\cdot a_3\cdot x-3\cdot a_4\cdot x^2+4\cdot a_5\cdot x^3=0$

and

$2\cdot a_3-6\cdot a_4\cdot x+12\cdot a_5\cdot x^2=0.$

In case that there is no solution to the last equation, there is no inflection point, one minimum and lately the equation above has only one positive solution. The possible inflection points are

$$x=\frac{6\cdot a_3 \pm \sqrt{36\cdot a_4{}^2-96\cdot a_5\cdot a_3}}{24\cdot a_5}.$$

They exist, if the discriminant

$36\cdot a_4^2-96\cdot a_5\cdot a_3$

is positive. Introducing a_3 up to a_5 leads to

$36\cdot(c_{0.25}\cdot d_{0.75}+0.6\cdot d_{0.75}-1.8\cdot d_1)^2>96\cdot(c_{0.25}\cdot d_1+0.6\cdot d_1)\cdot(c_{0.25}\cdot d_{0.5}+0.6\cdot d_{0.5}+1.2\cdot d_{0.75}+1.8\cdot d_1)$

$3\cdot(c'_{0.6}{}^2\cdot d_{0.75}{}^2-3.6\cdot c'_{0.6}\cdot d_1\cdot d_{0.75}+3.24\cdot d_1{}^2)>8\cdot(c'_{0.6}\cdot d_1\cdot d_{0.5}+1.2\cdot c'_{0.6}\cdot d_1\cdot d_{0.75}+1.8\cdot c'_{0.6}\cdot d_1{}^2)$,

whereby $c'_{0.6}=c_{0.25}+0.6$.

If the interest rate for the third quarter is below 150% per quarter.

$3\cdot d_{0.75}{}^2<8\cdot d_1\cdot d_{0.5}$

leading to

$3\cdot(-3.6\cdot c'_{0.6}\cdot d_1\cdot d_{0.75}+3.24\cdot d_1{}^2)>8\cdot(1.2\cdot c'_{0.6}\cdot d_1\cdot d_{0.75}+1.8\cdot c'_{0.6}\cdot d_1{}^2)$.

Rearranging yields to

$1.68\cdot d_1{}^2>8\cdot(1.2\cdot c'_{0.6}\cdot d_1\cdot d_{0.75}+1.8\cdot c_{0.25}\cdot d_1{}^2)+10.8\cdot c'_{0.6}\cdot d_1\cdot d_{0.75}$,

which is automatically false under the above interest rate condition.

Hence the defining equation has no inflexion point and therefore one positive survival probability.

Qued.

In case of a CDS with a maturity longer than one year, the CDS can be recombined out of

the present value of the payments for the first periods and a CDS for last period. In case the present value for the first payments is negative the arguments of above still hold. If the present value is positive, it has to be smaller than $-0.6\cdot d_{0.25+p}$ to ensure a solution.

Therefore result can be transferred to the multiperiod case.