The Determinants of Variance Swap Rate Changes

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Using the price information of S&P 500 equity index options, we investigate the determinants of synthetic variance swap rate changes for the March 1^{st}, 2006, to August 29^{th}, 2008, period. The purpose of this paper is to bridge two strands in the literature. One suggests that variance swaps should be driven by higher moments of the risk-neutral return distribution such as skewness and kurtosis. Our results indicate that they have a significant impact on long-term variance swap rates. However, they have rather limited explanatory power for short-term quotes. Another part of the literature identifies variance risk as the main driving factor. We back out implied variance of at-the-money VIX options as a simple proxy for variance risk and find a significant impact on both, short and long-term swap rates.

To determine implied moments of the risk-neutral return distribution we use the model-free metrics of Bakshi, Kapadia, and Madan (2003) and discuss their discretization biases. Contrary to previous work in the literature, we show that asymmetric integration intervals are not a source of bias and confirm our finding empirically. Instead, we suggest to include all observations over the complete range of nonzero option prices.

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1 Introduction

Previous empirical work has found that risk-neutral index return variance generally exceeds its counterpart under the physical measure.[1] Therefore, mean variance swap premia are higher than the average historical return variance would suggest.[2] Some authors argue that the entire differential should be attributed to variance risk as its absence leads to the equality between the risk-neutral and realized variance.[3] This interpretation is, however, challenged by a second strand in the literature. Its argumentation can be sketched as follows. As implied variance is negatively dependent on equity market returns, the difference can be interpreted as insurance premium against sudden market declines.[4] Investors' desire to protect portfolio values typically shifts probability mass to the negative risk-neutral tail of the conditional equity index return distribution (relative to its physical counterpart).[5] This is observable as implied skewness and kurtosis. Intuitively, just like the expected market return is corrected by the variance in the CAPM world, expected variance is in turn corrected by skewness and perhaps even higher moments of the risk-neutral return distribution (RND).[6]

Abundant empirical work investigates the question to what extent variance risk is priced in derivative contracts. Various studies calibrate structural models to option prices allowing for the existence of variance risk.[7] These specifications have the strongest assumptions. Their results are prone to model misspecification. Coval and Shumway (2001) investigate the properties of ATM zero-beta straddles and find negative variance risk premia. Bakshi and Kapadia (2003a, 2003b) find similar results by examining the statistical properties of delta-hedged option portfolio payoffs. The magnitude of the negative premium is higher for out-of-the-money options and therefore dependent on moneyness levels. Carr and Wu (2009) explore the correlation of

[1] See, e.g., Bakshi and Madan (2006) and the references therein.

[2] Let $Q_{t,T}$ be the realized return variance during the time interval $[t, T]$. A variance swap with fixed leg α^2 and constant conversion factor u^2 pays its holder at maturity $u^2 \times \left(Q_{t,T} - \alpha^2\right)$. For an extensive treatment, see Carr and Lee (2007a, 2007b).

[3] See Polimenis (2006) and the references therein.

[4] Empirical evidence for a negative dependence has been found in Jackwerth (2000), Bakshi and Kapadia (2003a, 2003b), Dennis, Mayhew, and Stiver (2006), Bollerslev, Gibson, and Zhou (2007), Bollerslev, Tauchen, and Zhou (2009), and Carr and Wu (2009).

[5] See, e.g., Bakshi and Madan (2006).

[6] See, e.g., Jiang and Tian (2005), Bakshi and Madan (2006), Polimenis (2006), Bollerslev, Gibson, and Zhou (2007), Carr and Wu (2009), and the references therein. The importance to include co-skewness and co-kurtosis in asset pricing models has been investigated, among others, in **?**, Christie-David and Chaudhry (2001), Bakshi, Kapadia, and Madan (2003), Vanden (2006), and Moreno and Rodriguez (2009).

[7] Examples are Hull and White (1987), Scott (1987), Wiggins (1987), Heston (1993), Bakshi, Cao, and Chen (1997), Guo (1998), Bates (2000), Chernov and Ghysels (2000), Pan (2002), Broadie, Johannes, and Chernov (2005), and Broadie, Chernov, and Johannes (2008).

synthetic variance risk premia with stock index returns and conclude that variance is a negative-beta asset class. Similarly, Ang, Hodrick, Xing, and Zhang (2006) find a negative relation between the performance of stock portfolios and their sensitivity to changes in the VIX index. Most related to our study are the studies of Wu (2005), Bollerslev, Gibson, and Zhou (2007), and Todorov (2009). They use high-frequency equity index (futures) return data and the VIX to investigate the dynamics of the variance risk premium. The general finding of these studies is that variance risk premia are negative and proportional to the variance rate level. Leippold, Egloff, and Wu (2007) calibrate a two-factor setting to model the term structure dynamics of OTC variance swap contracts.

Our study uses a more direct approach to measure variance risk. On February 24^{th}, 2006, the Chicago Board Options Exchange (CBOE) launched the first exchange-traded options written on the VIX. The underlying tracks the square root of a standardized synthetic 30 calendar day variance swap rate backed out from S&P 500 index options. We determine implied volatility levels of their ATM contracts to construct a simple measure for variance risk. During the past few years, substantial progress has been achieved in understanding the properties and price behavior of volatility spot and derivative markets.[8] First empirical evidence on VIX option pricing models, such as Whaley (1993), Grünbichler and Longstaff (1996), Detemple and Osakwe (2000), and Carr and Wu (2006), is investigated in Wang and Daigler (2009). Lin and Chang (2009) find that state-dependent jumps in the underlying price dynamics are important for pricing VIX options. Yet, to the best of our knowledge, our study is the first one exploiting the price information of VIX options to construct a measure for variance risk. Our results show a strong and significant impact on the changes of variance swap rates. In addition, we observe that higher moments in the risk-neutral return distribution have no significant impact on changes of variance swap rates with a short time to maturity. However, we identify a significant positive dependence for swap premia with longer terms to expiry.

To derive the moments of the risk-neutral return distribution, we use the model-free metrics of Bakshi, Kapadia, and Madan (2003). They require only mild assumptions on the implied dynamics of the underlying and can be backed out from observable S&P 500 index option prices. However, they are based on a continuum of available option strike prices. Therefore, Dennis and Mayhew (2002) discuss possible biases derived from discrete strike price availability. Contrary to their findings, we do not find evidence that asymmetric integration intervals are a potential source of distortion. We suggest to consider all observations over the complete support of the risk-neutral share price distribution for the calculation of the Bakshi, Kapadia, and Madan (2003) metrics.

[8] See, e.g., Dash and Moran (2005), Black (2006), Brenner, Ou, and Zhang (2006), Daigler and Rossi (2006), Windcliff, Forsyth, and Vetzal (2006), Dong (2007), Dash and Moran (2007), Pavlova and Daigler (2008), and the references therein.

Our article is organized as follows: Section 2 gives a short introduction to the moment measures of Bakshi, Kapadia, and Madan (2003). We show that the calculation of the VIX can generally be seen as a special case of this method. Section 3 describes our dataset and provides details on our construction of the variance risk measure. In addition, it contains an extensive discussion on the discretization bias of the Bakshi, Kapadia, and Madan (2003) metrics. The subsequent Section 4 identifies whether changes in variance risk or higher moments in the RND are the main driving force in variance swap rate changes. Finally, we conclude in Section 5.

2 Measuring RND Moments - The VIX and Its Derivatives

The fundamental approach of derivatives pricing (and even of asset pricing in general) is to derive the shape of the state-price density (SPD), which assigns to the realizations of the risk factors unique Arrow-Debreu prices.[9] Its existence and characterization is mainly derived by arbitrage-based reasoning or by preference-based approaches. In the former case, the fair value is determined by the discounted expected payoff under the risk-neutral density. In specifications that allow for dynamic replication of option values, prices are determined by the assumed SPD of the replicating portfolio. Models which do not treat options as redundant assets require additional restrictions on the risk-neutral distribution.[10] Empirical pricing fits of the suggested valuation frameworks critically depend on the validity of the presumed SPD's functional form.[11] Therefore, a second strand in the literature turns the perspective and intends to back out the risk-neutral density from observed option prices. Although Arrow-Debreu prices are not traded in financial markets, they can be approximated, as first suggested by Breeden and Litzenberger (1978), Banz and Miller (1978), and Ross (1976). According to this approach, they can be estimated by the second partial derivative of the option price O with respect to the strike price X, i.e. $\partial O^2 / \partial^2 X$, evaluated at the underlying's price S_T at maturity, i.e. $X = S_T$. Applying this across a continuum of exercise prices determines the SPD.

[9] Each strand of the literature has developed its own nomenclature for related approaches. Asset pricing models, for example, apply the concept in the form of stochastic discount factors or pricing kernels by transforming the physical risk factor distributions into equivalent martingale measures or risk-neutral densities. See, for example, Hansen and Richard (1987), as well as Hansen and Jagannathan (1991). The SPD also uniquely determines the state price deflator (See Duffie (2001), as well as Aït-Sahalia and Duarte (2003).)

[10] The two standard approaches are (1) to treat the additional risk source as idiosyncratic with zero price and (2) to introduce restrictions by the form of a representative agent's preference function. We refer to Bates (1991), Bakshi, Cao, and Chen (1997), and Bates (2000) for overviews on the most influential works.

[11] For example, Jackwerth and Rubinstein (1996) reject the validity of the lognormal distribution implied by the Black-Scholes model for a proper description of S&P 500 index option prices.

As Bakshi and Madan (2000) note, its knowledge principally solves the pricing problem of derivatives with arbitrary payoff functions.[12] Yet, as outlined in Jackwerth and Rubinstein (1996), as well as Jackwerth (1999), finite quote precision and noisy option values (for example due to bid-ask spreads or asynchronous price observation times) render the market incomplete and the SPD unidentifiable. As such, there exists more than one pricing measure.[13] Therefore, an overwhelming number of different techniques have been suggested for the estimation of the risk-neutral density. Most of the approaches have the common aim to find a parametric or nonparametric functional form that interpolates between available strike prices and extrapolates outside their range.[14]

In this study, we are merely interested in the moments of the risk-neutral SPX option density. Therefore, most of the approaches are unnecessarily cumbersome and in many cases too restrictive. Instead, we use the metrics of Bakshi, Kapadia, and Madan (2003), which are based on the work of Bakshi and Madan (2000).[15] They show that fair values of any twice-continuously differentiable payoff function $H(S_T)$ with bounded expectation, i.e.

$$E_t^{\mathbb{Q}}\left[H(S_T)\right] = \int_0^\infty \left|H(S_T)\right| q(S_T) dS_T < \infty,$$

under the risk-neutral measure $q(S_T)$ can be recovered from bonds, the underlying, and out-of-the-money option prices. As we will see later in this section, the calculation of the VIX is a special case of this technique.

For an outline of the concept, let $V_t(\tau)$, $W_t(\tau)$, and $Y_t(\tau)$ denote the time-t prices of derivatives with a quadratic, cubic and quartic return payoff with time to maturity $\tau = (T-t)/365$, i.e. $V_t(\tau) = E_t^{\mathbb{Q}}\left[e^{-r_f\tau}R_t(\tau)^2\right]$, $W_t(\tau) = E_t^{\mathbb{Q}}\left[e^{-r_f\tau}R_t(\tau)^3\right]$ and $Y_t(\tau) = E_t^{\mathbb{Q}}\left[e^{-r_f\tau}R_t(\tau)^4\right]$, where r_f denotes the matching risk-free interest rate. Following Bakshi, Kapadia, and Madan (2003), the skewness $\gamma_t(\tau)$ and kurtosis $\kappa_t(\tau)$ of the risk-neutral distribution can be written as

$$\gamma_{t,T} = \frac{E_t^{\mathbb{Q}}\left\{\left(R_t(\tau) - E_t^{\mathbb{Q}}\left[R_t(\tau)\right]\right)^3\right\}}{\left\{E_t^{\mathbb{Q}}\left[R_t(\tau) - E_t^{\mathbb{Q}}\left[R_t(\tau)\right]\right]^2\right\}^{3/2}} = \frac{e^{r_f\tau}W_t(\tau) - 3\mu_t(\tau)e^{r_f\tau}V_t(\tau) + 2\mu_t(\tau)^3}{\left[e^{r_f\tau}V_t(\tau) - \mu_t(\tau)^2\right]^{3/2}}$$

[12] In fact, the knowledge of the SPD translates into knowing its characteristic function (cf). Its multiplication with the cf of the payoffs provides a simple method to determine fair option prices by the discounted inverse. See also Duffie, Pan, and Singleton (2000) for affine models.

[13] Further empirical problems can be found in Jackwerth (1999) and the references therein.

[14] See, e.g., Jackwerth (1999), Aït-Sahalia and Duarte (2003), Yatchew and Härdle (2006), and the references therein.

[15] See also Carr and Madan (2001), and Bakshi, Kapadia, and Madan (2003).

and

$$\kappa_{t,T} = \frac{E_t^{\mathbb{Q}}\left\{\left(R_t(\tau) - E_t^{\mathbb{Q}}\left[R_t(\tau)\right]\right)^4\right\}}{\left\{E_t^{\mathbb{Q}}\left[R_t(\tau) - E_t^{\mathbb{Q}}\left[R_t(\tau)\right]\right]^2\right\}^2} = \frac{e^{r_f\tau}Y_t(\tau) - 4\mu_t(\tau)e^{r_f\tau}W_t(\tau) + 6e^{r_f\tau}\mu_t(\tau)^2V_t(\tau) - 3\mu_t(\tau)^4}{\left[e^{r_f\tau}V_t(\tau) - \mu_t(\tau)^2\right]^2},$$

where

$$\begin{aligned} V_t(\tau) &= \int_{S_t}^{\infty} \frac{2\left(1 - \ln\left(\frac{X}{S_t}\right)\right)}{X^2} C_t(\tau, X)dX + \int_0^{S_t} \frac{2\left(1 + \ln\left(\frac{S_t}{X}\right)\right)}{X^2} P_t(\tau, X)dX, \\ W_t(\tau) &= \int_{S_t}^{\infty} \frac{6\ln\left(\frac{X}{S_t}\right) - 3\left(\ln\left(\frac{X}{S_t}\right)\right)^2}{X^2} C_t(\tau, X)dX - \int_0^{S_t} \frac{6\ln\left(\frac{S_t}{X}\right) + 3\left(\ln\left(\frac{S_t}{X}\right)\right)^2}{X^2} P_t(\tau, X)dX, \\ Y_t(\tau) &= \int_{S_t}^{\infty} \frac{12\left(\ln\left(\frac{X}{S_t}\right)\right)^2 - 4\left(\ln\left(\frac{X}{S_t}\right)\right)^3}{X^2} C_t(\tau, X)dX \\ &+ \int_0^{S_t} \frac{12\left(\ln\left(\frac{S_t}{X}\right)\right)^2 + 4\left(\ln\left(\frac{S_t}{X}\right)\right)^3}{X^2} P_t(\tau, X)dX, \end{aligned} \tag{1}$$

and

$$\mu_t(\tau) = E_t^{\mathbb{Q}}\left[R_t(\tau)\right] \approx e^{r_f\tau} - 1 - \frac{e^{r_f\tau}}{2}V_t(\tau) - \frac{e^{r_f\tau}}{6}W_t(\tau) - \frac{e^{r_f\tau}}{24}Y_t(\tau).$$

$C_t(\tau, X)$ and $P_t(\tau, X)$ denote the time-t out-of-the-money call an put prices, respectively.

Having a closer look at (1), we find a very intuitive construction principle of the Bakshi, Kapadia, and Madan (2003) metrics. Consider calculating the value of a twice-continuously differentiable payoff $H_t(\tau, S)$.[16] Remember from Breeden and Litzenberger (1978) that it may be written as

$$H_t(\tau, S) = \int_0^{\infty} H(S_T = X)\frac{\partial^2 O(\tau, S_T = X)}{\partial X^2}\Big|_{S_T = X} dX,$$

where $\frac{\partial^2 O(\tau, S_T = X)}{\partial X^2}$ denotes the second derivative of an out-of-the-money option with respect to its strike price X. It can be interpreted as the price of an Arrow-Debreu pure security paying 1\$ in case of $S_T = X$ and zero otherwise. Applying integration by parts twice leads to

$$\begin{aligned} H_t(\tau, S) =& H_t(S_t)\frac{\partial P_t(\tau, X)}{\partial X}\Big|_0^{S_T} + H_t(\tau, S_t)\frac{\partial C_t(\tau, X)}{\partial X}\Big|_{S_T}^{\infty} - P_t(\tau, X)\frac{\partial H_t(\tau, S_T)}{\partial X}\Big|_0^{S_T} \\ & - C_t(\tau, X)\frac{\partial H_t(\tau, S_T)}{\partial X}\Big|_{S_T}^{\infty} + \int_0^{\infty} \frac{\partial^2 H_t(\tau, S_T)}{\partial X^2}\Big|_{S_T = X} O(\tau, S_T = X)dX \end{aligned}$$

Furthermore, put-call-parity delivers $\frac{\partial P_t(\tau, X)}{\partial X} - \frac{\partial C_t(\tau, X)}{\partial X} = e^{-r_f\tau}$. Inserting yields

$$\begin{aligned} H(\tau, S) = H(\tau, S_t) + (S_T - S_t)\frac{\partial H(\tau, S_T)}{\partial S_T}\Big|_{S_T = S_t} + \int_{S_t}^{\infty} \frac{\partial^2 H(\tau, S_T)}{\partial S_T^2}\Big|_{S_T = X}(S_T - X)^+ dX + \\ \int_0^{S_t} \frac{\partial^2 H(\tau, S_T)}{\partial S_T^2}\Big|_{S_T = X}(X - S_T)^+ dX. \end{aligned} \tag{2}$$

[16] See also Carr and Madan (2001).

Note that $P_t(\tau, X=0) = C_t(\tau, X=\infty) = \frac{\partial P_t(\tau, X=0)}{\partial X} = \frac{\partial C_t(\tau, X=\infty)}{\partial X} = 0$. Taking the risk-neutral expectation and inserting the derivatives yields the required result in (1). Equation (2) highlights several interesting issues. First, it reveals that any twice-continuously differentiable payoff function may be spanned by zero bonds, the stock, and its options.[17] Second, assuming a continuum of available strike prices, there exists a one to one correspondence between pure security prices and the risk-neutral value of the payoff function. Discrete strike price intervals lead to distortions in the Bakshi, Kapadia, and Madan (2003) metrics.[18] The sign of the bias cannot be determined ex ante for all payoff functions. It is clearly negative for $V_t(\tau)$ and $Y_t(\tau)$ as these functions are spanned by long derivative positions. Yet, as we consider a short position of a put portfolio in the calculation of $W_t(\tau)$, omitting nonzero Arrow-Debreu prices may cause a positive distortion of the metric. Third, the valuation principle allows to calculate the fair value of a variance swap rate $E_t^{\mathbb{Q}}\left[\sigma_\tau^2\right]$. Applying Equation (2) delivers

$$E_t^{\mathbb{Q}}\left[\sigma_\tau^2\right] = \frac{2}{\tau} E_t^{\mathbb{Q}}\left[-\ln\frac{F_t(\tau)}{S_t}\right] = \frac{2}{\tau}\int_0^\infty e^{r_f \tau}\frac{O_t(\tau, X)}{X^2}dX, \tag{3}$$

where $F_t(\tau)$ denotes the time-t forward value. This equation establishes the calculation of the risk-neutral variance under the presence of term structure smiles and skews. It constitutes the construction principle of CBOE's volatility index VIX. Its theoretical foundation is based on the log-contract theory, developed in Carr and Madan (1998), Demeterfi, Derman, Kamal, and Zou (1999), Britten-Jones and Neuberger (2000), and Jiang and Tian (2005).

The calculation of the VIX requires some approximations in practice due to discrete strike price observations. Its definition turns Equation (3) into

$$E_t^{\mathbb{Q}}[\sigma_\tau^2] = \frac{2}{\tau}\sum_{i=1}^{N}\frac{\Delta X_i}{X_i^2}e^{r_f \tau}O_t(\tau, X_i) - \frac{1}{\tau}\left[\frac{F_t(\tau)}{X_0} - 1\right]^2, \tag{4}$$

where $O_t(\tau, X_i)$ denotes the observable midquote price of the respective out-of-the-money call or put, i.e. $O_t(\tau) = O_t(\tau)^{bid} + 0.5 \cdot \left(O_t(\tau)^{ask} - O_t(\tau)^{bid}\right)$. X_0 is the first strike price below the forward index level. The last term of the approximation scheme is an adjustment factor turning all discretely sampled option values into out-of-the-money prices.[19] As Demeterfi, Derman, Kamal, and Zou (1999) show, the discretization method leads to systematic upward biases of future return variation estimates due to limited strike price availability. Britten-Jones and Neuberger (2000), Carr and Wu (2006), Carr and Lee (2007b), and Carr and Wu (2009) illustrate the variance tracking error induced by the presence of jumps.

Due to the availability of discrete time to maturity intervals, the definition of the VIX applies an additional

[17] See Bakshi and Madan (2000), Carr and Madan (2001), and Bakshi, Kapadia, and Madan (2003).

[18] See, e.g., Dennis and Mayhew (2002) and Hansis, Schlag, and Vilkov (2009). In a similar context see, e.g., Demeterfi, Derman, Kamal, and Zou (1999), and Carr and Wu (2006).

[19] See Carr and Wu (2006) for a complete derivation and further details of the approximation scheme.

approximation technique to calculate a constant 30 calender day bracket of implied volatility. Let N_{T_1} denote the number of minutes to the annualized expiration ($\tau_1 = T_1/365$) of the nearby contract with at least eight trading days and N_{T_2} the respective counterpart until the next adjacent contract maturity ($\tau_2 = T_2/365$). Linear interpolation leads to

$$\begin{aligned} \mathcal{V}_t &= 100 \cdot \sqrt{E_t^{\mathbb{Q}}\left[\sigma^2_{t,t+30}\right]} = \\ &= 100 \cdot \sqrt{\left[E_t^{\mathbb{Q}}[\sigma^2_{t,T_1}]\tau_1 \left(\frac{N_{T_2} - N_{30}}{N_{T_2} - N_{T_1}}\right) + E_t^{\mathbb{Q}}[\sigma^2_{t,T_2}]\tau_2 \left(\frac{N_{30} - N_{T_1}}{N_{T_2} - N_{T_1}}\right)\right] \cdot \frac{N_{365}}{N_{30}}} \end{aligned} \quad (5)$$

N_{30} and N_{365} denote the number of minutes to the end of the 30 and 365 calendar day brackets, respectively. The squared value of the VIX has two important economic interpretations.[20] First, it can be regarded as the risk-neutral expectation of the S&P 500 annualized 30 calendar day return variance, i.e. $\mathcal{V}_t^2 \cong E_t^{\mathbb{Q}}\left[\sigma^2_{t,t+30}\right]$.[21] Second, it is the value of a portfolio consisting of S&P 500 index options.

The remainder of this section provides a brief introduction to VIX derivatives. For better illustration purposes, we sketch their structure in Figure 1. Again, let $\mathcal{V}_t$ denote the time-t VIX closing level and $O_{t,T+30}$ denote the price of SPX options maturing in $T + 30$. Each equity index option series is associated with VIX options and futures, indexed by $\mathcal{F}_t(\mathcal{V}_T)$ and $\mathcal{O}_t(\mathcal{V}_T)$, respectively. Contracts expire exactly calendar 30 days prior to the corresponding SPX option tranches.[22] This convention enables to calculate the settlement price in T from a single SPX option series expiring in $T + 30$. It avoids the linear interpolation scheme of Equation (5). Note that VIX derivatives prices are associated with S&P 500 options that may not enter the calculation of the current VIX index level $\mathcal{V}_t$.

Squared implied volatility values of at-the-money VIX options with maturity T may serve as a measure for investor's uncertainty about $\mathcal{V}_t$-changes in period $[t, T]$, i.e. $Var_t^{\mathbb{Q}}(\mathcal{V}_T)$. Note, however, that this is only an approximation for variance risk: By Jensen's inequality $Var_t^{\mathbb{Q}}(\mathcal{V}_T) = Var_t^{\mathbb{Q}}(\sqrt{E_T^{\mathbb{Q}}\left[\sigma^2_{T,T+30}\right]}) \neq Var_t^{\mathbb{Q}}(E_T^{\mathbb{Q}}\left[\sigma^2_{T,T+30}\right])$, i.e. we cannot back out the variance of the variance swap rate itself, but its square root.

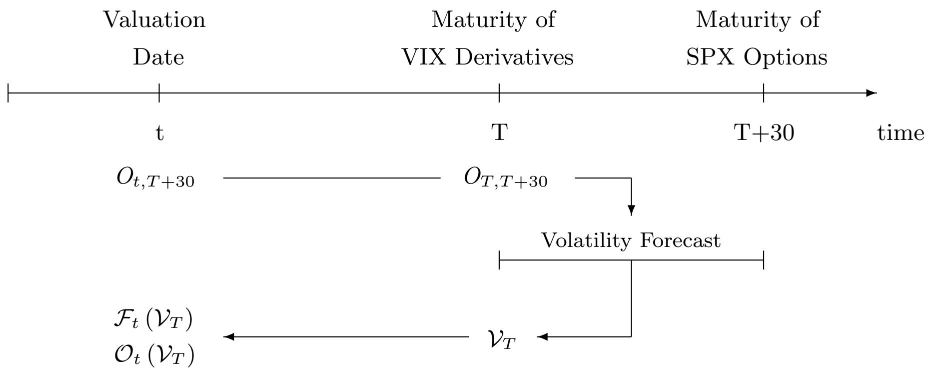
[20]See Carr and Wu (2006), as well as Carr and Lee (2007a, 2007b).

[21]Possible errors in this estimation may be induced by discretely observable SPX option strike prices and by jumps in the return dynamics of the S&P 500. For illustrations, see Demeterfi, Derman, Kamal, and Zou (1999), Carr and Wu (2006), and Carr and Wu (2009).

[22]VIX derivatives expire on Wednesday that is thirty days prior to the third Friday of the following calendar month (the expiration date of SPX options).

Figure 1: The Structure of VIX and SPX Derivatives

This figure illustrates the typical structure of VIX futures $\mathcal{F}_t(\mathcal{V}_T)$ and options $\mathcal{O}_t(\mathcal{V}_T)$ with maturity T and their associated S&P 500 index option prices ($O_{t,T+30}$). As the VIX ($\mathcal{V}_t$) is a measure for the annualized 30 day risk-neutral volatility, its derivatives mature exactly 30 days prior to the maturity of their associated SPX options ($T + 30$).



3 Data

This section introduces our dataset and outlines the calculation of the explanatory variables.

3.1.1 VIX Options

The Chicago Board Options Exchange (CBOE hereafter) introduced VIX options on February 24^{th}, 2006. We consider daily trade data of calls and puts of the March 1^{st}, 2006, to August 29^{th}, 2008, period including all available strike prices. Following Longstaff (1990), we include prices only if the total trading volume for a specific day is at least 40 closed contracts in order to avoid problems arising from thin markets. Furthermore, we exclude observations with a time to maturity of less than five calendar days as VIX futures do not converge to the VIX index.[23] As we want to keep effects deriving from bid-ask spreads as small as possible, option values are determined by the closing midquote prices, i.e. $\mathcal{O}_{i,t} = \mathcal{O}_{i,t}^{bid} + 0.5 \cdot \left(\mathcal{O}_{i,t}^{ask} - \mathcal{O}_{i,t}^{bid}\right)$.[24] Some bid prices are determined as zero. In these cases we apply the closing transaction price as the turnover volume is still at a very high level for most of these observations. Our sample comprises a total of 14133 call and 6394 put prices.

[23] See Pavlova and Daigler (2008), and Wang and Daigler (2009).

[24] Note, however, that our results are robust to different price definitions.

3.1.2 VIX Futures

We use daily VIX futures prices for the calculation of VIX option moneyness and Black-implied volatility levels. They are obtained from the CBOE Futures Exchange. As for the sample options, trading ceases at 3:15 p.m. (CST). Therefore, we do not face major problems deriving from asynchronous data observation times. On March 26^{th}, 2007, the quotation of these derivatives was re-scaled by the factor 10. We account for that simply by dividing the notations observed before that date by 10.[25] Every sample option price observation is then associated with a futures value with the corresponding time to maturity.

3.1.3 S&P 500 Options

Our SPX option dataset to calculate variance swap rates, as well as the skewness and kurtosis of the risk-neutral S&P 500 return distribution is obtained from OptionMetrics (provided through Wharton Research Data Services). It contains daily closing index levels, as well as bid and ask prices. Their settlement date is usually the third Friday of the contract month. In addition to the contracts opened previously, available exercise prices typically spread at 10-point intervals ranging in- and out-of-the-money relative to the current level of the forward price. The set of traded strike prices therefore depends on the stock market history. To be consistent with the literature[26], we eliminate observations (i) with zero bid prices, (ii) with higher bid- than ask-prices, and observations (iii) violating the arbitrage bound $C_t(\tau, X) \leq S_t$. All option values are determined by their closing midquote prices, i.e. $O_t = O_t^{bid} + 0,5 \cdot \left(O_t^{ask} - O_t^{bid}\right)$.[27]

3.1.4 Risk-Free Interest Rates

After experimenting with different interest rates including USD LIBOR and T-Bill rates, we apply modified put-call-parity regressions of SPX options, as proposed in Shimko (1993). Let $PV_t(D_\tau)$ denote present value of dividends paid from time t until the expiration of the derivative contracts. Then, the put-call-parity for European options can be written as

$$C_{it}^{bid}(\tau, X_i) - P_{it}^{ask}(\tau, X_i) = [S_t - PV_t(D_\tau)] - X_i e^{r_f \tau},$$

[25] Before March 26^{th}, 2007, VIX futures were written on the VBI index, which is derived by multiplication of the VIX index level by the factor 10.

[26] See e.g. Bakshi, Kapadia, and Madan (2003).

[27] This rule follows the calculation conventions of the VIX. Note, however, that the final settlement of VIX derivatives is based on a so-called "Special Opening Quotation". On the settlement day, the value of the underlying is calculated on the opening prices of the SPX options, which can either be bid or ask quotes. See Pavlova and Daigler (2008) for an extensive discussion on this issue.

where S_t denotes the S&P 500 index level.[28] Simple linear regressions over all strike prices and maturities provide estimates for the time-t term structures of implied dividends and interest rates. Note that we occasionally restrict the regression specification by $PV_t(D_\tau) \geq PV_t(D_{ttm<\tau})$ to ensure that $PV_t(D_\tau)$ is a non-decreasing function in τ, although violations of this restriction lead only to negligible biases of our results.[29]

3.2 Measuring Variance Risk

This section outlines our method to measure variance risk, i.e. $Var_t^{\mathbb{Q}}(\mathcal{V}_T)$. To fix ideas, let $s_t^2(\tau, X)$ denote the squared time-t Black-implied VIX return volatility, backed out from at-the-money VIX options with strike price X and time to maturity $\tau = (T-t)/365$. Then,

$$s_t^2(\tau, X) = \frac{1}{\tau} \ln \frac{Var_{t,X}^{\mathbb{Q}}(\mathcal{V}_T) + \mathcal{F}_t^2(\tau)}{\mathcal{F}_t^2(\tau)},$$

where $Var_{t,X}^{\mathbb{Q}}(\mathcal{V}_T)$ denotes the variance implied by the VIX option with strike price X and maturity T. Therefore,

$$\mathcal{F}_t^2(\tau)\left(e^{\tau s_t^2(\tau,X)} - 1\right) = Var_{t,X}^{\mathbb{Q}}(\mathcal{V}_T) = Var_{t,X}^{\mathbb{Q}}\left(\sqrt{E_T^{\mathbb{Q}}[\sigma_{T,T+30}^2]}\right). \quad (6)$$

Note that Equation (6) assumes $\mathcal{V}_T$ to follow the log-normal law which may empirically be rejected.[30] The time series plot of closing VIX levels and its implied standard deviation for the March 1st, 2006, to August 29th, 2008, period is outlined in Figure 2. Values for the standard deviation of $\mathcal{V}_{t+30}$ are determined as follows: We calculate $s_t^2(\tau, X)$ for all time-t observations with $0.9 \leq \mathcal{F}_t/X \leq 1.1$ and $20 \leq T-t \leq 40$. $\sqrt{Var_t^{\mathbb{Q}}(\mathcal{V}_{t+30})}$ is determined by linear interpolation. Its positive relation to 30 calendar day variance swap rates is also observable in the scatter plot of Figure 2(b).

Please insert Figure 2 around here.

3.3 Measuring Higher Implied Moments of S&P 500 Index Option Returns

This section describes our technique to estimate the moments of the risk-neutral S&P 500 return distribution. Following Bakshi, Kapadia, and Madan (2003), and Dennis and Mayhew (2002), we approximate the

[28] Please note that we use bid values for calls and ask prices for puts. This provides the implicit borrowing rate. Estimated lending rates sometimes have negative levels.

[29] We also experimented with SPX futures data. Yet, their crude grid of available maturities (quarterly) leads to noisy approximations of the interest rate structure.

[30] However, our results are robust to using $s_t^2(\tau, X)$ directly. This specification does not require the assumption of log-normality.

integrals of (1) with discrete strike prices. For illustration purposes, consider the cubic contract

$$W_t(\tau) = \sum_{j=1}^{S_t} w(X_j) C_t(\tau, X_j) \Delta X_j - \sum_{j=S_t}^{\max_j} w(X_j) P_t(\tau, X_j) \Delta X_j, \tag{7}$$

where $w(X_j) = \frac{6 \ln\left(\frac{X_j}{S_t}\right) \pm 3\left(\ln\left(\frac{X_j}{S_t}\right)\right)^2}{X_j^2}$. $C_t(\tau, X_j)$ and $P_t(\tau, X_j)$ denote the time-t call and put midquote prices with maturity τ and strike price X_j. $\Delta X_j = X_j - X_{j-1}$ is the difference between two adjacent strike prices. Dennis and Mayhew (2002) outline that the empirical implementation of the Bakshi, Kapadia, and Madan (2003) metrics might be biased to some extent. Equation (1) is based on a continuum of observable strike prices, whereas in reality we have only discrete strike price intervals. To account for this, we follow Hansis, Schlag, and Vilkov (2009) and use cubic splines to interpolate between observed strike prices inside the range of all included observations. We choose an interval spacing of 1$. This provides a reasonably fine grid of strike prices which leads to very robust values of the Bakshi, Kapadia, and Madan (2003) metrics.[31]
Dennis and Mayhew (2002) outline that a second source of bias might derive from asymmetry in the integration domain. Their reasoning is that the BKM metrics are based on weighted sums and differences of call and put option portfolios. Therefore, having more observations on calls than on puts could introduce a source of bias. Contrary to their argumentation, we think that excluding observations with nonzero mass of the risk-neutral probability distribution implies cutting off part of the implied return distribution. This can clearly be seen in the relationship of Equation (2). As such, omitting nonzero option prices in the calculation of the Bakshi, Kapadia, and Madan (2003) metrics is the actual source of bias. For illustration purposes, reconsider the example of Dennis and Mayhew (2002). We calculate Black Scholes out-of-the-money option prices for a strike price range of $X \in [1, 100]$, an annualized volatility of $\sigma = 0.2$, a time to maturity of one year, and a risk-free interest rate of $r_f = 0.07$. The stock price is $S_0 = 50$$. The results are outlined in Figure 3.

Please insert Figure 3 around here.

Although the return distribution of the Black Scholes model is symmetric, the terminal stock price S_T follows the lognormal law and is therefore asymmetric. For illustration purposes, it is depicted in Figure 3. As the Black Scholes return distribution follows the Gaussian law, we should observe zero skewness metrics. This is the case if we integrate over the complete domain. Defining the integration domain $I = [Z : 100]$, where $Z \in [1 : 40]$, the blue dotted line in Figure 3 indicates a positive bias, up to a level of 0.3 as we omit

[31] We found that our results are robust to the the use of narrower strike price intervals.

positive put prices in the short position of the portfolio. This result closely resembles the findings of Dennis and Mayhew (2002). We additionally calculate the Bakshi, Kapadia, and Madan (2003) skewness metric for the domains $I = [1 : Z]$, where $Z \in [60, 100]$. The resulting skewness values turn out to be negative as we cut off the part of the implied share price distribution which is covered by long call positions. Compared to the former calculations, the value of the bias rises more slowly and reaches a higher maximum of -0.34 for $Z = 60$. This shows that its source might not derive from asymmetric integration bounds.

To get a sense of whether the set of available strike prices completely covers the implied share price distribution, we evaluate whether option prices converge to zero for lower moneyness values. Specifically, we determine at each point in time of our sample period whether the observed tranches contain option prices with two consecutive bid quotes of zero.[32] If this is the case, we assume that the specific tranche covers the domain of the implied share price distribution. As options with zero bid prices might have a positive value, our approach is still an approximation. We then equally weigh all observations to determine average number of observations covering the relevant strike price range. Resulting values are reported in **Panel A** of Table 1.

Please insert Table 1 around here.

Our results show that the upper tail of the implied share price distribution is covered by the call option portfolio in 54.8 per cent of the cases. The lower tail has a coverage in 47.3 per cent of all observations. This implies that the **?** metrics are biased in about half of all calculations. These values are a bit disappointing as the sign and the magnitude of the distortion is also unclear. To get a sense whether these rather low values are due to the criterion to determine a covered share price distribution, we also calculate the fraction of observations whose option with the lowest moneyness has a bid value of zero. Obtained figures are given in brackets. They slightly rise to 64.8% and 53.8% for calls and puts, respectively. As such, our findings are quite robust. **Panel B** gives the corresponding values after sorting observations with respect to maturity. Tranches with a remaining time to maturity of less than 30 calendar days have two consecutive bid prices of zero in 90.6 and 95.7 per cent of the cases for the call and put portfolios, respectively. These values diminish to 23.8% and 6.1% for calls and puts with $230 \leq T - t \leq 250$. If we regard the coverage of the implied share price distribution assured if the option with the lowest moneyness has a bid price of zero, values increase

[32] We also experimented with a simulation study of the Bakshi, Cao, and Chen (1997) jump-diffusion model and found a reasonable coverage of the implied share price distribution. Yet, the results of this model are derived from the set of available strike prices and may therefore be biased themselves.

to 97.4 and 98.8 per cent for calls and puts with $T - t \leq 30$. As such, we conclude that limited strike price availability might not constitute a major source of bias for the Bakshi, Kapadia, and Madan (2003) metrics for short term observations.

In addition to the contracts opened previously, the Chicago Board Options Exchange introduces S&P 500 options with strike prices distributed around the corresponding forward index level. As such, the set of available options depends on the history of the index levels. It may grow substantially during and after volatile market periods. This might be one reason for the high coverage of tranches with a short time to maturity. Furthermore, coverage is not assured after strong market movements, e.g. after stock market downturns. Therefore, we calculate the six month SPX return prior to the observation time for each tranche. After sorting for returns, we recalculate our analysis for observations with the lowest and highest return percentile. Values are given in **Panel B** of Table 1. Regarding observations with the lowest percentile, 86.7 per cent of the call portfolios contain two consecutive bid prices of zero, whereas only 21.1 per cent of the put portfolios cover the implied share price distribution. These results are quite intuitive. However, if we regard the highest return percentile, we observe coverage fractions of 37.0% and 55.2% for calls and puts, respectively. These values are - more or less - of the same magnitude. That means that the Bakshi, Kapadia, and Madan (2003) skewness metric tends to have a positive bias during or after equity market downturns.

To determine integration bounds, we introduce a set of different rules. First, following the calculation method of the VIX, we include all out-of-the-money observations up to a point where we observe two adjacent bid prices of zero. Options with lower moneyness are excluded from our calculations. Second, we do not consider price observations with zero bid values. These values are typically extrapolated by our cubic spline method. Third, following Skiadopoulos, Hodges, and Clewlow (2000), we also eliminate observations with an option's vega of less than eight. The intuition for this can be sketched as follows. We are not able to determine the true prices of the contracts as we only observe bid- and ask-prices. This induces a measurement error ΔO of the true price O, as outlined, for example, in Harvey and Whaley (1991). The resulting bias in the implied volatility $\Delta\sigma$ rises with lower vega and can be approximated by $\Delta O/vega$.[33] We find that this filtering rule provides very robust results for implied return distribution moments.

Summary statistics for the unconditional distributions of implied return moments are given in Table 2.

Please insert Table 2 around here.

[33] See also Skiadopoulos, Hodges, and Clewlow (2000).

For notational simplicity, let $t = 0$ in the following. Regarding a standardized time to maturity of 30 calendar days, we find average variance swap rates of 376.576 basis points. The lower median of 277.083 and the positive skewness of 2.542 are indicative of positive outliers. These findings are consistent with previous results in the literature.[34] Different results can be found for a time to maturity of 150 days. Our sample shows average variance swap rates of 456.569 basis points. Their distribution is slightly negatively skewed with a median of 506.930 and a skewness of -0.139. Furthermore, we find a lower standard deviation of 226.458, compared to the distribution of short-term options (Std. of 355.795).

Please insert Figure 4 around here.

Please insert Figure 5 around here.

Negative skewness for equity index options has been found in previous studies.[35] We observe average Bakshi, Kapadia, and Madan (2003) skewness values of -1.674 and -1.605 for $T = 30$ and $T = 150$, respectively. These are comparable in magnitude. The time series plot for $T = 30$ is given in Figure 4(a). The metric seems to have only small variation with moderate negative spikes. Standard deviations of 0.575 and 0.620 for $T = 30$ and $T = 150$ confirm this finding. Regarding the scatter plot for $T = 30$ in Figure 4(b), we find that variance swap rates below 200 basis points are accompanied by skewness values between -3 and -1.2. Rising rates, however, imply a positive relationship between both metrics. The behavior of the Bakshi, Kapadia, and Madan (2003) kurtosis measure is more erratic. It has standard deviations of 3.475 and 3.537 and positive skewness values of 0.239 and 2.096, respectively. This is indicative of positive outliers, which can also be detected in the time series plot of Figure 5(a). It indicates that excess kurtosis values tend to decrease during erratic market situations with high variance swap rates. This is confirmed by the scatter plot in Figure 5(b) for $T = 30$. We find a pattern which is similar to the behavior of the skewness metric. Variance swap rates lower than 200 basis points are accompanied by excess kurtosis values between 4 and 18. Increasing rates, however, lead to a strong decrease in kurtosis values. Interestingly, we even find excess kurtosis values as low as 0.424 and 0.179 for $T = 30$ and $T = 150$. Jarque-Bera tests can be rejected for all three moments to a level of one per cent.

[34] See, e.g., Canina and Figlewski (1993), Fleming, Ostdiek, and Whaley (1995), Moraux, Navatte, and Villa (1998), Whaley (2000), Blair, Poon, and Taylor (2001), Simon (2003), as well as Corrado and Miller (2005). This list is by no means exhaustive.

[35] See, e.g., Bakshi, Cao, and Chen (1997), Bakshi, Kapadia, and Madan (2003), and the references therein.

4 The Impact of Implied SPX Return Moments on VIX Option Prices

This section matches the risk-neutral moments of S&P 500 returns and the implied volatility of VIX option prices. In particular, we test whether volatility swap premia are mainly driven by variance risk or by higher moments of the implied return distribution, such as skewness and kurtosis. In its most comprehensive setting, we estimate the model

$$\Delta z\left(\sqrt{E_t^{\mathbb{Q}}\left[\sigma_{t,T}^2\right]}\right) = \phi_1 \Delta z\left(Var_t^{\mathbb{Q}}(\mathcal{V}_T)\right) + \phi_2 \Delta z\left(E_t^{\mathbb{Q}}\left[\gamma_{t,T}\right]\right) + \phi_3 \Delta z\left(E_t^{\mathbb{Q}}\left[\kappa_{t,T}\right]\right) + \tilde{\varepsilon}_t, \tag{8}$$

where $E_t^{\mathbb{Q}}\left[\sigma_{t,T}^2\right]$, $E_t^{\mathbb{Q}}\left[\gamma_{t,T}\right]$, and $E_t^{\mathbb{Q}}\left[\kappa_{t,T}\right]$ denote the implied variance, skewness, and kurtosis for the period $[t, T]$. Note, that we use the square root of the variance swap rate to be consistent with the definition of the VIX. It is well-known in the literature that implied moments are highly persistent and have a high degree of auto-correlation.[36] Therefore, we consider first-order differences Δ of the estimates. These are z-standardized for better comparability of the estimated regressor values. The estimation procedure is as follows:

First, we determine $Var_t^{\mathbb{Q}}(\mathcal{V}_T)$ by (6) for all observations. At-the-money values for each calendar day are obtained by linear interpolation.[37] Second, we determine $E_t^{\mathbb{Q}}\left[\sigma_{t,T}^2\right]$, $E_t^{\mathbb{Q}}\left[\gamma_{t,T}\right]$, and $E_t^{\mathbb{Q}}\left[\kappa_{t,T}\right]$ using standard log-contract theory (Equation (4)), as well as the skewness and kurtosis metrics of Bakshi, Kapadia, and Madan (2003) outlined in the last section. To bracket the period $[t, T]$, we use the linear interpolation scheme of Carr and Wu (2006)

$$E_t^{\mathbb{Q}}\left[\sigma_{t,T}^2\right] = E_t^{\mathbb{Q}}\left[\sigma_{t,t_j}^2\right] + E_t^{\mathbb{Q}}\left[\sigma_{t,T+30}^2\right] \frac{N_T - N_{t_j}}{N_{T+30} - N_{t_j}},$$

where t_j is selected from all tranches i by $t_j = \min_i \{T - t_i | t_i \leq T\}$.[38] Again, N_j denotes the number of minutes to maturity of an SPX option tranche with j calendar days to maturity. We calculate $E_t^{\mathbb{Q}}\left[\gamma_{t,T}\right]$ and $E_t^{\mathbb{Q}}\left[\kappa_{t,T}\right]$ in a similar way.

Third, following Dennis and Mayhew (2002), we standardize our calculated values to fixed times to maturity of $T = 30$ and $T = 150$ calendar days for the two subsamples in order to make the moments comparable.[39]

[36] See, e.g., Bollerslev (2008), and the references therein.

[37] Our specification assumes $\mathcal{V}_t$ to be independent of the S&P 500 index (See, e.g., Brenner and Subrahmanyam (1988), and Carr and Lee (2007a, 2007b)). Using log-contract theory, we also experimented with the construction of synthetic variance swap rates. This procedure allows to drop the independence assumption. Yet, it results in crude estimations due to very large strike price intervals of VIX options.

[38] Note that each VIX option price observation with maturity T has a corresponding SPX option tranche with maturity $T+30$.

[39] As outlined in Bakshi, Kapadia, and Madan (2003), variance and especially skewness may not aggregate linearly across the time spectrum. This is especially true with nonzero implied auto-correlation in the time series of the moments. To keep biases induced by the standardization as small as possible, we choose different standardization maturities of $T = 30$ and $T = 150$.

Fourth, we calculate first differences of the four time series and standardize the resulting values. The obtained time series are used for the estimation of (8). The error covariance matrix is determined by the HAC consistent estimator of Newey and West (1987).

Please insert Table 3 around here.

Resulting values are outlined in **Panel A** of Table 3. Examining the results, we come to the following conclusions. First, the coefficient for $Var_t^{\mathbb{Q}}(\mathcal{V}_T)$ is positive and significant with values of 0.363 and 0.252 for $T = 50$ and $T = 150$, respectively. Variance swap rates rise with higher VIX option premia. This finding is consistent with previous results in the literature.[40] Second, the coefficients for $E_t^{\mathbb{Q}}[\gamma_{t,T}]$ have positive values of 0.139 and 0.406 for short and long-term contracts, respectively. Yet, only the coefficient of for $T = 150$ is significant. This finding si a bit counter-intuitive at first sight, especially with regard of the scatter plot in Figure 4(b). As outlined before, skewness values vary between 3 and -1.2 for variance swap rates lower than 200 basis points. The positive relation between both values becomes stronger only for quotes above 200 basis points. Third, the coefficient for $E_t^{\mathbb{Q}}[\kappa_{t,T}]$ is positive with a value of 0.697 for $T = 150$. The corresponding regressor value for short-term contracts is not significant with a value of -0.201. Fourth, regarding adjusted determination coefficients, we find values of 0.254 for $T = 30$ and 0.179 for $T = 150$.

To get a better understanding of the question which variables do best explain the evolution of variance swap rates, we estimate the model

$$\Delta z\left(\sqrt{E_t^{\mathbb{Q}}\left[\sigma_{t,T}^2\right]}\right) = \phi_1 \Delta z\left(Var_t^{\mathbb{Q}}(\mathcal{V}_T)\right) + \tilde{\varepsilon}_t.$$

Results are given in **Panel B** of Table 3. Again, coefficients for $Var_t^{\mathbb{Q}}(\mathcal{V}_T)$ have significant and positive values of 0.393 and 0.272 for $T = 30$ and $T = 150$. The adjusted determination coefficients have values of 0.154 and 0.074. **Panel C** shows the estimation results of the model

$$\Delta z\left(\sqrt{E_t^{\mathbb{Q}}\left[\sigma_{t,T}^2\right]}\right) = \phi_2 \Delta z\left(E_t^{\mathbb{Q}}[\gamma_{t,T}]\right) + \phi_3 \Delta z\left(E_t^{\mathbb{Q}}[\kappa_{t,T}]\right) + \tilde{\varepsilon}_t.$$

Coefficients for $E_t^{\mathbb{Q}}[\gamma_{t,T}]$ are 0.112 and 0.346 for $T = 30$ and $T = 150$, respectively. Yet, we only find the regressor of the long-term options to be significant. Similar conclusions can be drawn for the values of $E_t^{\mathbb{Q}}[\kappa_{t,T}]$. They are -0.261 and 0.664 for $T = 30$ and $T = 150$, respectively. Adjusted determination coefficients are 0.072 and 0.121. Compared to those in **Panel B**, we conclude that short-term variance swap

[40]See, e.g., Ammann and Süss (2009) and the references therein.

rates are mainly driven by variance risk. A significant impact of higher moments of the risk-neutral return distribution can only be found for long-term contracts.

5 Conclusion

This study demonstrates empirically that the dynamics of variance swap premia is driven by both, variance risk and higher moments of the risk-neutral return distribution. We find a significant impact of higher RND moments only for long-term swap rates. The impact of variance risk, however, is strongest for variance swap contracts with a short time to maturity. Summarizing, while short term rates are best explained by variance risk, rates with longer terms to expiry are mainly driven by higher moments of the implied return distribution.

Furthermore, we provide an extensive discussion on the metrics of Bakshi, Kapadia, and Madan (2003) and investigate systematic biases deriving from discrete option strike price intervals. Contrary to previous work, we do not find evidence that asymmetric integration intervals are a source of bias. Instead, we suggest to consider all observations with nonzero prices for the calculation of risk-neutral return distribution moments.

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Table 1: The Bias of the Bakshi, Kapadia, and Madan (2003) Metrics

This table evaluates whether discrete strike price availability constitutes a major source of bias for the Bakshi, Kapadia, and Madan (2003) metrics. Using all SPX option data for our sample period, we calculate the fraction of observations with two consecutive bid prices of zero. The analysis is done separately for out-of-the-money call and put tranches. **Panel A** shows values for the complete sample. Corresponding results for observations with a remaining time to maturity of $T \leq 30$ and $230 \leq T \leq 250$ are illustrated in **Panel B**. In addition, we sort observations with respect to the 6-month SPX return of the period prior to observation time t. The corresponding results are given in **Panel C** for the highest and for the lowest return percentile, respectively. The number in brackets state the fraction of observations whose option with the lowest moneyness has a bid price of zero.

	Calls	Puts
Panel A: Time to Maturity		
	0.548	0.473
	[0.648]	[0.538]
Panel B: Time to Maturity		
$T - t \leq 30$	0.906	0.957
	[0.975]	[0.988]
$230 \leq T - t \leq 250$	0.238	0.061
	[0.279]	[0.170]
Panel C: SPX Return		
Lowest return percentile	0.867	0.211
	[0.885]	[0.263]
Highest return percentile	0.370	0.552
	[0.455]	[0.620]

Table 2: Descriptive Statistics of Implied Return Distribution Moments

This table illustrates descriptive statistics of implied S&P 500 return moments for the time period of March 1^{st}, 2006, until August 29^{th}, 2008. We provide values for $T = 30$ and 150 calendar day brackets. They are given by linear interpolation of the Bakshi, Kapadia, and Madan (2003) metrics. Annualized variance swap rates are denoted in basis points ([bp]). The provided statistical measures are the mean, median, standard deviation, skewness, kurtosis, Jarque-Bera test statistics, as well as the first-order autocorrelation ($\rho_{t,t-1}$). p values are quoted in brackets.

	T=30			T=150		
	Variance[bp]	Skewness	Kurtosis	Variance[bp]	Skewness	Kurtosis
Mean	376.576	-1.674	7.073	456.569	-1.605	5.759
Median	277.083	-1.672	6.904	506.930	-1.474	5.140
Std.dev	355.795	0.575	3.475	226.458	0.620	3.537
Skewness	2.542	-0.104	0.239	-0.139	-0.987	2.096
Kurtosis	14.106	4.038	4.023	1.748	4.745	9.095
Minimum	60.070	-2.542	0.424	91.039	-3.903	0.179
Maximum	946.526	-0.867	12.562	981.013	-0.267	22.274
Jarque-Bera	6110.291	45.850	52.198	12.335	52.068	410.340
p value	0.000	0.000	0.000	0.002	0.000	0.000
$\rho_{t,t+1}$	0.468	0.147	0.125	0.825	0.478	0.353
p value	0.000	0.000	0.000	0.000	0.000	0.000

Table 3: The Dependence of Variance Swap Rates on Implied SPX Return Skewness and Implied Volatility Option Variance

This table outlines our empirical results regarding impact of skewness ($E_t^{\mathbb{Q}}\left[\gamma_{t,T}\right]$) and kurtosis ($E_t^{\mathbb{Q}}\left[\kappa_{t,T}\right]$) of the risk-neutral SPX return distribution, as well as VIX option implied variance $Var_t^{\mathbb{Q}}\left(\mathcal{V}_T\right)$ on synthetic variance swap rates $E_t^{\mathbb{Q}}\left[\sigma_{t,T}^2\right]$. We back out $\sigma_{t,T}^2$, $\gamma_{t,T}$, and $\kappa_{t,T}$ from the corresponding SPX options tranches. Furthermore, we determine $Var_t^{\mathbb{Q}}(\mathcal{V}_T)$ as the at-the-money Black-implied variance of the VIX option tranche with maturity T. To make observations comparable, we standardize the resulting moments to fixed times to maturity of $T = 30$ and $T = 150$. Based on these time series, we estimate the model

$$\Delta z\left(\sqrt{E_t^{\mathbb{Q}}\left[\sigma_{t,T}^2\right]}\right) = \phi_1 \Delta z\left(Var_t^{\mathbb{Q}}(\mathcal{V}_T)\right) + \phi_2 \Delta z\left(E_t^{\mathbb{Q}}\left[\gamma_{t,T}\right]\right) + \phi_3 \Delta z\left(E_t^{\mathbb{Q}}\left[\kappa_{t,T}\right]\right) + \tilde{\varepsilon}_t,$$

where Δ denotes the change in the weekly average of the respective moment. In order to make the regressors comparable, we z-standardize weekly differences. Obtained results are given in **Panel A**. Corresponding p-values are outlined in brackets. We denote adjusted determination coefficients with $\bar{R}^2$. In addition, we estimate two submodels

$$\Delta z\left(\sqrt{E_t^{\mathbb{Q}}\left[\sigma_{t,T}^2\right]}\right) = \phi_1 \Delta z\left(Var_t^{\mathbb{Q}}(\mathcal{V}_T)\right) + \tilde{\varepsilon}_t$$

and

$$\Delta z\left(\sqrt{E_t^{\mathbb{Q}}\left[\sigma_{t,T}^2\right]}\right) = \phi_2 \Delta z\left(E_t^{\mathbb{Q}}\left[\gamma_{t,T}\right]\right) + \phi_3 \Delta z\left(E_t^{\mathbb{Q}}\left[\kappa_{t,T}\right]\right) + \tilde{\varepsilon}_t.$$

Corresponding results are given in **Panels B** and **C**, respectively.

	ϕ_1	ϕ_2	ϕ_3	$\bar{R}^2$
Panel A: Complete Model				
$T = 30$	0.363	0.139	-0.201	0.254
p-value	[0.000]	[0.533]	[0.380]	
$T = 150$	0.252	0.406	0.697	0.179
p-value	[0.011]	[0.007]	[0.004]	
Panel B: Influence of $Var_t^{\mathbb{Q}}\left(\mathcal{V}_T\right)$				
$T = 30$	0.393			0.154
p-value	[0.000]			
$T = 150$	0.272			0.074
p-value	[0.025]			
Panel C: Influence of $E_t^{\mathbb{Q}}\left[\gamma_{t,T}\right]$ and $E_t^{\mathbb{Q}}\left[\kappa_{t,T}\right]$				
$T = 30$		0.112	-0.261	0.072
p-value		[0.658]	[0.320]	
$T = 150$		0.346	0.664	0.121
p-value		[0.016]	[0.015]	

Figure 2: Variance Swap Rates and their Implied Standard Deviation

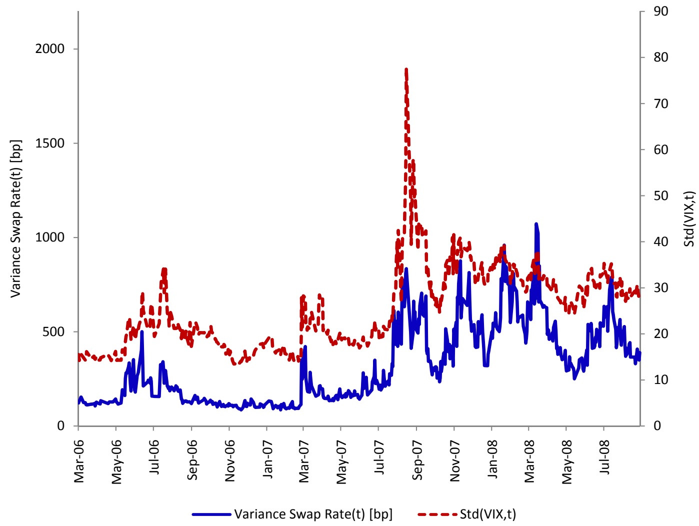
Figure (a) displays the time series of 30 calendar day variance swap rates for the March 1^{st}, 2006, to August 29^{th}, 2008, period. Its annualized implied standard deviations, i.e. $\sqrt{Var_t^{\mathbb{Q}}\left(\mathcal{V}_{t+30}\right)}$, are given by the dashed red line. Their calculation can be sketched as follows: Let $s_t^2(\tau, X)$ denote the squared time-t Black-implied VIX return volatility, backed out from at-the-money VIX options with strike price X and time to maturity $\tau = (T-t)/365$. Then,

$$s_t^2\left(\tau, X\right) = \frac{1}{\tau}\ln\frac{Var_{t,X}^{\mathbb{Q}}(\mathcal{V}_T) + \mathcal{F}_t^2(\tau)}{\mathcal{F}_t^2(\tau)},$$

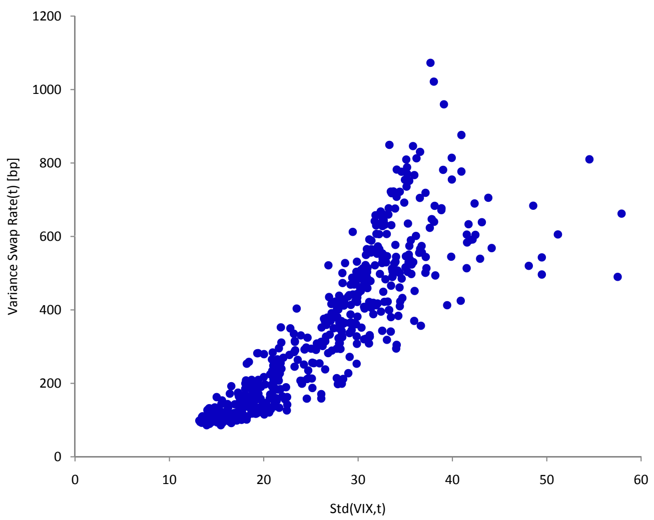
where $Var_{t,X}^{\mathbb{Q}}(\mathcal{V}_T)$ denotes the variance implied by the VIX option with strike price X and maturity T. $s_t^2(\tau, X)$ is the squared implied volatility of an option with time to maturity τ. Assuming the RND of $\mathcal{V}_T$ to follow the log-normal law, we can infer the corresponding implied volatility for $\mathcal{V}_T$ by

$$\mathcal{F}_t^2(\tau)\left(e^{\tau s_t^2(\tau,X)} - 1\right) = Var_{t,X}^{\mathbb{Q}}\left(\mathcal{V}_T\right).$$

We determine the average value of $Var_{t,X}^{\mathbb{Q}}\left(\mathcal{V}_T\right)$ of all time-t OOM observations with $0.9 \leq \mathcal{F}_t(\tau)/X \leq 1.1$ and $20 \leq T-t \leq 40$. The plotted series is defined as the square root of the obtained values. The corresponding scatter plot is given in **Figure (b)**.



(a) Time Series of Closing VIX Values and their Implied Standard Deviations



(b) Scatter Plot of Closing VIX Values and their Implied Standard Deviations

Figure 3: Biases Induced by the Integration Domain

This figure shows the bias of the Bakshi, Kapadia, and Madan (2003) skewness metric introduced by finite integration domains. We calculate Black Scholes option prices with strike prices of $[1, 100]$, a share price of $S_0 = 50$, time to maturity of one year, annualized volatility of $\sigma = 0.2$, and risk-free interest rates of $r_f = 0.07$. The risk-neutral skewness is then determined by the Bakshi, Kapadia, and Madan (2003) metric. The integration domain I is defined by $[Z : 100]$, for $Z \in [1 : 40]$ and $[0 : Z]$, for $Z \in [60 : 100]$. The resulting skewness values are illustrated by the dotted lines. Furthermore, the implied distribution of the terminal share price (S_T) is given by the red line.

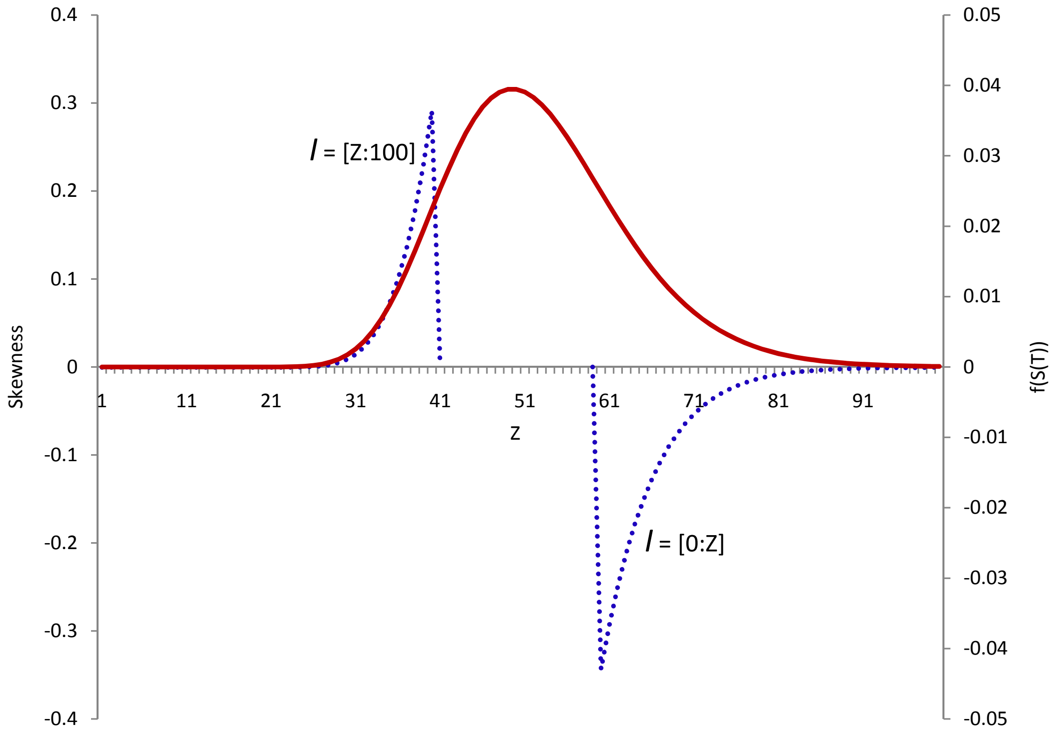
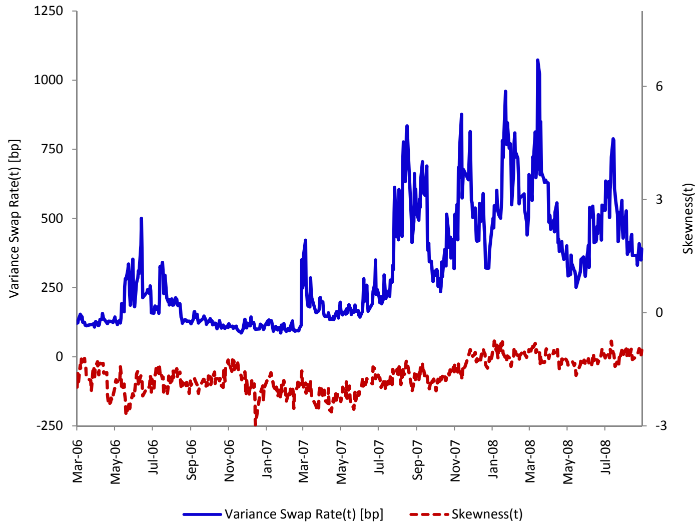
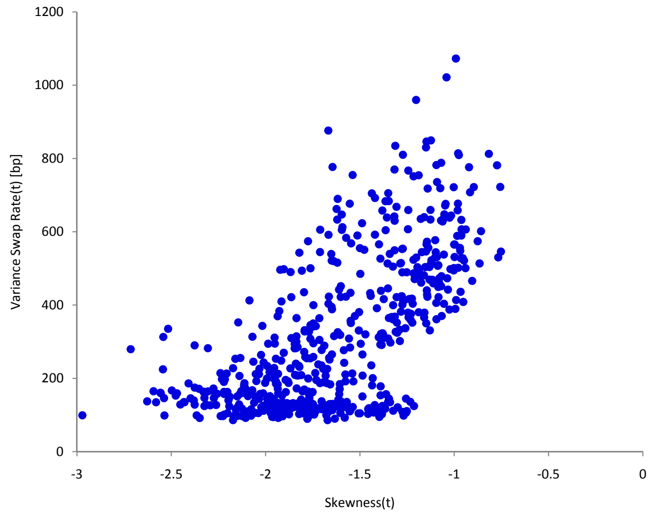


Figure 4: Variance Swap Rates and their Corresponding Implied Skewness

Figure (a) gives the time series plot of 30 calendar day variance swap rates levels and their implied skewness metric for the March 1^{st}, 2006, to August 29^{th}, 2008, period. The skewness values are backed out from S&P 500 index options by the discretized Bakshi, Kapadia, and Madan (2003) measure. To be consistent with the literature, we exclude pricing observations with a trading volume of less than 40 closed contracts on a specific trading day or a time to maturity of less than 16 calendar days. Furthermore, following Skiadopoulos, Hodges, and Clewlow (2000), we exclude observations with vega values of less than eight. Interest rates are determined by modified put-call-parity regressions proposed in Shimko (1993). **Figure (b)** gives the corresponding scatter plot.



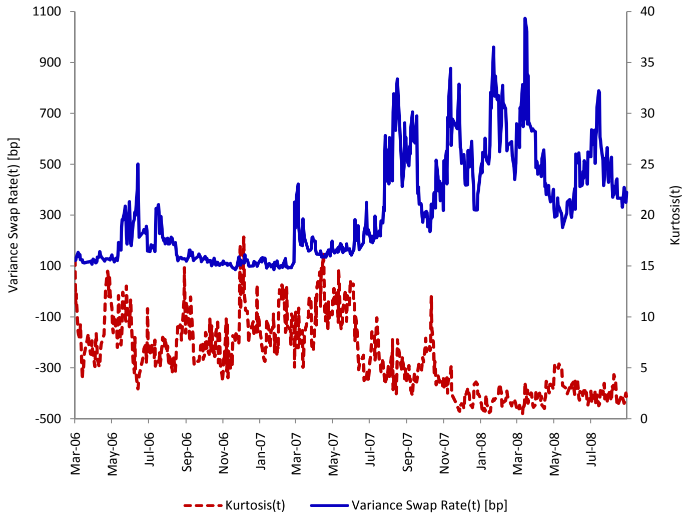
(a) Time Series of 30-Day Variance Swap Rates and their Corresponding Implied Skewness



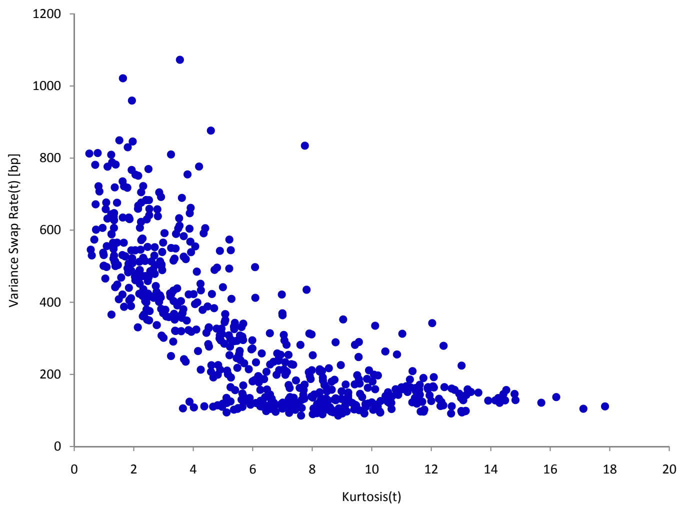
(b) Scatter Plot of 30-Day Variance Swap Rates and their their Implied Skewness Values

Figure 5: Variance Swap Rates and their Corresponding Implied Excess Kurtosis

Figure (a) plots 30 calendar day variance swap rates and their implied excess kurtosis for the March 1^{st}, 2006, to August 29^{th}, 2008, period. Values are backed out from S&P 500 index options by the discretized Bakshi, Kapadia, and Madan (2003) measure. To be consistent with the literature, we exclude pricing observations with a trading volume of less than 40 closed contracts on a specific trading day or a time to maturity of less than 16 calendar days. Furthermore, following Skiadopoulos, Hodges, and Clewlow (2000), we exclude observations with vega values of less than eight. Interest rates are determined by modified put-call-parity regressions proposed in Shimko (1993). **Figure (b)** gives the corresponding scatter plot.



(a) Time Series of 30-Day Variance Swap Rates and their Corresponding Implied Kurtosis



(b) Scatter Plot of 30-Day Variance Swap Rates and their their Implied Kurtosis Values