Optimal Responsible Investment

– A new application of conventional portfolio theory –

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Abstract

Numerous institutions are now engaged in Socially Responsible Investment or have signed the "UN Principles for Responsible Investment". Retail investors, however, are still lacking behind. This is peculiar since the sector constitutes key stakeholders in environmental, social and governmental standards.
This paper considers optimal responsible investment for a small retail investor. It extends conventional portfolio theory by allowing for a personal-value based investment decision. Preferences for responsibility are defined in the framework of mean-variance analysis and an optimal responsible investment model identified. Implications of the altered investment problem are investigated when the dynamics between portfolio risk, expected return and responsibility is considered. Relying on the definition of a responsible investor, it is shown how superior investment opportunities can emerge when the new dimension is incorporated into the investment decision.

Key words: Retail investors, Portfolio allocation, Preferences for responsibility, Socially responsible investment
JEL Classification: G11, D14

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1. Introduction

Socially Responsible Investment (SRI) is defined by the incorporation of environmental, social and governmental (ESG) factors into the investment decision.[2] SRI is a broad term, applied in this context for all investment activities categorized as responsible, ethical, sustainable, green, triple-bottom-line and alike. Their common goal is to simultaneously consider financial properties of an investment and its impact on society and environment.

The European Social Investment Forum, Eurosif, reports that the market for SRI until the beginning of 2007 was dominated by severe growth, mainly driven by institutional investors. Activities labeled responsible or sustainable are still very popular in a financial market currently comprehending the lessons of the crisis. Apart from long-term financial motives, it is presumed to be a reaction to an alteration of social norms: The general public requests a larger degree of corporate responsibility. The financial crisis has stressed the need for good corporate governance and an increase in environmental concerns has emerged from newly projected consequences of global warming. Preferences for non-financial goals such as social and environmental stability are therefore indeed held by the general public. Nonetheless, according to Eurosif (2008) the retail sector only modestly participates in the market for SRI. It is peculiar since this sector has the main direct interest in social and environmental issues.

The motivation to engage in responsible investment can differ severely for institutions and small retail agents: The former are often driven by long term risk management and good public relations while the latter seek to align personal values with investment holdings. Neoclassical financial theory typically relies on the rational value-maximizing paradigm and ignores potential personal benefits from investment decisions incorporating ESG factors. The retail SRI sector thereby holds a unique set of investor incentives, theoretically not well understood nor empirically investigated. Emerging research questions could be: Are small retail agents positioned to make informed responsible investment decisions? Are appropriate investment vehicles available, matching investor preferences for responsibility? And, within what modeling framework can this be studied?

It is the general message from the SRI mutual fund industry that responsibility comes at no extra cost. This statement has been investigated in a number of empirical studies and it is in fact commonly found that SRI mutual funds perform at least no worse than their counterparts. See Renneboog et al (2007) for an overview of literature on the topic. Considering a classical mean-variance analysis, this is a remarkable finding since imposing constraints restricts the investment universe and causes limitations to portfolio diversification.

Perhaps ESG factors do indeed hold extra financial information, at least to cover diversification costs. Another option is that SRI mutual funds focus primarily on

[2] European Social Investment Forum, www.eurosif.org.

keeping up with the market and only allow for the degree of responsibility which does not compromise with portfolio risk and expected return. The latter could give rise to concern about the level of responsibility actually held in the SRI mutual funds, i.e. fear of so-called "green-washing".[3] The aim of this study is not to evaluate the degree of responsibility in the SRI mutual fund industry. Rather, it is to investigate what difference the level of portfolio responsibility can make to the small retail investor. It is hypothesized that if an agent has preferences for responsibility which potentially can provide a private benefit from investing responsibly then a main-stream SRI fund might not be the most suitable investment vehicle.

In opposition to the implication of the rational value-maximizing paradigm, this study does not regard responsible investing as irrational. Instead it is considered a subjectively defined investor preference, similar to the case of risk averse investors. They weigh portfolio expected return and variance against their preference for certainty. In order to make a well-founded portfolio choice it is necessary that preferences for certainty and responsibility are determined in an integrated manner since they may constitute conflicting goals.

This paper suggests a theoretical framework for the portfolio selection of the small retail investor, allowing for a personal-value based decision. Preferences for responsibility are defined into the setting of the classical portfolio mean-variance analysis and a model for optimal responsible investment is given. It is further considered if structured products may facilitate retail SRI. The goal is to examine the implications of the modification of the investment problem; identify new challenges and draw attention to potential opportunities for obtaining improved investments holdings compared to value-neutral counterparts.

The majority of previous SRI studies have been dedicated to considerations on SRI performance and only limited attention has been given to theoretical research on investment decisions incorporating non-financial aspects. Related to this paper, Beltratti (2005) investigates in an equilibrium model the utility costs to the investing individual when discriminating against certain firms acting unethically. The effect on the market value of the unethical firm is also quantified. She finds that responsibility comes at a cost depending on market circumstances when applying a utility function recognizing only financial performance. The intention behind the discrimination is thereby not reflected in the applied preferences and the investing individual is acting irrationally.

Wisebrod (2007) proposes what might be common practice for some SRI mutual funds; a portfolio selection model where responsibility is only considered in the magnitude it does not affect the risk-return profile. It relies on choosing the so-called socially dominant portfolio from a subset of equally preferred optimal portfolios. Theoretically, this method is not attractive since in a com-

[3] See Hawken (2004), www.responsibleinvesting.org.

plete financial market the optimal investment problem has only one solution. Another practical proposal is given by Hallerbach et al. (2002): They suggest an interactive programming method to evaluate and re-allocate asset allocations cooperatively between advisor and investor. My model distinguishes itself from these approaches since it works with a classic setup in finance theory, readjusted by defining a responsible investor and giving appropriate assumptions.
The paper is organized as follows. Section 2 defines a SRI rating procedure and models preferences for responsibility. Section 3 identifies a portfolio selection problem subject to ratings and gives results on the dynamics of the portfolio characteristics; expected return, risk and responsibility. Section 4 comments on implementation, drawing a note on real rating data, and considers SRI structured products for retail investment. The final section concludes.

2. Preferences for Responsibility

In mean-variance analysis asset allocation simplifies to the consideration of the two first moments of the portfolio return distribution; expectation and variance. Risk averse investors balance expected portfolio return against their preference for certainty. Responsible investors are likewise suggested to consider the estimated responsibility level of a portfolio in conjunction with risk and return.
This section investigates the additional dimension in the asset allocation problem of the small retail investor: It defines responsibility rating within this context and suggests a formation of individual preferences necessary to determine a responsible optimal portfolio.

2.1. SRI rating

Let the market consist of N assets and consider K responsibility criteria included in the rating procedure. The outcome of a market screening is defined as a matrix $\mathbf{K} \in \mathbb{M}(N, K)$ where the entries k_{ij} lie in the interval $[-1, 1]$ for all $i \in \{1, .., N\}$ and $j \in \{1, .., K\}$. The number $k_{ij} \in [-1, 1]$ describes to what extent asset i meets screening criteria j,

$$\mathbf{K} = \begin{bmatrix} k_{11} & k_{12} & \dots & k_{1K} \\ k_{21} & k_{22} & \dots & k_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ k_{N1} & k_{12} & \dots & k_{NK} \end{bmatrix}.$$

The rating procedure is the translation of qualitative responsibility attributes of a particular asset into a real number in the continuum $[-1, 1]$. It has the intuitive interpretation as follows.

- $k_{..} = -1$ is the lowest standard; it represents destruction of the non-financial value expressed in the rating criteria. For a humanitarian criteria it could be weaponry production or an undertaking using child labor abusively.

- $k_{..} = 0$ represents the neutral case where the asset has no positive nor negative effect on the given criteria. Also, $k_{..} = 0$ applies when the criteria is not relevant for the given asset.

- $k_{..} = 1$ is the highest standard; it represents creation of the non-financial value expressed in the rating criteria. For environmental considerations it could be technology related to clean energy or pollution reduction.

The K responsibility criteria can generally be any set of values specified by the rating agency. Jantzi Research[4], as an example, considers more than 100 indicators grouped into the following focus areas: Community and society, customers, corporate governance, employees, environment, human rights, and controversial business activities. The latter includes alcohol, gaming, genetic engineering, nuclear power, pornography, tobacco, and weaponry. Each of the indicators are further divided into a number of micro-level factors why the total Jantzi Research matrix for ESG performance is very comprehensive.

2.2. Investor preferences

It is assumed that individual investors are equipped with a set of personally defined priorities regarding responsibility. They are referred to here as *preferences* for responsibility. It is expressed using a K-dimensional vector

$$\psi = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_K \end{bmatrix},$$

such that $\psi_i \in [0, 1]$ for all $i \in \{1, .., K\}$ represents how the given investor relates to a certain criteria. The priority is represented by a real number in the range from zero, non-important, to one, important.

As an example consider an investor who is somewhat interested in human rights (HR) yet very interested in environmental issues (E). For simplicity assume that except from these criteria the investor disregards non-financial values within the investment decision. The corresponding investor characteristic could then for example read

$$\psi' = \begin{bmatrix} \psi_{HR} \\ \psi_E \end{bmatrix} = \begin{bmatrix} 0.40 \\ 0.85 \end{bmatrix},$$

and ψ' constitute the preferences for responsibility for the given investor.

A neutral investor who wishes to ignore responsibility within the investment decision has the preference structure $\psi = \mathbf{0}$. The case of $\psi = \mathbf{0}$ corresponds to the traditional setup from the original Merton (1971) investment problem where

[4] www.jantziresearch.com.

the investor is indifferent to non-financial values.
Let a portfolio consist of a set of relative weights w_j for the assets $i \in \{1, .., N\}$;

$$\mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix}, \qquad \sum_{j=1}^{N} w_j = 1.$$

The restriction on $\mathbf{w}$ infers that the weights w_j represent relative weights of the portfolio of N assets. In order to apply investor specific responsibility preferences to the rating data, the following assumption is made.

Assumption 1. *Investor preferences for responsibility are* linear *across the K rating criteria.*

Assumption 1 implies that a weighting of the investor preferences for responsibility ψ with portfolio ratings and portfolio weights is feasible. In other words it allows for inner product operations.
The span of N investment opportunities rated $\mathbf{K}$ is said to have the *match* to a particular investor with preference ψ found as the inner product

$$s_N = \mathbf{K}\psi,$$

where $s_N \in [-1, 1]^N$. The vector s_N then represents the compatibility of investor interests to market opportunities. Likewise, a portfolio $\mathbf{w}$ of the N assets rated $\mathbf{K}$ has the match to a particular investor of

$$s = (\mathbf{K}\psi)^\top \mathbf{w} = s_N^\top \mathbf{w}, \qquad (1)$$

where $s \in [-1, 1] \subset \mathbb{R}$ is a single quantity summarizing the responsibility standard of a given portfolio to a particular investor. The summarized portfolio responsibility level defined by (1) is denoted the *SRI index* of the investment in the following.
This paper works with two distinct incentives to invest responsibly which can be seen as general for both retail and institutional investors. Investors engaged in SRI either obtain a source of *private benefit* from investing according to their personal values or alternatively they hold *private expectations* to the performance of responsible investments. The latter includes the view that responsibility data holds additional financial information not recognized in conventional financial markets.
It may require at least some market knowledge and speculative effort to form private expectations. Such expectations are likely to be held mainly by institutional investors while private benefits are expected to relate more strongly to the retail segment. The incentive from private benefits are mainly considered in this paper

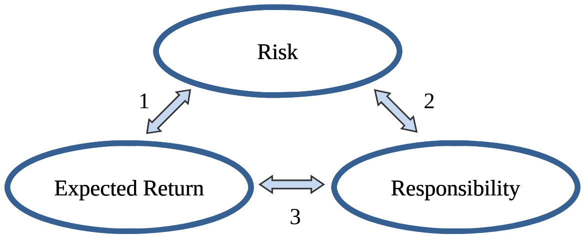

Figure 1: The new dimension of the investor preferences causes the necessity to quantify two out of three relations between the portfolio characteristics; risk, expected return and responsibility.

yet the private expectation structure is commented on in section 3.4 as well as a type of methodology is suggested.

2.3. Quantification of preferences

Within mean-variance analysis the relationship between risk and expected return is determined by a risk aversion parameter for the individual investor. It is the first indicator in Figure 1. In order to form a consistent view of the responsible portfolio characteristics it is necessary to define the second or the third indicator in Figure 1. The last relation will be implicitly given. That is, assuming the investor is rational and does not pursue contradicting goals. This section considers quantifications of the second and the third relation individually.

2.3.1. Utility function

In order to evaluate an investment with respect to risk, return and level of responsibility a utility scheme is introduced. It is designed to recognize utility from both financial return and from possible private benefits of responsibility.
Let the investment span a given period of time $t \in [0, T]$ such that the investment outcome is evaluated at maturity, $t = T$. The financial return of the investment in addition to the responsibility profile will determine the level of satisfaction. Let the utility function have the general form

$$u : \mathbb{R}_+ \times [-1, 1] \rightarrow \mathbb{R}^2 \tag{2}$$

The definition of a responsible investor relies on the presumption that the investment decision is affected by ESG criteria. In order to reflect this in investor preferences, it is necessary for the utility function to incorporate both financial return x and SRI index s of the portfolio defined by (1). Thus, the investment evaluation in (2) is the mapping $(x, s) \mapsto \mathbb{R}^2$.
For now let the total investment utility be decomposable such that

$$u = u_1 \circ u_2 \tag{3}$$

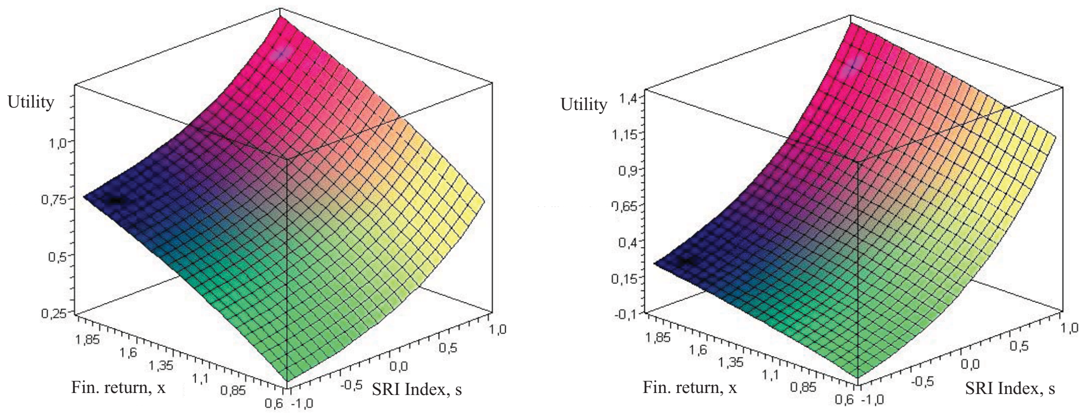

Figure 2: The graphs display utility evaluation of both financial return and responsibility of an investment. The left figure shows the case where both inputs are equally weighted ($\alpha = 0.5$) and the right figure shows the situation where profitability is weighted much higher than responsibility ($\alpha = 0.2$).

where $u_1 : \mathbb{R} \to \mathbb{R}$ describes utility from financial return and $u_2 : [-1, 1] \to \mathbb{R}$ describes utility from portfolio SRI index. An intuitive and simple example is to consider the affine combination: For $\alpha \in [0, 1]$

$$u(x, s) \quad = \quad (1 - \alpha)u_1(x) + \alpha u_2(s), \tag{4}$$

such that $u : (x, s) \mapsto \mathbb{R}$. The parameter α describes to which degree an investor appreciates financial return compared to responsibility.
To illustrate this form of investment evaluation consider, only as an example, the case where the evaluation of financial return is a simple logarithmic function, adjusted one unit on the y-axis in order to have a non-negative range

$$u_1(x) = \ln(x + 1). \tag{5}$$

Also, let the evaluation of responsibility be an exponential function, adjusted to be non-negative only for positive values of s

$$u_2(s) \quad = \quad \exp(s) - 1. \tag{6}$$

The affine combination suggested in (4) yields the utility functions as shown in Figure 2 where the appreciation levels $\alpha \in \{0.5, 0.2\}$ are displayed. It is the case where financial return and responsibility are equally weighted and the case where responsibility has only 20% weight. Here, the utility is a concave function in return and a convex function in responsibility. Note that the structure of u_2, however, has no empirical verification. The lack of evidence is presumably due to lack of data but indeed an interesting line of future research to pursue.

The SRI index rating s is assumed to be deterministic since in practice no monitoring feedback is typically available during time $t \in [0, T]$. It can be verified by the recognition that goals for responsibility are often long term, outliving $t = T$. Ideally, the SRI index could be modeled as stochastic or the responsibility standard could be equipped with an additional error term due to differences in rating methodologies over rating agencies.

2.3.2. Expected return and responsibility

Investor objectives regarding responsibility are initially formalized in the following definition.

Definition 1 (SRI preferences I). *An investor is said to have objectives regarding responsibility if the following relationship is fulfilled. For any $\bar{x} \in \mathbb{R}_+$:*

$$\forall \ \{s_1, s_2\} \in [-1, 1] \ \Big| \ s_1 \geq s_2 \ : \ u(\bar{x}, s_1) \geq u(\bar{x}, s_2).$$

I.e. a (strictly) higher level of responsibility will provide a (strictly) higher investor utility. It means that

$$u(\bar{x}, s_1) = u(\bar{x}, s_2) + \Delta_u,$$

where $\Delta_u > 0$ when $s_1 > s_2$, or alternatively that

$$u(\bar{x}, s_1) = u(\bar{x} + \Delta_r, s_2).$$

Here Δ_u denotes a utility premium and Δ_r denotes the equivalent financial return premium.

Consider the utility scheme presented in section 2.3.1, apply the definition above, and let $s_2 = 0$. For the affine combination with $\alpha \in [0, 1]$ it yields from (4) that

$$\begin{aligned} u(\bar{x}, s_1) &= u(\bar{x}, 0) + \Delta_u \\ &= (1 - \alpha)u_1(\bar{x}) + \alpha u_2(0) + \Delta_u \\ &= (1 - \alpha)u_1(\bar{x}) + \Delta_u, \end{aligned}$$

when $u_2(0) = 0$ by definition. It means that $\Delta_u = \alpha u_2(s_1)$ as expected. For a fixed level of financial return $\bar{x}$, Δ_u quantifies the utility gained by engaging in an investment with a responsibility level of s_1 as opposed to the corresponding neutral investment, $s_2 = 0$. Δ_u can be considered a utility responsibility premium since it is the amount of utility an investor is willing to give up in order to obtain the non-financial objective of responsibility level s_1. It is similar to the risk premium which quantifies what an investor is willing to give up in order to gain certainty.

The utility premium Δ_u can be expressed in terms of financial return, Δ_r. When u_1 is continuous and strictly increasing, u_1^{-1} exists. Usually these conditions apply

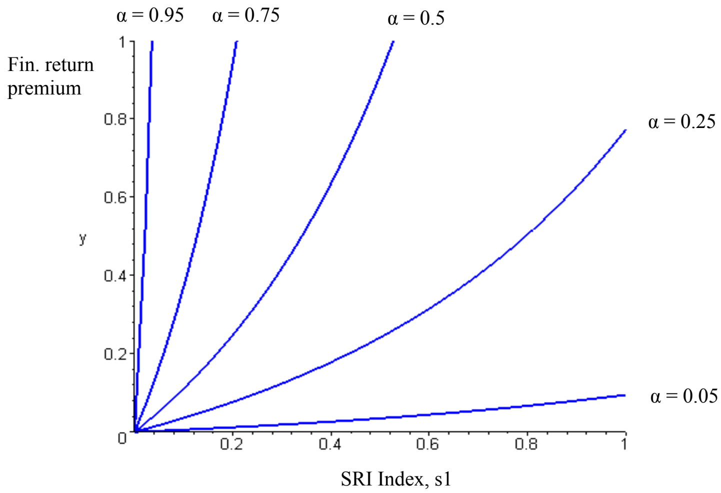

Figure 3: The figure plots the return premium Δ_r of a given level of responsibility, $s_1 \in (0,1]$. Different levels of responsible investors are expressed using the parameter α.

for utility functions. In that case, considering the affine utility function in (4) it is

$$\begin{aligned}\Delta_r &\equiv u_1^{-1}\left(\frac{\Delta_u}{1-\alpha}\right)\\ &= u_1^{-1}\left(\frac{\alpha u_2(s_1)}{1-\alpha}\right).\end{aligned}$$

If further u_1 and u_2 are defined as in the special case of (5)-(6), the responsibility premium is

$$\begin{aligned}\Delta_r &= u_1^{-1}\left(\frac{\alpha u_2(s_1)}{1-\alpha}\right)\\ &= \exp\left(\frac{\alpha u_2(s_1)}{1-\alpha}\right)-1\\ &= \exp\left(\frac{\alpha[\exp(s_1)-1]}{1-\alpha}\right)-1,\end{aligned}$$

for $s_1 \in (0,1]$ and $\Delta_r \in [0,1]$. Figure 3 shows the relationship for the financial return Δ_r the investor is willing to give up in order to obtain an investment profile with SRI index $s_1 \in (0,1]$ instead of a neutral investment, $s_2 = 0$. The responsibility characteristics of the investor is expressed in the figure for the parameters $\alpha \in \{0.05, 0.25, 0.5, 0.75, 0.95\}$.

2.3.3. Risk and responsibility

In order to measure risk aversion the certainty equivalent measure can be useful. It is defined as the return x^* for which the investor is indifferent between holding the uncertain investment yielding the stochastic return x or the the certain return x^*. For a given level of SRI index $\bar{s}$, the certainty equivalent x^* is determined from the equation

$$\mathbf{E}[u(x, \bar{s})] \quad = \quad u(x^*, \bar{s}),$$

such that x^* can be found from $u^{-1}(\mathbf{E}[u(x, \bar{s})])$ when the existence of u^{-1} can be established. The risk premium λ associated with an investment is

$$\lambda(x, \bar{s}) = \mathbf{E}[x] - x^*,$$

why $u(x^*, \bar{s}) = u(\mathbf{E}[x] - \lambda(x, \bar{s}), \bar{s})$ also holds.
An alternative suggestion to the modeling of preferences for responsible investments is to consider the relationship between risk aversion and responsibility. Instead of gaining direct utility from the level of SRI index, a higher SRI index level can be considered to lower the risk aversion of a given investor. The intuition is that the investor is more willing to take on risk when a potential downside is complemented by a high SRI index. In other words, in case of loss then the investment still had a cause compatible with the personal values of the investor and could be considered not a total waste. The suggestion leads to an alternative definition as follows.

Definition 2 (SRI preferences II). *For a general utility function u as defined in (2), an investor is said to have objectives regarding responsibility if the following relationship is fulfilled. For SRI index level $\bar{s} \in (0, 1]$ and for the certainty equivalents $x_1^*, x_2^* \in \mathbb{R}_+$:*

$$\left\{ \begin{array}{l} u(x_1^*, \bar{s}) = \boldsymbol{E}[u(x, \bar{s})] \\ u(x_2^*, 0) = \boldsymbol{E}[u(x, 0)] \end{array} \right\} \qquad \Longleftrightarrow \qquad x_1^* > x_2^*$$

That is, the investor has a higher certainty equivalent x^* when the investment is responsible ($\bar{s} > 0$) than for the neutral investment ($s = 0$).

The indifference curves of an investment having an expected return of $\mathbf{E}[x] = 1.00$ could have the different levels as shown in Figure 4. Here, the certainty equivalent is concave in SRI index level s.
Example: Consider the constant relative risk aversion utility function

$$u(x) = \frac{x^{1-\gamma'}}{1-\gamma'}.$$

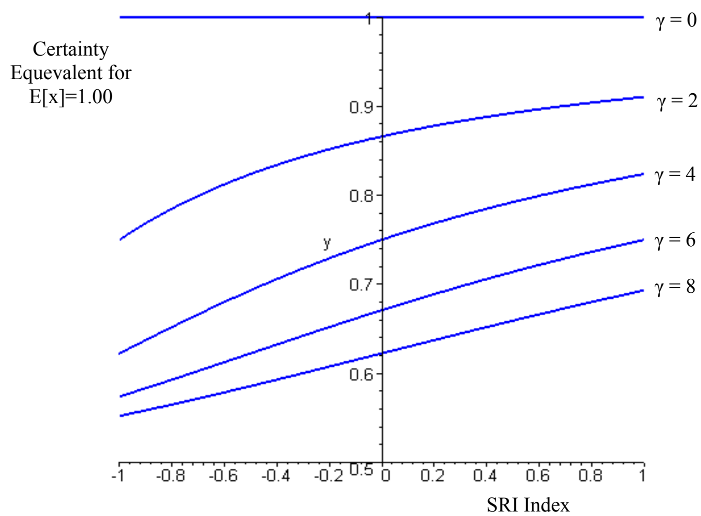

Figure 4: The figure shows the dependence of the certainty equivalent to the level of responsibility expressed using $s \in [-1, 1]$.

Let the risk aversion parameter γ' depend on the responsibility level s in the following manner

$$\gamma' = \frac{\gamma}{s+2},$$

where the denominator $(s+2)$ is chosen such that $s = -1$ implies $\gamma' = \gamma$. A very simple investment problem is as follows: An investor can engage either in a risk free bank account earning the deterministic outcome $\bar{x}$ or in a stock with the outcome $(1-\beta)\bar{x}$ with probability 0.5 and $(1+\beta)\bar{x}$ with probability 0.5. The certainty equivalent x^* is then found from

$$\frac{x^{*(1-\gamma')}}{1-\gamma'} = 0.5\left[\frac{[(1-\beta)\bar{x}]^{1-\gamma'}}{1-\gamma'}\right] + 0.5\left[\frac{[(1+\beta)\bar{x}]^{1-\gamma'}}{1-\gamma'}\right],$$

which implies

$$x^* = \bar{x} \cdot \left[\frac{(1-\beta)^{1-\gamma'} + (1+\beta)^{1-\gamma'}}{2}\right]^{\frac{1}{1-\gamma'}}.$$

Considering this expression for the certainty equivalent with the parameter value set

$$\{\ \bar{x} = 1.00,\ \beta = 0.5,\ \gamma \in \{0, 2, 4, 6, 8\},\ s \in [-1, 1]\ \},$$

yields the indifference curves in Figure 4.

3. The investment problem

The previous section defined a preference model recognizing potential private benefits from non-financial portfolio characteristics. The next step in the asset allocation is forming the investment problem. Initially this section provides the most general problem in order to demonstrate the necessity for further structure. Under restrictions it then restates the problem in terms of mean-variance analysis, leading to a readily applicable procedure.

3.1. The general case

Consider the general utility function in expression (2) where the utility mapping $u : (x, s) \mapsto \mathbb{R}^2$ has no further structure such as additivity. It has two objectives and the determination of trade-off between any of the three portfolio attributes; expected return, risk and responsibility, is implicitly given. The investment problem is a multi-objective system as follows.
Let X_t denote a stochastic N-dimensional return vector for the investment universe. At time $t = 0$ investor chooses the portfolio $\mathbf{w}$ in order to maximize the expected utility at maturity. It is

$$\begin{aligned} \max_{\mathbf{w}} \quad & \mathbf{E}_0 \left[u(\mathbf{w}^\top X_T, \mathbf{w}^\top s_N) \right] \\ \text{s.t.} \quad & \mathbf{w}^\top \mathbf{1} = 1. \end{aligned} \tag{7}$$

This problem has two competing objectives. As long as the utility function has the range $\mathbb{R}^2$, it is therefore impossible to determine how to invest optimally unless further structure is imposed. Solving the responsible investment problem ignoring a potential willingness to trade off expected return or certainty for responsibility is therefore not mathematically meaningful. The only exception is when the investor explicitly expresses that responsibility should only be considered in the magnitude where it does not interfere with financial performance. In that particular situation the procedure of Wisebrod (2007) for portfolio selection is sufficient.
Note that small retail agents choosing to access SRI through a fund or a responsibility index are likely to experience lack of information on the entity's attitude towards this trade-off. If the above multi-objective problem is solved with parameters chosen according to internal institutional policies then investors hold very limited knowledge on the actual level of responsibility in the product bought.

3.2. Mean Variance analysis and SRI

For X_t stochastic N-dimensional return vector for the investment universe of N assets let

$$\mathbf{E}[X_T] = \mu \qquad \text{and} \qquad \mathrm{Var}(X_T) = \Sigma,$$

where Σ is positive definite and the elements of μ are not all equal. Investing W_0 at time $t = 0$ in the portfolio $\mathbf{w}$ yields the payment W_T at maturity with variance

$$\mathrm{Var}(W_T) = W_0^2 \mathbf{w}^\top \Sigma \mathbf{w}.$$

Mean variance analysis is comprised in the fundamental problem

$$\min_{\mathbf{w}} \mathbf{w}^\top \Sigma \mathbf{w} \qquad \text{s.t.} \qquad \begin{cases} \mathbf{w}^\top \mu = \bar{\mu} \\ \mathbf{w}^\top \mathbf{1} = 1 \end{cases} . \tag{8}$$

It is solved using a simple Lagrangean approach. The result is the optimal portfolio weights

$$\hat{\mathbf{w}} = \Sigma^{-1}[\mu \ \mathbf{1}]A^{-1} \begin{bmatrix} \bar{\mu} \\ 1 \end{bmatrix},$$

where $A = [\mu \ \mathbf{1}]^\top \Sigma^{-1} [\mu \ 1]$. The minimum variance portfolio frontier is found as the relationship between the minimum variance $\bar{\sigma}^2$ and $\bar{\mu}$. Inserting the solution $\hat{\mathbf{w}}$ into the objective in (8), it simplifies to

$$\bar{\sigma}^2 = [\bar{\mu} \ \mathbf{1}]\mathbf{A}^{-1} \begin{bmatrix} \bar{\mu} \\ 1 \end{bmatrix}.$$

Let the methodology for finding the responsibility premium under portfolio analysis generalize to the level of individual assets. A slight change in notation applies; let Δ_r denote an N-dimensional vector where the i'th entry describes the responsibility return premium for the i'th asset. If the investor redeems a personal benefit from holding a responsible investment, i.e. $\Delta_r^\top \mathbf{1} \geq 0$, then adjusting the expectation of the portfolio return in the portfolio choice can describe such individual preferences for responsibility. That is, recall Definition 1 regarding responsibility and profitability and let the entries of the vector Δ_r describe the equivalent responsibility adjustment for each asset.
Now, consider the expected return μ adjusted for investor specific preferences with the potential private benefits of $\Delta_r^\top \mathbf{1}$. It is

$$\mu^{SRI} \quad = \quad \mu + \Delta_r \ = \begin{bmatrix} \mu_1 + \Delta_1 \\ \mu_2 + \Delta_2 \\ \vdots \\ \mu_N + \Delta_N \end{bmatrix}, \tag{9}$$

where Δ_r is backed out from the utility function recognizing private benefits, as defined in the previous section. Since Δ_r is the responsibility equivalent return premium describing the financial value of responsibility to the investor, μ^{SRI} can be seen as the private view on the investment universe held by the responsible investor. The readjusted mean-variance analysis is demonstrated through the

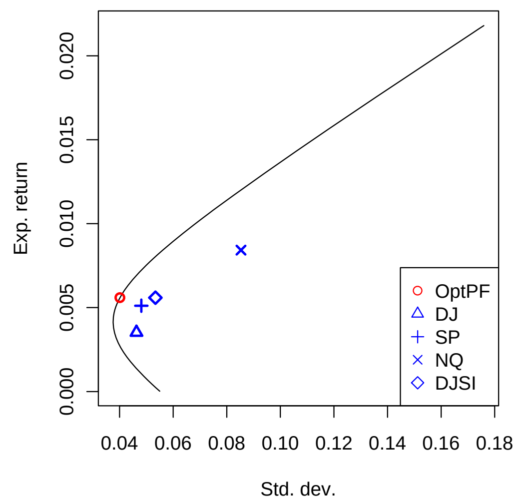

Figure 5: The efficient portfolio frontier for the neutral market. Individual indexes are shown as well as SI.

implementation of the responsible portfolio choice in the following index example.

3.3. Example: Sustainable index investment

The Dow Jones Sustainability Index World (SI) tracks the performance of the top 10% of the companies in the Dow Jones Global Total Stock Market Index leading the field in terms of corporate sustainability. The Dow Jones Sustainability Indexes Guide (2009) provides some details; its main focus is held on long term financial goals and managing sustainability costs and risks. The index thus appears to have purely financial motives for pursuing sustainability. Data on SI monthly price return is available from September 1999 onwards.

This example considers the broad market as comprised by the indexes; Dow Jones Industrial Average (DJ), Nasdaq Composite (NQ) and S&P500 Index (SP). Data on SI was drawn from the homepage of Dow Jones Sustainability Indexes[5] and remaining market data from the Yahoo Finance[6] database. Figure 5 plots the efficient portfolio frontier and the indexes in addition to the optimal portfolio found from the traditional mean-variance analysis framework with a required return (the parameter $\bar{\mu}$ in section 3.2) equal to the expected return on SI.

In this investment universe the price for being a strictly responsible investor can be quantified as the distance from the SI asset to the efficient frontier. It is either an expected financial return premium or an acceptance of additional standard deviation on investment return. In this particular case it amounts to approximately

[5]www.sustainability-index.com

[6]finance.yahoo.com

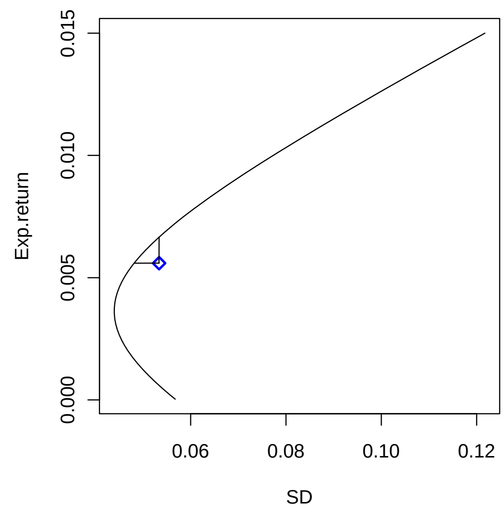

Figure 6: The responsible investor accepts a diversification cost that can either be expressed in expected return or standard deviation.

11 Bsp pr month of expected return or 0.5% standard deviation, see Figure 6. Consider the specification of μ^{SRI} from (9);

$$\mu^{SRI} = \begin{bmatrix} \mu_{DJ} \\ \mu_{SP} \\ \mu_{NQ} \\ \mu_{SI} + \Delta_{SI} \end{bmatrix}, \quad \text{for} \quad \Delta_{SI} \in \mathbb{R}_{+}.$$

It applies the assumption that the market indexes are personal-value neutral, i.e. $\Delta_i \equiv 0$ for $i \in \{\mathrm{DJ}, \mathrm{SP}, \mathrm{NQ}\}$. The empirical monthly rate of return expectations on the indexes are

$$\mu^{\top} = [\ 0.0035 \ \ 0.0051 \ \ 0.0084 \ \ 0.0056\ ].$$

Solving the traditional mean-variance problem yields some preliminary results and provides further intuition on the behavior of the minimum-variance portfolios. The statistical software **R** with the package *tSeries* is applied.[7]
For $\Delta_{SI} \in \{0.000, 0.001, .., 0.009\}$ the optimal portfolio is found and the corresponding portfolio standard deviation and expected return plotted. It is done for each of the following optimal portfolio classes; the global minimum variance portfolio, the tangency portfolio and a target-return portfolio. See Figure 7. The target-return which has to be defined for the latter mentioned portfolio is set to be the historical average of all the indexes. For the adjustments $\Delta_{SI} \in \{0.001, .., 0.009\}$

[7] **R** and *tSeries* can be downloaded from cran.r-project.org.

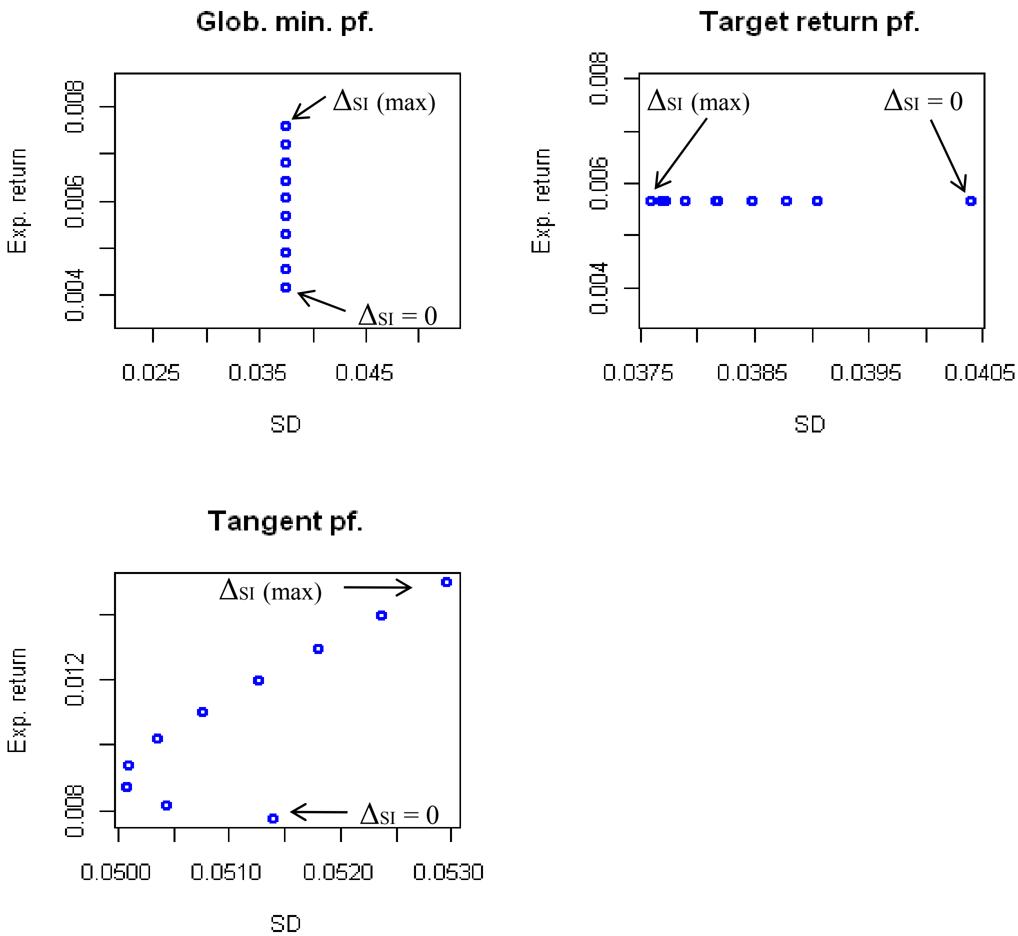

Figure 7: For different levels of SRI premium on SI, the charts show the resulting standard deviation and expected return on a set of minimum variance portfolios; the global minimum variance, the target-return portfolio, and the tangency portfolio.

the optimal portfolio characteristics are plotted; $\Delta_{SI} = 0$ and Δ_{SI} (max) are indicated and the spectrum in between is expected to be self explanatory.

Observe that the global minimum variance portfolio only gains a higher expected return when the SRI premium is raised on SI since the risk is minimized at all times. This applies for an investor with a fixed risk aversion and corresponds to Definition 1. The minimum variance portfolio subject to the target-return sees a shift in standard deviation as the premium grows higher since the target can be obtained increasingly easily with SI. Due to diversification considerations, however, there seems to be convergence of the extent to which the standard deviation can be lowered. The tangent portfolio shows that as the SRI premium is raised the portfolio becomes less risky at first, only to become much more risky at higher responsibility premiums. It has the intuitive interpretation that as the

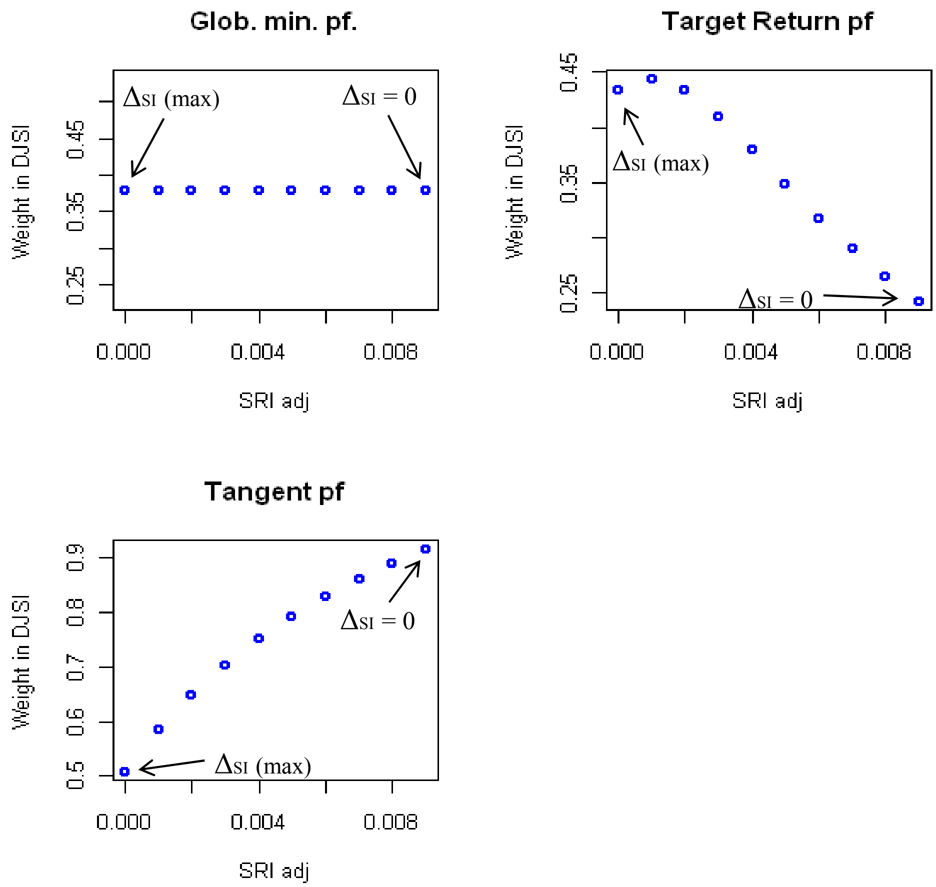

Figure 8: For different levels of SRI permium on SI, the charts show the resulting weights in SI in the minimum variance portfolios; global minimum variance, target return portfolio and tangency portfolio. The green point denotes no adjustment, i.e. $\Delta_{SI} = 0$

SRI premium rises it becomes worth it to pursue a less diversified portfolio.
Next, consider similar graphing of optimal portfolios but where the magnitude of SRI adjustment is plotted to the resulting weights in SI, see Figure 8. The global minimum variance portfolio holds a constant proportion of SI since its objective is to minimize risk alone. The target return portfolio, however, shows a behavior which may seem peculiar at first: The weight in the responsible asset starts with a small rise, only to fall over the increasing SRI premium thereafter. It is, since the target return can be obtained with only a small weight in the responsible investment and lighter gearing of the investment, less short sale of other assets. Finally, the tangent portfolio shows a larger weight in SI for increasing responsibility premium as expected.
The problem is solved with no short sales constraints and the optimal tangency

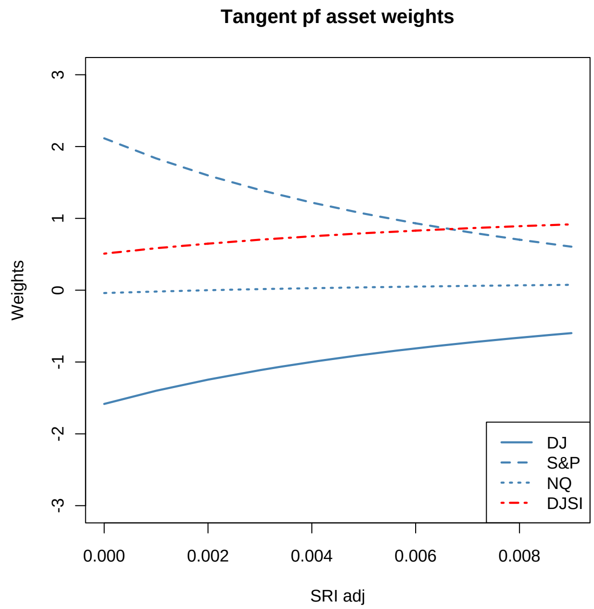

Figure 9: The weights of the four assets in the tangency portfolio changes over rising responsibility premium.

holding for no SRI adjustment is somewhat aggressive. Yet, it is seen that the tangency portfolio becomes less geared when the SI index has a high responsibility premium, see Figure 9. This is interesting since it means that when a responsible investor includes an asset with a high responsibility premium, the optimal tangent portfolio becomes less extreme. Loosely speaking, it enables an investor to benefit utility wise from a personal advantage over the market due to his particular preferences. This benefit is also seen in the next plot.

The total expected return can be divided into a financial and a non-financial part, the responsibility premium. See Figure 10 for the corresponding development over rising premium for the total expected return. Recall, however, from Figure 7 that the very stable expected financial return comes at a diversification cost why the tangency portfolio has high standard deviation for high levels of adjustment. This is not surprising since a high SRI premium can only occur when investors are very concerned about sustainability and hence put a great value to it.

In summary, using an index example this section demonstrated how loss of diversification is compensated for when responsibility premiums are sufficiently high. It also showed that the optimal tangent portfolio for high premiums is less geared and that the responsible investor has unique opportunities to use the personal value-based advantage to obtain utility gains. It is the exact purpose of undertaking an optimal responsible investment problem to be aware of this potential

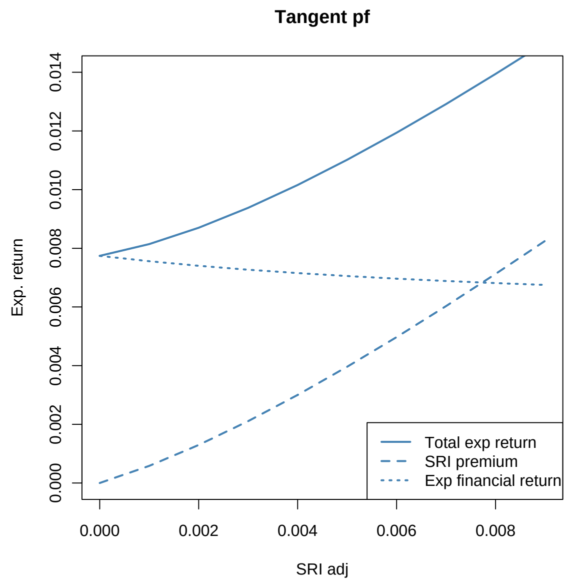

Figure 10: Here, the total expected return is divided up into a responsibility premium and the expected financial return.

and seek to fully use it.

3.4. Private views with the Black-Litterman model

It has been argued that incentives to engage in responsible investment can be divided into two groups; private benefits and private expectations. While the first mentioned has already been considered, this section notes how a general investor, institutional or retail, may incorporate private expectations regarding responsibility factors.

Integrating subjective market views into a portfolio decision has been studied by Black and Litterman (1992). They suggest a systematic method for specifying and incorporating investor views into the estimation of market parameters. Adjusted parameters are then applied in the portfolio optimization.

A major advantage of using the Black-Litterman model is that sufficient attention is drawn to correlation and diversification effects from imposing the subjective views. It contrasts that market sentiment based on non-financial data is often applied ad-hoc in practice. The method is reviewed and demonstrated briefly here since at least to my knowledge it has not been connected to expectations regarding responsibility in previous studies.

3.4.1. Model review

The Black-Litterman model builds on three main pillars: The semi-strong Market Efficiency assumption, the Capital Asset Pricing Model (CAPM), and the Bayes' Rule. Consider the N-dimensional return vector X where the joint

distribution is taken to be multivariate normal; $X \sim N(\mu, \Sigma)$. The model addresses the merge of investor views into the estimation of the market mean μ.
Let μ itself be stochastic and normally distributed and assume the dispersion is proportional to that of the market

$$\mu \sim N(\phi, \tau\Sigma).$$

The parameter ϕ will typically be determined by some established procedure such as the CAPM while τ is a technical parameter.
The subjective views of the investor is formed on the actual mean of the return for the holding period and should be formed as linear combinations of the asset returns held in the vector μ. The views take form as expected mean plus an error term

$$p_{i1}\mu_1 + p_{i2}\mu_2 + \ldots + p_{iN}\mu_N = q_i + \epsilon_i,$$

where $\epsilon_i \; N(0, \sigma^2)$. The confidence in each view is thereby controlled by the standard deviation σ. The views are assembled into a matrix P why the general view specification summarizes to

$$P\mu \;\sim\; N(\mu, \Omega),$$

where Ω is the diagonal matrix with entries $(\sigma_1^2, .., \sigma_N^2)$. Using Bayes' law the following result can be shown, see for example Cheung (2009) for the full derivation.
The posterior distribution of the market mean conditional on the subjective investor views are

$$\mu \mid_{q,\Omega} \sim \; N(\mu_{BL}, \Sigma_{BL}),$$

where

$$\begin{aligned} \mu_{BL} &= \left[(\tau\Sigma)^{-1} + P^{\top}\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\phi + P^{\top}\Omega^{-1}q\right] \\ \Sigma_{BL} &= \left[(\tau\Sigma)^{-1} + P^{\top}\Omega^{-1}P\right]^{-1} \end{aligned}$$

3.4.2. Example

Considering the index example from the previous section, preliminary results can be derived using the Black-Litterman technique. The view is established that the sustainable index will financially perform better than the market predicts. The view is thereby

$$1\mu_{SI} = (\mu_{SI} + \Delta_{SI}) + \epsilon.$$

The error term ϵ allows for a given uncertainty in the expressed view. Depending on agency or mutual fund rating information etc. it may be convenient to have some flexibility in the certainly of the view.
Private expectations to market performance has to be considered carefully when implemented. Institutional investors familiar with the Black-Litterman model

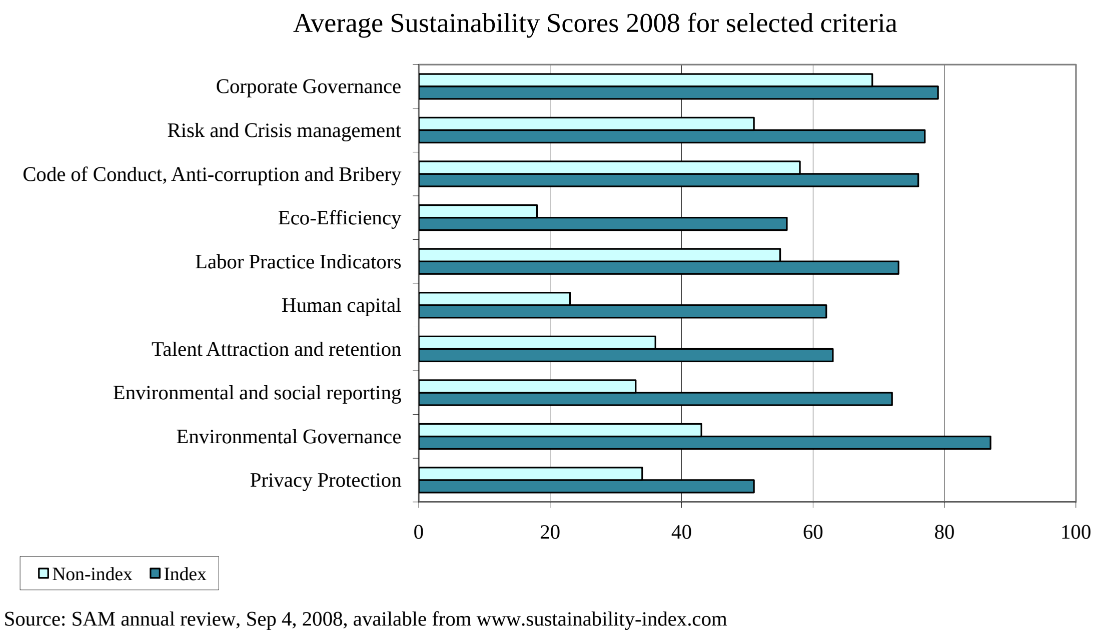

Figure 11: The Dow Jones Sustainability Index World displays the shown improvements in SRI index ratings compared to market average. Some criteria have large improvements while others are almost the same.

from ordinary investment practices may find it useful also with respect to expectations based on responsibility data. Yet the model and parameter specification can be complex, see discussion by Chincarini and Kim (2009).

4. Practical implementation

It can be very difficult for the small retail investor to access a responsible investment which has a verifiable good match to their preferences. This section touches upon access to rating data and gives some observations on these that may be crucial to the responsible investor. Thereafter it proposes structured products as possible alternatives to mutual fund or index investment.

4.1. Rating data

Rating data of the form presented in section 2.1 is offered by various agencies. One example is the already mentioned Jantzi Research which covers mainly the Canadian market. Responsibility rating data is costly due to the considerable research necessary to generate it. Here, some of the ratings underlying the SI index considered in the examples are presented. It offers further intuition on the diverse character of investments labeled responsible.
Figure 11 shows the average score for sustainability of index members compared

to the general market. For the small retail investor it is likely that the SI does not provide a particularly high aggregate portfolio rating or favorable match to individual preferences. Hawken (2004) criticized a number of mutual funds, promoting themselves as sustainable, green or responsible, when he argues that their portfolios largely resemble the general market. The index data in Figure 11 stresses that a responsible investment is a diverse concept why it may need further quantitative documentation in order to be theoretical meaningful. There is a clear difference in many aspects, in particular for eco-efficiency, human capital, social/environmental reporting and environmental governance. Note that the method for calculating these factors are also a black box to index investors, making it a challenging task to quantify the real responsibility utility for the investor.

4.2. SRI Structured Products

A structured investment product is generally defined as a pre-packed investment based on ordinary assets and derivatives or strategies. The assets can be stocks, bonds, treasury notes or likewise, while the derivative or strategy typically provides the product with its particular properties. At least theoretically, structured products can provide tailored risk mitigation and give payoff opportunities matching investor preference structures well. It is stressed that the a necessary condition for SRI structured products to be relevant is that the preservation of transparency in product potential, costs and structures is maintained. Two suggestions of potentially useful structures are briefly described.

4.2.1. Principal Protected Note

A very simple form of structured investment is the Principal Protected Note (PPN). Essentially, it consists of a stock and a derivative where the latter can provide insurance against a decline in the value of the stock. It could be as simple as a European put option. In that case the construction can be compared to a structured bond, basically consisting of a bond and a European call option.

The stock should be chosen according to investor preferences for responsibility and it is crucial that the stock provides a clear and transparent high SRI rating. The purpose is to avoid inferior investor preference matches such as the SI index might provide. For an environmentally concerned investor it could be clean technology research or CO_2 reducing activities.

The derivative can either provide the investor with a capital guarantee or it can give an enhanced profit in certain states. Note that the latter should mainly appeal to the risk tolerant investor and the option would likely be a type of barrier option.

4.2.2. Hybrids

Three types of hybrid securities could essentially be feasible; convertible bonds, income securities or payment-in-kind with warrants. They combine elements of debt and equity, providing a fixed or floating predictable rate of return

until a certain point in time where the security holder has a number of options, e.g. convert debt into equity. The convertible bond may be the most relevant since it is a product well know to the market.
This is a particular interesting structure since it can enable retail investors to access venture capital investments with high SRI ratings. Many innovative project regarding environmental issues are project potentially falling in the category. The downside is, however, that the risk analysis may be quite complex. Holding a large amount of convertibles in the same company is therefore not recommendable. For an investor with objectives as defined in Definition 2, i.e. who may relax risk aversion when responsibility standards are raised, hybrids can therefore seem particularly relevant.

5. Conclusion

The paper suggests a theoretical framework for the portfolio choice of the small retail investor, allowing for a personal-value based investment decision. Preferences for responsibility are defined into the common mean-variance analysis and a model for optimal responsible investment is given. Institutional investment is briefly considered in a Black-Litterman model and finally, structured products are suggested to potentially facilitate retail SRI.
Demonstration of the model using an index example shows how loss of diversification is compensated for when responsibility premiums are sufficiently high. It is also seen that the optimal tangent portfolio for high premiums is less geared and that the responsible investor has unique opportunities to use the personal value-based advantage to obtain utility gains.
The example demonstrated how important it is for the small retail investor to be aware of own priorities in order to obtain the best possible portfolio. It is necessary in order to take advantage of the potential personal benefits of responsible investment, normally considered irrelevant in the financial market. Thus, on partly un-familiar ground for conventional finance theory, opportunities emerge for improved retail investments compared to value-neutral counterparts.
Future studies could consider empirical evidence for the behavior of the responsible retail investor. It could shed some light on the formation of preferences for responsibility and utility functions incorporating an SRI index. It would be useful information for both the investment industry as well as theoretically interesting with respect to behavioral aspects of household finance. Understanding investor preferences to a higher degree would also give the opportunity to apply financial engineering in the process of creating the best possible responsible investment solution for the retail investor.
Indeed, it would be very interesting to know if under improved information on the subject of responsible investment the sector would participate in the market for SRI to a higher degree. After all, small retail investors in particular constitute the key stakeholders of environmental preservation and social welfare.

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