Valuing Venture Capital Contracts: An Option Pricing Approach*

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Abstract

This article derives the economic value of venture capital contracts using option pricing techniques. We identify the options embedded in model contracts as published by the National Venture Capital Association (NVCA) and show how they can be priced in interaction using Least Squares Monte Carlo simulation. The pricing model is calibrated using a dataset of deal terms in Silicon Valley collected by the law firm Fenwick & West LLP, as well as industry statistics from the NVCA. We obtain estimates of the value of individual terms and the total value of contracts in realistic scenarios, with multiple financing rounds and multiple investors.

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1 Introduction

Venture Capital (VC) investments are equity investments in young companies with high growth potential and accompanying high market and technology uncertainties. At the outset of an investment, it is very hard to predict market developments, time to liquidity, management performance and other factors influencing the future performance of such companies.[1] Short of robust estimates of company value at the time of contracting, Venture Capitalists (VCs) seek to mitigate this uncertainty by structuring complex contractual agreements. These agreements make incentive and control mechanisms contingent on the future development of the venture and on the occurrence of future events such as follow-on VC financing rounds or exit transactions. Thereby, they generate ex-ante agreed flexibility and influence the distribution of payoffs at exit.

According to real option theory, this type of flexibility under uncertainty generates economic value, which can be captured using option pricing techniques. These techniques have been extensively used in related fields of research, but their potential for the pricing of VC investment contracts is largely unexplored. A first category of related literature applies option theory to price real assets, including high-growth companies.[2] The methodologies employed for this purpose range from closed-form solutions such as the traditional Black and Scholes approximation methodology[3] to numerical techniques including Monte Carlo simulation, which is also used in this article.[4] This literature provides valuable guidance on the pricing of real options with exotic features including path-dependencies, uncertain parameter values, and American exercise rights, which also apply to options embedded in VC contracts.

Another related strand of literature assesses the value of options embedded in legal covenants. For the pricing of debt contracts, option-based approaches have become standard tools.[5] In the context of equity investment contracts, the application of option theory is less advanced. Chemla, Habib and Ljungqvist (2004) claim that each clause of a shareholder agreement can be viewed as an option, whether it is explicit (as in the case of put and call clauses), or implicit (as in the case of drag-along rights or catch-up clauses). In their study, the real option approach is essentially used to gain a better understanding of incentive and control mechanisms, but not to assess the economic value of contract features. This gap has been bridged for joint venture agreements by Juan, Olmos and Ashkeboussi (2007b), who present a pricing approach for clauses of

[1] See Wilmerding (2005).

[2] See Willner (1995), Berk, Naik and Green (1998), Schwartz & Gorostiza (2000), or Schwartz & Moon (2000).

[3] See Benaroch & Kauffman (1999), Trigeorgis & Panayi (1998), Kumar (1996, 1999), Perlitz, Peske and Schrank (1999).

[4] See Jagle (1999), Schwartz & Cortazar (1998), Schwartz & Gorostiza (2000).

[5] See Merton (1974), Ingersoll (1977), Black & Cox (1976), Anderson & Sundaresan (1996).

dynamic reallocation and clauses of termination and restatement of ownership interest. They show that these clauses generate exotic options such as compensation options and options with uncertain initial date, and price these options using Monte Carlo simulation and probability distribution modelling.[6]

Surprisingly, only a few academic articles have given systematic thought to the economic value of VC investment contracts. Woronoff and Rosen (2005) show that legal terms in VC contracts can significantly affect the distribution of value among various interested parties upon exit, and should therefore be quantified at the outset of the investment relationship. They suggest to capture the economic value of terms indirectly, via their impact on "effective valuation", which accounts for the expected distribution of payoffs among the parties at exit. However, they do not employ asset pricing techniques to quantify this effect. To our best knowledge, the only systematic analysis of VC contract values is provided by Cossin, Leleux and Saliasi (2002). Their framework addresses some of the major covenants found in VC contracts (i.e. liquidation preference, staging, conversion and anti-dilution) and prices them both in isolation and in interaction, using closed-form solutions and numerical analysis (based on finite differences).

The existing academic literature addressing VC contract design does not account for the economic value of contract terms, nor for the various interaction effects between individual terms.[7] Optimal contract theory focuses on the function of individual terms (i.e. on agency and moral hazard problems), whereas empirical studies analyze the formal specification of terms and their use in different countries.[8] As to practitioners, they often rely on "boiler plate" contracts and cannot assess precisely the impact of individual provisions on expected returns.[9]

In this article, we develop an option pricing model for VC contracts, which covers the majority of provisions contained in U.S. model documents and which accounts for interaction effects among these provisions. The model is calibrated using data on deal terms in Silicon Valley provided by the law firm Fenwick & West LLP as well as industry statistics from the NVCA. We then apply this pricing model to realistic contracting scenarios (with multiple financing rounds and multiple investors), taking the perspective of the initial ("Series A") investor, and obtain estimates of the value of individual terms and of the full contract. The methodology is based on advanced simulation techniques including Least Squares Monte Carlo Simulation and extensions developed by Gamba (2003), which are flexible enough to account for the specificities of options

[6]See also Juan et al. (2007a).

[7]See Cossin et al. (2002) and Hellmann (1998).

[8]Theoretical papers include Bergemann & Hege (1998, 2000), Admati & Pfleiderer (1994), Noldeke & Schmidt (1998), Bascha & Walz (2001); empirical results are presented, for example, in Gompers (1995), Kaplan & Stroemberg (2002a, 2002b), Lerner (1994), or Gompers and Lerner (1996).

[9]See Woronoff & Rosen (2005a) and Brobeck, Hale & Dorr (2003).

embedded in VC contracts (including American exercise rights, path-dependencies, uncertain parameters and a discontinuous underlying process).

This option pricing approach contributes to several streams of academic research. First, it extends the nascent research line on the economic value of VC contracts by covering new types of covenants (including exit rights and control rights) and by accounting for "trigger events" (i.e. follow-on VC financing rounds and exit events in the form of an IPO, a Company Sale or a Liquidation Event). It also provides the first application of Least Squares Monte Carlo simulation to this research line. In related fields of research on VC contract design, this approach can be used for the derivation and empirical testing of general optimality arguments which are based on economic contract value. Finally, it provides practitioners with a tool to estimate the relative value of individual terms and to optimize the outcome of negotiations in financing rounds. While the focus in this article is laid on U.S. practices, the pricing model can be adapted to practices in any other country using relevant industry statistics and model documents.

The remainder of our article is organized as follows. In Section 2, we present the general methodology used for pricing of VC contracts. In Section 3, we identify the types of options embedded in U.S. model contract terms and show how they can be priced using Least Squares Monte Carlo Simulation. In Section 4, we calculate contract values in realistic scenarios that account for multiple investment rounds with multiple investors. Section 5 summarizes the findings and proposes directions for future research.

2 General Methodology

This section describes the methodology developed in this article to identify and price options embedded in VC contracts.

2.1 Screening Methodology

Our analysis addresses deal terms that are representative of current structuring practices and industry norms. It focuses on terms which are negotiable among the contracting parties and/or which are value drivers. Value drivers are defined as terms which are either enforceable in practice, or difficult to enforce but valuable given their impact on the incentive and control structures of the parties. We also identify interaction effects among contract terms, to the extent that they influence the types and values of embedded options.

The article focuses on deal structures in the U.S.; it relies on model VC financing documents (hereafter "NVCA model documents") published by the NVCA (the trade association representing the U.S. venture capital industry). These documents are intended to reflect current customs as well as industry best practice and include alternative specifications of terms and extensive commentaries, which facilitate the identification of embedded options and help to distinguish between the formal specification of provisions in model contracts and their effective use in practice.[10]

2.2 Valuation Methodology

VC contracts provide investors with claims on the shareholders' equity of the investee company. Their key provisions become exercisable upon the occurrence of share issues (follow-on VC financing rounds) and exit transactions (IPO, sale of the company, liquidation).[11] Since VC investee companies are typically not listed, these events also represent the only objective (market-defined) pricing points of company valuation.[12] Therefore, we chose to model the path followed by equity value not as a continuous diffusion process, but as a jump process that allows for random changes in value upon the occurrence of follow-on VC financing rounds or exit events (hereafter collectively defined as "Pricing Events").[13]

Over time, the parties to VC contracts typically include multiple VCs with different series of Preferred shares and different contractual rights. As a result, contract values for various investors in the same company can differ significantly. The pricing model developed in this article calculates contract values from the perspective of a single investor (as opposed to the investors as a group), hereafter referred to as "VC_1". However, we account for the fact that certain contractual rights can be jointly held by multiple investors, in which case the options embedded in these rights are shared among multiple parties. Embedded options are either of European or American type, depending on whether or not the option holders have control over the timing of exit. They may have significant interactions, since multiple options can be triggered by the same Pricing Event, and the exercise of certain options may preclude that of other rights.[14] We assume that these options have two independent variables, namely *time* (t) and *share value* (or price per share) ($P(t)$). The variable t defines the discrete

[10]The initial model documents are the result of a consensus process among the members of the NVCA Model Document Working Group, which consists of leading VC lawyers and VC firms. The set of documents used for this article is the result of the second round of intense review, comment and revision by the working group performed in January 2008, as well as updates performed after this date as deemed appropriate by the working group. The documents diverge from current customs if necessary to avoid hidden legal traps as well as internal inconsistencies or redundancies.

[11]See Chemla et al. (2004); the importance of these events is documented in the NVCA model documents by the frequent use of the phrase "in the event of [...]".

[12]These pricing points can be assumed to be at or close to fair value, since contractual protective provisions of VCs and fiduciary duties of the Board of Directors prevent underpriced transactions.

[13]A similar approach is taken by Willner (1995) for the valuation of start-up ventures and in Pennings & Lint (1998) for the valuation of R&D projects.

[14]Notably, the conversion of Preferred Shares into Common Shares leads to the loss of all Preferred Rights.

time steps of the value process (reflecting the occurrence of Pricing Events). $P(t)$ is the underlying asset (subject to stochastic jumps upon the occurrence of Pricing Events) and the principal source of randomness affecting embedded options values. The payoff functions of embedded options are described as $\Pi(t, P(t))$.

In the presence of stochastic jumps, markets are incomplete and one cannot construct a riskless hedge portfolio to substantiate the use of risk-neutral valuation.[15] However, it can be assumed that the stochastic jumps in the share value of VC-funded companies between consecutive Pricing Events are uncorrelated with the market portfolio, and that VC investors are well diversified at the level of their portfolios.[16] Hence, the jump components generate only non-systematic risk which is fully diversifiable, and the systematic risk is zero. According to the Capital Asset Market Model, the expected return in this case equals the risk-free rate. This makes risk-neutral valuation applicable in our specific context.

To value the options embedded in VC contracts, we use Monte Carlo simulation. This technique relies on the fact that the distribution of the terminal underlying values is determined by the process generating future movements of the underlying, and invokes the risk-neutrality assumption to derive the option value.[17] It accommodates complex distributions of the underlying process (including stochastic jumps) as well as exotic option features (including path-dependencies and uncertain parameter values). The accuracy of the method is measured based on the standard deviation of the estimate and can be improved by increasing the number of simulations and by applying variance reduction techniques.[18]

For American options, we use the Least Squares Monte Carlo simulation (LSM) developed by Longstaff and Schwartz (2001). This method combines the forward simulation of share price paths from initial date to maturity with an assessment (at each time step from maturity to initial date) of the benefit of exercising versus holding, using a simple regression across stock prices.[19] To account for interaction effets among embedded options, we rely on the extension to the LSM derived by Gamba (2003).

The notation used in this article closely follows the notations presented in Longstaff and Schwartz (2001) as well as Gamba (2003). For the sake of brevity, European-type claims are treated as special cases of American claims.

[15]See Merton (1976).

[16]A similar line of argumentation is developed in Willner (1995), who assumes that the jumps in the value of start-up companies reflect new discoveries, which are not correlated with the market portfolio.

[17]Monte Carlo simulation was first applied to option pricing by Boyle (1976) and relies on the general derivative pricing paradigm of Black and Scholes (1973), Merton (1973), Harrison and Kreps (1979), Harrison and Pliska (1981), Cox, Ingersoll and Ross (1985), Heath, Jarrow and Morton (1992), and others.

[18]See Boyle (1977), Kind (2005), or Rodrigues and Rocha Armada (2006).

[19]The potential of LSM to solve real option valuation problems has been recognized in the extent literature; it is used, for example, in Schwartz and Moon (2000) to derive the option value of internet companies.

2.3 Model Specification

2.3.1 Underlying asset path

As described above, the underlying asset in our model is the share value of the portfolio company, which is adjusted discontinuously upon the occurrence of Pricing Events. Generally, VCs seek to exit their investment within a given timeframe, that is agreed upon by the parties at the time of contracting, typically between 3 and 8 years, depending notably on the stage of development of the company and the remaining lifetime of the VC fund. The maximum investment period is therefore the interval $[t_0, t_{max}]$ between the date of contracting (t_0) and the date agreed among the parties (at t_0) as the latest possible exit date (t_{max}). Over this time horizon, there are a random number of Pricing Events, and of jumps in the share price. These jumps are of random direction and random magnitude. Since adjustments in share value take place exclusively at Pricing Events, the share price remains constant between consecutive jumps. Therefore, we describe the path followed by the share price as a compound Poisson process, that is as a jump process with zero drift, exponentially distributed waiting times and stochastic jump amplitude:

$$dP(t) = P(t)dN$$

where dN equals 0 with probability $1 - \lambda dt$ and a jump size of J_i with probability λdt.[20]

The waiting time between consecutive jumps is assumed to be exponentially distributed with scale parameter θ.[21] The number of jumps (per time step) follows a homogeneous Poisson process with intensity parameter $\lambda = 1/\theta$. The expected number of price adjustments over the maximum investment period $[t_0, t_{max}]$ is then equal to $\lambda(t_{max} - t_0)$. Empirical findings of Ewens (2009) show that the mean "round-to-round holding period" (i.e. the waiting time between consecutive Pricing Events) equals 1.5 years. Therefore, in our model, $\theta = 1.5$ and $\lambda = 0.67$. Accordingly, the total number of information arrivals over the maximum investment period, equals $N = \lambda(t_{max} - t_0) = 0.67 * (t_{max} - t_0)$ with $t_{max} \in [3, 8]$. The last price jump occurs at time step $t_N = sup\{t_i : t_i \leq t_{max}\}$.

The jump amplitude is derived from a dataset compiled by the law firm Fenwick & West LLP, which is the basis for its quarterly report on "Trends in Terms of Venture Financings in Silicon Valley".[22] The data used for this article covers the main terms of

[20] See Baldwin (1979) or Pennings and Lint (1997) who develop similar jump processes, with stochastic jumps and deterministic (but non-zero) drift rate.

[21] The exponential distribution is mainly chosen for its no-memory property, implying that the probability of arrival of new strategic information does not depend on the arrival of past strategic information.

[22] The dataset is built using information from VentureSource (from Dow Jones) and ThomsonONE (from

VC financings in the San Francisco Bay Area over the period 2004 to 2008. The sample size of 2,168 financings represents nearly 40% of the total VC financings reported in the region over this period.[23] We have evaluated the data to specify the jump amplitude of our pricing model and to gain insights on the use of individual provisions, which are presented in later sections.

The Fenwick & West data indicates, for each sample financing, the direction of the change in the price per share as compared to the previous round (which allows for distinction between "up rounds" vs. "even rounds" vs. "down rounds"), as well as the magnitude of the price change (expressed as a percentage of the price per share at the previous round). We have evaluated the statistics on the types of rounds to derive the probability distribution of the jump direction, X_i, in our model:

$$X_i = \begin{cases} 1 & \text{with probability } p^u = 66.74\% \text{ (for up rounds);} \\ -1 & \text{with probability } p^d = 21.62\% \text{ (for down rounds);} \\ 0 & \text{with probability } p^e = 11.64\% \text{ (for even rounds).} \end{cases}$$

The statistics on the size of changes in the price per share are used to specify the jump amplitudes for the different types of jumps:

$$\begin{aligned} &E(x^u) = 85.82\% && \text{(average magnitude of upward jumps);} \\ &E(x^d) = 50.10\% && \text{(average magnitude of downward jumps);} \\ &E(x^e) = 0\% && \text{(average magnitude of even rounds, by definition).} \end{aligned}$$

These results show that the probability of positive jumps is higher than the probability of negative jumps, and that the magnitude of positive jumps is higher than the magnitude of negative jumps. Moreover, as illustrated by the Figures 1 and 2, the observed probability distributions of upward versus downward jump magnitudes follow different patterns. The differences between the distributions can be explained by the fact that downward jumps cannot exceed 100% (since shareholders have limited liability); in addition, they reflect lower competition level and higher bargaining power of investors in down rounds versus up rounds.[24] We therefore assume that a better fit to the data can be achieved by modelling the magnitudes of upward versus downward jumps using different distributions and perform separate goodness-of-fit tests to estimate the probability distribution function ("pdf") providing the best approximation to the observed data in each case.[25]

Thomson Reuters) as well as publicly available sources, and reviewed by senior lawyers to ensure that the financings as well as individual terms are classified and interpreted consistently across the dataset.

[23]See NVCA Yearbook 2009.

[24]As explained by Michael Patrick (Fenwick & West).

[25]A similar argument in favour of asymmetric jump models with mixed distributions has been used for listed stocks, on the basis that prices respond differently to the arrival of good news and bad news (see

Figure 2 indicates that the pdf for the magnitude of upward rounds resembles a Weibull distribution with shape parameter $k = 1$ and scale parameter γ (which is equivalent to the exponential distribution).[26] This translates into the following null hypothesis:

$$H_0 : Y_{x^u} \sim Wei(\gamma, 1)$$

The γ parameter of the Weibull distribution is estimated using maximum likelihood estimation. With k = 1, the maximum likelihood estimator for γ is obtained as follows[27]:

$$\hat{\gamma} = \sum_{1}^{n} x_i/n = 0.8582$$

with n being the number of sample rounds.
A chi-squared goodness-of-fit test is then used to test H_0. Appendix A shows that we cannot reject H_0 at the $\alpha = 0.10$ level of significance and hence it appears that the exponential distribution (or Weibull distribution with $k = 1$) well fits the magnitude of upward jumps.

A similar test is performed for the amplitudes of downward jumps. The distribution of observed jump sizes (see Figure 1) shows resemblance to a uniform distribution, which leads to the following null hypothesis:

$$H_0 : Y_{x^d} \sim U(n)$$

As shown in Appendix A, we cannot reject H_0 at the $\alpha = 0.10$ level of significance and it appears that the uniform distribution fits the magnitude of downward jumps.

Based on the findings on jump amplitudes and the number of jumps, we obtain the following underlying process:

$$P(t) = P(0) \prod_{i=0}^{N^u(t)} [J_i^u + 1] \prod_{i=0}^{N^d(t)} [J_i^d + 1],$$

respectively the following instantaneous return:

$$\frac{dP(t)}{P(t)} = J^u(t)dN^u(t) + J^d(t)dN^d(t)$$

Ramezani and Zeng (1998) and Dupoyet (2004)).

[26]We have chosen to use the Weibull distribution for testing purposes, as this allows to assess the fit against a broader range of distributions based on different choices of the shape parameter. The analysis in this section is restricted to the Weibull distribution with $k = 1$, which provided the best test results.

[27]See Cohen (1965).

The parameters are defined as follows:

- $J^u(t)$ is the percentage up-jump size conditional on an upward jump, defined as: $J^u(t) = x^u(t)$ with $x^u(t)$ distributed $Weibull(\gamma, 1)$;
- $J^d(t)$ is the percentage down-jump size conditional on a downward jump, defined as: $J^d(t) = -x^d(t)$ with $x^d(t)$ distributed $uniform(a, b)$;
- $N^u(t)$ and $N^d(t)$ are Poisson up and down jump counters with intensities λ^u and λ^d, where
  $\lambda^u = \lambda * p^u = 0.67 * 0.67 = 0.45$ and
  $\lambda^d = \lambda * p^d = 0.67 * 0.22 = 0.15$.

The weibull and uniform density functions for the up-jump and down-jump magnitudes are assumed to follow:

- $Y_{x^u}(x^u) = \frac{1}{\gamma} \exp^{-\frac{x^u}{\gamma}}$
  with $x^u \geq 0$, $\gamma = 0.8582$, $E(x^u) = \gamma$ and $\sigma^2_{x^u} = \gamma^2 = 0.7365$;
- $Y_{x^d}(x^d) = \frac{1}{b-a}$
  with $x^d \in [0, 1]$, $E(x^d) = 0.5$ and $\sigma^2_{x^d} = 0.0833$.

As mentioned earlier, the options embedded in VC contracts can be priced as if the expected growth rate for the underlying asset was the risk-free rate, r. Hence, we transform the physical jump model derived above into a (market diversified) risk-neutral process, by means of the following adjustment:[28]

$$\begin{aligned}
\frac{d(P(t))}{P(t)} &= [r + \lambda(-p^u E(x^u) + p^d E(x^d))]dt + J^u(t)dN^u(t) + J^d(t)dN^d(t) \\
&= 0.03 - (0.45 * 0.86) + (0.15 * 0.50) + J^u(t)dN^u(t) + J^d(t)dN^d(t) \\
&= -0.28 + J^u(t)dN^u(t) + J^d(t)dN^d(t)
\end{aligned}$$

where r is set equal to the Treasury-bill rate over the period 2004 to 2008 (3.05%). All other parameters remain unchanged.

2.3.2 Value process of embedded options

In this section, we derive a general value process for all options held by VC_1. Let there be one state variable $P(t)$, which represents the share value of the portfolio company, and which affects the value of the embedded option (from the perspective of VC_1). Consider the probability space $(\Omega, \mathcal{F}, P)$, where Ω is the state space of all possible paths ω of the state variables relevant for pricing the option, $\mathcal{F}$ is the sigma field of disjoint events at time T, and $\mathcal{P}$ is the probability measure corresponding to $\mathcal{F}$. Let the maturity of the option be defined as $T \in [0, t_{max}]$ and the time step of the last

[28]See Ramezani and Zeng (1998), Kou and Wang (2001), Kou (2002) and Dupoyet (2004).

jump in share value before maturity be $t_M = sup\{t_i : t_i \leq T\}$. Let there be O discrete stopping times, $t_0 \leq t_1 \leq t_2 \leq \cdots \leq t_O$, with $t_0 = 0$.[29]

We further define $\Pi(\omega, s; t, t_M)$ as the cash flows deriving from the option for VC_1 at time s (representing the VC's proceeds from the exit transaction), and $I_{vc1}(\omega, s; t, t_M)$ as the cumulated amount invested by VC_1 at time s, when the state path ω is realized, given that the option is not terminated at or before time t, and that the VC follows the optimal exercise strategy for all exercise dates s between time t and t_M, with $t < s \leq t_M$. S defines the exercise restrictions in the form of sets of time in which exercise of the option is allowed. These sets of time must concur with the time steps used for the simulation of the jump process (see Section 2.3.1). Note that the presence of a barrier condition may cause S to differ across paths because the option might be triggered in certain paths only, due to the evolution of the state variables in these paths. Let the value of the claim at t be $F(t, P(t))$.

If the contingent claim can be exercised exclusively at maturity T, it is a European-type claim. The value of this claim at any time t equals:

$$F(t, P(t)) = \frac{1}{n} \sum_{i=1}^{n} e^{-r(T-t)} E_t^*[\Pi(\omega_i; T, P(T)) - I_{vc1}(\omega_i; T, P(T))]$$

where $E_t^*[\cdot]$ is the expected value in a risk-neutral world, conditional on the information available at t, and n is the number of simulation paths.

If the contingent claim can be exercised at any time before t_{max}, it is an American-type claim; respectively, if exercise is restricted to certain dates in $[0, t_{max}]$, it is a Bermudan-type claim. In these cases, the value of the claim at any time t equals:

$$F(t, P(t)) = \frac{1}{n} \sum_{i=1}^{n} \max_{s_i \in S_i(\omega_i; t, T)} \{exp^{-r(s_i - t)} E_t^*[\Pi(\omega_i; s_i, P(s_i)) - I_{vc1}(\omega_i; s_i, P(s_i))]\}$$

where $S_i(\omega_i; t, T)$ is the set of possible exercise dates in $[t, T]$ for simulation path ω_i with regards to $\{\mathcal{F}_t\}$.

To derive embedded option values in the following sections, we will rely on a set of additional dependent variables, which are influenced by the independent variables of the model as well as by certain contractual provisions. These variables are defined in the following.

[29] These stopping times do not have to be equivalent to the time steps used for the simulation model (which reflect the dates of jumps in share value).

$N_{tot,c}$	total number of shares (fully diluted)
$N_{vc1,p}$	number of Preferred Shares held by VC_1
$N_{vc1,c}$	number of Common Shares receivable by VC_1 upon conversion
$N_{pref,p}$	number of Preferred Shares held by all VCs (Preferred Holders)
$N_{pref,c}$	number of Common Shares receivable by all VCs upon conversion
I_{tot}	total amount invested by all VCs
V_{pre}	pre-money valuation of the company (fully diluted)
V_{post}	post-money valuation of the company (fully diluted)
α	ownership percentage of VC_1 (fully diluted)
β_{pref}	ownership percentage of all VCs (fully diluted)
β_{com}	ownership percentage of all Common Stock holders
C_{vc1}	conversion price applicable to Preferred Shares held by VC_1

Variables defined on a "fully diluted" basis rely on the number of Common Shares *outstanding* and *deemed outstanding* after conversion of all convertible securities and after exercise of all stock options.

2.3.3 Modelling of pricing events

As derived above, stochastic jumps in the underlying process reflect the occurrence of Pricing Events in the form of follow-on VC financing rounds or exit transactions. Follow-on VC financing rounds are defined as issuances of additional convertible Preferred Shares to VC investors, while exit transactions include the following types of (mutually exclusive) events:[30]

- *Initial Public Offering (IPO):* a sale of common stock to the general investing public for the first time; or
- *Liquidation Event (LE):* voluntary or involuntary dissolution, liquidation or winding up of the corporation; or
- *Sale of the Company (CS):* This term covers both Stock Sales and Deemed Liquidation Events
  - *Stock Sale (or sale of control) (SS):* a transaction in which a person acquires from stockholders of the company shares representing more than fifty percent of the outstanding voting power;
  - *Deemed Liquidation Event (DLE):* a merger or consolidation, or the sale (or other disposition) of all or substantially all the assets of the corporation.

[30] See NVCA Certificate of Incorporation, C2.1 (pages 8,11-12,30-31).

The timing of follow-on VC financing rounds is defined by the jump model. The offer size of any follow-on round is assumed to be a fixed percentage of the company's valuation at that round. Let η be the percentage obtained by dividing the amount raised in any follow-on round k by the post-money valuation at that round. The post-money valuation at round k can then be derived from the total number of shares at the previous round $(k-1)$ as follows:

$$V_{post}(t_k) = \frac{N_{tot,c}(t_{k-1}) * P(t_k)}{(1-\eta)}$$

Accordingly, the total number of shares (fully-diluted) after this round is:

$$N_{tot,c}(t_k) = \frac{N_{tot,c}(t_{k-1})}{1-\eta}$$

From the statistics published in the NVCA Yearbook 2009, we derive that $\eta = 15\%$.[31]

To determine the timing of exit events, we take a different approach than for financing rounds. We assume that the occurrence of an IPO or a Liquidation is restricted to scenarios of success or failure of the portfolio company, while a Company Sale can be performed in all scenarios of company performance. Generally, an IPO only becomes attractive at very high levels of company valuation, at which the expected proceeds compensate for the high cost (direct and indirect expenses) and risk (lock-up period) of the transaction. A Liquidation, to the contrary, is most likely to occur in states of failure, when the company runs out of cash or when major technology or market uncertainties remain unsolved over time.[32] Therefore, we model the uncertain dates of a Liquidation (τ_{LE}) and of an IPO (τ_{IPO}) as *random times of class 1*, which are defined as follows[33]:

Definition 1 (Class 1 Random Time) *A random (jump) time τ is said to be Class 1 random time if τ is a stopping time of the filtration $\mathbb{F}$ generated by asset prices, that is if the event $t_i < \tau$ is $\mathcal{F}_t$-measurable for all $t_i \geq 0$.*

Since observing the asset price up to the i-th jump provides full knowledge about whether τ has occurred or not, the timing risk is embedded within asset price risk and no new uncertainty is added to the economy. Under the assumption that the IPO and the Liquidation are path-dependent events, the embedded options that are triggered by these events are effectively transformed into barrier options.

[31] This figure is obtained by dividing the average investment size of follow-on rounds by the average post-money valuation of follow-on rounds in the year 2008; see NVCA Yearbook 2009, pages 31, 41-42.

[32] As confirmed in the expert interview with Hassan Sohbi (Taylor Wessing).

[33] See Karoui and Martellini (2001).

Regarding the occurrence of Liquidation Events, we make the following assumption.

Assumption 1 (Liquidation Threshold) *The company goes out of business when the price per share falls below a certain minimum treshold, which is calculated using a percentage discount of $\delta = 75\%$ on the initial share value at t_0. Hence, the date of the Liquidation Event for all $t_i \geq 0$ is determined by $\tau_{LE} = \inf\{t_i : P(t_i) \leq h_{LE}\}$ with $h_{LE} = P(t_0) * (1 - \delta)$, $\delta = 0.75$.*

A definition of the liquidation threshold based on the Price Per Share is only justifiable in the absence of structural changes such as stock splits or reverse splits. Since structural changes occur rarely (i.e. in less than 7% of all Fenwick & West sample financings), and reflect formal rearrangements of the company's capital structure (generally not driven by changes in company performance), they will be excluded from our analysis of embedded options.

For IPOs, the threshold is derived from median pre-money valuations of VC-backed IPOs in the U.S. for the 10-year period 1999-2008.[34]

Assumption 2 (IPO Threshold) *The company goes public when the pre-offer valuation of the company exceeds \$200m. The IPO date is therefore obtained, for all $t_i \geq 0$, as $\tau_{IPO} = \inf\{t_i : V_{pre}(t_i) \geq h_{IPO}\}$ with $h_{IPO} = \$200m$.*

The offer amount of the IPO is set equal to 27% of the post-offer valuation. This percentage is derived from the NVCA Yearbook 2009 (by dividing median offer amounts by median post-offer valuations over the period 1999-2008). Hence, the post-offer valuation and the total number of shares after IPO are obtained as follows:

$$\begin{aligned} V_{post}(\tau_{IPO}) &= N_{tot,c}(t_{IPO-1}) * P(\tau_{IPO})/(1 - 0.27) \\ N_{tot,c}(\tau_{IPO}) &= N_{tot,c}(t_{IPO-1})/(1 - 0.27) \end{aligned}$$

Accounting for the threshold condition above, $V_{post}(\tau_{IPO}) \geq \$274m$, and the offer amount is equal or superior to $\$74m$.

If neither an IPO nor a LE have occurred before or at the last time step t_N (i.e. the date of the last jump in share value before t_{max}), we assume that a Sale of the Company is initiated by either the company (reflecting a Deemed Liquidation Event) or by the VCs (reflecting a Stock Sale) at t_N.[35]

[34] This figure is derived from the NVCA Yearbook 2009 statistics on venture capital exits.

[35] This presupposes that a trader buyer willing to pay fair company value can be found at this date. In reality, this may only be possible with a certain delay and/or with concessions regarding the offer price.

Assumption 3 (Sale condition) *The company initiates a Sale if no other exit transaction has occurred until the last jump date t_N. Hence, for all $t_i \geq 0$,*

$$\tau_{CS} = \begin{cases} t_N & \text{if } \tau_{IPO} = \tau_{LE} = 0, \text{ and} \\ 0 & \text{otherwise} \end{cases}$$

By combining the above assumptions, the uncertain exit date can be defined as follows:

$$\tau_{exit} = \begin{cases} \tau_{IPO} & \text{if } \tau_{LE} > \tau_{IPO} > 0, \text{ or if } \tau_{IPO} > 0 \wedge \tau_{LE} = 0 \\ \tau_{LE} & \text{if } \tau_{IPO} > \tau_{LE} > 0, \text{ or if } \tau_{LE} > 0 \wedge \tau_{IPO} = 0 \\ \tau_{CS} & \text{if } \tau_{IPO} = \tau_{LE} = 0 \end{cases}$$

with $h_{IPO} = \$200m$ and $h_{LE} = 0.25 * P(t_0)$.

3 Pricing Model

In this section, we analyze the options embedded in standard provisions and show how to assess their economic value while accounting for interaction effects. There are several alternatives for the categorization of VC contract terms. Notably, one can distinguish between initiatory rights and protective rights, or categorize the provisions based on the specific Pricing Event they relate to. We chose a categorization driven by methodology, based on the type of option pricing parameters influenced by each group of provisions. Hence, we differentiate between provisions that (a) define the payoff functions of embedded options, (b) impact the number of shares receivable by the investor upon conversion and (c) introduce American exercise rights.

3.1 Provisions Defining the Payoff Functions

The most common security used for VC financings in the U.S. is convertible preferred stock.[36] This instrument provides the VC with both downside protection and upside participation, via Liquidation Preference and (Optional and Mandatory) Conversion Rights. These provisions define the payoff functions of embedded options and are therefore analyzed together in this section. The analysis furthermore covers Piggyback Registration Rights, which generate value in interaction with Mandatory Conversion.

[36] Common equity (without preferred rights) is only issued exceptionally, for example within the scope of pre-IPO financing rounds (approx. 6 months before the IPO), where institutional types of investors would be offered common stock at a discount to existing shares.

3.1.1 Mandatory conversion and piggyback registration

Mandatory (or Automatic) Conversion is a "given" in VC contracts.[37] It specifies that convertible preferred stock automatically converts to common stock upon occurrence of certain trigger events, which typically include (a) a Qualified Public Offering (QPO), defined as a public offering that meets certain minimum conditions, or (b) a majority (or sometimes supermajority) vote of the holders of Preferred Stock.[38] Mandatory Conversion represents a protection mechanism for both the company and the VC. It protects the company by clearing the way towards an IPO in success scenarios (since underwriters typically require that the company has a single class of equity) and by mitigating the VC's incentive to "grandstand" (i.e. to take a company public prematurely in order to improve his reputation).[39] It protects the VC, since the company can only perform an IPO if the offering meets certain minimum requirements, or if it obtains the consent of a majority of investors. At the same time, this obligation to convert is costly to the VC, since he loses the rights attached to Preferred Shares; this cost of conversion is all the higher, the earlier the stage of the company and the more important the downside protection attached to the Preferred Shares. For this reason, VCs would only *voluntarily* agree on foregoing their Preferred rights in exceptional situations, for example if it allows them to gain Board Control (e.g. when the tie-breaking seat is filled by Common and Preferred Holders voting as a single class, on as-converted basis), or if conversion of existing Preferred Shares is required by an external investor as a prerequisite for the infusion of new funds. Hence, Automatic Conversion is rarely initiated by consent of the Preferred Holders, but in most cases triggered by a QPO event. Therefore, this section will focus on the trigger event in the form of a QPO.

[37]This is mentioned in the NVCA Yearbook 2009 and has been confirmed by Michael Patrick (Fenwick & West); in mathematical formulas, Mandatory Conversion will be abbreviated as "MC".

[38]Conversion can furthermore be forced upon the VC if he foregoes to (fully) exercise his Pre-emption Right; this scenario represents a special case of Pay-to-play provisions, which are covered in Section 3.2.3. Model definitions of the Mandatory Conversion provision are provided in the NVCA Certificate of Incorporation, C5.1 (pages 30-31) and the NVCA Term Sheet (pages 7-8).

[39]These mechanisms are described in further detail in Camp (2002), Black and Gilson (1998), and Gompers (1996).

Typically, a QPO is defined as an offering which meets the following minimum criteria:[40]

- minimum valuation (pre- or post-money): guarantees the investors a minimum return on investment;
- minimum proceeds (gross or net): guarantees that the offering compensates for the high IPO-related expenses.

Sometimes, the conditions also include a minimum public float and/or a minimum price per share. The minimum public float condition is indirectly covered by our minimum proceeds condition.[41] The minimum price per share condition represents a formal requirement, which is not necessarily related to company performance or to the scope and quality of a public offering (since the price per share may also change due to structural changes such as share splits). Therefore, the share price condition will not be considered in this section. As determined by the IPO threshold derived in Section 2.3.3, the pre-offer valuation of the company is at least \$200m and the gross proceeds from IPO amount to at least \$73m. The QPO conditions are usually defined at levels that are inferior to these IPO threshold levels. Therefore, any IPO event in our model meets the QPO requirements, and the date of QPO can be set equal to the IPO date, τ_{IPO}.

In terms of embedded derivatives, the Mandatory Conversion provision generates a forward contract on Common Shares, since the VC is *obliged* to convert his Preferred Shares into Common Shares in the event of an IPO. Upon conversion, the VC holds a position of α *long call with* $E = 0$, which represents the value of his Common Shares (on as-converted basis). Investors, management and employees usually agree on a so-called "lock-up period" (i.e. a period of at least 180 days after the IPO, during which they refrain from selling their shares to the public).[42] This avoids large sales of stock immediately after the IPO and allows the company to build interest among potential buyers of its shares. We assume that VCs sell their shares as soon as possible, that is immediately after expiration of this lock-up period. Under this assumption, the maturity of the forward contract is $T_{MC} = \inf\{t_i : t_i \geq \tau_{IPO} + 0,5\}$.

In the U.S., shares in a public company cannot be sold to the public unless they have been registered with the Securities and Exchange Commission (SEC) or are exempt from registration. Hence, Mandatory Conversion is only valuable in the presence of Piggyback Registration Rights, which entitle the VC to "piggyback" on any reg-

[40] See NVCA Certificate of Incorporation, C5.1 (pages 30-31) and Camp (2002).

[41] Under the assumptions that no shareholder (other than employees) holds less than 10% of total shares outstanding prior to the IPO and that all members of the pre-IPO shareholder base are subject to a lock-up period after IPO, the free float equals the gross proceeds to the company.

[42] The lock-up period is usually defined in a "Market Stand-Off Agreement" (see NVCA Investors' Rights Agreement, 2.11 (pages 17-18) and NVCA Term Sheet (page 10)).

istrations initiated by the company or any (demand) registration initiated by other investors.[43] Therefore, VCs invariably require Piggyback Registration Rights as a condition of funding for U.S.-based companies.[44] Piggyback Registration is not valuable in isolation, since registration is only feasible for Common Shares. However, if analyzed in interaction, Mandatory Conversion and Piggyback Registration generate a full-fledged forward contract:

Proposition 1 (Automatic Conversion Plus Piggyback Registration) *In the presence of Automatic Conversion and Piggyback Registration, the VC is effectively granted a forward contract on Registered Common Shares of the Company. The maturity of this contract is* T_{MC}.

This contract allows the VC to convert his Preferred Shares into Common Shares, register these shares with the SEC, and sell them to the public after expiration of the lock-up period, at market price. We assume that the VC aims at liquidating his position fully as soon as possible after the IPO, and that he will not speculate on the future development of the company. Therefore, the number of Common Shares sold after expiration of the lock-up period equals the total number of Common Shares receivable at the IPO date:

$$N_{vc1,c}(T_{MC}) = N_{vc1,c}(\tau_{IPO}).$$

Applying the conversion price in effect at the date of IPO, the number of Common Shares receivable upon conversion is obtained as follows:

$$N_{vc1,c}(\tau_{IPO}) = N_{vc1,p}(\tau_{IPO}) * \frac{P(t_0)}{C_{vc1}(\tau_{IPO})}$$

The variables $N_{vc1,p}(\tau_{IPO})$ and $C_{vc1}(\tau_{IPO})$ are affected by the terms of Pre-emption Rights, Anti-dilution Rights and Pay-to-play provisions as described in Section 3.2. The payoff function of the forward contract can be written as:

$$\Pi_{MC}(T_{MC}, P(T_{MC})) = N_{vc1,c}(\tau_{IPO}) * P(T_{MC})$$

[43]Demand Registration is covered in section 3.3.2, since it influences the exercise restrictions of embedded options, and not their payoff functions.

[44]For investments in non-U.S. companies, VCs do generally not pay major attention to Registration Rights at the time of contracting. However, if an IPO in the U.S. evolves as a likely exit scenario for a company over time, VCs would usually ask for the (a posteriori) inclusion of Registration Rights into their contract before voting in favour of an IPO.

Accordingly, the value of the contract at $t \leq T_{MC}$ is:

$$F_{MC}(t, P(t))$$
$$= e^{-r(T_{MC}-t)} E_t^* [\Pi_{MC}(T_{MC}, P(T_{MC})) - I_{vc1}(T_{MC}, P(T_{MC}))]$$

3.1.2 Liquidation preference

By definition, Convertible Preferred Stock provides for some type of *Liquidation Preference*, which grants the holders of Preferred Shares the right to receive a minimum value for their Preferred Stock in "preference" to the holders of other classes of stock upon ocurrence certain exit events.[45] As mentioned in Smith (2005), Liquidation Rights (almost) never confer the VC a contractual right to liquidate the portfolio company. Therefore, they should be understood as *protective* exit rights and not as *initiatory* exit rights. The major function of Liquidation Rights is to protect the VC against opportunistic liquidation by a controlling entrepreneur and to increase his incentive to force liquidation through exercise of other contractual rights (e.g. via board voting rights) in circumstances when the entrepreneur would like to maintain the status quo. The payment of the Liquidation Amount is effectively triggered by the occurrence of a Liquidation Event or the Sale of the Company.[46] This leads to the following proposition with regard to embedded derivatives:[47]

Proposition 2 (Liquidation Rights) *Liquidation Rights can be valued by means of a European option which becomes exercisable upon the occurrence of the earliest of a Liquidation Event or a Sale of the Company, that is at*

$$T_{LP} = \begin{cases} \tau_{LE} & \text{if } \tau_{IPO} > \tau_{LE} > 0, \text{ or if } \tau_{LE} > 0 \wedge \tau_{IPO} = 0; \\ \tau_{CS} & \text{if } \tau_{IPO} = \tau_{LE} = 0. \end{cases}$$

The payoff function of this claim can be derived from the definition of the "Liquidation Amount", as well as the degree of "Seniority" of the Preferred Holder.

[45]See NVCA Yearbook 2009.

[46]See NVCA Certificate of Incorporation C2.1 (pages 8-9) and NVCA Voting Agreement 3.4 (pages 8-9). The trigger event in the form of Stock Sale is not listed in the Charter, since this type of transaction may not be within the control of the corporation; however, the "Restrictions on Sales of Control of the Company" in the Voting Agreement ensure that the Preferred Holders receive the same share of exit proceeds in the event of a Stock Sale than in the case of a Deemed Liquidation Event.

[47]Note that "Liquidation Preference" is abbreviated as "LP" in mathematical formulas.

The *Liquidation Amount* usually contains the following elements:

- a *fixed amount* equalling a multiple of the initial investment (usually 1x to 3x); plus
- (if specified,) a *variable amount* conditional on total exit proceeds (also called "participating" feature); plus
- (if specified,) *dividends* defined as "cumulative" or "non-cumulative" dividends.

The degree of *Seniority* defines the ranking order among the different shareholders with regard to the payout of the Liquidation Amount. Thus, if Preferred Shares issued in later rounds rank *senior* to the shares issued in prior rounds, the holders of these shares are entitled to be paid out their Liquidation Amount before any payments are made on *junior* shares. By definition, Seniority only becomes relevant in the presence of multiple financing rounds. In the Fenwick & West dataset, 46% of all sample financings have senior Liquidation Preference.

Dividends payable to Preferred Holders are usually defined as *cumulative* or *non-cumulative* Dividends. Cumulative dividends (also called "accrued") grant the right to receive dividends that have cumulated over time (independently of whether they were declared or not) and which have not been paid out in full prior to the exit event. In the Fenwick & West dataset, less than 6% of all sample financings provide for cumulative dividends. Non-cumulative dividends (also called "declared but unpaid") are payable to owners of preferred stock only if they were declared by the Board of Directors and if the company has sufficient cash available. This type of dividends is rarely employed in VC contracting and not covered in the Fenwick & West database. To limit the complexity of the payout functions derived in this article, our model will abstract from dividend rights.[48]

To determine the payoff functions in this section, we assume that the company performs a single financing round with a single investor (VC_1), which implies that no additional preferences are issued after t_0. This assumption will be relaxed in Section 4, which assesses the value of contract provisions in realistic scenarios (multiple rounds, multiple investors).

No participation The simplest form of the Liquidation Right is generated by *non-participating* Preferred Stock. The Liquidation Amount is simply defined as a multiple (m) of the amount invested. Since we assume that there is only one round with one investor, the total amount invested by VC_1 at exit simply equals his initial investment $I_{vc1}(t_0) = N_{vc1,p}(t_0) * P(t_0)$. To simplify the notation, $I_{vc1}(t_0)$ will be abbreviated

[48]This simplification comes without a major loss in generality, since dividends are rarely used in the U.S. and since the share of dividend payments as a percentage of total exit proceeds is typically minor.

as I in the remainder of this section. At exit, the VC receives either his Liquidation Amount mI (if total proceeds are equal or superior to the Liquidation Amount), or the totality of exit proceeds $V(T_{LP}) = N_{tot,c}(T_{LP}) * P(T_{LP})$ (if total proceeds are inferior to the Liquidation Amount).[49] Hence, the payoff function of non-participating Preferred Stock can be replicated using the following option basket:

- 1 long call with $E = 0$;
- 1 short call with $E = mI$.

The total payoff from this option basket is:

$$\begin{aligned}\Pi_{LP}(T_{LP}, P(T_{LP})) &= \max\{V(T_{LP}), 0\} - \max\{V(T_{LP}) - mI, 0\}\\ &= V(T_{LP}) - \max\{V(T_{LP}) - mI, 0\}\\ &= \min\{V(T_{LP}), mI\}\end{aligned}$$

Full participation In the case of *full participation*, the VC does not only receive a multiple of his invested amount in preference to the holders of Common Shares, but also participates in exit values above this multiple on an as-converted basis. The participation feature has major effects on the incentive structures of both parties: while it deters the Entrepreneur from favouring mergers over public offerings, it prevents VCs from strategically vetoing a worthwhile merger proposal in hope for an uncertain public offering, as described in Smith (2005). The payoff function with full participation differs from the payoff function without participation for exit proceeds exceeding the Liquidation Preference (see Figure 3). At those levels, his payoff equals his pro rata ownership percentage (fully diluted) at exit, defined as follows:

$$\alpha(T_{LP}) = \frac{N_{vc1,c}(T_{LP})}{N_{tot,c}(T_{LP})}$$

Under our simplifying assumption, $\alpha(T_{LP}) = \alpha(t_0)$, hereafter abbreviated as α. The payoff function of Preferred Shares with full participation can be replicated using the following option basket:

- 1 long call with $E = 0$.
- $(1 - \alpha)$ short calls with $E = mI$. Hereby, the VC participates in exit proceeds above the Liquidation Preference on an as-converted basis (i.e. according to the fraction of total equity he would be holding if he converted his Preferred Shares into Common Shares).

[49] Under the assumption that the company issues no new shares in the exit transaction, we define $V(T_{LP}) = V_{post}(T_{LP}) = V_{pre}(T_{LP})$.

The total payoff function for full participation looks as follows:

$$\begin{aligned}\Pi_{LP}(T_{LP}, P(T_{LP})) &= \max\{V(T_{LP}), 0\} - (1-\alpha)\max\{V(T_{LP}) - mI, 0\} \\ &= \min\{V(T_{LP}), mI\} + \alpha \max\{V(T_{LP}) - mI, 0\}\end{aligned}$$

Capped participation In the presence of *capped participation*, the VC participates only up to a pre-specified cap, c, which is defined as a multiple of the invested amount (typically 3x to 5x). Figure 4 shows that capped participation generates the same payoff function as full participation for levels of total exit proceeds below $\frac{cI}{\alpha}$; For exit proceeds above this level, the VC's payoff stays cI. Consequently, the total payoff function for capped participation can be represented as a basket of the following options:

- 1 long call with $E = 0$
- $(1-\alpha)$ short call with $E = mI$
- α *short call with* $E = \frac{cI}{\alpha}$

The total basket yields the following payoff function:

$$\begin{aligned}\Pi_{LP}(T_{LP}, P(T_{LP})) &= \max\{V(T_{LP}), 0\} - (1-\alpha)\max\{V(T_{LP}) - mI, 0\} \\ &\quad -\alpha \max\{V(T_{LP}) - \tfrac{cI}{\alpha}, 0\} \\ &= \min\{V(T_{LP}), mI\} + \alpha \max\{V(T_{LP}) - mI, 0\} \\ &\quad -\alpha \max\{V(T_{LP}) - \tfrac{cI}{\alpha}, 0\}\end{aligned}$$

According to the Fenwick & West dataset, nearly 65% of all sample financings grant the investor participation in proceeds above the Liquidation Preference; in approx. 47% of the cases, this participation is capped. Liquidation multiples (i.e. preferences with $m > 1$) are found in 20% of the *senior* sample rounds.[50]

3.1.3 Optional conversion

The Optional Conversion Right enables holders of Convertible Preferred Stock (or Convertible Debt) to force the company to replace their Preferred Shares with Common Shares, at any time before expiration (i.e. the date of Redemption, a (Deemed) Liquidation Event, or a Qualified Public Offering), and at a preset conversion ratio.[51] Formally, Optional Conversion generates an American-type call option on Common Shares of the company.

[50] Fenwick & West has restricted the analysis of multiple sizes to senior rounds, since multiples are hard to justify and therefore infrequently used in pari passu rounds (i.e. only in cases where the company has so much existing senior Liquidation Preferences that a new investor cannot expect a sufficient return based on simple Liquidation Preference).

[51] A model provision for Optional Conversion is provided in the NVCA Certificate of Incorporation, C4.1 (pages 18-19) and the NVCA Term Sheet (page 5).

The payoff function can be described as a position of α *long call with* $E = 0$, which corresponds to the stake of Common Shares held after conversion. The number of Common Shares receivable after conversion is obtained as follows:

$$N_{vc1,c}(t_i) = N_{vc1,p}(t_i) * P(t_0)/C_{vc1}(t_i)$$

The variables $N_{vc1,p}(t_i)$ and $C_{vc1}(t_i)$ are affected by the terms of Pre-emption Rights, Anti-dilution Rights and Pay-to-play provisions as described in Section 3.2.

Although VCs are formally allowed to convert at any time before expiry, they will exercise this option only shortly before an anticipated exit transaction, when they can evaluate whether the exit proceeds receivable after conversion compensate for the loss of preferred rights. Thus, Optional Conversion is effectively exercised upon the occurrence of a Liquidation Event or a Sale of the Company.[52] Since this study focuses on the effective use of covenants (as opposed to their formal specification), we define the option embedded in Optional Conversion as follows[53]:

Proposition 3 (Optional Conversion) *The Optional Conversion Right generates a European-type call option on Common Shares of the company, which becomes exercisable upon the occurrence of the earliest of a LE or CS, that is at*

$$T_{OC} = T_{LP} = \begin{cases} \tau_{LE} & \textit{if } \tau_{IPO} > \tau_{LE} > 0, \textit{ or if } \tau_{LE} > 0 \wedge \tau_{IPO} = 0; \\ \tau_{CS} & \textit{if } \tau_{IPO} = \tau_{LE} = 0. \end{cases}$$

The payoff function generated by Optional Conversion is:

$$\Pi_{OC}(T_{OC}, P(T_{OC})) = N_{vc1,c}(T_{OC}) * P(T_{OC}) = \alpha(T_{OC}) * V(T_{OC})$$

The value of the option at $t \leq T_{OC}$ is:

$$F_{OC}(t, P_t) = e^{-r(T_{OC}-t)} E_t^* [\Pi_{OC}(T_{OC}, P(T_{OC})) - I_{vc1}(T_{OC}, P(T_{OC}))]$$

3.1.4 Interaction of optional conversion and liquidation rights

By definition, Convertible Preferred Stock generates *mutually exclusive* options, as Conversion into Common Shares brings about the loss of all Preferred rights. In the NVCA model documents, this mutual exclusiveness is described as follows[54]:

[52]Theoretically, Optional Conversion is also relevant in the event of a Public Offering which does *not* fulfill the conditions of a "Qualified Public Offering". However, these non-qualified offerings are excuded in our model, since an IPO always meets the QPO conditions.

[53]Note that "Optional Conversion" is abbreviated as "OC" in mathematical formulas.

[54]See NVCA Certificate of Incorporation, C4.3.3 (page 20).

> All shares of Series A Preferred Stock which shall have been surrendered for conversion shall no longer be deemed to be outstanding and all rights with respect to such shares shall immediately cease and terminate at the time of conversion, except the right of the holders to receive shares of Common Stock in exchange therefore and to receive payment of any dividends declared but unpaid thereon.

Hence, in anticipation of a Liquidation Event or a Company Sale, the VC has the choice between exercising his Optional Conversion Right or keeping his Preferred Shares, which translates into the following payoff alternatives:

- Convert his Preferred Shares into Common Shares and earn his pro rata stake of the exit proceeds; or
- Sell his Preferred Shares and receive the Liquidation Amount.

This tradeoff is illustrated in Figure 6 at the example of Non-Participating Preferred Stock. Optional Conversion yields payoffs below the Liquidation Amount (mI) for exit values below the Indifference Value (mI/α), while it yields payoffs above the Liquidation Amount for exit values above the Indifference Value. As derived earlier, the VC will make the conversion versus liquidation decision only shortly before the date of the Sale or Liquidation, when he has all information needed to compare the alternative payoffs. Thus, the Chooser option generated by Optional Conversion and Liquidation Rights is best described as a European-type option. The pricing algorithm can be derived based on Gamba (2002).

Let there be $H = 2$ mutually exclusive. These mutually exclusive options have payoffs $\Pi_1 = \min\{V(T_1), mI\}$ (from Liquidation), and $\Pi_2 = \alpha(T_2)V(T_2)$ (from Optional Conversion) and the same (event-triggered) maturity $T = T_{LP} = T_{OC}$. The values of the two mutually exclusive options are defined as $F_1(T, \Pi_T)$ and $F_2(T, \Pi_T)$.

Let $G(t, P_t)$ be the value of the opportunity to choose the best out of the options to convert or to obtain the liquidation amount. We define the control as (ξ), where ξ takes value in the set $\{1, 2\}$. The value of the opportunity to select the best option is

$$G(t, P_t) = \max_{\xi}\{e^{-r(T-t)}E_t^*[F_\xi(T, P_T)]\}$$

The optimal exercise decisions and the corresponding payoffs of the Chooser Option embedded in Liquidation (without Participation) and Optional Conversion Rights are provided in Table 3 presents.

In a multiple round, multiple investors setting, the tradeoff becomes more complex. The option holder still has the right to convert his Preferred Shares into Common, but his payoff from conversion will depend on whether or not the remaining investors decide to convert as well. If all Preferred Holders as a group decide to convert, VC_1 receives the pro rata share of the full exit proceeds. If the group of investors decides not to convert, VC_1 receives the pro rata share of the proceeds remaining after distribution of liquidation preferences to other investors. Hence, the decision to convert for VC_1 will depend on the conversion decision taken by the group of Preferred Holders.

3.2 Provisions Influencing the Number of Shares

A major consideration of VCs when investing in a company is to anticipate how new rounds of financing required to fund the growth of the company until the exit transaction will affect the value of their shareholdings.[55] Therefore, at the time of investment, VCs have to assess how the issuance of additional shares in future financing rounds will dilute their ownership in the portfolio company over time, and to seek contractual protection against this dilutive effect. Dilution may take the form of *percentage dilution* (i.e. a decrease in the percentage of the entity an investor owns), or *economic dilution* (i.e. a decrease in the economic value of his investment in the entity). While economic dilution has a *direct* impact on the value of an investor's holdings, percentage dilution may have an important *indirect* value impact by altering non-economic features such as veto rights and other control rights. As shown in the following, Pre-emption Rights are designed to protect the VC against percentage dilution, while Anti-dilution Rights protect him against economic dilution. In the presence of Pay-to-play provisions, Pre-emption and/or Anti-dilution Rights become contingent on the VC's participation in future rounds. Hence, protection from dilution comes at the cost of participation in future rounds.

3.2.1 Pre-emption rights

The Pre-emption Right (also called Right of First Offer) represents a so-called "informal staging" mechanism, as it allows the VC to expand his investment at any new financing round, whereby the terms are renegotiated at every round based on the future performance of the portfolio company and the bargaining power of the contracting parties.[56] More precisely, the VC may participate in future share issues up to his percentage interest in the company immediately before the round (or more if there are several rounds of allocation and other holders of this right do not fully exercise it), and is thereby fully protected against percentage dilution of his holdings.[57]

[55] See Wilmerding (2006).

[56] Due to their status of "insider" investors, existing investors usually have a substantial information advantage compared to external investors, which enables them to influence the terms of external rounds; they also have substantial bargaining power towards management, which allows them to lead internal rounds at attractive valuations if they can provide the necessary funding.

[57] A model description of the Pre-emption Right is provided in the NVCA Investors' Rights Agreement, 4.1(a),(b) and 4.3 (pages 24-27) and NVCA Term Sheet (page 11).

Assuming a single round of allocation, the VC can purchase any portion ρ_1 of newly offered securities, which is inferior or equal to his pro rata share at the date of the new issue (on a fully-diluted basis). In this case, the number of Preferred shares held after exercise of his Pre-emption right, for all $t_i < \tau_{exit}$, is defined as:

$$N_{vc1,p}(t_i) = N_{vc1,p}(t_{i-1}) + \rho_1(N_{pref,p}(t_i) - N_{pref,p}(t_{i-1}))$$

with $\rho_1 \in [0, \alpha(t_{i-1})]$ and $\alpha(t_{i-1}) = (N_{vc1,c}(t_{i-1}))/(N_{tot,c}(t_{i-1}))$.

In the case of multiple rounds of allocation, the VC's participation may exceed his pro rata share, up to a maximum portion ρ_2, which equals the total ownership percentage of existing investors (on a fully-diluted basis) immediately before the new issue (and reflecting the case where none of the remaining Preferred Holders would exercise his Pre-emption Right). In this scenario, the number of Preferred shares held by the VC after exercise of his Pre-emption Right equals:

$$N_{vc1,p}(t_i) = N_{vc1,p}(t_{i-1}) + \rho_2(N_{tot,p}(t_i) - N_{tot,p}(t_{i-1}))$$

with $\rho_2 \in [0, \beta_{pref}(t_{i-1})]$ and $\beta_{pref}(t_{i-1}) = (N_{pref,c}(t_{i-1}))/(N_{tot,c}(t_{i-1}))$.

3.2.2 Anti-dilution rights

Anti-dilution protection allows an investor to limit the economic dilution of his investment in a company without being required to commit more capital over time.[58] According to the Fenwick & West dataset, Anti-dilution protection is found in nearly 99% of the sample financings in Silicon Valley. Generally, anti-dilution clauses protect Preferred Holders against dilution resulting from the following corporate events:

- cheap issuances of additional common stock or deemed additional common stock (i.e. common stock purchase rights, warrants, or securities convertible into common stock);
- structural changes in equity securities, including stock dividends, stock splits and reverse stock splits, as well as other distributions[59]

Our analysis will focus on protection mechanisms against dilution from cheap issuances of additional equity securities, since structural changes are not related to the performance of the company, and their inclusion adds no value to the pricing model.

[58]See Woronoff & Rosen (2005).

[59]Including those related to mergers and consolidations; extraordinary distributions of cash and property; sales of all or substantially all of the company's assets, followed by a distribution of the sale proceeds in the form of cash or property; recapitalizations; and common stock buybacks.

Cheap issuances are issues of additional equity securities at a price per share inferior to the applicable conversion price in effect immediately prior to such issue. Hence, anti-dilution protection becomes applicable in "down rounds" and must be assessed separately for each financing round, as each round may reflect different initial conversion prices. Typically, there are two types of (conversion price) anti-dilution formulas: Full Ratchet and Weighted Average.

Full Ratchet Anti-dilution protection reduces the Preferred Holder's conversion price to the share price applicable at the new round[60]:

$$C_{vc1}(t_i) = P(t_i)$$

with
$C_{vc1}(t_i)$: Conversion Price applicable to the Preferred Shares held by VC_1 after full-ratchet anti-dilution adjustment at t_i; and
$P(t_i)$: Purchase Price paid in the new round at t_i.
This mechanism fully protect the investor against economic dilution from Initial Investment, as the securities receivable upon conversion after the adjustment will have the same aggregate value as the initial investment. At the same time, however, it fully shifts the costs of any decline in value to common shareholders. For this reason, this type of anti-dilution protection is used more rarely than weighted average protection, which is described in the following.

Weighted Average Anti-dilution protection reduces the Preferred Holder's conversion price to the weighted average price per share of securities issued both prior to and in the dilutive issuance, in accordance with the following formula:[61]

$$\begin{aligned} C_{vc1}(t_i) &= C_{vc1}(t_{i-1}) \times \frac{N_{tot,c}(t_{i-1}) + N^*_{new,c}(t_i)}{N_{tot,c}(t_{i-1}) + N_{new,c}(t_i)} \\ &= \frac{C_{vc1}(t_{i-1}) \times N_{tot,c}(t_{i-1}) + I_{new,p}(t_i)}{(N_{tot,c}(t_{i-1}) + N_{new,c}(t_i))} \end{aligned}$$

Hereby, the parameters are defined as follows:
$C_{vc1}(t_i)$: conversion price in effect after adjustment for dilution;
$C_{vc1}(t_{i-1})$: conversion price in effect immediately prior to the dilutive issue (after all prior adjustments);
$N_{tot,c}(t_{i-1})$: number of shares of Common Stock outstanding immediately prior to the dilutive issue of Common Stock;
$N_{new,c}(t_i)$: number of additional shares of Common Stock issued (or deemed issued) in such transaction;
$N^*_{new,c}(t_i)$: number of shares of Common Stock that would have been issued (or deemed

[60]See NVCA, Certificate of Incorporation, C4.4.4 (page 26), NVCA Term Sheet (page 7).
[61]See NVCA Certificate of Incorporation, C4.4.4 (pages 25-26) and NVCA Term Sheet (pages 6-7).

issued) if such shares had been issued at a price per share equal to $C_{vc1}(t_{i-1})$ (determined by dividing the aggregate consideration $I_{new,p}(t_i) = N_{new,c}(t_i) * P(t_i)$ received by the company in respect of such issue by $C_{vc1}(t_{i-1})$).

The relative price contributions to arrive at the new conversion price can be weighed in several ways, depending on which shares to consider as outstanding (and hence on which shares to include in $N_{tot,c}(t_{i-1})$)[62]:

- The "Broad-based" or "California" interpretation accounts for all common stock outstanding on a fully-diluted basis (i.e. assuming the exercise or conversion of all warrants, options, and convertible securities outstanding immediately prior to the dilutive issue), which means that the Preferred is seen to own less of the company prior to the dilutive issuance.
- The "Narrow-based" or "East Coast" interpretation excludes certain shares from the calculation of shares outstanding, for example all convertible securities that are out-of-the-money, or simply all shares of Common Stock issuable on conversion of options, warrants (and, potentially, even the Preferred Stock itself), whether they are in-the-money or not.

As the shares outstanding prior to the dilutive issuance are valued higher in the formula, the more shares considered outstanding (i.e. the broader the base) prior to the dilutive issuance, the better off the unprotected shareholders (the Common Holders) and the worse off the holders of protected convertible securities (the Preferred Holders). Thus, the narrow-based interpretation can be understood as the more investor-favorable approach, while the broad-based approach is more company-favorable.[63]

With the Weighted Average approach, losses from the value decrease are distributed among all holders of securities, as opposed to the holders of Common Shares exclusively. This may not be the accurate way, but the best the parties can hope for, as the true cause of the drop in value cannot be clearly determined. For this reason, Weighted Average is the most commonly used type of Anti-dilution protection in the U.S.[64] This is confirmed by the Fenwick & West dataset, as 94% of Anti-dilution provisions use the Weighted Average formula. Moreover, Full Ratchet provisions provided for in the initial contract often flip into Weighted Average provisions at later investment rounds.[65] The predominance of Weighted Average also applies to European contracting practices.[66]

[62] See NVCA Yearbook 2009.
[63] See Wilmerding (2005).
[64] See NVCA Yearbook 2009.
[65] See Bagley and Dauchy (2008).
[66] As confirmed by Hassan Sohbi (Taylor Wessing).

Independently of the chosen protection mechanism, the cost of anti-dilution protection granted to VCs is partially born by the founders and the management. Thus, in cases where the continued involvement of these groups is crucial, VCs only insist on exercising their anti-dilution rights up to a certain point, to prevent the management ownership percentage from dropping below a minimum level of $10 - 15\%$. Beyond this point, they will either expand the employee stock option plan or stop applying their Anti-dilution Rights, in order to prevent demotivation of the Common Holders.[67]

Anti-dilution provisions do not generate option value in isolation. However, they may alter embedded option values, since reducing the conversion price of Preferred Shares increases the number of Common Shares receivable by the VC upon conversion and consequently his final payoff. Technically, Anti-dilution Rights add strong path-dependency to the option pricing problem. The number of shares receivable upon conversion becomes dependent on a property of the path followed by the underlying asset, namely on whether the price per share paid at the new financing round is inferior to the conversion price in effect immediately prior to this round. This can be captured in our model by adding a new independent variable in the form of $C_{vc1}(t-1)$, which is the conversion price in effect immediately before the new issue. The value of embedded options must then be described as $F(t, P(t), C_{vc1}(t-1))$.

In practice, a company issues multiple series of Preferred Shares to multiple investors over time. The conversion price of shares belonging to a newly issued series is initially set equal to the purchase price of these shares, and then adapted over time as imposed by the Anti-dilution protection mechanism in place. Therefore, the conversion price variable must be tracked separately for each series of Preferred Shares.

3.2.3 Pay-to-play penalties and interaction effects

Generally, Pay-to-play provisions (also called "Play-or-lose") are clauses which penalize the VC for not fully participating in future "Qualified Financings" (unless a minimum percentage of Preferred Holders elects otherwise), by forcing him to convert all his Preferred Shares (or the applicable portion of his Preferred Shares)[68] into Common Shares or to forego certain preferred rights. Hereby, Qualified Financings are defined as Share Issues which result in a minimum amount of gross proceeds and the reduction of the conversion price applicable to the relevant series of Preferred Shares. The penalties may take different degrees, ranging from forfeiture of Dilution Protection, Registra-

[67] As commented by Hassan Sohbi (Taylor Wessing); this is also emphasized in Wilmerding (2005).

[68] This portion equals the number of shares obtained by multiplying the aggregate number of shares of Preferred Stock held by such holder immediately prior to the Qualified Financing by a fraction, the numerator of which is equal to the amount (if positive) by which such holder's Pro Rata Amount exceeds the number of Offered Securities actually purchased by such holder in such Qualified Financing, and the denominator of which is equal to such holder's Pro Rata Amount.

tion Rights or Pre-emption Rights, to Mandatory Conversion of Preferred Shares into Shadow Preferred Shares or Common Shares and loss of Board Rights.[69]

Pay-to-play provisions are found in only 12% of the sample financings in the Fenwick & West dataset. However, this figure does not cover increasingly used "pull up" provisions, which have a similar economic effect than pay-to-play provisions, but which are defined in separate contractual agreements and not in the charter.[70] In Europe, the contractual specification of Pay-to-play penalties is even less frequent, but both the (ex-ante) incentive effect and the (ex-post) penalty effect are often replicated informally, due to the bargaining power of incoming investors. On the one hand, incoming investors often require existing investors to participate in the new round as a key precondition to invest (and often even to look in depth at the investment opportunity) and/or as a major influence factor of contract terms (e.g. the lack of participation of old investors may cause a steep valuation discount or a loss of rights such as anti-dilution, veto rights, etc.). On the other hand, incoming investors will usually ask that their shares bear more senior rights than the shares of existing investors. This sometimes forces old investors to reduce their protection levels by foregoing certain rights, such as Liquidation Preference, Anti-dilution protection (usually for the consecutive financing round) and/or veto rights (indirectly, via shift in their percentage ownership).[71]

The most onerous version of Pay-to-play penalties in the U.S. is mandatory conversion into Common Stock, which represents 78% of the Pay-to-Play provisions found in the Fenwick & West sample rounds. In essence, this penalty ends any preferential rights tied to the converted shares, including the right to participate pro rata in future financings and the right to influence management decisions.[72]

The alternative version of the Pay-to-play penalty forces non-participating investors to convert their shares into so-called "shadow" Preferred, which differs from the original Preferred Stock in the following aspects:[73]

- the conversion price is fixed at the conversion price in effect immediately prior to the financing and will not be subject to any further anti-dilution adjustment (applying in the case of cheap issuances of additional Common Stock);
- the new series will not include an analogous provision on Special Mandatory Conversion; and
- the terms of this series may vary from the terms of the Series A Preferred Stock to the extent deemed necessary by the Board of Directors to accomplish the intent of this provision.

[69]See NVCA Term Sheet (page 8).
[70]This comment is provided in the Fenwick & West Quarterly Reports 2009.
[71]As commented by Hassan Sohbi (Taylor Wessing).
[72]See NVCA Certificate of Incorporation B 5A.1 (page 32).
[73]See NVCA Certificate of Incorporation B 5A.1 (pages 34-35).

Hence, the VC loses some, but not all preferred rights. For example, he may lose his anti-dilution protection and pre-emption right, but keep the remaining rights including protective provisions and board voting rights.

According to the NVCA, conversion into Common Stock is preferable to conversion into shadow Preferred because (a) it represents a harsher penalty (and is hence more effective at forcing the VC to participate in future rounds); (b) it facilitates future charter amendments, since the latter do not require the approval of common shareholders, whereas they may require the approval of the holders of a majority of shadow Preferred Stock (e.g. for Delaware law); and (c) it avoids the complexities associated with the creation of the shadow series of Preferred Stock.

When planning an investment, VCs broadly estimate how much funding the company will need to raise in the future and at which valuations, based on its cash-flow requirements and likely exit scenarios. Hence, they anticipate the potential dilution effect from future rounds and generally reserve a certain amount of funding (i.e. on average three or four times the first investment) to exercise their Pre-emption Rights.[74] Moreover, VCs are acutely aware of signalling effects: if existing investors do not fully participate in a future round, this deters potential new investors from entering the deal. Finally, in the presence of Pay-to-play penalties, VCs will not incur the risk of losing their Pre-emption Rights, their Anti-dilution Protection, or even all preferred rights. This is confirmed by the Fenwick & West dataset, since less than 5% of the sample rounds brought about reorganizations in the form of conversion of Preferred Shares into junior stock (whether Shadow Preferred or Common Stock). Based on these arguments, we make the following assumption:

Assumption 4 (Full Exercise of Pre-emption Rights) *The VC fully exercises his Pre-emption Right at any future financing round and hence maintains his pro rata ownership percentage as well as his preferred rights over time.*

Assuming full exercise of Pre-emption Rights by all holders of this right (and hence one single round of distribution), the number of shares held by VC_1 after financing round i equals, for all $t_i < \tau_{exit}$:

$$N_{vc1,p}(t_i) = N_{vc1,p}(t_{i-1}) + \alpha(t_{i-1})(N_{tot,p}(t_i) - N_{tot,p}(t_{i-1}))$$

with $\alpha(t_{i-1}) = N_{vc1,c}(t_{i-1})/(N_{tot,c}(t_{i-1}))$.

The total amount invested by the VC in round i is:

$$I_{vc1}(t_i) = I_{vc1}(t_{i-1}) + \alpha(t_{i-1})(N_{tot,p}(t_i) - N_{tot,p}(t_{i-1}))P(t_i)$$

[74]See NVCA Yearbook 2009; this was also confirmed by Hassan Sohbi (Taylor Wessing).

3.3 Provisions Introducing Exercise Flexibilities

This section covers contractual rights which allow the VC to directly or indirectly control the type and timing of exit. Control over exit is crucial for the VC business model, since different exit scenarios impact the size of exit proceeds, their allocation among the parties and final returns on investment.[75]

3.3.1 Shareholder and board voting rights

This section covers Board Voting Rights and Shareholder Voting Rights, which allow the VC to initiate and decide upon major corporate actions and are collectively defined as "Control Rights". Since Control Rights are prescribed by corporation law, we rely on the revised Model Business Corporation Act (MBCA), which provides the basis for corporation law in the majority of U.S. states.[76]

The Board of Directors is responsible for hiring, evaluating and firing top management, for advising and ratifying general corporate strategies and decisions in the ordinary course of its business, for filling vacancies on the board and, most importantly in our context, for initiating certain corporate actions including financing and exit events.[77] The composition of the Board of Directors varies from company to company. We present a general model, which is aligned with the NVCA model documents and results of empirical studies[78]:

- a specified number of board seats (n_p) are allocated to the holders of each series (or multiple series voting together) of Preferred Stock; and
- a specified number of board seats (n_c) are allocated to the holders of Common Stock; and
- any remaining board seats (n_r) are filled by
  - Directors elected by the holders of Preferred Stock and the holders of Common Stock *voting together as a single class*; *or*
  - independent Directors, which are *mutually agreed upon* by the Common and Preferred shareholders.

[75] See Hellmann (19989).

[76] The MBCA represents a *model* state incorporation statute that was prepared by the American Bar Association's Committee on Corporate Laws for adoption by state legislatures, with the purpose of improving the rationality of U.S. corporation law. It was completely revised in 1984.

[77] See Smith (2005).

[78] See Kaplan and Stroemberg (2003), Smith (2005). The NVCA model documents contains two conflicting provisions on the composition of the board: the Certificate of Incorporation prescribes that the remaining Directors be voted by Common and Preferred Holders together as a single class, while the Voting Agreement foresees that these seats be filled by independent directors. As a company would generally not want to have two conflicting provisions within the same contractual framework, we assume that this inconsistency between the documents is unintended, and present the described scenarios as two *alternative* scenarios.

Hence, total board size of a portfolio company can be described as $n_{tot} = n_p + n_c + n_r$. This structure translates into one of the following control scenarios:

- *Investor Control* where $n_p > n_c + n_r$;
- *Entrepreneur Control* where $n_c > n_p + n_r$;
- *Contingent Control* where $n_p = n_c$ and the tie-breaking vote(s) is (are) held by n_{r_v} directors that are elected by Common and Preferred Holders voting as a single class. Over time, this vote can tip the balance of power to one side or the other. At any specific point in time, however, only one party controls the board vote and Contingent Control is effectively equivalent to:
  - *Investor Control*, if $\beta_{pref}(t_i) > \beta_{com}(t_i)$; or
  - *Entrepreneur Control*, if $\beta_{com}(t_i) > \beta_{pref}(t_i)$

  where β_{pref} is the (fully-diluted) ownership percentage of Preferred Stock holders, and β_{com} the (fully-diluted) ownership percentage of Common Stock holders.
- *Joint Control* where $n_p = n_c$ and the tie-breaking vote(s) is (are) held by n_{r_i} independent Director(s), who act(s) in the interest of the company. Here, Board Control can shift towards the VC or the Entrepreneur at any vote, depending on who's interest is more closely aligned with the company interest for the item subject to vote.

Over time, board composition tends to move from Entrepreneur Control to Investor Control, as VCs usually gain additional board seats with each round of investment (either by bargaining or by acquiring a majority ownership stake). Our option pricing model focuses on the case of Contingent Control. This board structure reflects common practices in the U.S. (see Smith (2005)) and accommodates the automatic transfer of control over time, in alignment with changes in the shareholder structure. Under this assumption, knowledge about the shareholder structure provides full knowledge about the scenario of board control.

Stockholder voting rights Stockholder voting rights are the rights of holders of preferred and common stock to vote on major corporate actions within the scope of (ordinary or extraordinary) Shareholder Meetings.[79] Sharesholder decisions typically require simple majority of votes, or sometimes a supermajority (if so prescribed by the statute, articles of association, bylaws, or shareholder voting agreements).

[79] See NVCA Yearbook 2009.

Matters subject to vote in Sharesholder Meetings include, among others:

- Election and removal of directors (vote by classes of shares)
- Fundamental corporate changes (in the structure or business of the corporation), including:
  - Amendments of the Articles of Incorporation (e.g. increase in the number of authorized shares or reduction of dividend rights of Preferred Holders);[80]
  - Merger (excluding short-form mergers, provided the parent corporation holds 90% of the subsidiary's shares)
  - Consolidation
  - Share exchange
  - Sale of all or substantially all of the assets
  - (Voluntary) Dissolution and Liquidation
- Adoption and amendment of bylaws.

For simplification, we assume that the above matters are subject to simple majority vote. The party possessing simple voting majority can then decide on all matters subject to sharesholder vote including the exit strategy and timing. Accordingly, at any time step, $t_i \in [t_0, t_{max}]$, the shareholder structure defines one of the two following scenarios of Voting Control:

- VC Voting Control: the group of all Preferred Holders has more than 50% of the voting rights, that is, $\beta_{pref} > 50\%$;
- Founder Voting Control: the group of all Common Holders has more than 50% of the voting rights (e.g. in early stage investments), that is, if $\beta_{com} \geq 50\%$.

To assess the impact of Control Rights on embedded option prices, we make the simplifying assumption that Preferred Holders act in common interest with regard to exit decisions, i.e. they make these decisions as a group.[81]

As Voting Control also infers Board Control, the group of Preferred Holders gains full control over the exit event when it obtains majority ownership of the company (i.e. if $\beta_{pref} > 50\%$). The time step at which this switch of control occurs is defined as: $\tau_{contr} = \inf\{t_i : \beta_{pref} > 50\%\}$. This affects embedded options as follows:

[80] Since IPOs inevitably require an amendment of the company's Articles of Incorporation, the right to prevent such amendment provides effective control over the timing of the IPO. In Europe, legal frameworks typically prescribe that share issues, share transfers and IPOs be also listed as separate voting items. This was commented by Hassan Sohbi (Taylor Wessing).

[81] In practice, the interests of different Preferred Holders are not fully aligned, since they hold different Series of Preferred Stock with different rights, and usually represent multiple VC funds (with distinct life-times, IRR expectations, stage focus and industry focus). This conflict of interest among Preferred Holders could be accounted for in a game-theoretical analysis, which exceeds the scope of this paper but represents an interesting path for future research.

Proposition 4 (Direct Control Rights) *In the interval* $[\tau_{contr}, \tau_{max}]$*, the trigger events of embedded options are not path-dependent, but reflect the optimal exercise policies of the group of Preferred Holders with regard to the type and timing of exit.*

Board and Voting Control allows the group of Preferred Holders to choose among IPO or Sale of the Company, and to prepone or postpone the transaction to the optimal date (in the time interval providing this exercise flexibility). Formally, their choice can be described as follows:

- *Sale of the Company*: in this exit scenario, they may either convert their Preferred Shares into Common or not and sell them to the acquirer (or partner in case of a merger);
- *IPO*: if this option is available (i.e. if the pre-offer valuation of the company is equal or superior to $\$200m$, their shares are automatically converted into Common and sold to the public after expiration of the lockup period;
- *Wait or defer the exit*: if the payoff from continuation (i.e. from exiting at a later point in time) is superior to the proceeds from exiting at the current time step, the Preferred Holders will defer the exit until the optimal date.

If, at any time step in the given interval, the share value falls below the Liquidation threshold (see Section 2.3.3), the company is assumed to go bankrupt and the above options are not available. Depending on the optimal exit decision made by the group of Preferred Holders, the individual investor (i.e. VC_1 in our model) has the following options:

- *In the event of Sale of the Company*: if the Preferred Holders opt for conversion, the individual investor's shares will also convert and he will earn his pro rata share of total exit proceeds; if the Preferred Holders vote against conversion, the individual investor may choose between receiving his Liquidation Amount or converting into Common, whereby he would receive his pro rata share of the proceeds remaining after payout of the Liquidation Preferences of his co-investors;
- *In the event of an IPO*: the VC's shares automatically convert into common and are sold to the public after the expiration of the lockup period.

Let the value of the VC's claim at t be $F(t, X_t)$, with $F(T, X_T) = \Pi(T, X_T)$. The contingent claim can be exercised in the interval $[\tau_{contr}, \tau_{max}]$. In this case, the value of the claim at any stopping time $t \leq \tau_{contr}$ equals:

$$F(t, P_t) = \max_{s \in S(\tau_{contr}, \tau_{max})} \{e^{-r(s-t)} E_t^*[\Pi(s, P(s)) - I_{vc1}(s, P(s))]\}$$

where $S(\tau_{contr}, \tau_{max})$ is the set of exercise dates in $[\tau_{contr}, \tau_{max}]$
The optimal exercise decisions from the perspective of the group of Preferred Holders

and the corresponding payoffs are presented in Table 4. To compare the payoffs from alternative exit scenarios at any time step t_k, we discount the payoff from IPO (that would be obtained after expiration of the lock-up period) back to t_k. Hence,

$$\Phi_{MC}(t_k) = e^{-r(T_{MC}-t_k)}\Phi_{MC}(T_{MC})$$

with $T_{MC} = \inf\{t_i : t_i \geq \tau_{IPO} + 0,5\}$; $\Phi_{MC}(T_{MC})$ is the payoff that would be obtained by the group of Preferred Holders at T_{MC}.

3.3.2 Redemption, demand registration and drag-along rights

Redemption rights Redemption rights formally allow the VC to ask that their Preferred shares be repurchased by the Company at a pre-defined price (equal or superior to the original price per share paid at investment) if no exit event has taken place within a certain time period.[82] This time period is usually defined as a minimum of 5 years after the investment date, and aligned with the anticipated exit horizon.[83] The Redemption Right is the only term, besides Liquidation Preference, which defines a contracted payment on which the company could default. This is especially relevant in situations in which the management objectives differ from the exit strategy and timing originally agreed upon with the investors (e.g. if the company would have the opportunity to realize a liquidity event but the founder opposes the transaction to stay in his position rather than supporting the transaction), and in so-called "sideways situations", where the company generates sufficient revenues and cash flow to maintain operations, but was not able to realize attractive exit scenarios in the anticipated time horizon.

In practice, Redemption Rights raise a number of difficulties for the company and for investors. The company may find it harder to obtain new financing when redemption obligations are categorized as company debt. Unless the company is performing particularly well, it does generally not have enough cash available to buy the investors out, or it may not be legally permitted to redeem shares (by reason of restrictions under Delaware law and other state corporate law).[84] To limit these exercise risks, VCs sometimes require the company to create a *sinking fund* to ensure that sufficient capital would be available for the redemption, and/or they demand that certain *penalty provisions* apply if the company cannot pay the full Redemption Amount (e.g. payment of the redemption amount in the form of a one-year note to each unredeemed Preferred Holder, or entitlement of the Preferred Holders to elect a majority of the

[82] Model provisions are presented in the NVCA Certificate of Incorporation C6.1 (pages 37-38) and the NVCA Term Sheet (page 8).
[83] See NVCA Yearbook 2009.
[84] This explanation was provided by Michael Patrick (Fenwick & West).

Company's Board of Directors until the redemption amount is paid back in full).

In the Fenwick & West dataset, Redemption Rights are found in 27% of the sample financings. Due to the difficulties described above, VCs rarely exercise them. However, the mere threat of exercise provides VCs with additional leverage to force a liquidity event as soon as possible after the initially anticipated exit horizon. European investors rarely ask for a Redemption Right in the contract, and consider its exercise a measure of last resort. Instead, they rely on "Mandatory Exit Rights", which represent truly *initiatory* exit rights.[85] As Redemption Rights are rarely exercised, and, if exercised, do not guarantee payment of the full Redemption Amount, we do not model them as a separate embedded option. However, they generate value by indirectly allowing the Preferred Holders to initiate an exit after a certain time period (i.e. at the initial date of the redemption right), and influence the exercise restrictions of options embedded in other rights.

Demand registration Demand Registration Rights formally enable investors to require the company to register their shares for sale in a public offering upon the earliest of (a) a minimum number of years after the closing (3-5 years according to the NVCA model provision), or (b) the expiration of the lock-up period after the IPO. Such registration must meet certain minimum conditions, defined as minimum anticipated net proceeds (at least $\$1 to \$5$ million for Form S-3 Demand and $\$5 to \$15$ million for Form S-1 Demand) or a minimum percentage of outstanding Registrable Securities ($20\% - 100\%$ for Form S-1 Demand).[86] The VC typically asks for the right to request at least two registrations, while the company tries to limit this number as far as possible (since the costs are mostly born by the company and not the investor).

Theoretically, if the VC has the right to make a demand, he can indirectly force the company to initiate a separate public offering. De facto, the VC cannot impose an IPO upon the other parties, as the cooperation of the founders and the management is an essential element of any IPO process.[87] Moreover, the provision is not formally designed to provide investors with a right of initiation, since the effective date is usually postponed with each new investment round up to a point in time after the expected date of the IPO. For those reasons, Demand Registration should be interpreted as a protective exit right, which is rarely exercised in practice, but which provides investors with leverage in influencing the nature and timing of company-initiated registrations (via the impending deadline and not the impending threat of exit) or other exit events.[88]

[85] As commented by Hassan Sohbi (Taylor Wessing).

[86] The model provision is presented in the NVCA Investors' Rights Agreement 2.1 (pages 6-8) as well as the NVCA Term Sheet (pages 9-10).

[87] See Brobeck (2003).

[88] See Smith (2005).

Hence, as argued in the case of redemption rights, we will not model Demand Registration as a separate option. Instead, we account for the fact that they allow the Preferred Holders to initiate an exit event, if such event has not taken place by the date at which the Registration Right becomes effective.

Drag-along rights Drag-along rights (also called "Bring-along rights") constitute the contractual right of an investor to force all other shareholders to agree to a specific action, most commonly the Sale of the Company, and to participate in this transaction on the same terms as himself. This right is systematically granted to majority shareholders, and regularly also to VCs with minority shareholdings.[89] Formally, it enables the VC to initiate a Sale of the Company to a strategic buyer in a scenario where the exit through a public listing is difficult and where minority shareholders could block the transaction. Moreover, it allows him to offer the full company for sale (as opposed to his share only), which is usually a precondition to attract strategic buyers (who are unlikely to be interested in a minority stake), and to obtain a "premium" (i.e. an appreciation in the offer price resulting from a higher level of control over the company). In the U.S., Drag-along Rights are typically provided for in later rounds, where there is growing concern that the parties may have diverging economic interests in the exit transaction.[90]

In practice, it is hard to force an exit without full cooperation of the management, because the buyer often requires continuous involvement of management as a prerequisite of the transaction, and because the transaction can only be finalized once all parties agree upon fair company value, which is not possible in the presence of diverging subjective views. Thus, the initiation of a Sale of the Company by Preferred Holders is not as straightforward as implied by the legal terms, and drag-along rights are rarely exercised (in the sense that the holder goes to the Courts to enforce the exit transaction). However, VCs rely on the credible threat of exercising their Drag-along Right to indirectly force management to initiate exit via Sale of the Company or via IPO.[91]

Again, we do not model this right as a separate option, but assume that it reinforces the initiatory power of Preferred Holders with regard to an exit event, and measure its impact on the exercise restrictions of other embedded options. This is formally described in the next section.

[89]Model provisions are listed in the NVCA Voting Agreement 3.1,3.2 (pages 5-6) and in the NVCA Term Sheet (page 14).

[90]Drag-along rights are not covered by the Fenwick & West dataset.

[91]As clarified by Hassan Sohbi (Taylor Wessing).

Synthesis For illustrative purposes, we assume that Demand Registration, Drag-along Rights, as well as Redemption Rights become effective on the same date, t_{ant}, which reflects the anticipated exit date agreed upon by all parties at the time of contracting (with $t_{ant} < t_{max}$). Hence, the first time step with exercise flexibility induced by these rights is: $\tau_{ant} = \inf\{t_i : t_i \geq t_{ant}\}$. Similarly than for Control Rights, we assume that optimal exercise decisions represent consensus decisions of the group of Preferred Holders. The impact of these covenants on embedded option prices can be assessed as follows:

Proposition 5 (Exit Rights) *At any time step $t_k \in [\tau_{ant}, \tau_{max}]$, the trigger events of embedded options are not path-dependent, but reflect the optimal exercise policies of the group of Preferred Holders with regard to the type and timing of exit, provided that no exit event has taken place before or at this time step t_k.*

As opposed to Control Rights, Exit Rights only allow Preferred Holder to initiate an exit event, but not to defer the exit event. Hence, we impose an additional restriction upon the set of possible exercise dates, namely $t_k < \tau_{exit}$. The optimal exercise decisions and the corresponding payoffs of the Chooser Option obtained at each relevant time step and for each sample path are presented in Table 5.

4 Application of the Pricing Model

We apply the pricing model developed in the previous sections to compute option values in realistic scenarios. These scenarios account for the occurrence, over time, of multiple financing rounds with multiple investors and multiple series of Preferred Stock. The contract values are assessed from the perspective of a Series A investor at the time of his initial investment. This Series A investor is assumed (a) to be the sole investor at the initial financing round, and (b) to participate alongside new investors in all future rounds. The exit dates are defined as $t_{ant} = 6$ years (anticipated exit date) and $t_{max} = 8$ years (latest possible exit date). The price terms of the Series A financing are specified as follows:

- Pre-money Valuation: \$16m (fully diluted);
- Series A investment: \$4m (all paid out at the initial date);
- Price per Share: \$10.

It follows that the post-money valuation equals \$20m, the total number of shares (on a fully diluted basis) after the Series A financing equals 2m, and the ownership percentage of the Series A investor at t_0 is 20%. The timing of future rounds and the respective price terms are determined by our jump process.

For the definition of non-price terms, we differentiate between *basic* terms (and specifications), which are included in VC contracts by default because they represent the minimum level of protection acceptable to investors, and *negotiable* terms (and specifications), which are subject to negotiation between the parties. Whereas we use both types of terms to build our pricing scenarios, we perform an assessment of economic value only for the second category.

The basic terms are defined as follows:

- **Simple Liquidation Preference**
  The VC is granted a simple liquidation preference, without participation and without multiple.
- **Optional Conversion**
  The Optional Conversion right is exercised only in the context of an exit event, that is of a LE or CS (see Section 3.1.3); the decision to convert is taken individually (and either aligned with the collective decision of the group of Preferred Holders or not).
- **Mandatory Conversion upon QPO**
  We assume that this clause is triggered exclusively by the occurrence of a Qualified Public Offering (and not by a majority decision of Preferred Holders); since the IPO threshold level in our model exceeds the minimum conditions for a QPO (as explained in Section 3.1.1), any jump in valuation above the threshold level triggers mandatory conversion of Preferred Shares.
- **Piggyback Registration**
  The investor has the right to piggyback on any registrations initiated by the Company or the other investors (see Section 3.1.1).
- **Pre-emption Right**
  As derived in Section 3.2.3, we assume that the investor fully exercises his Preemption Rights at each new financing round to avoid percentage dilution of his investment.

For the definition of negotiable terms, we focus on the most commonly used provisions, which are derived from the Fenwick & West dataset (see Table 6). Typically, the Series A Investor is granted a liquidation preference with participation (either capped or full). In the majority of cases, he is also provided with Anti-dilution protection. Demand Registration is not covered in the dataset, but also represents a standard term in U.S. contracts.[92] On the contrary, Redemption Rights and Dividend Rights are found only in a minority of cases and will not be covered in the model scenarios. Considering the limited differences in terms across series, we assume that the rights granted to

[92]This was confirmed by Michael Patrick (Fenwick & West).

investors in follow-on rounds (i.e. financings of series B or higher) are the same as those granted to the Series A investor, with one exception: the Liquidation Preference of shares issued from the Series D financing onward is assumed to be senior to that attached to shares issued in series A, B or C financings (see above). Option values will therefore be assessed for the following terms and specifications:

- Liquidation Preference
  We compare the following term specifications: No Participation, Capped Participation (with the cap set at 3x the initial investment) and Full Participation; the preference is a simple preference, i.e. it equals the amount invested.

- Anti-dilution
  We compare the following term specifications: absence of Anti-dilution protection, weighted average Anti-dilution and full ratchet Anti-dilution; in the weighed average formula, the number of shares outstanding is defined according to the broad-based interpretation (i.e. accounting for all shares outstanding on a fully diluted basis).

- Demand Registration
  We distinguish between the absence of Demand Registration and the presence of Demand Registration (with initial date set equal to t_{ant}).

The basic terms and the negotiable terms collectively constitute the "base scenarios" used for the pricing of contract terms in this section.[93] We define three base scenarios:

- Company-favorable: contains only basic terms.
- Middle-of-the-Road: contains basic terms as well as a Liquidation Preference with full participation, weighted average Anti-dilution and Demand Registration.
- Investor-favorable: contains basic terms as well as a Liquidation Preference with full participation, full ratchet Anti-dilution and Demand Registration.

To obtain the value of an individual term (or alternative specifications of this term) for a particular base scenario, we add this term to the base scenario (respectively deduct it from the base scenario, or modify its specification), while keeping all remaining terms constant. The value of the term (or term specification) under analysis can then be assessed both directly, that is as the difference between the full contract value *with* versus *without* the term (or term specification), and indirectly, that is from the difference between the investor's expected share of proceeds *with* versus *without* the term (or specification).

We perform a Monte Carlo simulation over 5,000 sample paths for each pricing scenario.

[93]Practitioners could derive the base scenario directly from the Term Sheet, which sets out the initial terms and represents the starting point of negotiations.

To reduce the variance of simulation results, the streams of random numbers have been synchronized across all pricing scenarios.[94] The simulation results for the individual base scenarios are presented in Tables 7 to 9. A comparison of the results across all scenarios is provided in Table 10.

In the company-favorable base scenario (see Table 7), the investment contract generates an aggregate value of $4,854,488 for the Series A investor. This value is substantially increased in the presence of participating liquidation preference and anti-dilution protection. More specifically, the addition of capped participation increases full contract value by 12.79%, while full participation (i.e. without cap) increases it by 25.51%. The presence of Anti-dilution protection increases the value of the full contract by 5.24% (for weighted average), respectively by 48.00% (for full ratchet), as compared to the base scenario without any protection mechanism.

In the middle-of-the-road base scenario, the full contract generates a value of $6,326,496, which represents an increase of over 30% as compared to the company-favorable base scenario. Accepting a cap on participation would reduce this value by 9.76% (as compared to the base scenario with full participation), while giving up the participatory feature entirely would reduce it by 19.75% (as compared to the base scenario with full participation). In terms of Anti-dilution protection, moving from the weighted average formula to the full ratchet formula would increase contract value by 31.10%, whereas giving up the protection would reduce contract value by 4.18% (as compared to the base scenario with weighed average protection).

Finally, in the investor-favorable base scenario (see Table 9), the investment contract generates a total value of $8,293,867; this reflects an increase of over 70% as compared to the contract value in the company-favorable base scenario. Moving from full participation to capped participation would decrease contract value by 6.61%, while giving up the participatory feature would imply a loss of 13.80% in contract value. Accepting an Anti-dilution protection based on the weighted average formula would reduce contract value by 23.72%, while giving up the Anti-dilution protection entirely would reduce it by 26.91% (as compared to the base scenario of full ratchet protection).

In all three scenarios, the presence of Demand Registration has a minor and negative impact on contract value to the Series A investor, which is not surprising in the light of the model assumptions. On the one hand, the ownership share of Preferred Holders equals 20% after the Series A financing, and further increases by over 17% at any follow-on round. This implies that the combined ownership percentage of the group of Preferred Holders is expected to exceed 50% at the Series D financing, while

[94] See Hammersley and Handscomb (1964) or Kahn and Marshall (1953).

Demand Registration only becomes exercisable at the Series E financing.[95] Hence, when Demand Registration becomes exercisable, the group of Preferred Holders already controls the type and timing of exit (via their control rights). On the other hand, the exercise decision for Demand Registration is not taken by the Series A investor individually, but by the group of Preferred Holders. Accordingly, the optimal exercise policy of the group may diverge from the optimal exercise policy of the Series A investor, which implies a loss in contract value.

The values obtained for Anti-dilution rights substantiate the widespread criticism of the full ratchet protection. As explained in Section 3.2.2, this type of protection is rarely used by practitioners because it is highly penalizing to Common Holders, who have to bear alone the full cost of any reduction in company value. Effectively, the full ratchet protection generates disproportionately high value for the Series A investor, since it increases full contract value by 37% to 48% (for the company-favorable and the investor-favorable scenario, respectively).

Most importantly, the results show that there is considerable value in the structuring of VC contracts, since total contract values can be increased by up to 70% by amending exclusively the negotiable terms. Moreoever, they reiterate the relevancy of *effective valuation*, a concept of (implied) company valuation which accounts for the distribution of proceeds at exit (and not only for the investor's ownership percentage at the date of investment, which represents current practice).[96] In our base scenarios, the Series A investor acquires an initial ownership percentage of 20% (fully diluted) and fully participates in follow-on rounds, while his expected share of exit proceeds lies between 31.29% (in the company-favorable scenario) and 35.07% (in the investor-favorable scenario), in spite of economic dilution caused by the entrance of new investors in follow-on rounds. This divergence between (a) ownership percentage and (b) expected share of exit proceeds can be accounted for in the price terms of the contract. Based on an initial investment of $4m and the ownership of 20%, the *nominal* post-money valuation of the company is $20m (obtained by dividing the initial investment by the ownership percentage), whereas the *effective* valuation (obtained by dividing the initial investment by the expected share of proceeds) ranges between $16,394,370 (in the company-friendly base scenario) and $13,978,258 (in the investor-friendly scenario).

[95]Since the initial date of Demand Registration is set equal to 4 years after the initial investment and the waiting time between two consecutive pricing events is on average 18 months, the right becomes exercisable on average at the fourth jump date, that is the date of the Series E financing.

[96]This concept is introduced in Woronoff & Rosen.

5 Conclusion and Directions for Further Research

We develop an option pricing model to assess the economic value of VC contract terms. The model covers the majority of terms included in U.S. model documents, notably voting rights on the shareholder and board levels and "initiatory" exit rights (i.e. demand registration rights, drag-along rights and redemption rights). In realistic scenarios with multiple financing rounds and multiple investors, we compute total contract values and the relative value impact of individual terms on contract values and on the distribution of exit proceeds, from the perspective of a Series A investor.

Since it relies on Least Squares Monte Carlo simulation, our option pricing approach is flexible enough to cope with the specificities of VC contracts and the resulting exotic features of embedded options. The underlying asset of embedded options is modelled as a jump process to reflect the discontinuous adjustment of (market-based) VC portfolio company values at discrete events (i.e. at follow-on VC financing rounds and at the exit event). The model also copes with uncertain timing, path-dependencies and interaction effects that arise because VC contract terms are contingent on the occurrence of these events and may be held by several investors at a time. Furthermore, it accommodates American exercise rights that are generated when VCs gain control over the timing of the exit transaction. Finally, it accounts for the issuance of multiple series of preferred stock over time that may bear different contractual rights. The model parameters are derived from model legal documents and industry statistics, and do not require subjective estimation.

The simulation results show that VC contracts generate considerable economic value, and that this value is strongly influenced by negotiable provisions. By obtaining more investor-favorable terms at the outset of his investment, the Series A investor can increase his total contract value by up to 70% (i.e. the difference between our investor-favorable and company-favorable base scenarios). Hence, there is considerable value in the structuring of VC contracts. Moreover, since VC contract provisions generate non-linear payoff functions, the VC's expected share of proceeds at exit exceeds by far his ownership percentage at investment (i.e. by over 15% in our investor-favorable scenario). This deviation reiterates the importance of using the "effective value" concept for the calculation of the price terms in VC contracts; according to this concept, implied valuations should be based on the distribution of proceeds at exit, and not only on the VC's percentage ownership at investment. Effective company valuations lie between 18% (for the company-favorable base scenario) and 30% (for the investor-favorable base scenario) below the nominal valuations. Lastly, we find that the values of the full contract and of individual terms are strongly influenced by the chosen "base scenario" (i.e. on the combination of terms in the contract which represent the starting point of negotiations). This confirms the importance of interaction effects between the terms and reiterates the necessity of treating VC contracts as option baskets, as highlighted by Cossin et al. (2002).

The pricing framework developed in this article could be extended along the following dimensions. First, the framework could be adapted to legal environments and contracting practices in other countries or regions, based on historical data and model documents that apply to the specific geography. Secondly, it would be valuable to further explore *shared* option ownership in VC contracts by combining our discrete-time pricing approach with complex games. Notably, this would facilitate the pricing of tag-along rights and rights of first refusal, which remain uncovered in the academic literature. Thirdly, the model could be expanded to account for the impact of round direction on the size of price jumps and on the types of rights granted to investors in follow-on rounds. The statistics derived from the Fenwick & West dataset indicate that this impact is important, especially with regard to the seniority of shares issued in later rounds.

More generally, an option-based approach to VC contract design can be used to derive new optimality arguments, which are based on an assessment of economic value and which account for interaction effects among the terms. Empirical analyses of VC contracting practices across countries could focus on economic contract values and hence abstract from formal differences in contract design across countries. This would deepen the understanding of existing clauses and their interactions and help to further improve contract designs. The simulation model can also be used by practitioners to better evaluate alternative investment structures and optimize the outcome of contract negotiations.

A Chi-squared Goodness-of-Fit Tests

A.1 A Chi-squared Goodness-of-Fit Test Applied to the Distribution of Upward Jump Amplitudes

To estimate the probability distribution function providing the best approximation to the observed distribution of upward jump magnitudes, we test $H_0 : Y_{x^u} \sim Wei(\gamma, 1)$ using a chi-squared goodness-of-fit test. First, divide the sample space into 5 cells, namely $A_1, \ldots, A_5$. Let $p_j = P[Y \in A_j]$ and let o_j denote the number of observations that fall into the j-th cell. Under H_0 the expected number in the j-th cell equals $1204p_j$. H_0 is rejected at the 10% significance level if

$$X^2 = \sum_{j=1}^{5} \frac{o_j - e_j{}^2}{e_j} > \aleph^2_{0.90}(2) = 4.61.$$

Table 1 shows the observed and expected frequencies for the chi-squared test of a Weibull distribution with $k = 1$ and $\gamma = 0.8582$. As $X^2 = 3.97$, H_0 cannot be rejected at the $\alpha = 0.10$ level of significance and it thus appears that the exponential distribution (or Weibull distribution with $k = 1$) fits well the magnitude of upward jumps.

A.2 A Chi-squared Goodness-of-Fit Test Applied to the Distribution of Downward Jump Amplitudes

To estimate the probability distribution function providing the best approximation to the observed distribution of downward jump amplitudes, we test the $H_0 : Y_{x^d} \sim U(n)$ using a chi-squared goodness-of-fit test. $U(n)$ is defined as a discrete uniform distribution with n equalling the number of cells dividing the sample space ($n = 5$). Table 2 shows the observed and expected frequencies for the chi-squared test of a uniform distribution. Since $X^2 = 5.36 < \aleph^2_{0.90}(4) = 7.78$, H_0 cannot be rejected at the $\alpha = 0.10$ level of significance and it appears that the uniform distribution well fits the magnitude of downward jumps. If u is a value sampled from the standard uniform distribution, then the value $a + (b - a)u$ follows the uniform distribution parametrised by a (minimum value) and b (maximum value). Since the magnitude of downward jumps must be in the interval [0, 1], the minimum and maximum values of the uniform distribution are defined as $a = 0$ and $b = 1$.

B Tables

Table 1: Frequencies of the Magnitude of Upward Jumps

Magnitude	$\leq 20\%$	$\leq 160\%$	$\leq 180\%$	$\leq 200\%$	$> 200\%$
Observed Frequencies (o_j)	251	789	31	25	108
Observed Probabilities (p_j)	0.21	0.66	0.03	0.02	0.09
Expected Frequencies (e_j)	250	767	39	31	117

Table 2: Frequencies of the Magnitude of Downward Jumps

Magnitude	$\leq 20\%$	$\leq 40\%$	$\leq 60\%$	$\leq 80\%$	$> 80\%$
Observed Frequencies (o_j)	76	73	96	74	72
Observed Probabilities (p_j)	0.19	0.19	0.25	0.19	0.18
Expected Frequencies (e_j)	78	78	78	78	78

Table 3: Chooser Option Held by VC_1 from Liquidation and Conversion Rights

Provision	Payoff	Exercise Conditions	Exercise Restrictions
Conversion	$\Pi_{OC}(t_k)$	$\Pi_{OC}(t_k) \geq \Pi_{LP}(t_k)$	for $t_k = T_{LP} = T_{OC}$
Liquidation	$\Pi_{LP}(t_k)$	$\Pi_{LP}(t_k) \geq \Pi_{OC}(t_k)$	for $t_k = T_{LP} = T_{OC}$

Table 4: Chooser Option Held by the Group of VCs in the Presence of Control Rights

Exit Event	Action	Payoff	Exercise Conditions	Exercise Restrictions
IPO	conversion (automatic)	$\Phi_{MC}(t_k)$	$\Phi_{MC}(t_k) \geq \Phi_{LP}(t_k)$ and $\Phi_{MC}(t_k) \geq \Phi_{OC}(t_k)$ and $\Phi_{MC}(t_k) \geq W(t_k)$	for $t_k \in [\tau_{contr}, \tau_{max}]$ and $t_k < \tau_{LE}$ and $V_{pre}(t_k) \geq h_{IPO}$ and $T_{MC} \leq t_{max}$
Sale	convert (optional)	$\Phi_{OC}(t_k)$	$\Phi_{OC}(t_k) \geq \Phi_{LP}(t_k)$ and $\Phi_{OC}(t_k) \geq \Pi_{MC}(t_k)$ and $\Phi_{OC}(t_k) \geq W(t_k)$	for $t_k \in [\tau_{contr}, \tau_{max}]$ and $t_k < \tau_{LE}$
Sale	do not convert	$\Phi_{LP}(t_k)$	$\Phi_{LP}(t_k) \geq \Phi_{OC}(t_k)$ and $\Phi_{LP}(t_k) \geq \Phi_{MC}(t_k)$ and $\Phi_{LP}(t_k) \geq W(t_k)$	for $t_k \in [\tau_{contr}, \tau_{max}]$ and $t_k < \tau_{LE}$
No exit	continue	0	otherwise	$t_k \in [\tau_{contr}, \tau_{max}]$ and $t_k < \tau_{LE}$

Note: $W(t_i)$ is the continuation value at exercise date (t_k) and $\Phi(t_k)$ is the payoff function for the group of Preferred Holders.

Table 5: Chooser Option Held by the Group of VCs in the Presence of Exit Rights

Exit Event	Action	Payoff	Exercise Conditions	Exercise Restrictions
IPO	conversion (automatic)	$\Phi_{MC}(t_k)$	$\Phi_{MC}(t_k) \geq \Phi_{LP}(t_k)$ and $\Phi_{MC}(t_k) \geq \Phi_{OC}(t_k)$ and $\Phi_{MC}(t_k) \geq W(t_k)$	for $t_k \in [\tau_{ant}, \tau_{max}]$ and $t_k < \tau_{exit}$ and $V_{pre}(t_k) \geq h_{IPO}$ and $T_{MC} \leq t_{max}$
Sale	conversion (optional)	$\Phi_{OC}(t_k)$	$\Phi_{OC}(t_k) \geq \Phi_{LP}(t_k)$ and $\Phi_{OC}(t_k) \geq \Phi_{MC}(t_k)$ and $\Phi_{OC}(t_k) \geq W(t_k)$	for $t_k \in [\tau_{ant}, \tau_{max}]$ and $t_k < \tau_{exit}$
Sale	no conv.	$\Phi_{LP}(t_k)$	$\Phi_{LP}(t_k) \geq \Phi_{OC}(t_k)$ and $\Phi_{LP}(t_k) \geq \Phi_{MC}(t_k)$ and $\Phi_{LP}(t_k) \geq W(t_k)$	for $t_k \in [\tau_{ant}, \tau_{max}]$ and $t_k < \tau_{exit}$
No exit	continue	0	otherwise	$t_k \in [\tau_{ant}, \tau_{max}]$ and $t_k < \tau_{exit}$

Note: $W(t_i)$ is the continuation value at exercise date (t_k) and $\Phi(t_k)$ is the payoff function for the group of Preferred Holders.

Table 6: Frequency of Terms and Specifications by Series

	Series A	Series B				Series C				Series D				Series E+				Total Sample			
Provision / Specification	Total	Down	Even	Up	Total	Down	Even	Up	Total	Down	Even	Up	Total	Down	Even	Up	Total	Down	Even	Up	Total
Liquidation Preference																					
Seniority																					
Pari Passu	n/a	46%	44%	74%	67%	35%	35%	65%	56%	20%	25%	53%	40%	25%	39%	49%	39%	32%	36%	65%	54%
Senior	n/a	54%	56%	26%	33%	65%	65%	35%	44%	80%	75%	47%	60%	75%	61%	51%	61%	68%	64%	35%	46%
Multiple (if senior)																					
m <= 1x	n/a	74%	73%	93%	86%	78%	75%	87%	82%	72%	50%	86%	74%	70%	58%	90%	76%	73%	63%	89%	80%
1x < m <= 2x	n/a	21%	24%	7%	12%	15%	22%	13%	15%	26%	34%	13%	21%	21%	19%	9%	16%	21%	25%	10%	16%
m > 2x	n/a	4%	3%	1%	2%	7%	3%	0%	3%	3%	16%	1%	4%	10%	23%	1%	9%	6%	11%	1%	4%
Participation																					
Yes	59%	79%	73%	58%	62%	72%	76%	60%	64%	78%	65%	65%	69%	75%	73%	61%	68%	76%	71%	60%	65%
No	41%	21%	27%	42%	38%	28%	24%	40%	36%	22%	35%	35%	31%	25%	27%	39%	32%	24%	29%	40%	35%
Cap (if participating)																					
No	60%	49%	71%	50%	52%	44%	68%	50%	51%	60%	70%	49%	55%	64%	59%	52%	58%	54%	67%	50%	53%
Yes	40%	51%	29%	50%	48%	56%	32%	50%	49%	40%	30%	51%	45%	36%	41%	48%	42%	46%	33%	50%	47%
Anti-dilution																					
Protection																					
Yes	96%	98%	100%	99%	99%	96%	98%	99%	98%	100%	96%	98%	99%	98%	100%	97%	98%	98%	99%	99%	99%
No	4%	2%	0%	1%	1%	4%	2%	1%	2%	0%	4%	2%	1%	2%	0%	3%	2%	2%	1%	1%	1%
Formula																					
Weighted Average	98%	96%	88%	97%	96%	94%	88%	97%	96%	85%	82%	96%	91%	87%	86%	96%	91%	91%	86%	97%	94%
Full Ratchet	2%	4%	12%	3%	4%	6%	13%	3%	4%	15%	18%	4%	9%	13%	14%	4%	9%	9%	14%	3%	6%
Redemption																					
No	77%	66%	56%	78%	74%	66%	71%	74%	72%	69%	69%	73%	71%	74%	59%	76%	72%	69%	63%	76%	73%
Yes	23%	34%	44%	22%	26%	34%	29%	26%	28%	31%	31%	27%	29%	26%	41%	24%	28%	31%	37%	24%	27%
Dividends																					
No	96%	95%	90%	96%	95%	95%	98%	95%	95%	89%	90%	93%	92%	92%	82%	96%	92%	93%	90%	95%	94%
Yes	4%	5%	10%	4%	5%	5%	2%	5%	5%	11%	10%	7%	8%	8%	18%	4%	8%	7%	10%	5%	6%

Note: The frequencies are mean values over the time period 2004-2008, derived from the Fenwick & West dataset.

Table 7: Simulation Results for the Company-favorable Base Scenario

	Liquidation Preference			Anti-dilution Protection			Demand Registration	
	No Part.	Capped Part.	Full Part.	No Prot.	WA	FR	No	Yes
Base Scenario	X			X			X	
Contract Value	4,854,488	5,475,138	6,093,057	4,854,488	5,109,063	7,184,470	4,854,488	4,822,820
% change from base case	-	+12.79%	+25.51%	-	+5.24%	+48.00%	-	-0.65%
Term Value	0	620,650	1,238,569	0	254,575	2,329,982	0	-31,669
in % of contract value	-	11.34%	20.33%	-	4.98%	32.43%	-	-0.66%
Share of Proceeds	31.29%	33.40%	33.60%	31.29%	31.51%	32.61%	31.29%	31.27%
% change from base case	-	+6.74%	+7.39%	-	+0.71%	+4.22%	-	-0.05%
Eff. Company Value	16,394,370	15,102,773	14,899,460	16,394,370	16,164,087	15,264,605	16,394,370	16,401,166
% change from nom. value	-18.03%	-24.49%	-25.50%	-18.03%	-19.18%	-23.68%	-18.03%	-17.99%

Note: This table shows the simulation results for the company-favorable base scenario; in this scenario, the VCs hold a Liquidation Preference without participation and are granted neither Anti-dilution protection nor Demand Registration Rights. "Contract Value" reflects the option value of the full contract from the perspective of the Series A investor. "Term Value" reflects the option value of an individual term; it is provided as an absolute value (i.e. the difference between contract value *with* and *without* the term under analysis) and as a relative value (i.e. as a percentage of full contract value). The "Share of Proceeds" is defined as the Series A investor's expected share of total exit proceeds (with total proceeds equalling the fully-diluted company valuation at exit). "Eff. Company Valuation" is the *effective* post-money valuation of the portfolio company at t_0. The "% change from nominal value" is the deviation of effective valuation from nominal valuation, which equals $20m by assumption.

Table 8: Simulation Results for the Middle-of-the-Road Base Scenario

	Liquidation Preference			Anti-dilution Protection			Demand Registration	
	No Part.	Capped Part.	Full Part.	No Prot.	WA	FR	No	Yes
Base Scenario			X		X			X
Contract Value	5,077,093	5,709,135	6,326,496	6,062,143	6,326,496	8,293,867	6,357,689	6,326,496
% change from base case	-19.75%	-9.76%	-	-4.18%	-	+31.10%	+0.49%	-
Term Value	0	632,042	1,249,403	0	264,353	2,231,723	0	-31,193
in % of contract value	-	11.07%	19.75%	-	4.18%	26.91%	-	-0.49%
Share of Proceeds	31.50%	33.61%	33.82%	33.58%	33.82%	35.07%	33.84%	33.82%
% change from base case	-6.87%	-0.62%	-	-0.71%	-	+3.69%	+0.06%	-
Eff. Company Value	16,169,562	14,926,412	14,723,207	14,909,227	14,723,207	13,978,258	14,714,858	14,723,207
% change from nom. value	-19.15%	-25.37%	-26.38%	-25.45%	-26.38%	-30.11%	-26.43%	-26.38%

Note: This table presents the results for the middle-of-the-road base scenario, in which VCs hold a Liquidation Preference with full participation, weighted average Anti-dilution protection and Demand Registration Rights. "Contract Value" reflects the option value of the full contract from the perspective of the Series A investor. "Term Value" reflects the option value of an individual term; it is provided as an absolute value (i.e. the difference between contract value *with* and *without* the term under analysis) and as a relative value (i.e. as a percentage of full contract value). The "Share of Proceeds" is defined as the Series A investor's expected share of total exit proceeds (with total proceeds equalling the fully-diluted company valuation at exit). "Eff. Company Valuation" is the *effective* post-money valuation of the portfolio company at t_0. The "% change from nominal value" is the deviation of effective valuation from nominal valuation, which equals \$20m by assumption.

Table 9: Simulation Results for the Investor-favorable Base Scenario

	Liquidation Preference			Anti-dilution Protection			Demand Registration	
	No Part.	Capped Part.	Full Part.	No Prot.	WA	FR	No	Yes
Base Scenario			X			X		X
Contract Value	7,149,284	7,745,253	8,293,867	6,062,143	6,326,496	8,293,867	8,328,995	8,293,867
% change from base case	-13.80%	-6.61%	-	-26.91%	-23.72%	-	+0.42%	-
Term Value	0	595,969	1,144,583	0	264,353	2,231,723	0	-35,128
in % of contract value	-	7.69%	13.80%	-	4.18%	26.91%	-	-0.42%
Share of Proceeds	32.59%	34.82%	35.07%	33.58%	33.82%	35.07%	35.10%	35.07%
% change from base case	-7.07%	-0.71%	-	-4.24%	-3.56%	-	+0.09%	-
Eff. Company Value	15,269,902	14,183,097	13,978,258	14,909,227	14,723,207	13,978,258	13,972,006	13,978,258
% change from nom. value	-23.65%	-29.08%	-30.11%	-25.45%	-26.38%	-30.11%	-30.14%	-30.11%

Note: This table presents the results for the investor-favorable base scenario, in which VCs are granted a Liquidation Preference with full participation, full ratchet Anti-dilution protection and Demand Registration Rights. "Contract Value" reflects the option value of the full contract from the perspective of the Series A investor. "Term Value" reflects the option value of an individual term; it is provided as an absolute value (i.e. the difference between contract value *with* and *without* the term under analysis) and as a relative value (i.e. as a percentage of full contract value). The "Share of Proceeds" is defined as the Series A investor's expected share of total exit proceeds (with total proceeds equalling the fully-diluted company valuation at exit). "Eff. Company Valuation" is the *effective* post-money valuation of the portfolio company at t_0. The "% change from nominal value" is the deviation of effective valuation from nominal valuation, which equals \$20m by assumption.

Table 10: Comparison of Results from Alternative Contracting Scenarios

	Liquidation Preference			Anti-dilution Protection			Demand Registration	
	No Part.	Capped Part.	Full Part.	No Prot.	WA	FR	No	Yes
Company-favorable Base Scenario								
Terms in Base Scenario	X			X			X	
Contract Value	4,854,488	5,475,138	6,093,057	4,854,488	5,109,063	7,184,470	4,854,488	4,822,820
Term Value	0	620,650	1,238,569	0	254,575	2,329,982	0	-31,669
% change in contract value (with vs. without term)	-	+12.79%	+25.51%	-	+5.24%	+48.00%	-	-0.65%
Effective Company Value	16,394,370	15,102,773	14,899,460	16,394,370	16,164,087	15,264,605	16,394,370	16,401,166
% change (compared to nominal company value)	-18.03%	-24.49%	-25.50%	-18.03%	-19.18%	-23.68%	-18.03%	-17.99%
Middle-of-the-road Base Scenario								
Terms in Base Scenario			X		X			X
Contract Value	5,077,093	5,709,135	6,326,496	6,062,143	6,326,496	8,293,867	6,357,689	6,326,496
% change (compared to company-favorable base)	+4.59%	+17.61%	+30.32%	+24.88%	+30.32%	+70.85%	+30.97%	+30.32%
Term Value	0	632,042	1,249,403	0	264,353	2,231,723	0	-31,193
% change in contract value (with vs. without term)	-	+12.45%	+24.61%	-	+4.36%	+36.81%	-	-0.49%
Effective Company Value	16,169,562	14,926,412	14,723,207	14,909,227	14,723,207	13,978,258	14,714,858	14,723,207
% change (compared to nominal company value)	-19.15%	-25.37%	-26.38%	-25.45%	-26.38%	-30.11%	-26.43%	-26.38%
Investor-favorable Base Scenario								
Terms in Base Scenario			X			X		X
Contract Value	7,149,284	7,745,253	8,293,867	6,062,143	6,326,496	8,293,867	8,328,995	8,293,867
% change (compared to company-favorable base)	+47.27%	+59.55%	+70.85%	+24.88%	+30.32%	+70.85%	+71.57%	+70.85%
Term Value	0	595,969	1,144,583	0	264,353	2,231,723	0	-35,128
% change in contract value (with vs. without term)	-	+8.34%	+16.01%	-	+4.36%	+36.81%	-	-0.42%
Effective Company Value	15,269,902	14,183,097	13,978,258	14,909,227	14,723,207	13,978,258	13,972,006	13,978,258
% change (compared to nominal company value)	-23.65%	-29.08%	-30.11%	-25.45%	-26.38%	-30.11%	-30.14%	-30.11%

Note: This table summarizes the simulation results obtained from the three base scenarios. To facilitate the comparison of results across scenarios, the ratio "% change from base" is replace by "% change in contract value (with vs. without term)". This ratio reflects the percentage deviation of total contract value *with* the term under analysis from total contract value *without* the term.

C Figures

Figure 1: Observed magnitude of downward price changes

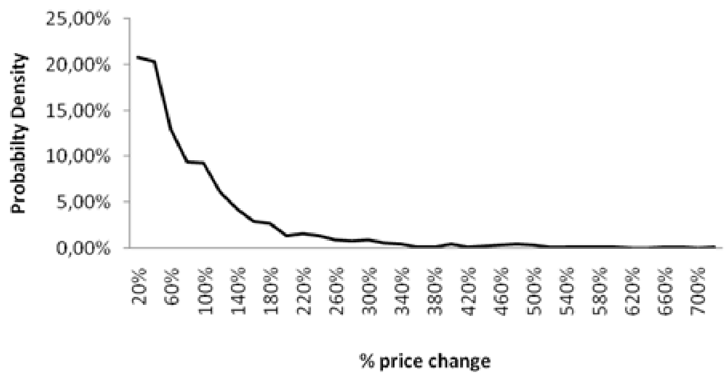

Figure 2: Observed magnitude of upward price changes

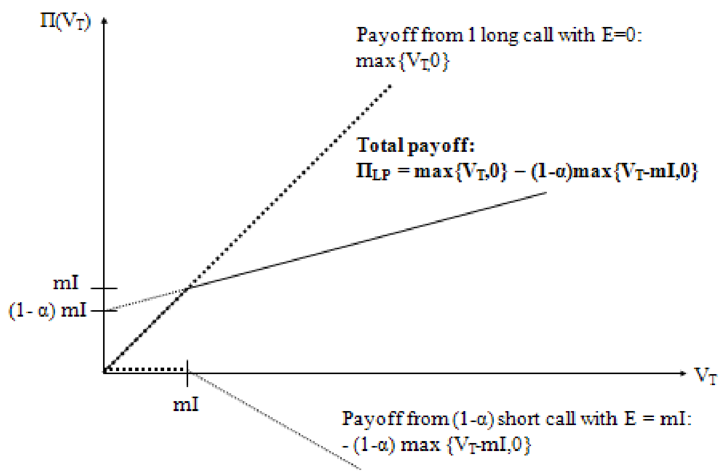

Figure 3: Payoff diagram for full participation

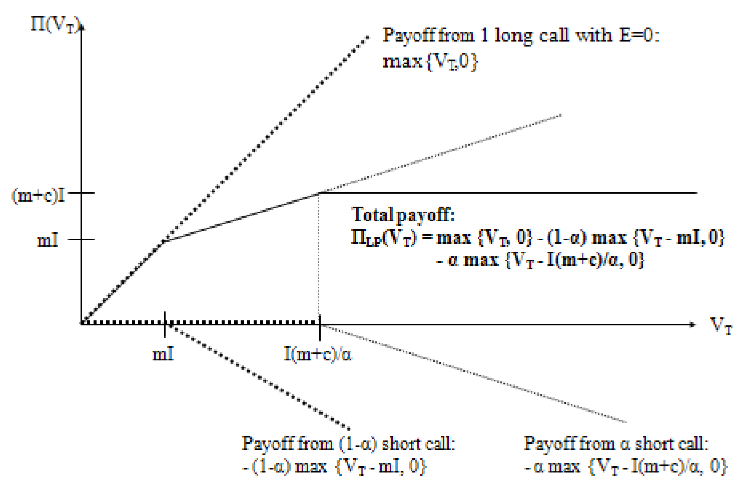

Figure 4: Payoff diagram for capped participation

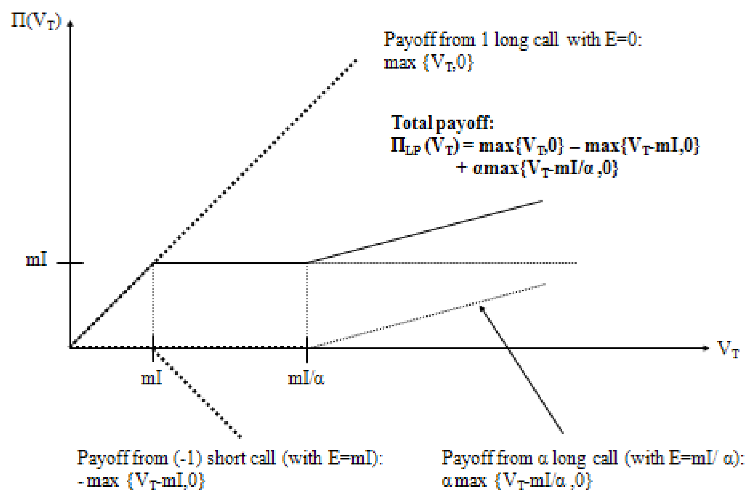

Figure 5: Payoff diagram for participation with catch-up

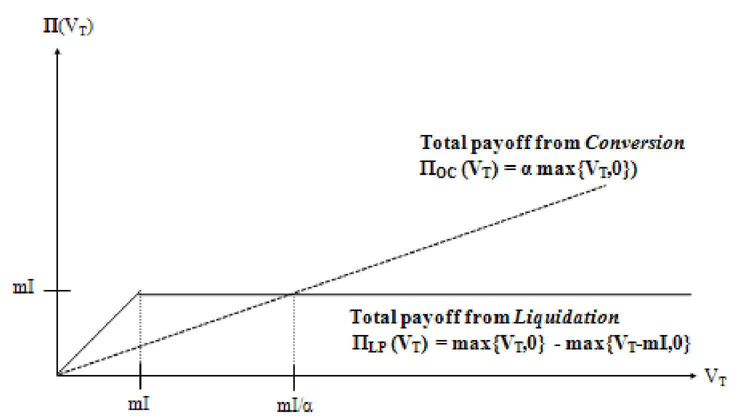

Figure 6: Payoff diagram for liquidation (without participation) versus optional conversion

Références

Admati, A., & Pfleiderer, P. (1994). Robust financial contracting and the role of the venture capitalist. *Journal of Finance*, *49*(2), 371-402.

Anderson, R., & Sundaresan, S. (1996). Design and valuation of debt contracts. *Review of Financial Studies*, *9*, 37-68.

Bagley, C., & Dauchy, C. (2008). *The entrepreneur's guide to business law* (2nd éd.). Thomson West.

Baldwin, C. Y., & Meyer, R. F. (1979). Liquidity preference under uncertainty: a model of dynamic investment in illiquid opportunities. *Journal of Financial Economics*, *7*(2), 347-374.

Bascha, A., & Walz, U. (2001). Convertible securities and optimal exit decisions in venture capital finance. *Journal of Corporate Finance*, *7*, 285-306.

Benaroch, M., & Kauffman, R. J. (1999). A case for using real options pricing analysis to evaluate information technology project investments. *Information Systems Research*, *10*(1), 70-86.

Bergemann, D., & Hege, U. (1998). Venture capital financing, moral hazard, and learning. *Journal of Banking and Finance*, *22*, 703-735.

Bergemann, D., & Hege, U. (2000). *The financing of innovation: Learning and stopping.* (Working Paper, Yale University and ESSEC Business School and CEPR)

Berk, J. B., Naik, V., & Green, R. C. (1998). *Valuation and return dynamics of new ventures.* (NBER Working Paper 6745)

Black, B., & Gilson, R. (1998). Venture capital and the structure of capital markets: banks versus stock markets. *Journal of Financial Economics*, *47*, 243-277.

Black, F., & Cox, J. (1976). Valuing corporate securities - some effects of bond indenture provisions. *Journal of Finance*, *31*(2), 351-367.

Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy*, *81*, 637-659.

Boyle, P. (1977). Options: A monte carlo approach. *Journal of Financial Economics*, *4*, 323-338.

Camp, J. J. (2002). *Venture capital due diligence: a guide to making smart investment choices and increasing your portfolio returns.* John Wiley and Sons.

Chemla, G., Habib, M. A., & Ljungqvist, A. (2004, July). *An analysis of share-*

holder agreements. (UBC, CNRS, and CEPR, University of Zurich and CEPR, and NYU Stern School and CEPR)

Cohen, A. C. (1965, November). Maximum likelihood estimation in the weibull distribution based on complete and on censored samples. *Technometrics*, *7*(4), 579-588.

Cossin, D., Leleux, B., & Saliasi, E. (2002, February). *Understanding the economic value of legal covenants in investment contracts: A real-options approach to venture equity contracts.* (Unpublished Working Paper, University of Lausanne, HEC)

Cox, J., Ingersoll, J., & Ross, S. (1985). A theory of the term structure of interest rates. *Econometrica*, *53*, 385-467.

Dupoyet, B. V. (2004, October). *Asymmetric jump processes: Option pricing implications.* (Working Paper, Florida International University)

Ewens, M. (2009, March). *A new model of venture capital risk and return.* (Working Paper, UCLA)

Gamba, A. (2003, December). *Real options: a monte carlo approach.* (Working Paper, University of Verona)

Gompers, P. A. (1995). Optimal investment, monitoring, and the staging of venture capital. *Journal of Finance*, *50*(5), 1461-1489.

Gompers, P. A. (1996). Grandstanding in the venture capital industry. *Journal of Financial Economics*, *42*, 133-56.

Gompers, P. A., & Lerner, J. (1996). The use of covenants: An empirical analysis of venture partnership agreements. *Journal of Law and Economics*, *39*, 566-599.

Grew, C., & Gent, D. (2003, Winter). *Full ratchet: The trans-atlantic venture capital review* (Rapport technique). Brobeck Hale and Dorr. (Issue Number Two)

Hammersley, J. M., & Handscomb, D. C. (1964). *Monte carlo methods.* Methuen, London : Chapman & Hall.

Harrison, J. M., & Kreps, D. M. (1979). Martingales and arbitrage in multiperiod securities markets. *Journal of Economic Theory*, *20*, 381-408.

Harrison, J. M., & Pliska, S. R. (1981). Martingales and stochastic integrals in the theory of continuous trading. *Stochastic Processes and their Applications*, *11*, 215-260.

Heath, D., Jarrow, R., & Morton, A. (1992). Bond pricing and the term structure of interest rates: A new methodology for contingent claims valuations.

Econometrica, *60*, 77-105.

Hellmann, T. (1998). The allocation of control rights in venture capital contracts. *Rand Journal of Economics*, *29*, 57-76.

Ingersoll, J. E. (1977). A contingent-claims valuation of convertible securities. *Journal of Financial Economics*, *4*(3), 289-231.

Jagle, J. J. (1999). Shareholder value, real options and innovations in technology-intensive companies. *R&D Management*, *29*(3), 271-287.

Juan, C., Olmos, F., & Ashkeboussi, R. (2007a). Compensation options in joint ventures: A real options approach. *The Engineering Economist*, *52*(1), 67-94.

Juan, C., Olmos, F., & Ashkeboussi, R. (2007b, March). Evaluation of international joint venture agreements: Real options in practice (case study). *International Journal of Business*, *12*(2).

Kahn, H., & Marshall, A. W. (1953). Methods of reducing sample size in monte carlo computations. *Journal of the Operations Research Society of America*, *1*, 263-271.

Kaplan, S. N., & Strömberg, P. (2002a, December). *Characteristics, contracts, and actions: Evidence from venture capitalist analyses.* (Working Paper, University of Chicago)

Kaplan, S. N., & Strömberg, P. (2002b). Financial contracting theory meets the real world: An empirical analysis of venture capital contracts. *Review of Economic Studies*, *70*, 281-315.

Karoui, N. E., & Martellini, L. (2001, November). *Dynamic asset pricing theory with uncertain time-horizon.* (Working Paper, USC Marshall School of Business)

Kind, A. (2005). Pricing american-style options by simulation. *Financial Markets and Portfolio Management*, *19*(1), 109-116.

Kou, S. (2002). A jump-diffusion model for option pricing. *Management Science*, *48*(8), 1086-1101.

Kou, S., & Wang, H. (2001, September). *Option pricing under a double exponential jump diffusion model.* (Working Paper, Columbia University and Brown University)

Kumar, R. L. (1996). A note on project risk and option values of investments in information technologies. *Journal of Management Information Systems*, *13*(1), 187-193.

Kumar, R. L. (1999). Understanding dss value: An options perspective. *OMEGA,* *The International Journal of Management Science*, *27*, 295-304.

Lerner, J. (1994). The syndication of venture capital investment. *Financial Management*, *23*, 16-27.

Longstaff, F. A., & Schwartz, E. S. (2001). Valuing american options by simulation: A simple least-squares approach. *The Review of Financial Studies*, *14*(1), 113-147.

Merton, R. (1973). Theory of rational option pricing. *The Bell Journal of Economics and Management Science*, *4*(1), 141-183.

Merton, R. C. (1974). On the pricing of corporate debt: The risk structure of interest rates. *Journal of Finance*, *29*(2), 449-470.

Nöldeke, G., & Schmidt, K. M. (1998). Sequential investment and options to own. *RAND Journal of Economics*, *29*(4), 633-653.

Pennings, E., & Lint, O. (1997). The option value of advanced r&d. *European Journal of Operational Research*, *103*, 83-94.

Perlitz, M., Peske, T., & Schrank, R. (1999). Real options valuation: The new frontier in r&d project evaluation? *R&D Management*, *29*(2), 125-140.

Ramezani, C. A., & Zeng, Y. (1998, December). *Maximum likelihood estimation of asymmetric jump diffusion processes: Application to security prices.* (Working Paper, California Polytechnic, University of Missouri)

Reuters, T. (2009). *National venture capital association yearbook* (Rapport technique). National Venture Capital Organization.

Rodrigues, A., & Rocha Armada, M. J. (2006, February). *The valuation of real options with the least squares monte carlo simulation method.* (Working Paper, University of Minho, Portugal)

Schwartz, E., & Cortazar, G. (1998). Monte carlo evaluation model of an undeveloped oil field. *Journal of Energy Finance & Development*, *3*(1), 73-84.

Schwartz, E., & Moon, M. (2000). Rational pricing of internet companies. *Financial Analysts Journal*, *27*, 62-75.

Schwartz, E., & Zozoya-Gorostiza, C. (2000). *Valuation of information technology investment as real options.* (UCLA)

Smith, G. (2005). The exit structure of venture capital. *UCLA Law Review*, *53*, 315-356.

Trigeorgis, L., & Panayi, S. (1998). Multi-stage real options: The cases of information technology infrastructure and international bank expansion. *The Quarterly Review of Economics and Finance*, *38*(Special Issue), 675-692.

Willner, R. (1995). Valuing start-up venture growth options. In L. Trigeorgis (Ed.), *Real options in capital investment: Models, strategies and applications* (p. 221-239).

Wilmerding, A. (2005). *Deal terms: The finer points of venture capital deal structures, valuations, term sheets, stock options and getting deals done.* Aspatore Books.

Woronoff, M. A., & Rosen, J. A. (2005a). Effective vs. nominal valuations in venture capital investing. *NYU Law & Business Journal*, *2*, 101-131.

Woronoff, M. A., & Rosen, J. A. (2005b). Understanding anti-dilution provisions in convertible securities. *Fordham Law Review*, *74*.